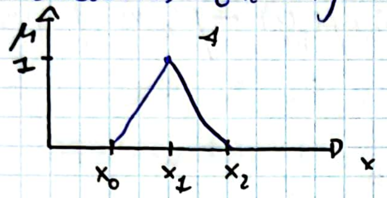
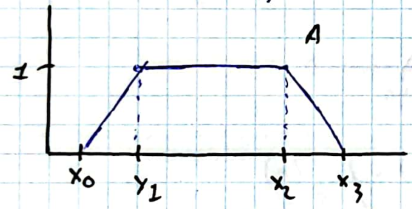
- Borroso
- Lógica borrosa
  - Conjunto clásico mediante función de pertenencia
    - Etiqueta lingüística A
    - Variable de entrada $x \in \mathbb{R}$
    - Función de pertenencia $\mu_A(x)$ Binaria
  - Conjunto borroso
    - La función ya no es binaria, $\mu \in [0,1]$

- Funciones triangulares



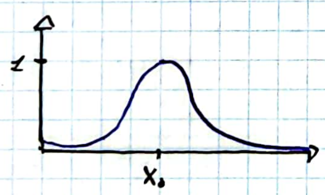
$$\mu_A(x) = \begin{cases} \dfrac{x - x_0}{x_1 - x_0} & x_0 \le x \le x_1 \\ \dfrac{x_2 - x}{x_2 - x_1} & x_1 \le x \le x_2 \\ 0 & \text{Resto} \end{cases}$$

- Funciones trapezoidal



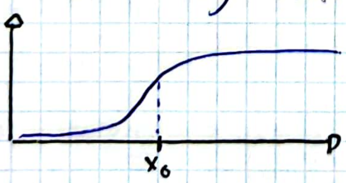
$$\mu_A(x) = \begin{cases} \dfrac{x - x_0}{x_1 - x_0} & x_0 \le x \le x_1 \\ 1 & x_1 \le x \le x_2 \\ \dfrac{x_3 - x}{x_3 - x_2} & x_2 \le x \le x_3 \\ 0 & \text{Resto} \end{cases}$$

- Funciones gausianas



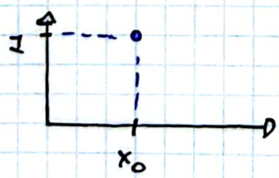
$$\mu_A(x) = e^{-\frac{1}{2}\left(\frac{x - x_0}{\sigma^2}\right)^2}$$

- Función sigmoidal



$$\mu_A(x) = \frac{1}{1 + e^{-k(x - x_0)^2}}$$

- Función singleton



$$\mu_A(x) = \begin{cases} 1 & x = x_0 \\ 0 & \text{Resto} \end{cases}$$

- Operaciones sobre conjuntos borrosos

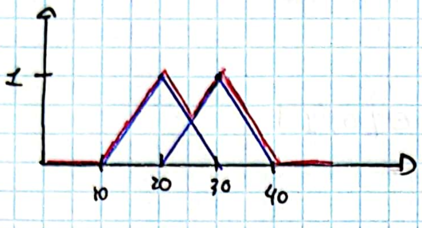
- Unión

$A \to \mu_A(x)$

$B \to \mu_B(x)$

$$A \cup B \to \mu_{A \cup B}(x) = S\{\mu_A(x), \mu_B(x)\}$$

- S-norma Max



$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$$

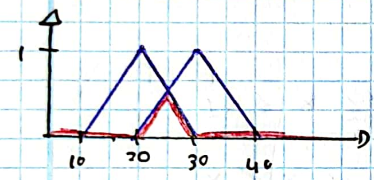
- S-norma suma probabilística

$$\mu_{A \cup B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$$

- Intersección

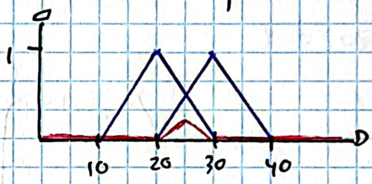
$$A \cap B \to \mu_{A \cap B}(x) = T\{\mu_A(x), \mu_B(x)\}$$

- T-norma min



$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

- T-norma producto



$$\mu_{A \cap B}(x) = \mu_A(x) \cdot \mu_B(x)$$

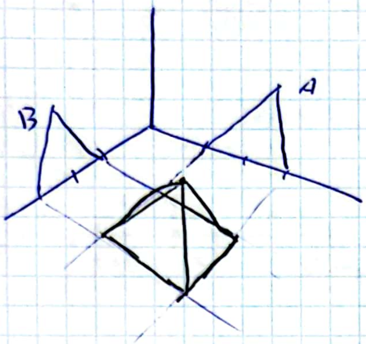
- Negación

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

$$A \cap B = \overline{\bar{A} \cup \bar{B}} \qquad A \cup B = \overline{\bar{A} \cap \bar{B}}$$

- Operadores definidos en diferentes variables
  - Extensión cilíndrica (Relación borrosa)

$$R(A,B) \rightarrow \mu_{R(A,B)}(x,y) = T\{\mu_A(x), \mu_B(x)\}$$



  - Composición de conjuntos
    - Opera con al menos 1 var en común

$A \rightarrow \mu_A(X,Y)$

$B \rightarrow \mu_B(Z,X)$

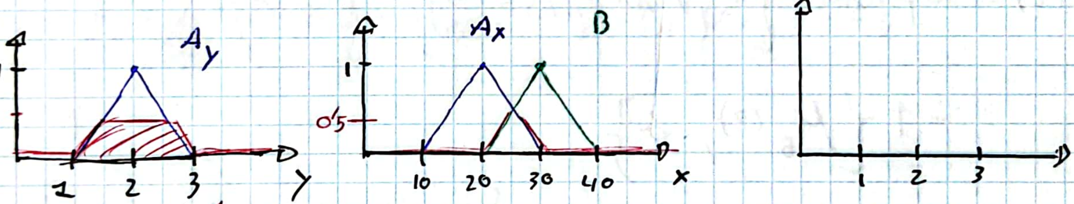
$$A \circ B \rightarrow \mu_{A \circ B}(Z,Y) = \underset{x}{\text{supremo}}\left\{T\left\{\mu_A(x,y), \mu_B(z,x)\right\}\right\} =$$

$$= T\left\{\mu_{A_Y}(y), \alpha\right\}$$

$$\alpha = \underset{x}{\text{supremo}}\left\{T\left\{\mu_{A_X}(x), \mu_B(x)\right\}\right\}$$

---

ej:



T- min

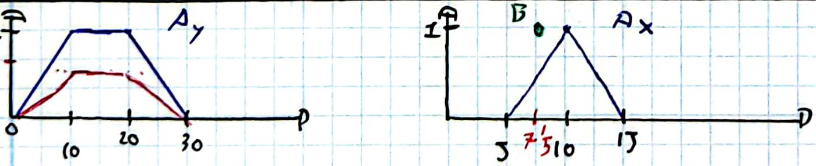
$$\alpha = \underset{x}{\text{supremo}}\left\{T\left\{\mu_{A_X}(x), \mu_B(x)\right\}\right\} = 0'5$$

$A \rightarrow \mu_A(X,Y)$

$B \rightarrow \mu_B(X)$

---

ej:



$A \rightarrow \mu_A(X,Y)$

$B \rightarrow \mu_B(X)$

T- prod

$$\alpha = \underset{x}{\text{supremo}}\left\{T\left\{\mu_{A_X}(x), \mu_B(x)\right\}\right\} = \text{supremo}\left\{1 \cdot 0'5\right\} = 0'5$$

$$\mu_{A \circ B}(y) = T\left\{\mu_{A_Y}(y), \alpha\right\}$$

• Proposición borrosa

- Axioma   x es A   // 'variable' es 'conj. borroso'
- Proposiciones condicionales

$$\text{si } \underbrace{x \text{ es } A}_{\substack{\text{Antecedente} \\ \text{(simple ó} \\ \text{compuesta)}}} \rightarrow \underbrace{z \text{ es } B}_{\substack{\text{consecuente} \\ \text{(simple)}}}$$

• Mecanismo diferencia borroso

R: si x es A → z es B

A: x es A'

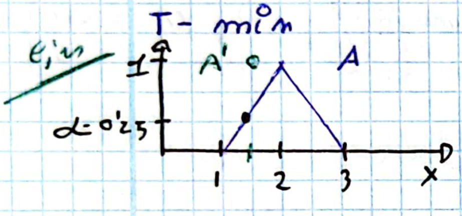
---

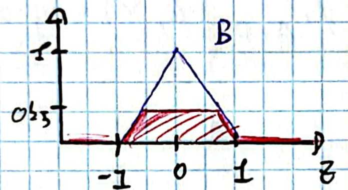
z es B'

$$B' = (A \cap B) \circ A'$$

T- min

Ej:





$$\mu_{A'}(x) = \begin{cases} 1 & x = 1'25 \\ 0 & \text{Resto} \end{cases}$$

$$\mu_{B'}(z) = \underset{x}{\text{supremo}} \left\{ t\left\{\mu_A(x), \mu_B(z)\right\}, \mu_{A'}(x) \right\} =$$

$$= t\left\{\mu_B(z), \alpha\right\}$$

$$\alpha = \sup_x \left\{ t\left\{ \mu_{Ax}(x), \mu_{A'}(x) \right\} \right\} = 0'25$$

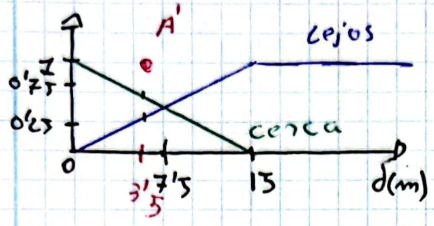
$$\mu_{B'}(z) = \min\left\{\mu_B(z), 0'25\right\}$$

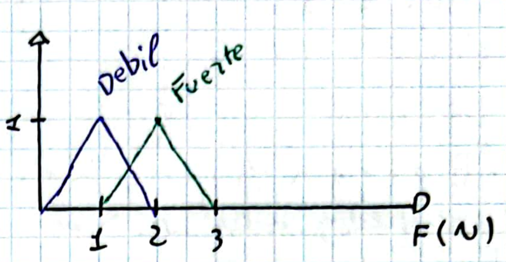
Ej:

R_1: si d es cerca → F es fuerte
R_2: si d es Lejos → F es debil

A: d es A'

F es B'





$B' = B_1' \cup B_2'$

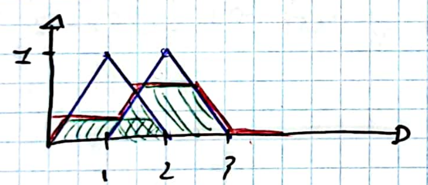
$B_1' = (\text{cerca} \cap \text{fuerte}) \circ A'$
$B_2' = (\text{Lejos} \cap \text{debil}) \circ A'$

$\mu_{B_1'}(F) = \min \left\{ \mu_{Fuerte}(F) , \alpha_1 \right\}$

$\mu_{B_2'}(F) = \min \left\{ \mu_{debil}(F) , \alpha_2 \right\}$

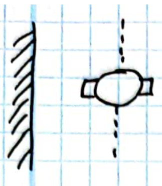
$\alpha_1 = \sup_{D} \left\{ \mu_{cerca}(d), \mu_{A'}(d) \right\} = 0'75$

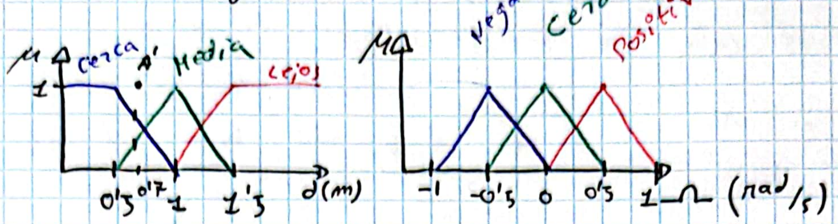
$\alpha_2 = \sup_{D} \left\{ \mu_{Lejos}(d), \mu_{A'}(d) \right\} = 0'25$



Ej

$\Omega > 0 \Rightarrow$ gira der





T min

S Max

R_1: si d es Media $\rightarrow$ Ω es cero

R_2: si d es cerca $\rightarrow$ Ω es positiva

R_3: si d es Lejos $\rightarrow$ Ω es Negativa

A: d es A'

Ω es B'

$B_1' = (\text{Medio} \cap \text{cero}) \circ A'$

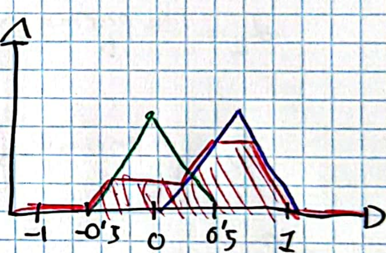
$B_2' = (\text{cerca} \cap \text{positivo}) \circ A'$

$B_3' = (\text{Lejos} \cap \text{Negativo}) \circ A'$

$\mu_{B_1'}(\Omega) = \min(\mu_{cero}(\Omega), \overset{0'4}{\alpha_1}) =$

$\mu_{B_2'}(\Omega) = \min(\mu_{pos}(\Omega), \overset{0'6}{\alpha_2}) =$

$\mu_{B_3'}(\Omega) = \min(\mu_{Neg}(\Omega), \overset{0}{\alpha_3}) =$



R: si x es A $\underline{\text{AND}}$ y es B → z es C

↳ si es AND ⇒ $\cap$
si es OR ⇒ $\cup$

$\vdots$

---

z es C'

$$C' = \sum C_i'$$

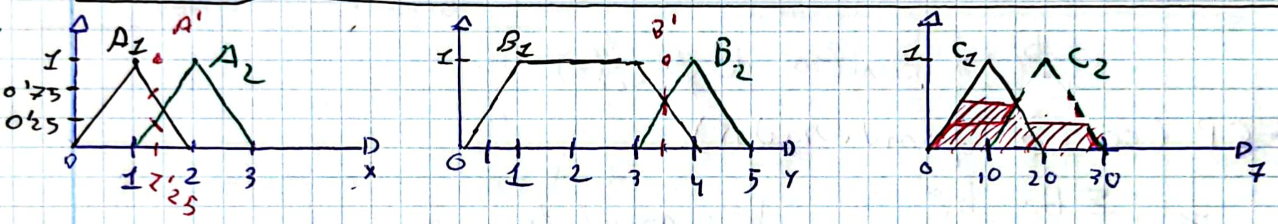
$$C_i' = \underbrace{\left(\left(A_i \cap C_i\right) \circ A'\right)}_{C_{i_1}'} \cap \underbrace{\left(\left(B_i \cap C_i\right) \circ B'\right)}_{C_{i_2}'}$$

$$\mu_{C_{i_1}'}(z) = T\left\{\mu_{C_i}(z),\ \alpha_i\right\}$$

$$\alpha_i = \begin{matrix}T\\ ó\\ S\end{matrix}\left\{\alpha_{x_i},\ \alpha_{y_i}\right\}$$

↳ $\alpha_{x_i} = \underset{x}{\text{supremo}}\left\{T\left\{\mu_{A_i}(x),\ \mu_{A'}(x)\right\}\right\}$

Ejm



si x es A_1 AND y es B_1 → z es C_1 T-Min
si x es A_2 AND y es B_2 → z es C_2 S-Max
x es A'
y es B'

---

z es C'

$$C' = C_1' \cup C_2'$$

$$\mu_{C_1'}(z) = \min\left(\mu_{C_1}(z),\ \alpha_1\right)$$

$$\alpha_1 = T\left(\alpha_{x_1},\ \alpha_{y_1}\right) = \min\left(0'75,\ 0'25\right) = \underline{\underline{0'25}}$$

$$\alpha_{x_1} = \underset{x}{\text{supremo}}\left\{\min\left\{\mu_{A_2}(x),\ \mu_{A'}(x)\right\}\right\} = \min\left(0'75,\ 1\right) = 0'75$$

$$\alpha_{y_1} = 0'25$$

$$\mu_{C_2'}(z) = \min\left(\mu_{C_2}(z),\ \alpha_2\right)$$

0'25

- control borroso

1) tipo mamdani

$$x \longrightarrow \boxed{\text{Borrificador}} \longrightarrow \boxed{} \longrightarrow \boxed{\text{Desborrificador}} \longrightarrow z$$

- tipos Desborrificador

- CoA (center of area)

$$Z = \frac{\int \mu_{c_i}'(z) \cdot z \, dz}{\int \mu_{c_i}'(z) \, dz}$$

- CoG (center of gravity)

$$Z = \frac{\sum_{i=1}^{m} b_i \; A(\mu_{c_i}'(z))}{\sum_{i=1}^{m} A(\mu_{c_i}'(z))}$$

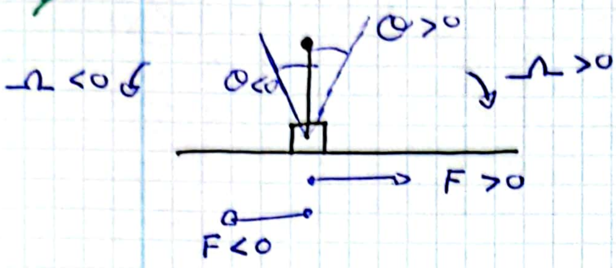
$A(\mu_{c_i}'(z)) \to \int \mu_{c_i}(z) \, dz$

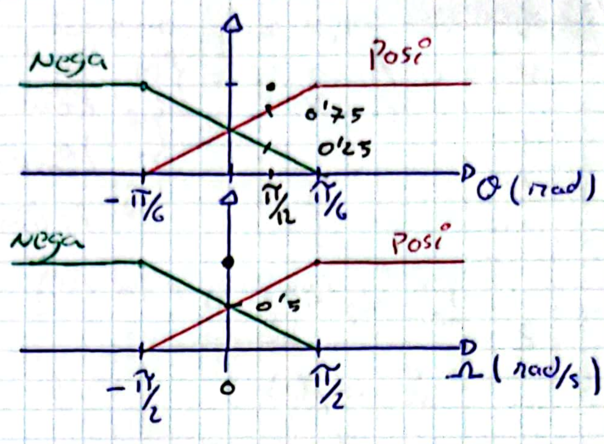
$b_i \to$ centro de $\mu_{c_i}'(z)$

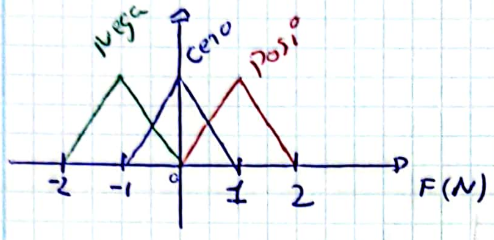
- CP (centros ponderados)

$$Z = \frac{\sum b_i \cdot h_i}{\sum h_i} \qquad h_i = \underbrace{\max(\mu_{c_i}'(z))}_{\alpha_i}$$

ejer. Péndulo invertido





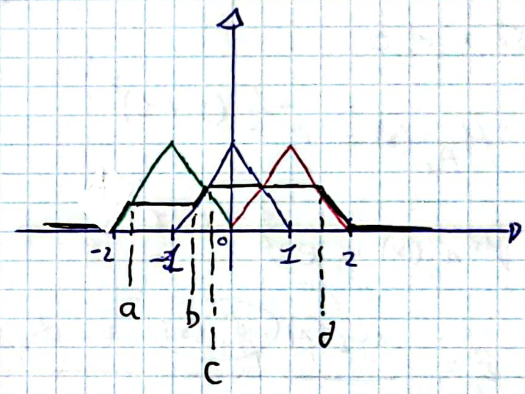


Θ \ Ω	Nega 0'5	Posi 0'5
Nega 0'25	Nega $\alpha_1 = 0'25$ R_1	Cero $\alpha_2 = 0'25$ R_2
Posi 0'75	Cero $\alpha_3 = 0'5$ R_3	Posi $\alpha_4 = 0'5$ R_4

T: min

S Max

D CPA



$$\mu_c'(F) = \begin{cases} F+2 & -2 \le F \le a \\ 0'25 & a \le F \le b \\ F+1 & b \le F \le c \\ 0'5 & c \le F \le d \\ 2-F & d \le F \le 2 \\ 0 & \text{Resto} \end{cases}$$

CP

$$F = \frac{\sum b_i \cdot h_i}{\sum h_i} = 0'16$$

$b_1 = -1$ $h_1 = 0'25$

$b_2 = 0$ $h_2 = 0'25$

$b_3 = 0$ $h_3 = 0'5$

$b_4 = 1$ $h_4 = 0'5$

CF

$$z = \frac{(0'25 \cdot -1) + (0 \cdot 0'25) + (0 \cdot 0'5) + (0'5 \cdot 1)}{0'25 + 0'25 + 0'5 + 0'5} = 0'16$$

2/ Sugeno

si x es $A \rightarrow z = f(x)$
- constantes
- Lineales
- Polinomiales

$$z = \frac{\sum \alpha_i(x) \cdot f_i(x)}{\sum \alpha_i(x)}$$

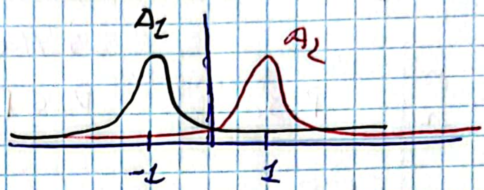
$\hookrightarrow \alpha_i(x) = \mu_{A_i}(x)$

---

ej:

si x es $A_1 \rightarrow z = 1$

si x es $A_2 \rightarrow z = -1$



si x es 0

$$\mu_{A_1}(x) = e^{-\frac{1}{2}(x+1)^2}$$

$$\mu_{A_2}(x) = e^{-\frac{1}{2}(x-1)^2}$$

$$z_0 = \frac{\alpha_1(x) \cdot f_1(x) + \alpha_2(x) \cdot f_2(x)}{\alpha_1(x) + \alpha_2(x)} =$$

$$= \frac{e^{-\frac{1}{2}(x+1)^2} \cdot 1 + e^{-\frac{1}{2}(x-1)^2} \cdot -1}{e^{-\frac{1}{2}(x+1)^2} + e^{-\frac{1}{2}(x-1)^2}} = 0$$

- si antecedentes compuestos $\Rightarrow \alpha_1(x) = T(\quad , \quad)$

$$\alpha_i(x,y) = T\{\mu_{A_i}(x), \mu_{B_i}(y)\} = \underbrace{\mu_{A_i}(x) \cdot \mu_{B_i}(y)}$$

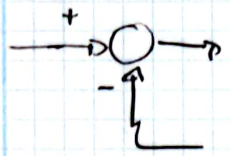
1)

x_2 \ x_1	NB 0	N 0	Z 0	P 0'5	PB 0'5
N 0	N 0	Z 0	N 0	P 0	PB 0
Z 0'25	NB 0	N 0	Z 0	P 0'25	PB 0'25
P 0'75	NB 0	N 0	P 0	Z 0'5	P 0'5

- Aplicar Sugeno para $x_1 = 60$ y $x_2 = 30$ T-Min S-Max

$$\text{Sugeno} = \frac{\sum \alpha_i \cdot f_i(x)}{\sum \alpha_i} : \frac{(0'25 \cdot 10) + (0'25 \cdot 20) + (0'5 \cdot 0) + (0'5 \cdot 10)}{0'25 + 0'25 + 0'5 + 0'5}$$

todas las casillas pero como muchas son 0 no las pongo



$\underbrace{(\text{Deseo } 0) - (\text{Actual } 10)}_{+0 \; - 10}$

ej: T-producto