JOHANN WOLFGANG GOETHE UNIVERSITÄT FRANKFURT AM MAIN

www.uni-frankfurt.de

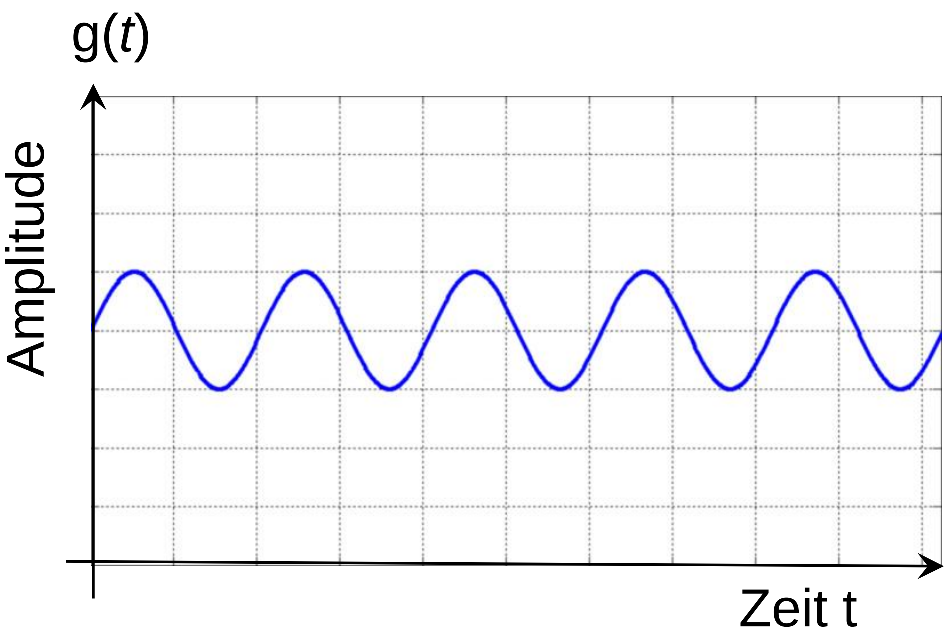
Diskrete Fourier-transformation

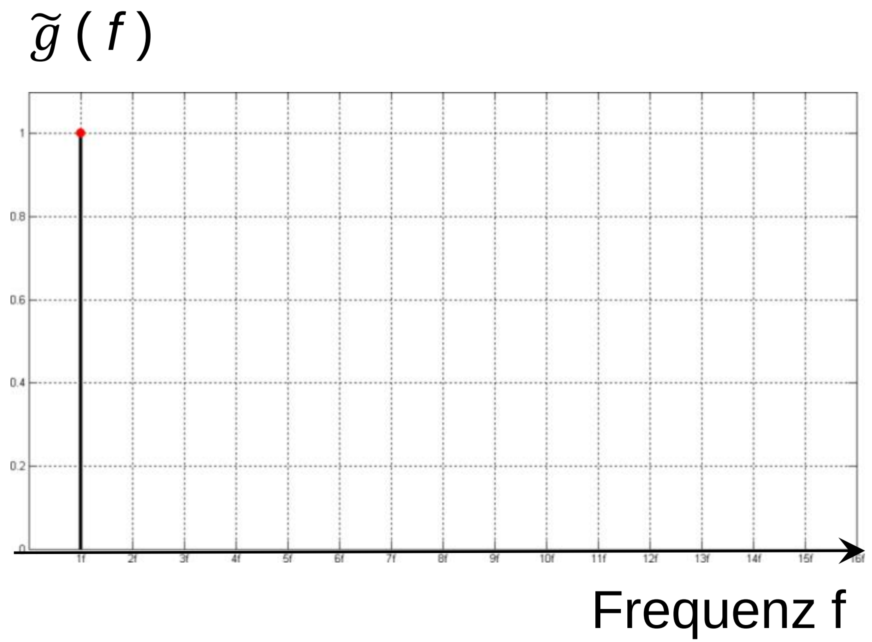
Praktikum Adaptive Systeme

Signale

- **Einzelsignal: Darstellungsformen**

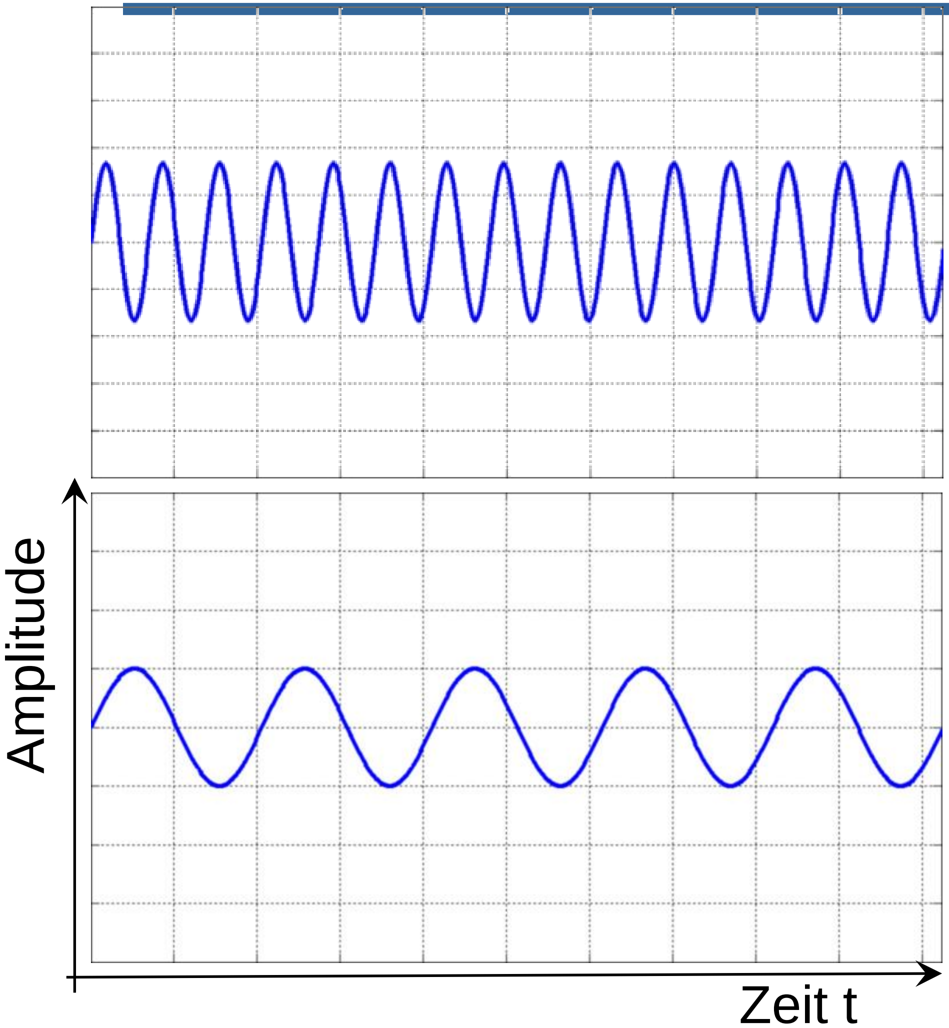
Zeit t variabel,   Frequenz f fest





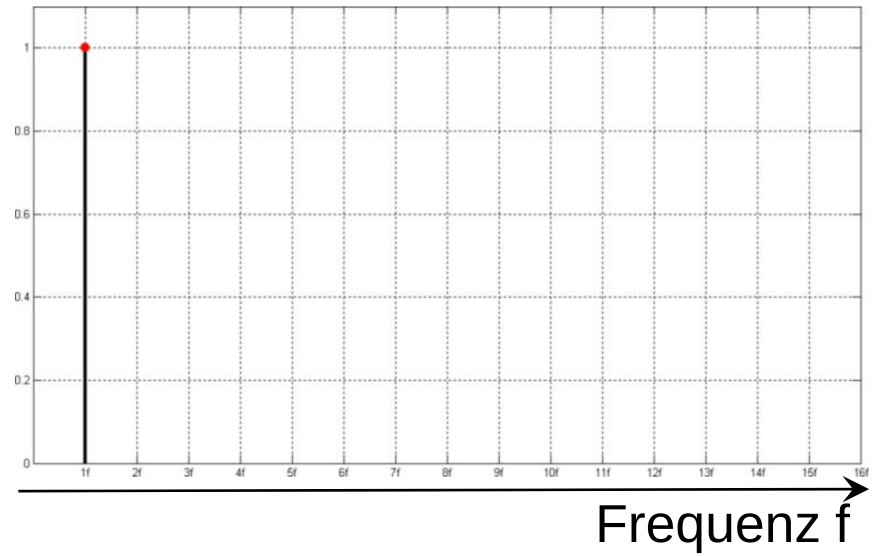
$g(t) = \sin(t \cdot f)$

Signalmischung



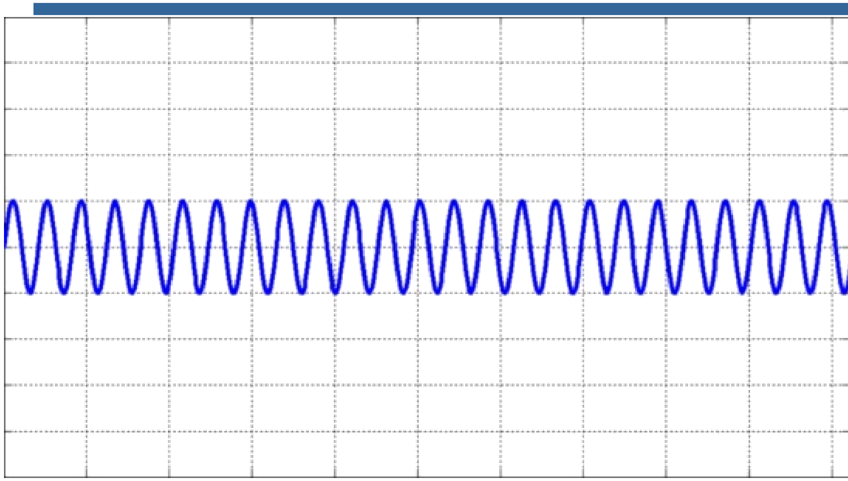
$$\frac{1}{3}\sin(t \cdot 3f)$$

$\tilde{g}(f)$

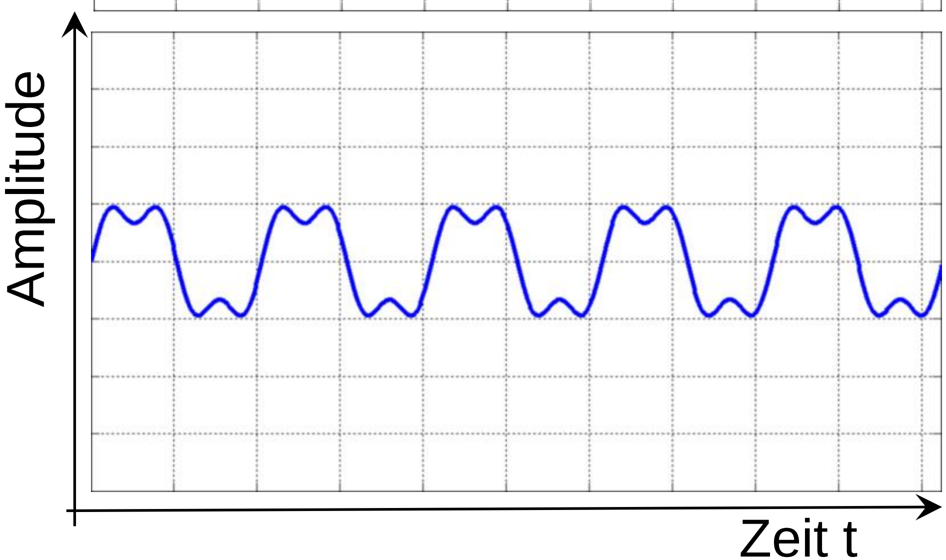


g(t) = sin($t \cdot f$)

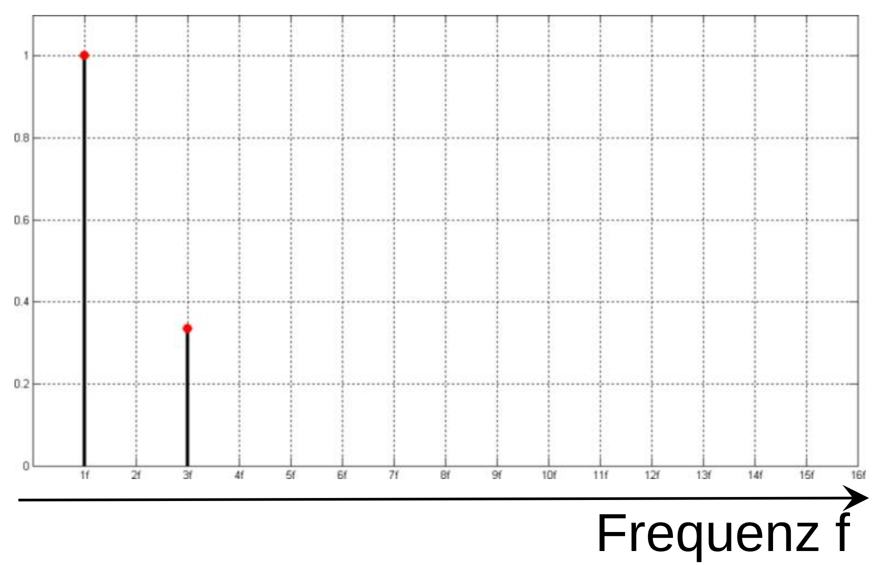
Signalmischung



$$\frac{1}{5}\sin(t \cdot 5f)$$

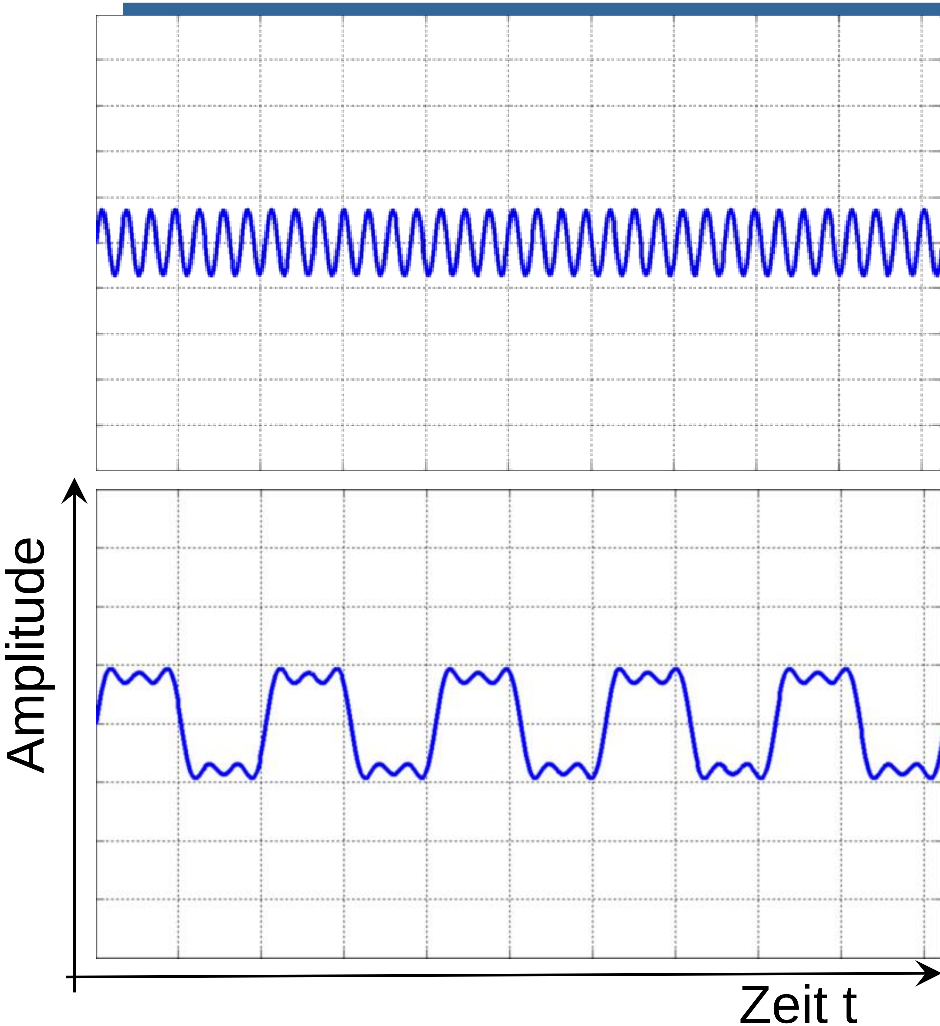


$\tilde{g}(f)$



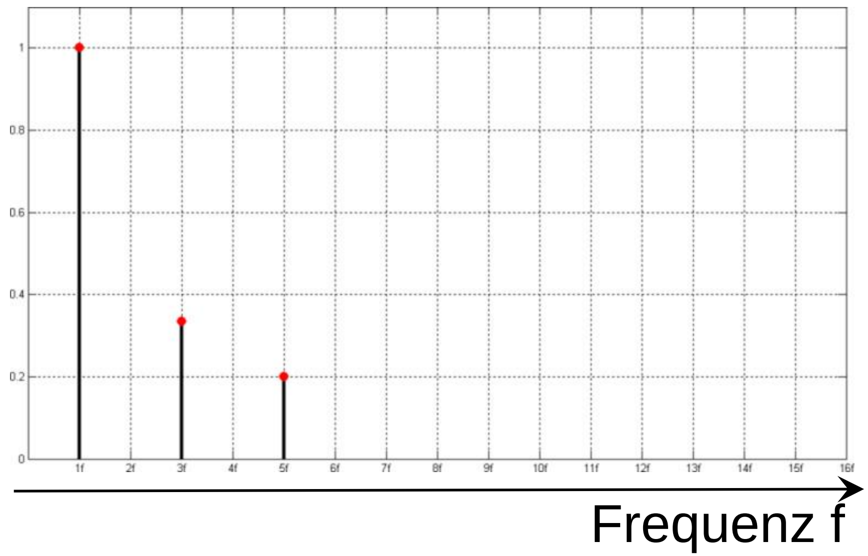
g(t) = sin($t \cdot f$) + 1/3 sin($t \cdot 3f$)

Signalmischung



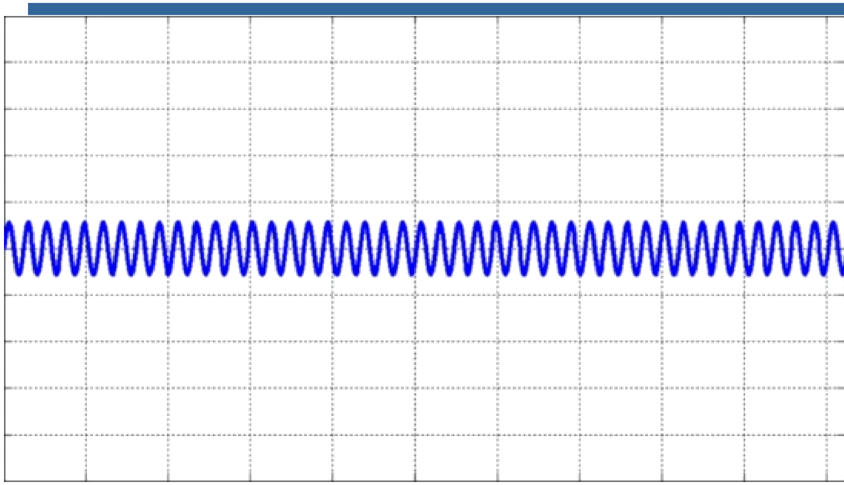
$$\frac{1}{7}\sin(t \cdot 7f)$$

$\tilde{g}(f)$

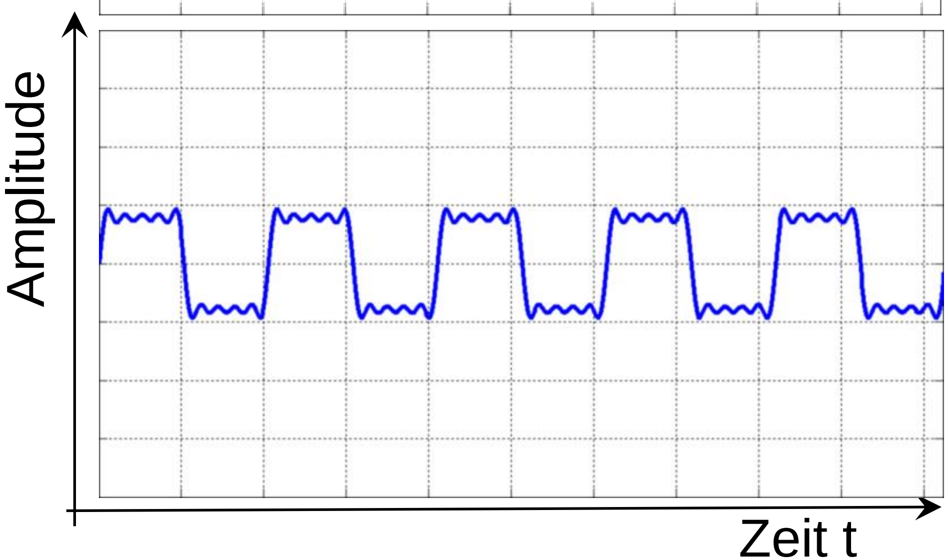


g(t) = sin($t \cdot f$) + 1/3 sin($t \cdot 3f$) + 1/5 sin($t \cdot 5f$)

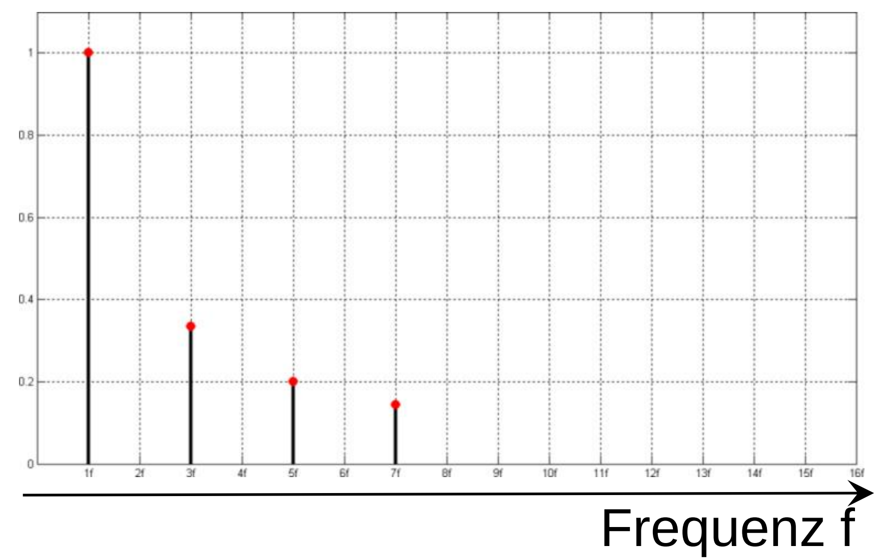
Signalmischung



$$\frac{1}{9}\sin(t \cdot 9f)$$

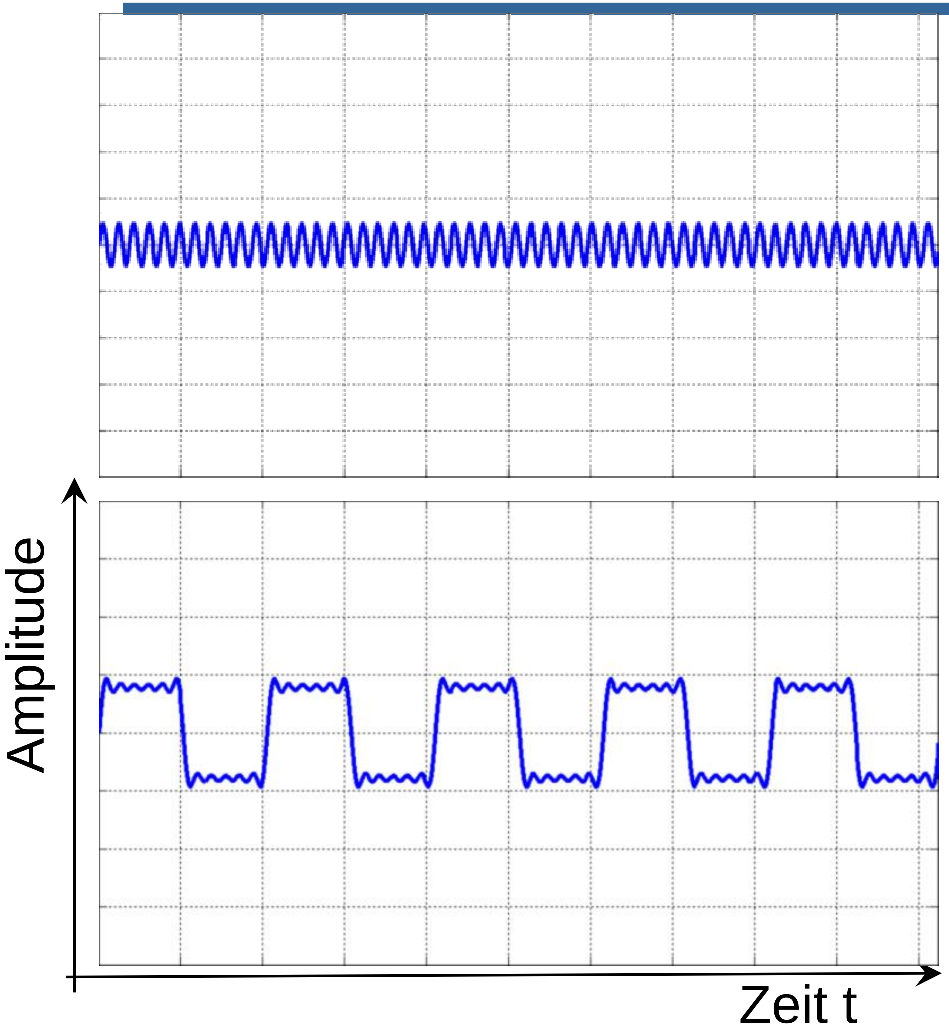


$\tilde{g}(f)$



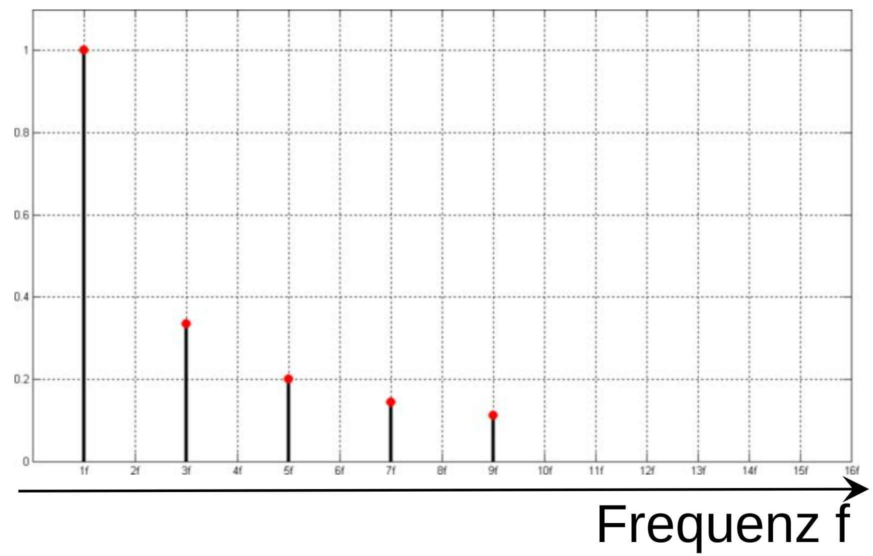
g(t) = sin($t \cdot f$) + 1/3 sin($t \cdot 3f$) + 1/5 sin($t \cdot 5f$) + 1/7 sin($t \cdot 7f$)

Signalmischung



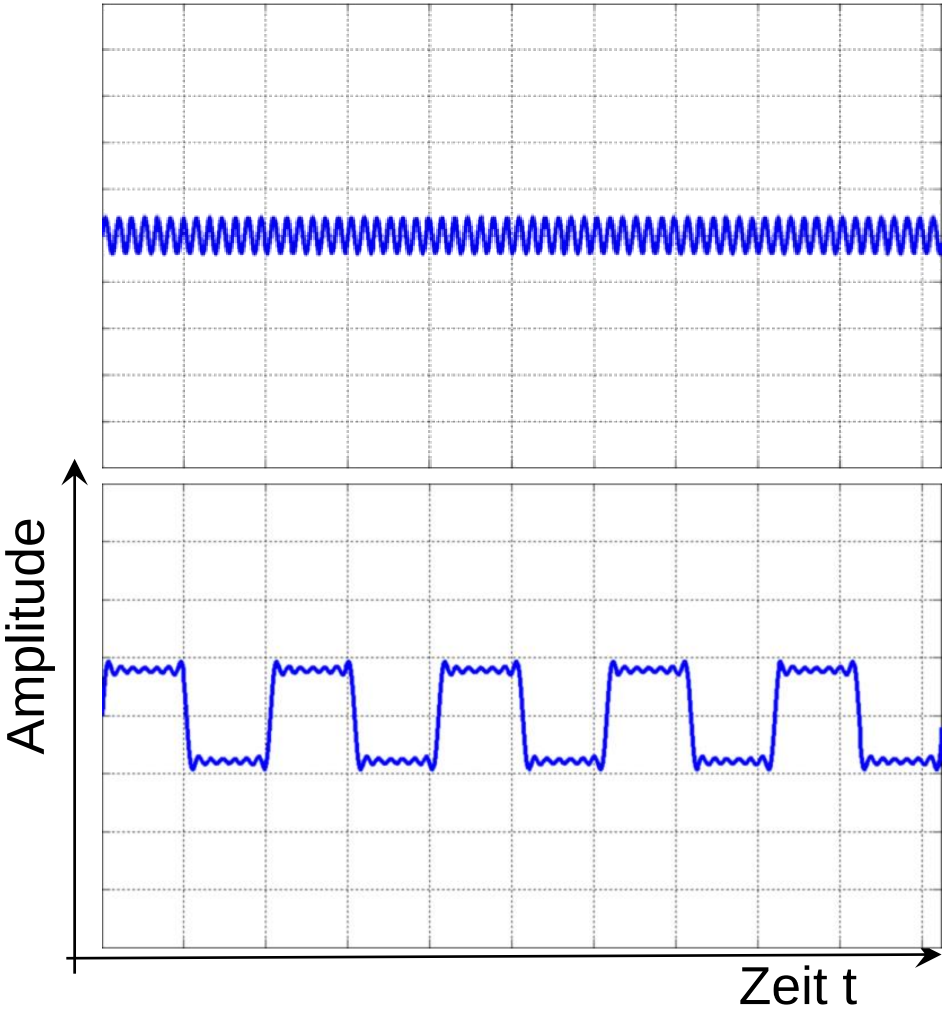
$$\frac{1}{11}\sin(t \cdot 11f)$$

$\tilde{g}(f)$

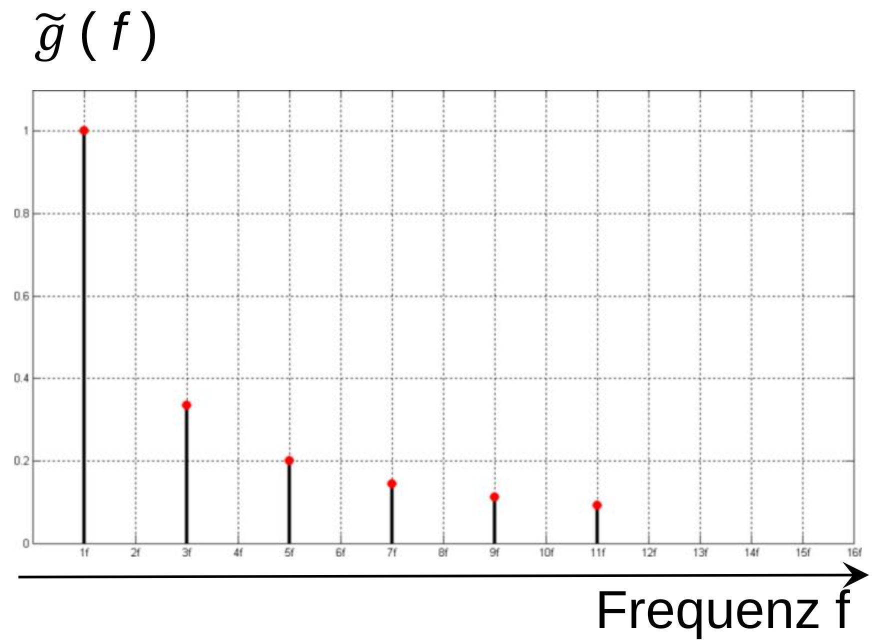


g(t) = sin($t \cdot f$) + 1/3 sin(t·3f) + 1/5 sin(t·5f) + 1/7 sin(t·7f) + 1/9 sin(t·9f)

Signalmischung



$$\frac{1}{13}\sin(t \cdot 13f)$$



g(t) = sin($t \cdot f$) + 1/3 sin(t·3f) + 1/5 sin(t·5f) + 1/7 sin(t·7f) + 1/9 sin(t·9f) + 1/11 sin(t·11f)

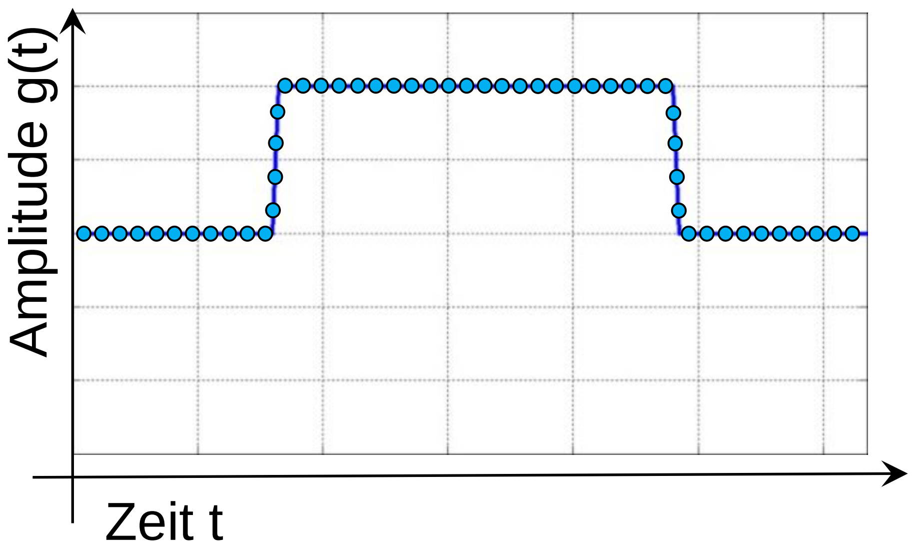
Darstellung als diskrete Signalmischung

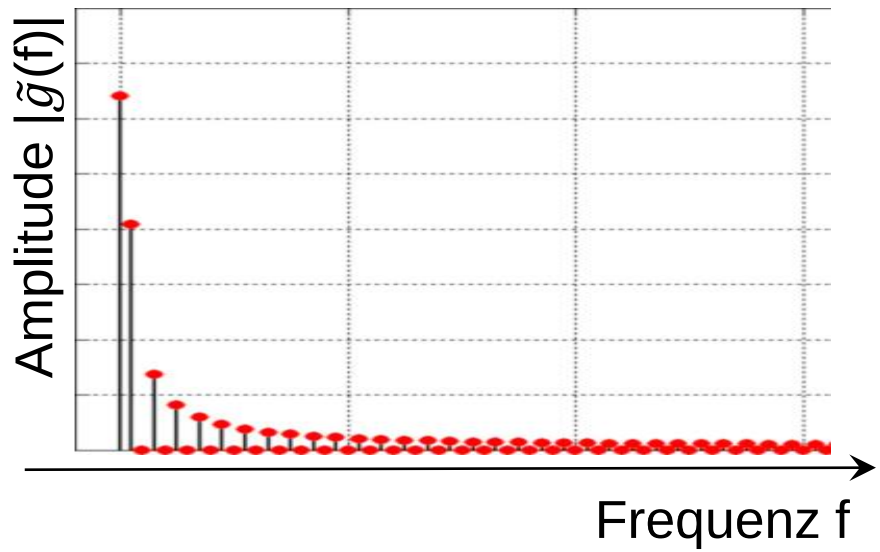
Sehr viele Komponenten:

n Zeitpunkte $g(t_k)$, *k=0..n-1*

n Frequenzpunkte $\tilde{g}(f_j)$, *j=0..n-1*

Transformation $g(t_k) \rightarrow \tilde{g}(f_j)$ ***– wie?***





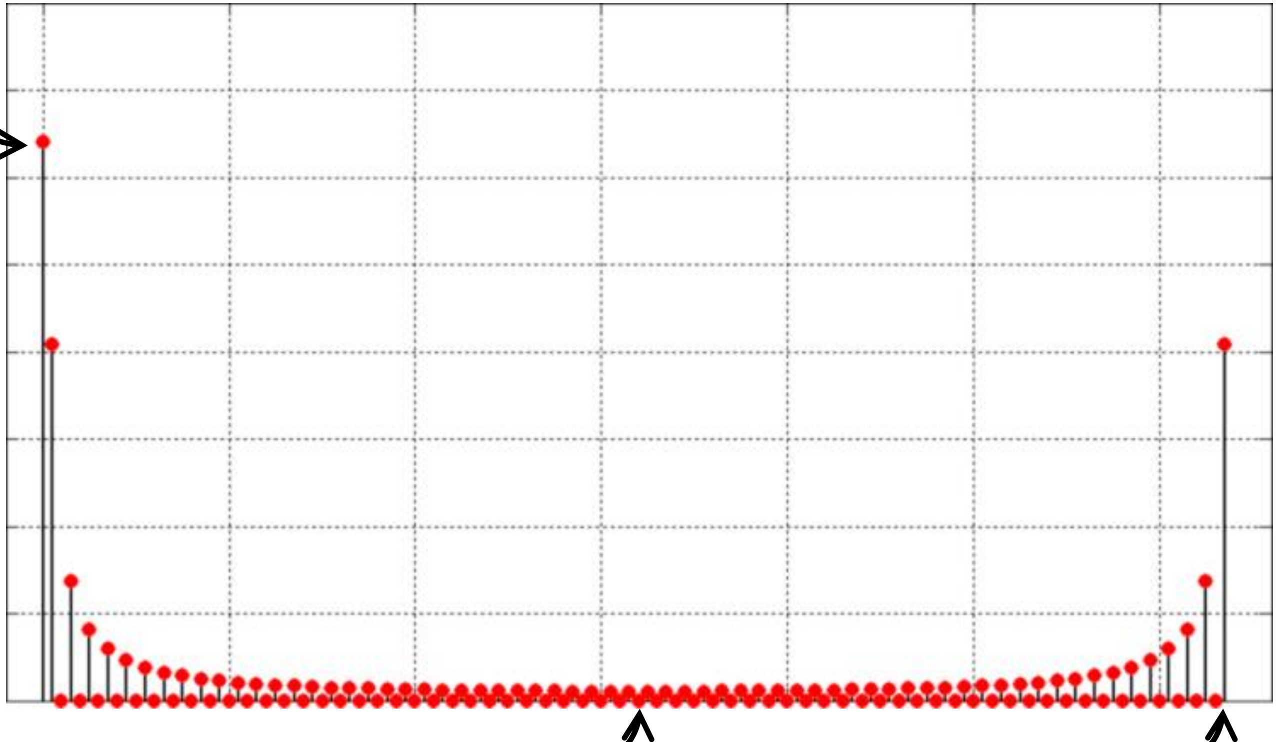
Diskrete Fouriertransformation

- Die Zeit zwischen zwei Samples $\Delta t = t_{i+1}-t_i$ entspricht mit $f_s = 1/\Delta t$ der Samplefrequenz.
- Transformation eines Feldes von *n* Zahlen (2-Potenz) auf n Zahlen, z.B. *n* Amplitudenwerte auf *n* Frequenzwerte mit der max. Frequenz f_s.
- Nach dem Nyquist-Theorem sind nur Frequenzanteile eines Signals darstellbar mit Frequenzen $< f_s/2$
- Also sind nur die (komplexen) Zahlenwerte bis n/2 nutzbar.

Power-Spektrum

JOHANN WOLFGANG GOETHE
UNIVERSITÄT
FRANKFURT AM MAIN

Bei f = 0 →
Mittelwert $\langle g(t) \rangle$



$\Delta f = f_s/n$

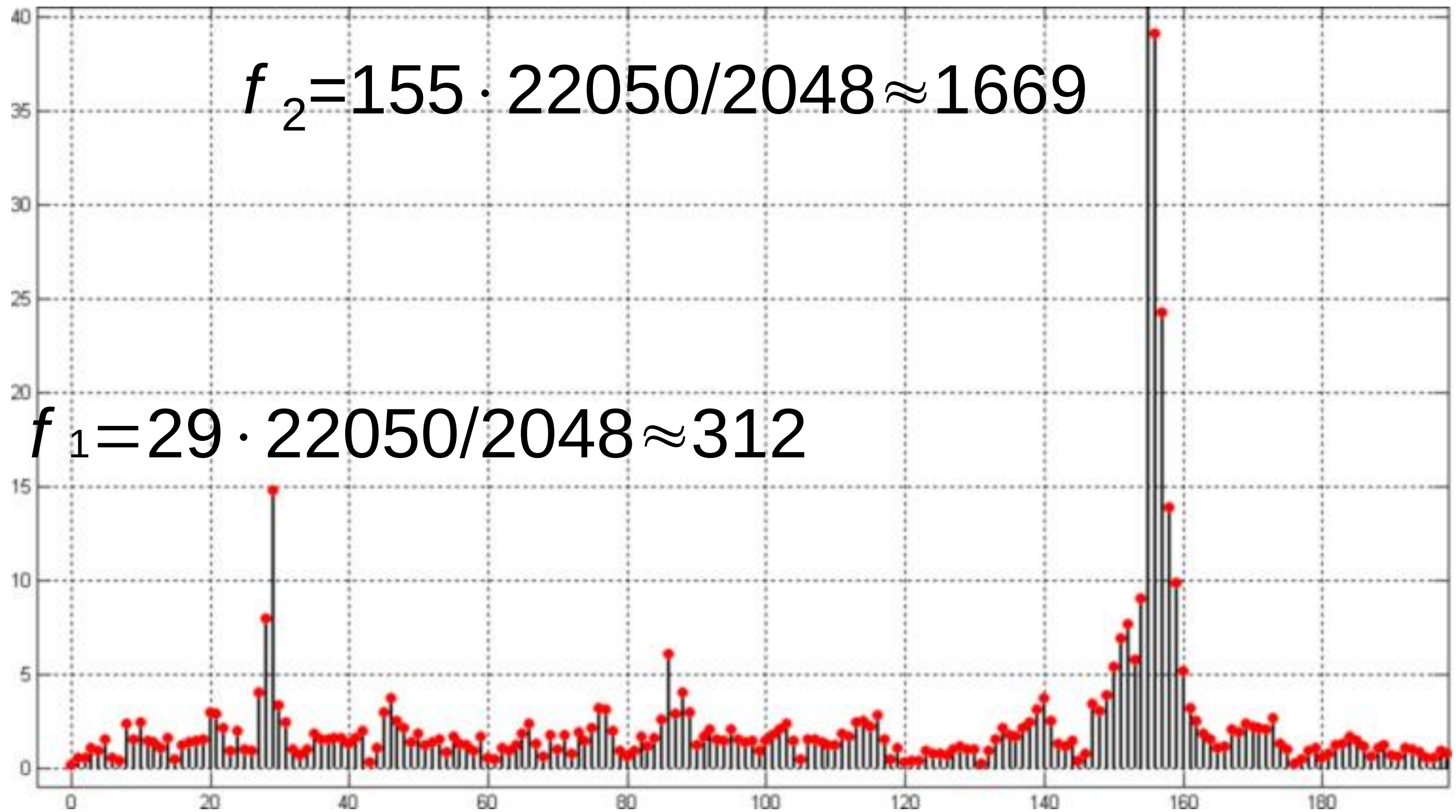
Max. Frequenz
$f_{max} = f_s/2$

f_s

Beispiel Audiosignal

Samplefrequenz f_s = 22,05 kHz, DFT auf den ersten 2048 Werten

Peaks auf Indizes 29 und 155 – Welche Frequenzen?



$$f_2 = 155 \cdot 22050/2048 \approx 1669$$

$$f_1 = 29 \cdot 22050/2048 \approx 312$$

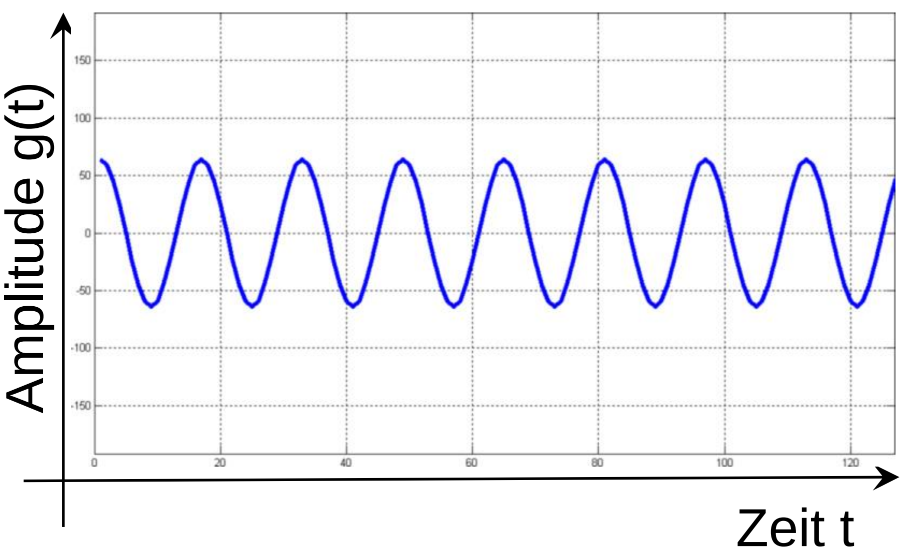
Komplexe Signaldarstellung 1

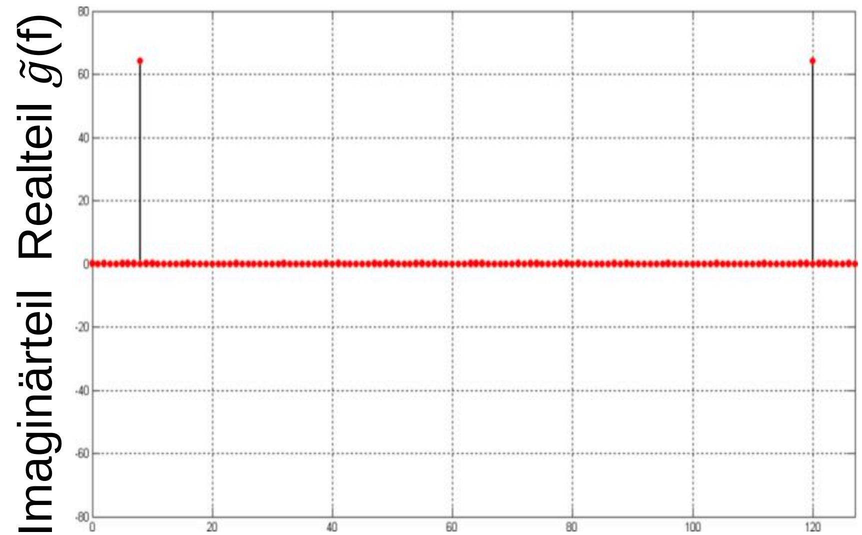
Darstellung

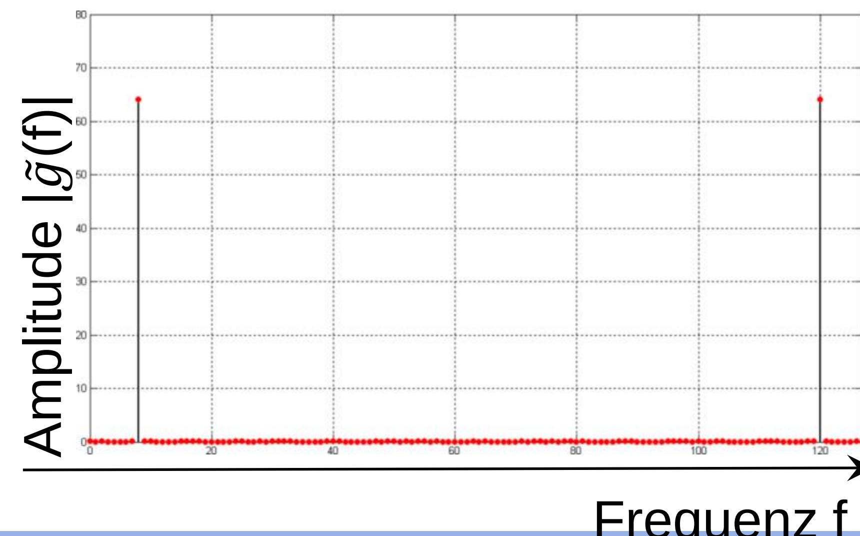
Ortsraum/Frequenzraum

$g(t) = a \cos(t \cdot f + \varphi)$,

$a=64$, $f=8 \cdot 2\pi/n$, $\varphi=0$





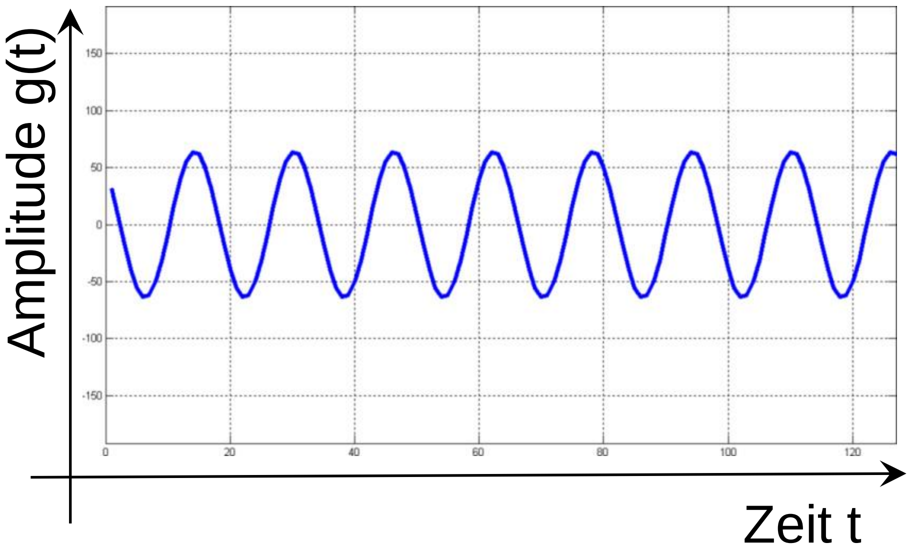


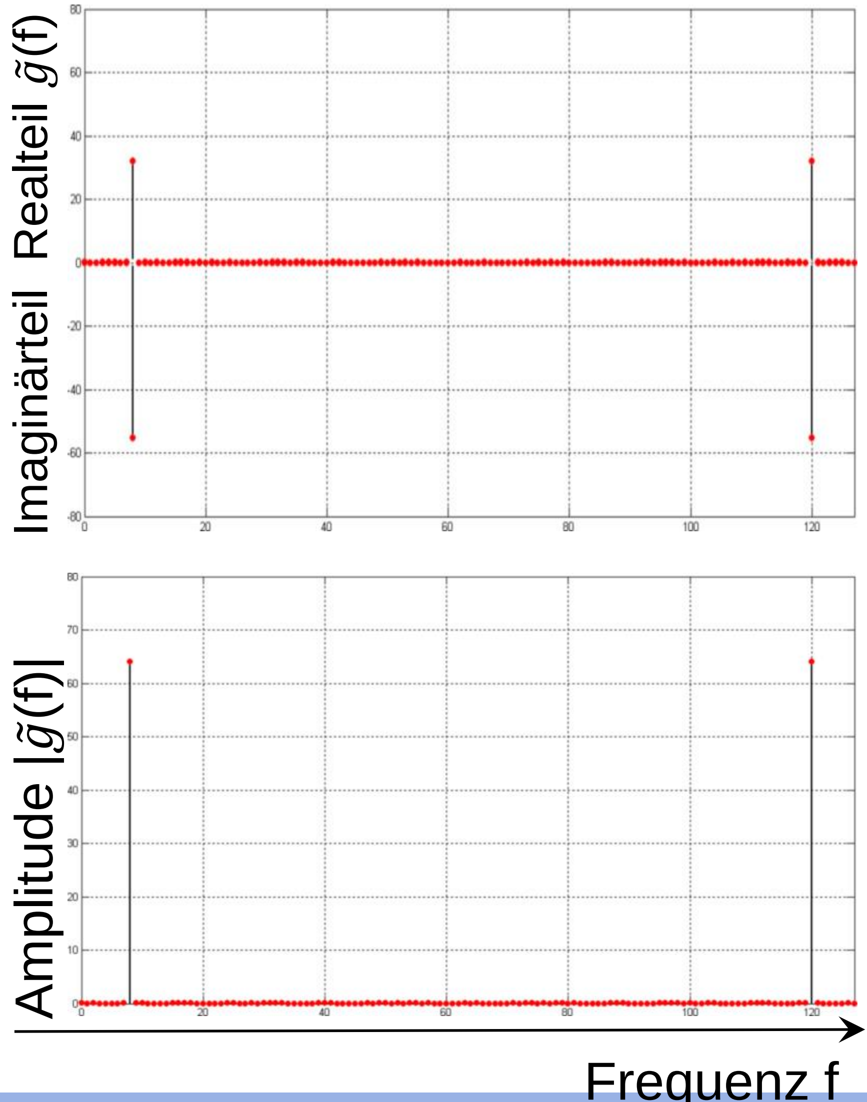
Komplexe Signaldarstellung 2

Einfluss der Phase φ

g(t) = a cos(t·f + φ),

a=64, f=2π8/n, φ=**π/3**



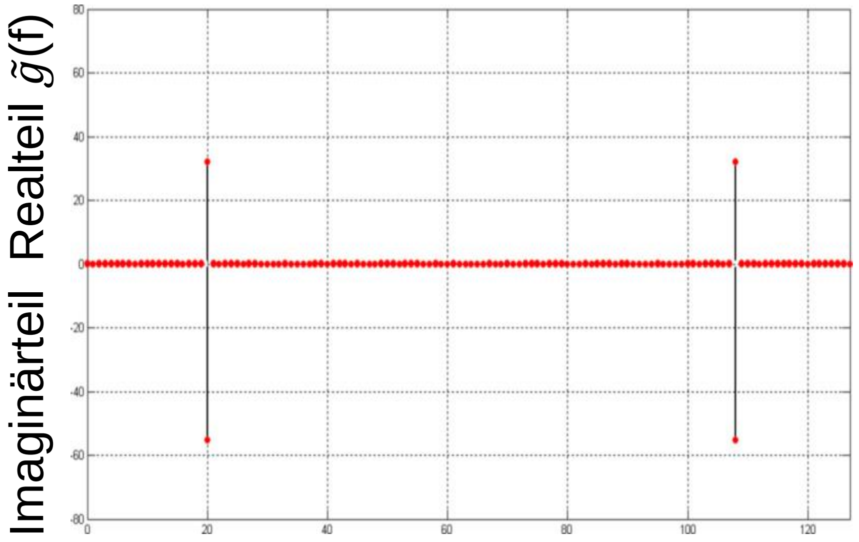


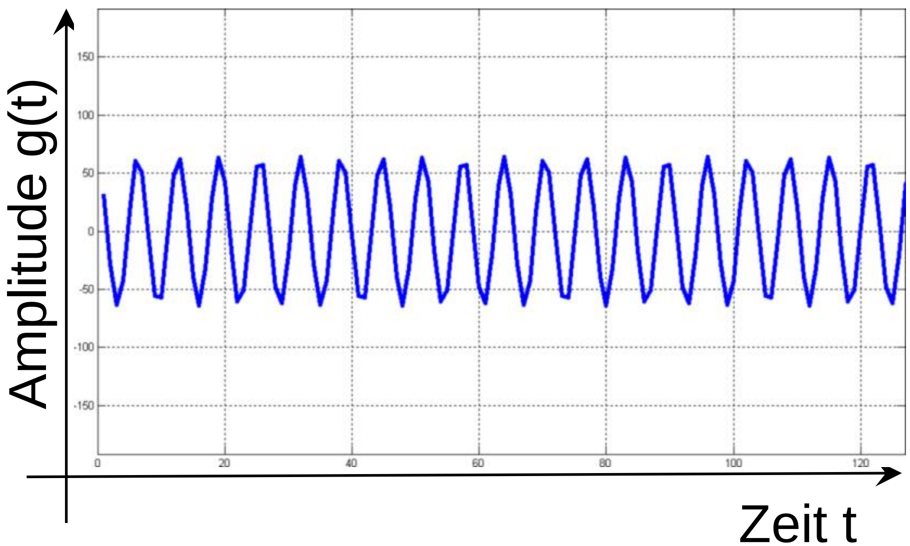
Komplexe Signaldarstellung 3

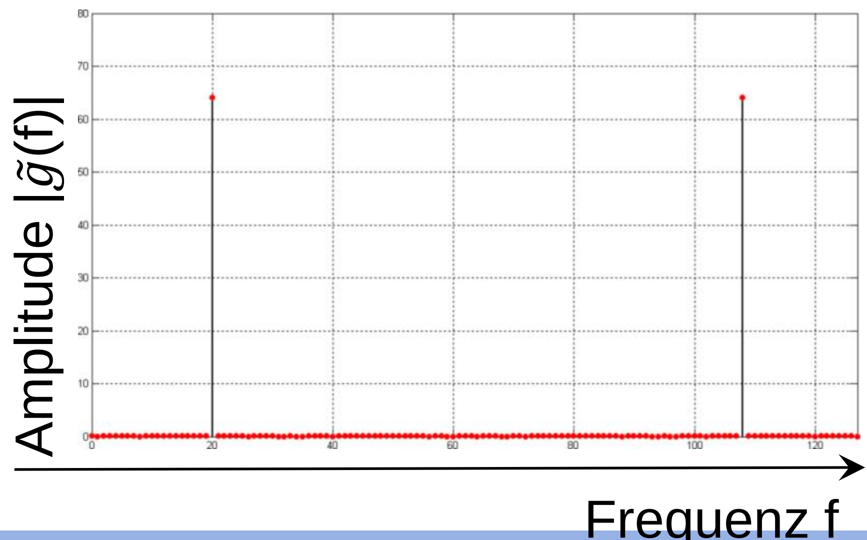
Einfluss der Frequenz f

$g(t) = a \cos(t \cdot f + \varphi)$,

a=64, f=**20**·2π/n, φ=π/3





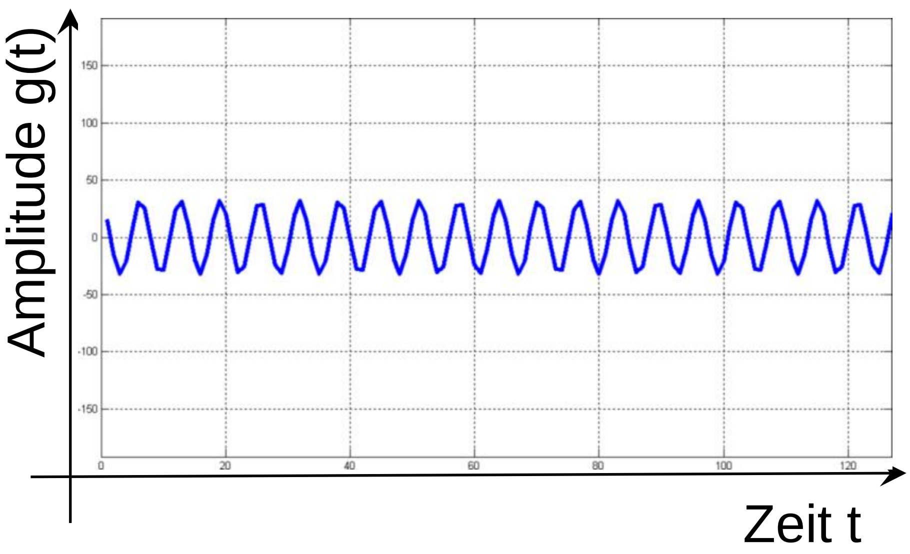


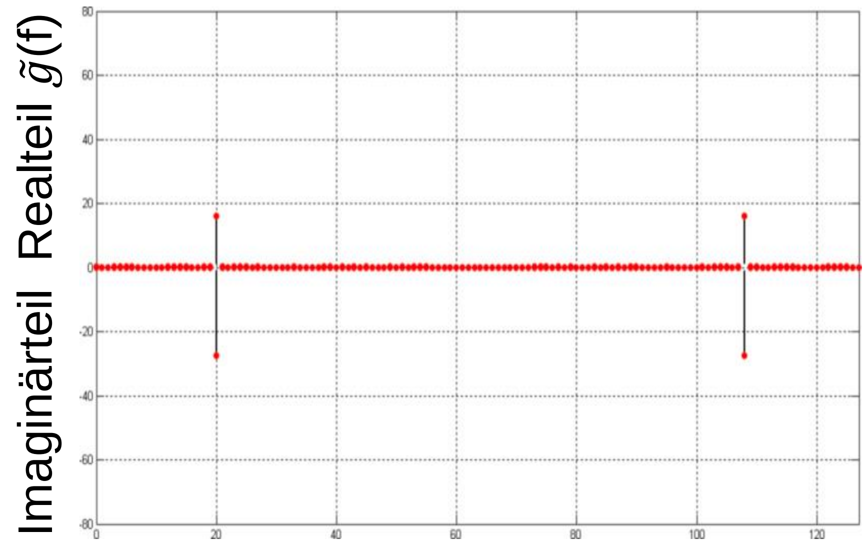
Komplexe Signaldarstellung 4

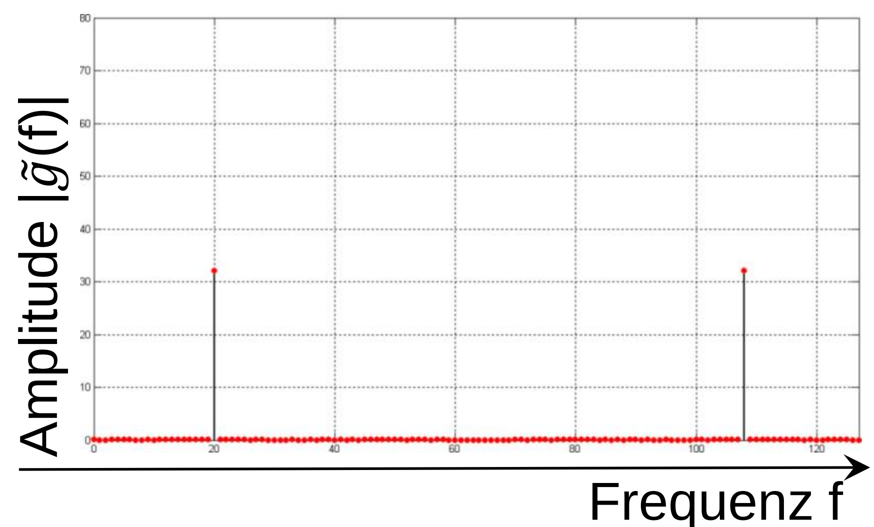
Einfluss der Amplitude a

g(t) = a cos(t·f + φ),

a=**32**, f=2π8/n, φ=π/3







Formalisierung 1

- Veränderung der Amplitude im Ortsraum = Veränderung der Amplitude im Frequenzraum
- Phasenverschiebungen im Ortsraum = Beschreibung durch 2 Komponenten („Realteil“ und „Imaginärteil“):

$$\cos(f\cdot t + \varphi) = \cos(f\cdot t)\cos(\varphi) - \sin(f\cdot t)\sin(\varphi)$$

$$= \alpha\cdot\cos(f\cdot t) - \beta\cdot\sin(f\cdot t)$$ *getrennt zu führen*

- Beschreibung durch 2-dim.Vektor (Komplexe Zahl):

$$\exp(i\cdot f\cdot t) = \cos(f\cdot t) + \boldsymbol{i}\cdot\sin(f\cdot t)$$

Darstellung als diskrete Signalmischung

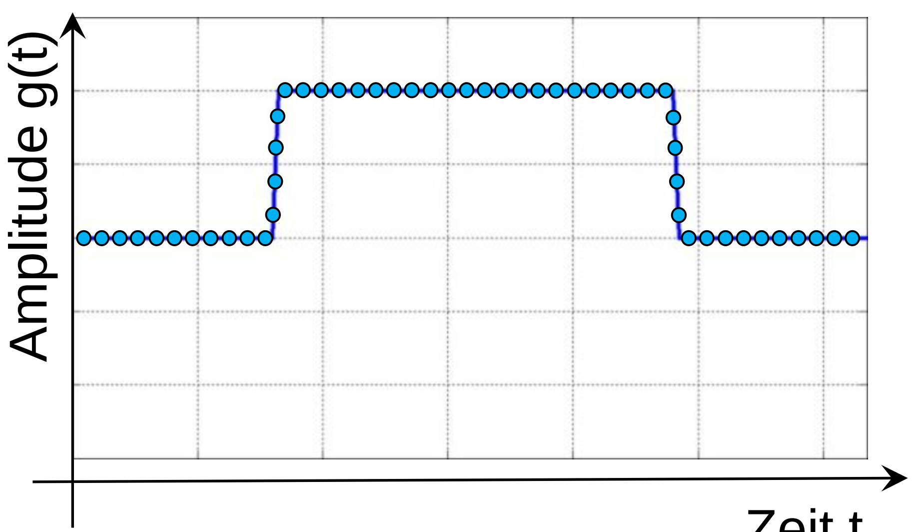
Sehr viele Komponenten:

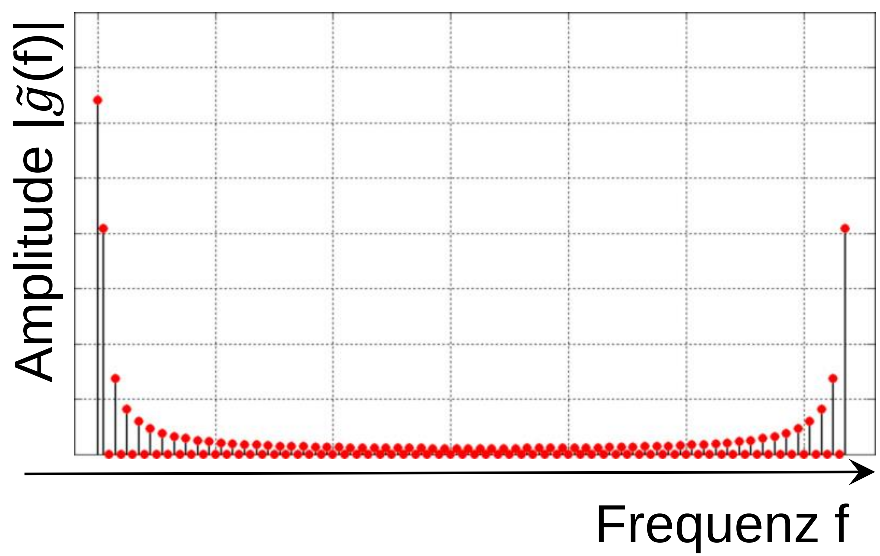
n Zeitpunkte $g(t_k)$, *k=0..n-1*

$$g(t_k) = \frac{1}{2}\sum_{j=0}^{n-1} \tilde{g}(f_j)\, exp(2\pi i \frac{jk}{n})$$

n Frequenzpunkte $\tilde{g}(f_j)$, *j=0..n-1*

$$\tilde{g}(f_j) = \frac{2}{n}\sum_{k=0}^{n-1} g(t_k)\, exp(-2\pi i \frac{jk}{n})$$





Formalisierung 2

- **Lineare Transformation:** Basis1 → Basis2

x = **Ax** mit **x** = Komponenten in Basis 1
x = Komponenten in Basis 2
A = Matrix-Zeilen = Basisvektoren 2

- **Fouriertransformation**

Zusammenfassung aller Komponenten im Vektor

$$\begin{pmatrix} \hat{x}_0 \\ \vdots \\ \hat{x}_{n-1} \end{pmatrix} = \begin{bmatrix} \frac{2}{n}\mathrm{D}(0,0) & \cdots & \frac{2}{n}\mathrm{D}(n-1,0) \\ \vdots & \ddots & \vdots \\ \frac{2}{n}\mathrm{D}(0,n-1) & \cdots & \frac{2}{n}\mathrm{D}(n-1,n-1) \end{bmatrix} \begin{pmatrix} x_0 \\ \vdots \\ x_{n-1} \end{pmatrix}$$

$$mit\ D(j,k) = \exp\left(-2\pi i \frac{jk}{n}\right)$$

Fragen ?