Chapter 1

Stretched Beam Equation

Solve the Equation:

$$(EAu_x)_x + \bar{f}Ax = 0 \tag{1.1}$$

For initial conditions:

1.
$$u(0) = g_1, u(L) = g_2$$

2.
$$u(0) = g_1, u_x(L) = \frac{h}{EA}$$

For reference, the values are defined as:

$$\begin{split} E &= 10^{11}~Pa,\, A = 10^{-4}~m^2,\, f = 10^{11}~Nm^{-4},\, L = 0.1~m\\ g_1 &= 0~m,\, g_2 = 0.001~m,\, h = 10^6~N \end{split}$$

Solution to initial condition (1):

$$u(x) = -\frac{\bar{f}}{6E}x^3 + \left(\frac{g_2 - g_1}{L} + \frac{\bar{f}L^2}{6E}\right)x + g_1 \tag{1.2}$$

Solution to initial condition (2):

$$u(x) = -\frac{\bar{f}}{6E}x^3 + \left(\frac{h}{EA} + \frac{\bar{f}L^2}{2E}\right)x + g_1$$
 (1.3)

Chapter 2

Langrange Basis Functions

Lagrange Polynomial is used for the basis function in domain.

$$N^{A}(\xi) = \frac{\prod_{\substack{B=1\\B\neq A}}^{N} (\xi - \xi_{B})}{\prod_{\substack{B=1\\B\neq A}}^{N} (\xi_{A} - \xi_{B})}$$
(2.1)