

Solve the Equation:

$$(EAu_x)_x + \bar{f}Ax = 0 \quad (1)$$

For initial conditions:

1. $u(0) = g_1, u(L) = g_2$
2. $u(0) = g_1, u_x(L) = \frac{h}{EA}$

For reference, the values are defined as:

$$E = 10^{11} \text{ Pa}, A = 10^{-4} \text{ m}^2, f = 10^{-4} \text{ N}, L = 0.1 \text{ m} \\ g_1 = 0 \text{ m}, g_2 = 0.001 \text{ m}, h = 10^6 \text{ N}$$

Solution to initial condition (1):

$$u(x) = -\frac{\bar{f}}{6E}x^3 + \left(\frac{g_2 - g_1}{L} + \frac{\bar{f}L^2}{6E}\right)x + g_1 \quad (2)$$

Solution to initial condition (2):

$$u(x) = -\frac{\bar{f}}{6E}x^3 + \left(h + \frac{\bar{f}L^2}{2E}\right)x + g_1 \quad (3)$$

Lagrange Polynomial is used for the basis function in domain.

$$N^A(\xi) = \frac{\prod_{\substack{B=1 \\ B \neq A}}^N (\xi - \xi_B)}{\prod_{\substack{B=1 \\ B \neq A}}^N (\xi_A - \xi_B)} \quad (4)$$