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## *Improved Elephant Herding Optimization and Application*

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### 12.1 Introduction

Elephants show complex social and emotional bonding with their family groups as compared to many of the animals. These families are headed by an old and experienced female called a ‘matriarch’ or ‘leader’, whereas, the male elephants prefer to live a solitary life hanging around with their male friends, which could

be from other herds or families. Interestingly, elephants have a powerful communication link with other herds or elephants by seismic waves travelling through or just over the ground. A herd can send the information about food, water, and sometimes warning of hungry predators or natural disasters as well. The other elephants can sense these signals through their feet or ear flapping and take action accordingly. The swarming or herding behaviour of elephants promotes the elephant herding optimization (EHO).

In 2015, Wang et al. [1, 2] proposed the EHO method inspired by the herding behaviour of elephants. It is a swarm intelligence based meta-heuristic optimization method. In this method, two behaviours of elephants are modelled into some set of mathematical equations. One is the clan updating operator, in which all the elephants of a clan update their positions according to their matriarch position and the matriarch also updates its own position by following the positions of that clan. The second operator is male elephant separation operator. In this operation, the male elephant is separated from the clan and a new position is obtained irrespective of the clan and leader elephant.

After its development, the EHO is applied to solve the diversified real-life optimization problems. A multi-level optimal image thresholding is determined in [3], as optimal threshold value determination has been found to be a hard optimization problem. In [4], the EHO method is applied to determine the optimal parameters tuning of the support vector machine. In order to establish best possible monitoring, of all required targets, with minimum number of static drones, the optimal drone deployment problem is solved in [5] by using the EHO method. Due to the large number of control points, a unmanned aerial vehicle path planning problem is solved in [6] by using the EHO method. In Correia et al. [7], the EHO algorithm is applied to solve an energy-supported source determination problem of wireless sensors networks. An EHO-based PID controller is designed in [8] for load frequency of power systems. The EHO, while suggesting some improvements, is also applied in [9] to solve the mixed-integer, non-linear and non-convex optimization problem of distributed generation (DG) allocation in distribution networks.

In this chapter, the modified EHO is applied to solve the optimal economic dispatch of microgrids composed of multiple distributed energy resources (DERs). The chapter is organized as follows. In Section 12.2, the brief of EHO algorithm is presented for basic understanding of the method. In Section 12.3, some modifications in standard EHO are discussed in order to improve the performance of this method. The application of modified EHO is applied to solve the optimal economic dispatch problems of microgrids to minimize the daily operating cost of the systems, followed by conclusions.

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## 12.2 Original elephant herding optimization

In this section, the standard version of EHO is briefly discussed. To solve the global optimization problems, the swarm-intelligence behaviour of elephant

herding is modelled into some set of mathematical equations based on three idealized rules. These rules are as follows, 1) The elephant herding is composed of pre-defined clans and each clan consists of a pre-defined fixed number of elephants; 2) In each generation, a fixed number of male elephants leave the clans to live independent life or with other males in the area; and 3) The elephants' population of each clan live or move under the leadership of their respective matriarch. As mentioned, the EHO method has two operators, discussed below.

### 12.2.1 Clan updating operator

This operator is specifically designed to update the position of elephants in each clan, which is influenced by the position of the leader elephant of that clan. The position of  $j$ th elephant of  $c$ th clan, except best and worst elephants, can be updated as

$$p_{jc}^{t+1} = p_{jc}^t + \alpha \times (p_{best} - p_{jc}^t) \times r \quad (12.1)$$

where,  $p_{jc}^t$  represents the position of  $j$ th elephant of clan  $c$  in  $t$ th generation. The  $p_{best}$ ,  $\alpha$  and  $r$  denote the position of the matriarch, scaling factor varies between 0 to 1, and a random number following the uniform distribution respectively. The position of matriarch is not updated by using (12.1) therefore, is updated as

$$p_{jc}^{t+1} = \beta \times p_{center,c} \quad (12.2)$$

$$p_{center,c} = \frac{\sum_{j=1}^{n_c} p_{jc}^t}{n_c} \quad (12.3)$$

where,  $\beta \in [0, 1]$  and  $n_c$  represent the scaling factor and number of elephants in  $c$ th clan respectively.

### 12.2.2 Separating operator

The male elephants are likely to live a solitary life when they grow up; therefore, a male separating operator is designed in EHO. In this operator, the elephant individual having worst fitness is removed from the population and a new elephant is generated as

$$p_{worst,jc}^{t+1} = p_{min,c} + rand \times (p_{max,c} - p_{min,c} + 1) \quad (12.4)$$

where,  $p_{min,c}$ ,  $p_{max,c}$ , and  $rand$  are the minimum and maximum allowed position limits of clan  $c$ , and uniformly distributed random number respectively.

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## 12.3 Improvements in elephant herding optimization

It has been observed that the standard version of EHO has some inherent limitations. In this section, some modifications are discussed to improve the performance of EHO algorithm.

### 12.3.1 Position of leader elephant

In standard EHO, the positions of leader elephants are updated according to the mean position, i.e.  $p_{center,c}$ , of the clan, as expressed in (12.2). In [9, 10], it has been investigated that by doing this, the clan is misguided as a result of that searching ability of the method. In order to overcome this limitation, the position of matriarch in (12.2), is updated as

$$p_{jc}^{t+1} = p_{best} + \beta \times p_{center,c} \quad (12.5)$$

### 12.3.2 Separation of male elephant

In EHO, when the male elephant leaves its family group, a new born baby is added to the clan in order to keep the number of elephants constant in a clan. The new born babies are placed at a randomly generated position, as suggested in (12.4). However, it has been observed from the herding behaviour of elephants that females always keep their babies near to fittest females groups, rather placing them, at random positions. In this improvement, the new born baby will be placed near the fittest female elephant of the clan. The position of the new elephant is updated as in [9]

$$p_{worst,jc}^{t+1} = \mu \times p_{fittest,jc} \quad (12.6)$$

where,  $\mu$  and  $p_{fittest,jc}$  represent the random number that varies from 0.9 to 1.1 and fittest female elephant  $j$  of clan  $c$  respectively.

### 12.3.3 Chaotic maps

The chaotic map is also one of the approaches to improve the performance of swarm intelligence based techniques. In this approach, the random numbers or values are replaced by chaotic maps. It is found that chaotic maps provide a random number without repetitions and ergodicity which thus improves the solution searching ability of swarm-based methods. In [11], two unlike one-dimensional maps are considered to improve the performance of the EHO method namely, chaotic circle and sinusoidal maps. The chaotic circle map is defined as

$$\theta_{k+1} = \theta_k + \Omega - \frac{K}{2\pi} \sin(2\pi\theta_k) \quad (12.7)$$

where,  $\theta_{k+1}$  is computed in mod 1. To generate the chaotic sequence between zero and 1, the value of the circle map parameters are as follows:  $K = 0.5$  and  $\Omega = 0.2$ . The chaotic sinusoidal map can be expressed as

$$\theta_{k+1} = A \cdot \sin(\pi\theta_k) \quad (12.8)$$

where,  $A \in (0, 1]$  and  $x \in (0, 1)$ . In chaotic EHO, the maps explained in (12.7) and (12.8) are used to produce chaotic sequence numbers which replace the random numbers used in (12.1) and (12.4).

### 12.3.4 Pseudo-code of improved EHO algorithm

The pseudo-code of improved EHO (IEHO) is presented in Algorithm 16. In order to explain the implementation of the algorithm, all the steps of this algorithm are discussed in this section. At the start of IEHO, a function  $OF(.)$  is prepared which produces the fitness value. The parameters of IEHO such as number of clans  $N$ , number of elephants in each clan,  $n_c$ , are initialized in step-2. The number of clans will be equal to the number of variables to be optimized, whereas, the number of elephants is the population size of the swarm. In step-3, the algorithm parameters are initialized such as  $\alpha$ ,  $\beta$ , maximum number of generations  $G_{max}$  etc. The upper and lower limits of all  $N$  variables are provided in step-4, i.e.,  $[p_{min,c}, p_{max,c}] \forall c = 1, 2, \dots, N$ . In order to initialize the algorithm, an array ( $n_c \times N$ ) of random but feasible population of elephants is generated in steps from 5 to 11, the fitness of each individual is calculated in step 10. For chaotic EHO, *rand* in step-8 is replaced by chaotic sequence number, as discussed in Section 12.3.3.

In step 12, the best and worst elephants are determined based on calculated fitness. The generation of EHO starts from step-14 and the new position of each elephant is updated in one of the steps from  $\{19, 22, 24\}$ , followed by their fitness calculation in step 28. In improved EHO, the steps 19 and 24 are replaced by (12.5) and (12.6) respectively, discussed in Sections 12.3.1 and 12.3.2. For chaotic EHO, the random numbers ‘*rand*’ will be replaced by the chaotic number sequence. In step 30, the new best and worst elephant are identified and then the matriarch elephant is updated if its fitness is better than the previous best elephant.

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**Algorithm 16** Pseudo-code of IEHO.

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- 1: determine the objective function  $OF(.)$
- 2: set the number of clans  $N$ , where  $c \in [1, N]$ , and number of elephants in each clan  $n_c$
- 3: determine the values of scaling factors  $\alpha$ ,  $\beta$ , and maximum number of generations,  $G_{max}$
- 4: set the lower and upper bounds for each variable/clan  $c$ ,  $[p_{min,c}, p_{max,c}]$
- 5: randomly generate the positions for all elephants in each clan, as follows
- 6: **for** each  $j$ -th elephant **do**
- 7:   **for** each  $c$ -th clan **do**
- 8:      $pp_{jc} = p_{min,c} + (p_{max,c} - p_{min,c}) \cdot rand$
- 9:   **end for**
- 10:    $Fitness_j = OF(pp_j)$
- 11: **end for**
- 12: determine the best and worst elephants with their locations *bestloc* and *worstloc*
- 13: set generation  $t = 1$ ;
- 14: **while**  $t \leq G_{max}$  **do**
- 15:   update the position of elephants in all clans, as follows

```

16:   for each  $j$ -th elephant do
17:     for each  $c$ -th clan do
18:       if any( $(j \neq bestloc) \& (j \neq worstloc)$ ) then
19:          $pp\_new_{jc} = pp_{jc} + \alpha \cdot (pbest_c - pp_{jc}) \cdot rand$ 
20:       else if  $j = bestloc$  then
21:          $pp_{center,c} = mean(pp_c)$ 
22:          $pp\_new_{jc} = \beta \cdot pp_{center,c}$ 
23:       else if  $j = worstloc$  then
24:          $pp\_new_{jc} = p_{min,c} + (p_{max,c} - p_{min,c}) \cdot rand$ 
25:       end if
26:     end for
27:     evaluate the fitness of new individual  $j$  as
28:      $Fitness_j = OF(pp\_new_j)$ 
29:   end for
30:   determine the new best and worst elephants
31:   if Is new best better than previous best then
32:     replace the best individual with new one
33:   end if
34:   reset old population  $pp = pp\_new$ 
35:   reset iteration  $t = t + 1$ 
36: end while
37: return the  $pbest$  as a result

```

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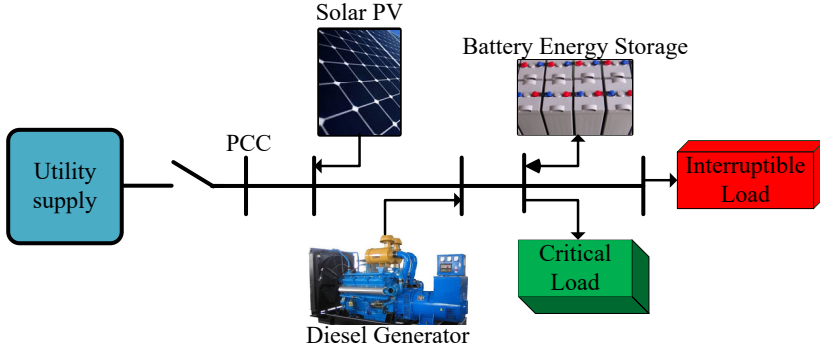
The old population of elephants will be replaced by new population for the next generation, if any. The best solution will be printed at the end of the generation.

## 12.4 Application of IEHO for optimal economic dispatch of microgrids

### 12.4.1 Problem statement

A simple microgrid system is shown in Fig. 12.1, composed of a diesel generator (DG), battery energy storage system (BESS), and solar photovoltaic (PV) along with some interpretable and critical loads. The total load demand  $P_d(h) = 2\text{MW}$ , PV power generation  $P_{pv}(h) = 748\text{kW}$ , available state of charge in BESS  $E_{bess}(h) = 2\text{MWh}$ , scheduled power of utility  $P_{sch}(h) = 1\text{MW}$  are the given data in hour  $h$ . The utility allows a maximum of 20% overdraw (OD) and under-draw (UD) of scheduled power in any hour. Determine the optimal dispatch of these energy resources for the minimum operating cost of hour  $h$ , expressed as

$$C(h) = C_{dg}(h) + C_{utility}(h) \quad (12.9)$$



**FIGURE 12.1**  
Microgrid system.

$$C_{dg}(h) = 0.3312 + 0.0156P_{dg}(h) + 0.0003P_{dg}^2(h) \text{ \$} \quad (12.10)$$

$$C_{utility}(h) = (P_{sch}(h) + |P_{sch}(h) - P_{utility}(h)|) \times E_{utility}(h) + (|P_{sch}(h) - P_{utility}(h)|) \times E_{utility}^{OD/UD} \quad (12.11)$$

s. t.

#### Diesel generator limits

$$10 \text{ kW} \leq P_{dg}(h) \leq 100 \text{ kW} \quad \forall h \quad (12.12)$$

#### BESS power dispatch limits

$$-1000 \text{ kW} \leq P_{bess}(h) \leq 1000 \text{ kW} \quad \forall h \quad (12.13)$$

#### Power balance/equality constraint

$$P_{utility}(h) = P_d(h) - P_{pv}(h) - P_{bess}(h) - P_{dg}(h) \quad \forall h \quad (12.14)$$

#### Utility supply limits

$$0.8P_{sch} \text{ kW} \leq P_{utility}(h) \leq 1.2P_{sch} \text{ kW} \quad \forall h \quad (12.15)$$

#### Utility power constraint

$$P_{utility}(h) \geq 0 \quad \forall h \quad (12.16)$$

#### Energy balance constraints of BESS

$$E_{bess}(h+1) = E_{bess}(h) - \left[ \frac{\sigma P_{bess}(h)}{\eta_{dis}} + \eta_{ch}(1 - \sigma) \cdot P_{bess}(h) \right] \quad \forall h \quad (12.17)$$

where,  $P_{dg}(h)$ ,  $P_{bess}(h)$ ,  $P_{utility}(h)$ ,  $E_{utility}(h)$ ,  $E_{utility}^{OD/UD}$ ,  $\eta_{ch}/\eta_{dis}$ ,  $\alpha$ , and  $\sigma$  represent the DG power generation, BESS power generation, utility power supply, power selling price of utility, OD/UD penalty price, charging/discharging efficiencies of BESS, and binary decision variable for charging of BESS respec-

tively. The values of other parameters, used in the study, are  $E_{utility} = 0.033$  \$/kWh,  $E_{utility}^{OD/UD} = 10$  \$/kWh, and  $\eta_{dis/ch} = 0.90$  etc.

### 12.4.2 Application of EHO to solve this problem

In order to solve this microgrid economic dispatch problem, we need to create an optimization model and then EHO can be applied to solve this problem. For doing this, we need to identify the optimization variables, i.e. clans, for EHO. From the problem statement, it can be observed that there could be three optimization variables for a given hour  $h$ , such as DG power generation ( $P_{dg}$ ), battery power dispatch ( $P_{bess}$ ) and utility power supply ( $P_{utility}$ ) etc. The other parameters cannot be considered as optimization variables because we do not have control over these. For example, solar PV generation  $P_{pv}$  which depends on environmental factors and load demand  $P_d$  depends on consumers, which are already given in the problem statement. It may also be observed that PV and BESS are owned by microgrid and no fuel charges are given for these, as they might have fixed annual O&M costs, therefore they not considered in cost calculation of (12.9).

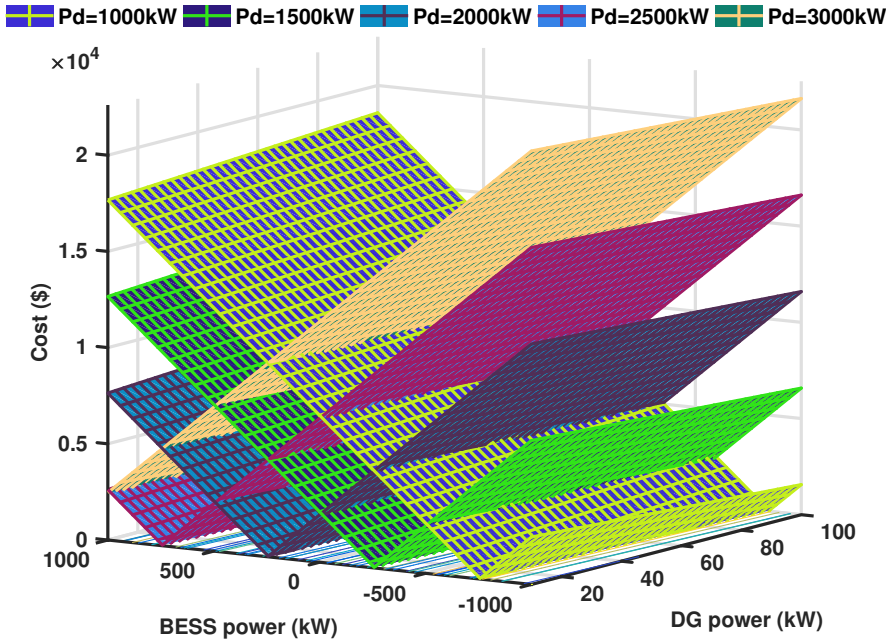
Furthermore, it may also be observed that  $P_{utility}$  can also be determined from (12.14), if other variables are known. Therefore, two variables [ $P_{dg}$   $P_{bess}$ ] are needed to optimize for the minimum operating cost of the microgrid system. The number of clans in EHO will be  $N = 2$  with their lower [ $p_{min,dg}$   $p_{min,bess}$ ] and upper [ $p_{max,dg}$   $p_{max,bess}$ ] bounds being [10 –1000] and [100 1000] respectively, see (12.12) and (12.13). A contour plot of the microgrid's unconstrained operating cost varying with BESS and DG dispatches is shown in Fig. 12.2. The figure shows that the dispatch of different distributed resources and utility grid depends on the load demand.

### 12.4.3 Application in Matlab and source-code

In this section, the EHO method discussed in Section 12.3.4 is applied to determine the minimum operating cost of the microgrid. The Matlab source-code of OF(.) for the same is presented in Listing 12.1. In this listing, steps 2 to 10 initialize the problem constants or parameters. Step 11 expresses the quadratic cost of DG, as defined in (12.9). Similarly, step 12 calculates the demand and supply mismatch of the microgrid as expressed in (12.14) and we did not consider the  $P_{utility}$  as an optimization variable. Basically, it would be the excess load demand which is not supplied by local generation such as PV, DG and BESS or excess generation if demand is less than the generation.

However, it may be observed that microgrid schedules its hourly demand with utility, i.e.  $P_{sch}$ . Technically, the utility allows the microgrid to maintain scheduled demand with a maximum error margin of 20%. Therefore, the microgrid has to fulfill the promised constraints expressed in (12.15) and (12.16). In steps 14 to 19, these constraints are expressed. Here, we used a penalty based approach in which high microgrid cost is assigned in step 18, if these



**FIGURE 12.2**

Operating cost of microgrid at different load demand (unconstrained).

constraints are violated. The penalty based approach smoothly rejects the bad solution (not satisfying constraints) without or less affecting the initialization of algorithm. These constraints can also be maintained by directly checking and eliminating such solutions in the optimization process.

```

1 function [FIT] = OF_Microgrid(pop)
2 Ebess=2000; % BESS sos level in this hour kWh
3 Psch=1000; % utility scheduled demand of hour in kW
4 Pd = 2000; % kW Load demand
5 Ppv = 748; % kW solar power
6 Euti = 0.033; % $/kWh utility energy price
7 Eod_ud=10; % OD/UD penalty price $/kWh
8 neta=0.90; % BESS charging/discharging efficiency
9 X = pop(1); % Dispatch of diesel generator (kW)
10 Y = pop(2); % Dispatch of BESS (kW)
11 C_dg = 0.3312 + 0.0156*X + 0.0003*X^2; %Running cost of DG
12 P_utility = Pd-Ppv-Y-X; % Power to/from utility grid
13 % constraints on utility power
14 if P_utility > 0 && P_utility > 0.8*Psch && P_utility < 1.2*Psch
15 P_utility = P_utility;
16 else
17 % High penalty for not satisfying equality and non-equality
   constraints of utility power
18 P_utility = randi([1e6 1e10],1,1);
19 end
20 %Cost calculation of power drawing from the grid...
21 C_utility = (Psch + abs(Psch-P_utility))*Euti + abs(Psch - P_utility)
   *Eod_ud;

```

```

22 %update the state of charge of BESS
23 if Y>=0
24 sigma=1;
25 else
26 sigma=0;
27 end
28 Ebes_new = Ebess -((sigma*Y)/neta + neta*(1-sigma)*Y);
29 %Total running cost of microgrid (To be minimized)
30 FIT = C_dg + C_utility;

```

### Listing 12.1

Definition of microgrid cost function  $OF(.)$  in Matlab.

In steps 23 to 27, a binary decision variable  $\sigma$  is generated in order to update the state of charge of the battery for the next hour, i.e.  $h+1$ , followed by energy update in step 28. Finally, the total controlled operating cost is expressed in step 30. The function  $F(.)$  is minimized by applying the IEHO method and the optimal dispatch of DG and BESS are determined. The minimum operating cost and optimal dispatch of DG and BESS obtained by IEHO are expressed as

$$P_{dg} = 25.7692 \text{ kW}$$

$$P_{bess} = 226.0874 \text{ kW}$$

$$\text{minimum } C = 35.3705 \$$$

The other calculated variables are as follows

$$\text{SOC of BESS for next hour, } E_{bess}(h+1) = 1748.80 \text{ kWh}$$

$$\text{Power supplied by utility, } P_{utility} = 1000.14 \text{ kW.}$$

From the results, it has been observed that all the constraints are satisfied by the proposed model and IEHO effectively determined the minimum operating cost of microgrid system.

## 12.5 Conclusions

In this chapter, the EHO algorithm is discussed along with some suggested improvements in order to improve the performance of the standard variant of the method. For its implantation, the pseudo-code is discussed point-wise and then a real-life microgrid optimization problem is formulated and solved to demonstrate its competitiveness to solve an engineering optimization problem. The Matlab source-code of the problem is also explained. The performance

**TABLE 12.1**

Some of the performance parameters of IEHO for solving the microgrid economic dispatch problem.

Best fitness	Worst fitness	Mean fitness	Standard deviation
35.3705	40.5471	35.9114	1.5981

parameters of the EHO method are presented in [Table 12.1](#). The tables shows the best fitness, worst, and mean fitnesses along with standard deviation. It has been observed that the method performs well for the microgrid economic dispatch problem.

---

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