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Ant Colony Optimization, Modifications, and Application

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1.1 Introduction

Ant colony optimization (ACO) is a robust and very effective optimization technique, proposed by Marco Dorigo in the year 1992 [1]. This is a population based meta-heuristic technique, inspired by the biological characteristic of ants. Like humans, ants too prefer to live in colonies and work together. They exhibit very interesting behaviors; for instance while a single ant has very limited capabilities, the whole ant colony system is highly organized [2]. The ant colony travels through the shortest path between their nest and food source. Ants basically have low visibility but good senses. They communicate with each other by the help of organic substances known as '*pheromone trails*'. This is a chemical released by an ant while traveling on the ground, to provide clues to fellow ants especially when traveling with food. The neighboring ants sense this chemical and follow the path with a high level of pheromone concentration.

Basically, ACO is one of the class of model based search (MBS) techniques [3, 4], with a characteristic to employ a specified probabilistic model and without reforming the model configuration during the run. The chapter briefly describes the fundamental characteristics of artificial ants and their mathematical representations adopted in standard ACO. This method iteratively simulates the behavior of ants with an aim to find the shortest path between food and nest. Furthermore, some improved variants of ACO are discussed which overcome some of the limitations observed in its standard variant.

1.2 Standard ant system

1.2.1 Brief of ant colony optimization

The most popular, successful and standard version of the ACO technique is based on the ant system [1]. It has been observed that the capabilities of a single ant are very limited but an ant system has very complex behavior. Each ant is collectively cooperating in this ant system without knowing their cooperative behavior [2]. By laying the pheromone on the ground, they are helping their fellows to take that path. The purpose of the ACO technique is not to simulate the complete ant colony but instead to use the collective behavior of ants to develop an optimization tool for complex real-life optimization problems. Fig. 1.1 demonstrates the shortest path finding behavior of ants [5].

In Fig. 1.1(a), ants follow the shortest path between nest and food source. In order to demonstrate the path seeking behavior of ants, a block is placed in this path, as shown in Fig. 1.1(b). Abruptly, ants scatter to find the path to their food source or nest. Now they have two paths to follow: paths through

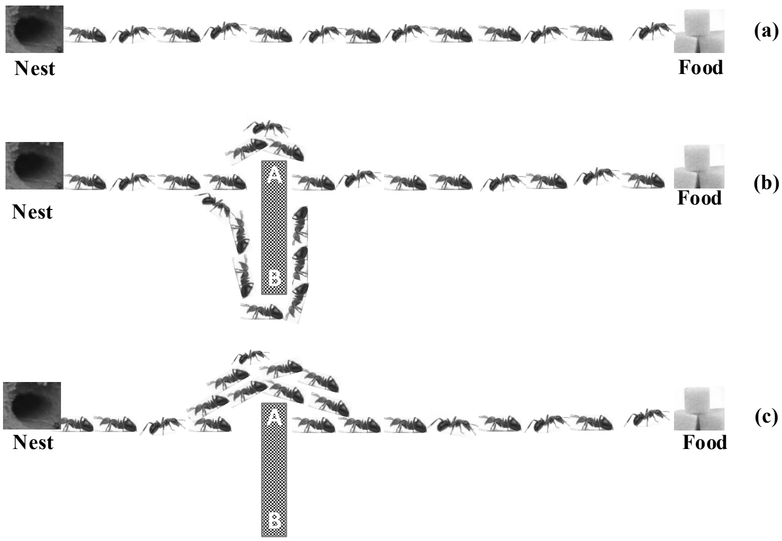

FIGURE 1.1
Ant system: a) ants follow the shortest path between nest and food source, b) a block interrupts the ant path and ants again seek the shortest path, c) ants again follow the shortest path after some time.

edge $\boldsymbol{A}$ and $\boldsymbol{B}$. The ants which are going through edge $\boldsymbol{A}$ will take less time to return than ants following edge $\boldsymbol{B}$. Therefore, more pheromone concentration will be deposited on the path through edge $\boldsymbol{A}$. Due to this increasing pheromone concentration all ants will follow this shortest path, as shown in Fig. 1.1(c). This discussed behavior of the ant system is adopted in the ACO algorithm.

The brief description based on this working mechanism of the ant system in the ACO algorithm is as follows:

1. initially, a frame is designed with edges and vertexes;
2. n_a artificial ants of the ant colony system are randomly initialized;
3. ants are arbitrarily set on vertexes of the frame; and
4. considering the fact that initial concentration of pheromone trails on all the edges is set to a small value, i.e., $\tau_0 \in [0, 1]$.

At each solution building stage, every ant cumulatively contributes to the pheromone trail by adding her share of organic substance on the partially constructed frame of solution. Suppose the kth ant is starting its journey from vertex i during the tth building phase, as shown in Fig. 1.2.

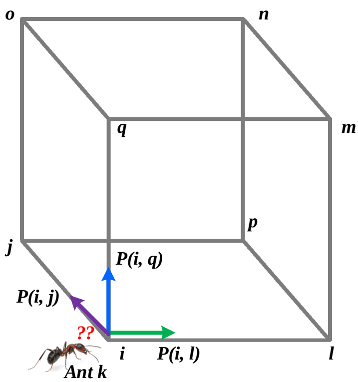

FIGURE 1.2
Decision making on ant system frame.

The ant k pursues a random walk from the vertex i to the next or nearby possible vertexes (j, l, q) for constructing an expedient solution cumulatively. The next vertex to travel is probabilistically chosen according to the transition probabilities of other vertexes such as $P_k(i,j)$, $P_k(i,l)$, and $P_k(i,q)$, with respect to the present vertex i of ant k. This transition probability of ant k to choose the next vertex j from its original vertex i is determined by using the random proportional state transition law [1], mathematically expressed as

$$P_k(i,j) = \begin{cases} \frac{[\tau(i,j)]^\alpha \cdot [\eta(i,j)]^\beta}{\sum_{j \in allowed} [\tau(i,j)]^\alpha \cdot [\eta(i,j)]^\beta}, & \text{if } j \in allowed \\ 0, & \text{othewise} \end{cases} \tag{1.1}$$

where, $\tau(i,j)$ represents the quantity of pheromone trail on the edge that links the vertex i and j. Further, $\eta(i,j)$ is a heuristic value also called the *desirability* or *visibility* of the ant to the building solution on the edge connecting vertexes i and j. It is set as the inverse of connection cost or distance between these vertexes i.e. $\eta(i,j) = 1/d(i,j)$. It is generally suggested for promoting the cost effective vertex of the frame which has a large quantity of pheromone concentration.

$\alpha \in (0,1]$ and $\beta \in (0,1]$ are known as regulating parameters that help to control the relative significance of pheromone versus heuristic values.

When all the ants of the ant system have completed their journey, the pheromone concentration is updated on the edges by the pheromone global updating rule, defined as

$$\tau(i,j) \leftarrow (1-\rho) \cdot \tau(i,j) + \sum_{k=1}^{n_a} \Delta\tau_k(i,j) \tag{1.2}$$

where, $\rho \in (0,1]$ represents the fractional amount of evaporated pheromone between two steps or iterations of the ACO algorithm. The value of ρ is set to be very small to avoid unlimited pheromone deposition on the path. $\Delta\tau_k(i,j)$ represents the amount of pheromone deposited by ant k on path (i,j), measured in per unit length and expressed as

$$\Delta\tau_k(i,j) = \begin{cases} \frac{Q}{L_k}, & \text{if ant } k \text{ travels through path } (i,j) \\ 0, & \text{otherwise} \end{cases} \tag{1.3}$$

here, Q is a constant and L_k represents the total travel length of ant k. The value of constant Q is generally set to zero.

1.2.2 How does the artificial ant select the edge to travel?

In this section, we demonstrate how an artificial ant selects the shortest path to travel. Figure 1.3 shows the mathematical expressions and explains the phenomena of an ant selecting its traveling edge. In this figure, a frame has been designed with four vertexes labeled *star*, *rectangle*, *circle*, *triangle*. The cost and pheromone graph of the edges are given in sub-figure (A) which can be treated as input data. Suppose an ant is starting its journey from the star vertex. Sub-figure (B) shows that currently the ant is standing at vertex *star*. It is willing to travel in any one direction out of three available vertexes, i.e. *rectangle*, *circle*, *triangle*. To make the decision, the transition probability explained in (1.1) should be calculated for each possible traveling

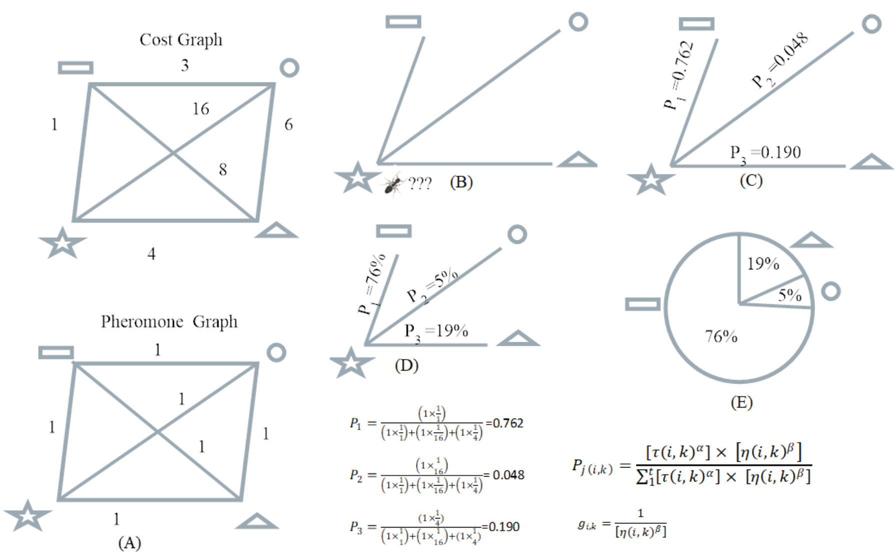

FIGURE 1.3
Mathematical and graphical representations of rules adopted by an artificial ant to select path in ACO algorithm.

path available to this ant. Now, the transition probability of each possible traveling edge is determined and presented in sub-figure (C). Similarly sub-figures (D) and (E) represent the percentage chances of an ant to travel on these edge. It can be observed from this mathematical analysis that the ant on edge *star* has the highest probability to travel towards the *rectangle* vertex. Similarly, the same approach will be used to select the future path.

1.2.3 Pseudo-code of standard ACO algorithm

The pseudo-code of the standard ACO is presented in Algorithm 1.

Algorithm 1 Pseudo-code of standard ACO.

determine the objective function $OF(.)$
set the ant population n_a, maximum number of iterations $iter_{max}$, constant and algorithm parameters such Q, τ_0, α, β, ρ, etc.
determine the distances $d(i, j)$ between vertexes from where ants start their journey and visibility $\eta(i, j) = 1/d(i, j)\ \forall\ i, j$
set iteration $t = 0$
while $t < iter_{max}$ **do**
    $t = t + 1$
    for each k-th ant **do**
        select a random vertex or point i to start the journey of kth ant.
        determine transition probability $P_k(i, j)$ of ant k to move from vertex i to j, by using (1.1)
        apply roulette wheel selection criteria on $P_k(i, j)\ \forall\ i, j$ to select the next vertex or point of travel for ant k ▷ make sure ant does not revisit any node or vertex
        determine the fitness value $OF(.)$ for ant k
        preserve the best ant, $best\_ant$ and its fitness, $best\_fit$
    end for
    update the pheromone concentration $\tau(i, j)\ \forall\ i, j$ visited by all ants, by using (1.2)
    return the best ant, $best\_ant$ and best fitness, $best\_fit$
end while

1.3 Modified variants of ant colony optimization

In this section, popularly known variants of ACO algorithms are discussed.

1.3.1 Elitist ant systems

The first improved variants of the ACO algorithm were proposed in [6]. One of these adopted an elitist strategy known as elitist ant system (EAS). It provides reinforcement to the solution components belonging to the best ant found since the start of the algorithm. In EAS, the global best solution pheromone, in each iteration, is deposited on its trail for all the ants of the colony, irrespective of their visit to this trail. As explained in [8], suppose the global best solution is represented by s^{gb}; then a substance quality $n_e/OF(s^{gb})$ will be added on this trail, where n_e and $OF(s^{gb})$ denote the number of elitist ants and solution cost of s^{gb}. In [6], it has been investigated that an adequate number of elitist ants should be chosen in the elitist ant system to determine the best path in fewer numbers of iterations. Alternatively, the ACO technique can converge to the suboptimal solution with premature convergence if an excessive number of elitist ants are present in the ant system.

1.3.2 Ant colony system

Another improved variant of ACO is known as ant colony system (ACS), introduced in [7]. The ACS has the following additional features which make it different from the ant system.

1. It has a state transition rule which straightway balances the exploration of new edges and exploitation of the superior solution and accumulated information of the problem.
2. In this technique, the global updating rule is applied only to the path traveled by the best ant of the colony.
3. while ants construct the solution, a local pheromone updating rule is also employed.

In ACS, n_a ants are initially placed on randomly selected vertexes of a constructional graph or cities. Repeatedly, each ant starts its tour by using the state transition rule, i.e., stochastic greedy rule. A local pheromone updating rule is also employed for each ant, on visited edges, to modify the concentration of its deposited pheromone. Like in the ant system, the ant will choose the next path by using the visibility heuristic and pheromone concentration. The edge with the highest pheromone intensity will always be preferred by the ants in ACS as well.

Suppose the kth ant is currently at vertex or city i in the tth step of construction; then the probability of this ant to travel towards the next edge or city j is determined by using the state transition rule known as the random-proportional rule [7], expressed as

$$j = \begin{cases} \arg\max_{l \in Z_k(i)} \{[\tau(i,l)] \cdot [\eta(i,l)]^{\beta}\}, & \text{if } q \leq q_0 \text{ (exploitation)} \\ B, & \text{othewise (biased exploitation)} \end{cases} \tag{1.4}$$

where, $Z_k(i)$ represents the set of the node/vertexes/cities that remain to visit by ant k currently placed at vertex i. Further, q and q_0 are the uniformly distributed random number and a parameter $0 \leq q_0 \leq 1$ respectively. B is a random variable selected according to the probability expressed in (1.1). The state transition rule consequent from (1.1) and (1.4) is known as the *pseudo-random-proportional rule*. It helps to achieve the most cost effective vertexes with high concentration of pheromone trail. The adequate value of q_0 provides the balance between exploitation and exploration to the ACO algorithm.

In ACS, the best ant will only be allowed to deposit the pheromone on shorted visited edges to make the search faster. This rule is applied when all ants complete their journey. In this rule, the pheromone intensity on the edges is updated as

$$\tau(i,j) \leftarrow (1-\rho)\cdot\tau(i,j) + \rho\cdot\Delta\tau(i,j) \tag{1.5}$$

where,

$$\Delta\tau(i,j) = \begin{cases} 1/J_{gb}, & \text{if } (i,j) \in S_{best} \\ 0, & \text{othewise} \end{cases} \tag{1.6}$$

where, S_{best} and J_{gb} represent the global best tour and its length respectively, from the start of the trail.

While building the solution for every step, ants change the pheromone concentration by applying the local updating rule, defined as

$$\tau(i,j) \leftarrow (1-\zeta)\times\tau(i,j) + \zeta\cdot\Delta\tau(i,j) \tag{1.7}$$

where, ζ is a parameter varying between 0 to 1.

1.3.3 Max-min ant system

Fundamentally, the ACO method inspired by the food seeking behavior of an ant colony has shown poor performance for large and complex benchmark problems. In order to overcome some of the limitations of the standard ant system, Stutzle T. and Hoos HH. [8] have proposed a new model of ACO technique known as max-min ant system (MMAS). It is inspired by the conventional ant system model that claims to be an effective method to solve the quadratic assignment problem (QAP), the traveling salesman problem [7]. The MMAS has some unique advantages over AS, given below.

- The MMAS deploys a strategy in which only the best solution found during the iteration will construct the pheromone trail. It means only a single ant which is having the best solution can add the organic substance on the path; it can be the best ant of the current iteration or the best ant found so far from the beginning of the trial.

- Initially, the highest possible intensity of pheromone, τ_{max} is considered on all the edges/paths, for better exploration of the solution.

- the range of possible pheromone trails for each solution element is limited between τ_{min} and τ_{max} to avoid search stagnation of the algorithm.

As discussed in the conventional ACO, suppose n_a number of ants are randomly placed at vertexes of a construction framework. Initially, the value of the pheromone trail on all the connecting edges is set at upper limit τ_{max}. Similar to the traditional model of ACO, every ant of the colony travels randomly to build its solution. From its current vertex i, each ant k will select the next suitable vertex j by using the transition probability, $P_k(i, j)$, defined in (1.1).

In MMAS, when all the ants of the colony have obtained their respective solution or the program has completed one iteration, the best solution of pheromone trail is upgraded by applying the rule of global update, expressed below

$$[\tau(i,j) \leftarrow (1-\rho)\cdot\tau(i,j) + \Delta\tau^{best}(i,j)]_{\tau^{\min}}^{\tau^{\max}} \tag{1.8}$$

The mathematical operator $[z]_b^a$ is defined as

$$[z]_b^a = \begin{cases} a, & \text{if } z > a \\ b, & \text{if } z < b \\ z, & \text{otherwise} \end{cases} \tag{1.9}$$

Moreover, the $\Delta\tau^{best}(i, j)$ is determined as

$$\Delta\tau^{best}(i,j) = \begin{cases} 1/g_{best}, & \text{if } (i,j) \text{ belongs to the best tour} \\ 0, & \text{otherwise} \end{cases} \tag{1.10}$$

here, g_{best} is representing the tour length of the best ant. This may be the best tour solution obtained in the current iteration or the best solution sought from the beginning of the trial. Furthermore, the value of upper and lower boundaries, i.e. τ^{max} and τ^{min}, are problem specific [8].

1.3.4 Rank based ant systems

Bullnheimer *et al.* [9], proposed a rank based ACO algorithm. In this system, all the obtained solutions are ranked based on their length and only a fixed number of ants which are having the best solutions in that iteration are allowed to update the *pheromone trial*. Furthermore, the quantity of deposited pheromone is weighted for every obtained solution. This system helps to achieve a higher concentration level of pheromone on the shortest path as compared to other corresponding paths.

1.3.5 Continuous orthogonal ant systems

A continuous orthogonal ant system was also developed by Hu *et al.* [10]. In this model, a new mechanism of pheromone deposition was suggested that enables ants to seek a possible solution by an effective and collaborative manner. By the help of an orthogonal framework, the ants in a feasible region can effectively explore their selected area rapidly while improving the global solution searching capability of the algorithm. The elitist strategy is also adopted to preserve the most valuable solutions.

1.4 Application of ACO to solve real-life engineering optimization problem

1.4.1 Problem description

Reactive power compensation has always been an important concern for power system operators to maintain the desired node voltage profile and stability in small to large-scale power systems. Shunt capacitors are traditionally deployed in power distribution networks. The optimal integration of shunt capacitors (SCs) can provide reduced real and reactive power losses while improving the node voltage profile in distribution systems. The optimal SC allocation is a complex combinatorial optimization problem aiming to minimize power delivery loss and node voltage deviation subjected to various system and distribution network operator (DNO) constraints.

1.4.2 Problem formulation

In this section, we formulate a optimal SC allocation problem for a benchmark 33-bus test distribution system. The bus and line data of this system is obtained from [11]. It is a 12.66 kV network with total real and reactive power loads of 3715 kW and 2300 kVar respectively. In this problem, the objective function for power loss minimization is expressed as

$$F = \sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_{ij}\big(P_iP_j + Q_iQ_j\big) + \beta_{ij}\big(Q_iP_j - P_iQ_j\big) \tag{1.11}$$

where,

$$\alpha_{ij} = \frac{R_{ij}\cos(\delta_i - \delta_j)}{V_iV_j} \text{ and } \beta_{ij} = \frac{R_{ij}\sin(\delta_i - \delta_j)}{V_iV_j} \tag{1.12}$$

subjected to:

$$P_i = V_i \sum_{j=1}^{N} V_j Y_{ij} \cos(\theta_{ij} + \delta_j - \delta_i) \quad \forall\ i \tag{1.13}$$

$$Q_i = -V_i \sum_{j=1}^{N} V_j Y_{ij} \sin(\theta_{ij} + \delta_j - \delta_i) \quad \forall\ i \tag{1.14}$$

$$V^{\min} \le V_i \le V^{\max} \qquad \forall\ i \tag{1.15}$$

$$I_{ij} \le I_{ij}^{\max} \qquad \forall\ i,j \tag{1.16}$$

$$n_i^{sc} Q_{bank}^{SC} \le Q^{\max} \qquad \forall\ i \tag{1.17}$$

$$\sum_{i=1}^{N} \sigma_i n_i^{sc} Q_{bank}^{SC} \le \chi \sum_{i=1}^{N} Q_{D_i} \tag{1.18}$$

Equations (1.13) to (1.18) express the constraints known as nodal real and reactive power balances, node voltage, feeder current, allowed SC capacity at a single node and total system constraints respectively. Here, P_i, Q_i, Q_{D_i}, V_i, δ_i, n_i^{sc}, and σ_i denote the real power injection, reactive power injection and demand, voltage magnitude and angle, number of SC banks and binary decision variable of SC deployment respectively, all at node i. The R_{ij}, Y_{ij}, θ_{ij}, I_{ij} and $I_{ij}^{\max}$ are respectively representing the resistance, impedance matrix element, impedance angle, current and maximum allowed current limit of branch connecting nodes i and j. Furthermore, the constants N, χ, $V^{\max}$, $V^{\min}$, Q_{bank}^{SC}, $Q^{\max}$ represent the total number of nodes in the system, nominal to peak demand conversion factor (usually, $\chi = 1.6$), maximum and minimum allowed node voltage limits in per units, VAr capacity of a capacitor bank and maximum allowed reactive power compensation at any single node respectively.

1.4.3 How can ACO help to solve this optimization problem?

Before solving this optimization problem, we apply some engineering knowledge to make it simple. In a 33-bus system, the peak reactive power load of this system would be equal to $\chi \sum_{i=1}^{N} Q_{D_i} = 1.6 \times 2300 = 3680$ kVAr. In this problem, a $Q_{bank}^{SC} = 300$ kVAr capacitor bank is considered. Generally, three nodes are found to be optimal for this system; therefore each node can accommodate a maximum of $\left(\chi \sum_{i=1}^{N} Q_{D_i}\right)/(3.Q_{bank}^{SC}) = 3680/(300 \times 3) \approx 4$ banks.

Now, we use the graph representation of search space for ants. One of the suitable graphical structures for the optimal SC allocation problem is shown in Fig. 1.4. This graph is designed for N number of SCs and corresponding number of capacitor banks to be deployed at these nodes. In C_1, '1' represents

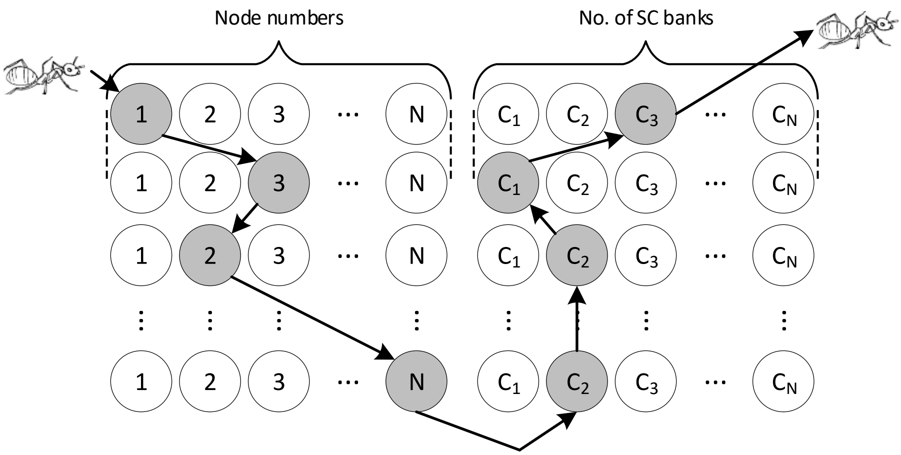

FIGURE 1.4
Proposed graph for SC allocation problem.

the one capacitor bank, i.e. 300 kVAr. In this problem, we deploy three capacitors on three different sites. Therefore, no ant should revisit any of the nodes in site selection; however the same number of capacitor banks can be obtained for two different nodes while selecting sites. A max-min ant system is adopted to solve this problem and simulation results are presented in the following section.

1.4.4 Simulation results

The optimal sites and sizes of the SCs obtained by the ACO algorithm based on max-min ant system are presented in Table 1.1. The convergence characteristic of the ACO method is also shown in Fig. 1.5.

TABLE 1.1
Simulation results of optimal SC deployment in 33-bus distribution system.

Cases	Optimal sites of SCs	Optimal capacity of SCs (kVAr)	Real power loss (kW)
Uncompensated network	–		202.6650
Optimally compensated network	15, 23, 30	300, 300, 1200	134.7107

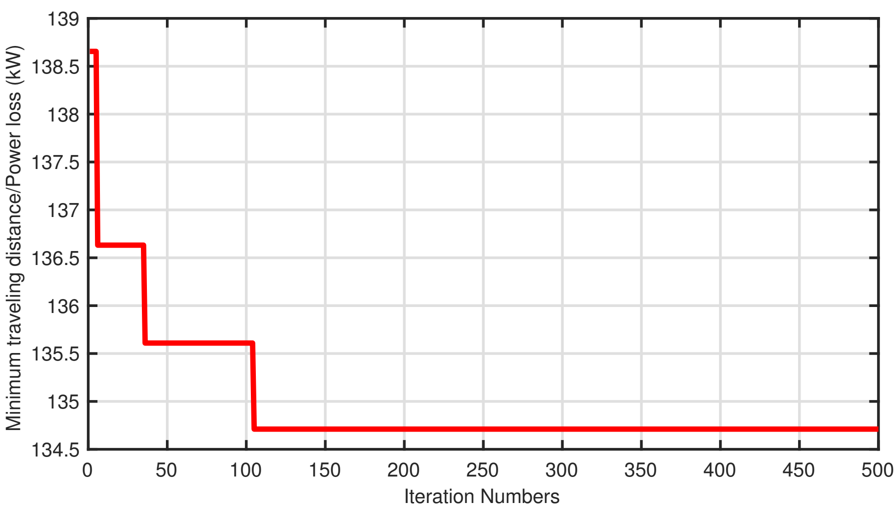

FIGURE 1.5
Convergence characteristic of ACO for best solution search.

1.5 Conclusion

In this chapter, the basic ACO technique and its popularly known modified variants are discussed. The standard ACO algorithm is also explained for better understanding of this meta-heuristic approach. Furthermore, some of the limitations of the standard ACO method and advantages of modified variants are discussed. The improved versions overcome some of the limitations observed in the standard ACO variant and help to build better solution frameworks. At the end, the application of the ACO technique is explained by solving a real-life optimal SC allocation problem of a 33-bus distribution system for real power loss minimization. A MMAS based ACO method has been used to solve this optimization problem.

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