



陈省身数学研究所

Chern Institute of Mathematics

Higgs bundles, non-Abelian Hodge correspondence and related topics

November 03–08, 2023

Chern Institute of Mathematics Nankai University, Tianjin



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Organizing Committee 组织委员会

Li, Qiongling 李琼玲 (南开大学)

He, Ling 何玲 (天津大学)

Sun, Hao 孙浩 (华南理工大学)

Service Guide 服务指南

1. 11月03日会议报到, 地点: 南开大学八里台校区嘉园宾馆;
11月04-07日会议报告, 地点: 南开大学八里台校区省身楼**216**;
11月08日离会.
2. 会议参会代表会议期间在嘉园用餐.

1. Registration on November 03, location: **Jiayuan Hotel, Balitai Campus, Nankai University**;
Talks for the conference from November 4th to 7th, location: **Xingshen Building 216, Balitai Campus, Nankai University**;
Leave on November 08.

2. During the conference, the participants will dine in **Jiayuan**.

Contact 会议联系人

:

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Directions 位置与交通

本次会议报告均安排在陈省身数学研究所(省身楼)216教室, 会议住宿地点为南校门总理像右侧的嘉园宾馆(以下简称嘉园). 陈省身数学研究所位于天津市南开区卫津路94号南开大学(八里台校区)校内, 南校门总理像左侧. 注意, 学校南门只在特殊时间打开(早上07:00-09:00/下午17:00-19:00), 其余学校大门均为全天开放. 根据学校目前的规定, 所有人员进校需提供身份证件或护照确认入校申请. 同时, 出租车不能进校. 建议您从东门或西南门入校, 走路到嘉园宾馆大概十分钟路程。

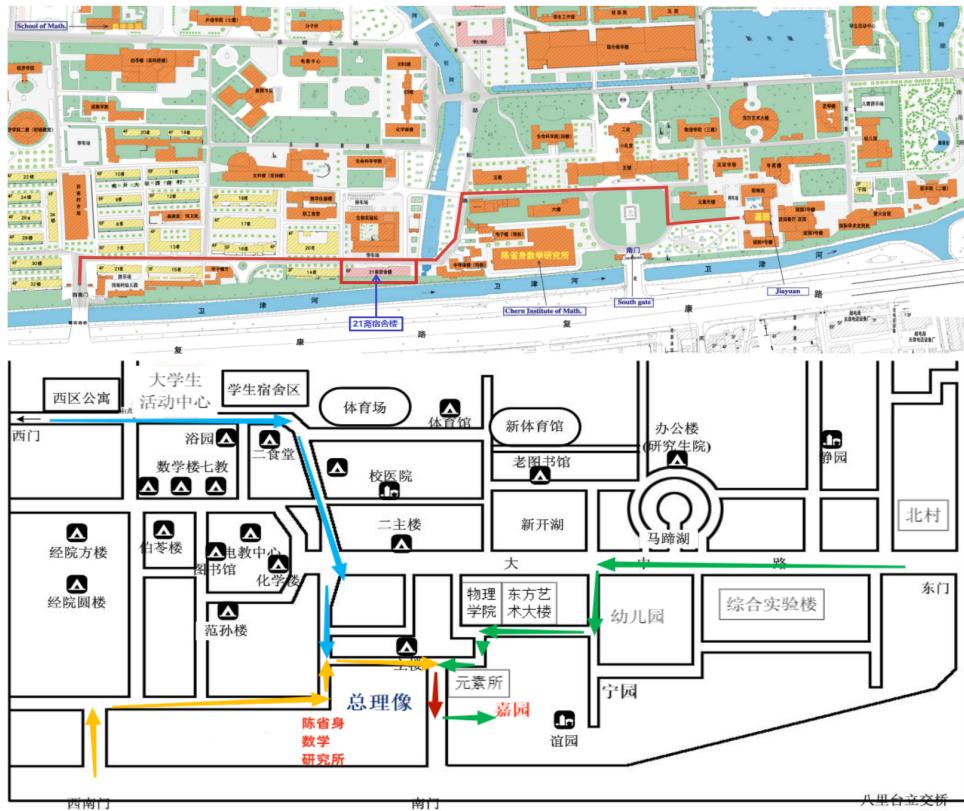


图 1: 各个校门至省身楼与嘉园的路线图

我们为与会者提供如下几种交通线路:

1. 打出租车或者网约车 至南开区卫津路南开大学东门, 或南开区复康路南开大学西南门, 建议您直接打到西南门, 离嘉园宾馆较近.
2. 天津滨海国际机场 乘坐地铁2号线(曹庄方向)至“天津站”换乘地铁3号线(南站方向)至“天塔站”下车, 步行至南开大学, 步行线路见图 1.
3. 天津站 乘坐地铁3号线(南站方向)至“天塔站”下车步行至南开大学, 或乘坐公交8路、832路至八里台(南开大学东门), 845路至手表厂(南开大学西南门)下车.
4. 天津南站 乘坐地铁3号线(小淀方向)至“天塔站”下车步行至南开大学.
5. 天津西站 乘坐地铁1号线(双林方向)至“营口道站”换乘地铁3号线(南站方向)至“天塔站”下车, 步行至南开大学; 或在西站南广场乘坐公交859路至八里台(南开大学东门)下车.

Program 会议日程

November 03, Friday Registration 报到

November 04, Saturday

Time 时间	Speaker 报告人	Title 报告题目	Chair 主持人	
07:20-08:20	Breakfast 早餐			
08:20-08:30	Opening Ceremony 开幕式			
08:30-09:20	Mochizuki, Takuro	Asymptotic behaviour of the Hitchin metric of the moduli space of Higgs bundles	Li, Qiongling 李琼玲	
09:20-09:40	Tea break 茶歇			
09:40-10:30	Chen, Gao 陈杲	Metric asymptotics on the irregular Hitchin moduli space		
10:30-10:50	Tea break 茶歇			
10:50-11:40	He, Siqi 何思奇	Rank one symmetric differentials over projective variety		
12:00-13:00	Lunch 午餐			
14:00-14:50	Fan, Huijun 范辉军	LG/CY correspondence between tt^* -geometries	He, Ling 何玲	
14:50-15:10	Tea break 茶歇			
15:10-16:00	Wei, Chuanhao 魏传豪	Kodaira-type vanishings via Non-abelian Hodge Theory		
16:00-16:20	Tea break 茶歇			
16:20-17:10	Huang, Pengfei 黄鹏飞	Around irregular nonabelian Hodge theory		
18:00-20:00	Banquet 晚宴			

November 05, Sunday

Time 时间	Speaker 报告人	Title 报告题目	Chair 主持人	
07:20-08:20	Breakfast 早餐			
08:30-09:20	Zuo, Kang 左康	Constructing abelian schemes of GL_2 -type over 4-punctured complex projective lines via p -adic Hodge theory and Langlands correspondence	Sun, Hao 孙浩	
09:20-09:40	Photo & Tea break 合影&茶歇			
09:40-10:30	Xu, Jinxing 许金兴	A higher-dimensional Chevalley restriction theorem for orthogonal groups		
10:30-10:50	Tea break 茶歇			
10:50-11:40	Wang, Bin 王彬	Springer Correspondence and mirror symmetry for Sp/SO Hitchin Systems		
12:00-13:00	Lunch 午餐			
14:00-14:50	Sheng, Mao 盛茂	p -adic Simpson correspondence via exponential twisting	Xu, Bin 许斌	
14:50-15:10	Tea break 茶歇			
15:10-16:00	Shentu, Junchao 申屠钧超	Kollar's package with coefficients		
16:00-16:20	Tea break 茶歇			
16:20-17:10	Zhang, Zili 张子立	Non-abelian Hodge correspondence and the P=W conjecture		
18:00-19:00	Dinner 晚餐			

November 06, Monday

Time 时间	Speaker 报告人	Title 报告题目	Chair 主持人
07:20-08:20	早餐		
08:30-09:20	Zhang, Xi 张希	The non-Abelian Hodge correspondence on the non-Kähler case	
09:20-09:30	茶歇		
09:30-10:20	Wan, Xueyuan 万学远	Curvature of the base manifold of a Monge-Ampère fibration and its existence	
10:20-10:40	茶歇		
10:40-11:30	Shen, Zhenghan 沈正晗	On the existence of harmonic metrics on non-Hermitian Yang-Mills bundles	Li, Jiayu 李嘉禹
11:30-11:40	茶歇		
11:40-12:30	Xu, Yan 徐言	Classification of constantly curved holomorphic 2-spheres of degree 6 in the complex Grassmannian $G(2, 5)$	
12:40-13:40	午餐		
18:00-19:00	晚餐		

November 07, Tuesday

时间	报告人	报告题目	Chair 主持人
07:20-08:20	早餐		
08:30-09:20	Nie, Xin 聂鑫	Cyclic Higgs bundles and minimal surfaces in pseudo-hyperbolic spaces	
09:20-09:40	茶歇		
09:40-10:30	Heller, Lynn	Loop group methods for the non-abelian Hodge correspondence	Hu, Zhi 胡智
10:30-10:50	茶歇		
10:50-11:40	Dai, Song 戴嵩	Some estimates on Higgs bundles over Riemann surfaces	
12:00-13:00	午餐		
18:00-19:00	晚餐		

Title and Abstract 报告题目和摘要

Asymptotic behaviour of the Hitchin metric of the moduli space of Higgs bundles

Takuro Mochizuki

Kyoto University

Abstract: The moduli space of stable Higgs bundles of degree 0 is equipped with the hyperkähler metric, called the Hitchin metric. On the locus where the Hitchin fibration is smooth, there is the hyperkähler metric called the semi-flat metric, associated with the algebraic integrable systems with the Hitchin section. As predicted by Gaiotto, Moore and Neitzke, the difference between the metrics along the curve $(E, t\theta)$ decays in an exponential way. In this talk, we shall discuss how to improve the exponential rate in the rank 2 case.

Metric asymptotics on the irregular Hitchin moduli space

陈昊 Gao Chen

中国科学技术大学

Abstract: In 1987, Hitchin constructed a complete hyperkähler metric on the moduli space of Higgs bundles, which can be generalized to accommodate singularities. In this talk, we consider Higgs bundles with irregular singularities over the Riemann sphere. We construct a generic ray in the moduli space and study the asymptotic behavior of the Hitchin metric along this ray. Using the techniques developed by Biquard-Boalch, Fredrickson-Mazzeo-Swoboda-Weiss, and Mochizuki, we show that the Hitchin metric is exponentially close to a simpler semi-flat metric. In dimension four, we obtain an explicit asymptotic formula for the semi-flat metric, which is of type ALG or ALG*. This is a joint work with Nianzi Li.

Rank one symmetric differentials over projective variety

何思奇 Siqi He
中国科学院

Abstract: Rank one symmetric differentials, a concept introduced by Taubes, play a significant role in gauge theory and differential geometry. In this talk, we'll dive into the world of rank one symmetric differentials over projective varieties. We'll explore how rank one symmetric differentials are connected to Higgs bundle and a recent proposal by Chen-Ngo. Furthermore, we will explain how rank one symmetric differentials could play a role in the Simpson integral conjecture. We will discuss a new proof using Finsler metric rigidity to prove characteristic rigidity and integrality for arithmetic varieties with rank bigger than one. This talk is based on collaborated work with J.Liu and another work with J.Liu and N.Mok.

LG/CY correspondence between tt*-geometries

范辉军 Huijun Fan
北京大学

Abstract: tt*-geometry structure was found by physicists in the 1980's, and defined and developed later in mathematics at the beginning of 90's. It is an integrable structure mixed with the holomorphic and anti-holomorphic parts, and has close connections with Higgs bundles, Frobenius manifolds and other interesting structures. It is believed that it can be applied to more important occasions. The tt* geometrical structures of Calabi-Yau manifolds have been built long ago in the name of "special geometry". In this talk, I will explain my construction of tt*-geometry for Landau-Ginzburg model via geometrical analysis method long time ago and formulate very recent results building the explicit LG/CY isomorphism between tt* geometrical structures for projective CY hypersurfaces. The latter work appears in arxiv: 2210.16747.

Kodaira-type vanishings via Nonabelian Hodge Theory

魏传豪 Chuanhao Wei
西湖大学

Abstract: In the past decade, T. Mochizuki has completed the spectacular theory of mixed Twistor D-modules. In this talk, I will first briefly introduce this result. Then, I will show that Kodaira-type vanishing still holds under the setting of mixed Twistor D-modules, which generalizes Saito vanishing under the setting of mixed Hodge Modules. I will also introduce a relative version and a version of Kawamata-Viehweg vanishing with Q-divisors, under this general setting. Both of them seem to be new even in the mixed Hodge Modules setting.

Around irregular nonabelian Hodge theory

黄鹏飞 Pengfei Huang
Heidelberg University

Abstract: Nonabelian Hodge theory studies a correspondence between Higgs bundles and local systems, via the intermediate objects: integrable connections. In this talk, we will first give a quick review of the classical theory. Then we will talk about integrable connections with irregular singularities and show a nonabelian Hodge correspondence for them. This correspondence is applicable to general complex reductive groups as the structure group. Finally we will provide a construction of moduli spaces of various filtered local systems. Based on some recent joint work with Hao Sun.

Constructing abelian schemes of GL_2 -type over 4-punctured complex projective lines via p -adic Hodge theory and Langlands correspondence

左康 Kang Zuo
武汉大学

Abstract: This is a joint work with Jinbang Yang. We construct infinitely many non-isotrivial abelian schemes of GL_2 -type over 4-punctured complex projective lines with bad reduction of type- $(1/2)_\infty$ via p -adic Hodge theory and Langlands correspondence. Recently Lin-Sheng-Wang proved conjecture on the torsionness of zeros of Kodaira-Spencer maps of those type abelian schemes. Based on their theorem we show the set of those type schemes is exactly parameterized by torsion sections of the universal family of elliptic curves. We note that, recently Lam-Litt gave a totally different construction of those abelian schemes by applying Katz's middle convolution functor. It could be highly interesting to compare these two very different constructions.

A higher-dimensional Chevalley restriction theorem for orthogonal groups

许金兴 Jinxing Xu
中国科学技术大学

Abstract: The classical Chevalley restriction theorem asserts that for a semisimple complex Lie group G , the ring of G -invariant polynomials on the Lie algebra \mathfrak{g} is isomorphic through restriction to the ring of Weyl group invariant polynomials on the Cartan subalgebra. In studying the Hitchin morphism of principal G -Higgs bundles over higher dimensional varieties, Chen and Ngo conjectured a multi-variable generalization of the Chevalley restriction theorem, and they proved the GL and Sp cases. We then prove the orthogonal group case and our treatment can apply to the GL and Sp cases in a uniform way. In this talk I will explain the main ideas of the proof and the backgrounds from Hitchin morphism and algebraic invariant theory. This is a joint work with Lei Song and Xiaopeng Xia.

Springer Correspondence and mirror symmetry for Sp/SO Hitchin Systems

王彬 Bin Wang
香港中文大学

Abstract: Starting from special nilpotent orbits in $\mathrm{Sp}_{2n}/\mathrm{SO}_{2n+1}$ which are related by Springer correspondence, we construct various Hitchin systems on curves with marked points. We resolve associated planar singularities and apply it to analyze the corresponding affine Spaltenstein fibers. As a result, we obtain (Strominger-Yau-Zaslow) mirror symmetry for those Hitchin systems. This is a joint work with Xiaoyu Su, Xueqing Wen and Yaoxiong Wen.

p -adic Simpson correspondence via exponential twisting

盛茂 Mao Sheng
中国科学技术大学

Abstract: In this talk, I shall explain the work of G. Faltings on a p -adic Simpson correspondence, using exponential twisting. This reinterpretation of the correspondence makes clearer analogy between the p -adic Simpson correspondence of Faltings and the char p Simpson correspondence due to Ogus-Vologodsky. This is a joint work with Zhaofeng Yu.

Kollar's package with coefficients

申屠钧超 Junchao Shentu
中国科学技术大学

Abstract: Kollar proved in 1986 a package of theorems on the pushforward of dualizing sheaves (torsion freeness, injectivity theorem, vanishing theorem, decomposition theorem). They have been generalized into two directions: 1) Kollar's package for dualizing sheaves twisted by a multiplier ideal sheaf; 2) Kollar's package for Hodge theoretic objects such as Hodge modules. In this talk I will survey the resent progress on this topic and explain how to unify the above results by using L2 analytic method, and show how to establish the Kollar's package in the context of nonabelian Hodge theory. This is a joint work with C. Zhao.

Non-abelian Hodge correspondence and the P=W conjecture

张子立 Zili Zhang
同济大学

Abstract: For a complex projective curve C and a reductive group G , the character variety M_B and the moduli of Higgs bundles M_{Dol} are canonically homeomorphic via the non-abelian Hodge correspondence and hence the cohomology groups of them are naturally identified. The geometric structures of the moduli spaces induce various filtrations in the cohomology groups. De Cataldo-Hausel-Migliorini conjectured in 2012 that the Perverse filtration (P) of M_{Dol} is identical to the Hodge-theoretic Weight filtration (W) of M_B ; the P=W conjecture. We will introduce the background recent progress of the P=W conjecture.

The non-Abelian Hodge correspondence on the non-Kähler case

张希 Xi Zhang
南京理工大学

Abstract: The non-abelian Hodge correspondence was established by Corlette-Donaldson-Hitchin-Simpson, it states that, on a compact Kähler manifold, there is a one-to-one correspondence between the moduli space of semisimple flat complex vector bundles and the moduli space of poly-stable Higgs bundles with vanishing Chern numbers. In this talk, we consider this correspondence on the non-Kähler case. The key to obtain the extended non-abelian Hodge correspondence lies in our recent works on semi-stable Higgs bundle and the existence of Poisson metric on semisimple projectively flat bundle. Finally, we give sThese works are joint with Chao Li, Jiayu Li, Yanci Nie, Changpeng Pan and Chuanjing Zhang.

Curvature of the base manifold of a Monge-Ampère fibration and its existence

万学远 Xueyuan Wan

重庆理工大学

Abstract: In this talk, we will consider the Monge-Ampère fibration, a relative Kähler fibration that satisfies a homogenous Monge-Ampère equation. There are two generalized Weil-Petersson metrics on its base manifold. For one metric, we present a specific curvature formula, demonstrating its non-positive holomorphic bisectional curvature, along with several other curvatures capped negatively. We'll also prove the conditions under which a holomorphic vector bundle over a compact Kähler manifold corresponds to this Monge-Ampère fibration. Furthermore, we link relative Kähler fibrations to Higgs-flatness of associated infinite rank Higgs bundles. We conclude by highlighting classical Monge-Ampère fibration examples. This work is joint with Xu Wang.

On the existence of harmonic metrics on non-Hermitian Yang-Mills bundles

沈正晗 Zhenghan Shen

南京理工大学

Abstract: In this talk, we will study the non-abelian Hodge correspondence to vector bundles with arbitrary Chern class. We will talk about the non-Hermitian Yang-Mills (NHYM for short) bundles over compact Kähler manifolds. We will show that the existence of harmonic metrics is equivalent to the semisimplicity of NHYM bundles. This talk is based on a joint-work with Dr. Changpeng Pan and Prof. Xi Zhang.

Classification of constantly curved holomorphic 2-spheres of degree 6 in the complex Grassmannian $G(2, 5)$

徐言 Yan Xu

南开大学

Abstract: Up to now the only known example in the literature of constantly curved holomorphic 2-sphere of degree 6 in the complex $G(2, 5)$ has been the first associated curve of the Veronese curve of degree 4. By exploring the rich interplay between the Riemann sphere and projectively equivalent Fano 3-folds of index 2 and degree 5, we prove, up to the ambient unitary equivalence, that the moduli space of generic (to be precisely defined) such 2-spheres is semi-algebraic of dimension 2. All these 2-spheres are verified to have non-parallel second fundamental form except for the above known example. This is a joint work with Professor Q.S. Chi and Z.X. Xie.

Cyclic Higgs bundles and minimal surfaces in pseudo-hyperbolic spaces

聂鑫 Xin Nie

东南大学

Abstract: Higgs bundles are well-known to have associated harmonic maps into symmetric spaces. We will discuss situations where this harmonic map is the Gauss map of a minimal surface in a pseudo-hyperbolic space. The study of such minimal surface in relation with classical Teichmüller theory (essentially $\text{SL}(2, \mathbb{R})$ -Higgs bundles) was mainly due to Mess, Bonsante and Schlenker; while a generalization to cyclic $\text{SO}(2, n)$ -Higgs bundles is more recently given by Collier-Tholozan-Toulisse. We will explain a further generalization to cyclic $\text{SO}(n+1, n)$ -Higgs bundles, which allows us to cover the exceptional Lie group.

Loop group methods for the non-abelian Hodge correspondence

Lynn Heller

北京雁栖湖应用数学研究院

Abstract: The non-abelian Hodge correspondence is a real analytic map between the moduli space of stable Higgs bundles and the deRham moduli space of irreducible flat connections mediated by solutions of the self-duality equation. In my talk I will explain how to construct such solutions for strongly parabolic $\mathfrak{sl}(2, \mathbb{C})$ Higgs fields on a 4-punctured sphere with parabolic weights $t \sim 0$ using loop groups methods through an implicit function theorem argument. The methods and computations are based on Deligne's interpretation of the twistor space via λ -connections. As a first application of this approach, I identify the rescaled limit hyper-Kähler moduli space at $t = 0$ as the Eguchi-Hanson space, and I show that the hyper-Kähler metric can be expanded to arbitrary order in terms of multiple-polylogarithms. Finally, the geometric properties lead to some identities involving multiple-polylogarithms. This talk is based on joint work with Sebastian Heller and Martin Traizet.

Some estimates on Higgs bundles over Riemann surfaces

戴嵩 Song Dai

天津大学

Abstract: Given a Riemann surface, the non-Abelian Hodge theory roughly speaking builds a correspondence between Higgs bundles and harmonic maps. In this talk, under the background of the moduli space of the Higgs bundles, we discuss some estimates of the geometric quantities from harmonic maps. We focus on two kinds of estimates: domination and boundedness. We will also show some applications of the estimates. The main results in this talk are joint with Qiongling Li.