# Transform Order by Reversals : ILP vs SAT-solver

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For Integer Linear Programming, we have the following conditions:
**1.**
For each $k$ from 1 to $n-1$:

$$NOP(k) + \sum_{(p,q):p<q} R(p,q,k) = 1$$

For an equivalent dnf, we see that there is one and only one 1 among all these values.

$$(NOP(k) \wedge \neg R_1 \wedge \neg R_2 \wedge ......) \vee (\neg NOP(k) \wedge R_1 \wedge \neg R_2 \wedge ......) \vee ......$$

We want a cnf that is not too complicate. One of them is true and none of the pairs can be both true. [1]

$$(NOP(k) \vee R_1 \vee R_2 \vee ...) \wedge (\neg NOP(k) \vee \neg R_1) \wedge (\neg NOP(k) \vee \neg R_2) \wedge (\neg R_1 \vee \neg R_2) \wedge ...$$

**2**
For $i$ from 1 to $n$:

$$\sum_{q=1}^{n} X(1,i,i,q) = 1$$

For an equivalent cnf, we see that there is one and only one 1 among all these values.

$$(X_1 \vee X_2 \vee X_3 \vee ...) \wedge (\neg X_1 \vee \neg X_2) \wedge (\neg X_1 \vee \neg X_3) \wedge (\neg X_2 \vee \neg X_3) \wedge ...$$

**3**
For $i$ from 1 to $n$:

$$\sum_{p=1}^{n} X(n-1,i,p,Q_2(i)) = 1$$

For an equivalent cnf, we see that there is one and only one 1 among all these values.

$$(X_1 \vee X_2 \vee X_3 \vee ...) \wedge (\neg X_1 \vee \neg X_2) \wedge (\neg X_1 \vee \neg X_3) \wedge (\neg X_2 \vee \neg X_3) \wedge ...$$

**4**
For each node $q$ at a level $k$ from 2 to $n-1$:

$$\sum_{p=1}^{n}[\sum_{i=1}^{n} X(k-1,i,p,q)] = 1$$

For an equivalent cnf, we see that there is one and only one 1 among all these values.

$$(X_1 \vee X_2 \vee X_3 \vee ...) \wedge (\neg X_1 \vee \neg X_2) \wedge (\neg X_1 \vee \neg X_3) \wedge (\neg X_2 \vee \neg X_3) \wedge ...$$

[1] https://math.stackexchange.com/questions/2554805/writing-in-cnf-that-only-one-statement-can-be-true

**5**

For each node $p$ at a level $k$ from 2 to $n-1$:

$$\sum_{q=1}^{n}[\sum_{i=1}^{n} X(k,i,p,q)] = 1$$

For an equivalent cnf, we see that there is one and only one 1 among all these values.

$$(X_1 \vee X_2 \vee X_3 \vee ...) \wedge (\neg X_1 \vee \neg X_2) \wedge (\neg X_1 \vee \neg X_3) \wedge (\neg X_2 \vee \neg X_3) \wedge ...$$

**6**

For each integer $i$ from 1 to $n$:

$$\sum_{q=1}^{n} X(k-1,i,q,p) = \sum_{q=1}^{n} X(k,i,p,q)$$

We know that there is at most one 1 (XNOR). The tested format is in the ipynb file. Suppose $n = 5$.

$(X_1 \vee X_2 \vee X_3 \vee X_4 \vee X_5 \vee \neg X_6) \wedge (X_1 \vee X_2 \vee X_3 \vee X_4 \vee X_5 \vee \neg X_7) \wedge ...(X_6 \vee X_7 \vee X_8 \vee X_9 \vee X_{10} \vee \neg X_1)...$

**7**

For each pair $(k, w)$ with $k$ from 1 to $n-2$, and $w$ from 1 to $n$:

$$\sum_{i=1}^{n}[X(k,i,w,w)] \leq NOP(k) + \sum_{\{p,q\}:w=(p+q)/2} R(p,q,k) + \sum_{p,q<w} R(p,q,k) + \sum_{p,q>w} R(p,q,k)$$

It's an implication. The tested format is in the ipynb file. Suppose n=5.

$(X_6 \vee X_7 \vee X_8 \vee X_9 \vee X_{10} \vee \neg X_1) \wedge (X_6 \vee X_7 \vee X_8 \vee X_9 \vee X_{10} \vee \neg X_2) \wedge (X_6 \vee X_7 \vee X_8 \vee X_9 \vee X_{10} \vee \neg X_3)...$

**8**

For each level $k$ from 1 to $n-2$, and each (unordered) pair of positions $(w, z)$:

$$\sum_{i=1}^{n} X(k,i,w,z) \leq \sum_{p=a-h<q=b+h:h\leq \min(a-1,n-b)} R(p,q,k)$$

It's an implication. The tested format is in the ipynb file. Suppose n=5.

$(X_6 \vee X_7 \vee X_8 \vee X_9 \vee X_{10} \vee \neg X_1) \wedge (X_6 \vee X_7 \vee X_8 \vee X_9 \vee X_{10} \vee \neg X_2) \wedge (X_6 \vee X_7 \vee X_8 \vee X_9 \vee X_{10} \vee \neg X_3)...$

**9**

For each $k$ from 1 to $n-1$:

$$NOP(k) \leq NOP(k+1)$$

It's an implication.

$$(NOP(k+1) \vee \neg NOP(k))$$