A Time-scaled Interpolation Reduced Order Model

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MORTech Sevilla, November 10, 2017

Outline

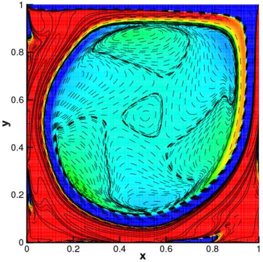
Introduction

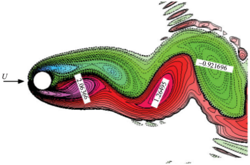
Time-scaling ROM

Applying Time scaling interpolation ROM

Outlook and conclusion

Motivation





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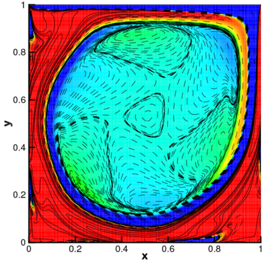

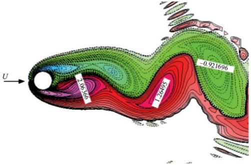

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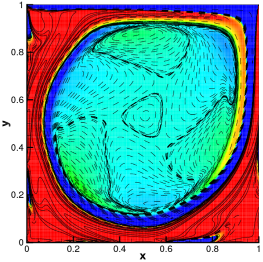

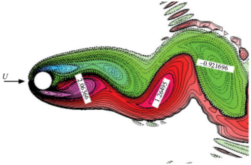

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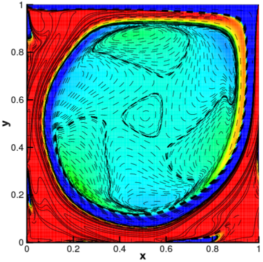

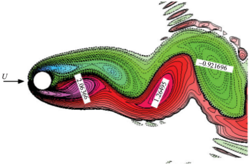

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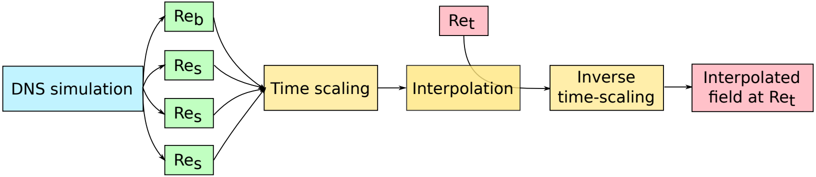
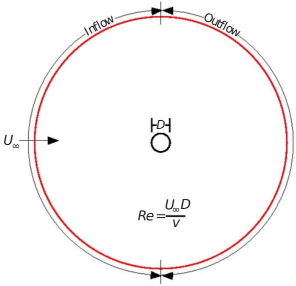


Flow past a circular cylinder



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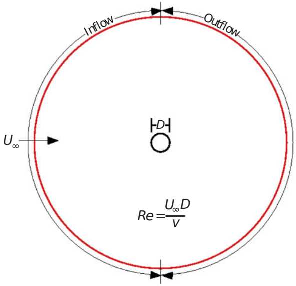

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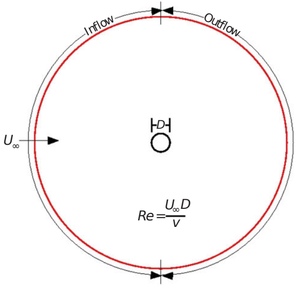

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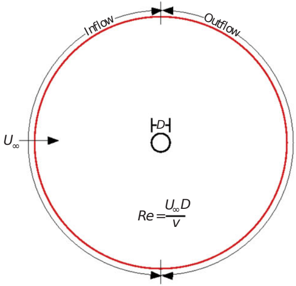

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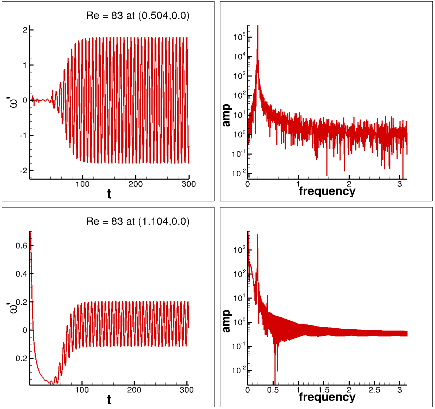

Time series and associated FFT at $Re = 83$ at two wake points $(0.504, 0.0)$ and $(1.104, 0.0)$.

Need for time-scaled interpolation

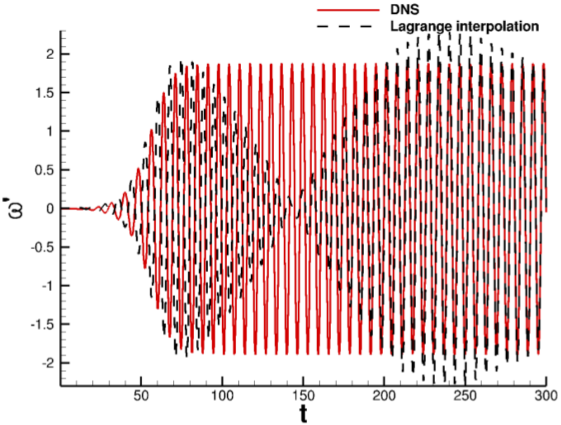

FIGURE – Direct interpolation of DNS vorticity time series at point (0,0.504) for flow past a circular cylinder.

Time scales in FPCC

- Strouhal number :

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- Proposed power low scaling :

$$\frac{St(Re_s)}{St(Re_b)} = \left(\frac{Re_b}{Re_s}\right)^n$$

TABLE – **Scaling Constant and Base Re_b for Different Range of Re_s for FPCC**

Re Range	Scaling Constant (n)	Basic Re (Re_b)
$55 - 68$	-0.49 ± 0.02	60
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Having fixed n on a segment, time scaling reads :

$$t_s = t_b \left(\frac{Re_b}{Re_s} \right)^n + t_0(Re_b, Re_s)$$

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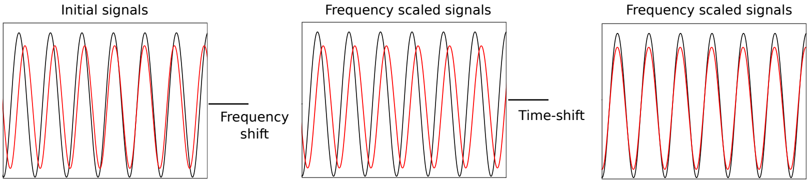

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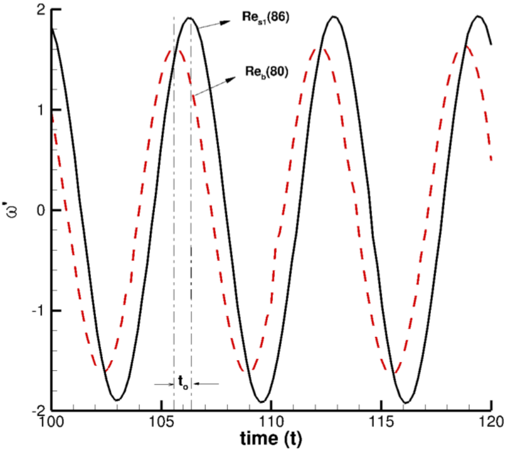

Disturbance vorticity at a point (0.504, 0.0) with t_b and t_s for $Re_b = 80$ and $Re_s = 86$.

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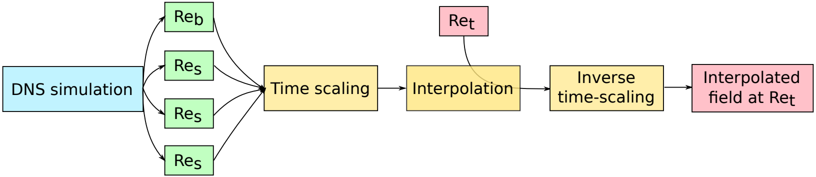
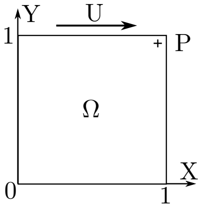


Schematic of the time-scaled interpolation ROM

2D Singular Lid Driven Cavity problem



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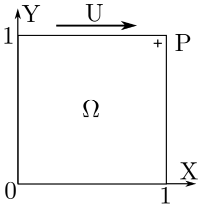

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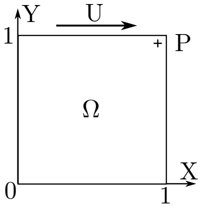

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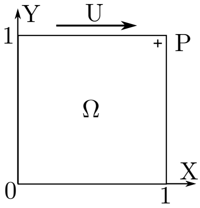

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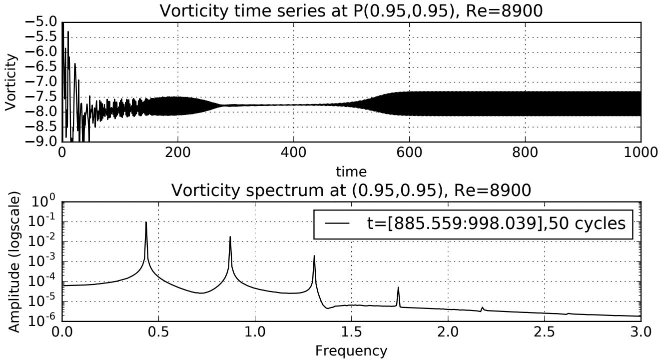

LDC Flow behavior : characterisation

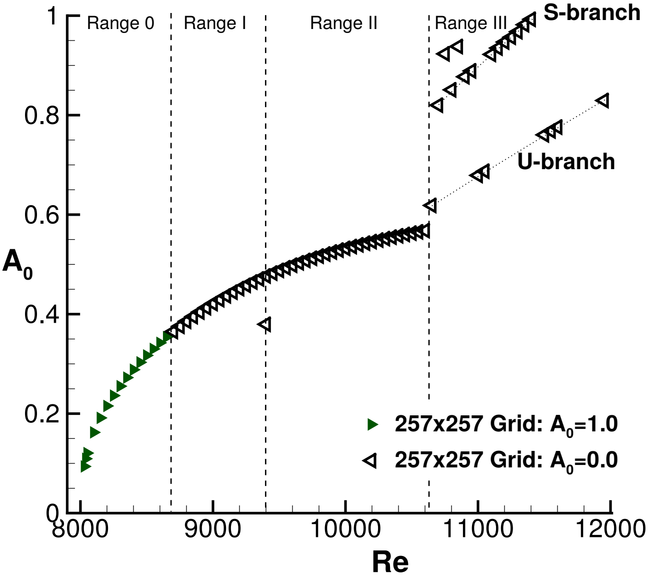

FIGURE – Bifurcation sequence explained via equilibrium amplitude variation with Re

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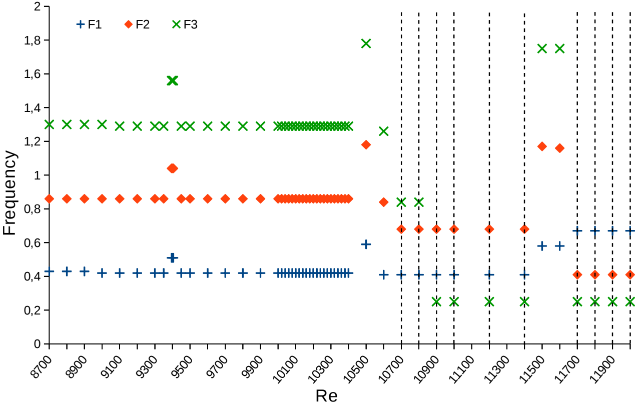

FIGURE – Leading frequencies variation with Re

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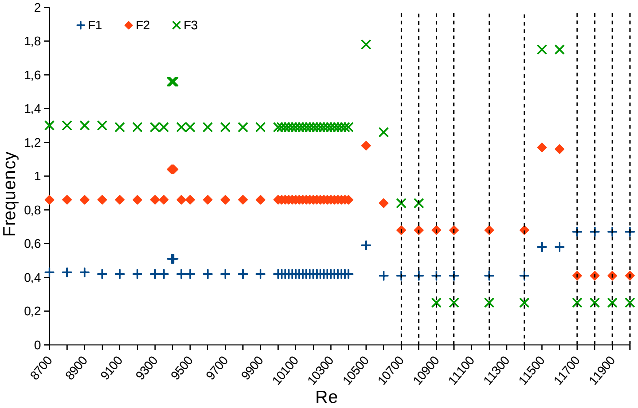

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The main frequency is constant per segments $\Longrightarrow n = 0$

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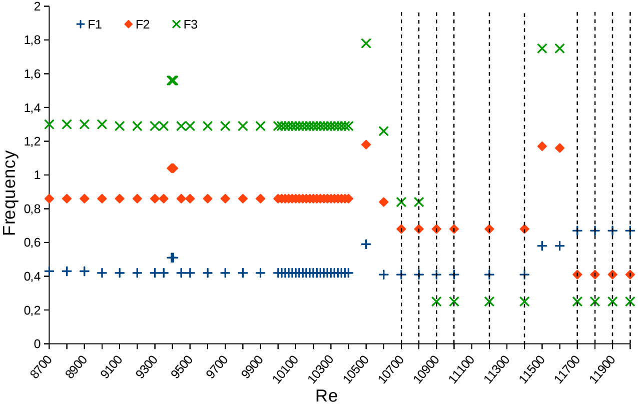

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Time scaling becomes a **time-shifting** :

$$t_s = t_b + t_0(Re_b, Re_s)$$

Time-shifting interpolation ROM in action on LDC flow

- Limit cycle interpolation ($t \in [1900 : 1940]$), narrow range
- $Re_t = 10040$
- Donor Re $\{Re_b = 10000, 10020, 10060, 10080\}$

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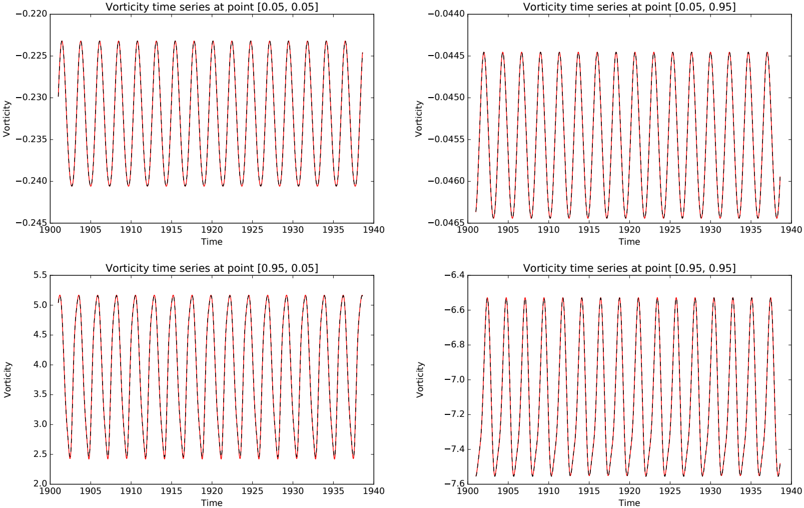

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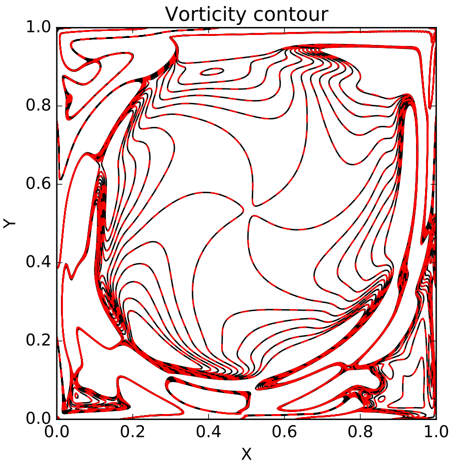

Disturbance vorticity contour at $t = 1900.199$

- With time-shifting RMS Error is 7.1×10^{-4}
- Without time-shifting RMS Error is 8.1×10^{-2}

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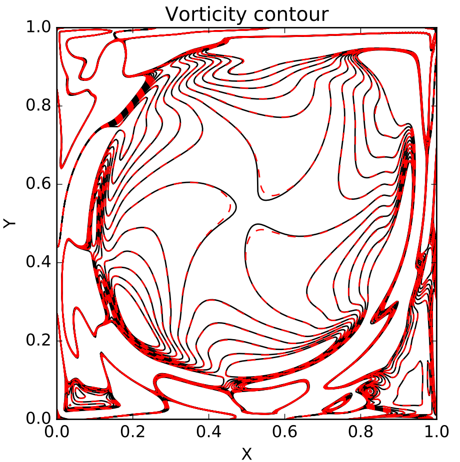
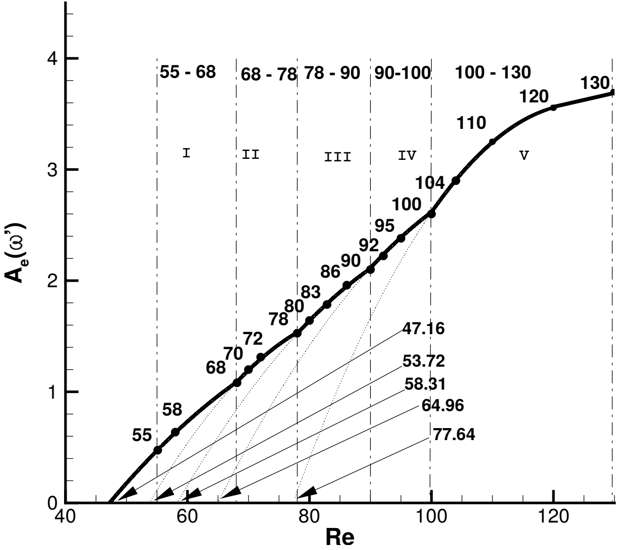
- Interpolation across two ranges, limit cycle
- $Re_t = 9600$
- Donor Re $\{Re_b = 9350, 9500, 9800, 10000\}$



Disturbance vorticity contour at $t = 1900.199$

- With time-shifting RMS Error is 5.6×10^{-4}
- Without time-shifting RMS Error is 1.7×10^{-2}

FPCC Flow behavior



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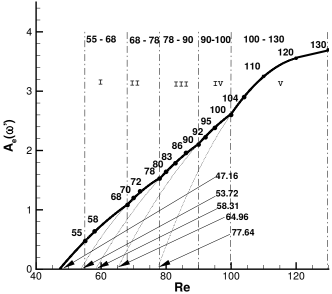

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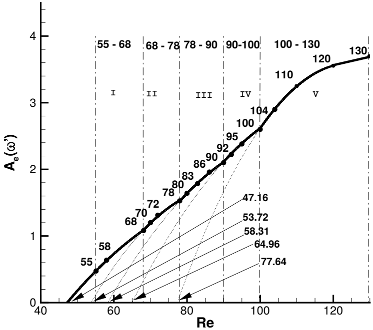

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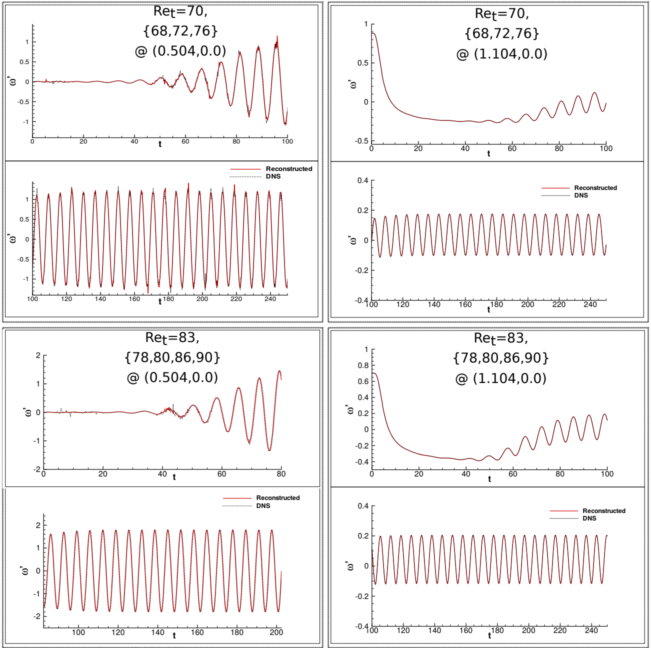
$\longrightarrow$ The full time scaling method is required :

$$t_s = t_b \left(\frac{Re_b}{Re_s} \right)^n + t_0(Re_b, Re_s)$$

- The whole time and space domain can be interpolated.

Time-shifting interpolation ROM in action on FPCC



Time-shifting interpolation ROM in action on FPCC

TABLE – RMS Error estimates of interpolation for $Re = 83$

Cases	Re of Donor Points	Interpolation Error (%)
I	(78,80,86,90)	4.3
II	(72,80,86,90)	4.3
III	(68,80,86,90)	4.4
IV	(55,80,86,90)	6.2
V	(55,80,86,130)	14
VI	(55,68,72,86)	131
VII	(55,68,72,130)	852

Conclusion

Time-scaling ROM is :

- Physics based ROM for supercritical flows (internal/external)
- Allows Lagrange interpolation directly on DNS data
- Only 3-4 snapshots required

Future work :

- Compare with EIM/DEIM, Grassmann manifold interpolation
- Apply to POD representation

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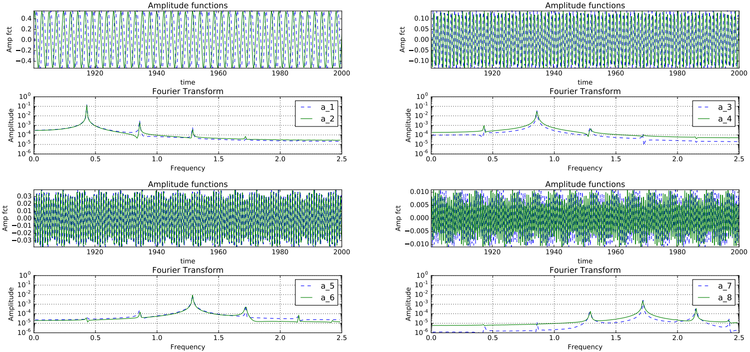

Limit cycle modes time series and FFT, Re=9800

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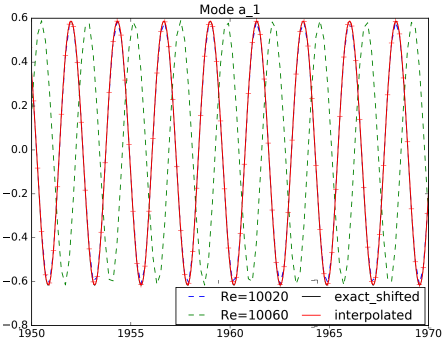

Interpolation of first amplitude mode, Re=10400

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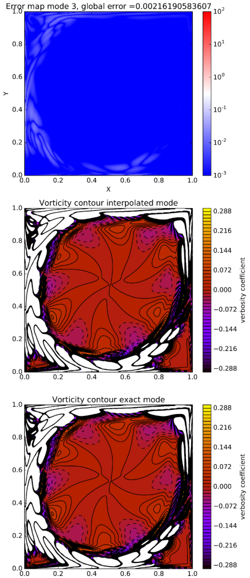

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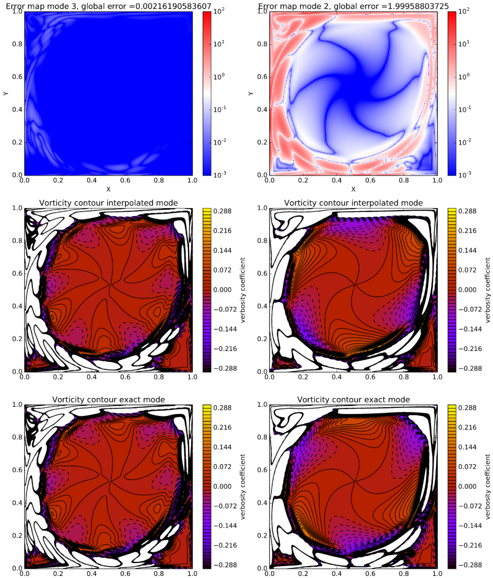

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- Reconstruction of the interpolated sum must be dealt with carefully
  - 1 by 1 comparison (with time scaling) works : $e \approx 10^{-3}$
  - Many competing St for inverse time scaling
  - Impossible to reconstruct t_0, need to find another way

THANK YOU FOR YOUR ATTENTION