

# Deriving the full EOM

## Equation of Motion

Airway pressure is predicted mathematically by the equation of motion:

$$P_{\text{vent}} + P_{\text{mus}} = \boxed{V_T/C_{RS}} + \boxed{R_{\text{aw}} \times \dot{V}_I} + \text{PEEP} + \text{PEEPi} + \text{inertance} \quad (1)$$

$$R_I = (PIP - \boxed{P_{\text{plat}}})/\dot{V}_I$$

$$R_E = \boxed{(P_{\text{plat}} - \text{PEEP})}/\dot{V}_{\text{EXH}}$$

$$P_{\text{plat}} = \frac{(V_T \times PIP) - (V_T \times \text{PEEP})}{V_T + \boxed{\tau_E} \times \dot{V}_I} \quad (2)$$

This approach has the advantage of being able to be used in spontaneous breathing modes such as pressure support, but has the disadvantage of requiring a computerized algorithm to make the necessary calculations.

$$\tau_E = \text{exhaled } V_T/\dot{V}_{\text{EXH}}$$

# Expiratory time constant for determinations of plateau pressure, respiratory system compliance, and total resistance

## Hypothesis:

Expiratory time constant ( $T_E$ ) can be used to determine Ppl<sub>t</sub>, Crs and R<sub>tot</sub>. Since  $T_E$  contains information regarding the mechanical properties of the respiratory system, namely elastance and resistance.

## Material and Methods

$T_E$  is expressed in seconds, and one  $T_E$  represents the time required for the lungs to reach 63% of its equilibrium value.  $T_E$  was measured 0.10 to 0.50 seconds after beginning of exhalation, using available V<sub>t</sub> and flow measurements. Specifically, the slope of the least square fit between V<sub>t</sub> and Flow constituted  $T_E$ . A straight line between the linear fit is necessary, as it ensures relaxation of the patients respiratory muscles.

## Derivation of equations

### ***R<sub>tot</sub>***

For R<sub>tot</sub>, we again start with the EOM, disregarding P<sub>mus</sub>

$$P_{aw} - PEEP = V_T / Crs + R_{tot} \times \text{Inhaled flow}$$

Multiply V<sub>t</sub>/Crs by R<sub>tot</sub>/R<sub>tot</sub>:

$$P_{aw} - PEEP = V_T \times R_{tot} / Crs \times R_{tot} + R_{tot} \times \text{Inhaled flow}$$

Substitute  $T_E$  for R<sub>tot</sub>\*Crs on the right side gives:

$$P_{aw} - PEEP = V_T \times R_{tot} / T_E + R_{tot} \times \text{Inhaled flow}$$

Factoring the right side gives:

$$P_{aw} - PEEP = R_{tot} (V_T / T_E + \text{Inhaled flow})$$

Divide both sides by V<sub>t</sub>/T<sub>E</sub>:

$$R_{tot} = \frac{P_{aw} - PEEP}{\frac{V_T}{T_E} + \text{Inhaled flow}}$$

$T_E$  is traditionally the product of  $C_{rs}$  and  $R_{tot}$ .

$C_{rs}$

Equation of Motion  
(disregarding  $P_{mus}$ )

$$P_{aw} - PEEP = (V_T / C_{rs}) + R_{tot} * \text{InspFlow}$$



Multiply both sides by  $C_{rs}$

$$C_{rs}(P_{aw} - PEEP) = V_T + (R_{tot} * \text{InspFlow}) C_{rs}$$



Substitute  $R_{tot} * C_{rs}$  for  $T_E$

$$C_{rs}(P_{aw} - PEEP) = V_T + \text{InspFlow} * T_E$$



Divide by  $(P_{aw} - PEEP)$

$$C_{rs} = \frac{V_T + \text{InspFlow} * T_E}{P_{aw} - PEEP}$$

**$P_{plat}$**

Plateau pressure = (tidal volume / static compliance  
) + PEEP

$$P_{plat} = V_t / C_{rs} + PEEP$$



Substitute  $C_{rs}$  by

$$C_{rs} = (V_t + T_e * \text{Inhaled Flow}) / (P_{aw} - PEEP)$$

$$P_{plat} = (V_t / (V_t + T_e * \text{Inhaled Flow} / (P_{aw} - PEEP))) + PEEP$$



Move  $(P_{aw} - PEEP)$  up to the numerator

$$P_{plat} = (V_t * P_{aw}) - (V_t * PEEP) / (V_t + T_e + \text{Inhaled Flow})$$