Independent View

Quantitative Test

Introduction

Below you will find a list of quantitative problems of various difficulty. You're not expected to solve all problems. The test was designed to give you a variety of challenges to choose from, depending which ones you think present your skills in the best way. It is better to have one solved problem than several half-solved.

You may use any possible source of information for inspiration and hints, except another person coding the problems for you.

You have maximum **three hours** to work on the selected problems. Even if none of the assignments is solved after three hours, we would like to see the entire source code that you worked on. This will form the basis for the following interview where you are expected to explain and discuss any choices that have been made.

All solutions must be programmed with **Python**.

Most problems on that list don't have "one proper" solution and we don't want you to stress too much over finding it. Any attempt at solving the problem that is a working program is a valid solution.

Biased coin game

You have a biased coin, but you don't know exactly how biased it is. The probability of it falling heads is any number between 0 and 1. You have 1000 USD in your pocket when Gary offers you to play a game – you can bet any increment of 1 USD to bet on one side of the coin. Afterwards, Gary throws the coin in the air and if it lands on the side you've chosen, your bet is doubled. If it doesn't, Gary takes the amount you've bet.

Write a program that plays this game to maximize your net worth growth (or a logarithm of net worth) in terms of times the coin was flipped.

You don't have to find the optimal solution. Just find the best solution you can in reasonable time.

Bacterial colony

A bacterial colony consists of individual bacteria. One of the following happens with each bacterium each second:

- The bacterium dies.
- The bacterium remains without a change.
- The bacterium splits in two bacteria.
- The bacterium splits in three bacteria.

Each of the above happens with equal probability 25%. Your task is to estimate probability that the bacterial colony, which initially consists of one bacterium, will never die.

Game show

You participate in a game show. The game consists of *n* rounds. During each round, the game show host proposes you a prize of a known value (in USD). You have two choices:

- You can take the prize and end the game.
- You can reject the prize and continue the game. If you reject the prize, which was proposed to you during the last round, you will not get any price (you can think about it as you get a prize of value 0 USD).

It is known that the values of proposed prizes are independent and identically distributed random variables. It is also known that the values of proposed prizes have binominal distribution with parameters 40 and 0.7.

Here is an example of a possible game show scenario. Bob participates in a game show which consist from 3 rounds (n=3).

- During the first round Bob is proposed a price of value 15 USD. Bob rejects the price. Therefore the second round starts.
- During the second round Bob is proposed a price of value 20 USD. Bob accepts the price. Since Bob accepted the price, the game ends, and there is no third round.

Your task is to write a program, which for a given number of rounds *n* outputs optimal game strategy. The optimal game strategy maximizes expected value of the received prize. The optimal strategy can be described as an array *A* of prize values. If the value of the prize proposed on the round *i* is grater or equal then *A[i]* than you take the prize, otherwise you continue the game.

Shape Scanner

Given a square matrix MxM, which contains only entries of either 0 or 1, find out whether the entries of 1 form a circle or an unrecognizable shape. It is up to you to decide where exactly the boundary between “circles” and “unrecognizable shapes” is. If it’s a circle also calculate the coordinates of the center and its radius. Otherwise calculate only the center of mass of all ones.

Portfolio optimization

The trading signals for three assets A, B, and C are

$$\mu = [0.47, \quad 0.33, \quad 0.58]$$

The three assets have a correlation matrix as follows:

$$Q = \begin{pmatrix} 1 & 0.54 & 0.19 \\ 0.54 & 1 & 0.12 \\ 0.19 & 0.12 & 1 \end{pmatrix}$$

The portfolio optimization problem to solve x is:

$$max_x \, \mu^T x,$$

with the constraints:

$$x^T Q x = 1 \text{ and } x^T \iota = 0$$

where ι is a 3 by 1 vector of 1, and x is a 3 by 1 vector.

1. Solve the above problem for x by any optimization packages you might find suitable.
2. Derive the analytical solution for x, and test if the numerical solution from 1) matches the analytical one.