2016 CAS Exam 9
Comprehensive Practice Exams (with Solutions)

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Version: November 6, 2015

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Practice Exam 1

This practice exam is intended to contain exam-style questions for a representative subset of the Learning Objectives.

Section Weights

Questions from each part of the syllabus are included, with weights generally consistent with the weights specified on the syllabus.

Time Limit

The actual exam has a time constraint that typically poses a real challenge for students. Nonetheless, I have decide to not specify a time limit for the practice exams in order to encourage you to carefully consider each of these questions and take whatever time is needed to fully formulate your responses. It would be a mistake to skip practice questions due to time constraints.

Points Per Question

Points have been allocated for each question, but the vast majority of the questions are worth just one point. This is because I have tried to make most of the questions comparable in terms of difficulty and time needed to perform calculations, so I did not vary the points much.

How to Grade Yourself

You should target scoring 85% or more to fully prepare for the actual exam.

1. **(1 point)** Suppose that the risk-free rate of return were 4% and an investor has a utility function given by $U = E(r) - .5A\sigma^2$ with $A = 3$. Would the investor prefer to invest in the risk-free asset or in an asset with an expected return of 16% and a standard deviation of 30%?

Solution:

Here, the utility value for the risky asset would equal $U_{risky} = .16 - .5(3)(.30^2) = .025$. Similarly, the utility for the risk-free asset is $U_{risk-free} = .04 - .5(3)(0) = .04$. The investor will prefer the investment with the highest utility, so in this case they prefer the risk-free asset.

2. (1 point) An investor has a log utility function $U = \ln(W)$ where W represents his total wealth. He has initial wealth of \$100,000 and is presented with a risky gamble which gives him a 40% chance of winning \$30,000 and a 60% chance of losing \$10,000. Will he accept the gamble if he is seeking to maximize his expected utility?

Solution:

If the gamble is rejected his expected utility is:

$$E(U) = \ln(100,000) = 11.513$$

If the gamble is accepted his expected utility would be:

$$E(U) = (.40)\ln(130,000) + (.60)\ln(90,000) = 11.555$$

Since the gamble increases his expected utility he would accept the gamble.

3. **(1 point)** You were given the following information regarding the expected returns, standard deviations and covariances for three risky assets.

	E(r)	Std Dev
Asset 1	9.00%	13.58%
Asset 2	16.71%	38.17%
Asset 3	9.39%	29.01%

Table 1: Covariance Matrix

	Asset 1	Asset 2	Asset 3
Asset 1	0.018	0.038	0.023
Asset 2	0.038	0.146	0.051
Asset 3	0.023	0.051	0.084

After identifying an optimal risky portfolio, P, you decide to invest 30% of your assets in the risk-free asset and 70% in this risky portfolio. If the Sharpe ratio of your complete portfolio is 28.3%, what is the Sharpe ratio of the optimal risky portfolio, P?

Solution:

You are given a lot of information which could be used to do a lot of calculations, but you are not given the risk-free return. Without this information, none of these calculations can be completed. However, we know that all of the points along the CAL have the same Sharpe ratio and since the optimal risky portfolio P is one of the points on the CAL, its Sharpe ratio must be 28.3%.

4. **(1 point)** Bodie, Kane and Marcus developed two methods for identifying the optimal portfolio of risky assets — the Markowitz procedure and a specific algorithm that relies on a single index model using a diversified portfolio such as the S&P 500 as the index. Discuss the advantages and disadvantages of each of these approaches.

Solution:

The Markowitz approach, which is also called the full covariance approach, identifies the portfolio of risky assets with the highest possible Sharpe ratio but depends upon accurate estimates of not just the expected returns and standard deviations of each stock in the market but also the covariances for each pair of stocks. Given that the true covariance matrix is not observable, using estimates of each of these covariances can result in *wacky weights* in the estimated optimal portfolio.

The algorithm using the index model also achieves the goal of identifying the portfolio with the highest Sharpe ratio, but it does so under the assumption that the covariances are driven entirely by a common factor and therefore only has to estimate the expected returns and betas for each asset. The covariance is then just $Cov(i,j) = \beta_i \beta_j \sigma_m^2$ as the error terms in the index model are assumed to be uncorrelated.

In addition to decreasing the number of data points that need to be estimated and enforcing consistency among the estimates of the covariances, this approach also allows for the separation of macroeconomic analysis that can be used to estimate the expected market return and security analysis that is used to estimate the alphas for each asset.

The downside to this approach though is that it assumes the residual error terms are uncorrelated and so to the extent there is some correlation among the assets this information will be lost and the resulting Sharpe ratio may be lower than what you would get with the full covariance approach.

5. **(1 point)** List three reasons why it is hard to test the CAPM.

Solution:

1. you cannot observe the true market portfolio,
2. it is difficult to estimate the model parameters (expected return for the market portfolio, betas) accurately, and
3. the true model parameters are unlikely to remain constant over time.

6. **(1 point)** You have estimated the following model of expected excess returns using Arbitrage Pricing Theory:

$$E(R_i) = 3\% + \beta_1(2\%) + \beta_2(4\%)$$

A diversified portfolio, Portfolio A, can be constructed that has an expected excess return of 12% and factor sensitivities of $\beta_1 = 1.2$ and $\beta_2 = .8$. Explain how you would exploit an arbitrage opportunity.

Solution:

According to the model the expected excess returns for this portfolio should be 8.6%. Since you believe the expected excess returns are 12% you would want to purchase Portfolio A and at the same time sell short another portfolio (a factor mimicking portfolio) that had the same factor sensitivities $\beta_1 = 1.2$ and $\beta_2 = .8$.

This would leave you with an aggregate portfolio with a positive expected return but no sensitivity to either factor, thereby eliminating all risk. Since this portfolio would also have no net initial cost you would try to exploit this as much as possible. Attempts to buy large quantities of this mispriced portfolio and sell large quantities of the factor mimicking portfolio would drive up the price of the former and drive down the price of the latter. When everything settles down (in equilibrium) the expected return on Portfolio A should be consistent with the expected return of the APT model.

7. **(1 point)** Two of your colleagues are having a discussion about market efficiency and the effect that behavioral biases might have on this. Describe the arguments that each one could make based on the picture below, which shows the cumulative abnormal returns for portfolios sorted by the magnitude of their earnings surprises before and after the announcement of their earnings.

Figure 1: BKM Figure 11.5

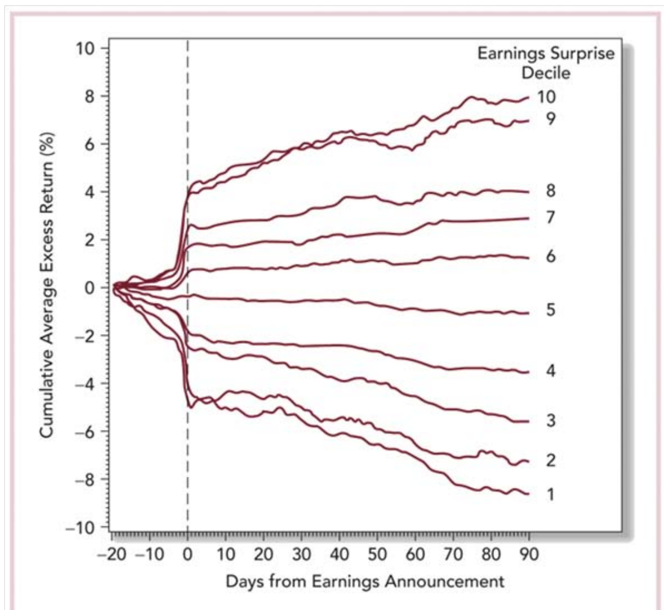

FIGURE 11.5 Cumulative abnormal returns in response to earnings announcements

Source: Reprinted from R.J. Rendleman Jr., C. P. Jones, and H. A. Latané, "Empirical Anomalies Based on Unexpected Earnings and the Importance of Risk Adjustments," *Journal of Financial Economics* 10 (1982), pp. 269–287. Copyright 1982 with permission from Elsevier Science.

Solution:

One of the main points of the efficient markets hypothesis is that prices should instantaneously react to new information. This diagram supports this because prices (and hence abnormal returns) quickly rose for firms with positive surprises and fell for firms with negative surprises. In addition, the median firms whose earnings were closer to what was expected the abnormal returns were essentially zero, suggesting that the prices already reflected the expected earnings.

However, the responses to surprises, although in the right direction, are a bit sluggish. This is even more the case for negative surprises than for positive surprises. This suggests that investors are slow to fully reflect the new information in prices, perhaps because of behavioral biases that cause them to be slow to modify their beliefs in the face of new information.

The slow response to both positive and negative earnings surprises suggests that a simple trading strategy of buying the positive surprise firms and selling the negative surprise firms would have been profitable. Such easy opportunities to earn positive risk-adjusted returns should not be possible if the market were efficient.

8. (1 point) Lists and briefly describe three limits to arbitrage.

Solution:

Three broad categories are:

1. Fundamental Risk — All strategies to exploit an apparent arbitrage opportunity likely contain some residual risk, such as the time it might take for the mispricing to be corrected in the market and the potential for mispricing to get worse before they get better.

2. Implementation Costs — Often times, mispricings cannot be fully exploited due to transaction costs or other limitations on short selling, for instance.

3. Model Risk — What if you are wrong? All opportunities to exploit identified mispricings will carry some risk and therefore may limit the willingness of arbitrageurs to try to exploit it fully.

9. **(1 point)** The Chen, Roll and Ross model and the Fama-French 3-Factor model are both presented in the BKM textbook as examples of multi-factor equilibrium return models. Contrast the way in which the factors were formed in the two models.

Solution:

The Chen, Roll and Ross (CRR) model was developed by identifying factors that appeared to be plausible sources of risk for which investors would demand risk premiums. For instance, declines in GDP may be a sign that economic conditions are generally bad (income low, employment low, etc.) and so assets whose returns are positively correlated with changes in GDP (returns are low when GDP declines) would be risky and investors would want to be paid extra to assume that risk. Having identified factors based on various economic variables, CRR then collected historical data and estimated how well differences in expected return across portfolios could be explained by differences in their sensitivities to these factors.

Fama and French used a very different approach. They started with a universe of risky assets and formed three portfolios (market portfolio, SMB portfolio and HML portfolio) that together seemed to do an excellent job of explaining the variation in returns across different portfolios. They speculated that the SMB and HML portfolios were serving as proxies for some unidentified risk factor but did not initially attempt to identify what the risk factors were. They later were able to justify their empirical results by telling a story about why the SMB and HML premiums might be related to underlying risk factors, but that came later.

10. **(1 point)** Assume that the one-year zero rate is 5.5% and the two-year zero rate is 6.5%, both continuously compounded. What is the forward rate, continuously compounded, between time periods one and two (the one year rate one year from now)?

Solution:

On a continuous basis, the forward rate, R_F, is defined as the value that makes the following relationship hold:

$$e^{R_2 T_2} = e^{R_1 T_1} e^{R_F (T_2 - T_1)}$$

From this, it is easy to solve for R_F:

$$\begin{aligned} R_F &= \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1} \\ &= \frac{6.5\%(2) - 5.5\%(1)}{2 - 1} \\ &= 7.5\% \end{aligned}$$

11. **(1 point)** You are the Chief Investment Officer overseeing the bond investments of your firm. Your only liability is a series of three equal annual cash flows of \$250,000, which have a present value of \$692,721 given that the annually compounded risk-free interest rates are 4.08% for all maturities. If you wanted to have a net worth of zero today and also wanted that to be immunized against the effect of changes in interest rates, what zero coupon bond would you invest in? Specify the maturity and the face value of the bond and assume that bonds of any maturity and any face value can be obtained.

Solution:

To achieve the objective we want the dollar value and the Macaulay duration of the asset to be the same as the dollar value and Macaulay duration of the liability.

Begin with the liability's Macaulay duration. We can calculate that directly as follows:

Time	CF	PV(CF)	T*PV(CF)
1	250.00	240.20	240.20
2	250.00	230.78	461.57
3	250.00	221.74	665.21
		692.72	1,366.98
		Macaulay Duration	1.973

Since the Macaulay duration of zero coupon bonds is just equal to their maturity, we know we want a zero coupon bond with 1.973 years to maturity. For the face value, we know we want the present value to match the present value of the liability and so:

$$692,721 = \frac{FV}{1.0408^1.973} \Rightarrow FV = \$749,600$$

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12. (1 point) The strategy of bond portfolio immunization against interest rate changes is often much more difficult to apply in practice because durations of assets and liabilities change over time. But besides the challenges of continually rebalancing an immunized portfolio, what are three other reasons why simplistic examples of the effect of immunization are unrealistic.

Solution:

Three reasons are:

1. It assumes that interest rates only change by small amounts,
2. it assumes parallel shifts in the term structure and
3. it ignores inflation.

13. **(1 point)** Suppose an insurer is established with initial surplus of $S = 50$. They write policies on January 1 that have up-front expenses of $E = 25$, expected claims that will be paid in full at the end of the year equal to $L = 75$ and charge total premium of $P = 101.19$. If all of the assets are invested in risk-free securities yielding $y = 5\%$ with a duration of 1.0 and we ignore risk margins when determining the present value of the liabilities, what is the modified duration of the surplus?

Solution:

For the duration, note that the invested assets of the firm will be:

$$A = 50 + 101.19 - 25 = 126.19$$

Because the asset duration was assumed to be equal to 1.0, the dollar duration of the assets is then 126.19.

The present value of the liabilities is $75/1.05 = 71.43$ and the modified duration is:

$$D_L = \frac{1}{1+y} = .9523$$

resulting in a dollar duration for the liabilities of $(71.43)(.9523) = 68.027$.

The dollar duration of surplus is then $126.19 - 68.027 = 58.16$. To get the duration of surplus we have to divide the dollar duration by the current economic value of the surplus. Immediately after writing the policies, the current economic value is:

$$\begin{aligned} C &= S + P - E - \frac{L}{1+y} \\ &= 50 + 101.19 - 25 - \frac{75}{1.05} \\ &= 54.76 \end{aligned}$$

This gives us a duration of surplus:

$$D_S = \frac{58.16}{54.76} = 1.062$$

Note that in the reading the duration of the liability is referred to as simply being approximately 1.0. Using 1.0 rather than 0.9523 would cause the numbers to differ slightly.

14. (2 points) Suppose you had a three-year semi-annual coupon bond with \$100 face value paying a coupon of 7% and trading at a continuously compounded yield of 6.8%. Assume the continuously compounded risk free yield is 5% and in the event of default the recovery is 40% of the face value.

Assume that defaults can only occur midway through each year (just before the coupon is paid). Also assume that the (unconditional) probability of default in year two is twice the (unconditional) probability of default in year one and the (unconditional) probability of default in year three is three times the (unconditional) probability of default in year one.

Determine the probability of default in each year.

Solution:

We will first determine the present value of the expected default losses based on the difference between the risk-free value of the bond and the actual bond price (based on its reported yield). We will then set that equal to a direct calculation of the present value of the expected default losses based on the unknown year one default probability, Q, and solve for Q.

The bond pays \$3.5 in interest every six months and \$103.5 at maturity, and therefore its risk free price is:

$$\begin{aligned} G &= 3.5e^{-.05(.5)} + 3.5e^{-.05(1)} + 3.5e^{-.05(1.5)} \\ &\quad + 3.5e^{-.05(2)} + 3.5e^{-.05(2.5)} + 103.5e^{-.05(3)} \\ &= 105.33 \end{aligned}$$

Using the 6.8% bond yield, the market price of the bond is:

$$\begin{aligned} B &= 3.5e^{-.068(.5)} + 3.5e^{-.068(1)} + 3.5e^{-.068(1.5)} \\ &\quad + 3.5e^{-.068(2)} + 3.5e^{-.068(2.5)} + 103.5e^{-.068(3)} \\ &= 100.22 \end{aligned}$$

This tells us that the total expected present value loss from default is:

$$G - B = 105.33 - 100.22 = \$5.11$$

Since default can only occur at the .5, 1.5 or 2.5 year points, we need to know the amount of *loss* at each of those points. The loss is defined as the risk-free value of the bond at that point less the recovery in the event of default.

Let's start with the risk free values of the bond at $T = .5$ right before the coupon is paid.

We have future cash flows of \$3.5 right now, then \$3.5 for 4 more periods and \$103.5 at maturity. The value of this using the risk free yield of 5.0% is:

$$
\begin{aligned}
F_1 &= 3.5 + 3.5e^{-.05(.5)} + 3.5e^{-.05(1)} + 3.5e^{-.05(1.5)} + 3.5e^{-.05(2.0)} + 103.5e^{-.05(2.5)} \\
&= 107.995
\end{aligned}
$$

The following table shows the same calculations for time periods $T = 1.5$ and $T = 2.5$ as well:

Time	CF	T = 0.5	T = 1.5	T = 2.5
0.5	3.50	3.500	0.000	0.000
1.0	3.50	3.414	0.000	0.000
1.5	3.50	3.329	3.500	0.000
2.0	3.50	3.247	3.414	0.000
2.5	3.50	3.167	3.329	3.500
3.0	103.50	91.338	96.021	100.945
	Risk-Free Value	107.995	106.264	104.445
	Recovery	40.000	40.000	40.000
	LGD	67.995	66.264	64.445

Notice that we could use our CAS approved calculators for these three calculations, but only if we are careful to convert the 5% continuously compounded rate to a semi-annually compounded rate:

$$1 + r = e^{.05*.5} \Rightarrow r = 2.5315\%$$

Here, r is the six-month rate. Then, set to get the values except for the coupon payment about to be received we can use the following: $N = 5$, $I/Y = 2.5315\%$, $PMT = 3.5$, $FV = 100$ and then compute the present value to get $PV = 104.495$. Add in the 3.5 coupon about to be paid to get 107.995. Similar calculations are done for the other two values.

Using these risk-free values at the time of default, the loss given default (LGD) reflects the risk free value at each point less the recovery, as shown in the table.

Multiplying the LGD at each possible default time by the default probability and the risk free discount factor, we get a total present value of expected loss equal to $359.8857Q$:

Time	Probability	LGD	PV Factor	E[PV(LGD)]
0.5	Q	67.995	0.9753	66.3162(Q)
1.5	2Q	66.264	0.9277	61.4760(2Q)
2.5	3Q	64.445	0.8825	56.8725(3Q)
				359.8857Q

To find Q we simply set $359.8857Q = 5.11$ and solve for $Q = 1.42\%$.

15. **(1 point)** An insurance company is in run-off and its sole source of future income comes from its marketable securities that have a present value of \$100 million and a volatility of $\sigma = 27\%$ (assume they have no other assets besides these).

The company currently has loss reserves equal to \$95 million that it intends to pay out according to the following pattern: 25% in year 1, 40% in year 2, 35% in year 3. You believe that the loss reserves represent an accurate estimate of the expected claim payments but believe that due to the potential volatility of the actual claim payments as well as the volatility of the invested assets there is a possibility that the assets will be worth more than the claims that will be paid and are interested in purchasing the equity (stock) of this company.

You see the equity as essentially being a *call option* on the net assets of the firm and want to use the Merton model of equity as a call option to value the equity. Describe whether this can be done and, if so, what additional information would be needed. If it cannot be done, explain what aspects of this situation differ from the basic implementation of the Merton model discussed in Hull.

Solution:

The Merton model cannot be used in this situation, at least not without substantial refinement relative to the basic version discussed in Hull.

The liability of an insurance company is a lot like the *debt* of any other company, so conceptually the Merton model is applicable. However, in the version presented in Hull the debt has a fixed face value and a single maturity date. In this case, the liability value is uncertain and, even if it were fixed, does not have a single maturity date.

Notice that in practice almost no companies will satisfy the strict assumption of a fixed amount of debt due on a single date, so when people use the Merton model they typically define a more general *default point* that reflects some combination of all of their short-term and long-term debt and focus on the probability that the assets values fall below that default point at different time horizons.

16. **(2 points)** Determine the spread for a 3-year $10 million notional CDS on a reference bond with a conditional default probability equal to 4% per annum, assuming that defaults only occur at the *middle* of each annual period. In the event of default, the recovery on the reference bond is 60% of its face value and the CDS payoff is equal to the difference between the face value of the bond and the bond recovery.

Assume the risk-free rate is 4% on a continuously compounded basis and determine the CDS spread assuming the spread is paid annually *at the end of each year* and that the accrued spread payment is paid if default occurs before the end of the year.

Solution:

Notice that the notional value of the swap is $10 million, but since the CDS spread is quoted as a percentage of the notional value it is always easiest to calculate the spread assuming $1 notional value.

Step 1: Calculate the Unconditional Default Probabilities and Survival Probabilities

Table 2: Default and Survival Probabilities

Time (Years)	Unconditional Default Probability	Survival Probability
1	0.0400	0.9600
2	0.0384	0.9216
3	0.0369	0.8847

Step 2: Determine Protection Buyer's Spread Payments

Here, we need to account for both the annual payments that are made so long as the bond has not defaulted and the accrual payments (one-half of the spread) that will be made in the event of default.

Table 3: Swap Spread Payments – Annual

Time (Years)	Survival Probability	Expected Payment	Discount Factor	PV of Expected Payment
1	0.9600	0.9600s	0.9608	0.9224s
2	0.9216	0.9216s	0.9231	0.8507s
3	0.8847	0.8847s	0.8869	0.7847s
Total				2.5578s

Table 4: Swap Spread Payments – Accrual

Time (Years)	Default Probability	Accrual Payment	Expected Payment	Discount Factor	PV of Expected Payment
0.5	0.0400	.5s	0.0200s	0.9802	0.0196s
1.5	0.0384	.5s	0.0192s	0.9418	0.0181s
2.5	0.0369	.5s	0.0184s	0.9048	0.0167s
Total					0.0544s

The total expected spread payments are therefore $2.5578s + .0544s = 2.6122s$.

Step 3: Determine the Protection Seller's CDS Payments

Table 5: CDS Payments

Time (Years)	Default Probability	Recovery Rate	Expected Payment	Discount Factor	PV of Expected Payoff
0.5	0.0400	60%	0.0160	0.9802	0.0157
1.5	0.0384	60%	0.0154	0.9418	0.0145
2.5	0.0369	60%	0.0147	0.9048	0.0133
Total					0.0435

Step 4: Solve for s

$$2.6122s = .0435 \Rightarrow s = .0166$$

That is, for each dollar of notional value the CDS spread is $s = 166$ basis points. For \$10 million in notional then the spread payment is \$166,495.

17. **(1 point)** Your company is considering issuing a catastrophe bond to replace traditional high-layer excess of loss reinsurance. You are concerned that when Hurricane Katrina occurred your company's losses were larger than the estimates obtained for an identical event in your catastrophe model and that more than 5 years later you still have open and unpaid claims. How can those concerns be addressed so that they don't impact the spread you have to pay on the bond?

Solution:

One way to address this is to use an index-based trigger for the catastrophe bond that establishes covered losses in any one of the following ways:

1. *Industry index loss triggers* rely on aggregate industry losses from specified events (e.g. hurricane, earthquake) in specified geographies.

2. *Modeled loss triggers* apply an agreed upon catastrophe risk model, using the physical parameters of the actual events, to the insured's exposure data and pay according to the model's estimated losses rather than actual losses.

3. *Parametric triggers* pay fixed dollar amounts based upon the physical characteristics of catastrophe events, such as wind speed for hurricanes or magnitude of earthquake.

The main concern with these triggers is that the company will retain the basis risk between their actual losses and the covered losses in the bond. This will complicate the evaluation of the pricing relative to traditional reinsurance.

With respect to the lengthy claim development tail, this is actually an issue in all catastrophe bonds because the bonds have to have a stated maturity (perhaps with some optional extension periods) and so at some point the coverage will have to cease. Nonetheless, some index triggers alleviate this risk because final loss determination can be significantly expedited in that case.

18. **(1 point)** Describe why some managers of publicly traded firms might want to hedge more of the firm's financial risks than its shareholders would want to hedge. Give an empirical example of an instance when this behavior was observed.

Solution:

The shareholders are likely to own a diversified portfolio of shares and therefore the risks of any one firm are naturally managed through diversification. However, the managers of the firm are unlikely to be as diversified — they likely have a relatively large proportion of their wealth tied up in the firm's stock, options on the firm's stock and future compensation. So the managers will be more concerned with the specific risks of the firm than the typical shareholders are. Hence, they'll be more inclined to hedge the firm's risks.

In a survey of gold mining firms conducted by Peter Tufano he found that the larger the managers' equity stakes (in the form of actual shares owned), the more they chose to hedge.

19. **(1 point)** You have prepared a report summarizing the aggregate risks faced by an insurance company and have used a 99%, one-year Value at Risk as a risk measure, which quantifies the size of aggregate net loss that will be exceeded only 1% of the time. Your boss has asked you to consider alternative risk measures that reflect not just the probability of a loss larger than some amount but also the size of the losses. You are considering using Conditional Tail Expectation (CTE), Expected Policyholder Deficit (EPD) and Below Target Risk (BTR). Describe how these three measures relate to each other, identifying specific differences between them.

Solution:

The CTE measures the average value of all loss scenarios that are worse than some specified percentile of the loss distribution. For instance, the 99% CTE would reflect the average of the worst 1% of the scenarios.

The EPD is similar in that it calculates an average of values beyond some threshold, but there are three differences compared to CTE. First, it looks specifically at the threshold where the liabilities exceed the value of the assets rather than a specific percentile. Second, it includes only the *shortfall* between the liabilities and the assets, which means it is not well defined unless it includes both assets and liabilities. And third, it treats non-shortfall scenarios as having a zero shortfall, so the CTE can be considered a conditional mean value (conditional on the loss exceeding the chosen percentile) whereas the EPD is unconditional.

The BTR was one example that Culp, Miller and Neves provided for a shortfall risk measure. It was not well defined, but it can be thought of as a more general version of the EPD that can measure the risk relative to any defined threshold. Whereas EPD is defined as the risk relative to a zero value for the assets less liabilities, BTR can use any threshold. Like both CTE and EPD, the BTR reflects the size of the loss and not just its probability.

20. (1 point) An insurer has a fixed liability of $\$5,000$ due in one period and $\$250$ in capital. All of its assets are invested in risky securities with the following distribution of ending asset values at the end of the period:

Ending Assets	Probability
10,000.00	0.20
5,000.00	0.70
2,500.00	0.10

The EPD Ratio is currently equal to 5%. Determine the amount of capital needed so that the EPD ratio is only 2% assuming that all of the assets are invested in the same risky securities and therefore have the same total return distribution.

Solution:

Consider the basic calculation of the EPD ratio, as shown below:

Scenario	Beg Assets	Return	Ending Assets	Loss	Probability	Claims Paid	Deficit
1	5,250	190.48%	10,000	5,000	0.20	5,000	0
2	5,250	95.24%	5,000	5,000	0.70	5,000	0
3	5,250	47.62%	2,500	5,000	0.10	2,500	2,500
Mean							250
						EPD	250
						Exp Loss	5,000
						EPD Ratio	5.00%

Notice that we were told the initial capital was $\$250$, which is how I know that we start with $\$5,250$ in assets. The return distribution wasn't given, but it was determined based on the starting and ending assets in each scenario. It is needed because we have to alter the starting assets in order to determine the revised starting capital level.

In the original 5% EPD ratio case there was only a deficit in one scenario. Since we want the EPD ratio to be lower, we know that we will have to have a deficit in only one scenario. This makes the algebra for solving for the starting assets very easy. We just solve for the amount of assets so that in the scenario that has a 10% probability (Scenario 3) the ending assets result in a deficit of 100:

$$.10max(0, A * 47.62\% - 5,000) = .02(5,000) \Rightarrow A = 8,400$$

Given the expected losses of $\$5,000$ and starting assets of $\$8,400$, the capital needed would be $\$3,400$.

21. **(1 point)** You are trying to allocate capital to a particular line of business using a target 2.5% EPD ratio. The line of business has the following characteristics:

- Expected losses (claims) are $\$2,000$
- Standard deviation of losses is $\$500$
- Losses are lognormally distributed

Assume assets are fixed and use the following table of EPD ratios to estimate how much capital would be allocated to this lines of business if the capital allocations for all lines were based upon their stand-alone risk measures. To keep the calculations to a minimum, do not interpolate within the table and just use the closest values.

	σ			
c	0.150	0.200	0.250	0.300
50.00%	0.02%	0.19%	0.67%	1.49%
47.50%	0.03%	0.24%	0.78%	1.66%
45.00%	0.04%	0.30%	0.91%	1.86%
42.50%	0.05%	0.37%	1.05%	2.07%
40.00%	0.08%	0.45%	1.21%	2.32%
37.50%	0.11%	0.55%	1.40%	2.58%
35.00%	0.15%	0.68%	1.62%	2.88%
32.50%	0.20%	0.83%	1.87%	3.21%
30.00%	0.28%	1.01%	2.15%	3.57%
27.50%	0.38%	1.23%	2.47%	3.98%
25.00%	0.50%	1.48%	2.83%	4.42%
22.50%	0.67%	1.79%	3.24%	4.91%
20.00%	0.89%	2.15%	3.71%	5.44%
17.50%	1.17%	2.57%	4.23%	6.03%
15.00%	1.52%	3.06%	4.81%	6.67%

Solution:

Because the assets are fixed, the tables above simply use Butsic's EPD ratio formulas for lognormally distributed liabilities. When Butsic used this formula, he approximated the lognormal σ parameter using the coefficient of variation, $k = .25$.

When c, the capital to liability ratio is 27.5% and $\sigma = .25$, the EPD ratio is 2.47%, which is close to the target of 2.5%. Normally we would want to linearly interpolate, but here you were told not to bother with that, so the capital for this line of business would be 27.5% of the expected loss, or \$550.

Notice too that in some cases you might be given a table containing $1 + c$ rather than c as in this case.

22. **(1 point)** List and briefly describe the four main methods used in Goldfarb's Risk-Adjusted Performance Measurement paper to allocate capital to different risk sources.

Solution:

The four methods are:

- Proportional Allocation Based on a Risk Measure — This method simply calculates stand-alone risk measures for each risk source (market risk, reserve risk, Line A underwriting risk, Line B underwriting risk) and then allocates the total risk capital in proportion to the separate risk measures.

- Incremental Allocation (Merton-Perold) — This method determines the impact that each risk source has on the aggregate risk measure and allocates the total risk capital in proportion to these incremental amounts.

- Marginal Allocation (Myers-Read Method) — This method determines the impact of a small change in the risk exposure for each risk source (e.g. amount of assets, amount of reserves, premium volume) and allocates the total risk capital in proportion to these marginal amounts. One particular method demonstrated in the paper is the Myers-Read method.

- Co-Measures Approach — This method determines the contribution each risk source has to the aggregate risk measure.

23. **(1 point)** Many insurance pricing models focus solely on the underwriting cash flows and ignore the equity commitments of the investors (they take the product market viewpoint rather than the financial market viewpoint). Describe why this is a problem and identify the characteristic of insurance companies that makes the investors' perspective important.

Solution:

If rates do not include an adequate provision for returns to equity investors, for instance by carefully tracking the flow of equity capital to and from investors, it isn't possible to ensure that enough capital can be attracted to the industry to satisfy society's demand for insurance.

And the reason the equity flows are important in the first place is that in the insurance industry the capital committed by investors is primarily invested in marketable securities. This subjects investors to double taxation of the investment income, which is something that could be easily avoided if investors simply invested their capital directly in the same marketable securities. This imposes an additional cost that must be considered in the rates.

24. **(2 points)** Ferrari notes that in some sense an insurer's reserves reflect borrowed funds (borrowed from the policyholder) that can be invested. Describe what Ferrari says the "cost" of this borrowing is and how it relates to the typical cost of funds for non-insurers who borrow from their debt investors? Then show how this cost of funds impacts the total return on surplus for an insurer with the following characteristics:

- Written premium is \$75 million
- Reserves are equal to \$150 million
- Surplus is \$50 million
- Combined ratio is 104%
- Invested assets earn an 8% return

Solution:

The cost of the borrowed funds is the underwriting loss, which relative to premium is 4% in this case (the underwriting profit is one minus the combined ratio). However, relative to reserves this cost is actually 2%.

This 2% cost of funds differs from the cost of actual borrowing because in the latter case the interest paid is usually fixed, whereas in this case the cost is really just an expected value. Actual costs could be much higher or lower.

We can calculate the total return on surplus using Ferrari's second formula which reflects the investment income on reserves and the "cost" of borrowing these funds. We just need to be careful to use the underwriting profit relative to reserves, which is $U/R = -2\%$ here.

$$\begin{aligned}
\frac{T}{S} &= \frac{I}{A} + \frac{R}{S}\left(\frac{I}{A} + \frac{U}{R}\right) \\
&= 8\% + (3.0)\,(8\% + (-2\%)) \\
&= 8\% + 18\% \\
&= 26\%
\end{aligned}$$

25. (1 point) Roth's calculation of the required return on surplus for mutual insurers was similar to the calculation for stock insurers but it excluded considerations for dividends and paid-in capital changes. What other adjustment(s) did he say were appropriate?

Solution:

Because the mix of business for mutual and stock insurers is very different, with mutual companies writing much more short-tailed personal lines business, Roth adjusted the expected inflation rate accordingly.

He didn't use stock-specific and mutual-specific historical premium and reserve growth estimates, but assuming you could get comfortable with the reliability of the data you could do that as well.

26. **(1 point)** When discussing his Present Value Offset Method Robbin suggests an approximate way to partially account for taxes. What is the method he suggests and what does he say is missing from the calculation?

Solution:

He suggests approximating the taxes by discounting the payment patterns at after-tax rates. This, however, only captures the tax on investment income from the present value loss provision and is therefore only part of the overall income tax. A more elaborate model would be needed to fully capture the effect of taxes.

27. **(2 points)** You are given the values below for the mean, standard deviation and variances for two reinsurance policies, A and B.

	A	B	Total
Mean	358	588	946
Std Dev	701	2,930	3,398
Variance	491,347	8,587,110	11,543,353

If the correlation between the two policies is $\rho = .6$, the target return on marginal surplus is 12% and required capital is calculated at the 97.5th percentile, what would be the renewal risk load for each policy using the Marginal Variance method as discussed in Mango's paper, along with Mango's suggested value for the MV risk load multiplier?

For convenience, note that $\Phi^{-1}(97.5\%) = 1.96$.

Solution:

First, when using the Marginal Variance method requires a risk load multiplier. Mango adjusts the Kreps Marginal Surplus risk load multiplier so that the MV risk load multiplier, λ is consistent by dividing by the portfolio standard deviation:

$$\begin{aligned}\lambda &= \frac{yz/(1+y)}{\text{Portfolio Standard Deviation}} \\ &= \frac{.12(1.96)/1.12}{3,398} \\ &= 0.0000618\end{aligned}$$

And now, the two renewal risk loads can be calculated as follows:

MV Renewal Risk Load for A		MV Renewal Risk Load for B	
Portfolio Variance	11,543,353	Portfolio Variance	11,543,353
Variance of B	8,587,110	Variance of A	491,347
Marginal Variance from A	2,956,244	Marginal Variance from B	11,052,007
MS Multiplier $yz/(1+y)$	0.210	MS Multiplier $yz/(1+y)$	0.210
λ	0.0000618	λ	0.0000618
MV Renewal Risk Load for A	**183**	MV Renewal Risk Load for B	**683**

28. (2 points) You are considering writing a reinsurance contract on a risk with the following characteristics:

- claim costs and expenses are lognormally distributed
- expected claim costs are $500,000
- standard deviation of claim costs equals $1,250,000
- All losses are paid at the end of one year.

You intend to follow an investment strategy that involves investing solely in risk-free assets at a rate of 4% and you want the expected return for shareholders from the combined reinsurance and investment strategy to at least equal the expected return on a risky investment equal to $y = 8\%$.

You have determined that in order to satisfy constraints on the riskiness of the combined reinsurance and investment strategy that you need the variance, in dollars, of this strategy to be no greater than the variance of a strategy of investing the capital directly in the risky investment. The standard deviation of the risky investment is 13%.

Determine the risk load that should be charged based on this strategy and this constraint.

Solution:

Under this constraint, we want the dollar standard deviation of the combination of the reinsurance contract and the risk-free investment to be less than or equal to the dollar standard deviation of the same invested capital invested in the risky asset. This tells us how much capital, A, has to be invested:

$$A\sigma_y \geq \sigma_L$$

At the equality, this gives us:

$$\begin{aligned} A &= \frac{\sigma_L}{\sigma_y} \\ &= \frac{1,250,000}{.13} \\ &= 9,615,385 \end{aligned}$$

From this, the risk load formula is the amount which gives an expected return on the combined reinsurance and risk-free investment strategy equal to the expected return on the risky investment. This leads to:

$$\begin{aligned} R &= \frac{y - r_f}{1 + r_f} A \\ &= \frac{.08 - .04}{1.04}(9,615,385) \\ &= 369,822 \end{aligned}$$

Cumulative Normal Distribution (Positive x)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Cumulative Normal Distribution (Negative x)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Practice Exam 2

This practice exam is intended to contain exam-style questions for a representative subset of the Learning Objectives.

Section Weights

Questions from each part of the syllabus are included, with weights generally consistent with the weights specified on the syllabus.

Time Limit

The actual exam has a time constraint that typically poses a real challenge for students. Nonetheless, I have decide to not specify a time limit for the practice exams in order to encourage you to carefully consider each of these questions and take whatever time is needed to fully formulate your responses. It would be a mistake to skip practice questions due to time constraints.

Points Per Question

Points have been allocated for each question, but the vast majority of the questions are worth just one point. This is because I have tried to make most of the questions comparable in terms of difficulty and time needed to perform calculations, so I did not vary the points much.

How to Grade Yourself

You should target scoring 85% or more to fully prepare for the actual exam.

1. **(1 point)** Suppose that you have two risky assets, one with an expected return of 20% and a standard deviation of 25% and the other with an unknown expected return and a standard deviation of 30%. If we knew that an investor with an index of risk aversion $A = 5$ was indifferent between these two risky assets, what is the return on the second risky asset? Assume that the investor's utility curve is given by the equation $U = E(r) - .5A\sigma^2$.

Solution:

The fact that the investor is indifferent between these two assets tells us that the utility values are identical. In the case of the first asset, would equal:

$$U = .20 - .5(5)(.25^2) = .0438$$

Setting the utility of the second asset equal to 4.38% and solving for $E(r)$ gives us:

$$\begin{aligned} E(r) &= U + .5A\sigma^2 \\ &= 4.38\% + .5(5)(.30^2) \\ &= 26.88\% \end{aligned}$$

2. (2 points) You are planning to split your portfolio between two risky index funds with the following characteristics:

	Stock Fund	Bond Fund
Expected Return	11%	7%
Standard Deviation	13%	6%
Correlation Coefficient	0.20	

The risk free rate is 3%. What is the Sharpe ratio of your portfolio if it is invested 70% in the stock fund and 30% in the bond fund?

Solution:

Using these weights, the expected return and standard deviation are:

$$E(r_p) = w_S E(r_S) + w_B E(r_B)$$
$$= 9.8\%$$
$$\sigma_p = \sqrt{w_S^2\sigma_S^2 + w_B^2\sigma_B^2 + 2w_S w_B \sigma_S \sigma_B \rho}$$
$$= 9.62\%$$

From this, we can calculate the Sharpe ratio as:

$$S = \frac{E(r_p) - r_f}{\sigma_p} = 0.707$$

3. **(1 point)** You have identified your optimal risky portfolio and calculated the expected return is 11% and the standard deviation is 14%. If the risk-free rate is 4% and your utility curve is given by $U = E(r) - 2\sigma^2$, determine the proportion of your complete portfolio that is invested in the risk-free asset.

Solution:

To determine the proportion of assets, y, invested in the risky portfolio we write the utility as a function of y and find the value of y that maximizes the utility.

Here, $E(r) = y(11\%) + (1 - y)(4\%)$ and $\sigma = y(14\%)$, which gives the following for the utility:

$$U = .04 + .07y - 2y^2(.14^2)$$

Setting the derivative with respect to y equal to zero and solving for y^*:

$$0 = .07 - .0784y^* \Rightarrow y^* = 89.29\%$$

This is the proportion invested in the risky assets, so the portion invested in the risk-free asset is $1 - y^* = 10.71\%$

4. **(1 point)** You have used a single index model, with the S&P 500 portfolio as the index, and identified the following estimates of the regression coefficients for Portfolio A:

$$R_A = .01 + 1.1R_{S\&P500} + \epsilon$$

The R-squared of this regression is .8 and the standard deviation of the excess returns for Stock A was 20%. The standard deviation of the excess market returns was 15%.

Describe how you would use a tracking portfolio to capture the alpha in this portfolio and discuss the risk in the aggregate portfolio. Assume that you can purchase $1 million of Portfolio A.

Solution:

A tracking portfolio is the combination of the S&P 500 index and the risk-free asset that has the same beta as Portfolio A. In this case, to get a beta of 1.1 we would need to borrow 10% of the value of Portfolio A and invest 110% of the value of Portfolio A in the S&P 500 index in order to have a portfolio iwth a beta of 1.1. Because we are using the market portfolio and the risk free asset here, the alpha of this tracking portfolio and the residual error of the tracking portfolio would both be zero.

To capture the pure alpha in Portfolio A and not take any market risk we could buy Portfolio A and sell short the tracking portfolio. We would do this by *shorting* an amount of the S&P 500 equal to 110% of the value of Portfolio A, *investing* 10% of this in the risk free asset and using the remaining proceeds to buy Portfolio A. This gives a net portfolio with a zero beta and so the expected return (in dollars) is just the $1 million invested in Portfolio A times the alpha of 1%, or $10,000.

Because there is no exposure to the market, the market risk is zero. However, we do still have residual risk (the tracking error) so this net portfolio does not represent a risk-free profit opportunity. Here, I gave you the R-squared and the standard deviation of the returns. The R-squared is the percentage of the variance in Y explained by X, which allows you to determine that the variance of the error term is:

$$(1 - R^2) * (.2^2) = .008$$

So the standard deviation of the residual is 8.94%.

Notice that ideally what we would want to do is to find a more diversified portfolio than Portfolio A that also has a positive alpha. With more diversification we would expect the residual standard deviation to be lower and then we would really be able to isolate the alpha. In addition, note that in this example our net investment is zero and so we could still take our funds and invest in a completely different asset class, such as bonds, and then earn the bond return plus the equity market alpha. This is known as a *portable alpha* strategy.

5. **(1 point)** What are the key assumptions in the standard CAPM that are modified in each of the following alternative models?

- Zero Beta CAPM
- Intertemporal CAPM (ICAPM)
- Consumption CAPM

Solution:

The following are the main points:

- Zero Beta CAPM – Assumes that investors cannot borrow or lend at the risk-free rate, which leads to a model similar in form to CAPM but with a zero-beta portfolio (a portfolio uncorrelated with the market portfolio) used in place of the risk-free rate.

  This causes the SML to have a higher intercept and a lower slope compared to the standard CAPM, which is consistent with the empirical data

- Intertemporal CAPM (ICAPM) – The ICAPM assumes that investors care about risk and return over multiple periods and recognizes the betas, market risk premiums and other factors that could impact their consumption over time can change, which is itself another source of risk.

  This causes investors to bid up the price of assets that can hedge these risks (i.e. assets whose returns are negatively correlated with these risks) and bid down their expected returns. Similarly, they pay less for assets that are correlated with these risks and demand higher expected returns. This results in a model in which returns are a function of betas and risk premiums for various risk factors besides just the market risk. The apparent positive alphas for low beta stocks and negative alphas for high beta stocks

- Consumption CAPM – While CAPM assumes that the only thing investors care about is their end of period wealth and that this is driven entirely by the returns on the market portfolio, the Consumption CAPM focuses directly on the real reason people want to accumulate wealth — to buy things. Returns on risky assets will then depend on how they are correlated with consumption instead of market returns. The Consumption CAPM is really just a way of collapsing the multiple factors in the ICAPM into a single factor.

6. **(1 point)** You have estimated the following model of expected excess returns using Arbitrage Pricing Theory:

$$E(R_i) = 3\% + \beta_1(2\%) + \beta_2(4\%)$$

Portfolio A can be constructed that has an expected excess return of 12% and factor sensitivities of $\beta_1 = 1.2$ and $\beta_2 = .8$. According to the APT model, the expected excess returns on this portfolio should be only 8.6%, suggesting an arbitrage opportunity. Is it necessarily true that, in equilibrium, the expected return of Portfolio A will equal 8.6%?

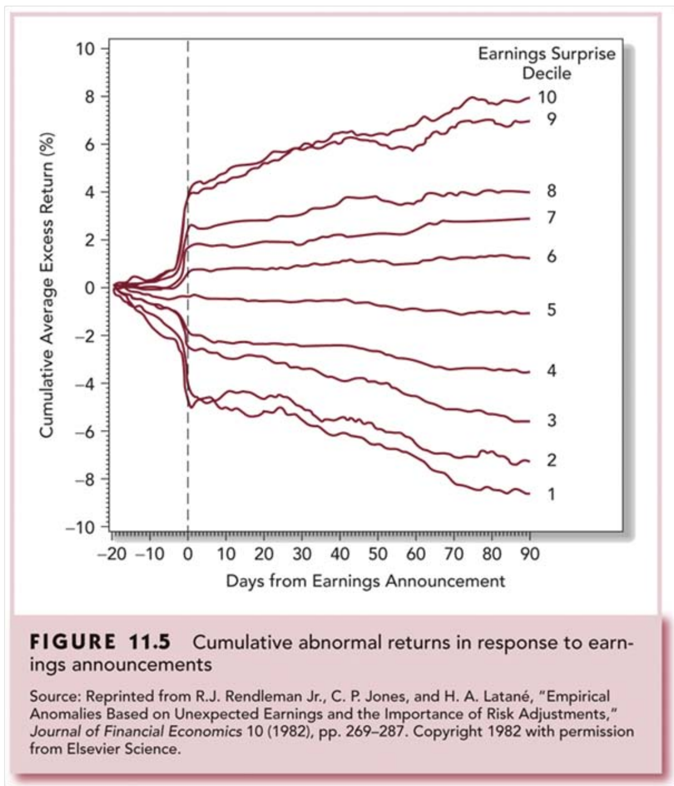
Solution:

First, the APT model technically applies only to diversified portfolios for which we can ignore the idiosyncratic or residual risk. Since we were not told how well diversified this portfolio is, there is a possibility that there is a large degree of residual variance in the returns that would limit the willingness of investors to try to fully exploit this arbitrage opportunity. As a result, differences between the expected return on the portfolio and the APT model may persist.

Second, even if the arbitrage opportunity were fully exploited, all that we can say for sure is that the expected excess returns will be equal. This will come about because the expected return on Portfolio A will fall **and** the expected returns on the factor mimicking portfolio used to exploit the arbitrage opportunity will rise. That is, the APT equation may change and so we cannot say that Portfolio A's expected return will be equal to the current APT model results.

7. **(1 point)** You have shown the following diagram to a colleague and told her that you intended to exploit the apparently slow response to earnings surprises by buying stocks with positive surprises and selling stocks with negative surprises to earn a positive expected return with no initial investment. She responds that you should be careful because the positive and negative cumulative returns depicted might just reflect compensation for assuming market risk and so your net returns won't be risk-free. Describe why she is mistaken about her main point and also describe why she may be right about the fact that your returns will not be risk free.

Figure 2: BKM Figure 11.5



Solution:

She is mistaken because the graph depicts cumulative *abnormal* returns that have already been adjusted to reflect each stock's sensitivity to general market movements and the returns on the overall market. These are returns that have been stripped of their market risk already.

However, any such adjustment from returns to abnormal returns requires a model of "normal" returns and so it is possible that if the wrong model is used there could still be systematic risks (exposures to common factors) that are driving the seemingly abnormal returns.

8. **(1 point)** List three real-world instances where there appeared to be violations of the Law of One Price. Comment briefly on how these violations might be explained.

Solution:

The three discussed in the text are:

1. Siamese Twin Companies — Royal Dutch and Shell's share prices were linked by a simple formula that split the combined profits 60%/40% to the two firms, suggesting that Royal Dutch's price should always be 1.5 times Shell's price. Yet over long periods of time this was actually not the case, with deviations ranging from -5% to 17%.

   The fundamental risk that the mispricing could get even more out of whack made it difficult at times for these price differentials to be exploited.

2. Equity Carve Outs — In 2002 3Com sold 5% of their Palm subsidiary to new investors and arranged for existing 3Com shareholders to receive the other 95% of Palm's shares (specifically, they were to receive 1.5 shares of Palm for each share of 3Com they owned). As a result, 3Com's price should have been at least 1.5 times the price of Palm prior to the distribution of the Palm shares to 3Com's shareholders, plus the value of all of 3Com's other businesses. However, for an extended period of time, 3Com's stock price was actually less than the price of Palm.

   Apparently investors were overvaluing Palm and or undervaluing 3Com, making it possible for an arbitrage profit by purchasing 3Com and shorting Palm. However, it was actually quite difficult and expensive to short Palm at the time, due to the very small volume of Palm shares outstanding and the fact that they were already sold short.

3. Closed-End Funds — The market value of closed-end mutual fund shares often reflects a substantial discount from the net asset value (NAV), which suggests that if the fund were to simply sell all of its holdings, the investors would receive a windfall since the holdings would be sold for the NAV.

   Researchers have found that the discounts on different funds tend to move together and are correlated with small stock returns, suggesting that all might be impacted by variations in investor sentiment. In addition, while it may be theoretically plausible to buy the funds trading at a discount and sell (short) the funds trading at a premium, eventually earning an arbitrage profit when prices converge to NAV, this would carry significant fundamental risk since the premiums and discounts could widen. And finally, the premiums and discounts could simply reflect a rational analysis of the dividends, expenses and alphas. Even minor amounts of expenses in excess of the fund's alpha can lead to substantial discounts.

9. **(1 point)** The current term structure of zero coupon bond yields is upward sloping, with longer-term yields higher than shorter term yields. Explain why this might be the case under three alternative theories of the term structure.

Solution:

Three theories and their explanations are as follows:

- Expectations Hypothesis – This says that forward rates are equal to the expected future short rates. The yield curve will therefore slope upwards if investors expect short-term rates to rise in the future.

- Liquidity Preference Theory – This says that investors demand a risk premium for assuming liquidity risk. If the term structure is upward sloping then this would indicate that investors in general want to invest short term and so they are demanding a positive liquidity premium for investing long term.

- Segmentation Theory – This says that the short-term and long-term bond markets function independently, with short-term borrowers and lenders negotiating short term rates and long-term borrowers and lenders negotiating long-term rates. Rates in each market will therefore depend on supply and demand in each market. An upward sloping yield curve may simply reflect relatively more long term borrowers than long term investors at that moment.

10. **(1 point)** Assume that the one-year zero rate is 5.5% and the two-year zero rate is 6.5%, both continuously compounded. Describe how you can earn the 7.5% forward rate between the time periods $T_1 = 1$ and $T_2 = 2$ on \$10,000 using only these two zero coupon bonds and then determine the value of a forward rate agreement that allows you to earn the forward rate of 8% on an annually compounded basis during the same time period on the same \$10,000 in notional value. Assume you can either buy or sell either bond in any quantity (i.e. you can borrow or lend at the zero coupon rates).

Solution:

The strategy to earn the forward rate is to borrow the present value of \$10,000 for one year and use the proceeds to simultaneously invest for two years. This will result in no net cash flows now, a negative payment of \$10,000 (the repayment of the loan with interest) at time T_1 and the receipt of interest and principal payment at time T_2.

The amount to borrow now will be $10,000e^{-.055(1)} = 9,464.85$ and with that amount invested at 6.5% for two years you will receive $9,464.85e^{.065(2)} = 10,778.84$ at maturity. Note that the interest earned during time periods one and two will be $\ln(10,778.84/10,000) = 7.5\%$, the forward rate.

To value the FRA that gives you 8% on an annual basis, notice that you could earn 7.5% on a continuously compounded basis on your own, so the FRA is only worth whatever *extra* interest it affords you, on a present value basis.

We first convert the forward rate to an annually compounded rate for comparison to the FRA rate $R_K = 8\%$, which can be found as $R_F = e^{.075} - 1 = 7.788\%$. Then, the value of the FRA is:

$$V_{FRA} = 10,000(.08 - .07788)e^{-.065(2)} = 18.58$$

11. **(2 points)** You are the Chief Investment Officer overseeing the bond investments of your firm. Your only liability is a series of three equal annual cash flows of $250,000, which have a present value of $692,707 given that the continuously compounded risk-free rates are 4% for all maturities. You currently have $1 million in very short-term Treasury bills, which gives you a net difference between the value of your assets and the value of your liabilities of $307,293.

    Your CEO has asked you to consider investing in longer term risk-free securities instead of keeping all of the investments in Treasury bills so that if interest rates fell and the value of the liabilities rose the value of your assets would rise too, preferably by an equal amount.

    Determine the maturity of an appropriate zero coupon bond that would satisfy your CEO's objectives.

Solution:

If you want to ensure that the value of your assets and the value of your liabilities rise or fall by the same amounts when interest rates change, you want the product of their modified durations and their present values to be equal.

In the case of the liability it is easy to find the modified duration is equal to 1.973, as follows:

Time	CF	PV(CF)	T*PV(CF)
1	250.00	240.20	240.20
2	250.00	230.78	461.56
3	250.00	221.73	665.19
		692.71	1,366.95
		Duration	1.973

With continuously compounded rates and a zero coupon bond, the modified duration will just be equal to the maturity of the bond and so now we just need to find an appropriate maturity. It would be wrong to set the maturity equal to 1.973 in this case because the dollar value of the assets exceeds the dollar value of the liabilities and so if rates change we don't want their percentage changes in value to be the same but rather the dollar value of the changes in value to be the same.

So we want to solve the following:

$$MVA * D_A = MVL * D_L$$

$$D_A = \frac{692,707(1.973)}{1,000,000}$$

$$= 1.367$$

You should invest the full \$1 million in a zero coupon bond with a maturity of 1.367 years.

12. **(1 point)** The Chief Investment Officer (CIO) of your insurance company has announced an interest rate risk management strategy which aims to match the durations of the company's assets and liabilities. The liabilities have a present value of $500 million and the average time to payment of the liabilities is equal to 3.5 years, so he purchases a portfolio of risk-free coupon bonds, mortgage pass through bonds and stocks that have a weighted average Macaulay duration equal to 3.5.

After implementing this strategy, interest rates shift up 1% across all maturities and the company's surplus falls considerably. Surprised that duration matching did't immunize the surplus against changes in value from changes in interest rates, the CIO asks you to explain what may have gone wrong. Give four possible reasons, with one related to the magnitude of the surplus, one related to the choice of assets, one related to the liabilities and one related to the convexity of the portfolio.

Solution:

Simple "duration matching" strategies can fall apart for many reasons:

- Regarding surplus, in all solvent companies the value of their assets exceeds the value of the liabilities and so simply matching asset and liability durations will leave the duration of the surplus as a non-zero (positive) value. As Noris showed:

$$D_S = \frac{MVA\ D_A - MVL\ D_L}{MVS}$$

- Regarding the choice of assets, it is important to take into account the fact that simple calculations of Macaulay duration will be inadequate when the asset cash flows are sensitive to changes in interest rates. Both the mortgage pass through bonds and the stocks have cash flows that vary with interest rates — in the case of mortgage pass throughs the effective durations are larger than the Macaulay durations because prepayments speed up when rates fall and slow down when rates rise; in the case of stocks the effect of inflation on both rates and the nominal value of dividends cause their effective durations to be lower than might be presumed based on their "weighted average time to payment".

- Regarding the liabilities, although the weighted average time to payment was specified, we can't say anything about the effective duration of the liabilities without more information. But we might assume that they are lower than simple Macaulay durations because they are likely sensitive to the effects of inflation, so their value will generally fall less when rates rise.

- Finally regarding convexity, we know that duration is only a useful measure of the change in value of assets and liabilities when the change in rates is very small. In this case, rates moved by 1%, which is a large increase, and so to know how much our asset portfolio and liabilities would change in value we would need to include what is referred to as the convexity adjustment.

13. **(1 point)** Suppose an insurer is established with initial surplus of $S = 50$. They write policies on January 1 that have up-front expenses of $E = 25$, expected claims that will be paid in full at the end of the year equal to $L = 75$. They price their business in the same way suggested by Panning using a target return of $k = .10 + y$, where $y = 5\%$ is the risk free rate, which results in a premium of $P = 101.19$. If they write the same business in the future in perpetuity, but the volume is reduced each year according to a retention rate of $cr = 95\%$, what is the franchise value for this firm and the total economic value of the firm?

Solution:

The franchise value is the present value of the net premium, expense and claim cash flows in perpetuity. While you can calculate each component separately, you can also use the following substitution:

$$d = \frac{cr}{1+y} = \frac{.95}{1.05} = 0.90476$$

and calculate the total franchise value as:

$$\begin{aligned} F &= \left[P - E - \frac{L}{1+y}\right]\left[\frac{d}{1-d}\right] \\ &= [101.19 - 25 - 71.43]\left[\frac{0.90476}{1-0.90476}\right] \\ &= 45.24 \end{aligned}$$

Note that you could also use Panning's other formula for the franchise value after substituting in for P and d in terms of y:

$$F = \frac{(cr)S(a + (b-1)y)}{(1+y)(1+y-cr)} = 45.24$$

To get the total economic value, we add in the current economic value which represents the value of the surplus and the present value of the premiums, loss and expense for the business just written:

$$\begin{aligned} C &= S + P - E - \frac{L}{1+y} \\ &= 50 + 101.19 - 25 - \frac{75}{1.05} \\ &= 54.76 \end{aligned}$$

The total economic value is therefore $54.76 + 45.24 = 100$.

14. (2 points) The continuously compounded risk-free yield is 5% for all maturities and you are trying to determine the price for a three-year semi-annual coupon bond with \$100 face value that pays a promised coupon of 7% but which contains default risk.

Assume the risk-neutral conditional probability of default in each year, given that the bond has not yet defaulted, is 3%, that defaults can only occur midway through each year (just before the coupon is paid) and that in the event of default the recovery is 40% of the face value.

Determine the price of this bond.

Solution:

The price of the bond should equal its risk-free value less the present value of the expected default losses. We can calculate the present value of expected default losses using the unconditional default probabilities at each possible default date $T = .5, 1.5, 2.5$, the loss given default and the risk-free present value factor (which we can use here because we were given the risk-neutral default probabilities).

Notice that this is just the same calculation that the textbook uses to calculate the annual probability of default except now we are given information about the default probability and we just have to solve for the price.

For the default probabilities, we want to use the unconditional default probabilities and were given the conditional default probabilities. The conditional and unconditional default probabilities are the same for the first period, so $q_1 = 3\%$.

For the second period, we solve:

$$3\% = \frac{q_2}{1 - q_1} \Rightarrow q_2 = 2.91\%$$

And for the third period:

$$3\% = \frac{q_3}{1 - (q_1 + q_2)} \Rightarrow q_3 = 2.823\%$$

The loss given default is defined as the risk-free value of the bond at the time of default point less the recovery in the event of default.

Let's start with the risk free values of the bond at $T = .5$ right before the coupon is paid. We have future cash flows of \$3.5 right now, then \$3.5 for 4 more periods and \$103.5 at maturity. The value of this using the risk free yield of 5.0% is:

$$\begin{aligned} F_1 &= 3.5 + 3.5e^{-.05(.5)} + 3.5e^{-.05(1)} + 3.5e^{-.05(1.5)} + 3.5e^{-.05(2.0)} + 103.5e^{-.05(2.5)} \\ &= 107.995 \end{aligned}$$

The following table shows the same calculations for time periods $T = 1.5$ and $T = 2.5$ as well:

Time	CF	T = 0.5	T = 1.5	T = 2.5
0.5	3.50	3.500	0.000	0.000
1.0	3.50	3.414	0.000	0.000
1.5	3.50	3.329	3.500	0.000
2.0	3.50	3.247	3.414	0.000
2.5	3.50	3.167	3.329	3.500
3.0	103.50	91.338	96.021	100.945
	Risk-Free Value	107.995	106.264	104.445
	Recovery	40.000	40.000	40.000
	LGD	67.995	66.264	64.445

Notice that we could use our CAS approved calculators for these three calculations, but only if we are careful to convert to the 5% continuously compounded rate to a semi-annually compounded rate:

$$1 + r = e^{.05*.5} \Rightarrow r = 2.5315\%$$

Here, r is the six-month rate. Then, set to get the values except for the coupon payment about to be received we can use the following: $N = 5$, $I/Y = 2.5315\%$, $PMT = 3.5$, $FV = 100$ and then compute the present value to get $PV = 104.495$. Add in the 3.5 coupon about to be paid to get 107.995. Similar calculations are done for the other two values.

Using these risk-free values at the time of default, the loss given default (LGD) reflects the risk free value at each point less the recovery, as shown in the table.

Multiplying the LGD at each possible default time by the default probability and the risk free discount factor, we get a total expected present value of loss equal to 5.384:

Time	Probability	LGD	PV Factor	E[PV(LGD)]
0.5	3.000%	67.995	0.9753	1.989
1.5	2.910%	66.264	0.9277	1.789
2.5	2.823%	64.445	0.8825	1.605
				5.384

Now we can calculate the price of the bond, which will reflect the risk-free value of the promised cash flows less the expected present value of default losses.

$$\begin{aligned} G &= 3.5e^{-.05(.5)} + 3.5e^{-.05(1)} + 3.5e^{-.05(1.5)} \\ &\quad + 3.5e^{-.05(2)} + 3.5e^{-.05(2.5)} + 103.5e^{-.05(3)} \\ &= 105.33 \end{aligned}$$

This gives us a bond value of $B = 105.33 - 5.384 = 99.946$.

15. **(1 point)** Explain the role of Fannie Mae and Freddie Mac in the mortgage and mortgage-backed securities market, being careful to identify a distinction between the role they originally played and the role they assumed in the years leading up to the 2007-2009 financial crisis.

Solution:

Fannie Mae and Freddie Mac are quasi-government agencies that were established to provide funds and enhance liquidity of the mortgage market. Originally they would simply borrow money from investors and use those funds to buy mortgages issued by banks, thereby absorbing the credit risk associated with such loans to finance home purchases.

When mortgage backed securities, such as mortgage pass throughs and collateralized mortgage obligations were developed in the 1970's, Fannie and Freddie were the primary issuers. They would typically buy mortgages that met specific underwriting criteria from the issuing banks, bundle them into (diversified) pools and sell securities backed by these pools to investors. In so doing, Fannie and Freddie would provide guarantees against default of the underlying mortgages (for a fee). The result was that the interest rate and prepayment risk would be passed along to investors but Fannie and Freddie would retain the default risk.

Prior to the last decade, the vast majority of mortgage default risk was held by Fannie Mae and Freddie Mac, with implicit backing by the US government, and trillions of dollars of mortgage backed securities that retained no default risk were issued.

By 2006 however, banks began to issue substantial volumes of mortgages to borrowers who did not meet the Fannie Mae and Freddie Mac underwriting criteria, with the standards eventually declining considerably. For this growing "subprime" market, Fannie Mae and Freddie Mac generally did not provide default guarantees. Instead, investors assumed both the default risk and the interest rate/prepayment risk.

Despite the fact that they were not providing credit guarantees on subprime mortgages, as they did for conforming mortgages, Fannie and Freddie were under tremendous pressure from the US government to "support" the goal of increasing homeownership in the U.S. and eventually they became significant investors in subprime mortgage backed securities. That is, even when they did not guarantee the default risk for the entire pools of subprime mortgages, they did assume much of this risk through their investment directly in the securitized tranches.

Whether by purchasing mortgages and holding them to maturity, issuing mortgage backed securities and providing credit insurance or by investing directly in subprime mortgage backed securities that contained (significant) default risk, Fannie and Freddie have played a critical role in lowering mortgage rates for homeowners and, in many respects, played a key role in fueling the housing price bubble that deflated so painfully beginning in 2006.

16. **(2 points)** Determine the spread for a 3-year \$2 million notional binary CDS on a reference bond with conditional default probability equal to 4% per annum, assuming that defaults only occur at the *middle* of each annual period. In the event of default, the recovery on the reference bond is 60% of its face value and the CDS payoff is equal to the notional value.

Assume the risk-free rate is 4% on a continuously compounded basis and determine the CDS spread assuming the spread is paid annually *at the end of each year* and that the accrued spread payment is paid if default occurs before the end of the year.

Solution:

In the case of a binary CDS we merely have to reflect the fact the CDS payoff is a fixed dollar amount, in this case \$2 million. Since the CDS spread is quoted as a percentage of the notional value, it is easiest to calculate the spread assuming \$1 of notional value and then reflecting a fixed \$1 payment in the event of default. Otherwise, we calculate the spread in the same way as in a standard CDS.

Step 1: Calculate the Unconditional Default Probabilities and Survival Probabilities

Table 6: Default and Survival Probabilities

Time (Years)	Unconditional Default Probability	Survival Probability
1	0.0400	0.9600
2	0.0384	0.9216
3	0.0369	0.8847

Step 2: Determine Protection Buyer's Spread Payments

Here, we need to account for both the annual payments that are made so long as the bond has not defaulted and the accrual payments (one-half of the spread) that will be made in the event of default.

Table 7: Swap Spread Payments – Annual

Time (Years)	Survival Probability	Expected Payment	Discount Factor	PV of Expected Payment
1	0.9600	0.9600s	0.9608	0.9224s
2	0.9216	0.9216s	0.9231	0.8507s
3	0.8847	0.8847s	0.8869	0.7847s
Total				2.5578s

Table 8: Swap Spread Payments – Accrual

Time (Years)	Default Probability	Accrual Payment	Expected Payment	Discount Factor	PV of Expected Payment
0.5	0.0400	.5s	0.0200s	0.9802	0.0196s
1.5	0.0384	.5s	0.0192s	0.9418	0.0181s
2.5	0.0369	.5s	0.0184s	0.9048	0.0167s
Total					0.0544s

The total expected spread payments are therefore $2.5578s + .0544s = 2.6122s$.

Step 3: Determine the Protection Seller's CDS Payments

Table 9: CDS Payments

Time (Years)	Default Probability	CDS Payoff	Expected Payment	Discount Factor	PV of Expected Payoff
0.5	0.0400	1.0000	0.0400	0.9802	0.0392
1.5	0.0384	1.0000	0.0384	0.9418	0.0362
2.5	0.0369	1.0000	0.0369	0.9048	0.0334
Total					0.1087

Step 4: Solve for s in Dollars

$$2.6122s = .1087 \Rightarrow s = 4.1624\%$$

17. (1 point) What are the key considerations in determining the loss triggers to be used for a catastrophe bond?

Solution:

The choice of loss triggers is a matter of balancing the trade-off between moral hazard, transparency for the investors (which ultimately impacts investor demand, ease of execution, etc.) and basis risk for the insured.

Indemnity triggers generally eliminate the basis risk for the insured, but require much more extensive disclosure to investors, require much more investor due diligence and complicate claim settlement after an occurrence. The latter issue can be important because of the need to ensure that the SPR is on automatic pilot with a contractual maturity date of the bond.

Index triggers are easier for investors to evaluate — they are not subject to the claims handling practices of the insured and therefore the risk analysis is simplified. Index-based bonds are also not impacted by changes in risk exposure, premium volume, etc., so the overall due diligence process is simplified and execution risk is minimized. However, this comes with increased basis risk, since it is possible that the index losses do not mirror the company's actual claim experience.

18. **(1 point)** Describe why some managers of publicly traded firms might want to hedge less of the firm's financial risks than its shareholders would want to hedge. Give an empirical example of an instance when this behavior was observed.

Solution:

In most firms managers face asymmetric benefits from risk-taking — if their bets pay off they are more likely to stand out and be rewarded but if their bets go wrong the shareholders suffer most of the downside consequences. This gives managers a strong incentive to take risk.

In a survey of gold mining firms conducted by Peter Tufano he found that when the managers held large call option positions, as opposed to direct equity stakes, they had a strong incentive to increase the volatility of the firm's results and thus hedged less.

19. **(2 points)** Culp, Miller and Neves cite four well-known derivatives disasters involving P&G, Orange County, Barings and Metallgesellschaft:

- P&G's treasury department lost hundreds of millions of dollars using fixed for floating interest rate swaps that contained an embedded bet on the movement of the term structure.
- Orange County's treasurer lost $1.5 billion through the use of leveraged inverse floating rate bonds (and other similar products) when rates rose more than expected.
- Barings lost over a billion dollars when a "rogue" trader used put options and future options to bet on the Nikkei index, seemingly without approval and without reporting the transactions or their status to his supervisors.
- Metallgesellschaft lost over a billion dollars when oil and gas prices fell and the short-term futures contracts being used to hedge long-term fixed price contracts lost money and could not be rolled over.

What reasons do they give in each case why the use of VaR would have been unlikely to prevent these disasters?

Solution:

Some of the reasons included the following:

- P&G — Had P&G used VaR for their entire swap book and monitored the additional VaR for each new transaction, they might have avoided the large bet they made on interest rates. However, VaR was never intended to be used at the transaction level and it is unlikely that a firm focused on cash flow risk management would have had the monitoring systems in place to approve swap transactions based on the VaR.
- Orange County — It is argued that the Orange County treasurer was intentionally taking a bet on interest rates in order to improve his portfolio returns. Since VaR focuses only on the downside without any mechanism to assess the risk in relation to the potential gains, this risk measure would have been unlikely to motivate any change in strategy.
- Barings — Knowing the VaR of Leeson's trading book might have prevented Barings' losses, but unfortunately Barings completely lacked any internal controls or information systems that would have passed this information to senior management. Leeson himself was responsible for monitoring his portfolio and it is probably safe to assume that he would not have reported his VaR to his superiors.
- Metallgesellschaft — Again, VaR could have provided quantitative information about the size of the rollover and basis risk, but MGRM was in the business of taking these

risks and appeared to be doing it knowingly. It seems unlikely that whatever additional information contained in the VaR would have altered their strategy. More importantly, VaR is a value risk measure and what MGRM really needed was a better understanding of the cash flow risks associated with their strategy.

20. **(1 point)** An insurer has a fixed liability of $\$100,000$ due in one period and $\$20,000$ in capital. All of its assets are invested in risky securities with the following distribution of ending asset values at the end of the period as shown in the table below.

After being asked to estimate the EPD ratio for the company you produce the exhibit below:

Scenario	Beg Assets	Return	Ending Assets	Loss	Probability	Claims Paid	Deficit
1	120,000	120.00%	144,000	100,000	0.60	100,000	0
2	120,000	90.00%	108,000	100,000	0.30	100,000	0
3	120,000	70.00%	84,000	100,000	0.10	84,000	16,000
Mean							1,600
						EPD	1,600
						Exp Loss	100,000
						EPD Ratio	1.60%

Determine the capital needed to obtain an EPD ratio of 5% assuming that all of the assets are invested in the same risky securities and therefore have the same total return distribution.

Solution:

Using the information in the table just solve for the starting assets that, under the same return distribution, produce the target 5% EPD ratio.

The algebra to solve for the starting assets is complicated by the function $max(0, L - A)$ used to determine the deficit by scenario. Since we want the EPD ratio to be higher, we know we need to lower the assets so that we have larger deficits, but we have to assume we know which scenarios will have deficits, solve for the assets and then confirm that our assumption was right.

If we assume there will only be a deficit in Scenario 3 and solve the 5% EPD ratio we would get assets of $\$71,428$, as shown below:

$$\text{EPD Ratio} = 5\% = \frac{.6(0) + .3(0) + .1[L - A(.70)]}{100,000} \Rightarrow A = 71,428$$

The algebra was easy because we simply assumed the deficits were zero in the first two scenarios. However, this amount would actually result in deficits in all three scenarios. This means that our assumption was wrong.

So we redo the calculation assuming a deficit in Scenarios 2 and 3:

$$\text{EPD Ratio} = 5\% = \frac{.6(0) + .3[L - A(.90)] + .1[L - A(.70)]}{100,000} \Rightarrow A = 102,941$$

Given the liabilities, this would result in starting capital of $\$2,941$.

21. (1 point) Cummins calculates the EPD Ratio using the Black-Scholes option pricing formula for a put on the asset to liability ratio. What is the formula he uses to calculate the volatility of this ratio based on the asset and liability volatilities?

Solution:

The volatility parameter that Cummins uses is technically the standard deviation of the natural log of the asset to liability ratio, A/L.

$$\mathrm{Var}\left[\ln\left(\frac{A}{L}\right)\right] = \mathrm{Var}[\ln(A) - \ln(L)] = \sigma_A^2 + \sigma_L^2 - 2\sigma_A\sigma_L\rho$$

22. (1 point) You have simulated the aggregate risk profile for a P&C insurer, producing 10,000 iterations. The 5 scenarios with the worst total loss results are shown in the following table:

Scenario	Market	Reserves	Line A	Line B	Total
5,043	-3,407,081	13,140,377	7,607,985	788,471	18,129,751
2,097	494,425	8,169,822	3,734,913	8,695,665	21,094,825
1,229	-1,311,004	-1,203,142	3,238,333	16,924,158	17,648,345
7,593	779,323	12,180,298	3,188,429	4,994,583	21,142,632
7,143	-779,922	2,587,705	5,675,660	10,386,216	17,869,658

Determine the 99.97% Co-CTE for Line B.

Solution:

The 99.97% Co-CTE is calculated by first sorting the scenarios based on the total losses and then calculating the average losses for Line B in the scenarios that are worse than the 99.97th percentile. Since there are 10,000 scenarios, we are interested in the $1 - 99.97\% = 0.03\%$ worst scenarios. That is, we are interested in the average of the worst 3 scenarios.

The following table shows the sorted scenarios. The Co-CTE is the average of the Line B results for the top three scenarios, or $4,826,240.

Scenario	Line B	Total
7,593	4,994,583	21,142,632
2,097	8,695,665	21,094,825
5,043	788,471	18,129,751
7,143	10,386,216	17,869,658
1,229	16,924,158	17,648,345

23. **(1 point)** You are trying to use Feldblum's and Robbin's IRR method to determine the risk loads in insurance rates and notice the importance of the amount and timing of the equity flows, including the initial requirements as well as the release of the initial committed surplus over time.

Briefly discuss some methods for setting the initial surplus and for determining how this is released over time.

Solution:

Using industry premium to surplus ratios, industry reserve to surplus ratios or regulatory risk-based capital requirements are all common methods, though often adjustments are made to reflect the fact that regulatory requirements reflect *minimum* capital requirements.

Once the initial surplus estimated, the release of that surplus over time is often based on premium to surplus ratios or reserve to surplus ratios. A better approach is to capture the relative risk over time and release surplus as the risk dissipates, in which case the release pattern will vary considerably from line to line.

24. (1 point) You have used Ferrari's formula for the total shareholder return using estimated prospective rates of return and leverage ratios. When discussing the resulting estimate with your senior management they conclude that as long as the investment return is larger than the underwriting loss, any increase additional premium written results in greater shareholder value. Show the formula they must have been thinking of when they said this and explain why their statement may not be correct.

Solution:

The equation that is relevant here is:

$$\frac{T}{S} = \frac{I}{A} + \frac{R}{S}\left(\frac{I}{A} + \frac{U}{R}\right)$$

Although it is true that this rate of return calculation increases as the ratio of R/S increases, so long as the investment return is greater than the underwriting loss and all other ratios remain constant, this does not mean that shareholder value is created. The main issue is that writing more business may involve assuming more risk and so higher expected returns may have to be discounted at higher rates that reflect this risk and the impact on shareholder is ambiguous. It could even be negative.

In addition, increasing the volume of business written may cause the other ratios, such as the investment return or the underwriting profit to decline.

25. **(1 point)** What two reasons does McClenahan give for preferring that insurance rate regulation rely on return on sales as opposed to return on equity as a profitability measure?

Solution:

His two reasons are that return on sales (or return on premium) is easy to interpret and it does not require the use of allocated surplus.

26. (1 point) Robbin's Risk-Adjusted Cash Flow method is presented in the paper as:

$$PV(P;i_f) = PV(L;i_r) + PV(FX;i_f) + PV(FIT;i_f)$$

But when discussing the Present Value Cash Flow Return model he suggested that an approximate way to account for taxes would be to apply the tax rate to the present value underwriting profit and to the investment income on equity or surplus.

Show what the Risk-Adjusted Cash Flow formula would be if this approximation for taxes were used.

Solution:

Following the same approach and assuming only fixed, not variable, expenses:

$$PV(P) = PV(L) + PV(FX) + t[PV(P) - (PV(L) + PV(FX))] + tPV(II_S)$$

$$PV(P) - t[PV(P) = PV(L) - tPV(L) + PV(FX) - tPV(FX) + tPV(II_S)$$

$$PV(P)(1-t) = PV(L)(1-t) + PV(FX)(1-t) + tPV(II_S)$$

$$PV(P) = PV(L) + PV(FX) + \frac{tPV(II_S)}{1-t}$$

In those equations, the losses (L) are discounted at a risk-adjusted rate, the other amounts are discounted at a risk-free rate, and II_S represents the investment income on the *surplus* only.

Note that this is the formula that I used in the numerical problems in my notes. It is alluded to in the paper but not shown explicitly, so test questions are unlikely to use it. I included it here to force you to think a bit about how the methods are related and what each of the terms means.

27. (2 points) You are given the following event loss table for two risks, X and Y. Calculate Mango's Covariance Share for each risk using the same allocation as he uses in his paper.

Table 10: Occurrence Size of Loss Distribution

Event	$p(i)$	$1 - p(i)$	Loss for X	Loss for Y	Total Loss
1	2.0%	98.0%	25,000	200	25,200
2	1.0%	99.0%	15,000	500	15,500
3	3.0%	97.0%	10,000	3,000	13,000
4	3.0%	97.0%	8,000	1,000	9,000
5	1.0%	99.0%	5,000	2,000	7,000
6	2.0%	98.0%	2,500	1,500	4,000

Solution:

Mango's Covariance Share calculation applies a loss weighting *by event* to allocate the contribution of the covariance to the total portfolio risk. The calculation is as follows:

Table 11: Calculation of Covariance Share

Event	Loss for X	Loss for Y	Weight X	Weight Y	Cov Share X	Cov Share Y
1	25,000	200	99.2%	0.8%	194,444	1,556
2	15,000	500	96.8%	3.2%	143,710	4,790
3	10,000	3,000	76.9%	23.1%	1,343,077	402,923
4	8,000	1,000	88.9%	11.1%	413,867	51,733
5	5,000	2,000	71.4%	28.6%	141,429	56,571
6	2,500	1,500	62.5%	37.5%	91,875	55,125
					2,328,401	**572,699**

28. **(2 points)** You are trying to determine a risk load for a reinsurance policy following Kreps' put option strategy, which involves investing the premium and capital funds in a risky asset but using some of the funds to purchase a put option that allows you to sell the risky assets for the amount the assets would have grown to if invested at the risk-free rate.

The risk-free rate is 3% and the following reflects the distribution of the returns on the risky asset, with an expected rate of return of 4%:

Risky Asset Return	
Probability	Return
0.125	-5%
0.250	0%
0.250	3%
0.250	8%
0.125	15%

Determine the expected put-protected rate of return, i.

Solution:

The put-protected rate of return reflects the fact that the put allows us to sell our assets for at least the risk-free rate of return. Setting the rate of return in each of the scenarios shown above to be the maximum of 3% or the return on the risky asset, we can calculate an expected put-protected rate of return of $i = 5.75\%$.

Cumulative Normal Distribution (Positive x)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Cumulative Normal Distribution (Negative x)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Practice Exam 3

This practice exam is intended to contain exam-style questions for a representative subset of the Learning Objectives.

Section Weights

Questions from each part of the syllabus are included, with weights generally consistent with the weights specified on the syllabus.

Time Limit

The actual exam has a time constraint that typically poses a real challenge for students. Nonetheless, I have decide to not specify a time limit for the practice exams in order to encourage you to carefully consider each of these questions and take whatever time is needed to fully formulate your responses. It would be a mistake to skip practice questions due to time constraints.

Points Per Question

Points have been allocated for each question, but the vast majority of the questions are worth just one point. This is because I have tried to make most of the questions comparable in terms of difficulty and time needed to perform calculations, so I did not vary the points much.

How to Grade Yourself

You should target scoring 85% or more to fully prepare for the actual exam.

1. **(1 point)** Suppose your utility function were given by the equation $U = E(r) - .5A\sigma^2$ with risk aversion coefficient $A = 4$.

Your total wealth is $\$1,000,000$ which you hold entirely in cash. You face a contingent liability with a 4% probability of having to pay $\$500,000$ and a 96% probability of not having to pay anything.

Calculate the premium you would be willing to pay for an insurance policy against this contingent liability such that your utility is the same with or without the insurance policy.

Solution:

Notice that the utility function has been specified in terms of the expected *return* and its variance, so we don't want to use the dollar amounts. Instead, convert the outcomes to reflect a 4% probability of a -50% return and a 96% probability of a 0% return. This gives an expected return of -2% and a variance of:

$$\begin{aligned}\text{Variance}(r) &= E(r - \bar{r})^2 \\ &= p(-50\% - (-2\%))^2 + (1-p)(0 - (-2\%)) \\ &= 0.0096\end{aligned}$$

Without the insurance policy, your utility is:

$$U = E(r) - .5A\sigma^2 = -.02 - .5(4)(.0096) = -0.0392$$

With the insurance policy, your return is guaranteed to be 0% with no standard deviation, but for the price of the insurance policy. If the premium paid is v (as a percentage of your wealth), then your utility is simply $-v$.

Setting these equal, we see that the fair premium is such that $-v = -0.0392$ or $v = 3.92\%$. In dollars, this suggest that you would be willing to pay $\$39,200$ for a policy that protects you against a loss with an expected value of $\$20,000$.

2. **(2 points)** You are planning to split your portfolio between two risky index funds with the following characteristics:

	Stock Fund	Bond Fund
Expected Return	11%	7%
Standard Deviation	13%	6%
Correlation Coefficient	0.20	

The risk-free rate is 3% and you are able to borrow or lend at this rate. Determine the weights for the stock fund and the bond fund that maximize the Sharpe ratio of your complete portfolio.

Solution:

The Sharpe ratio is maximized when you hold the optimal risky portfolio whose weights are:

$$w_S = \frac{[E(r_S) - r_f]\sigma_B^2 - [E(r_B) - r_f]\sigma_{SB}}{[E(r_S) - r_f]\sigma_B^2 + [E(r_B) - r_f]\sigma_S^2 - [E(r_S) - r_f + E(r_B) - r_f]\sigma_{SB}}$$

In this case, the risk premiums and the covariance are:

$$E(r_S) - r_f = 8\%$$

$$E(r_B) - r_f = 4\%$$

$$\sigma_{SB} = .00156$$

The resulting weights are therefore:

$$\begin{aligned} w_S &= \frac{(.08)(.06^2) - (.04)(.00156)}{(.08)(.06^2) + (.04)(.13^2) - (.08 + .04)(.00156)} \\ &= 29\% \\ w_B &= 71\% \end{aligned}$$

3. **(1 point)** Suppose the universe of risky assets consists of a large number of stocks, each of which has an expected return of 12%, a standard deviation of 50% and a common correlation coefficient of .4. The risk-free rate is 4%. What is the systematic risk in this security universe?

Solution:

The systematic risk can be defined as the standard deviation of the optimally diversified portfolio — it is the amount of risk that remains after achieving the most diversification possible.

As the number of assets in the portfolio, N, gets large, the portfolio variance approaches the average covariance of the assets in the portfolio. In this case, the average covariance is $\rho * \sigma^2 = .4(.5^2) = .1$, and therefore the portfolio standard deviation will approach $\sqrt{.1} = .316$. This is the systematic risk in this stock universe.

4. **(1 point)** You have collected the historical monthly returns in excess of the risk free rate for Stock A, Stock B and the S&P 500 for the past 60 months and estimated the following regression coefficients:

Table 12: Regression Results

$R_i = \alpha + \beta\, R_m + \epsilon$	$\hat{\alpha}$	$\hat{\beta}$	$\sigma^2_{\epsilon_i}$
Stock A	0.005	0.92	0.002
Stock B	-0.002	1.15	0.001

You also calculated the historical variance of the excess market return as $\sigma^2_m = .04$. Calculate the variances of returns for Stock A and Stock B as well as their covariances.

Solution:

The regression represents a single index model and so we can get the variances and covariances as follows:

$$
\begin{aligned}
\sigma^2_A &= \beta^2_A \sigma^2_m + \sigma^2(e_A) \\
&= .92^2(.04) + .002 \\
&= 0.0359 \\
\sigma^2_B &= \beta^2_B \sigma^2_m + \sigma^2(e_B) \\
&= 1.15^2(.04) + .001 \\
&= 0.0539 \\
\sigma^2_{AB} &= \beta_A \beta_B \sigma^2_m \\
&= (.92)(1.15)(.04) \\
&= .04232
\end{aligned}
$$

5. **(1 point)** What are the key assumptions in the standard CAPM that are modified in the CAPM with Non-Traded Assets and Labor Income and how does the resulting model compare to the empirical findings about the relation between expected returns and betas?

Solution:

The CAPM assumes that investors' only source of wealth is their investment in the market portfolio and the risk free rate, which ignores the existence of *i*) private businesses that are not actively traded and thus not part of the market portfolio and *ii*) investors' income from their employment, which is a key source of wealth.

Including those in the model means that investors will avoid holding risky assets that are highly correlated with their own individual non-traded assets and/or labor income. This has the effect of increasing the betas of low-beta stocks and decreasing the betas of high-beta stocks, which eliminates that apparent positive alphas for low-beta stocks and negative alphas for high-beta stocks.

6. **(1 point)** One important assumption underlying the Capital Asset Pricing Model (CAPM) is that every investor follows the Markowitz portfolio construction procedure. Discuss what assumption was made regarding how investors behave in order to develop the Arbitrage Pricing Theory.

Solution:

The APT assumes only that arbitrage opportunities do not persist in equilibrium and that there are no limits to the amount of an asset that could be bought or sold. Because true arbitrage opportunities are risk free and can therefore be exploited with no up-front investment, all that is required is that *someone* is able to identify and exploit them. There are no other restrictions on how all other investors form their portfolios.

7. **(1 point)** What is the behavioral critique of conventional financial theory?

Solution:

The premise of behavioral finance is that conventional financial theory ignores how people really make decisions under uncertainty. First, there is experimental evidence that suggests that people commonly make errors inferring probability distributions. Second, even when they know the probability distributions, they often make systematically sub-optimal decisions.

These are interesting observations, but their impact on security markets is less clear. If these sorts of errors lead to mispriced securities, then we should expect arbitrageurs to take advantage of these errors. Their actions would eventually cause the errors to be corrected. So if these errors do impact market prices, there must be some other limits to the actions of the arbitrageurs.

8. (1 point) You are trying to test the standard CAPM and have performed a first-pass regression using 60 months of historical return data to estimate market betas for five portfolios of stocks whose average returns during the sample period were as follows:

Table 13: Average Return in Excess of Risk Free Rate

Portfolio A	5.0%
Portfolio B	8.5%
Portfolio C	3.0%
Portfolio D	7.0%
Portfolio E	6.0%
Average Portfolios A-E	5.9%
Market Portfolio	5.5%

You then run the following second-pass regression:

$$\overline{r_i - r_f} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{\sigma^2_{ei}} + \epsilon$$

Describe what values you should expect for $\gamma_0, \gamma_1, \gamma_2$ if the standard CAPM is valid. Your answers should specify a specific numerical value in each case.

Solution:

If the CAPM is valid, then you would expect to have:

- γ_0 should equal zero, indicating no expected average excess return except that which is due to the beta on the market excess return
- γ_1 should equal the average excess return on the market, or 5.5% given the historical data shown in the question
- γ_2 should equal zero, indicating that there is no average excess return from other factors or sources of idiosyncratic risk

9. **(1 point)** You are given the following annually compounded yields on zero-coupon risk-free bonds. Determine the forward rate between years two and three.

Maturity (Years)	Yield
1	4.00%
2	4.40%
3	4.60%
4	4.80%

Solution:

Solve the following for the forward rate, f:

$$(1 + y_3)^3 = (1 + y_2)^2(1 + f)$$

$$1.046^3 = 1.044^2(1 + f)$$

$$f = 5.001\%$$

10. **(1 point)** Calculate the modified duration and convexity of a 2-year, 5% coupon bond with annual coupons and a $1,000 face value if the yield on the bond is currently 5% on a continuously compounded basis.

Solution:

The modified duration for a bond with continuous compounding is given by the formula,

$$D = \frac{\sum t_i c_i e^{-yt_i}}{B}$$

T	Cash Flow	PV(CF)	T*PV(CF)
1	50	47.56	47.56
2	1,050	950.08	1,900.16
		997.64	1,947.72
		Duration	**1.952**

The formula for convexity is as follows when you have continuous compounding:

$$C = \frac{\sum t_i^2 c_i e^{-yt_i}}{B}$$

T	Cash Flow	PV(CF)	$T^2 PV(CF)$
1	50	47.56	47.56
2	1,050	950.08	3,800.32
		997.64	3,847.88
		Convexity	**3.857**

11. (1 point) Assume you own a bond with $1,000 in face value, 9% coupons paid annually and 5 years to maturity. Interest rates are currently 8% compounded annually and the term structure is flat. What is the total value of this bond, including reinvestment of coupons, two years from today if rates are unchanged? What if rates immediately increase to 9% compounded annually?

Solution:

The future value will include the future value of the coupons plus the value of the bond two years from today. The first part is found by noting that we will receive coupons of $90 in one year, which will be reinvested at 8% for one year, and then we will receive another $90 in two years. The future value of these two coupon payments is:

$$\text{Future Value of Coupons} = 90(1.08) + 90 = 187.20$$

1025.77097 187.2 1212.97097 For the value of the bond in two years, note that this is now just a 3-year bond with a 9% coupon, so its value is easily found to be $1,025.77. The total future value is $1,212.97.

If rates increase to 9%, we simply redo those calculations, noting that the reinvestment of coupons is now done at 9% and the present value of the bond at 2 years is now found by discounting the remaining cash flows at 9%. These two values are easy to show to equal $188.10 and $1,000 for a total of $1,188.10.

12. **(1 point)** An insurer has assets with a market value of \$1,000 and a Macaulay duration of 2.0. Their liabilities have a market value of \$750 and a Macaulay duration of 2.0 as well. What is the Macaulay duration of their surplus?

Solution:

The basic relationship between the surplus, asset and liability durations is:

$$
\begin{aligned}
D_{\text{MVS}} &= \frac{D_{\text{MVA}}\text{MVA} - D_{\text{MVL}}\text{MVL}}{\text{MVS}} \\
&= \frac{2(1,000) - 2(750)}{250} \\
&= 2.0
\end{aligned}
$$

An alternative way to see this is:

$$D_{\text{MVS}} = \frac{\text{MVA}}{\text{MVS}} \left(D_{\text{MVA}} - D_{\text{MVL}}\right) + D_{\text{MVL}}$$

From this, it should be obvious that when $D_{\text{MVA}} = D_{\text{MVL}}$ then the surplus duration will equal the liability duration, which in this case is equal to 2.

13. (1 point) Suppose an insurer is established with initial surplus of $S = 50$. They write policies on January 1 that have up-front expenses of $E = 25$, expected claims that will be paid in full at the end of the year equal to $L = 75$. They price their business in the same way suggested by Panning using a target return of $k = .10 + y$, where $y = 5\%$ is the risk free rate, which results in a premium of $P = 101.19$. If they write the same business in the future in perpetuity, but the volume is reduced each year according to a retention rate of $cr = 95\%$, what is the duration of the firm's franchise value?

Solution:

Recall that the formula given by Panning for the franchise value in terms of all of the variables specified is:

$$F = \frac{(cr)S(a + (b-1)y)}{(1+y)(1+y-cr)}$$

Taking the derivative of the franchise value formula with respect to the risk free rate, y, is tedious, but when that is done the duration of franchise value turns out to be:

$$\begin{aligned} D &= \frac{a-b+1}{(1+y)(a+by-y)} + \frac{1}{1+y-cr} \\ &= \frac{.1-1+1}{(1.05)(.1+(1)(5\%)-5\%)} + \frac{1}{1+5\%-95\%} \\ &= 10.952 \end{aligned}$$

Notice that if you didn't remember this duration formula (and who would, really!), you could have approximated the duration using the same approximation used for bond duration:

$$D \approx \frac{F_{-} - F_{+}}{2F(\delta y)}$$

Here, we just set $\delta y = .1\%$ and plug in the values for the parameters in the basic franchise value formula to get:

$$\begin{aligned}
F &= \frac{(cr)S(a+(b-1)y)}{(1+y)(1+y-cr)} \\
&= \frac{(.95)(50)(.1+(1-1)(.05)}{(1.05)(1.05-.95)} \\
&= 45.23810 \\
F_- &= \frac{(.95)(50)(.1+(1-1)(.049)}{(1.049)(1.049-.95)} \\
&= 45.73861 \\
F_+ &= \frac{(.95)(50)(.1+(1-1)(.051)}{(1.051)(1.051-.95)} \\
&= 44.74758
\end{aligned}$$

And then the duration is:

$$D \approx \frac{F_- - F_+}{2F(\delta y)} = \frac{45.73861 - 44.74758}{2(45.23810)(.1\%)} = 10.953$$

14. **(1 point)** The continuously compounded risk-free yield is 5% for all maturities and you are trying to determine the yield spread for a three-year semi-annual coupon bond with \$100 face value that pays a promised coupon of 7% but which contains default risk. You observe that the price of the bond is \$99.946. What is the continuously compounded yield spread?

Solution:

Recall that the yield of a bond is just the rate that is used to discount all of the promised cash flows so that the present value of those cash flows is equal to the actual bond price.

If we write out the equation we see that it is a very messy algebra problem when written in terms of the continuously compounded yield, y:

$$99.946 = 3.5e^{-y(.5)} + 3.5e^{-y(1)} + 3.5e^{-y(1.5)} + 3.5e^{-y(2)} + 3.5e^{-y(2.5)} + 103.5e^{-y(3)}$$

On the exam you wouldn't be able to do this. But, you can solve for the semi-annually compounded yield, y_{SA} instead and then just convert it to a continuously compounded yield. Set $N = 6$, $PV = -99.946$, $PMT = 3.5$ and $FV = 100$ and then compute $I/Y = 3.5101$.

To convert this to a continuous rate we solve:

$$e^{y(.5)} = 1.035101 \Rightarrow y = 6.9\%$$

This gives us a yield spread (relative to the risk-free rate) of 1.9%.

15. **(1 point)** A synthetic CDO has been created using a pool of three credit default swaps on three corporate names. Each corporate name has a 5% probability of default and each credit default swap pays $100 in the event of default. For the CDO, the following tranches are created:

- Junior: 0 - 33.33%
- Mezzanine: 33.34 - 66.66%
- Senior: 66.67-100%

Compare the probability of default for the junior tranche in the case where the three corporate names have independent default risk and the case where the three corporate names have perfectly correlated default risk.

Solution:

Since the junior tranche suffers a loss if ANY of the three credit default swaps have to make a payment, the probability of a default on the junior tranche is the same as the probability that there are any defaults among the three corporate names.

In the case of independent default risk, the junior tranche default probability will be one minus the probability that none of them default, or $p_j = 1 - .95^3 = 14.26\%$.

In the case of perfectly correlated default risk, either all three corporate names default, with 5% probability, or none of them default, with 95% probability. That means the default probability for the junior tranche is 5%.

Notice that the junior tranche default probability declines as the correlation rises.

16. **(2 points)** One year ago you entered into a 3-year $10 million notional credit default swap and agreed to pay a swap spread of 1.7% on an annual basis (at the end of each year) and to pay the accrued portion of the spread if default occurred before the end of the year. The reference bond can only default in the middle of each year. In the event of default the reference bond will have a recovery rate of 60% of its face value and the CDS payoff will be equal to the difference between the face value of the bond and the bond recovery.

Based on the current market price of the bond, you believe that the conditional probability of default in each of the next two years is 5%. Assume the risk-free rate is 4% on a continuously compounded basis for all maturities and determine the value of the CDS contract to you at this point in time.

Solution:

The calculations here are identical to the calculations you would do to solve for the CDS spread, but now the spread is known and we just want to determine the present value of the payments we expect to receive on the CDS net of the known payments we have to make.

Step 1: Calculate the Unconditional Default Probabilities and Survival Probabilities

Table 14: Default and Survival Probabilities

Time (Years)	Unconditional Default Probability	Survival Probability
1	0.0500	0.9500
2	0.0475	0.9025

Step 2: Determine Protection Buyer's Spread Payments

Table 15: Swap Spread Payments – Annual

Time (Years)	Survival Probability	Spread Payment	Expected Payment	Discount Factor	PV of Expected Payment
1	0.9500	1.70%	0.01615	0.9608	0.01552
2	0.9025	1.70%	0.01534	0.9231	0.01416
Total					0.02968

Table 16: Swap Spread Payments – Accrual

Time (Years)	Default Probability	Accrual Payment	Expected Payment	Discount Factor	PV of Expected Payment
0.5	0.0500	0.0085	0.00043	0.9802	0.00042
1.5	0.0475	0.0085	0.00040	0.9418	0.00038
Total					0.00080

The total expected spread payments are therefore .02968 + .00080 = .03048.

Step 3: Determine the Protection Seller's CDS Payments

Table 17: CDS Payments

Time (Years)	Default Probability	Recovery Rate	Expected Payment	Discount Factor	PV of Expected Payoff
0.5	0.0500	60%	0.0200	0.9802	0.0196
1.5	0.0475	60%	0.0190	0.9418	0.0179
Total					0.0375

Step 4: Determine the Value The net value to us, as a percent of the \$10 million in notional, is:

$$.0375 - .03048 = .00702$$

The resulting value in dollars is \$70,200.

17. **(1 point)** Various regulatory, tax and securities law issues have impeded the growth of the catastrophe bond and risk-linked securities markets. Describe some of the concerns in this regard raised by Cummins.

Solution:

He notes the following:

1. Regulatory Considerations — Cummins would like a more flexible regulatory approach and more clarity on the statutory treatment of risk-linked securities in general.

2. Tax Issues — The key tax considerations are those that affect investors in the bonds. In the case of U.S. investors, despite the fact that the investments are structured as bonds, interest income is generally taxed as if it were a dividend.

3. Securities Laws — Catastrophe bonds are generally sold in the private markets through private placements to accredited or qualified investors only. This limits the amount of publicly available information about catastrophe bonds and reduces the ability of academics to study the market.

18. **(1 point)** Company ABC's CEO has resisted recommendations from his CFO and other senior executives to institute a more extensive program to hedge financial risks that are not within the control of the management team, arguing that it is not in the shareholders best interest for the firm to engage in active risk management. What impact might more hedging have on managers' incentives that could be expected to benefit the shareholders?

Solution:

The volatility caused by financial risks that are outside of the control of the company's managers results in a disincentive for the managers to own large equity stakes in the firm. Allowing the managers to hedge their risks more can offset this disincentive, causing them to own more shares in the firm and thus allowing the shareholders the additional benefit of stronger managerial incentives to focus on increasing the value of the firm.

19. (1 point) Culp, Miller & Neves discuss certain characteristics shared by value risk managers that make VaR a more appropriate risk measure for them than it is for cash flow risk managers?

Solution:

The three characteristics mentioned are:

- Value risk managers are interested in the value of their positions over short periods of time. The short time horizons imply that VaR measurement can be accomplished reliably with minimal concern about changing portfolio composition over the risk horizon.

- Value risk managers need to limit and control their exposures.

- Many value risk managers are involved in agency transactions where they act primarily as intermediaries rather than take their own proprietary positions (i.e. they act on behalf of others rather than for their own account or with their own capital).

20. (1 point) Your company has liabilities that are normally distributed with an expected value of $\$10,000$ and a standard deviation of $\$3,000$. If the asset values are fixed and your total capital is $\$4,000$, what is the expected policyholder deficit?

Solution:

The EPD Ratio under a normal distribution assumption is:

$$d_L = \frac{D_L}{L} = k\phi\left(-\frac{c}{k}\right) - c\Phi\left(-\frac{c}{k}\right)$$

Using the values given,

$$\begin{aligned}
k &= \text{coefficient of variation} = 3,000/10,000 = .30 \\
c &= \text{capital to expected loss ratio} = 4,000/10,000 = .40 \\
c/k &= .4/.3 = 1.33 \\
d_L &= .3\phi(-1.33) - .4\Phi(-1.33) = 1.27\% \\
EPD &= 127
\end{aligned}$$

21. (2 points) You are trying to use the Merton-Perold capital allocation method along with a 2% EPD ratio as the basis for the determination of the capital required for a company. The only two lines of business in this company have a correlation coefficient of $\rho = .15$ and have the following means and standard deviations:

	Line 1	Line 2
Mean	2,000	5,000
Std Dev	200	1,000

Determine the volatility parameter, σ, for the total company. For calculations involving σ parameters, round your values to two decimal places (e.g. 0.40) and feel free to use approximations if needed.

Solution:

To avoid lengthy calculations of the σ parameters we can follow Butsic's lead and approximate this using the coefficient of variation, k.

So first we just calculate the overall standard deviation:

$$\text{Std. Dev} = \sqrt{200^2 + 1,000^2 + 2(.15)(200)(1,000)} = 1,049$$

Then, we can either use the coefficient of variation as an estimate of the volatility for the total firm, $\sigma \approx k = 1,049/7,000 = 14.98\%$. Or, we could calculate the actual lognormal sigma parameter based on the relationship:

$$\sigma = \sqrt{ln(1 + k^2)} = 14.90\%$$

Either way, the question said to round to two decimal places, so the result would be 15%.

As an alternative calculation, note that we could also have used the approach shown in my notes that parallels what Cummins did in the paper and used For Lines 1 and 2, these values are $k_1 = .10$ and $k_2 = .20$.

To get the total volatility parameter, we use the weights based on the relative mean loss, so $w_1 = 2,000/7,000 = 28.57\%$ and $w_2 = 71.43\%$.

The total volatility is then:

$$\begin{aligned}\sigma_T^2 &= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho \\ &= (.2857)^2(.1)^2 + (.7143)^2(.2)^2 + 2(.2857)(.7143)(.1)(.2)(.15) \\ &= .0224 \\ \sigma_T &\approx .15\end{aligned}$$

Notice that if you didn't use the coefficient of variation k as an approximation then the volatility parameter could have been calculated directly as $\sigma = \sqrt{ln(1+k^2)}$, which would have given $\sigma_1 = 9.98\%$ and $\sigma_2 = 19.80\%$. Rounding to two decimal places produces the same values as the coefficient of variation.

22. **(1 point)** Describe how *economic profit* is calculated in Goldfarb's Risk-Adjusted Performance Measurement paper for the purposes of calculating an *ex post* RAROC?

Solution:

In the paper, the profit is measured at the end of one year. The premium, net of up-front expenses, is adjusted to reflect one period of investment income on the net premium and then the claims are subtracted on a discounted basis, with the discounting being done to the end of the year for all payments expected to be made beyond that time horizon.

23. (1 point) Your company has a total of $\$100,000$ in surplus and you write two lines of business with the following characteristics:

	Line A	Line B
Premium	50,000	200,000
Expected Loss Ratio	90%	60%
Expected Loss	45,000	120,000
Avg Time to Payment of Claims (Years)	2	3

Assume the company is in a steady state environment, with the same business being written (same premiums and loss ratios) for the past several years. If surplus is allocated in proportion to reserves, what would be the allocated surplus to each line of business?

Solution:

In a steady state, with Line A losses paid on average in two years, every year the loss reserves for Line A will be equal to two times the expected loss or $\$90,000$. Similarly, the Line B reserves will on average be $\$360,000$. If we allocate the $\$100,000$ in surplus in proportion to loss reserves, we will allocated 20% or $\$20,000$ to Line A and $\$80,000$ to Line B.

24. **(1 point)** You are given the following information for the P&C insurance industry for 2010:

Beginning Surplus	301.0
Ending Surplus	365.0
Stockholder Dividends Paid	20.0
Paid-in Surplus	35.0

Calculate the rate of return on surplus using Roth's calculations.

Solution:

Roth recommends calculating the profit for the industry inclusive of all sources of profit, so he looks at the total change in surplus during the year *before* reductions from dividends paid or additions from paid-in surplus.

Using the amounts shown, the calculation of the income is $dS = 49$ and the return on surplus is 16.3%, as shown below:

Beginning Surplus	301.0
Ending Surplus	365.0
Increase in Surplus	64.0
Add Back Dividends Paid	20.0
Subtract Paid-in Surplus	-35.0
Change in Surplus (dS)	49.0
Return on Surplus	16.3%

25. **(1 point)** Describe McClenahan's recommended approach for regulators to determine whether insurance rate targets are reasonable, not excessive and not inadequate? Is this approach likely to lead to much different results than targets which seek to ensure that shareholders' returns are commensurate with the risk?

Solution:

McClenahan argues that regulators should assess the reasonableness of rates relative to desired market characteristics such as the size of the residual market, the degree of competition and the degree of product diversity and innovation. That is, as insurers compete in the market they will respond through underwriting standards and premium volume to their own assessments of the fairness of rates (from their point of view). Insurers unwilling to write a particular class of risks is a clear indication that, in the eyes of the insurers, rates are inadequate.

Notice that McClenahan's approach ultimately depends on the behavior of the insurers themselves. If the insurers evaluate premiums using return on equity benchmarks, then regulating rates based on simpler return on sales measures may actually not be very different from using return on equity benchmarks. But it will be simpler for regulators.

Note also that when reading the papers on pricing insurance from the investors' perspective, we are typically quantifying *minimum* risk loads sufficient to attract capital and just satisfy shareholders. The implicit recommendation is to charge as much as the policyholder will pay, but no less than the premium determined in this manner. On the other hand, McClenahan's recommendations relate to how regulators should establish *maximum* rates allowed.

26. (1 point) You are evaluating the underwriting profit provision for a given line of business with the following cash flows at the beginning of each period:

Year	Premium	Losses	Expenses	Federal Income Taxes
0	100	0	30	
1	0	50		3
2	0	20		2
3	0	10		1

In this table, the federal income taxes reflect income taxes paid on both underwriting income and investment income.

You have decided to increase the premium and want to set the underwriting profit provision using Robbin's Risk-Adjusted Cash Flow method. You have estimated the following:

Risk-Free Rate	3.00%
Expected Equity Market Risk Premium	6.00%
Equity Beta	0.90
Liability Beta	-0.40

Calculate the premium that should be charged as of the *end* of the year using this method and the Capital Asset Pricing Model. What underwriting profit provision should be charged for this policy?

Solution:

The basic equation for the method is:

$$PV(P; i_f) = PV(L; i_r) + PV(FX; i_f) + PV(FIT; i_f)$$

We can use the risk-free rate to discount the premium, expense and income tax flows (though note that it is reasonable to risk-adjust some or all of those cash flows too). To discount the losses though, we use a risk-adjusted rate that reflects the *negative* underwriting beta:

$$i_r = 3\% + (-.4)[6\%] = .6\%$$

The present value discount factors, at the end of the year, are:

Year	Risk Free	Risk-Adjusted
0	1.030	
1	1.000	1.000
2	0.971	0.994
3	0.943	0.988

And then the premium is:

$$P = 79.76 + 30.90 + 5.88 = 116.55$$

Notice this is the premium as of the end of the year. In the paper, it is implied that the premium as of today would be discounted at the risk-free rate, so $P_0 = 116.55/1.03 = 113.15$.

This is how some old exam questions have been solved, but some people (me, for instance) think that all cash flows should be discounted back to today using the appropriate rates, which would mean the risk-adjusted rate for the liabilities. If we discount to today using the risk-free rate the underwriting profit provision is:

$$U = 1 - 110/113.15 = 2.79\%$$

And it is 4.34% if the losses are discounted to today at the risk-adjusted rate and everything else is discounted to today at the risk-free rate.

27. (2 points) An insurer prices two risks, X and Y, using Mango's Covariance Share risk load methodology, resulting in covariance share for X that is 4.066 times the covariance share for Y.

Their Marginal Variance risk load multiplier assumes a required return of $y = 20\%$ and a surplus requirement corresponding to the 97.73% loss level, or $z = 2.00$.

If the variance of the loss distribution for X is 19,619,900, the variance of the loss distribution for Y is 377,959 and the covariance is 1,450,550, what is allocation of the total risk load, in dollars, to each of these risks?

Solution:

To solve this, we need to determine the total risk load that would be charged using the marginal variance method. From the information given, it is easy to calculate the total portfolio variance as 22,898,959, which is the sum of the individual variances and two times the covariance. The marginal variance method uses a risk load multiplier that depends on y, z and the portfolio standard deviation (4,785.29), which gives us:

$$\begin{aligned}\lambda &= \frac{yz/(1+y)}{\text{Portfolio Standard Deviation}} \\ &= \frac{.20(2.00)/1.20}{4,785.29} \\ &= 0.0000697\end{aligned}$$

This would result in a total portfolio risk load of 1,595.

To determine the allocation of this to each risk, we were told that the covariance shares differed by a multiple of 4.066, but what are the amounts? We know that the two covariance share terms have to add up to two times the covariance, or $2,901,100$, so that tells us that:

$$4.066\text{CovShare}_Y + \text{CovShare}_Y = 2,901,100 \Rightarrow \text{CovShare}_Y = 572,661$$

And from this, $\text{CovShare}_X = 2,328,439$.

Now it is easy to calculate the two risk loads using the same $\lambda = .0000697$ and the sum of the stand-alone variances and the respective covariance shares:

	X	Y
Variance	19,619,900	377,959
Cov Share	2,328,439	572,661
Total	21,948,301	950,658
Lambda	0.0000697	0.0000697
Risk Load	1,529	66

28. (1 point) A colleague of yours is considering using Kreps' methodology for setting risk loads on reinsurance policies and notes that when considering the swap strategy and the put strategy, both of them result in the same minimum assets at the end of the period but the put strategy allows for the potential for additional upside gains if the risky asset returns are higher than the risk-free rate and the swap strategy returns are capped at the risk-free rate. As a result, the put strategy appears to be superior and she doesn't understand why anyone would ever consider the swap strategy. Why is she incorrect that the put strategy is necessarily superior?

Solution:

While the put strategy rate of return has a minimum value equal to the risk-free rate, it invests less of the funds because it has to use some of them to buy the put option. So when comparing the returns on the sum of the premium and shareholder capital provided, the put strategy downside is actually capped at a rate *below* the risk-free rate. The potential in the put strategy to earn higher than the risk-free rate is therefore offset by the lower minimum total return.

To clarify, assume you use the swap strategy and determine that the premium should be $P = \$10$ and the shareholder capital should be $A = \$90$. If the risk-free rate is 4% then the minimum ending value is $1.04(10 + 90) = 104$. That is also the maximum ending value.

But suppose the put cost associated with a particular risky asset is \$5. Now, instead of investing \$100 we will only have \$95 to invest and therefore while the rate of return cannot be lower than the risk free rate of 4%, the ending asset value in this case is actually only $1.04(95) = 98.8$. As a result, the put strategy does have its downside risk reduced, but not as much as the swap strategy does. While there is upside potential in the put option strategy, it isn't clear that it is necessarily "better".

Cumulative Normal Distribution (Positive x)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Cumulative Normal Distribution (Negative x)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Practice Exam 4

This practice exam is intended to contain exam-style questions for a representative subset of the Learning Objectives.

Section Weights

Questions from each part of the syllabus are included, with weights generally consistent with the weights specified on the syllabus.

Time Limit

The actual exam has a time constraint that typically poses a real challenge for students. Nonetheless, I have decide to not specify a time limit for the practice exams in order to encourage you to carefully consider each of these questions and take whatever time is needed to fully formulate your responses. It would be a mistake to skip practice questions due to time constraints.

Points Per Question

Points have been allocated for each question, but the vast majority of the questions are worth just one point. This is because I have tried to make most of the questions comparable in terms of difficulty and time needed to perform calculations, so I did not vary the points much.

How to Grade Yourself

You should target scoring 85% or more to fully prepare for the actual exam.

1. **(1 point)** An investor's complete portfolio consists of two components: a risky portfolio with an expected return of 15% and a standard deviation of 20%, and a risk-free asset with expected return equal to 5%. If the standard deviation of the complete portfolio is 30%, what is the expected return on the complete portfolio.?

Solution:

We can use the standard deviation of the complete portfolio to determine the proportion of his portfolio that is invested in the risky portfolio:

$$\sigma_c = y\sigma_p$$
$$30\% = y(20\%)$$
$$\Rightarrow y = 1.50$$

Knowing that the investor has 150% of his portfolio value in the risky portfolio and has therefore borrowed 50% of his portfolio value, the expected return is:

$$\begin{aligned} E(r_c) &= yE(r_p) + (1-y)r_f \\ &= r_f + y[E(r_p) - r_f] \\ &= 5\% + 1.50[15\% - 5\%] \\ &= 20\% \end{aligned}$$

2. **(1 point)** You are planning to split your portfolio between two risky index funds with the following characteristics:

	Stock Fund	Bond Fund
Expected Return	11%	7%
Standard Deviation	13%	6%
Correlation Coefficient	0.20	

You cannot borrow or lend and you want your portfolio to have an expected return of 10%. What weights in the stock and bond funds would allow you to achieve this with the lowest possible total portfolio standard deviation?

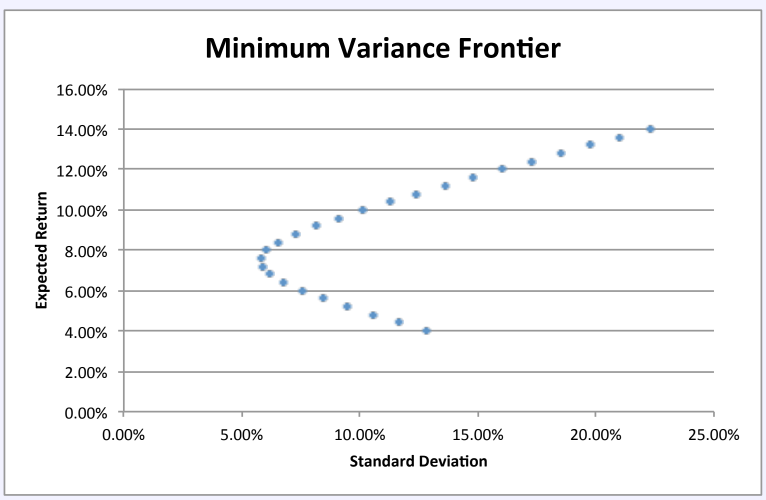
Solution:

When you cannot borrow or lend, and there are only two risky assets to choose from, all of your risk and return combinations will plot along a minimum variance frontier given by the following two equations:

$$E(r_p) = w_S E(r_S) + w_B E(r_B)$$

$$\sigma_p = \sqrt{w_S^2 \sigma_S^2 + w_B^2 \sigma_B^2 + 2 w_S w_B \sigma_S \sigma_B \rho}$$

When plotted in this case, the minimum variance frontier will look as follows:



Each point in this case reflects a different set of weights and in each case the point represents the lowest possible standard deviation for any level of expected return. This means that all we need to do is solve for the weight that gives us a 10% expected return:

$$
\begin{aligned}
10\% =& w_S 11\% + (1 - w_S)7\% \\
\Rightarrow w_S =& 75\% \\
w_B =& 25\%
\end{aligned}
$$

In this case, plugging in these weights gives a portfolio standard deviation of 10.16%, which visually is consistent with the graph shown.

3. **(1 point)** Suppose that the universe of risky assets from which you can choose from is such that the expected returns are all 10%, the standard deviations are all equal to 20% and the correlation coefficients for any pair of assets is .4. You currently have \$100 invested in one of these assets in your portfolio and are considering diversifying your risk by adding additional stocks to your portfolio. Describe what happens to the standard deviation of your portfolio in dollar terms if you add \$100 to each of four additional assets and what you would have to do in order to cause the standard deviation of your portfolio of five risky assets to decrease relative to your original standard deviation.

Solution:

You don't technically have to do any calculations here.

The *descriptive* answer is that adding four additional assets to the portfolio will cause the standard deviation to increase in proportion to the square root of the number of assets so that the ratio of expected return to standard deviation (or if the risk-free rate were subtracted from the expected return the Sharpe ratio) will increase. This is the effect of risk pooling, which does not necessarily decrease risk (it would if the correlations were negative) but does improve the risk-return relationship.

To decrease risk, what you would have to do is keep the size of your investment portfolio constant at \$100 and spread that investment over more assets, investing \$20 in each of five assets. This is known as risk sharing and it will have the effect of both decreasing the risk and improving the risk-return relationship.

To see this with numbers, we can use the following formula in this simple case where the variances and covariances are all equal:

$$\sigma = \sqrt{\frac{1}{N} \text{ Avg Variance} + \frac{N-1}{N} \text{ Avg Covariance}}$$

For N=1 this is simply $\sigma = 20\%$ and in dollar terms since the portfolio is worth \$100 this is $\sigma_\$ = \$20$. Since the expected return in dollars is 10%, this is a ratio of return to standard deviation of .5.

For N=5 this formula gives us $\sigma = 14.4\%$. In dollar terms, if we increase the size of the portfolio to \$500 this gives us a standard deviation in dollar terms of $\sigma_\$ = \$72$. The expected return is now \$50 though so the ratio of expected return to standard deviation has increased to 0.694.

But if we had kept the total portfolio value the same the standard deviation in dollars would have been $\sigma_\$ = \$14.4$ and the ratio of expected return to standard deviation would have been 0.694.

Notice that the ratio of expected return and standard deviation are the same under either risk pooling or risk sharing, but in the latter case the standard deviation actually decreases.

4. **(1 point)** You have collected the historical monthly returns in excess of the risk free rate for Stock A, Stock B and the S&P 500 for the past 60 months and estimated the following regression coefficients:

Table 18: Regression Results

$R_i = \alpha + \beta\ R_m + \epsilon$	$\hat{\alpha}$	$\hat{\beta}$	$\sigma^2_{\epsilon_i}$
Stock A	0.005	0.92	0.002
Stock B	-0.002	1.15	0.001

Discuss why you might not want to use these regression results along with estimates of the expected excess market return to estimate the expected return next month for Stock A or Stock B.

Solution:

It is certainly true that you could take the expectation of both sides of the regression equation and write it as:

$$E(R_i) = \alpha + \beta\ E(R_m)$$

However, we shouldn't always assume that the historical beta is the best estimator of the future beta, in part due to estimation error and in part due to the fact that betas are unlikely to remain constant over time. In fact, betas tend to regress towards the mean (1.0) over time.

One way to address this is to use a Bayesian credibility adjustment to the estimator with a prior estimate of the average beta for all stocks of 1.0. For instance, in one example in the textbook they calculated an adjusted beta as:

$$\text{Adjusted } \beta = \frac{2}{3}\hat{\beta} + \frac{1}{3}(1.0)$$

Another way to address this is to model the relationship between past betas and future betas.

5. **(1 point)** What are the key assumptions in the standard CAPM that are modified in the model discussed by Bodie, Kane and Marcus which includes Liquidity Betas?

Solution:

CAPM ignores transaction costs such as bid-ask spreads and liquidity costs such as the impact that an investor's buying or selling will have on the price or the costs associated with the inability to trade quickly.

Some of the effect of this will be mitigated through self-selection (investors who plan to trade frequently will tend to buy stocks with low transaction costs and investors who plan to trade infrequently will tend to buy the stocks with higher transaction costs), but the more important effect is that *changes* in liquidity will introduce additional sources of risk. In essence, investors will evaluate the covariance of net returns on an asset with net returns on the market:

$$\text{Adjusted } \beta = \frac{Cov(r - c, r_m - c_m)}{Var(r_m - c_m)}$$

When expanded, this term results in the standard market beta plus three additional liquidity betas.

6. **(1 point)** Describe the concept of market efficiency, including the three major forms.

Solution:

If a market is efficient then prices will reflect all currently available information. There are three versions of this:

- Weak Form — Current prices reflect all information that can be obtained from examining past price history, trading volume, etc. All of this information is easily available and virtually costless to obtain, so if this provided any insight into future prices it would be reflected in the current prices.
- Semi-Strong Form — All publicly available information regarding the prospects for the firm is reflected in the stock price.
- Strong Form — All information, including information known only by company insiders, is reflected in the current prices.

7. **(1 point)** Identify three main categories of behavioral issues which arguably impact market efficiency and describe their implications.

Solution:

The three major categories are:

1. *Information processing errors* make it difficult for investors to accurately assess probabilities and evaluate risk.

2. *Behavioral biases* make it difficult for investors to rationally evaluate risky opportunities using a simple rule to maximize their expected utility. Instead, they fall back on simple rules of thumb or are influenced by more subtle factors, such as regret avoidance.

3. *Limits to arbitrage* make it difficult for investors to exploit the mistakes made by others, allowing market prices to deviate from their fundamental value for long periods of time.

8. **(1 point)** A colleague is considering using the CAPM to evaluate expected returns on risky assets but is concerned about a number of academic studies that have identified problems such as:

- the failure of statistical tests of the slope and intercept of the Security Market Line
- numerous instances of anomalies in average returns relative to CAPM beta
- reports of an equity premium puzzle

Provide a response to each of these concerns that would support the argument that the CAPM may still be a valid model of expected returns on risky assets.

Solution:

While there is growing evidence of the need for additional risk factors besides the market return in CAPM, the three concerns listed above are not necessarily fatal to the CAPM, for the following reasons:

- the use of a poor proxy for the theoretical market portfolio could be a driver in the poor statistical test results
- anomalies (at least some of them) could be the result of data mining, trolling through years of data to find spurious results with no economic significance
- survivor bias could play a role in the high average return on the US stock market, which lies at the heart of the equity premium puzzle

9. (1 point) You are given the following continuously compounded yields on zero-coupon risk-free bonds. Determine the forward rate between years three and four.

Maturity (Years)	Yield
1	4.00%
2	4.40%
3	4.60%
4	4.80%

Solution:

Solve the following for the forward rate, R_F:

$$
\begin{aligned}
e^{4R_4} &= e^{3R_3}e^{(4-3)R_F} \\
R_F &= \frac{4R_4 - 3R_3}{4-3} \\
&= 5.4\%
\end{aligned}
$$

10. (1 point) ABC is an industrial company that is looking to borrow at floating rates of interest that will be correlated with the cyclicality of their business. Another company, XYZ, with a lower credit rating prefers not to introduce uncertainty with respect to their interest obligations and would prefer to borrow at fixed rates. The following table shows the interest rates that each company would have to pay if they borrowed in each of the floating rate and the fixed rate markets:

Table 19: Borrowing Costs for ABC and XYZ

	ABC	XYZ
Fixed Rate	7.20%	9.4%
Floating Rate	LIBOR + .2%	LIBOR + 1.2%

Assume that you represent a financial institution that is willing to enter into swaps with either ABC or XYZ and that you will charge a 10 basis point (.1%) fee in each case. Determine a set of swaps that you could enter into with each of ABC and XYZ that will make each of them better off and afford them an equal amount of savings relative to borrowing solely on their own.

Solution:

To see whether there is any opportunity here we need to see where each company has a comparative advantage:

Table 20: Determining Comparative Advantage

	ABC	XYZ	ABC's Absolute Advantage
Fixed Rate	7.20%	9.40%	2.20%
Floating Rate	LIBOR + .2%	LIBOR + 1.2%	1.00%
Comparative Advantage			**1.20%**

Notice that ABC can pay lower rates in either market, however they pay "more less" in the fixed rate market. This gives them a comparative advantage in the fixed rate market.

Using the swap market, ABC can borrow in the fixed rate market, where they have a comparative advantage and then enter into a swap with XYZ to convert their payments to floating rate payments. The two parties will then be able to split the savings equal to the comparative advantage.

Or, as in this case, they could each enter into swaps with a financial intermediary and the only difference is that the fees they pay to the intermediary reduce the savings available for them to split. Here, there are 120 basis points that can be saved but if the intermediary wants to earn 20 basis points in total, ABC and XYZ will split the remaining 100 basis points. The question directed us to assume they will split it equally.

1. ABC borrows in the market where it has the comparative advantage (fixed market) and enters into a swap to receive the fixed rate and pay the floating rate.

2. ABC determines what it would have paid in the floating rate market (LIBOR + .2%) and subtracts the negotiated share of the savings (50 basis points) to determine what it wants to ultimately pay (LIBOR - .30%).

3. ABC enters into a swap where they receive the full fixed rate payment they have to make (7.2%) and pays the target floating rate payment (LIBOR - .30%).

4. The terms of the swap can be simplified by noting that if we add .3% to both the fixed and floating payments the swap is equivalent to receiving 7.5% fixed and paying LIBOR.

We then do the same for XYZ, but note that we can avoid these steps if we want because we know the intermediary wants to have the same terms with XYZ as with ABC, except for a net fee of 20 basis points in total and so we already know the terms of the XYZ swap have to be pay 7.7% fixed and receive LIBOR.

But let's do the detailed steps just to be sure.

1. XYZ borrows in the market where it has the comparative advantage (floating market) and enters into a swap to receive the floating rate and pay the fixed rate.

2. XYZ determines what it would have paid in the fixed rate market (9.4%) and subtracts the negotiated share of the savings (50 basis points) to determine what it wants to ultimately pay (8.9%).

3. XYZ enters into a swap where they receive the full floating rate payment they have to make (LIBOR + 1.2%) and pays the target fixed rate (8.9%).

4. The terms of the swap can be simplified by noting that if we subtract 1.2% from both the fixed and floating payments the swap is equivalent to receiving LIBOR and paying a fixed rate of 7.7%.

11. **(1 point)** You have a liability of \$25 million that is due in 3 years and have chosen to fund it with a risk-free coupon bond. Describe how you would choose from among a variety of coupon bonds of varying coupon rates and maturities so that you are immune to small, instantaneous increases in interest rates and will still be able to meet your liability obligation in three years.

Solution:

When you are concerned about the value of a bond portfolio at a future date (in this case you want to be sure that you can pay the liability in 3 years), you should ensure that the Macaulay duration of the bond portfolio matches the Macaulay duration of the liability, or more generally, the target date you are concerned about. It the Macaulay duration of the bond in this case is 3 years then increases in interest rates will result in additional reinvestment income from the coupons that just offsets the loss in the value of the bond at the target date.

12. (1 point) An insurance company has outstanding reserves as of 12/31/2010 equal to $100 million which it expects to pay out according to the following payment pattern:

Year	% Paid
1	50%
2	30%
3	20%

If the risk-free rates are 4% for all maturities and the current yield on the company's investment portfolio is 5% (both on an annually compounded basis), how would Feldblum calculate the Macaulay duration of the loss reserves?

Solution:

Feldblum merely calculated the Macaulay duration the same way it is done for corporate bonds, except that he made two specific assumptions. First, he assumed that all payments were made in the middle of the year. Second, he assumed that the proper rate to use to value the liability and calculate the duration was the investment portfolio yield rather than the risk free rate.

Note that Feldblum used the company's investment portfolio yield only because he thought it was better to use highly rated bond yields rather than "risk-free" yields (in a more recent paper he recommends LIBOR rates). The company's assets are not really relevant to the determination of the liability duration unless they happen to have the same maturity and risk profile as the liabilities.

The calculations are as follows:

Time	% Paid	Amount Paid	PV Factor	PV Paid	T*PV Paid
0.5	50%	50	0.9759	48.80	24.40
1.5	30%	30	0.9294	27.88	41.82
2.5	20%	20	0.8852	17.70	44.26
				94.38	110.48
				Macaulay Duration	1.17

13. (1 point) An insurance company currently prices its policies using a target return of 10% above the risk free rate, or $k = 10\% + y$ where $y = 5\%$ is the risk free rate. It also expects to write the same business in perpetuity but with a 95% retention rate each year. It is trying to minimize the effect that changes in interest rates will have on the total economic value of the firm and is considering two options.

One option is to reduce its asset duration to zero by shifting into very short term risk-free investments. The other option is to alter its pricing strategy so that it charges more premium as interest rates change. Specifically, it is considering establishing the target return on surplus of $k = a + by = 5.28\% + 1.945y$.

Assuming the following calculations have been made, which strategy will produce the lowest amount of interest rate sensitivity for the total economic value of the firm?

	Amount	Modified Duration
Assets	126.19	1.000
Liabilities (PV)	71.43	0.952
Current Economic Value of Surplus	54.76	1.062
Franchise Value	45.24	10.952

Solution:

The information given can be used to calculate the economic value of the firm (54.76 + 45.24 = 100.00) and its duration:

$$\text{Duration of Economic Value} = \frac{(1.062)(54.76) + (10.952)(45.24)}{100.00} = 5.5363$$

If we could reduce the asset duration to zero, we could recalculate the duration of the current economic value of surplus as $[126.19(0) - 71.43(0.952)]/54.76 = -1.24$ and from this we could recalculate the duration of the total economic value as 4.275. This is a modest reduction.

However, notice that if we altered the pricing strategy, the current target return would remain at $k = 15\%$ but now future premiums would be sensitive to changes in rates and so the duration of the franchise value will fall. The exact formula for the duration of the franchise value is:

$$\begin{aligned} D &= \frac{a - b + 1}{(1 + y)(a + by - y)} + \frac{1}{1 + y - cr} \\ &= \frac{5.28\% - 1.945 + 1}{(1.05)(5.28\% + 1.945(.05) - .05)} + \frac{1}{1 + .05 - .95} \\ &= 1.507 \end{aligned}$$

Then, using this the duration for the total economic value would be 1.264.

In this case, changing the firm's pricing policy is more effective than changing the asset duration to zero.

14. **(1 point)** You own a corporate bond with $\$1,000$ face value, 6% coupons paid semi-annually and 4 years to maturity. You know that in the event of default the recovery rate will be 40% of the par value of the bond and are trying to calculate the loss given default (LGD) in the event that default occurs at time $T = 3$, right before the coupon is paid. What is the LGD in this case if the risk-free rate is 7% on a continuously compounded basis.

Solution:

The LGD is the difference between the risk-free value of the bond and the recovery amount at the time of default. Right before the coupon is paid at time $T = 3$ the present value of the remaining coupons and principal amount is:

$$G = 30 + 30e^{-.07(.5)} + 1030e^{-.07(1.0)} = 1,019.33$$

The LGD is then \$619.33.

15. **(1 point)** Your boss has come to you asking you to explain the role that Collateralized Debt Obligations (CDOs) played in the 2007-2009 credit crisis. He has heard that investors significantly underpriced the risks in structured finance tranches associated with subprime mortgages, but doesn't fully understand why this may have been the case. How would you describe the causes of this underpricing?

Solution:

There are three points to note:

- Sensitivity to Assumptions for CDOs – One point to note is that CDOs, and in particular CDOs of tranches of asset-backed securitizations or CDO-squareds, had default probabilities and expected cash flows that could be substantially more sensitive to assumptions than was often recognized. Any intentional or unintentional bias in the underlying asset default and recovery assumptions would have resulted in materially biased default and recovery rates for the securitized tranches.

- Special Characteristics of Subprime Mortgage Bonds – For CDOs that contained subprime mortgages, a key issue was how sensitive these underlying assets were to home price appreciation. Particularly for the so-called liar loans or the NINJA (no income, no job or assets) loans, in the absence of home price appreciation these mortgages were almost certain to default. This significantly impacted not only the reliability of default rate assumptions tied to recent historical default rates, but it also caused the underlying asset default rates to be much more highly correlated.

- Pricing for Systematic Risk – The effect of securitization, and particularly resecuritization, is to load the senior-most tranches with systematic risk. In this case, bonds that effectively serve as economic catastrophe bonds ought to be priced with substantial risk margins by most investors. Investors relying solely on ratings, which don't provide any indication of the degree of systematic risk, and yield spreads for more plain-vanilla bonds, would almost certainly underprice the risk associated with these tranches.

16. **(1 point)** You had been asked to determine the spread on a 3-year CDS on a zero coupon bond. After observing the market price of the bond and making the assumption that the recovery rate in the event of default would be 40%, you estimate a constant conditional default probability of 2% per year and determine the CDS spread should be $s = 1.7\%$.

    You later found out that market participants believe that the recovery rate in the event of default would actually be zero. Explain, without doing calculations, how this would affect your estimate of the fair credit default swap spread.

    Solution:

    Had the calculations been done with the recovery rate assumptions used by other market participants you would have calculated a higher default probability because the key determinant of both of these variables is the market price of the bond, which is a known amount. Had you then used these higher probabilities and the revised recovery rate to price the credit default swap, the calculations would have been approximately the same.

    In other words, the pricing of CDS contracts depend on both the default rate and the recovery rate assumptions. So long as you take the market price as given, changing the assumption regarding recovery rates alters the default probability assumption but not the pricing of the CDS when the same default and recovery assumptions are used.

17. **(1 point)** What factors does Cummins suggest could further facilitate the growth of the catastrophe bond market?

Solution:

He notes the following in his conclusion:

1. Improved reporting of insured losses could facilitate the creation of more robust loss indices, particularly outside of the U.S.

2. If regulatory capital requirements acknowledged the counterparty credit risk associated with reinsurance recoveries then fully collateralized catastrophe bonds would get a boost.

3. Personal lines insurance rates should be deregulated and more credit should be given to insurers who lock-in multi-year reinsurance coverage.

4. ERISA rules impacting catastrophe bonds should be explored.

18. (1 point) Stulz argues that Value at Risk (VaR) cannot be used over longer time horizons and recommends a different risk measure. Describe his concerns about VaR and his recommended alternative.

Solution:

Since VaR focuses on outcomes in the 99th percentile, longer time periods mean that there is substantially less data available to estimate model parameters or test the model. In addition, VaR as it is traditionally implemented relies on normal distributions.

He instead prefers Cash Flow Simulations that can identify if and when firms fall into financial distress (e.g. when their credit rating falls below some level). This allows them to see the path of firm value during a period of time, rather than just the distribution of ending values. It also facilitates sensitivity analysis and Monte Carlo simulations that can easily incorporate non-normal distributions and serial correlations.

19. (1 point) You own a portfolio of two newly issued bonds, one rated Aa and one rated Baa. Both bonds are zero coupon bonds maturing in 20 years and both have a face value of \$10 million. In the event of default the recovery rate on both bonds is 40% and the recovery is paid at the scheduled maturity date even if default occurs sooner. The two bonds have default correlation of .30.

You have also collected the following historical default rates for bonds with similar ratings:

Table 21: Average Cumulative Default Rates (%), 1970-2003

	1	2	3	4	5	10	20
Aa	0.02	0.03	0.06	0.15	0.24	0.68	2.70
Baa	0.20	0.57	1.03	1.62	2.16	5.10	12.59

You are trying to simulate the default experience for these bonds so you generate the random numbers shown below using a Gaussian copula for one specific simulation trial (out of $10,000$ total trials):

	Aa	Baa
Independent Uniform Random Numbers	0.060	0.220
Independent Normal Random Numbers	-1.555	-0.772
Dependent Normal Random Numbers	-1.555	-1.203
Dependent Uniform Random Numbers	0.060	0.114

Determine the cash received at maturity for the entire portfolio in this particular simulation trial.

Solution:

The table of random numbers contains the steps needed to apply the Gaussian copula model to generate two dependent uniform random numbers, as shown in the last row.

Using these numbers, we simulate a default event if the dependent random numbers are less than or equal to the respective default rates at each point in time.

For the Aa bond, we generated a random number of $U_{Aa} = .06 = 6\%$. Now we just look and see what year of default corresponds to this cumulative value and find that there is only a 2.7% probability of default before year 20 and so in this trial we would say that this bond did not default and that we receive the full face value at maturity, or \$10 million.

For the Baa bond, our random number is $U_{Baa} = .114 = 11.4\%$. Again, we scan the historical default rates and see that this value occurs between years 10 and 20 and conclude that this bond defaulted prior to maturity. At maturity, we therefore collect only the 40%

recovery rate, or $4 million.

In this simulation trial, we would have collected $14 million.

20. (1 point) Your company has liabilities that are log-normally distributed with an expected value of $\$10,000$ and a standard deviation of $\$3,000$. If the asset values are fixed and your total capital is $\$4,000$, what the expected policyholder deficit?

Solution:

For risky liabilities and fixed assets, the formula for the EPD ratio is:

$$d_L = \Phi(a) - (1+c)\Phi(a-k)$$

where, k is the coefficient of variation ($k = .30$), c is the capital to expected loss ratio ($c = .40$) and $a = \frac{k}{2} - \frac{\ln(1+c)}{k} = -0.97157$.

$$d_L = \Phi(-0.97157) - (1.40)\Phi(-0.97157 - .3) = 2.31\%$$

The question asked for the EPD, so multiplying by the expected loss we have $EPD = 232$.

21. (1 point) You are using the Merton-Perold method with a 2% EPD ratio target to allocate capital between two lines of business that have a correlation coefficient of 60% and the following means, standard deviations, coefficient of variation and lognormal sigma (volatility) parameter on a stand-alone basis and in total:

	Line 1	Line 2	Total
Mean	2,000.00	5,000.00	7,000.00
Std Dev	400.00	1,000.00	1,280.62
Coefficient of Variation	20.00%	20.00%	18.29%
Sigma	19.80%	19.80%	18.12%

Using the table of EPD ratios below determine the amount of capital allocated to each line of business using the Merton-Perold method. Round your volatility parameter to two decimal places but linearly interpolate within the table of EPD ratios if necessary.

c	$\sigma = .18$	$\sigma = .20$
30.00%	0.66%	1.01%
27.50%	0.83%	1.23%
25.00%	1.04%	1.48%
22.50%	1.29%	1.79%
20.00%	1.60%	2.15%
17.50%	1.97%	2.57%
15.00%	2.41%	3.06%

Solution:

The MP method starts with the total capital requirement and then determines the impact of removing one line of business from the total.

Here, using the $\sigma = 18\%$ column of the table we can see that the 2% EPD ratio target is achieved when $c = 17.33\%$ (with linear interpolation). This is the capital to liability ratio, so the total capital required would be $C_T = 17.33\% * 7,000 = 1,213$.

If we remove Line 1, then the firm only has Line 2 and on a stand-alone basis that would require capital as a percentage of expected loss equal to $c = 21.02\%$ based on the $\sigma = .20$ column of the table and interpolating. This is equivalent to a capital amount of $1,051$. Since removing Line 1 reduces the capital required to achieve a 2% EPD ratio by $1,213 - 1,051 = 162$, this is the amount allocated to Line 1.

The same calculations for Line 2 produce a capital allocation of 792.

22. **(1 point)** A new company was formed to write two lines of business and it was determined that, in order to satisfy a variety of constraints and objectives, a total of \$10 million of capital would be needed to support the market, credit and prospective underwriting risks. This amount represents the 99.5% CTE for the whole company. Using the following information regarding the Co-CTE risk measures and the *actual* results (as of the end of the year) for these two lines of business calculate the *ex post* RAROC for each line.

	Line A	Line B
99.5% Co-CTE	3,000,000	6,000,000
Premium	6,400,000	6,400,000
Expense Ratio	10.00%	15.00%
Discounted Loss Ratio	90.00%	81.00%
Investment Income Yield	5.00%	5.00%

Solution:

First, we calculate the economic profit for each line of business, as of the end of the year. For each line the net premium is invested for one year and the resulting economic profits for Line A and Line B are $288,000$ and $528,000$, respectively.

The RAROC calculation will now depend on how we allocate capital to these two lines. We are told that the total capital needed is \$10 million but that reflects all of the risks of the company. For RAROC purposes, we only want to include the capital allocated to the lines of business for their respective underwriting risk, which in this case we can reasonably use their respective Co-CTE amounts.

This gives RAROC for Line A of $288,000/3,000,000 = 9.6\%$ and RAROC for Line B of $528,000/6,000,000 = 8.8\%$.

23. **(1 point)** Robbin's IRR on Equity Flows method for setting the underwriting profit provision is usually calculated using Statutory income statements and balance sheets, as well as GAAP income statements and balance sheets. Nonetheless, Robbin says that the IRR on Equity Flows "is not a GAAP return or a Statutory return." What did he mean by this?

Solution:

When using this method it is important to recognize that both the Statutory and GAAP balance sheets must be sound and so both accounting standards are used to ensure that the equity flows reflect the constraints of both systems. But there is indeed only one measure of the actual flows between the company and its shareholders. We apply both sets of rules to establish the constraints but then estimate only one IRR.

It is also worth noting that because of the simplifications Robbin used in his paper, he didn't actually have any difference between the Statutory and GAAP equity flows. He only considered the effect of the Deferred Acquisition Costs and assumed that those had to be added to the required Statutory surplus to arrive at the required GAAP equity. This exactly offset the impact on the flows to investors from the UW income.

24. (1 point) What two reasons are given by Roth why regulators should not allocate surplus, and by extension the investment income on surplus, to lines of business or states?

Solution:

The appropriate amount of aggregate surplus is unique to each insurer based on all of its risks, so leverage ratios can vary considerably by insurer. And since all of the surplus an insurer stands behind all of its risks, it cannot be allocated to line or state in a realistic fashion.

25. (1 point) Briefly describe how Robbin calculates the company's expected after-tax investment yield in his CY Offset, PV Offset and CY ROE methods.

Solution:

Using the most recent calendar year data he uses actual dividend rates, in the case of stocks, or actual interest yields, in the case of bonds, along with realized capital capital (he excludes unrealized capital gains). To get this on an after-tax basis he performs the calculations separately by asset class so that differences in tax treatment (e.g. the dividends received deduction) can be reflected.

Robbin acknowledges that new money rates, or current yields on bond investments, are more theoretically sound, but determining expected returns including capital gains on bonds or expected returns on equities, real estate, etc. can be very difficult to do and to justify.

26. **(1 point)** You are trying to determine the premium to charge for a line of business using Robbin's Present Value Offset method and are given the following facts:

Undiscounted Expected Loss	100,000
Fixed Expense	3,000
Variable Expense (% of Premium)	15%
Annual Investment Yield	4.00%
Traditional Underwriting Profit Provision	5%

The reference line and the reviewed line loss payment patterns are as follows, with all payments made in the middle of each year:

Year	Reference Line	Reviewed Line
1	80%	60%
2	20%	20%
3	0%	20%

Calculate the underwriting profit provision using the Present Value Offset Method.

Solution:

The easiest (and clearest) way to solve this is to write the expression for the full premium in terms of the present value loss, expense and adjusted profit provision in dollars:

$$P = L[PV(X)] + FX + VX(P) + U^0(P) + L[1 - PV(X^0)]$$

Here, we can calculate the present value of the reference line payment pattern, using the 4% investment yield, is $PV(X^0) = 0.973$ and the present value of the reviewed line payment pattern is $PV(X) = 0.958$.

Then, the formula above is:

$$\begin{aligned} P &= L[PV(X)] + FX + VX(P) + U^0(P) + L[1 - PV(X^0)] \\ P(1 - VX - U^0) &= L[PV(X)] + FX + L[1 - PV(X^0)] \\ P &= \frac{100,000(.958) + 3,000 + 100,000(1 - .973)}{1 - 15\% - 5\%} \\ &= 126,901 \end{aligned}$$

Note that it would be tempting to try to use the formula for the adjusted underwriting profit provision given in the reading:

$$U = U^0 - (PLR)[PV(X^0) - PV(X)]$$

but this would require knowledge of the permissible loss ratio and hence the premium we are trying to calculate.

Given the result we just calculated for the premium, we could indeed calculate the loss ratio as 78.8% and then use that to find the adjusted underwriting profit provision:

$$U = 5\% - (78.8\%)[.958 - .973] = 3.834\%$$

Then, the premium is simply:

$$\begin{aligned} P &= \frac{L + FX}{1 - VX - U} \\ &= \frac{103,000}{1 - 15\% - 3.834\%} \\ &= 126,901 \end{aligned}$$

27. (1 point) Kreps proposed setting the risk load for a reinsurance policy and the amount of capital raised to support the policy in such a way so that, for the investor, the expected return of writing reinsurance and investing the premium and capital funds is no less than the expected return from investing directly in risky assets and that two risk constraints are satisfied. Briefly describe the two alternative financial strategies used by Kreps for investing the assets.

Solution:

Kreps' goal is to use a financial strategy, along with writing a reinsurance contract, that ensures that the total funds invested are not exposed to the risk of earning less than the risk free rate of return. His two methods are:

1. Swap Risk-Free Investment for Risky Investment — Invest the funds, comprising the premiums charged (net of expenses) plus the required capital investment, in risk free investments rather than in risky assets.

2. Purchase Put Options — Invest in risky assets but also purchase a put option with a strike price equal to the value of the funds grown at the risk free rate. This eliminates the downside risk from the investment in risky assets.

28. (1 point) Suppose you were going to use Kreps' put option strategy in the case where the risk-free rate is 3.6%, the risky investment yield is 5.4% and its standard deviation is 8.4%. What would be the cost of the put option per dollar of invested assets, where the invested assets are denoted F?

Solution:

Under this strategy, the put option has a strike price equal to the forward value of the invested assets. As a consequence, in this special case the formula for the value of the put option is relatively simple:

$$\text{Option Cost} = F\left[\Phi\left(\frac{\sigma\sqrt{t}}{2}\right) - \Phi\left(\frac{-\sigma\sqrt{t}}{2}\right)\right]$$

As a rate, we just divide by the amount invested to get:

$$r = \Phi\left(\frac{\sigma\sqrt{t}}{2}\right) - \Phi\left(\frac{-\sigma\sqrt{t}}{2}\right)$$

But there's a complication associated with calculating this from the inputs given. It is tempting to use 8.4% as the value of the σ variable above. However, that quantity given is the yield standard deviation, σ_y, and not the lognormal distribution parameter we need.

Reflecting the assumption that $(1+y)$ is lognormally distributed, we can calculate the σ parameter as:

$$\sigma = \sqrt{\ln\left(1 + \left(\frac{\sigma_y}{1+y}\right)^2\right)} = 0.0796$$

From this, the put option cost as a rate is:

$$\begin{aligned} r &= \Phi\left(\frac{\sigma\sqrt{t}}{2}\right) - \Phi\left(\frac{-\sigma\sqrt{t}}{2}\right) \\ &= \Phi\left(\frac{0.0796}{2}\right) - \Phi\left(\frac{-0.0796}{2}\right) \\ &= 0.0318 \end{aligned}$$

Notice that there are two approximations to keep in mind, which might be useful:

$$\sigma \approx \frac{\sigma_y}{1+y} \approx \frac{.084}{1.054} = .0797$$

$$r \approx \frac{1}{\sqrt{2\pi}}\sigma\sqrt{t}[1 - \frac{\sigma^2}{24}] \approx .4\sigma\sqrt{t} = .4(.0796) = .03185$$

These might be able to save you some time on the exam.

Cumulative Normal Distribution (Positive x)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Cumulative Normal Distribution (Negative x)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Practice Exam 5

This practice exam is intended to contain exam-style questions for a representative subset of the Learning Objectives.

Section Weights

Questions from each part of the syllabus are included, with weights generally consistent with the weights specified on the syllabus.

Time Limit

The actual exam has a time constraint that typically poses a real challenge for students. Nonetheless, I have decide to not specify a time limit for the practice exams in order to encourage you to carefully consider each of these questions and take whatever time is needed to fully formulate your responses. It would be a mistake to skip practice questions due to time constraints.

Points Per Question

Points have been allocated for each question, but the vast majority of the questions are worth just one point. This is because I have tried to make most of the questions comparable in terms of difficulty and time needed to perform calculations, so I did not vary the points much.

How to Grade Yourself

You should target scoring 85% or more to fully prepare for the actual exam.

1. **(1 point)** An investor is considering how to allocate his investments between a risk-free asset with an expected return of 4% and a risky portfolio with an expected return of 16% and a standard deviation of 25%. How should he allocate his investments to these two choices so that he maximizes his utility? Assume he has a utility function of the form $U = E(r) - .5A\sigma^2$ and a coefficient of risk aversion $A = 4$.

Solution:

To maximize the utility, we simply take the derivative of the utility function with respect to y and set it equal to zero. In order to write the utility function in terms of y though, we need to first plug in the formulas for $E(r)$ and σ in terms of y into our formula for utility.

In symbols,

$$\begin{aligned} U &= E(r) - .5A\sigma^2 \\ &= r_f + y[E(r_p) - r_f] - .5A(y\sigma_p)^2 \end{aligned}$$

The derivative is then equal to:

$$\frac{dU}{dy} = [E(r_p) - r_f] - 2(.5)Ay\sigma_p^2 = 0$$

which can be solved for the optimal allocation to the risky portfolio as:

$$\begin{aligned} y^* &= \frac{[E(r_p) - r_f]}{A\sigma_p^2} \\ &= \frac{16\% - 4\%}{(4)(25\%)^2} \\ &= 48\% \end{aligned}$$

2. **(1 point)** You are planning to split your portfolio between two risky index funds with the following characteristics:

	Stock Fund	Bond Fund
Expected Return	11%	7%
Standard Deviation	13%	6%
Correlation Coefficient	0.20	

You can borrow or lend at the risk-free rate of $r_f = 3\%$ and you want your portfolio to have an expected return of 10%. What weights in the stock and bond funds would allow you to achieve this with the lowest possible total portfolio standard deviation?

Solution:

When you can borrow or lend, the lowest standard deviation for any level of expected return can be achieved by combining the *optimal risky portfolio* with the risk-free asset. Since the question asked only for the weights in the stock and bond funds, the solution is found by simply identifying the weights in the optimal risky portfolio:

$$w_S = \frac{[E(r_S) - r_f]\sigma_B^2 - [E(r_B) - r_f]\sigma_{SB}}{[E(r_S) - r_f]\sigma_B^2 + [E(r_B) - r_f]\sigma_S^2 - [E(r_S) - r_f + E(r_B) - r_f]\sigma_{SB}}$$

In this case, the risk premiums and the covariance are:

$$E(r_S) - r_f = 8\%$$

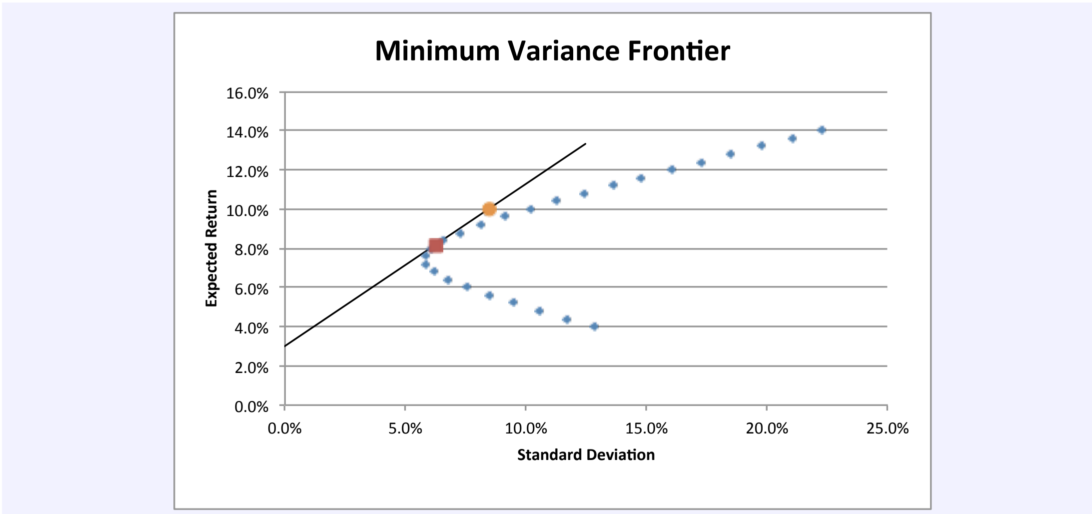
$$E(r_B) - r_f = 4\%$$

$$\sigma_{SB} = .00156$$

The resulting weights are therefore:

$$\begin{aligned} w_S &= \frac{(.08)(.06^2) - (.04)(.00156)}{(.08)(.06^2) + (.04)(.13^2) - (.08 + .04)(.00156)} \\ &= 29\% \\ w_B &= 71\% \end{aligned}$$

The graph shows this optimal risky portfolio, which is the tangent point, as well as the complete portfolio that achieves the target 10% expected return. As you can see, the standard deviation of this complete portfolio (8.44%) is lower than the standard deviation of the point on the minimum variance frontier at the same level of expected return.


Minimum Variance Frontier
Expected Return
Standard Deviation
16.0%
14.0%
12.0%
10.0%
8.0%
6.0%
4.0%
2.0%
0.0%
0.0%
5.0%
10.0%
15.0%
20.0%
25.0%

3. (1 point) Assume a Single Index Model for stock returns applies with a common factor, M, and the following parameters for Stock 1 and Stock 2:

$$R_1 = .02 + 1.1\ R_M + e_1$$

$$R_2 = -.01 + .9\ R_M + e_2$$

Further assume the standard deviation for the common factor is $\sigma_M = .25$ and that the standard deviation of the firm-specific errors are $\sigma_{e_1} = .40$ and $\sigma_{e_2} = .30$.

What is the covariance of returns between Stock 1 and Stock 2?

Solution:

The single index model assumes that returns are driven by a common factor, R_M above. With firm-specific random components e_i independent, the covariance between the two stocks is:

$$\sigma_{ij} = \beta_i \beta_j \sigma_M^2 = (1.1)(.9)(.25^2) = 6.19\%$$

4. **(1 point)** What are the differences between the Capital Allocation Line, the Capital Market Line and the Security Market Line?

Solution:

The Capital Allocation Line (CAL) shows the relationship between risk and return for various combinations of any arbitrary risky portfolio and the risk-free asset.

The Capital Market Line (CML) is one particular CAL where the risky portfolio is the *market portfolio*. If all investors use the same estimates and all follow the Markowitz procedure, they will all choose to use the market portfolio, levered up or down with the risk free asset, to form their complete portfolios and so the CML will depict the relationship between expected return and standard deviation for *any* portfolio held by investors.

The Security Market Line is the relationship between the beta for any asset and its expected return, which holds so long as the assumptions underlying the CAPM hold.

5. **(1 point)** You have been asked to estimate the expected return on a well diversified portfolio of risky assets. You intended to use the Capital Asset Pricing Model (CAPM) for this task until your boss expressed doubts about the validity of that model. He is concerned that the key assumptions underlying that model seem unrealistic and has asked you to consider using Arbitrage Pricing Theory (APT) instead. He is particularly concerned with the reliance of the CAPM on the Markowitz portfolio selection model.

   Identify three assumptions behind the CAPM that relate specifically to the Markowitz portfolio selection procedure that might be legitimate concerns and identify three assumptions that are needed to develop the APT model instead.

Solution:

There are a number of assumptions underlying the CAPM that do not relate specifically to the Markowitz procedure, such as the assumption that investors care only about their wealth at the end of a single period and that their wealth consists solely of marketable securities. However, three assumptions specifically related to the Markowitz procedure are:

- all investors use the Markowitz procedure to optimize the relationship between the mean and variance of returns for their investment portfolio
- all investors can borrow or lend, without limitation, at the risk-free rate
- all investors have identical and perfect information, without measurement error, regarding the means, variances and covariances of asset returns

Instead of making any of these assumptions, the APT assumes the following:

- a *factor model* describes the actual returns for all risky assets such that return surprises consist of exposure to n common factors across all assets and an idiosyncratic component that is unique to each asset and which can be diversified away in large portfolios
- arbitrage opportunities do not exist in equilibrium
- investors can buy or sell any risky asset in any quantity without limitation, including selling short

6. **(1 point)** Describe how both technical analysis and fundamental analysis can affect market efficiency.

Solution:

Technical Analysis involves the study of past stock prices in the search for recurring and predictable patterns. Since past price information is readily available, intense competition to uncover patterns will likely eliminate the trends the moment they become evident. In other words, if technical analysis were capable of discovering trends, people would quickly attempt to exploit the trends by either buying before the price rises or selling before the price falls. This will eliminate the lag from the start of the trend to the final resolution and effectively eliminate the pattern that was being detected. In the end, patterns will not be readily visible and the market will appear to be efficient in the sense that prices will reflect the information content (whatever it may be) in past prices.

Fundamental Analysis involves the valuation of risky assets based on constantly updated estimates of all future cash flows and their associated risks. By carefully forecasting the earnings, cash flow and dividend prospects for the firm and taking full consideration of all available information, fundamental analysts hope to uncover instances where stock prices deviate from this fundamental value and to exploit those differences. Because investors using this approach are always trying to identify underpriced or overpriced stocks, buying the underpriced ones and selling the overpriced ones, at any point in time it will be quite difficult to find obvious examples of underpriced or overpriced securities. This will make the market appear efficient in the sense that prices will reflect relevant information available.

7. (1 point) Briefly describe four examples of information processing errors.

Solution:

The four discussed in the textbook are:

1. Forecasting Errors — People tend to give too much weight to recent evidence and tend to produce extreme forecasts. This could lead to excessive earnings forecasts and cause high P/E stocks to subsequently under-perform.

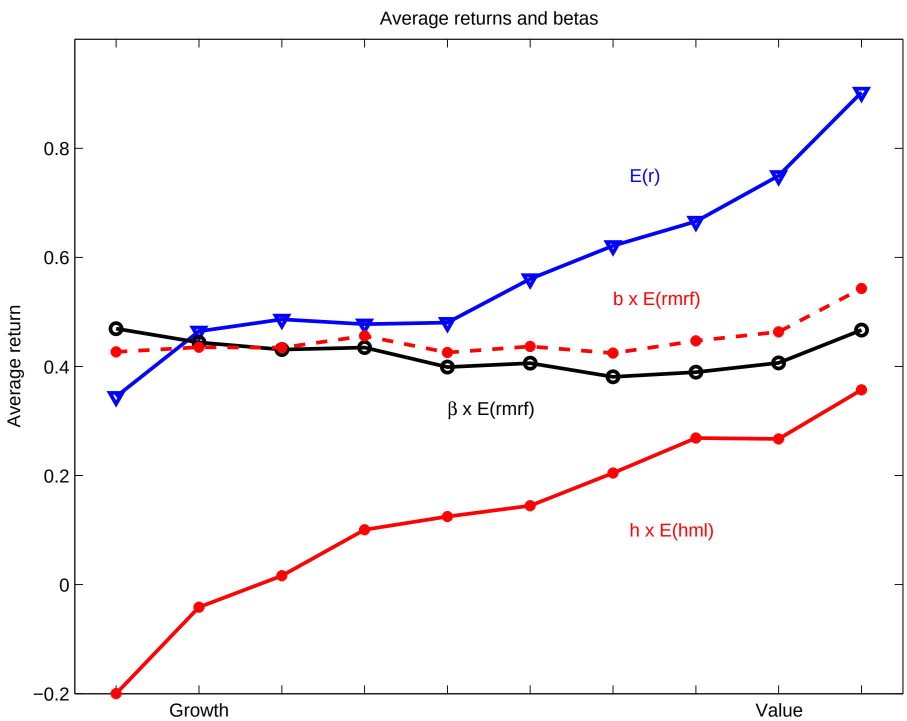
2. Overconfidence — People tend to exhibit extreme overconfidence and overestimate their abilities.

3. Conservatism — Investors are often slow to update their prior beliefs in the face of new information.

4. Sample Size Neglect and Representativeness — People tend to infer patterns from limited information.

8. (1 point) The graph below shows the average returns on ten portfolios, ranked based on their book-to-market ratio (with low book-to-market portfolios considered "growth" and high book-to-market portfolios considered "value"). It also shows the expected returns you would have expected based on the standard CAPM, depicted as the product of the CAPM beta and the excess market return over the risk free rate and denoted $\beta x E(rmrf)$ in the graph. The relatively flat line based on the CAPM is inconsistent with the pattern of the average returns (denoted $E(r)$ ion the graph), which seem to rise steadily as the book-to-market ratios rise.

Also shown in the graph is the product of the portfolios' sensitivity to the Fama-French HML index, denoted h, and the average return on the HML index, denoted $E(hml)$. The pattern of this line, denoted $hxE(hml)$ seems to match up very well with the average returns on the portfolios, supporting the idea that average returns are driven more by this HML factor than by the CAPM market portfolio factor.



Provide both a risk-based and behavioral interpretation of these results.

Solution:

The risk-based interpretation is that the HML index serves as a proxy for a risk factor that investors care about such that portfolios or assets with high HML factor loadings, high h, have high average returns. For instance, the average risk premiums on the HML index seem to be positively correlated with business cycles (changes in the growth rate of the overall economy) so a high *h* loading could indicate assets or portfolios that are very sensitive to changes in the macro-economy or more are more sensitive to the effects of recessions. This sensitivity may serve as an additional risk factor that investors demand higher returns to be exposed to. The key to this argument is that high book-to-market firms, so-called value firms, are riskier because they are more sensitive to a business cycle factor than low book-to-market firms, so-called growth firms, and it is this additional source of systematic risk that results in expected risk premiums.

The behavioral interpretation is that investors naively or erroneously bid up the price (pay too much) for growth stocks relative to value stocks. This causes their returns to lag behind the returns on value stocks that were not subject to this error, producing a positive return differential between them. Evidence that this occurs comes from the fact that low book-to-market firms have higher *past growth rates but not higher future growth rates than those of high book-to-market firms, suggesting investors are making errors of representativeness (assuming high past growth means high future growth). In addition, investors tend to, on average, be more disappointed with earnings announcements of growth stocks as compared to value stocks, further suggesting undue optimism.*

9. (2 points) You are given the following information for two risk-free bonds that pay coupons annually:

	Maturity (Years)	Annual Coupon	Face Value	Price
Bond A	1	4%	1,000.00	1,000.00
Bond B	2	5%	1,000.00	1,009.36

If the expectations hypothesis holds, determine the expected future one-year rate one year from today on an annually compounded basis.

Solution:

If the expectations hypothesis holds then the forward rate is equal to the expected one year rate (the short rate) one year from now.

To find the forward rate, we need the one-year zero coupon rate and the two-year zero coupon rate.

The one year rate is easy because it is simply the yield on the one-year bond. Although it has a coupon payment, there is just a single cash flow equal to the coupon and the face value in one year. So we just solve the following for the one-year yield, y_1:

$$1,000 = \frac{1,040}{1 + y_1} \Rightarrow y_1 = 4\%$$

Notice that because the price was equal to the par value, we could have known immediately that the yield was equal to the coupon rate and avoided any calculations.

Next, we need to find the two-year zero coupon yield. This is the rate that sets the price of Bond B equal to the present value of its cash flows when the one-year rate is used for the cash flow at the end of year one and the two-year rate is used for the cash flow at the end of year two:

$$1,009.36 = \frac{50}{1.04} + \frac{1050}{(1 + y_2)^2} \Rightarrow y_2 = 4.513\%$$

Finally, we can calculate the forward rate as:

$$(1 + y_2)^2 = (1 + y_1)(1 + f) \Rightarrow f = 5.028\%$$

10. **(1 point)** Three months ago you entered into an interest rate swap whereby you agreed to pay a fixed rate of 4% with semi-annual compounding and receive floating rate payments equal to 6-month LIBOR, also with semi-annual compounding. The notional principal of the swap was $10 million, the maturity of the swap was 18 months, with the next payment dates in 3 months, 9 months and 15 months.

At the inception of the swap, the 6-month LIBOR rate was 4.4% with semi-annual compounding and currently the continuously compounded LIBOR zero rates are 5% for all maturities.

Determine the current value of the swap in terms of the values of fixed and floating rate bonds, ignoring any complications resulting from day count or business day conventions.

Solution:

To value the swap, you can view this as a long position in a floating rate bond and a short position in a fixed rate bond.

For the fixed rate bond, the coupons are 4% and the principal is $10 million and so the cash flows are simply $4\%/2(10,000,000) = 200,000$ in 3 months and 9 months and then $10,200,000$ in 15 months. The value of this is found by discounting at the continuous LIBOR rates:

$$\begin{aligned} B_{fix} &= 200,000e^{-.05(.25)} + 200,000e^{-.05(.75)} + 10,200,000e^{-.05(1.25)} \\ &= 9,972,168 \end{aligned}$$

For the floating side, the rate for the next payment has already been set at $4.4\%/2 = 2.2\%$ and the cash flow is simply $2.2\%(10,000,000) = 220,000$. We also know that on the next reset date, the floating rate bond will be worth par, or $10 million. Therefore, the value today is simply:

$$\begin{aligned} B_{fl} &= 10,220,000e^{-.05(.25)} \\ &= 10,093,045 \end{aligned}$$

The resulting swap value is then $B_{fl} - B_{fix} = 120,877$.

11. (1 point) You are the Chief Investment Officer for your company and are concerned about the interest rate sensitivity of your reported net worth. You currently have liabilities valued at \$250 million with a modified duration of 4.0 and a bond portfolio valued at \$300 million with a modified duration of 4.5. Within the bond portfolio, \$250 million is invested in a long maturity bond with a modified duration of 5 and \$50 million is invested in a coupon bond with a modified duration of 2.0.

Identify a strategy that you could follow to lower the duration of your asset portfolio without selling either of the bonds in the portfolio.

Solution:

Lowering the duration of the bond portfolio would be easy if we could sell one or both of these bonds and replacing them with lower duration bonds. However, this is not an option here and so instead we can enter into an interest rate swap, agreeing to pay a fixed rate of interest and receive a floating rate of interest, that would effectively convert the bond(s) into floating rate bonds with duration of approximately zero. The magnitude of the duration depends on the frequency of the coupon reset dates, since on each reset date the bond will be worth par but between reset dates the price can fluctuate slightly.

Note that if we enter into a swap at the current market swap rate we will not require any up-front investment and will not require us to sell either of our existing bonds.

12. (1 point) You are the Chief Investment Officer for a P&C insurer that writes both property and liability lines of business. You have been charged with establishing an investment policy that reduces the interest rate sensitivity of the company's overall surplus. What arguments could be given to support the assertion that both property and liability reserve durations are low?

Solution:

Property reserves are paid out quickly and so they have low durations.

Liability reserves are inflation sensitive and thus have interest rate sensitivity comparable to low duration assets.

13. **(1 point)** Panning suggests that rather than altering the composition and duration of the invested asset portfolio firms could use their pricing strategy, as defined by how sensitive their target return on surplus is to changes in the risk free rates, to effectively manage the interest sensitivity of the total economic value of the firm, including the franchise value. What does he say is key virtue of this approach with respect to rating agencies and regulators?

Solution:

The concern is that the strategies that would have to be used to manage the duration of the total economic value through changes in the asset duration would likely lead to apparent duration mismatch and to the use of complex derivative securities that would possibly even appear to increase risk. This occurs because the franchise value is invisible to the rating agencies and regulators and therefore its effect is ignored.

A key advantage of using pricing strategy instead results from the fact that implementing a pricing strategy is nearly as invisible to these external audiences as the franchise value it is intended to protect.

14. (1 point) Explain why default probabilities calculated using historical data and default probabilities implied by either current bond prices or equity prices (based on the Merton Model) are generally different. Identify which estimates are typically larger and provide an explanation for these differences.

Solution:

Default probabilities implied by bond or equity prices are *risk-neutral probabilities* which reflect both the true default probability as well as one or more risk adjustments that are impossible to disentangle from the default probability.

Risk-neutral estimates are larger because of one or more of the following sources of positive risk-adjustments:

- Liquidity Premium — Corporate bonds are relatively illiquid and so there may be a liquidity risk premium, as discussed in Altman.
- Conservatism — Bond traders rely on subjective default probabilities that exceed the historical estimates, allowing for scenarios worse than what had been experienced over the time period used to measure the historical default probabilities.
- Systematic Risk and Contagion — The fact that default rates fluctuate, perhaps systematically, gives rise to a source of risk that traders want to be compensated for.
- Skewness — Bond returns are skewed (limited upside, large downside) and therefore it is more difficult to diversify the risk in a bond portfolio.

15. (1 point) Prior to 2007, policymakers often stated that the rise of securitization has made the world safer by transferring risks, especially credit risks, to the capital market participants most capable of measuring, managing and absorbing those risks. But the CDO market grew so substantially that when problems arose in that market the impact was bigger than most people thought was possible. List five reasons why the demand for CDOs (and related products) was so high in the years leading up to the credit crisis of 2007-2009.

Solution:

Five reasons given by Coval, et. al. were:

1. Seemingly Attractive Yields
2. Extreme Market Optimism
3. Too Little Appreciation for Fragility of Ratings
4. Perverse Incentives for Rating Agencies
5. Perverse Incentives for Banks

16. **(1 point)** The credit crisis of 2007-2009 began with higher than expected default rates in the relatively small subprime mortgage market, but quickly turned into a global crisis in which it became impossible to borrow money (i.e. obtain credit) in almost any market. Briefly outline the chain of events which led to this.

Solution:

Many descriptions exist, but Hull outlined the crisis as follows (in Chapter 8 of his Eighth Edition):

- The housing price bubble distorted the default experience on mortgages.
- Lenders began to relax their lending standards (liar loans, NINJA loans, adjustable rate and other exotic payment provisions, high LTV ratios)
- Housing bubble bursts in 2006/2007, causing a surge in subprime defaults, losses in junior tranches of CDOs containing subprime mortgages and huge mark-to-market losses on senior tranches.
- Downgrades of AAA-rated CDO tranches caused urgent needs to post collateral, which was obtained through fire sales of just about everything and massive price declines in many markets with no subprime connection.
- Large losses for banks holding securities intended to go into CDOs and for banks that had been dependent on rolling over short-term financing, which was suddenly unavailable because of the inability to post high-quality collateral.
- With the securitization market dead and liquidity scarce, spreads increased for all forms of debt at all maturities.

17. (1 point) Stulz argued that companies should directly manage financial risk, as opposed to leaving it up to their shareholders to do on their own, only if it can reduce real costs of financial distress. What does he say the goal of the company's risk management activities should be and what costs does he say this might reduce?

Solution:

Stulz argues that the goal of risk management should be to reduce the likelihood of "lower tail outcomes", which are events that could cause the company to become financial weak and lead to one or more of the following costs:

- Bankruptcy Costs, which include both the direct costs (e.g. legal expenses) of actually filing for bankruptcy as well as indirect costs such as the loss of flexibility or the inability to raise funds for new investments;
- Payments to Stakeholders, which includes employees (who might demand higher wages for the risk of losing their jobs) or customers (who might be reluctant to do business with the firm if there's a risk that the products will not be supported);
- Taxes, which are effectively higher when earnings are volatile because losses only result in tax credits that can be used to offset future profits.

18. **(1 point)** Stulz argues that firms that decide to take risks need to risk-adjust their expected returns. What method does he suggest can be used to do this?

Solution:

Stulz suggests managing risk taking activities using abnormal returns — i.e. returns in excess of the risk free rate — as a measure of the expected profitability of certain activities. Selective risk management then can be accomplished by allocating capital on a risk-adjusted basis and limiting capital at risk accordingly. To measure the risk-adjusted capital allocation, he suggests using the cost of new equity issued to finance the particular activity.

19. (1 point) You own a portfolio of two AA-rated bonds whose default correlation is .30.

You have used historical ratings data to estimate the following rating transition probabilities for Aa-rated bonds:

Table 22: Credit Rating Transition Probabilities (%)

	Rating at Year-End							
Initial Rating	AAA	AA	A	BBB	BB	B	CCC	D
AA	0.66	91.72	6.94	0.49	0.06	0.09	0.02	0.01

You are trying to simulate the rating for each of these bonds at the end of one year so you generate the random numbers shown below using a Gaussian copula for one specific simulation trial (out of $10,000$ total trials):

	Bond 1	Bond 2
Independent Uniform Random Numbers	0.945	0.220
Independent Normal Random Numbers	1.598	-0.772
Dependent Normal Random Numbers	1.598	-0.257
Dependent Uniform Random Numbers	0.945	0.399

Determine the rating for each bond in this particular simulation trial.

Solution:

The table of random numbers contains the steps needed to apply the Gaussian copula model to generate two dependent uniform random numbers, as shown in the last row.

To determine the end-of-period rating we can restate the rating transition table to show the cumulative probability that the bond has a rating better than or equal to the rating in each column:

Table 23: Cumulative Transition Probabilities

Initial Rating	AAA	AA	A	BBB	BB	B	CCC	D
AA	0.66	91.72	6.94	0.49	0.06	0.09	0.02	0.01
Cumulative	0.66	92.38	99.32	99.81	99.87	99.96	99.98	100.00

For the first bond, the simulated value is $U_1 = 0.945$. Moving across the cumulative row in the table above, we see that we have gone past the AA column but not past the A column, so we would conclude that for this trial the bonds is rated A.

For the second bond, the simulated value is $U_2 = 0.399$ and using the same logic the rating for this trial would be AA.

20. (1 point) You have used a 1.25% EPD ratio target in order to establish capital requirements for your bond portfolio, stock portfolio and loss reserves. There are no other sources of risk for this company. Use the following information and Butsic's square root rule to approximate the total capital required.

Capital Required for 1.25% EPD Ratio			
Bonds	200		
Stocks	400		
Loss Reserves	1,200		
Correlation Matrix			
	Bonds	Stocks	Loss Reserves
Bonds	1.00	0.25	0.50
Stocks	0.25	1.00	0.20
Loss Reserves	0.50	0.20	1.00

Solution:

The square root rule for non-independent risk elements calculates the total capital required as:

$$C = \sqrt{\sum C_i^2 + \sum_i \sum_{j \neq i} \rho_{ij} C_i C_j}$$

This is a simple formula, except for the fact that we have to reverse the signs of the correlation coefficients for elements on different sides of the balance sheet, so we will use correlation coefficients as follows:

Correlation Matrix			
	Bonds	Stocks	Loss Reserves
Bonds	1.00	0.25	-0.50
Stocks	0.25	1.00	-0.20
Loss Reserves	-0.50	-0.20	1.00

$$C = \sqrt{200^2 + 400^2 + 1,200^2 + 2(.25)(200)(400) + 2(-.50)(200)(1,200) + 2(-.20)(400)(1,200)}$$

$$= 1,117$$

21. (1 point) Cummins reviews several capital allocation methods, including using Value at Risk and the EPD Ratio one line of business at a time. What weakness do these methods share that can be corrected with either the Merton-Perold or the Myers-Read method and how do these two alternatives differ from each other?

Solution:

Both the Merton-Perold and Myers-Read methods are *marginal* methods in the sense that they reflect the impact of a particular line on the entire firm. Both can be used with the EPD ratio as the capital standard, but they differ in how the measure the marginal impact.

The Merton-Perold approach measures the impact on the capital required to achieve a target EPD ratio from completely adding or removing an entire line of business, whereas the Myers-Read method measures the impact of a small change in the size of the business.

The effect of this difference is that the marginal capital amounts under Merton-Perold do not add up to the total capital requirement under the same standard (i.e. the same target EPD ratio). The Myers-Read method does lead to additive capital allocations. Since most decisions are more likely to involve adding or removing small amounts of a business (writing one more policy, non-renewing one account) Cummins seems to prefer the Myers-Read method.

22. (1 point) You are writing a line of insurance for which the fixed expenses will be $\$320,000$, the expected investment yield is 5% and the expected discounted claim costs (discounted to the end of the year) is $\$5,862,400$. You have also determined that a fair risk-based allocation of capital for this line of business is $4,225,340$ and that this allocation is independent of the amount of premium charged. Determine the premium that would achieve a 25% expected RAROC.

Solution:

Using P to represent the unknown premium, we can set the RAROC equal to 25% and solve for P:

$$\begin{aligned} 25\% &= \frac{[P - 320,000](1 + 5\%) - 5,862,400}{4,225,340} \\ P &= \frac{25\%(4,225,340) + 320,000(1.05) + 5,862,400}{1.05} \\ &= 6,909,271 \end{aligned}$$

23. (3 points) You are trying to determine whether the premium you plan to charge for a policy meets your internal rate of return (IRR) target of 15% and have made the following assumptions:

Premium	100
Loss	60
Expense	30
Investment Yield	5%
Initial Surplus (% of Written Premium)	40%
Surplus (% of Loss Reserves)	50%

Note: The initial surplus requirement is 40% of written premium. Beginning at the end of the first year on-going surplus requirements are 50% of loss reserves.

In addition, you have the following information:

- All of the premium is earned in the first year.
- All of the premium is paid at inception.
- The losses are paid at the end of the first, second and third years according to the following pattern: 20%, 50%, 30%.
- All of the expenses are incurred at inception (rather than deferred).
- All of the expenses are paid at the beginning of the first and second year according to the following pattern: 50%, 50%.
- There are no income taxes.

Calculate the IRR for this policy.

Solution:

The tedious part of this problem is to prepare the income statement, cash flow statement and balance sheet. The earned, incurred and paid amounts are as follows:

Year	Earned Premium	Paid Premium	Incurred Loss	Paid Loss	Incurred Expense	Paid Expense	UW Income
0	0	100	0	0	30	15	-30
1	100	0	60	12	0	15	40
2	0	0	0	30	0	0	0
3	0	0	0	18	0	0	0

From this we can prepare the balance sheet, noting that the invested assets plus the premiums receivable (which are zero here) has to equal the sum of the surplus and all of the reserves. Investment income is calculated as 5% of the beginning invested assets (and not the average assets in this case because the question clearly said that the losses are paid at the end of the year rather than evenly throughout the year):

Year	UEPR	Loss Reserve	Expense Reserve	Surplus	Total Liab and Surplus	Premium Receivable	Invested Assets	Investment Income
0	100	0	15	40	155	0	155	
1	0	48	0	24	72	0	72	7.75
2	0	18	0	9	27	0	27	3.60
3	0	0	0	0	0	0	0	1.35

And then the total flows to and from the shareholders can be calculated. The inflows consist of the up-front expenses (which someone has to contribute because they are fully expensed and therefore reduce capital) plus the initial surplus requirement. The outflows consist of the UW profit each period and the decrease in the surplus required:

Year	UW Income	Investment Income	Total Income	Change in Surplus	Equity Flow
0	-30	0.00	-30.00	40	-70.00
1	40	7.75	47.75	-16	63.75
2	0	3.60	3.60	-15	18.60
3	0	1.35	1.35	-9	10.35

Finally, the IRR is calculated using the equity flow column. Using your financial calculator (this is for the TI BA II Plus, but the others would be similar), you would enter the following:

- Clear the cash flow registry by pressing 2nd CE/C.
- Enter the initial cash flow by pressing CF and then −70.0 ENTER.
- Enter the cash flow amounts and the number of times each cash flow occurs. In this case, we will use the default frequency of 1 for each cash flow and just enter the amounts. For the next cash flow, enter 63.75 ENTER and then ↓ ↓. The first down arrow accepts the default payment frequency of 1 and the second one moves to the next cash flow registry entry.
- Enter the remaining cashflows as $xx.xx$ ENTER ↓ ↓
- Then compute the IRR with the entries IRR CPT

You should get an IRR equal to 22.59%.

24. (1 point) A stock P&C insurer's financial statements showed the following amounts (in millions of dollars):

Income Statement Items		
Net Underwriting Gain or Loss	(2,500)	
Net Investment Income	3,200	
Net Realized Capital Gains or Losses	300	
Federal Taxes	180	
Balance Sheet Items	Beginning	Ending
Net Unrealized Capital Gains or Losses	8,500	8,700
Non-Admitted Assets	120	130
Liability for Reinsurance	400	440

Calculate the total economic income as recommended by Roth.

Solution:

The calculation of the net income should be easy, just add the income items and subtract the taxes. The tricky part is how to reflect the balance sheet reconciliation items since Roth is vague about the signs of those items in his exhibit. In some cases he adds the increase during the year and sometimes he adds the decrease during the year.

Net Underwriting Gain or Loss	(2,500.00)
plus Net Investment Income	3,200.00
plus Net Realized Capital Gains or Losses	300.00
minus Federal Taxes	180.00
= Net Income	820
Change in Net Unrealized Capital Gains or Losses	200
Change in Non-Admitted Assets	-10
Change in Liability For Reinsurance	-40
Total Economic Income	970

To clarify the signs of the entries above, note the following:

- Change in Net Unrealized Capital Gains or Losses – Increases are not included in the measurement of net income but they do appear in the valuation of the assets, so the increase has to be added to surplus to reconcile the balance sheet.

- Change in Non-Admitted Assets – When you use cash to buy a non-admitted asset such as furniture, your admitted assets decline but there is no income statement impact. As a result, the increase in the non-admitted "asset", which doesn't appear on the balance sheet, has to be subtracted from the surplus account to reconcile the balance sheet.

- Change in Liability For Reinsurance (Schedule F Penalty) – Increases in this provision do not impact the measurement of net income, but they are added to a "liability" entry on the balance sheet and therefore have to be removed from the surplus account.

  For example, assume we earn \$100 of premium, incur \$80 of gross loss and cede \$30 to an unauthorized reinsurer. Net income will show \$50 of income. However, when calculating the balance sheet entries, the asset rise by \$100 and the liabilities rise by \$80 since we can't recognize the \$30 cession (it will appear as \$50 net loss reserve and \$30 provision for reinsurance on the balance sheet). This causes surplus to rise by only \$20. In order to reconcile the \$50 of net income with the actual change in surplus we have to subtract the \$30 increase in the provision for reinsurance.

25. **(2 points)** Robbin discussed a Present Value Cash Flow model for determining underwriting profit provisions which he specified as:

$$PV(\Delta EQ; r) = PV(TCF; i)$$

where ΔEQ is the change in equity or surplus each period, TCF is the total cash flow from underwriting and investment of the surplus, r is the target return on equity, i is the discount rate for the total cash flows.

He further noted that the present value of the change in equity (the left-hand side) is equal to the present value of the stream of equity balances multiplied by the target return, r.

Discuss why this method is conceptually identical to the "EVA" method in Goldfarb's Risk-Adjusted Performance Measurement paper and identify three ways in which the Robbin formula differs from the EVA approach.

Solution:

Ignoring any formulas, note that this method is simply saying that if you estimate the "cost" in terms of the required rate of return on equity each period and multiply that by the equity balances, the present value essentially represents the total profit the shareholders want to earn on the capital they provide, taking into account the fact that they are making a multi-period capital commitment. That is the left-hand side.

Then, the underwriting profit is simply set so that the present value of all of the underwriting cash flows, including investment of the surplus, is at least equal to that target profit.

But that is conceptually the same as what the EVA method does. The EVA method multiplies the allocated capital each period by the target risk-adjusted return on capital (RAROC) and sets the premium so that the economic profit (reflecting discounting losses) is at least equal to that amount.

But there are some subtle differences:

- In the EVA approach presented in the RAROC paper, we used risk-adjusted allocated capital rather than actual surplus, and so the dollar amounts may vary in practice relative to what Robbin presented.
- Robbin's total required profit is calculated on a present value basis (as of the end of the first year) using the target return as the discount rate, whereas the EVA approach as it was presented in the RAROC paper discounted those amounts by a risk-free rate.
- The EVA approach as presented in the RAROC paper did not explicitly include the investment income on surplus in the measurement of economic profit, as Robbin

does, but it also was careful to define the target return on capital as an amount in excess of the risk-free rate, which is equivalent to assuming the surplus is invested risk free.

26. (2 points) A reinsurer is trying to determine the risk loads for two property-catastrophe reinsurance policies:

	A	B	Total
Mean	358	588	946
Std Dev	615	2,393	2,698
Variance	377,959	5,724,740	7,279,465

They have estimated the correlation coefficient between the two policies is $\rho = .4$, the target return on marginal surplus is 20% and the required capital is calculated at the 99.9th percentile. Using the *build-up* case where each risk is added sequentially to the portfolio, with A first and B second, and the Marginal Surplus method presented in Mango's paper, determine the risk loads for each policy.

Solution:

To apply the Marginal Surplus method, we can use the following formula where y is the required return on marginal surplus, z is the standard normal z-value at the target 99.9th percentile, S is the standard deviation of the portfolio before adding the risk and S' is the standard deviation of the portfolio after adding the risk.

$$\text{MS Risk Load} = \frac{yz}{1+y}(S' - S)$$

To use this formula, we can use the standard normal CDF table to find the value for $z = \Phi^{-1}(99.9\%) = 3.09$.

To determine the risk loads, start with policy A. Since this is the first one in the portfolio, its marginal standard deviation is the same as its stand-along standard deviation. Plugging in the values:

$$\text{MS Risk Load for } A = \frac{.20(3.09)}{1.20}(615) = 317$$

For B, we do the same but now the marginal standard deviation is the difference between the complete portfolio standard deviation, $S' = 2,698$ and the original standard deviation before adding B, or $S = 2,698 - 615 = 2,083$. The risk load is then:

$$\text{MS Risk Load for } B = \frac{.20(3.09)}{1.20}(2,083) = 1,072$$

27. **(1 point)** Kreps proposed setting the risk load for a reinsurance policy and the amount of capital raised to support the policy in such a way so that, for the investor, the expected return of writing reinsurance and investing the premium and capital funds is no less than the expected return from investing directly in risky assets and that two risk constraints are satisfied. Briefly describe the two risk constraints imposed which help to determine the amount of capital that must be raised to support the policy.

Solution:

The two constraints are:

- Safety Constraint — The safety constraint requires that the funds available to pay claims at the end of the year are at least equal to a specified loss safety level. This can be thought of as a constraint the policyholder might demand.

- Investment Variance Constraint — The variance constraint requires that the expected return of the combined reinsurance and investment strategy be no more volatile than a direct investment of the capital raised in the risky assets. This can be thought of as a constraint the shareholder (capital provider) might demand.

28. (2 points) You are trying to apply Kreps' put option strategy, where you invest in a risky asset with an expected return of y but also purchase a put option with a strike price equal to the value of the funds invested at the risk-free rate.

You have determined the following:

- Expected loss, $\mu_L = 1$ million
- Loss standard deviation, $\sigma_L = 2$ million
- Loss safety level at the 99.9th percentile, $s = 22,548,347$
- Risk-free rate, $r_f = 3.6\%$
- Risky asset expected return, $y = 5.3\%$
- Risky asset standard deviation, $\sigma_y = 8.4\%$
- Put option cost, $r = .0318$
- Put-protected expected return, $i = 7.84\%$
- Standard deviation of put-protected investment, $\sigma_i = 5.64\%$
- Capital required under safety constraint, $A = 21,340,147$
- Capital required under variance constraint, $A = 32,266,602$
- Covariance between investment return and insurance losses is zero.

Determine the risk load that would be charged under the put option strategy.

Solution:

Because so much information was provided, the formulas we need are fairly simple.

If we denote the net funds invested in the risky asset as $F = P + A - rF$ where P is the premium charged, A is the amount of capital raised and r is the option cost, then we can write both the safety constraint and the variance constraint, at the equality, as:

$$\text{Safety: } (1 + r_f)F = s$$

$$\text{Variance: } (A\sigma_y)^2 \geq (F\sigma_i)^2 + (\sigma_L)^2$$

We know that both the safety constraint and the variance constraint have to be satisfied. We could solve for A under both constraints, but luckily for us the two respective values for A were already given and the larger of the two is the one under the variance constraint, $A = 32,266,602$.

From the formula above, we can then solve for F under the variance constraint:

$$(A\sigma_y)^2 = (F\sigma_i)^2 + (\sigma_L)^2$$

$$(32,266,602(8.4\%))^2 = (F(5.64\%))^2 + (2,000,000)^2$$

$$\Rightarrow F = 32,433,913$$

To get the risk load, we just have to note that the funds invested are equal to:

$$F = \frac{\frac{\mu_L}{1+r_f} + R + A}{1+r}$$

Solving for the risk load gives us $R = 232,343$

Notice that because I gave you so many of the values it was easy to calculate R from the value of F I derived earlier. Normally we would have to jointly solve for A and R and so a more involved relationship between R and A would have to be used, which is derived by setting the expected return to the providers of the capital equal to the expected return on the risky asset.

Cumulative Normal Distribution (Positive x)

x	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Cumulative Normal Distribution (Negative x)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Practice Exam 6

This practice exam is intended to contain exam-style questions for a representative subset of the Learning Objectives.

Section Weights

Questions from each part of the syllabus are included, with weights generally consistent with the weights specified on the syllabus.

Time Limit

The actual exam has a time constraint that typically poses a real challenge for students. Nonetheless, I have decide to not specify a time limit for the practice exams in order to encourage you to carefully consider each of these questions and take whatever time is needed to fully formulate your responses. It would be a mistake to skip practice questions due to time constraints.

Points Per Question

Points have been allocated for each question, but the vast majority of the questions are worth just one point. This is because I have tried to make most of the questions comparable in terms of difficulty and time needed to perform calculations, so I did not vary the points much.

How to Grade Yourself

You should target scoring 85% or more to fully prepare for the actual exam.

1. **(1 point)** What three characteristics do U.S. Treasury Bills have that make them an appropriate choice as the risk-free asset?

Solution:

They are free of default risk, their short term nature makes them insensitive to interest rate risk and their exposure to inflation uncertainty is minimal.

2. **(2 points)** You are the Chief Investment Officer of an insurance company and you are planning to split your portfolio between two risky index funds with the following characteristics:

	Stock Fund	Bond Fund
Expected Return	9%	4%
Standard Deviation	13%	6%
Correlation Coefficient	0.10	

Your utility function is given by the following equation with a risk aversion parameter of A=5,

$$U = E(r) - .5A\sigma^2$$

If you are not allowed to borrow, what is your preferred allocation between the stock and bond funds?

Solution:

The calculation here simply requires you to determine the optimal weights on the two risky assets given your utility function. In the book they show you the resulting formula in the case where one asset is risk-free, but in this case we need a slightly more complicated calculation.

We can write the expected return and variance of our portfolio subject to the weights w_s and $w_b = 1 - w_s$:

$$E(r) = w_s(.09) + (1 - w_s)(.04)$$

$$\sigma^2 = w_s^2(.13^2) + (1 - w_s)^2(.06^2) + 2w_s(1 - w_s)(.1)(.13)(.06)$$

Writing the utility function as $U = E(r) - .5(5)\sigma^2$ we then just plug in the formulas for $E(r)$ and σ^2, take the derivative with respect to w_s and set it equal to zero. Solving for the weights then gives us $w_s = 67.7\%$ and $w_b = 32.3\%$.

3. (1 point) Bodie, Kane and Marcus compare the construction of a portfolio using the Markowitz procedure and using a single index model and note that the latter provides a convenient way to reduce the number of parameters that have to be estimated. Under what conditions would these two approaches result in the same optimal portfolio?

Solution:

Under the single index model, the covariance between any two stocks is $\sigma_{ij} = \beta_i\beta_j\sigma_M^2$ and the firm-specific noise components are uncorrelated across different assets. If this assumption is correct then the single index model and the Markowitz procedure will result in the same optimal portfolio. Otherwise, if there is any additional covariance between assets then the Markowitz procedure will take advantage of this and expand the efficient frontier.

4. **(1 point)** An employee has come to you asking for help in understanding the difference between a Single Index Model of asset returns with the aggregate market portfolio as depicted by the S&P 500 as the index and the Capital Asset Pricing Model. Describe one similarity and two differences in these models.

Solution:

Both models reflect a linear relationship between the return on a particular asset and the return on aggregate market portfolio:

$$\text{CAPM: } E(r_i) - r_f = \beta[E(r_m) - r_f]$$

$$\text{Single Index Model: } r_i - r_f = \alpha + \beta[r_m) - r_f] + \epsilon$$

Although they appear similar, the CAPM is a depiction of the equilibrium relationship between the unobservable *expected* returns on the asset and the market while the Single Index Model is a representation of the *actual* returns.

Further, the single index model does not impose any restriction on the value for α or the variance of the residual term ϵ while the CAPM specifically states that once returns are reduced by the risk free rate (depicted as excess returns) the relationship is linear with no intercept and no residual source of variance.

5. **(1 point)** Describe the difference between a *factor model* of asset returns and Arbitrage Pricing Theory.

Solution:

A factor model is an assumed relationship between the actual return on a risky asset and its expected return which assumes that the deviations (or shocks) come from two sources — a return surprise resulting from a common set of factors that affect all assets and an idiosyncratic return surprise that is unique to each asset and which can be diversified away in large portfolios.

$$r_i = E(r_i) + \sum_j \beta_j [F_j + \epsilon$$

A model such as this serves as the basis for Arbitrage Pricing Theory (APT). However, APT is a statement about the expected return on a risky portfolio of assets that must hold if, in equilibrium, there are no arbitrage opportunities. In symbols, if RP_j is the risk premium on a portfolio that has exposure to *only* factor j with a beta of $\beta_j = 1$ then for any diversified portfolio:

$$E(r_i) = r_f + \sum_j \beta_j E(RP_j)$$

In other words, APT assumes a factor model exists and then determines a relationship between the expected returns and factor exposures (betas) of all tradeable portfolios that must hold in order to avoid the existence of arbitrage opportunities.

6. **(1 point)** If markets are weak-form efficient we should not be able to use historical price information to earn abnormal returns. Discuss one anomaly that seems to show that indeed, over intermediate horizons such as 3–12 months, there are exploitable and recurring patterns in stock prices and also identify at least one reasons why exploiting this could be risky.

Solution:

The momentum effect has been identified as an important and persistent anomaly. It appears that when you rank stocks based on their returns over the most recent past (3–12 months) portfolios of the best performers tend to continue to outperform and portfolios of the worst performers tend to continue to underperform, over the subsequent 3–12 months. A strategy of buying winners and selling losers tends to earn abnormal returns after adjusting for risk.

However, there is also a longer term reversal pattern and so taking advantage of this requires risk that the reversal will occur before you liquidate your portfolio.

7. (1 point) Briefly describe four examples of behavioral biases.

Solution:

The four discussed in the textbook are:

1. Framing — Decisions tend to be influenced by the way they are framed. For instance, people tend to be risk averse when it comes to gains (would prefer a certain gain to a gamble) but are risk seeking when it comes to losses (would prefer a gamble to a certain loss).

2. Mental Accounting — A specific type of framing is when people segregate certain decisions. For example, some gamblers may make different decisions when playing with the house's money versus when they have to dip into their own funds.

3. Regret Avoidance — People tend to blame themselves less when their choices are more conventional, which causes people to shy away from out of favor stocks. This could explain why higher returns are required to entice investors to buy firms with high book to market ratios.

4. Prospect Theory — This theory argues that investors often exhibit loss aversion. Their utility does not depend on the level of wealth, but rather on changes in wealth. This means that people do not get less risk averse as their wealth rises and they are more risk-seeking rather than risk averse when it comes to losses.

8. **(1 point)** Historical average returns on the overall US stock market have been much higher than most economic models of consumption growth and investor risk aversion suggest should have been the case. This fact is often referred to as the Equity Premium Puzzle. Summarize the academic debate over this puzzle.

Solution:

The findings can be grouped into three categories.

First is the argument that there really is no puzzle at all. Historical returns on the stock market have been high not because investors are irrationally too risk averse and demanded high expected returns, but rather **they just got lucky and had high realized stock market returns relative to bonds due to changes in market valuations and a long period of terrible bond returns.**

Second is a series of arguments that suggest the data showing excess returns that are too high is biased due to i) survivorship bias in the data because it reflects only the US market, ii) the benchmark being used for what a *reasonable return is often used the standard CAPM, which ignores key additional risk factors such as labor income risk and liquidity risk.*

And third, the behavioral argument is that indeed investors are irrational and their myopic loss aversion results in high returns.

9. **(1 point)** Assume you have a 2-year, 5% coupon bond with annual coupons and a $1,000 face value. Determine the Macaulay duration and the modified duration of the bond assuming the current yield is 5% on an annually compounded basis.

Solution:

The Macaulay duration is just the weighted average time to each payment, with the weights equal to the PV of each cash flow as a percent of the total price.

As shown below, the bond price is $\$1,000$ and the Macaulay duration is 1.952. Modified duration is then $1.952/1.05 = 1.859$.

Period	T	Cash Flow	PV(CF)	T*PV(CF)
1	1	50	47.62	47.62
2	2	1,050	952.38	1,904.76
			1,000.00	1,952.38
			Macaulay Duration	**1.952**
			Modified Duration	**1.859**

This could also have been approximated by calculating the price when the yields are 5%, 4.9%, 5.1% and using the duration approximation:

$$\text{Modified Duration} = \frac{P_- - P_+}{2P\Delta y}$$

Using your financial calculators, it is easy to show that the bond prices at 5%, 4.9% and 5.1% are given as:

$$P = 1{,}000.00$$

$$P_- = 1{,}001.86$$

$$P_+ = 998.14$$

From this, we can find the duration to be:

$$\begin{aligned}\text{Modified Duration} &= \frac{P_- - P_+}{2P\Delta y} \\ &= \frac{1{,}001.86 - 998.14}{2(1{,}000)(.001)} \\ &= 1.859\end{aligned}$$

10. **(1 point)** Three months ago you entered into an interest rate swap whereby you agreed to pay a fixed rate of 4% with semi-annual compounding and receive floating rate payments equal to 6-month LIBOR, also with semi-annual compounding. The notional principal of the swap was \$10 million, the maturity of the swap was 18 months, with the next payment dates in 3 months, 9 months and 15 months.

At the inception of the swap, the 6-month LIBOR rate was 4.4% with semi-annual compounding and currently the continuously compounded LIBOR zero rates are as follows:

Table 24: Continuously Compounded LIBOR Rates

Maturity	LIBOR
0.25	5.0%
0.75	5.5%
1.25	6.0%

Determine the current value of the swap using the same risk-neutral method that is used to value forward rate agreements.

Solution:

We can value each piece of the swap by assuming the forward rates are realized and calculate the present value of the net cash flows.

For the first payment under the swap, we know that we will pay a fixed rate and receive the floating rate that was established at inception (4.4% on a bond equivalent basis). Therefore we know that on the first payment date the net payment to us will be $220,000 - 200,000 = 20,000$. The present value of this is $20,000e^{-.05(.25)} = 19,752$.

For the second payment, we first need to determine the LIBOR forward rate. Using the continuously compounded LIBOR rates given for the 3 month and 9 month periods, we know that if R_F is the forward rate then:

$$F = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1} = \frac{.055(.75) - .05(.25)}{75 - .25} = 5.75\%$$

But note that this is a continuously compounded rate and the swap payments are made using the semi-annual compounded rate. So we need the semi-annually compounded forward LIBOR rate, which we can find easily by noting that $e^{.0575(.5)} = 1 + (r/2)$ where r is the semi-annually compounded forward rate, or $r = 2(e^{.0575(.5)} - 1) = 5.833\%$.

Then, the second payment of the swap is simply the difference in the cash flows received and paid, assuming that the forward rate is realized. The present value of this is then simply:

$$PV(CF_2) = \left(\frac{5.833\%}{2}10,000,000 - \frac{4\%}{2}10,000,000\right)e^{-.055(.75)} = 87,968$$

Finally, the third payment is done in the exact same way. The continuously compounded forward rate is 6.75%, which is equivalent to 6.865% on a semi-annual basis. The net payment is therefore \$143,250 and the present value of this is $\$143,250e^{-.06(1.25)} = 132,899$.

The total swap value is then $19,752 + 87,968 + 132,899 = 240,619$.

11. (1 point) You are the Chief Investment Officer for your company and are concerned about the interest rate sensitivity of your reported net worth. You currently have liabilities valued at \$250 million with a modified duration of 4.0 and a bond portfolio valued at \$300 million with a modified duration of 4.5. Within the bond portfolio, \$250 million is invested in a long maturity bond with a modified duration of 5 and \$50 million is invested in a coupon bond with a modified duration of 2.0.

You are trying to lower the duration of your asset portfolio to 4.0 without selling the long maturity bond and have identified a bond that could be purchased using some of the proceeds from a sale of the short maturity bond. Describe how this could be accomplished.

Solution:

Note that if we were to replace the entire \$50 million short maturity bond with a floating rate bond with a duration of approximately zero we would still have an overall bond portfolio duration of:

$$D^* = (250/300)(5) + (50/300)(0) = 4.17$$

In order to lower the duration further, we would need to own a *negative duration* bond, which we could achieve by purchasing a mortgage-backed security Interest Only (I/O) tranche. This bond has a negative duration because when interest rates fall, the underlying mortgages in the pool will tend to be prepaid early and cash flows from interest payments will decline. This causes the present value of the cash flows to fall, rather than rise, when interest rates fall. This is the opposite effect for a fixed-coupon bond and so we say that the duration is negative.

12. **(1 point)** You have been asked to address the effective duration of your portfolio of stocks. Give one argument for why stocks have a high duration and one argument for why they might actually have a low duration.

Solution:

The traditional view of stock durations was that they were generally high (greater than 10) because if dividends are fixed (or grow at a constant rate) then changes in interest rates will have a large impact on the present value of all future dividends.

However, in general dividends are themselves impacted by changes in interest rates, especially those caused by changes in inflation rates. The effect will vary by the type of company, but in general the effective duration of stocks will be much lower than the traditional view because dividends will rise when interest rates rise.

13. **(1 point)** Assume that the continuously compounded risk free rate (based on the LIBOR/swap rate) for a five-year zero-coupon bond is 5% and the yield spread for a five-year zero coupon bond rated BB+ is 0.6%. If the face value of the bond is \$100 and the recovery rate in the event of default is 70%, what is the unconditional probability of default through year 5, Q_5?

Solution:

First, the price of the risk-free bond is:

$$G = \$100e^{-.05(5)} = \$77.880$$

The price for the corporate bond is:

$$B = \$100e^{-.056(5)} = \$75.578$$

This means that investors are willing to pay \$2.302 more for a risk free bond than a corporate bond with credit risk. This amount represents the present value of the expected loss from default.

Another way to calculate the present value of the expected loss from default is:

$$100(1 - 70\%)Q_5 e^{-.05(5)}$$

Setting this equal to \$2.302 and solving for Q_5 gives us $Q_5 = 9.85\%$.

Notice that we could also have calculated this directly by setting the bond price equal to the risk-free present value of the expected bond cash flows, treating Q_5 as the risk-neutral default probability:

$$100e^{-.056(5)} = [(1 - Q_5)(100) + Q_5(70\%)(100)]e^{-.05(5)}$$

Solve for $Q_5 = 9.85\%$.

14. **(1 point)** You are trying to assess the probability of default for a company that has a fixed liability of \$10 million due in exactly 3 years and have gathered the following information:

- The total market value of the company's stock is $\$3,000,000$
- The market value of the company's liability is $\$8,719,866$
- The volatility of the company's stock is 60%
- The volatility of the company's assets is 18.63%
- The current risk free rate is 3% on a continuously compounded basis

Determine the probability the company will default, in whole or in part, on its liability using the Merton Model.

Solution:

This is a basic application of the Merton model, with all of the information needed provided except for the market value of the company's assets. But given the market values of the debt and equity, we know that the market value of the assets is the sum of those, or $\$11,719,866$.

In the Merton model, $1 - N(d_2)$ is the probability of default:

$$\begin{aligned}
d_2 &= d_1 - \sigma_V\sqrt{T} \\
&= \frac{\ln(V/D) + (r + \frac{1}{2}\sigma_V^2)T}{\sigma_V\sqrt{T}} - \sigma_V\sqrt{T} \\
&= \frac{\ln(V/D) + (r - \frac{1}{2}\sigma_V^2)T}{\sigma_V\sqrt{T}} \\
&= \frac{\ln(11,719,866/10,000,000) + (.03 - \frac{1}{2}(18.63\%)^2)(3)}{18.63\%\sqrt{3}} \\
&= .6094
\end{aligned}$$

And then,

$$\text{Prob of Default} = 1 - N(.6094) = 27.11\%$$

Note that since we were given so many of the values, we could have also solved for $N(d_2)$

directly from the two equations that hold under the Merton Model:

$$E = VN(d_1) - De^{-rT}N(d_2)$$
$$\sigma_E E = N(d_1)\sigma_V V$$

The second equation could be used to solve for $N(d_1)$:

$$N(d_1) = .60(3,000,000)/(.1863(11,719,866) = .8244$$

Then plug that into the first equation to solve for $N(d_2)$:

$$N(d_2) = [3,000,000 - 11,719,866(.8244)]/10e^{-.03(3)} = .7289$$

Finally, $1 - N(d_2) = 27.11\%$ just as before.

15. **(1 point)** Prior to 2007, policymakers often stated that the rise of securitization has made the world safer by transferring risks, especially credit risks, to the capital market participants most capable of measuring, managing and absorbing those risks. But the CDO market grew so substantially that when problems arose in that market the impact was bigger than most people thought was possible. Describe two factors that made it possible for the supply of CDOs to be as significant as it was in the years leading up to the credit crisis of 2007-2009.

Solution:

There are arguably many reasons for this, but two in particular are:

- Underpricing of the senior tranches of CDOs, due to factors such as investors' failure to properly reflect the impact of correlation on the default risk of those tranches and investors' failure to properly price for the systematic risk in those tranches, made it possible to pay enough to the junior tranche investors to entice them into the market. The junior tranches contained substantial risk and it is possible that had the senior tranches been priced properly it would not have been possible to attract sufficient investor demand for the junior tranches and many CDOs would have never been issued.

- Although the supply of high-coupon collateral (corporate bonds, mortgage backed securities, etc.) for CDO pools was limited, the ability to create synthetic CDOs using credit default swaps (CDS) instead of physical (cash) bonds for the collateral pool allowed a nearly unlimited amount of CDOs to be created, in many cases reusing the same underlying collateral because the pool didn't need to own the actual bond and could instead just write a credit default swap with the given bond as the reference security.

16. **(1 point)** In the wake of the credit crisis of 2007-2009 there have been numerous attempts to add regulatory, legal and collateral restrictions on the multi-trillion dollar credit default swap (CDS) market as a way to prevent future similar problems in the credit markets. Briefly describe the role that credit default swaps played in the credit crisis.

Solution:

During the credit crisis the single-name CDS market, in which counterparties swap the risk associated with the default of individual companies, worked well and, if anything, aided in the proper functioning of the financial markets because it allowed risk to be transferred and because it helped signal which companies were most at risk of defaulting.

However, CDS also allowed the dollar value of the "bets" being taken on individual corporate names or specific structured finance bonds to far exceed the actual size of those bond markets. With a virtually unlimited ability to manufacture CDO's containing synthetic exposure to, for instance, the subprime mortgage market, through the use of CDS rather than actual bonds, the size of the CDO market grew significantly. When problems surfaced in that particular market the crisis was far worse than it would have been had the CDO market been limited by the size of the actual issuances of corporate and structured finance bonds.

A related issue is that because investors could effectively "purchase" the super-senior tranches of complex CDOs by simply writing credit default swaps on those tranches instead of actually purchasing bonds, they were able to assume far more risk than the financial resources at their disposal might indicate was prudent. Instead of having to come up with the funds to purchase a $5 billion super-senior bond, they only had to have highly rated marketable securities that they could use to pledge as collateral in the CDS and, importantly, only to the extent that their counterparties demanded that the collateral be posted.

17. (1 point) In surveys conducted regarding financial risk management, which of the following findings are reported by Stulz?

- Most firms hedge net companywide risks rather than worry about the risks associated with executed transactions.
- Small firms have more volatile cash flows and so they hedge more than large firms.
- Firms often engage in selective hedging based in part on their own views of risk in different markets.
- Firms are reluctant to give up gains in order to avoid moderate losses.

Solution:

The first two statements are false and the last two statements are true.

Stulz reported that most firms hedge the risks associated with executed transactions or near-term exposures, not companywide risks, and that large firms tended to hedge more than small firms. Firms seemed willing to give up large potential gains only to avoid very large losses.

18. (1 point) Culp, Miller and Neves have argued that Value at Risk (VaR) is a risk measure that has great appeal for trading firms but warn that it isn't very useful in helping firms that intentionally assume risks make important decisions. What are the three features of VaR that they find useful and what is the limitation they identify?

Solution:

VaR has proven to be a useful risk measure for trading firms because if offers a consistent risk measure across different markets, business and products; it is probability based; and it allows measurement over a common time horizon. These characteristics make the measure useful as an exposure monitoring tool, such as for policing external money managers or setting collateral requirements for customers.

But because VaR only focuses on the potential losses without any consideration given to the potential gains, it is not a useful measure for balancing the risk-reward trade-offs of proposed transactions.

19. (1 point) Hull discusses the fact that to use the Gaussian copula to simulate defaults for a large portfolio of bonds, you would need to begin with an entire correlation matrix and use a Choleski decomposition of the entire matrix. As a simplification, he suggests using a single-factor model such that the correlation among any two pairs of bonds is determined based on the sensitivity of each bond to this single factor.

As a simplification, he suggests using a single-factor model such that the correlation among any two pairs of bonds is determined based on the sensitivity of each bond to this single factor. In the case where the default probability is the same for all bonds, the correlation between any two bonds is the same, and the number of bonds in the portfolio is infinitely large, then X% of the time the percentage of bonds that default will be less than the following quantity:

$$V(X,T) = \Phi\left[\frac{\Phi^{-1}[Q(T)] + \sqrt{\rho}\Phi^{-1}(X)}{\sqrt{1-\rho}}\right]$$

Use this formula to estimate the 99%, one-year Credit Value at Risk for a \$100 million portfolio of bonds, if each bond has a 2% one-year default probability, the recovery rate is 70% and the Gaussian copula correlation coefficient is $\rho = .40$.

Solution:

Using the formula given, we can be 99% certain that the percentage of defaults will be less than:

$$\begin{aligned} V(X,T) &= \Phi\left[\frac{\Phi^{-1}[Q(T)] + \sqrt{\rho}\Phi^{-1}(X)}{\sqrt{1-\rho}}\right] \\ &= \Phi\left[\frac{\Phi^{-1}[.02] + \sqrt{.4}\Phi^{-1}(99\%)}{\sqrt{1-.4}}\right] \\ &= \Phi(-0.7519) \\ &= 22.605\% \end{aligned}$$

Therefore, the 1-year, 99% Credit VaR is equal to:

$$(\$100 \text{ million})(.22605)(30\%) = \$6.7815 \text{ million}$$

20. (1 point) Cummins argues that the NAIC's RBC method should not be used to allocate capital. Briefly summarize seven reasons given for this and identify two reasons why the NAIC RBC approach is still important to understand.

Solution:

The weaknesses he identified in his introductory remarks included:

- the charges are inaccurate,
- they are based on book values rather than market values,
- they ignore some important risk sources such as interest rate risk and certain derivatives exposures,
- they are based on industry data for the typical insurer.

Later, he also noted the following additional weaknesses:

- there was little theoretical foundation for the NAIC model,
- the reserve and underwriting charges reflect worst case scenarios rather than variances,
- correlations among the firm's businesses was not adequately addressed.

Nonetheless, the framework the NAIC used does reflect most of the risk sources and the regulatory capital requirements do serve as legitimate constraints regardless of the results of other approaches.

21. (1 point) Both the Cummins Capital Allocation reading and the Feldblum IRR reading identify a frictional cost associated with an insurance company holding capital. Identify the frictional cost that both authors mention as well as two others identified by Cummins.

Solution:

Both authors identify double taxation of investment income on capital as a frictional cost. Cummins also mentions agency costs associated with management not acting in the interest of shareholders and regulatory costs associated with the impact of investment restrictions and other regulatory constraints.

22. **(3 points)** You are trying to use Feldblum's IRR method to price an insurance policy with $20,000 in expected loss that will be paid as a lump sum in 2 years. At the moment you write the policy you have to hold all of the written premium in an unearned premium reserve, which will be earned fully during the first year. At the end of the year you will hold loss reserves until the claims are paid.

Assume that you have to maintain surplus equal to 50% of the total (undiscounted) reserves at any point in time, that you can release surplus only at the start of each period and that your up-front expenses are equal to $3,000.

If the assets do not earn any investment return (just to keep things simple) and you charged a premium of $25,000 what would be the IRR for the shareholders?

Solution:

In this particular problem, we know that the initial premium of $25,000 results in an unearned premium reserve of $25,000 and a surplus requirement at inception of $12,500. At the end of the first period, the premium is fully earned and then only the $20,000 has to be held in loss reserves, at which time the surplus only has to be $10,000.

To determine the amount that the shareholders have to contribute at inception, notice that they have to contribute enough so that they pay the up-front expenses *and* still have surplus of $12,500. That is, they have to contribute $15,500.

Then, at the end of the first period, they get to remove a total of $7,500. This is because they start the first period with a total of $37,500 in assets after paying the expenses and they only need to hold $30,000 in assets at the start of the second period. The release is equal to their profit in the period and the change in surplus (just as was defined by Robbin).

Then, at the end of the second period, they get to release the remaining $10,000 in surplus after paying the $20,000 in claims (at least that is what they expect to happen).

Writing the equation for the IRR then:

$$0 = -15,500 + \frac{7,500}{1 + IRR} + \frac{10,000}{(1 + IRR)^2}$$

Solving for IRR by hand is doable with the quadratic equation, but it should be done with your financial calculators. Enter the three cash flows and then compute the IRR as follows:

- Clear the cash flow registry by pressing $\boxed{\text{2nd}}$ $\boxed{\text{CE/C}}$.
- Enter the initial cash flow by pressing $\boxed{\text{CF}}$ and then $-15,500$ $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$.
- Enter the cash flow amounts and the number of times each cash flow occurs. In

this case, we will use the default frequency of 1 for each cash flow and just enter the amounts. For the next cash flow, enter $7,500$ $\boxed{\text{ENTER}}$ and then $\boxed{\downarrow}$ $\boxed{\downarrow}$. The first down arrow accepts the default payment frequency of 1 and the second one moves to the next cash flow registry entry.

- Enter the remaining cashflow as $10,000$ $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ $\boxed{\downarrow}$

- Then compute the IRR with the entries $\boxed{\text{IRR}}$ $\boxed{\text{CPT}}$

You should get an IRR equal to 8.08%.

23. **(1 point)** The following information is applicable for a policy rated using the IRR method and annual evaluation points.

- Premium: 836,754
- Expected loss: 690,000
- Ratio of expense to premium: 20%
- Investment yield (pre-tax): 7%
- Federal income tax rate on UW income: 35%
- Effective federal income tax rate on investment income: 20%
- Premium is collected on Jan 1 and earned evenly throughout the year
- Expenses are paid on Jan 1 and incurred when paid
- Losses are paid on Dec 31 and incurred as premium is earned
- Required surplus is 25% of written premium throughout the year

Compare the IRR on the equity flows under two alternative assumptions with respect to when taxes are paid. Under Scenario A, assume that the UW profit or loss is calculated at inception and again on Dec 31. Under Scenario B, assume that the UW profit or loss is only calculated on Dec 31 based on cumulative UW profit or loss through that date. In both cases, UW taxes are incurred and paid whenever UW profit or loss is calculated. UW losses receive a tax benefit (negative tax payments).

Solution:

Scenario A is the typical way this problem is done, based on the presentation in Robbin. The figures below show these calculations:

Time	Premium Earned	Premium Paid	Loss and LAE Incurred	Loss and LAE Paid	Expenses Incurred	Expenses Paid	UW Income
0	0	836,754	0	0	167,351	167,351	-167,351
1	836,754	0	690,000	690,000	0	0	146,754

Time	UEP	Reserves Loss and LAE	Expenses	Surplus	Total Liab & Surplus	Premium Receivable	Invested Assets
0	836,754	0	0	209,188	1,045,942	0	1,045,942
1	0	0	0	0	0	0	0

Time	Pre-tax Income UW	Pre-tax Income Investment	Taxes Paid	Decrease in Surplus	Equity Flow
0	-167,351	0	-58,573	-209,188	-317,966
1	146,754	73,216	66,007	209,188	363,151

From the equity flow column, it is easy to calculate the IRR:

$$0 = -317,966 + \frac{363,151}{1 + IRR} \Rightarrow IRR = 14.21\%$$

Under Scenario B, the deferral of the tax benefit from the up-front underwriting loss means that more capital has to be contributed at inception by the investors to pay the expenses, reducing the IRR to 12%, as shown in the cash flow items below:

Time	Premium Earned	Premium Paid	Loss and LAE Incurred	Loss and LAE Paid	Expenses Incurred	Expenses Paid	UW Income
0	0	836,754	0	0	167,351	167,351	-167,351
1	836,754	0	690,000	690,000	0	0	146,754

Time	UEP	Reserves Loss and LAE	Expenses	Surplus	Total Liab & Surplus	Premium Receivable	Invested Assets
0	836,754	0	0	209,188	1,045,942	0	1,045,942
1	0	0	0	0	0	0	0

Time	Pre-tax Income UW	Pre-tax Income Investment	Taxes Paid	Decrease in Surplus	Equity Flow
0	-167,351	0	0	-209,188	-376,539
1	146,754	73,216	7,434	209,188	421,724

$$0 = -376,539 + \frac{421,724}{1 + IRR} \Rightarrow IRR = 12.00\%$$

Note: This problem shows the importance of reading the wording of the question carefully. On both the 2008 and 2010 exams, this problem was asked and used wording to suggest *that they wanted you to follow Scenario B, even though Robbin follows Scenario A.*

24. (1 point) Consider the following data for the past five years for the P&C insurance industry and determine whether Roth would argue that the industry has been earning a fair and reasonable return.

Year	Return on Surplus	Shareholder Dividends	Paid-in Surplus
2005	10.50%	2.8	-2.0
2006	12.30%	4.4	-4.1
2007	10.40%	4.9	-5.0
2008	8.80%	5.5	-3.5
2009	12.10%	5.6	-0.9

Solution:

Although actual industry returns seemed to be high (over 10% in most years), we know that insurance profits are hard to measure accurately and likely to be volatile from year to year. It is not possible to infer from the historical return data whether investors expected to earn fair and reasonable returns each year.

However, the dividend and paid-in surplus data show that on balance shareholders were removing capital from the industry every year through dividends and reductions in paid-in capital (e.g. share repurchases). This suggests that investors did not expect to earn fair and reasonable returns each year.

25. **(1 point)** An insurer is trying to determine an appropriate underwriting profit provision and has the following business characteristics:

- Total policyholder supplied funds are 110% of premium
- Premium to surplus ratio is 2.0
- After-tax investment yield is 4.00%
- Effective corporate tax rate on underwriting income (or loss) is 35%
- GAAP Equity and Statutory Surplus are equal to each other

Use Robbin's Calendar Year ROE method to determine the underwriting profit margin needed to obtain a 16% CY ROE.

Solution:

$$ROE = [(1 - t_u) \cdot U + i_{AT} \cdot (PHSF)](\text{P:S Ratio}) + i_{AT}$$

$$16\% = [(1 - .35)U + 4\%(110\%)](2.0) + 4\%$$

$$\Rightarrow U = 2.46\%$$

26. **(2 points)** You are given the values below for the mean, standard deviation and variances for two reinsurance policies, A and B.

	A	B	Total
Mean	358	588	946
Std Dev	701	2,930	3,398
Variance	491,347	8,587,110	11,543,353

If the correlation between the two policies is $\rho = .6$, the target return on marginal surplus is 12% and required capital is calculated at the 97.5th percentile, what would be the renewal risk load for each policy using the Marginal Surplus method as discussed in Mango's paper?

Solution:

First, to calculate the marginal surplus multiplier we will need to know the z-value for the 97.5th percentile of the standard normal CDF, or $z = \Phi^{-1}(97.5\%) = 1.96$.

The renewal risk load is simply the risk load that would be calculated if the policy in question were assumed to be the *last* policy added.

For A's renewal risk load, we need to assume that B is already in the portfolio and A is being added to it. In this case,

$$\begin{aligned}\text{MS Risk Load for } A &= \frac{yz}{1+y}(S' - S)\\ &= \frac{.12(1.96)}{1.12}(3,398 - 2,930) = 98.1\end{aligned}$$

For B's renewal risk load, we need to assume that A is already in the portfolio and B is being added to it. In this case,

$$\begin{aligned}\text{MS Risk Load for } B &= \frac{yz}{1+y}(S' - S)\\ &= \frac{.12(1.96)}{1.12}(3,398 - 701) = 566.28\end{aligned}$$

27. (2 points) You are considering writing a reinsurance contract on a risk with lognormally distributed claim costs, the mean of which is \$5,000,000. All losses are paid at the end of one year.

You intend to follow an investment strategy that involves investing solely in risk-free assets at a rate of 4%. You have determined that in order to satisfy constraints on the riskiness of the combined reinsurance and investment strategy that you need to raise \$15,000,000 from investors.

What premium would you have to charge so that the investors have an expected return that is greater than or equal to the return they could earn on a risky investment with a mean return of $y = 8\%$?

Solution:

The expected return for the investors, which Kreps called the IRR, is the rate which sets the following equality:

$$0 = -A + \frac{(1 + r_f)(P + A) - \mu_L}{(1 + E(\text{IRR}))}$$

where P is the premium charged, A is the capital raised from investors and r_f is the risk-free rate of return.

Setting the IRR equal to 6% and solving for P gives us:

$$\begin{aligned} E(\text{IRR}) &= \frac{(1 + r_f)(P + A) - \mu_L}{A} - 1 \\ 8\% &= \frac{(1.04)(P + 15{,}000{,}000) - 5{,}000{,}000}{15{,}000{,}000} - 1 \\ P &= 5{,}384{,}615 \end{aligned}$$

Also notice that we could have solved for the risk load using the formula provided in the paper for a known quantity A:

$$\begin{aligned} R &= \frac{y - r_f}{1 + r_f} A \\ &= \frac{.08 - .04}{1.04}(15{,}000{,}000) \\ &= 576{,}923 \end{aligned}$$

From that, we can calculate the premium as:

$$P = \frac{5{,}000{,}000}{1.04} + 576{,}923 = 5{,}384{,}615$$

Cumulative Normal Distribution (Positive x)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Cumulative Normal Distribution (Negative x)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Practice Exam 7

This practice exam is intended to contain exam-style questions for a representative subset of the Learning Objectives.

Section Weights

Questions from each part of the syllabus are included, with weights generally consistent with the weights specified on the syllabus.

Time Limit

The actual exam has a time constraint that typically poses a real challenge for students. Nonetheless, I have decide to not specify a time limit for the practice exams in order to encourage you to carefully consider each of these questions and take whatever time is needed to fully formulate your responses. It would be a mistake to skip practice questions due to time constraints.

Points Per Question

Points have been allocated for each question, but the vast majority of the questions are worth just one point. This is because I have tried to make most of the questions comparable in terms of difficulty and time needed to perform calculations, so I did not vary the points much.

How to Grade Yourself

You should target scoring 85% or more to fully prepare for the actual exam.

1. (1 point) Assume that the average investor has a utility function of the form $U = E(r) - .5A\sigma^2$, with coefficient of risk aversion $A = 2.7$. What proportion of the average investor's investment portfolio will be invested in risk-free assets?

Assume the risk-free rate is 4%, the expected return on the aggregate portfolio of risky assets matches the historical risk premium on the S&P 500 equal to 8.4%, and that the standard deviation of the aggregate risky portfolio is 20.5%.

Solution:

If the average investor follows a utility maximization strategy, then we know that they will select an allocation to risky assets, y, that maximizes their utility:

$$\begin{aligned} y^* &= \frac{E(r_p) - r_f}{A\sigma_p^2} \\ &= \frac{.084}{(2.7)(.205^2)} \\ &= .75 \end{aligned}$$

This means that 25% will be invested in the risk-free portfolio.

2. **(1 point)** You have evaluated the expected returns and standard deviations for several risky portfolios and have decided to invest in a portfolio of risky assets with an expected return of 8.9% and a standard deviation of 10.5% and in the risk-free asset. If the risk-free rate is 6%, what is the equation for the Capital Allocation Line?

Solution:

The equation for a line in *slope-intercept* form is given as $Y = mX + b$. The Capital Allocation Line (CAL) is the line that goes through two particular points: the risk-free portfolio and the investor's selected portfolio of risky assets.

Here, $Y = E(r)$, $X = \sigma$, m = slope = $[r_p - r_f]/\sigma_p$ and $b = r_f$. Therefore, the equation for the line is:

$$\begin{aligned} E(r) &= r_f + \frac{r_p - r_f}{\sigma_p}\sigma \\ &= .06 + \frac{.089 - .06}{.105}\sigma \\ &= .06 + .276\sigma \end{aligned}$$

3. (1 point) You are using the single index model to identify the portfolio with the highest Sharpe ratio and have identified two stocks with the following statistics:

	Stock A	Stock B
α	0.02	0.01
σ_e	0.04	0.02
β	1.20	0.80

Identify the weights of Stock A and Stock B in the *active risky portfolio* and describe what additional information would be needed to determine the weights for these two stocks in the aggregate portfolio.

Solution:

The first step is to determine the weights in proportion to the ratios of the stocks' respective alphas and the **variance** of the noise term:

	Stock A	Stock B
α	0.02	0.01
σ_e	0.04	0.02
β	1.20	0.80
σ^2	0.0016	0.0004
$\frac{\alpha}{\sigma_e^2}$	12.50	25.00
Scaled Weight	0.33	0.67

Notice that the weights, before being rescaled to add to 1.0, use the variance of the noise term in the denominator and not the standard deviation. But this shows that the active portfolio will have one-third invested in Stock A and two-thirds invested in Stock B.

The next step is determine how much to invest in this active portfolio and how much to invest in the market portfolio. We don't have enough information to do this because we would need to know the expected risk premium for the market portfolio and the standard deviation of the market portfolio.

More subtly, even if we had the missing information for the market portfolio we would not be able to determine the overall weights of these two stocks unless we also knew their weights in the market portfolio.

4. **(1 point)** If all of the assumptions of the Capital Asset Pricing Model apply, what would be the expected return for a stock that has a standard deviation of 30% and a correlation coefficient with the market of .8 if the expected equity market return is 9%, the risk free rate is 4% and the standard deviation of the market return is 20%?

Solution:

To use CAPM we first calculate the beta:

$$\beta = \frac{\rho \sigma_i}{\sigma_M}$$

$$= .8(.3)/(.2)$$

$$= 1.20$$

From this, the CAPM tells us that:

$$E(r) = r_f + \beta\ [E(r_M) - r_f] = 4\% + 1.2 * [9\% - 4\%] = 10\%$$

5. **(1 point)** The following portfolios have expected returns and factor sensitivities as shown below:

Portfolio	E(r)	β_1	β_2
A	12%	0.80	1.20
B	9%	0.40	1.60
C	5%	0.00	0.00
D	???	1.20	.80

Assume that returns on risky assets follow a two-factor model and determine, using Arbitrage Pricing Theory, the expected return for Portfolio D. Assume all portfolios are sufficiently diversified so that there is no idiosyncratic (residual) risk in any of them.

Solution:

The assumption that returns follow a two-factor model tells us that, in the absence of arbitrage, expected returns must follow the following equation:

$$E(r_i) = r_f + \beta_1 E(RP_1) + \beta_2 E(RP_2)$$

So given the expected returns and betas for three portfolios (A, B and C) we simply have to solve three equations for the three unknowns on the right-hand side.

Since Portfolio C has betas of zero, this depicts the risk free rate of return. Now we have just two equations with two unknowns:

$$12\% = 5\% + .8E(RP_1) + 1.2E(RP_2)$$

$$9\% = 5\% + .4E(RP_1) + 1.6E(RP_2)$$

Luckily this is fairly easy to solve because if we multiply the second equation by -2 and add them together we get one equation with one unknown and can solve for $E(RP_2) = .5\%$. Then plugging into either equation we can get $E(RP_1) = 8\%$.

Now, to determine the expected return for Portfolio D we just use the resulting equation:

$$E(r_D) = 5\% + 1.2(8\%) + .8(.5\%) = 15\%$$

6. **(1 point)** Researchers have discovered several anomalies with respect to equity market returns that suggest market inefficiencies, including the January Effect, the Small Firm Effect, the Neglected Firm Effect and the Book to Market Effect. Describe why all of these might be very closely related.

Solution:

Positive abnormal returns (even after adjusting for differences in market betas) have been observed in returns in the first week(s) in January relative to other time periods, in small firms (as measured by market value), in neglected firms and in firms with high ratios of book to market (so-called value stocks).

Most of the January effect is associated with small firms and so those two supposed anomalies are really manifestations of the same thing. In addition, neglected firms tend to also be the small firms and so that too may be the same thing. What we are left with is really a small firm effect and a high book to market ratio effect.

The small firm effect has largely disappeared, but to the extent it remains it could be closely related to the book to market effect because firms whose stock price has fallen considerably will tend to be both small and to have high ratios of book value to market value.

But the high book to market effect seems to be real in the data. Portfolios of firms with high ratios of book to market (value firms) do better than portfolios of firms with low book to market ratios (growth firms). The question is whether it reflects mispricing of firms based solely on readily available information or if it reflects compensation for a risk factor.

7. **(1 point)** You work for a private insurance company whose stock is not actively traded in the public markets and therefore there is no readily available estimate of what the firm is worth. You have been asked to estimate the value of your company and plan to use the values of your closest competitors as a guide. Your boss is concerned about the appropriateness of using market prices of competitors' publicly traded stock. He is aware of numerous behavioral biases that make him believe that investors are incapable of accurately forming estimates of future cash flows or properly valuing them after adjusting for risk. Provide one argument for why market prices may still be useful even in light of the behavioral biases and describe a plausible counter-argument that your boss could make.

Solution:

The main argument for trusting market prices even in light of the well documented inability for investors to make good decisions under uncertainty is that so long as there are *some* investors who don't make these errors they will be able to take advantage of the mispricings caused by the other investors. These actions will make the market efficient, in equilibrium.

But the counter-argument is that there are often limits to arbitrage that limit the ability or willingness of informed investors to exploit market inefficiencies:

1. Fundamental Risk — All strategies to exploit an apparent arbitrage opportunity likely contain some residual risk, such as the time it might take for the mispricing to be corrected in the market and the potential for mispricing to get worse before they get better.

2. Implementation Costs — Often times, mispricings cannot be fully exploited due to transaction costs or other limitations on short selling, for instance.

3. Model Risk — What if you are wrong? All opportunities to exploit identified mispricings will carry some risk and therefore may limit the willingness of arbitrageurs to try to exploit it fully.

8. **(1 point)** Jaganathan and Wang (JW) found that changes in aggregate labor income can serve as an important additional source of systematic risk when included within the standard CAPM, such that portfolios whose returns are positively correlated with measures of aggregate labor income have higher expected returns. Explain this finding from the point of view of investors whose wealth (and hence consumption) is determined by their investment portfolio as well as income from their job.

Solution:

If investors are worried about fluctuations in their future labor income, they will want to hedge their risk by buying portfolios that perform well when aggregate labor income (as a proxy for their own labor income) declines. These investors are willing to pay more for such a portfolio that hedges their labor income risk, accepting a lower return on portfolios that are negatively correlated with aggregate labor income. Similarly, they will pay less for portfolios whose cash flows are low at the same time that labor income declines, demanding a higher rate of return on portfolios that are positively correlated with aggregate labor income.

9. **(1 point)** You are trying to determine the one year and two year continuously compounded zero coupon bond yields based on the following information regarding coupon bonds with annual coupons:

Maturity (Years)	Annual Coupon Rate	Face Value	Price
1	4.0%	1,000	980
2	5.0%	1,000	970

Solution:

Use the bootstrap procedure to solve for the zero coupon yields using the prices of coupon bonds by iteratively solving the basic bond price equation:

$$B = \sum_{i=1}^{n} CF_i e^{-y_i t_i}$$

Start with the one year bond that has a coupon payment and a principal payment at time $t = 1$ to get the one-year yield, y_1:

$$980 = 1,040e^{-y_1(1)} \Rightarrow y_1 = 5.942\%$$

Then, using this we can use the two-year bond price to solve for y_2:

$$970 = 50e^{-.05942(1)} + 1050e^{-y_2(2)} \Rightarrow y_2 = 6.452\%$$

10. (1 point) A colleague of yours is trying to determine the duration of a five year interest rate swap in which your company has agreed to pay 6.5% fixed and receive LIBOR+.2% on a notional value of $100 million. The current market swap rate is 6.3%, the LIBOR rate for the next swap payment has just been set at 6.2%, all swap payments are made on a semi-annual basis and the next payment date is in exactly 6 months. Without doing any calculations, describe how you would advise him to calculate the duration of the swap.

Solution:

Using the same trick used to value a swap, we can pretend that each side pays the notional amount at maturity and view the swap from your company's perspective as a long position in a floating rate bond and a short position in a fixed rate bond.

Since we know that the value of a floating rate bond that pays LIBOR flat is always worth its par value on the floating rate reset dates, we know that in between reset dates the duration of the floating rate bond is just the duration of a zero coupon bond with a maturity date equal to the next swap payment date. We also know that it is easy to calculate the duration of a fixed coupon bond.

However, in this case we need to be careful because the swap doesn't pay LIBOR flat and so what we just said above doesn't apply. To make it apply, we can convert the terms of the swap to a swap that pays 6.3% fixed and receives LIBOR flat.

Then, the swap duration is just an average of the fixed and floating rate sides whereby the weights are the dollar values (negative in the case of the fixed rate bond because we are short that bond). But notice a subtle complication - the current swap rate is the same as the terms of this swap, which tells us that the value of the swap is zero. We don't want to use our typical duration measure, which reflects *percentage* changes in value as interest rates change, for an instrument that has zero value. Instead, we need to use *dollar durations* and keep the measures in units of dollars rather than percentages.

11. (1 point) Suppose the term structure is upward sloping with one-year annual zero coupon yields equal to 5% and two-year annual zero coupon yields equal to 6%. Compare the expected one year holding period returns from investing in either a one-year zero coupon bond or a two-year zero coupon bond, each with a face value of $1,000, if the term structure is unchanged at the end of the year. What happens if we assume that the expectations hypothesis holds?

Solution:

The prices of the one-year and two-year zero coupon bonds will be as follows:

$$B_1 = \frac{\$1{,}000}{1.05} = \$952.4$$

$$B_2 = \frac{\$1{,}000}{1.06^2} = \$890$$

If we invest in the one-year zero, then we will pay $952.4 and receive $1,000 in one year, resulting in a 5% holding period return. If we buy a two-year zero, we will pay $890 and then at the end of one year the bond will have one more year until maturity and will have a price of:

$$B = \frac{\$1{,}000}{1.05} = \$952.4$$

This is a holding period return of:

$$\text{HPR} = \frac{952.4}{890} - 1 = 7\%$$

It appears as though the return from the longer term bond is higher than the return from the shorter term bond. This effect is known as riding the yield curve, since it is due to the fact that as time passes we are discounting the bond at lower yields.

However, that assumed that the term structure was unchanged. If the expectations hypothesis holds, then we should expect the differences between the 5% one-year yield and the 6% two-year yield to be due to expected changes in the one year rate. So we should expect the one-year rate one-year from now will not still be 5%.

We can determine the expected rate one year from now (assuming the expectations hypothesis holds) from the relationship:

$$1.06^2 = (1.05)[1 + E(r)]$$

and solve for $E(r) = 7\%$.

So we should really expect to pay \$890 today for the two-year bond and for it to be worth:

$$\frac{\$1{,}000}{1.07} = 934.5$$

at the end of the year. This results in a holding period return of:

$$\text{HPR} = \frac{934.5}{890} - 1 = 5\%$$

This is the same holding period return from investing in a one-year bond.

12. **(1 point)** Suppose an insurer is established with initial surplus of $S = 50$. They write policies on January 1 that have up-front expenses of $E = 25$ and expected claims that will be paid in full at the end of the year equal to $L = 75$. They have a target return on surplus of $k = a + by$ with parameters $a = 10\%$ and $b = 1$. Assume the current risk free rate is $y = 5\%$ on an annually compounded basis.

Use Panning's premium formula to determine what they would charge and what the current economic value of the firm's surplus is immediately after writing the policies.

Solution:

Panning uses the following formula for the premium:

$$P = \frac{S(k - y) + L}{1 + y} + E$$

In this case, the pricing strategy parameters a and b suggest a target return on surplus of:

$$k = 10\% + 1(5\%) = 15\%$$

This leads to premium equal to:

$$P = \frac{50(.15 - .05) + 75}{1.05} + 25 = 101.19$$

Immediately after writing the policies, the current economic value is:

$$\begin{aligned} C &= S + P - E - \frac{L}{1 + y} \\ &= 50 + 101.19 - 25 - \frac{75}{1.05} \\ &= 54.76 \end{aligned}$$

13. (2 points) Based on current bond prices you have estimated the cumulative probability of default for a corporate bond that matures in 4 years to be $Q(4) = 2.4\%$ and the cumulative probability of default for a corporate bond issued by the same company that matures in 5 years to be $Q(5) = 3.1\%$. Determine the annual default intensity (or annual hazard rate) for this bond in the fifth year (i.e. between time $T = 4$ and $T = 5$) and compare that to the average instantaneous default intensity (or the average instantaneous hazard rate) during the fifth year.

Solution:

The default intensity or hazard rate is a conditional probability of default in the fifth year, which is measured as the unconditional probability of default in that year divided by the probability of surviving to the start of that year:

$$h = \frac{Q(5) - Q(4)}{1 - Q(4)} = \frac{3.1\% - 2.4\%}{1 - 2.4\%} = 0.71721\%$$

Instead of using an annual rate, we could also assume an instantaneous conditional default probability at each point in time and then write the cumulative unconditional default probability as:

$$Q(t) = 1 - e^{-\bar{\lambda}(t)t}$$

Solving for the average instantaneous default intensity through 4 years we get $\bar{\lambda}(4) = 0.006073173$ and through 5 years we get $\bar{\lambda}(5) = 0.006298133$. Both of these are average instantaneous rates from time zero, not within any one year, so we can get the average between times $T = 4$ and $T = 5$ as:

$$\bar{\lambda} = \frac{0.006298133(5) - 0.006073173(4)}{5 - 4} = 0.71980\%$$

Notice that the difference between the annual default intensity ($h = 0.71721\%$) and the instantaneous default intensity during the year ($\bar{\lambda} = 0.71980\%$) is really just a difference in units, much like annual vs. continuous interest rates. Converting from one to the other is a bit trickier, but the equation is as follows:

$$e^{-.071980} = 1 - 0.71721\%$$

14. (1 point) The Merton Model for default risk of a bond relies upon the fact that the equity of a firm can be characterized as a call option on the firm's assets with a strike price equal to the face value of the firm's debt. Identify a specific assumption that Merton makes with regard to the value of the firm's assets and a specific assumption that Merton makes with regard to the company's debt that make it possible to then infer the probability of default on the debt from the market value of the firm's equity and the standard Black-Scholes option pricing formula.

Solution:

With respect to the assets, Merton assumes that the market value of the assets follows geometric Brownian motion:

$$dV = \mu V\, dt + \sigma V\, dz$$

This assumption, along with Ito's Lemma, then allows him to develop a relationship between the equity volatility and the asset volatility.

With respect to the debt, in order for him to use the standard Black-Scholes formula he has to have a fixed strike price on a specific maturity date. This means he has to assume that all of the company's debt is in the form of a zero coupon bond with a fixed maturity date. Then default occurs if and only if the asset value at the time of maturity is less than the face value of the debt.

15. **(1 point)** Prior to the credit crisis of 2007-2009 many insurance company investment portfolio managers were aggressively adding senior tranches of CDOs to their portfolios. In many cases, they were relying upon three factors that made these attractive:

- AAA-ratings from S&P, Moody's and Fitch
- studies that showed that the cash flows on senior tranches of CDOs were not very sensitive to changes in default and correlation assumptions
- attractive yield spreads compared to comparably rated corporate bonds

Given that P&C insurers face substantial risk that they may need to sell bonds prior to maturity in order to pay claims sooner than expected (e.g. in response to a property catastrophe event), why should they have been less willing to own senior tranches of CDOs?

Solution:

There are several issues that should have concerned them:

- Senior tranches of CDOs were difficult to rate and so it was inappropriate to rely solely on S&P's analysis, especially because they were paid by the issuer for their ratings, which may have given them some incentive to bias their ratings upwards. In addition, ratings for CDOs were highly dependent on correlation assumptions for the bonds in the pool. This is something that S&P, Moody's and Fitch had little expertise in evaluating.
- Although it is true that the cash flows of senior tranches of CDOs are not very sensitive to assumptions, except in extreme scenarios, their *ratings* are very sensitive to assumptions. Purchasing this tranche exposes you to potentially large swings in the price of the bond in the event the rating agency's assumptions change and the bond is downgraded.
- The AAA-rating and the yields on other AAA-rated bonds doesn't tell you anything about the appropriate yield on the CDO tranche. The latter is far more sensitive to systematic risk and so it's yield spread relative to other AAA-rated debt should be much higher.

16. **(1 point)** Your boss has asked you to evaluate whether catastrophe bonds could be used in place of traditional excess of loss reinsurance. Identify five ways in which catastrophe bonds differ from traditional reinsurance and whether each is an advantage or a disadvantage.

Solution:

The five key differences for catastrophe bonds relative to traditional reinsurance are:

i. Collateral — The policy limit is collateralized in a catastrophe bond, so there is essentially no credit risk. This is an advantage relative to traditional reinsurance.

ii. Multi-Year Coverage — Catastrophe bonds are typically issued for multiple risk periods (3 year periods), which offers both advantages and disadvantages. It locks in pricing and capacity, both of which tend to change adversely at exactly the wrong time. But it does complicate coordination of the cat bond layer with the rest of a company's reinsurance program as well as the basis risk analysis.

iii. Risk-remote Layers — Catastrophe bonds are typically issued for more risk remote layers (roughly in the range of 1% - 5% expected losses), so they aren't an effective replacement for most of the reinsurance capacity usually sought.

iv. Indemnity vs. Index Basis — Most cat bonds offer coverage on an index basis, with payouts determined based on aggregate industry losses, modeled losses or parametric factors. This can be a disadvantage because it introduces *basis risk* in that a company's actual losses may not be covered.

v. Pricing — In theory investors can view catastrophe bonds as zero-beta, diversifying risks relative to their traditional asset portfolios and could possibly offer coverage with lower risk premiums, though so far in practice cat bonds prices have been competitive with traditional rates (especially after considering the additional time and expense associated with issuing cat bonds.

17. **(1 point)** Company X is a highly rated firm with only a minimum amount of debt outstanding. As a result, they are unlikely to experience much costs associate with financial distress. This is why the CEO has always resisted incurring costs to hedge their financial risks. But the CFO believes that they should hedge more of their financial risks. What argument could the CFO make that Stulz would agree with?

Solution:

Although the firm can afford to experience some volatility caused by unhedged financial risks, if it were to hedge its financial risks it could operate with less equity and more debt in its capital structure without a net change in its overall risk profile. The shift to more debt in the capital structure would result in tax savings as well as a strengthening of the incentives for its management to act in the interest of its shareholders. These benefits could potentially exceed the added direct costs of the hedging program.

18. **(1 point)** Culp, Miller and Neves recommend three risk measures that offer some advantages over Value at RIsk (VaR), especially in light of their finding that having a VaR system in place would not have prevented some of the more notorious derivatives disasters. List and briefly describe these alternative risk measures.

Solution:

The three alternatives are:

- Cash Flow at Risk - Focusing on cash flows and not just a market value can help to ensure that transactions and/or financial instruments do not result in significant liquidity risks over their lives.

- Risk Based Capital - Measuring expected profitability on a risk-adjusted basis, such as through the use of a risk-based capital measure as a basis for a risk-adjusted rate of return on capital (RAROC), would allow the risks and returns of a transaction to be evaluated jointly, rather than rely solely on a measure of the downside risk.

- Shortfall Risk - Rather than measure a single (arbitrary) point on the profit and loss distribution it may be preferable to use a measure that incorporates outcomes that are below some target return or, more generally, below some other meaningful quantity (e.g. planned quarterly profit). Anchoring on a meaningful number and then focusing on either the probability or the amount of deviations from that can produce a risk measure that is easier to interpret.

  Two specific examples of shortfall risk measures are provided. One, which is referred to as the Below Target Probability (BTP), is just the probability of having a shortfall relative to the specified target. The other, referred to as Below Target Risk, measures the average value of outcomes below the target, which is closely related to the CTE and EPD risk measures.

19. (2 points) You own a portfolio consisting of a single AA-rated zero coupon bond with a face value of \$10 million and maturity of 10 years. The current zero coupon risk-free yield curve, continuously compounded, and the current credit spreads by rating are as shown below:

Table 25: Risk Free Zero Coupon Yields (Continuously Compounded)

		Yield Spreads					
Term	Risk Free Yield	AAA	AA	A	BBB	BB	B
1	4.0%	0.1%	0.2%	0.3%	0.4%	0.5%	0.6%
2	4.1%	0.1%	0.2%	0.3%	0.4%	0.5%	0.6%
3	4.2%	0.1%	0.2%	0.3%	0.4%	0.5%	0.6%
4	4.3%	0.1%	0.2%	0.3%	0.4%	0.5%	0.6%
5	4.4%	0.1%	0.2%	0.3%	0.4%	0.5%	0.6%
10	4.5%	0.1%	0.2%	0.3%	0.4%	0.5%	0.6%
20	4.6%	0.1%	0.2%	0.3%	0.4%	0.5%	0.6%

Assume the expectations hypothesis holds with respect to the risk-free term structure and that the yield spreads by rating will not change over the course of the next year.

Further, assume that the following table accurately depicts the bond rating transition probabilities over the course of the next year:

Table 26: Credit Rating Transition Probabilities

	Rating at Year-End							
Initial Rating	AAA	AA	A	BBB	BB	B	CCC	D
AA	0.66	91.72	6.94	0.49	0.06	0.09	0.02	0.01

You now want to simulate, using $10,000$ trials, the value at risk for this portfolio and separate the effect of credit risk from the effect of interest rate risk. In one of the trials you generated a uniform random number equal to $U = .935$. Determine the credit-related losses, in dollars, for this portfolio over the year (in this particular simulation trial).

Solution:

There are three steps. First, determine the current value for the bond. Second, for this trial determine the end-of-period credit rating for the bond. Third, determine the end-of-period value for the bond given the expected yields at the end of the year. The change in value due to credit ratings and credit spreads is then the credit-related loss.

To get the current value, we simply use the term structure and the .2% spread for AA-rated bonds:

$$B_0 = 10,000,000e^{-(4.5\%+.2\%)(10)} = 6,250,023$$

Next, to determine the end-of-period rating we can restate the rating transition table to show the cumulative probability that the bond has a rating better than or equal to the rating in each column:

Table 27: Cumulative Transition Probabilities

Initial Rating	AAA	AA	A	BBB	BB	B	CCC	D
AA	0.66	91.72	6.94	0.49	0.06	0.09	0.02	0.01
Cumulative	0.66	92.38	99.32	99.81	99.87	99.96	99.98	100.00

Because the simulated value $U = .935$ is greater than the amount shown in the AA column but less than the amount shown in the A column, we conclude that for this trial the bond will end the year rated A.

To value the bond at that point, we need to reflect two factors. On is that the risk-free yield curve might be different, and given the expectations hypothesis we would expect the 9-year rate one year from now to equal the one-year forward 9-year rate, or:

$$R_F = \frac{R_{10}(10) - R_1(1)}{10 - 1} = 4.56\%$$

Then reflecting the A-rated bond spread of 0.3%, we would get the new value of the bond as:

$$B_1 = 10,000,000e^{-(4.56\%+.3\%)(9)} = 6,459,456$$

It would be tempting to stop there and say that the credit losses are measured as the difference in the bond prices, but notice that even if the credit rating didn't change the bond price would have gone up because of the passage of time and the movement of the risk-free rates. In this case though, the question specifically wanted the credit-related losses, so we should really calculate the credit loss as the difference between B_1 above and the value the bond would have been without any credit-related changes. In this case, using the new risk-free forward rate but the original credit spread we would get:

$$B_1' = 10,000,000e^{-(4.56\%+.2\%)(9)} = 6,517,853$$

The credit-related losses are really just the difference between these last two values, or $58,397$.

20. **(1 point)** Butsic derived a closed form EPD ratio formula in the case where liabilities were normally distributed and the assets were fixed, as follows:

$$\text{EPD Ratio} = \Phi(a) - (1+c)\Phi(a-k)$$

where,

$$a = \frac{k}{2} - \frac{\ln(1+c)}{k}$$

Cummins used the EPD ratio as well, but used a different formula. Explain the assumptions that Cummins made that are different from Butsic's assumptions and which resulted in a different formula for the EPD ratio.

Solution:

Cummins used a more general formula that assumed that both assets and liabilities were stochastic and assumed the asset-to-liability ratio was lognormally distributed with volatility equal to:

$$\sigma^2 = \sigma_A^2 + \sigma_L^2 - 2\sigma_A\sigma_L\rho$$

Notice too that while Butsic assumed that the σ parameter of the lognormal distribution could be approximated with the coefficient of variation, Cummins uses the actual values for σ.

Cummins' EPD ratio was then calculated using the Black-Scholes formula for a put option on the asset-to-liability ratio with a strike price equal to 1.0.

21. **(1 point)** Your firm is considering adopting some of the capital allocation methods discussed in the Cummins Capital Allocation reading and you have been asked to briefly summarize the major conclusions of the Cummins paper. Describe why Cummins thinks it is important to allocate capital and list some of his conclusions regarding the various methods.

Solution:

Cummins argues that capital allocation will lead to better pricing, underwriting and strategy decisions and will lead to shareholder value creation for the winning firms. The key reason for this is that capital allocation allows firms to allocate the frictional cost of holding capital to lines of business.

In reviewing various methods, he states that the expected policyholder deficit (EPD) is better than Value at Risk (VaR), even though both might be useful to calculate. He also thinks it is better to estimate EPD and VaR at different thresholds rather than at just a single point.

He prefers marginal capital allocation methods (Myers-Read and Merton-Perold) because they allow recognition of diversification benefits. He prefers the Myers-Read to the Merton-Perold method in part because the results by line add to the total capital required.

Some additional comments made include the fact that capital allocation must reflect both the asset and liability risks and in particular the covariance between them, the duration and maturity of the liabilities should be reflected in the capital allocation and the decision making system should dictate the data needs, not the other way around.

22. **(1 point)** Historically, P&C insurance profit loads used rating bureaus reflected a flat 5% of written premium? What were some weaknesses of this approach.

Solution:

The problems noted by Feldblum were:

- Didn't consider the time value of money.
- Didn't respond to changes in the competitive market environment to capture current return expectations of, for instance, investors.
- Used sales as a rate base, which doesn't take into account the equity provided from the investors (owners). As a result, there is no way to ensure that capital can be attracted to support the business and no way to reflect the frictional costs of holding capital, such as the double taxation of investment income.

23. (1 point) An insurer has \$100 million of surplus and writes business at a 1.20:1 insurance exposure ratio (premium to surplus) and has an insurance leverage factor of 2.0. If their combined ratio is 98% and they earn a 4% return on invested assets, what is their total shareholder return?

Solution:

We can use Ferrari's formula in terms of the reserve to surplus and premium to surplus ratios:

$$\frac{T}{S} = \frac{I}{A}\left(1 + \frac{R}{S}\right) + \frac{U}{P}\frac{P}{S}$$

In that expression, the term $1 + R/S$ is what Ferrari called the insurance leverage factor, so we have:

$$\begin{aligned}\frac{T}{S} &= \frac{I}{A}\left(1 + \frac{R}{S}\right) + \frac{U}{P}\frac{P}{S} \\ &= 4\%(2.0) + (1 - 98\%)(1.20) \\ &= 10.4\%\end{aligned}$$

Notice that the key here is to recall that Ferrari referred to the whole term $1 + R/S$ as the insurance leverage factor, rather than just the reserve to surplus ratio.

24. **(1 point)** You are given the following facts about the insurance industry's historical and recent performance. Using Roth's approach, determine the prospective required return for mutual insurers and the required return for stock insurers.

Historical Industry Performance (Stocks and Mutuals)		
Claim and Expense Inflation Rate	4.0%	
Written Premium Growth Rate (inflation-adjusted)	1.5%	
Reserve Growth Rate (inflation-adjusted)	4.0%	
Premium to Surplus Ratio	1.5	
Reserve to Surplus Ratio	2.0	
Current Year Data (in $ billions)	Dollars	% of Surplus
Beginning Surplus	450.00	
Ending Surplus	485.00	
Change in Surplus	35.00	7.78%
Stockholder Dividends Paid	10.80	2.40%
Paid-in Surplus	6.75	1.50%

Solution:

Roth defines the required return as the amount needed to account for the required surplus change plus, in the case of stock insurers, the expected dividends and changes in paid-in surplus.

Start with the required surplus change, which is the amount needed so that the industry can continue to provide capacity after taking into account inflation, growth in demand and growth in reserves.

For the inflation we use the 4% historical amount and for the growth in demand we use the growth in inflation-adjusted premiums, which is 1.5%.

For the reserve growth, we can see that the historical reserve growth was 4%, but that reflects both old and new business. To get the reserve growth from existing business only, we subtract the premium growth rate to get 2.5%.

The net required surplus growth rate is then 4% + 1.5% + 2.5% = 8%.

Notice that we don't use the leverage ratios provided since when premiums or reserves grow at a given rate, if the premium to surplus ratio and the reserve to surplus ratio remain constant then the surplus just changes proportionately and so the growth rate in surplus from these two causes is the same as the respective premium and reserve growth rates.

The 8% just calculated is the required surplus growth rate in surplus, which would also reflect the required return on surplus for mutual insurers assuming that the same inflation rate, premium growth rate and reserve growth rates are appropriate.

But for stock insurers we also need to reflect the need to pay dividends and include anticipated changes in paid-in capital. For these two amounts we can use the current year figures of 2.4% and 1.5% to see that the total surplus change required is 8.9%, which reflects the 8% surplus growth rate plus the 2.4% dividend rate less the 1.5% paid-in capital amount.

25. **(2 points)** An insurer is using Robbin's PVI/PVE method and has projected the following GAAP equity balances:

Quarter End	Ending Equity
0	60.00
1	50.00
2	40.00
3	30.00
4	20.00

Using Robbin's annualization adjustment and a 6% annual discount rate, calculate the PVE denominator used in this method.

Solution:

To calculate PVE we do not calculate the present value of the equity flows. Instead, we calculate the present value of the *average equity balances*. Before the annualization adjustment, this calculation is as follows:

Quarter End	Equity	Avg Equity Balance	PV Factor	PVE
0	60.0			
1	50.0	55.0	0.985	54.2
2	40.0	45.0	0.971	43.7
3	30.0	35.0	0.956	33.5
4	20.0	25.0	0.942	23.6
			3.854	154.9

The PVE amount shown above, 154.9, is in this case distorted because it was calculated using quarterly balances. To obtain a PVI/PVE ratio that is properly annualized, Robbin adjusts the present value of the equity balances by dividing by the sum of the quarterly present value factors, which in this case is:

$$\sum_{j=1}^{4} 1.015^{-j} = 3.854$$

This gives an adjusted denominator of $154.9/3.854 = 40.19$.

Note that in this calculation I purposefully calculated both the present value of the equity balances and the present values for the annualization adjustment by discounting the first *entry for one period, the second entry for two periods, etc. This is not technically correct as it reflects one extra period of discounting for each entry. But this is the way Robbin calculates the values in his numerical example and so I wanted to show a consistent calculation. The extra discounting doesn't distort the final answer because the numerator and the denominator are treated consistently. However, if the data were annual and did not reflect the quarterly annualization adjustment, the equity balances would be discounted too much.*

26. **(2 points)** Mango points out that neither the Marginal Surplus (MS) nor Marginal Variance (MV) pricing methods are *renewal additive.* Describe what he means by this and describe for each method how the sum of the renewal risk loads for each risk in the portfolio compares to the risk loads that would be obtained using the same methods applied to the entire portfolio as if the policies were being written simultaneously.

Solution:

A risk load method is considered *renewal additive* if the risk loads charged for each policy, as if it was being renewed with all other policies already in the portfolio (as if it were the last policy in), add up to the risk load that would be charged if all policies were written simultaneously.

In the case of the MS method, the sum of the renewal risk loads is less than the risk load for the whole portfolio. This is due to the fact that the marginal standard deviations do not sum to the total standard deviation (as the result of the square root operator being sub-additive).

In the case of the MV method, the sum of the renewal risk loads are greater than the risk load for the whole portfolio. This is due to double-counting of the covariance term in the marginal variance calculation, with each policy being penalized for the total contribution that covariance makes to the total risk.

Recall that for a portfolio consisting of risks A and B, the total variance is:

$$\text{Total Portfolio Variance} = \text{Var}(A) + \text{Var}(B) + 2\ \text{Covariance}(A, B)$$

When calculating the renewal risk loads, the marginal variances are given as:

$$\begin{aligned}\text{Marginal Variance from A} &= \text{Total Portfolio Variance} - \text{Var}(B)\\ &= [\text{Var}(A) + \text{Var}(B) + 2\ \text{Covariance}(A, B)] - \text{Var}(B)\\ &= \text{Var}(A) + 2\ \text{Covariance}(A, B)\end{aligned}$$

And similarly,

$$\text{Marginal Variance from B} = \text{Var}(B) + 2\ \text{Covariance}(A, B)$$

So in this case, both of the marginal variances include the term 2 Covariance(A, B), which causes the covariances to be counted multiple times.

27. **(2 points)** You are considering writing a reinsurance contract on a risk with the following characteristics:

- claim costs and expenses are lognormally distributed
- expected claim costs are $\$500,000$
- standard deviation of claim costs equals $\$1,250,000$
- All losses are paid at the end of one year.

You intend to follow an investment strategy that involves investing solely in risk-free assets at a rate of 4% and you want the expected return for shareholders from the combined reinsurance and investment strategy to at least equal the expected return on a risky investment equal to $y = 8\%$.

You have determined that in order to satisfy constraints on the riskiness of the combined reinsurance and investment strategy that you need to have a loss safety level equal to the 99th percentile. That is, you need to ensure that you have sufficient funds to pay claims as high as the 99th percentile of the claim cost distribution.

If $\Phi^{-1}(.99) = 2.33$, determine the amount of capital that would have to be provided by investors to support this reinsurance policy and the risk load that would be charged under this particular constraint.

Solution:

The first step is to calculate the loss safety level. To do that, we need the lognormal μ and σ parameters:

$$\sigma = \sqrt{ln(1 + CV^2)} = \sqrt{ln(1 + (1,250,000/500,000)^2)} = 1.4075$$

$$\mu = ln(\text{mean}) - \frac{\sigma^2}{2} = 12.13$$

From this, we can calculate the 99th percentile as:

$$e^{\mu+2.33\sigma} = 4,932,183$$

To find the amount of capital, we know that the expected return on the combined reinsurance and investment strategy must equal the yield on the risky asset, so:

$$y = \frac{(1 + r_f)(P + A) - \mu_L}{A} - 1$$

This can be rewritten as:

$$(1 + y)A + \mu_L = (1 + r_f)(P + A)$$

And then to satisfy the safety constraint, we need the ending value of the invested premium and capital to at least equal the loss safety level, s:

$$(1 + r_f)(P + A) \geq s$$

At the equality, and plugging in the left-hand side of the previous equation for the left-hand side of this equation gives us:

$$\begin{aligned}(1 + y)A + \mu_L &= s \\ A &= \frac{s - \mu_L}{1 + y} \\ &= \frac{4,932,183 - 500,000}{1.08} \\ &= 4,103,873\end{aligned}$$

Using this value for A at the equality, the risk load is found plugging in the present value of the expected claims plus the risk load for the premium in the earlier formula:

$$\begin{aligned}(1 + y)A + \mu_L &= (1 + r_f)(P + A) \\ (1 + y)A + \mu_L &= (1 + r_f)(\frac{\mu}{1 + r_f} + R + A) \\ R &= \frac{y - r_f}{1 + r_f}A \\ &= \frac{.08 - .04}{1.04}(4,103,873) \\ &= 157,841\end{aligned}$$

Cumulative Normal Distribution (Positive x)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Cumulative Normal Distribution (Negative x)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000