CHAPTER SIX

INVESTMENT-EQUIVALENT REINSURANCE PRICING

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SUMMARY

Reinsurance pricing is usually described as market-driven. In order to have a more theoretical (and practical) basis for pricing, some description of the economic origin of reinsurance risk load needs to be given. A special-case algorithm is presented here which allows any investment criteria of return and risk to be applied to a combination of the reinsurance contract and financial techniques. The inputs are the investment criteria, the loss distributions, and a criterion describing a reinsurer's underwriting conservatism. The outputs are the risk load and the time-zero assets allocated to the contract **when it is priced as a stand-alone deal**. Since most reinsurers already have a book of business and hence contracts mutually support each other, the risk load here can be regarded as a reasonable maximum. The algorithm predicts the existence of minimum premiums for rare event contracts, and generally suggests reduction in risk load for pooling across contracts and/or years. Three major applications are (1) pricing individual contracts, (2) packaging a reinsurance contract with financial techniques to create an investment vehicle, and (3) providing a tool for whole book management using risk and return to relate investment capital, underwriting, and pricing.

I. INTRODUCTION

There has been an evolution over the last few years toward looking at an insurance or reinsurance enterprise as a whole, rather than seeing underwriting, investments, dividend policy, and so forth as a set of disjoint pieces. Whereas in modern financial theory various approaches to the interaction of risk and reward are reasonably well developed, for reinsurance in particular the very measurement of risk has been (and arguably still is) more of an art than a science. It is generally agreed that surplus creates capacity and writing business uses up surplus—but there is no agreement on how that happens.

This paper proposes a possible model for the special case where the contract is priced on a stand-alone basis, i.e., it is the reinsurer's only business. The risk loads (and hence pricing) derived here are maximal because reinsurers generally have an ongoing book of business. This book is mutually supporting, in that usually not all of it goes bad at the same time. Pricing on a stand-alone basis is equivalent to assuming that the whole book is fully correlated. In some sense, stand-alone pricing will in general result in larger risk loads than are actually needed.

Although the give and take of the market will in the end determine what prices are actually charged for contracts, both insurer and reinsurer can use an economic pricing model to help decide whether to write the contract, since for the insurer the decision not to reinsure externally is the decision to self-reinsure. The intent of this paper is to present a paradigm that will allow the combination of a reinsurance arrangement and suitable financial techniques to be thought of as an investment alternative. This allows a firm's investment criteria to be applied.

What will actually be done is to assume investment criteria in the form of a target mean return and risk measure thereon, and to obtain from the paradigm the necessary risk load and putatively allocated assets for the reinsurance arrangement.

The paradigm is as follows: when the reinsurer accepts a contract, it arranges to have available at every time of loss sufficient liquifiable assets to cover possible losses up to some safety level. These assets arise from premium and assets allocated from surplus, both of which are invested in appropriate financial instruments. The reinsurer wishes to have at least as favorable return and risk over the period of the contract as it would when doing its target investment with the underlying allocated assets.

Note that this is not—at least to the author's knowledge—how reinsurers currently do their pricing, nor is it advocated (except in special circumstances) as an operating procedure for reinsurers. It is meant as a way of deriving risk loads by relating them to investment criteria. At the same time, it is grounded in notions which make intuitive sense. Certainly in the real world reinsurers had better plan to have assets available to pay losses; otherwise they are planning for bankruptcy. **This paradigm essentially looks at risk load as an opportunity cost and represents it as a (partially offset) cost of liquidity**. This is not to say that this is the only way of looking at risk loads—but it is a simple and intuitive one.

The **loss safety level** is essentially a measure of reinsurer company conservatism. Again, it is intuitive that some measure of company conservatism must be present in a risk load paradigm. The more conservative the company, the higher the safety level and the less probable it is that the safety level will be exceeded. Higher safety levels will typically result in more expensive contracts.

A mundane example of a safety level occurs when a person decides to build a house in snow country. The question is, how strong to build the roof for snow load? If it is a cabin for only a few years, perhaps building to survive the 10 year storm will be enough. If it is meant for the grandchildren, perhaps the 200 year storm is more appropriate. It is, of course, more expensive to build it stronger. In any case some level <u>is</u> chosen depending on the builder's criteria.

The safety level used in the examples here will be the amount of loss associated with a previously chosen probability, such as the 99.9% level, i.e., the loss associated with a one thousand year return time. In some circumstances (see Section II.3) the full amount of the contract may be the appropriate safety level. There are, of course, other possibilities than a probability level. One such would be to choose a safety level of loss high enough such that the average value of the excess loss over that level is an acceptably small fraction of the mean loss. Another is that the average excess over the safety level times the probability of hitting the safety level is below some value. Whereas it would be interesting to examine various choices in the context of different management styles, for the present purposes the essential remark is that any quantifiable measure can be used.

Clearly, a risk load paradigm must involve the cost of capital—and more specifically measures of **investment return and risk** for comparison to the capital markets. A *reductio ad absurdum* shows the argument: if capital were free and freely available, insurance, much less reinsurance, would be unnecessary since a firm in temporary trouble would simply borrow to overcome difficulties. The measure of investment risk used here will be the standard deviation (or variance). Equally possible would be to use one of the more sophisticated strictly downside measures, such as a semi-variance or the average value of the (negative) excess of return below some trigger point such as the risk-free rate. Especially in the cases here where very large losses may generate negative results, such a downside risk measure may be desirable. These measures do not give pretty formulae, but are easily used numerically. Again, any quantifiable measure is feasible.

There are two types of **financial techniques** that will be considered. Please note that other techniques are possible; these are just two of the simplest. The first is where the reinsurer takes the capital that it would have put into the target investment (which could be, for example, corporate bonds), and puts it into a risk-free instrument such as government securities. This will be referred to as a **swap**. Even though such terminology is not technically correct, it carries the right flavor. The cost associated with this is basically the loss of investment income, but there is also a gain in that risk is reduced.

This technique will result in simple formulae[1], but in various examples it often turns out to create a higher risk load, and hence to be more expensive (to the cedent) and therefore less competitive than the second type of technique: buying **"put" options**. These options are the right to sell the underlying target investment at a predetermined strike price at maturity (we only consider European options). Here the strike price will be what investment in risk-free securities would have brought, so that the reinsurer is buying the right to sell the target investment at a return not less than the risk-free rate.

The Black-Scholes[2] formula is used to price the option. The distribution of investment returns underlying this formula is assumed for the reinsurer's target investment. The cost of these options will contribute to the risk load, but this is partly offset because the options both increase the return and decrease the variance of the target investment.

This treatment will not include the effects of reinsurer expenses, nor of taxes. However these could be put in, especially in the simulation models described in the latter part of

[1]For the variance measure of investment risk. As remarked earlier, other measures will in general not give simple formulae.

[2]See the discussion of Black-Scholes in, e.g., the CAS Part 5 reading, "Principles of Corporate Finance - 4th Edition" by Brealey and Myers (McGraw-Hill, 1991) page 502 ff..

the paper. For the taxes, one would have to make some assumptions as to whether the contract would affect any possible Alternative Minimum Tax situation. Probably this could best be treated by looking at the reinsurer's whole underwriting book and investment structure with and without the contract of interest. This is a can of worms which the author prefers not to open in this paper.

In Section II, the paper will first discuss the case of a single loss payment at the end of one year. In Section II.1, the swap is treated and in Section II.2, the option. These simple discussions will illustrate the general principles, so that they will hopefully not be obfuscated by the details of the subsequent development. For readability of the paper, technical details are relegated to appendices. In Section II.3 the limiting case of a high excess layer is presented, where it is shown that a minimum premium results. This is in accord with actual market behavior. In Section III the single payment case is extended to arbitrary known time of loss. Section III.1 is a numerical example, and Section III.2 is some general remarks on pooling and other subjects. The principal remark here is that whereas this paradigm may be used in the pricing, it is probably not either necessary or desirable that the reinsurer actually carry out the actions modeled by the paradigm for an individual contract.

The multiple payment case is illustrated in Section IV with a spreadsheet example. In this case, there are no longer simple formulae available, and simulation modeling must be explicitly used. Section IV.1 discusses the extension of the loss safety constraint. Section IV.2 describes the spreadsheet at average values—the analog of taking the mean of the stochastic equation, as was done in Section II. Section IV.3 gives an example and discussion of a full stochastic run. Section IV.4 has various comments on the spreadsheet. Section V contains some general remarks, principal among which is that the risk loads considered here are extreme: actual book pricing should be less.

II. SINGLE PAYMENT AT ONE YEAR

The principal determinants of interest here are

- s = the dollar safety level associated with the loss distribution.
- L = the amount of the loss.
- μ_L = the mean value of the loss.
- σ_L = the standard deviation of the loss.
- r_f = the risk-free rate.
- y = the yield rate of the target investment.
- σ_y = the standard deviation of the investment yield rate.
- P = the premium net to the reinsurer after expenses.

Quantities derived from the above are

- A = the assets allocated by the reinsurer.
- F = the funds initially invested: premium and assets less option cost, if applicable.
- R = the risk load in the premium: the premium less the discounted expected loss.

The premium in all cases is the risk load plus the expected loss discounted at the risk-free rate. Note that this premium does not include any reinsurer expenses. For a single payment at one year,

(1) $$P = R + \frac{\mu_L}{1 + r_f}$$

The constraints of the paradigm may now be stated as (1) the investment result from F as input must be at least s, and (2) the standard deviation of the overall result must be no larger than σ_y.

Although the fundamental cash flow relations are stochastic, it is possible in this section to obtain explicit formulae for the mean and variances involved, and hence get explicit forms for the risk load. In Section IV, the mean is easily obtained, but the variance of the final result of the fundamental cash flow will have to be determined by simulation.

II.1 SWAP CASE

At time zero the reinsurer has an inflow of P and an outflow of

(2) $$F = (P + A).$$

Since the investment is in risk-free securities, at the end of the year the reinsurer has an inflow of $(1 + r_f)F$ and an outflow of the loss L. The internal rate of return (IRR) on these cash flows is defined by the fundamental stochastic relation

(3) $$(1 + IRR)A = (1 + r_f)F - L$$

where both L and IRR are stochastic variables. Taking the mean value of this equation and asking that the mean value of the IRR be the yield rate y gives

(4) $$(1 + y)A = (1 + r_f)F - \mu_L$$

which may be expressed as[3]

(5) $$R = \frac{(y - r_f)}{(1 + r_f)} A.$$

[3]For readability, derivations of more than one line are done in Appendix 3.

Another equation is needed to solve the system, and there are two other constraints that must be satisfied, a loss safety constraint and an investment variance constraint. In general, it is clear that by making the asset base large enough the fractional variability of results can be made as small as desired and the funds available as large as desired. Hence there is always a solution. Both constraints may be phrased as placing lower limits on the allocated assets, so satisfying the more restrictive will satisfy both.

For the safety constraint, requiring the funds available at the year end to be greater than or equal to the safety level gives

$$(1+r_f)F \geq s. \tag{6}$$

Combining Eqs. (4) and (6) to eliminate F

$$A \geq \frac{(s-\mu_L)}{1+y} \tag{7}$$

and consequently from Eqs. (5) and (7) the risk load at the equality is

$$R = \frac{(y-r_f)(s-\mu_L)}{(1+r_f)(1+y)} \tag{8}$$

and from Eq. (1) the premium before expenses is

$$P = R + \frac{\mu_L}{1+r_f}. \tag{9}$$

This is the result for the safety constraint.

For the variance constraint, since there is no variability in the investment return (because it is risk-free) the standard deviation of the IRR is given from Eq. (3) as

$$A\sigma_{IRR} = \sigma_L. \tag{10}$$

The investment constraint is that the IRR should have variance less than or equal to that of the target investment, which gives

$$A \geq \sigma_L / \sigma_y \tag{11}$$

and using Eq. (5) again

(12) $$R = \frac{(y - r_f)}{(1 + r_f)}(\sigma_L / \sigma_y).$$

Given typical values for the loss distribution and the target investment, the latter is likely to be the more stringent constraint. This will be true when

(13) $$(s - \mu_L)/\sigma_L < (1 + y)/\sigma_y.$$

For a one in a thousand safety level and a normal distribution, the number on the left is around 3. For more positively skewed distributions, it will be larger; but in the work of the author it is seldom as large as 5 for typical reinsurance layers. However, in the example used later of an unlimited cover on a lognormal with coefficient of variation 2, the ratio on the left is over 10. The unlimited cover is a mathematical convenience for illustration rather a realistic contract, at least since pollution losses became noticeable. Plausible values for the ratio on the right are easily up around 12 for bonds and higher than 5 for equities.

II.2 OPTION CASE

At time zero the reinsurer will receive the premium, but keep the initial assets invested in the target investment. It will also buy an option to sell the target investment at the end of the year for the value that the risk-free technique would have achieved. By doing so it has obtained an instrument that eliminates that portion of the investment return distribution which lies below the risk-free rate. This will have the effect both of increasing the mean return from the investment and decreasing its standard deviation.

Let

- r = the rate (cost per dollar of investment protected) of a put option.
- I = investment return
- i = mean investment return (determined in Appendix 2).

The value of r depends upon the underlying investment parameter σ, which is determined by y and σ_y and defined in Appendix 1. For small values of the ratio of σ_y to $(1+y)$, it is approximately true that

(14) $$\sigma = \frac{\sigma_y}{(1 + y)}$$

and

(15) $$r = \frac{1}{\sqrt{2\pi}}\sigma\left(1 - \frac{\sigma^2}{24}\right).$$

However, the examples below use the exact formula from Appendix 1. At time zero the reinsurer has an inflow of P and an outflow of $(P+A)$. However, the funds available for investment have decreased by the cost of the option. Specifically, Eq. (2) becomes

$$F = P + A - rF \tag{16}$$

so

$$F = \frac{(P+A)}{(1+r)}. \tag{17}$$

Since the investment is now in risky securities (hedged at the bottom end to not drop below the risk-free rate), at the end of the year the reinsurer has an inflow of $(1+I)F$ and an outflow of the loss, L. The internal rate of return on these cash flows is defined by a fundamental stochastic relation similar to Eq. (3):

$$(1+IRR)A = (1+I)F - L. \tag{18}$$

Again, requiring that the mean value of IRR be the target yield rate gives

$$(1+y)A = (1+i)F - \mu_L. \tag{19}$$

This does not simplify easily, but fundamentally we have two unknowns—R and A—and this is one equation relating them. The other equation will come from whichever is the more restrictive constraint, as before.

The loss safety constraint on the funds available is again

$$(1+r_f)F \geq s. \tag{6}$$

It should be noted that the actual funds available are likely to be larger than this, since r_f represents the minimum value of the realizable investment return, thanks to the option. Combining Eqs. (6) and (19) to eliminate F, the allocated assets are

$$A \geq \frac{1}{1+y}\left\{\frac{(1+i)}{(1+r_f)}s - \mu_L\right\}. \tag{20}$$

This is larger than in the swap case since $i > y > r_f$.

The expression for the risk load at equality is[4]

(21) $$R = \frac{1}{(1+r_f)(1+y)}\left[s\{(1+y)(1+r) - (1+i)\} - \mu_L(y - r_f)\right].$$

For $i = r_f$ and $r = 0$ the results of the previous section are, of course, obtained in the above two formulae.

In order to express the investment variance constraint it is necessary to decide the correlation between the loss and the investment return. The linkage by inflation suggests that there may be a negative correlation—if inflation rises, typically claims costs rise and bond values fall. In the interest of simplicity the assumption will be made that the correlation is zero, although there is no essential complication induced by taking a non-zero value. The standard deviation of the investment return is derived in Appendix 2 and written as σ_i. When the variance of the *IRR* is required to be that of the target investment, there results

(22) $$(A\sigma_y)^2 = (F\sigma_i)^2 + (\sigma_L)^2.$$

The value of the initial fund F from the equation for the mean may be substituted into this, resulting[5] in a quadratic equation for A of the form

(23) $$-aA^2 + 2bA + c = 0$$

with

(24) $$a = \sigma_y^2(1+i)^2 - \sigma_i^2(1+y)^2$$

(25) $$b = L(1+y)\sigma_i^2$$

(26) $$c = L^2\sigma_i^2 + \sigma_L^2(1+i)^2.$$

All three coefficients are positive, the last two because of their explicit construction and the first because the option both decreases the variance and increases the mean of the investment return compared to the target values.

The positive solution is

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[4]See Appendix 3.

[5]See Appendix 3. The forms corresponding to a non-zero correlation are also given there.

$$A = \frac{b + \sqrt{b^2 + ac}}{a} \tag{27}$$

and[6]

$$R = A\frac{(1+r)(1+y) - (1+i)}{1+i} + L\left[\frac{1+r}{1+i} - \frac{1}{1+r_f}\right]. \tag{28}$$

In the limit as $\sigma_i \to 0$ the solution for A goes back to the ratio of standard deviations; with $i = r_f$ and $r = 0$ the risk load returns to the earlier form found in the swap case, as it should.

II.3 HIGH EXCESS LAYER AND MINIMUM PREMIUM

An interesting application of these formulae is in the case of a high excess layer or any similar finite rare event cover. A non-zero rate on line (ratio of premium to limit) is predicted even for cases where the loss probability goes to zero.

For simplicity, take the loss distribution to be binomial: There is a probability, p, of hitting the layer, and if it does get hit it is a total loss. The safety level, s, is taken to be the limit (total amount payable) of the layer. Note that the 99.9% level is not an appropriate way to get the safety level (especially for p <0.001), but there is still in fact an intuitive value.

The mean loss μ_L is ps and the variance of the loss is $p(1-p)s^2$. As the probability p gets smaller, corresponding to higher and higher layers, in both the swap and option cases the variance constraint gives A and R both going to zero as $\sqrt{p}$. However, the safety constraint in both cases is linear in p with a non-zero intercept. In the option case, the rate on line[7] *(ROL)* in the limit as p goes to zero is

$$ROL = \frac{(1+y)(1+r) - (1+i)}{(1+r_f)(1+y)}. \tag{29}$$

This is obtained by setting $L = 0$ in Eq. (21) and recognizing ROL as the ratio of R to S.

As usual, the swap version may be obtained by letting $r = 0$ and $i = r_f$, which results in

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[6] See Appendix 3.

[7] That is, the ratio of premium to limit.

(30) $$ROL = \frac{1}{1+r_f} - \frac{1}{1+y} = \frac{y-f}{(1+r_f)(1+y)}.$$

The latter form suggests that the minimum ROL is of the order of the real target return—i.e. the excess of the return over the risk-free rate. However, often the option from Eq. (29) will produce a smaller number. For the investment values such as are used below it is typically on the order of half as large. As the investment standard deviation gets small the swap ROL stays the same (of course) and the option ROL gets small because the option cost gets small and the mean investment return approaches the target yield. It is important to remember that this is all in the limit where the $p = 0$, so that the variance constraint is always satisfied. For small but currently reinsured probabilities—say in the range from 1% to 0.1%—as the target standard deviation of investment is made small the variance constraint will eventually become dominant.

In the market, a minimum rate on line is generally justified by underwriters as a charge for using surplus. This approach is consistent with that view, and also allows quantification of the charge.

III SINGLE PAYMENT AT ARBITRARY TIME

If all the returns in the preceding are interpreted as total return up to time t, then the formulae hold without modification. When we wish to express the returns in terms of the equivalent annualized returns, the results hold after the following replacements are made:

(31) $$(1+i) \rightarrow (1+i)^t$$

(32) $$(1+y) \rightarrow (1+y)^t$$

(33) $$(1+r_f) \rightarrow (1+r_f)^t$$

The forms for the option rate and the standard deviations given in Appendix 2 contain the time dependence.

III.1 NUMERICAL EXAMPLE

For any one-payment situation, the recommended procedure is as follows:

1. Calculate the four risk loads and allocated assets - safety and variance constraints for the option and swap cases.
2. Find for each financial technique which constraint has the larger allocated assets—this is the dominant one.

3. Compare the dominant risk loads for different techniques and choose the smaller—this is the preferred[8] solution.

This whole calculation is easily put on a spreadsheet. For the specific example, the following annualized values have been taken: yield rate y = 5.3%; standard deviation of the yield rate σ_y = 8.4%; risk-free rate r_f = 3.6%. The loss distribution is taken lognormal with mean of \$1M (million) and a standard deviation of \$2M. The safety level of loss is taken as the 99.9% level, \$22,548,702. Parenthetically, for a one-year interval this makes the left-hand side of Eq. (13) 10.8, while the right-hand side is 12.5, suggesting that variance will be the dominant constraint for the swap. For a two-year interval, the right-hand side changes to 8.9 and safety is dominant in the swap. The large value of the left-hand side is due to the fact that this is an unlimited contract.

As an example of the recommended procedure, the following results can be derived from the formulae in the preceding sections for a time of two years, and are incorporated in Table 1 below:

	SWAP		**OPTION**	
constraint	variance	safety	variance	safety
assets	\$15,963,111	\$19,434,097	\$23,024,033	\$20,737,421
risk load	\$528,184	\$643,031	\$316,332	\$283,248

For the swap, the safety is dominant; for the option the variance is dominant. Of the two, the option risk load is smaller, and hence preferred.

TABLE 1

VALUES FOR THE OPTION TECHNIQUE

Time	1	2	3	4
Option rate	3.18%	4.49%	5.50%	6.35%
Risk load	\$ 235,225	\$ 316,332	\$ 399,548	\$ 502,444
Risk-loaded premium	\$ 1,200,476	\$ 1,248,042	\$ 1,298,882	\$ 1,370,526
Total premium	\$ 1,379,857	\$ 1,434,531	\$ 1,492,967	\$ 1,575,317
Allocated assets	\$ 32,522,839	\$ 23,024,033	\$ 20,095,065	\$ 19,446,192
Initial investment	\$ 32,685,050	\$ 23,228,830	\$ 20,278,801	\$ 19,574,132
Determining constraint	variance	variance	safety	safety
Safety value	3,087 years	1,309 years	1,000 years	1,000 years
Annualized std/target std	100%	100%	97%	93%

[8] Preferred from the point of view of the cedent, and preferred from the point of view of offering competitive advantage to the reinsurer - less charge for the same return and risk. On the other hand, the reinsurer may prefer to charge more if the market will bear it. Of course, a higher market rate can always be recast as a more profitable target investment return.

For example, in the second column of the table, time is taken as two years. Following the formulae and notation of the appendices, the investment $2\mu = 9.69\%$ and $\sigma\sqrt{2} = 11.26\%$ at two years. The target investment mean and standard deviation are 10.88% and 12.53% as calculated from the lognormal formulae. The option rate is 4.49%. The mean and standard deviation of the option-protected investment are 14.21% and 8.95%, respectively, higher and lower than the target, as previously advertised. The investment minimum value is 7.33%, the risk-free cumulative return.

The calculated risk loads and asset values are given above for both the option and the swap, and the option variance is chosen.

Please note again that any form of loss distribution could have been used, including underwriter's intuition or simulation result. All that is needed for this choice of risk load is the mean, standard deviation, and safety level. Reinsurer expenses, needed to calculate total premium from risk loaded premium, are taken as 13% of the total.

The table also lists the safety level implied by the chosen asset allocation, and the ratio of the standard deviations of the annualized yield to the target standard deviation. Whichever is not the determining constraint is, or course, more than satisfied. It is noteworthy that as the contract period becomes longer, the safety constraint becomes the more restrictive. In numerical explorations this seems generally to be true.

III.2 POOLING AND OTHER REMARKS

It is an intuitive expectation that the total risk load may be reduced by **pooling**. Pooling over contracts will be considered here; over years after the multiple payments section. The one-year contract from Table 1 has a risk load of $235,225. If there are two contracts combined into a single contract then the fixed percentage safety level used here on the combined contract is certainly less than the sum of the individual safety levels, unless the contracts are fully correlated[9]. Specifically, taking the approximation that the sum of two uncorrelated lognormals may for these purposes be represented by a lognormal, the safety level for the combined contract is $29,455,245, which is only 65.3% of the sum.

The risk load for the combined contract over one year is $331,156, which is 70.4% of the sum of the individual risk loads. This risk load results from the option variance constraint. However, one may question whether some other investment risk measure might have given a different result. The author knows of no general theorem, but experimentation has given consistent pooling.

More intuitively, both the safety levels and investment risk measures will be primarily sensitive to the tail of the loss distribution. When two contracts are imperfectly correlated, the bulk of the tail results from one or the other of the contracts going bad,

[9] Or effectively taken as such, as in the high excess example.

and not both. The effect generally is to shorten the tail relative to the mean, making measures which depend on extreme values take on less dangerous significance.

In what sense is the combination of reinsurance contract and swap/option is priced as **an equivalent investment**? A glance at the values in Table 1 shows that it is possible that if the loss is very bad, say at the 0.001% level, then the final result at the end of the time period will be negative. That is, the reinsurer will lose all the premium and allocated assets, and still have to put in more money to fulfill the contract. At the very least, this result cannot be from a lognormal distribution.

Nevertheless, it is convenient to express the mean and standard deviation of the result in terms of those of a geometric Brownian motion investment that gives the same final values. This allows a direct comparison with the original investment possibility.

To the extent that whatever investment risk measure is used is valid for general distributions, a comparison can always be made.

Should a reinsurer actually **follow through on the indicated financial technique** for each contract? Almost surely not, unless it is very conservative or this is the only contract. The latter could be the case for a specialty reinsurer set up for a single contract; for example, for a large catastrophe contract. In general, a method relating investment criteria to reinsurance contracts could be useful when specifically engineered deals are made to connect reinsureds and investors looking for new opportunities. Considering the hunger of capital for uncorrelated risks, this kind of bundling would seem natural.

This procedure takes as input the financial targets and safety criterion and produces as output the risk load and the allocated assets. It is also possible to **take the financial targets and allocated assets as input** (more the financial point of view). The two constraints then become requirements on the loss distribution. The corresponding risk loads will emerge. Knowing the desired loss characteristics and the necessary risk loads, market knowledge can be used to do selective underwriting and keep the overall distribution within acceptable risk levels at the target rate of return. This point of view is really more applicable to the book as a whole, and requires a treatment of multiple payments.

IV MULTIPLE PAYMENTS

When there are multiple loss payments possible, the same basic paradigm is used but needs a more complex formulation. In the single payment case, simultaneously enforcing the rate of return (through the mean value of the stochastic equation) and the safety constraint gave an easy solution. This will also be done here. In contrast to the single payment case, the variance constraint cannot be conveniently calculated. However, for any given level of allocated assets the variance constraint can be evaluated. If it is not satisfied, the level of allocated assets can be increased until it is, since general considerations have shown that this is always possible.

The general procedure will be: (1) Express the fundamental stochastic process on a spreadsheet. It is now more than a simple equation because of the interaction of the fund levels at different times, but it is still easily expressed. (2) Define the safety levels. (3) Use those to define what funds are needed and what options need to be bought. The swap case will not be shown in the example, but is an easier problem and follows the same procedure. (4) Find the risk load corresponding to the target return for the indicated safety constraint by putting all the stochastic variables at their mean values. (5) Simulate to see if the variance constraint is satisfied. (6) If it is not, then add "excess capital" and simulate again. (7) Repeat step (6) until both constraints are satisfied.

The procedure will be illustrated by a two-year example. The same investment parameters are used as before and the loss has mean $2M spread over two years, in a 50-50 ratio.

IV.1 LOSS SAFETY CONSTRAINT

A procedure for the safety constraint will be illustrated by reference to Table 2 below.

TABLE 2

DEVELOPMENT OF SAFETY CONSTRAINT

Time	1	2
Loss mean	$ 1,000,000	$ 1,000,000
Discounted loss mean	$ 965,251	$ 931,709
Loss std	$ 2,000,000	$ 2,000,000
Loss mu	13.0108	13.0108
Loss sigma	1.2686	1.2686
Individual safety level	$ 22,548,702	$ 22,548,702
Cumulative mean	$ 1,000,000	$ 2,036,000
Cumulative std	$ 2,000,000	$ 2,879,789
Cumulative sigma	1.2686	1.0482
Cumulative safety	$ 22,548,702	$ 29,991,527
Discounted safety	$ 21,765,156	$ 27,943,389
Initial investments	$ 21,765,156	$ 6,178,233
Cumulative option rate	3.20%	4.49%
Option cost on initial investment	$ 691,386	$ 277,474

The first is the mean of each loss, taken to happen at year-end. The next line is the mean value discounted back to time zero at the risk-free rate. The next line is the standard deviation of each loss. For ease of replicability, the partial payment distributions here are taken as lognormal with the coefficient of variation = 2, as before. The "mu" and "sigma" parameters for the lognormals are in the next two lines. The corresponding thousand-year levels are shown subsequently.

The years are also taken as uncorrelated with each other.[10] There is, however, a slight twist in this calculation: The dedicated reader will have noticed that the mean of the cumulative distribution for year two is not just the sum of the individual years one and two. The time value of money for the loss in year one must be accounted for with an appropriate rate to be comparable to a loss in year two, and to be able to add them. Since the reinsurer can think of this as borrowing from itself, the rate taken is the risk-free rate. In the swap case, this is obvious, since the securities held are risk-free. In the option case this still seems appropriate, since the lower limit which will be realized is the risk-free rate. Similarly, the standard deviation is also inflated to give the cumulative value.

In this example, for the purpose of estimating safety levels the cumulative loss distributions, which are the sum of lognormal distributions, are assumed themselves lognormal and the parameters calculated. Simulation runs show that the resulting safety levels are close enough to use.

The cumulative safety levels, however arrived at, are discounted back to zero at the risk-free rate. Thinking in terms of the swap, one could just take the largest[11] of these numbers as the initial fund required. This will fulfill the guarantee to have the safety level liquid at all times. It will also mean that much of the time there will be (expensive) excess liquidity available.

For the option case it will be useful to think of different parts of the initial investment as relating to different time periods. Consider the year two cumulative safety level. Part of it will come from the year one level, increased by investment income at the risk-free rate. The next row, "initial investments", shows the amounts to be invested at time zero in order to have their cumulative value be the safety levels at different times. In the example here the second entry is just the difference between the discounted safety levels.

Options are considered to be purchased separately at time zero on each part of the total at, of course, different costs. The next row "cumulative option rate" shows the rates[12] for options out to the various times, and the last row is the dollar costs of these options.

---

[10] This is only a convenience. The mean and standard deviations of these cumulative loss distributions are easily calculated with correlation.

[11] In the current example, the largest discounted cumulative safety level is the last. However, in the next table and example is given where it is the first.

[12] These are the same rates as in Table 1.

It is useful to look also at another example, which is only changed by having the losses come in at 95% and 5% in the first and second year respectively. This is shown in Table 2A below:

TABLE 2A

DEVELOPMENT OF SAFETY CONSTRAINT

Time	1	2
Loss mean	\$ 1,900,000	\$ 100,000
Discounted loss mean	\$ 1,833,977	\$ 93,171
Loss std	\$ 3,800,000	\$ 200,000
Loss mu	13.6526	10.7082
Loss sigma	1.2686	1.2686
Individual safety level	\$ 42,842,533	\$ 2,254,870
Cumulative mean	\$ 1,900,000	\$ 2,068,400
Cumulative std	\$ 3,800,000	\$ 3,941,877
Cumulative sigma	1.2686	1.2381
Cumulative safety	\$ 42,842,533	\$ 44,098,350
Discounted safety	\$ 41,353,796	\$ 41,086,848
Initial investments	\$ 41,353,796	\$ 0
Cumulative option rate	3.20%	4.49%
Option cost on initial investments	\$ 1,313,633	\$ 0

IV.2 STOCHASTIC SPREADSHEET AT AVERAGE VALUES

With the preceding as preparation, Table 3 can be constructed, which describes the spreadsheet with all stochastic variables at average values. The two-period hedged investment return is 14.21%. The risk load is obtained by asking[13] that it be an amount such that when all the stochastic processes are at their average values the desired target results.

[13] This can be done by trial and error, but is more easily done by "Goal Seek" or its equivalent in the spreadsheet.

TABLE 3

STOCHASTIC PROCESSES AT THEIR AVERAGE VALUES

time	0	1	2
assets	**$27,172,116**	$28,612,238	$30,128,686
excess investment	**$0**		
risk load	**$329,782**		
premium	**$2,226,742**		
option cost	**$984,364**		
invested	**$28,414,494**		
Fund 01	**$22,236,261**	$23,978,898	
Fund 02	**$6,178,233**	**#N/A**	$7,056,055
loss 1		$1,000,000	
funds available		$22,978,898	
desired Fund 12		**$15,364,507**	
actual Fund 12		$15,364,507	$16,568,610
option rate 1 to 2		**3.177%**	
Fund12 option cost		$488,065	
funds released		$7,126,326	$7,504,021
loss 2			$1,000,000
result			$30,128,686

The assets allocated are given at the top. The method to get them is described below. The target investment value given the assets at time 0 is $30,128,686. The first row shows what would have happened on average if the assets had simply been invested.

The "excess investment" is the amount above that required for the safety constraint. At the moment, it is zero.

The "premium" is the sum of the discounted mean losses plus the risk load. The option cost is the cost of options on the investment. It is the sum of the costs shown in Table 2, plus a piece to be described later in this section. "Invested" is assets plus premium less the option cost. It is also the sum of "Fund01" and "Fund02". The Fund01 is the investment at time 0 to be used at time 1; similarly for Fund02.

In order to discuss Fund01, it is necessary to describe the process envisioned and the options which will be bought at different times. At time zero, the safety levels are evaluated and their option costs determined as in Table 2. There are two different options: one assures the initial investment needed for the cumulative safety level at time 1 will reach it at time 1. The other assures that the <u>difference</u> of the discounted cumulative safety levels will reach its desired value at time 2. There is nothing yet to assure that the funds invested to reach the cumulative safety level at time one will actually grow at the risk-free rate from time 1 to time 2.

Let "desired Fund12" be the amount at time 1 which when hedged will grow from time 1 to time 2 so that the sum of it and the mature Fund02 will be the cumulative safety level at time 2 evaluated at time 1, as seen from time zero. Specifically for the example, at time 1 loss 1 has already come in so only loss 2 is relevant. The safety level desired is the individual safety level of $22,548,702. The $6,178,233 invested will mature to $6,631,072, so the difference is $15,917,629. Discounted back one period, the desired Fund12 is $15,364,507. The projected option cost of desired Fund12 is $488,065, which is paid at time 1. The option cost discounted to time zero is $471,105.

Thus, Fund01 contains both the initial investment for the safety level at time 1 of $21,765,156 and the Fund12 option cost of $471,105. The cost of the one year option bought to cover Fund01 is then $691,386 from Table 2 plus the option cost on $471,105. The latter is the extra piece of the option cost referred to earlier.

At this point, Fund02 only contains the entry from Table 2 for the difference in the discounted cumulative safety levels. This is everything at time zero.

At time 1, Fund01 has grown to $23,978,898 and loss 1 has come in at $1,000,000. After paying the loss, the "funds available" of $22,978,898 exceed the desired Fund12 of $15,364,507 so the "actual Fund12" can be equal to the desired. The "Fund12 option cost", paid at this time, to take it to time 2 is the aforementioned $488,065. The funds available less actual Fund12 less its option cost is $7,126,326, which is the "funds released" to the unhedged target investment.

At time 2, the mature Fund02 is available. It will on average[14] have grown faster than the risk-free rate, as will the actual Fund12, so that their sum here will exceed the safety level of $22,548,702. Summing to the bottom (subtracting loss 2) the final result of the contracts is seen. As mentioned earlier, at the average values used here the risk load is chosen so that this final result is the same as if the assets had simply been invested, shown in the top line.

[14] It is to be noted that the two year average hedged investment will be at a larger rate than the compounded one year average, because of the possibility of very low returns one year being offset by high returns the next.

IV.3 STOCHASTIC SPREADSHEET AND VARIANCE

When the stochastic variables are not at their average values, the general flow is the same. Table 4 below shows a fairly atypical sample simulation, in that only 5.8% of the time will the available funds be less than the desired Fund12.

TABLE 4

SAMPLE SIMULATION

time	0	1	2
assets	**$27,172,116**	$32,968,038	$35,108,232
excess investment	**$0**		
risk load	**$329,782**		
premium	**$2,226,742**		
option cost	**$984,364**		
invested	**$28,414,494**		
Fund 01	**$22,236,261**	$26,979,345	
Fund 02	**$6,178,233**	**#N/A**	$7,982,699
loss 1		$15,149,162	
funds available		$11,830,183	
desired Fund 12		**$15,364,507**	
actual Fund 12		$11,465,959	$12,210,298
option rate 1 to 2		**3.177%**	
Fund12 option cost		$364,225	
funds released		$0	$0
loss 2			$1,118,584
result			$19,074,413

The values in the table that do not change over different simulations are shown in **bold**, such as the assets at time 0. In this particular simulation, the investments did very well. The first row shows what would have happened if the assets had simply been invested. However, the good investment returns were not able to offset the large loss at time 1 completely, so that the final result is well under the target value of $30,128,686.

The actual Fund12 needs more discussion. The simplest way to do the actual Fund12 would be to make it equal to the desired Fund12. In this case, that would mean re-allocating investments from elsewhere. However, the whole spirit here has been to allocate the investments up front, and not put more in until they were exhausted. The usefulness of a safety level is that is makes explicit, at least indirectly, the minimum funds to be allocated. Unless the safety level is 100%, there is always the possibility (which will occur in some simulations) that more will be required, but the intent is to run with what was allocated as long as possible.

Hence, the rule followed in this spreadsheet is that the actual Fund12 is the desired Fund12 if there are enough funds available at time 1, or whatever positive fund can be generated from the funds available (which is the case here). In most simulations, there will be more than enough funds available to generate the desired Fund12, and funds will be released back to the reinsurer to be invested in the target investment. In some simulations, there will be positive funds available, and they are used to create Fund12 and pay for the option; but no funds are released.

In a few simulations (about 0.6% in the example) with rather large losses, there will be negative funds available. In this situation Fund12 will be zero and the funds released will be negative. The interpretation is that the reinsurer will have to supply funds from the target investment. It is assumed that the Fund02 will not be available until time 2; but in any case it is earning on average more than the target anyway and it would not be profitable to cash it in.

With the spreadsheet defined, it is now possible to run simulations and evaluate the risk measure—here the standard deviation. With the example parameters the variance of the final result is 13.28%. This corresponds for a lognormal investment to a standard deviation of 8.91%[15].

If the standard deviation is smaller than the target value, then the safety constraint is the more restrictive. If not, then adding excess investment over the full time horizon will reduce the variance while still satisfying the safety constraint. Here, the target is 8.4%, so some "excess investment" must be added to reduce the variance. A first approximation can be obtained by noting that the standard deviation must be reduced about 6% which suggests increasing the investments by 6%. Accordingly, the excess investment was set to $1,800,000. The risk load that corresponds is a slight increase to $342,513. In the fund development, this excess investment lives in Fund02, and is option protected. The revised spreadsheet gives an investment-equivalent standard deviation of 8.31%. Since this is below the target, it is acceptable[16].

IV.4 COMMENTS ON THE SPREADSHEET

There are other possible ways of setting up the spreadsheet. **A general rule that should be satisfied is if the payment stream is essentially zero except at one time, the results for the risk load and the allocated assets should reduce to the single-payment case.**

A corollary is that funds must be able to be released: consider the case of a ten year contract with 99.9% of the payments in the first year. If the cumulative safety level as

[15] On 200,000 simulations. The values for 10,000 and 40,000 simulations were 8.16% and 9.00%. The size of this variation indicated the need for many more simulations.

[16] In order to go further, as the difference from the target gets smaller a much larger number of simulations needs to be done. The simulation uncertainty must be smaller than the difference between the result and the target. Preferably much smaller.

seen at time 0 were maintained for the full ten years, then when the first year loss is not at an extreme value the safety level maintained would be far too high—and therefore too expensive to maintain. The safety level must be revisited after each loss, and funds released when appropriate. On the other hand, if the first year loss *is* near an extreme value to take the subsequent safety levels at the values they would have had for a small first-year loss is to allow for a much more stringent safety condition than was originally intended.

It is for these reasons that the procedure was defined as above and the actual Fund12 is not always the desired Fund12 as defined. A more sophisticated version for the case where the safety level is done on straight probability levels for loss would be to use conditional probabilities and take into account how much probability of the 99.9% level the first loss had used up, followed by the second, and so on. However, the current version is relatively simple and probably accurate enough, especially given the parameter uncertainty inherent in the various aspects of the problem.

There is a difficulty with the current approach to the actual Fund12. It is that when we take average value of the stochastic inputs to the spreadsheet, it does not give average values to the spreadsheet and the resulting risk load is slightly low. The reason is that the average value of actual Fund12 is slightly below the desired Fund12, and not at it which is the result seen in Table 3. This difficulty can be overcome either by going to the simple version of the actual Fund12 treatment or by using the simulation results on the rate to readjust the risk load.

The above procedure for the two-period case may be extended to multiple periods, each with its own loss distribution. The desired funds are the results of the original safety criterion as seen at each period in time. Thus, for a four year problem there will be successively Funds01,02,03,04; desired Funds12,13,14; actual Funds12,13,14; desired Funds23,24; actual Funds23,24; desired Fund34; and actual Fund34. The discounted option costs for the desired FundNM is in Fund0N and any excess investment is in Fund04.

V GENERAL REMARKS

Generalizations: For convenience the losses are taken to happen at the end of each year, although there is no essential difficulty in generalizing to arbitrary times. Also, since simulations are being run any measures of risk and return which can be defined on individual results can be used.

A few words about **IRR and future value**: In the single payment case the IRR was used because it is unequivocally defined, and provides a natural way of talking about returns. It was not actually necessary to look at the IRR and only the end result need have been considered. In the multiple payment case the IRR may not even be definable as a real number. This is particularly obvious when the final value is negative because of large losses, but can also happen otherwise. In order to consider the end value (future value of the cash flows) it is necessary to set up some description of the investment policy on the released funds. The target investment is the obvious choice.

An inessential simplification used here is to ignore the fact that the **spot rates** for risk-free investment depend upon the length of time considered, usually rising with time. For example, incremental losses could be discounted back to time zero using the different spot rates. Here only one single risk-free rate is assumed to apply, for all times of the contract. However, if a reinsurer so desires, then the calculations can be straightforwardly reformulated to include the current spot rates and the view of what the future values of the spot rates are likely to be over the contract period.

It is intuitive that there should be a reduction from **pooling over years**, even allowing for the increased cost of liquidity of the later contract. The example in Tables 3 and 4 is two uncorrelated contracts, and the risk load of $342,513 is less than the twice the single contract value of $235,225 and comparable to the $331,156 for a two simultaneous contracts. Again, the author knows of no general theorem, but experimentation seems to indicate that pooling over time is usually present for uncorrelated contracts.

In real-world scenarios, however, the individual years of multi-year contracts may well have some correlation simply because they are from the same firm or exposures. In the simulation environment, there is no difficulty evaluating the overall contract if one has some idea of the correlation.

The pricing here is **extreme pricing** in that each contract is priced as a stand-alone entity, whereas in reality each contract is supported by the whole surplus of the reinsurer. A more accurate treatment of the actual risk load needed to satisfy investment criteria would be to consider the whole book with and without the proposed contract. Perhaps a satisfactory compromise would be to scale the extreme risk load contemplated here by the ratio of the overall portfolio risk charge to the sum of the extreme risk loads.

Many thanks are due to Gary Venter, the reviewer, and Mike Steel for valuable input and discussions.

APPENDICES

Appendix 1

The form of the Black-Scholes formula for the price of a European call option on a security is[17]

$$\text{call price} = \Phi(\Delta_1)P_0 - \Phi(\Delta_2)PV(E)$$

where $PV(E)$ = present value of the exercise price discounted at the risk-free rate,

P_0 = price of the security at time zero,

$$\Delta_1 = \frac{\ln\left(\frac{P_0}{PV(E)}\right)}{\sigma\sqrt{t}} + \frac{\sigma\sqrt{t}}{2}$$

$$\Delta_2 = \Delta_1 - \sigma\sqrt{t}$$

where σ is a parameter of the distribution of the underlying security and is $\Phi(x)$ the cumulative distribution function for the normal distribution, that is

$$\Phi(x) = \int_{-\infty}^{x} \frac{e^{-z^2/2}}{\sqrt{2\pi}} dz.$$

This function is available in at least one standard spreadsheet program.

The option is the right to buy the underlying security at the exercise price at the time t. The logarithm of the value of the security is assumed to follow a normal distribution with parameters μt and $\sigma\sqrt{t}$ for the mean and standard deviation, respectively[18]. Given the expected annual yield rate y and its standard deviation σ_y, then

$$\sigma^2 = \ln\left\{1 + \left(\frac{\sigma_y}{1+y}\right)^2\right\}$$

[17]Brealey and Myers, op. cit., page 502

[18]This is known as a geometric Wiener process or geometric Brownian motion process. See the development of Black-Scholes in "Stochastic Methods is Economics and Finance" by Malliaris and Brock (North-Holland, 1982) on pages 220-223, and the discussion of the Brownian motion on pages 36-38, especially equation (7.13) and the development leading to it.

and

$$\mu = \ln(1+y) - \frac{\sigma^2}{2}$$

The price for a put option, which is actually what is of interest here, is given by put-call parity as

$$\text{put price} = \text{call price} + PV(E) - P_0$$

In the case of interest here $PV(E) = P_0$ since we want the exercise price to be the growth at the risk-free rate. Hence the put price is equal to the call price, and for either option the

$$\text{option cost} = P_0\Phi\left(\frac{\sigma\sqrt{t}}{2}\right) - P_0\Phi\left(-\frac{\sigma\sqrt{t}}{2}\right)$$

so the

$$\text{option rate} = \Phi\left(\frac{\sigma\sqrt{t}}{2}\right) - \Phi\left(-\frac{\sigma\sqrt{t}}{2}\right)$$

Or,

$$\text{option rate} = \sqrt{\frac{2}{\pi}} \int_0^{\sigma\sqrt{t}/2} e^{-z^2/2} dz.$$

The exponential may be expanded to first order in a Taylor series to get the approximation quoted, which is actually rather good for the order of magnitude of numbers used here.

Appendix 2

As stated in appendix 1, the probability density function for the investment value (which is 1+return) is lognormal with parameters μt and $\sigma\sqrt{t}$. That is,

$$f(x) = \frac{1}{\sigma x\sqrt{2\pi t}} \exp\left\{\frac{-(\ln(x) - \mu t)^2}{\sigma^2 t}\right\}.$$

The investment hedged with the option to time t has the characteristics (f is the risk-free rate)

$$\begin{aligned} \text{investment} &= x && \text{for} \quad x \geq (1+f)^t \\ &= (1+f)^t && \text{for} \quad x < (1+f)^t \end{aligned}.$$

What is needed are the moments of the investment; in particular its mean and standard deviation.

Define

$$\begin{aligned} F_n &= \int_0^{(1+f)^t} x^n f(x) dx \\ &= \Phi\left(\zeta - n\sigma\sqrt{t}\right)\exp\left\{n\mu t + n^2\sigma^2 t/2\right\} \end{aligned}$$

where

$$\zeta = \sqrt{t}\left\{\frac{\ln(1+f) - \mu}{\sigma}\right\}.$$

In general,

$$\begin{aligned} \text{moment}(n) &= \int_0^{\infty} \text{investment}^n f(x) dx \\ &= (1+f)^{nt} \int_0^{(1+f)^t} f(x) dx + \int_{(1+f)^t}^{\infty} x^n f(x) dx \end{aligned}$$

Using the results for F_n above, the moment of order n of the investment is

$$\begin{aligned} \text{moment}(n) &= (1+f)^{nt} F_0 + \exp\left(n\mu t + n^2\sigma^2 t/2\right) - F_n \\ &= (1+f)^{nt}\Phi(\zeta) + \exp\left(n\mu t + n^2\sigma^2 t/2\right)\left[1 - \Phi\left(\zeta - n\sigma\sqrt{t}\right)\right] \end{aligned}$$

The mean value is just moment(1) and the variance of the investment is {moment(2) - moment(1)^2}.

APPENDIX 3

Derivation of Eq. (5): Substitute for μ_L and F in Eq. (4):

Eq. (1) may be solved for μ_L as

$$\mu_L = (1+r_f)(P-R) \tag{A.1}$$

Substitute F from Eq. (2) and L from Eq. (A.1) into Eq. (4):

$$\begin{aligned}(1+y)A &= (1+r_f)(P+A)-(1+r_f)(P-R)\\ &= (1+r_f)A+(1+r_f)R\end{aligned} \tag{4}$$

Solving for R gives Eq. (5).

Derivation of Eq. (21):

Eq. (17) can be written

$$(1+r)F = P+A = A+\frac{\mu_L}{(1+r_f)}+R$$

from Eq. (1). Rearranging to solve for R, and subsequently using Eq. (6) for F and Eq. (20) for A,

$$\begin{aligned}R &= (1+r)F-A-\frac{\mu_L}{(1+r_f)}\\ &= (1+r)\frac{S}{1+r_f}-\frac{1}{1+y}\left(\frac{1+i}{1+r_f}S-\mu_L\right)-\frac{\mu_L}{1+r_f}\\ &= \frac{S}{1+r_f}\left[(1+r)-\frac{1+i}{1+y}\right]+\mu_L\left[\frac{1}{1+y}-\frac{1}{1+r_f}\right]\\ &= \frac{1}{(1+r_f)(1+y)}\left[S\left\{(1+y)(1+r_f)-(1+i)\right\}-\mu_L\left\{y-r_f\right\}\right]\end{aligned}$$

Derivation of Eqs. (23)-(26):

Eq. (19) can be written

$$F = \frac{(1+y)A + \mu_L}{1+i}$$

Substituting for F in Eq. (22) gives

$$A^2\sigma_y^2 = \left[(1+y)^2 A^2 + 2A\mu_L(1+y) + \mu_L^2\right]\frac{\sigma_i^2}{(1+i)^2} + \sigma_L^2$$

Multiplying through by the denominator and collecting terms,

$$0 = A^2\left[(1+y)^2\sigma_i^2 - \sigma_y^2(1+i)^2\right] + 2A\mu_L(1+y)\sigma_i^2 + \mu_L^2\sigma_i^2 + \sigma_L^2(1+i)^2$$

This is Eqs. (23)-(26). If there is a correlation ρ_{iL} between investment and loss, then this equation becomes

$$0 = A^2\left[(1+y)^2\sigma_i^2 - \sigma_y^2(1+i)^2\right] + 2A(1+y)\sigma_i\left[\mu_L\sigma_i + \sigma_{iL}(1+i)\right]$$
$$+ \mu_L^2\sigma_i^2 + \sigma_L^2(1+i)^2 + 2\mu_L\sigma_i\sigma_L(1+i)\rho_{iL}$$

Derivation of Eq. (28):

By substituting for F from Eq. (17) into Eq. (19)

$$(1+y)A = (1+i)\frac{P+A}{1+r} - \mu_L$$

Multiplying through by the denominator and using Eq. (1) for P,

$$A(1+y)(1+r) = (1+i)\left(R + \frac{\mu_L}{1+r_f} + A\right) - \mu_L(1+r)$$

Rearranging terms,

$$(1+r) = A\left[(1+y(1+r) - (1+i)\right] + \mu_L\left[(1+r) - \frac{1+i}{1+r_f}\right]$$

Eq. (28) for R results immediately.

Errata for "Investment-Equivalent Reinsurance Pricing"

Actuarial Considerations Regarding Risk and Return In Property-Casualty Insurance Pricing, Chapter 6

—— April 2007 ——

Many of the errata result from an early version where the mean value of the loss was denoted L, and then subsequently changed to μ_L when L became the random variable.

Page 85: Equation (25) reads

$$b = L(1+i)\sigma_i^2$$

It should read

$$b = \mu_L(1+i)\sigma_i^2$$

Page 85: Equation (26) reads

$$c = L^2\sigma_i^2 + \sigma_L^2(1+i)^2$$

It should read

$$c = \mu_L^2\sigma_i^2 + \sigma_L^2(1+i)^2$$

Page 85: the line after Equation (29) reads

"This is obtained by setting L = 0 in Eq. (2 1) and recognizing ROL as the ratio of R to S."

It should read

"This is obtained by setting $\mu_L = 0$ in Eq. (2 1) and recognizing ROL as the ratio of R to s."

Page 86: Equation (28) reads

$$R = A\frac{(1+r)(1+y)-(1+i)}{1+i} + L\left[\frac{1+r}{1+i} - \frac{1}{1+r_f}\right]$$

It should read

$$R = A\frac{(1+r)(1+y)-(1+i)}{1+i} + \mu_L\left[\frac{1+r}{1+i} - \frac{1}{1+r_f}\right]$$

Page 87: Equation (30) reads

$$ROL = \frac{1}{1+r_f} - \frac{1}{1+y} = \frac{y-f}{(1+r_f)(1+y)}$$

It should read

$$ROL = \frac{1}{1+r_f} - \frac{1}{1+y} = \frac{y-r_f}{(1+r_f)(1+y)}$$

Page 103: The sentence after Equation (A.1) begins

Substitute F from Eq. (2) and L from Eq. (A.l) …

It should read

Substitute F from Eq. (2) and μ_L from Eq. (A.l) …

Page 104: The last equation of appendix C, which reads

$$(1+r) = A[(1+y(1+r)-(1+i)] + \mu_L\left[(1+r) - \frac{1+i}{1+r_f}\right]$$

It should read

$$(1+i)R = A[(1+y)(1+r)-(1+i)] + \mu_L\left[(1+r) - \frac{1+i}{1+r_f}\right]$$