CAS Exam 9 Study Kit

"Solvency Measurement for Property-Liability Risk-Based Capital Applications"

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ABSTRACT

Regulators have recently adopted a risk-based capital formula for property-liability insurers. This article develops practical methods for setting risk-based capital standards using the expected policyholder deficit as the solvency measure. The analysis considers the stochastic nature of insurance risk, using market valuation to remove measurement bias, and finds that a proper time horizon is the period between risk-based capital evaluations. The present value of the expected policyholder deficit is shown to be equivalent to a financial option implicitly given by the policyholders. Finally, covariance of risk elements is considered, deriving a simple square root rule.

Introduction

The recent failure of several large life insurers, following the disastrous experience of the savings and loan industry, has pushed solvency oversight to the top of the regulatory agenda. In late 1990, the National Association of Insurance Commissioners began a mission to establish risk-based capital formulas for both life and property-liability insurance, as well as model laws to institute the capital requirements.

Formula-driven capital requirements are not new to insurance. For about 40 years, European authorities have used various formulas to set solvency margins (see Byrnes, 1986, or Daykin et al., 1987, for an extensive overview of international approaches to solvency regulation). In the United States, detailed risk-based capital formulas for other financial institutions (banks and savings and loans) have been adopted and are now undergoing a phase-in period.

This article shows how risk can be quantified in establishing risk-based capital for property-liability insurers. The first section discusses the roles of parties to the insurance contract, establishing why the risk-based capital concept is economically useful. The second section defines capital and risk, lead-

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ing to the expected policyholder deficit as the relevant risk measure for solvency analysis. A balance sheet model relates capital levels to the size of the expected deficit, providing results for the commonly-used normal and lognormal distributions.

The third section introduces the time dimension with a discussion of bias. Market valuation, for both assets and liabilities, is used to remove bias in risk measurement. This section next describes diffusion processes to show how insurance risk is time-dependent and then shows that a proper time horizon for solvency determination is the period between risk-based capital evaluations. The model is completed by taking the present value of the policyholder deficit and showing that this measure is equivalent to a financial option implicitly given by the policyholders.

The fourth section shows that the degree of correlation between risk elements is a critical factor in properly setting capital levels. Using linear approximation, a simple square root rule incorporates the correlation; its application is illustrated with a hypothetical balance sheet. The final section discusses implications and applications of the results.

Economic Basis for Risk-Based Capital

The purpose of insurance is to spread the costs of unforeseen economic loss over a wide base of policyholders. In turn, the main purpose of solvency regulation is to ensure that the promised insurance protection is available to an acceptable degree of certainty.

The solvency of an insurer is intimately linked to the condition of its balance sheet. Capital, the excess of assets over obligations, represents the owners' stake, or equity in the firm. Under statutory accounting principles (SAP), capital is called surplus; under generally accepted accounting principles (GAAP), capital is called equity.

An insolvency occurs when obligations exceed assets, a condition called *technical insolvency.* Usually at this point regulators will have intervened to place the company in conservatorship or will have severely curtailed its operations (theoretically, however, an insurer could operate beyond the point of technical insolvency if payment of losses and expenses sufficiently lagged cash inflows). The capital providers lose their entire stake in the insurer, and the holders of the obligations, mostly policyholders, take over the assets.

In general, risk-based capital is the theoretical amount of capital needed to absorb the risks of conducting a business. For insurance, it is the amount of capital necessary to assure the major parties to an insolvency that the danger of failure is acceptably low. The standard for this low expectation will be addressed below.

For assessing the consequences of insolvency, the primary parties to the insurance contract are the policyholder and the equityholders—providers of capital to the insurance firm. Note that the policyholder can also be a capital supplier in the case of a mutual insurer. Both the insurer and the regulator are intermediaries. Third-party claimants also have a stake in the success of the insurer.

In a perfectly efficient market, solvency regulation would not be necessary. Consumers would know the likelihood of their insurer's becoming insolvent, with the price of the policy being adjusted to reflect the expectation that not all claims would be fully paid. Also, insurers could adjust their capital levels to reflect their customers' preferences for more or less protection at higher and lower prices. The result would likely be a range of capitalization from high to low leverage, with increasing degrees of policyholder security.

In the real world, however, solvency regulation exists despite the availability of considerable public information regarding the financial strength of insurers and a strong demand for solvency. Many policyholders, especially business customers, actively research their insurers' financial soundness. However, not all policyholders can efficiently obtain this information. Thus, regulators have determined that solvency protection is desirable, particularly for the less informed personal lines customers (although without the protection afforded by regulation, market mechanisms would arise to disseminate solvency information).[1]

Given the existence of solvency regulation, it is reasonable to assume that regulators should provide the public with a minimum level of protection from the adverse effects of insurer insolvency. Through a risk-based capital program, regulation can provide that minimum level, with additional security provided by the competitive insurance market.

Desirable Features of a Risk-Based Capital Method

In order to ensure equity for all parties to the insurance contract (policyholders, claimants, capital providers, and insurers) and to be practicable, the risk-based capital method should satisfy several criteria. First, the solvency standard should be the same for all classes of the above parties (e.g., personal vs. commercial insureds, second- vs. third-party claimants, and primary insurers vs. reinsurers).[2]

Second, the risk-based capital (RBC) should be objectively determined; that is, two insurers with the same risk measures should have identical risk-based capital. Also, a single insurer should obtain the same results under different

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[1] A reviewer has argued that the "consumer ignorance" hypothesis as a basis for regulation is not compelling. First, guaranty funds offer limited protection, requiring insurers to provide more private capital. Second, firms place a high value on ratings from private organizations like Standard & Poor's; there is currently a trend toward production of information regarding financial strength. Third, consumers are not considered ignorant when purchasing corporate securities, which entail risks similar to insurance failure.

[2] There is an issue as to whether the policyholder, in view of the price paid for the policy, actually anticipated potential nonpayment of claims. Some would argue that a customer accepting the contract for a low price has implicitly self-insured, with the premium saved being the expected value of the shortage. This objection could be partially met by establishing different insolvency standards for the different classes. However, since an insurer's insolvency affects *all* of its policyholders, this scheme would be impractical, requiring companies to insure only one class.

regulatory jurisdictions using the same RBC method. The objectivity criterion dictates that the risk-based capital method can be expressed as a mathematical formula incorporating financial data from insurers.

Third, the method must be able to discriminate between quantifiable items of meaningful value that differ materially in their riskiness. For example, if stocks are significantly riskier than bonds, and the amounts of these two assets are known for each insurer, then the risk-based capital method should incorporate the distinction. Each such distinct item is defined as a *risk element*. As shown below, when discussing the effect of time, a risk element must be a balance sheet quantity.

These features will be useful in the development of appropriate solvency measures for a risk-based capital program. The goal of this article—to determine how much capital is needed for the entire insurer—is accomplished by evaluating each risk element singly and then combining the capital amounts of all risk elements.

Expected Deficit as a Measure of Insolvency Risk

In solvency analysis, the relevant risk is that obligations (primarily reserves) may exceed assets, both items being balance sheet quantities. For a balance sheet item, risk is present when the future realization of the item can be one of several values, but the particular outcome is currently unknown.

To clarify the discussion, I use a simplified model with a parallel numerical example; both will be extended to incorporate additional features.

The value of assets is denoted by $\tilde{A}$; assets are cash (the realizable value is certain). The loss reserve $\tilde{L}$ is the unpaid loss, whose realizable value is a random variable. The capital $\tilde{C}$ equals assets minus the loss reserve. The realizable value of capital is a random variable.[3]

For simplicity, we initially assume that the passage of time does not affect value (i.e., interest is zero), other assets and liabilities are zero (e.g., receivables and tax liability), there are no other transactions (taxes, expenses, etc.), losses include loss adjustment expenses, and losses are valued at the beginning of the year and paid at the end of the year.

To determine how much capital is needed to provide the minimum security standard mentioned earlier, we must define the level of protection. The usual measure of risk with respect to insurance solvency is the *probability of ruin*. Although this measure may appear reasonable from the internal perspective of insurance management (whose employment opportunities depend more on the fact of insolvency than on its degree), it is inadequate for public policy.[4]

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[3] Random variables are denoted by tildes; their expectations are denoted by plain type. Notice that since the loss is a random variable, the capital is also a random variable.

[4] Classical risk theory, which has guided the development of European solvency margins (e.g., Beard, Pentikäinen, and Pesonen, 1984) has ignored the severity of ruin. Even the simulation modeling by Daykin et al. (1987), which provides an extensive individual insurer approach to risk-based capital, casts its results in terms of ruin probabilities. A possible reason why so little

To illustrate, suppose that two insurers each have the same beginning balance sheets. Assets of \$13,000 equal unpaid losses of \$10,000 plus capital of \$3,000. However, their unpaid losses have different probability distributions, producing three possible end-of-year results for each insurer (see Table 1). The payoff to policyholders is limited to the insurer's assets of \$13,000. Both insurers have a 20 percent chance of becoming insolvent under Scenario 3, but Insurer B's policyholders are clearly worse off. They will on average forfeit 0.2(18,000 – 13,000) = \$1,000 of their claim payments. Insurer A's policyholders, on the other hand, will forego an expected 0.2(13,100 – 13,000) = \$20 of their claim payments. Clearly, the probability-of-ruin criterion is inadequate to express the policyholders' exposure to loss. It is not sufficient merely to consider the probability of ruin—its *severity* must also be appreciated.

Table 1
Two Insurers with Same Balance Sheet but Different Unpaid Loss Distribution—Asset Amount Is Certain

	Asset Amount	*Loss Amount*	*Capital Amount*	*Probability*	*Claim Payment*	*Deficit*
Insurer A						
Scenario 1	13,000	6,900		0.2	6,900	0
Scenario 2	13,000	10,000		0.6	10,000	0
Scenario 3	13,000	13,100		0.2	13,000	100
Expectation	13,000	10,000	3,000		9,980	20
Insurer B						
Scenario 1	13,000	2,000		0.2	2,000	0
Scenario 2	13,000	10,000		0.6	10,000	0
Scenario 3	13,000	18,000		0.2	13,000	5,000
Expectation	13,000	10,000	3,000		9,000	1,000

This example suggests that a reasonable measure of insolvency risk is the expected value of the difference between the amount the insurer is obligated to pay the claimant and the actual amount paid by the insurer.[5] We will call this difference the *policyholder deficit*.

Using the expected policyholder deficit (EPD) risk measure, we can consistently measure insolvency risk in such a way that a standard minimum level

attention has been given to ruin severity in the insurance literature is that the ruin concept originated with the study of gambler's ruin (by Bernoulli and others in the eighteenth century). In the classical gambling problem, one could not bet more than one's stake, so if ruin occurred, the gambler could not default. With insurance, however, the management can risk *more* than the owners' stake (capital) and so the depth of ruin is important.

[5] Here we ignore guaranty fund or other external sources of recoupment such as another insurer or the federal government. However, the value of insolvency insurance can be determined by methods outlined in this article. Cummins (1988) develops risk-based guaranty fund premiums using similar principles. Note that a more complete measure of insolvency cost is the *present value* of the deficit, a feature added below.

of protection is applied to all classes of policyholders and insurers. The EPD measure can apply equally to all risk elements, whether assets or liabilities. To adjust to the scale of different risk element sizes, the ratio of the expected policyholder deficit to expected loss—or the *EPD ratio*—is used as the basic measure of policyholder security. The EPD ratio is denoted by d. The respective EPD ratios for Insurers A and B are 0.002 and 0.100.

For a discrete loss size probability distribution, when assets are certain, the expected policyholder deficit is

$$D_L = \sum_{x>A} p(x)(x-A). \tag{1}$$

where $p(\bullet)$ = the probability density for losses ($0 \le x < \infty$). The EPD ratio is $d_L = D_L/L$.

Expected Policyholder Deficit with Asset Risk

We now extend the preceding numerical example to risky assets (see Table 2). Insurer C has a known loss of \$5,000 about to be paid, but it has \$6,300 of assets whose year-end value is uncertain (for this example it is assumed that the expected future value of the assets equals their current value). Here the policyholders will come up short 10 percent of the time, when assets turn out to be worth \$3,000. The deficit in this case is \$2,000, giving an expected policyholder deficit of \$200 and an EPD ratio of 0.04. This protection level occurs with a 0.206 ratio of capital to assets.

Table 2
Insurer with Asset Risk—Unpaid Loss Amount Is Certain

	Asset Amount	*Loss Amount*	*Capital Amount*	*Probability*	*Claim Payment*	*Deficit*
Scenario 1	12,000	5,000		0.1	5,000	0
Scenario 2	6,000	5,000		0.8	5,000	0
Scenario 3	3,000	5,000		0.1	3,000	2,000
Expectation	6,300	5,000	1,300		4,800	200

For a discrete probability distribution of asset values, where losses are certain, the EPD is the expectation of assets being less than losses:

$$D_A = \sum_{L>y} q(y)(L-y), \tag{2}$$

where $q(\bullet)$ = the probability density for the asset value ($0 \le y < \infty$).

Setting Capital to a Common EPD Ratio

Now suppose that the regulator wishes to set a capital standard so that the expected policyholder deficit is the same for all insurers, at 5 percent of the expected losses. This amounts to \$500 for Insurers A and B and \$250 for C. In order to satisfy this requirement, it is assumed that the amount of the expected loss is given for each insurer, and the level of beginning assets is adjusted to reach the desired capital. The added or subtracted assets are the same type as the original assets, so that the probability distribution of ending asset values, relative to the beginning assets, stays constant. This homogeneity condition ensures that, like losses, the assets have the same risk (per unit) regardless of the amount of capital.[6]

The capital for Insurer B must be increased in order to meet the 5 percent expected policyholder deficit mark, while capital for A and C must be reduced to reach this standard. Table 3 summarizes this calculation, using Scenario 3 (the two favorable scenarios do not contribute to a policyholder deficit).

Table 3
Balance Sheets and Capital Requirements for Three Hypothetical Insurers with Assets Adjusted to Produce 5 Percent Expected Policyholder Deficit Ratio

	Insurer A	*Insurer B*	*Insurer C*
Scenario 3 Probability	0.2	0.2	0.1
Beginning Assets	10,600	15,500	5,250
Ending Assets	10,600	15,500	2,500
Expected Loss	10,000	10,000	5,000
Capital	600	5,500	250
Loss Amount	13,100	18,000	5,000
Claim Payment	10,600	15,500	2,500
Deficit	2,500	2,500	2,500
Expected Deficit	500	500	250
Capital/Expected Loss	0.060	0.550	
Capital/Assets			0.048

Note: The original beginning assets for Insurer C are \$6,300 with the Scenario 3 ending assets at 10/21 of this amount, or \$3,000. When the assets are reduced to produce the 5 percent expected policyholder deficit ratio, the 10/21 proportion is maintained, giving the above \$2,500 Scenario 3 ending assets.

Using the EPD with Continuous Probability Distributions

For a general discrete distribution of loss or asset values, there is no simple formula that will directly produce the amount of capital required to reach a given expected policyholder deficit value. One must iterate by varying the beginning assets. For continuous distributions, which better represent the distribution of actual realized balance sheet items, it is possible to obtain an explicit relationship between the EPD and capital.

Assume that assets are certain and liabilities (losses) are uncertain. From our earlier notation, we have $\tilde{A} = \tilde{L} + \tilde{C}$. Let $c = C/L$ be the capital per unit of

[6] This requirement allows the risk-based capital formula to apply a constant factor to an asset having a given risk. If, for example, to reduce a given EPD ratio, the added assets were riskless, then the asset risk per unit of assets would decline. An alternative to adding/subtracting assets is reducing/increasing the losses. This procedure will produce a different size for the balance sheet, but the assets, losses, and capital will be in the same proportions as when assets are modified.

expected loss. The expected loss is $L = \int_0^{\infty} xp(x)dx$, where $p(\bullet)$ is the probability density for the losses, and $0 \le x < \infty$.

The expected policyholder deficit here is analogous to the discrete version. It is the expectation of losses exceeding assets, or

$$D_L = \int_A^{\infty} (x-A)p(x)dx. \tag{3a}$$

Notice the similarity of this expression to the cost of excess loss coverage (x denotes the size of loss and A the retention). The excess loss cost is the sum of each possible loss amount exceeding the retention, weighted by the probability of occurrence.

For certain losses and uncertain assets, the expected policyholder deficit is also similar to its discrete counterpart. It is the expectation of assets being less than losses:

$$D_A = \int_0^{L} (L-y)q(y)dy, \tag{3b}$$

where $q(\bullet)$ = the probability density for the assets ($0 \le y < \infty$). From the balance sheet definition, the expected value of assets is $A = (1+c)L$.

The Appendix applies these formulas to derive the EPD ratios for the important case of normally distributed risk elements. Equations (4a) and (4b) express the results in terms of the ratio of capital to the expected value of the risk element, and its coefficient of variation (CV), or the ratio of the standard deviation of the risk element to its mean.

$$d_L = \frac{D_L}{L} = k\phi\left(\frac{-c}{K}\right) - c\Phi\left(\frac{-c}{k}\right) \tag{4a}$$

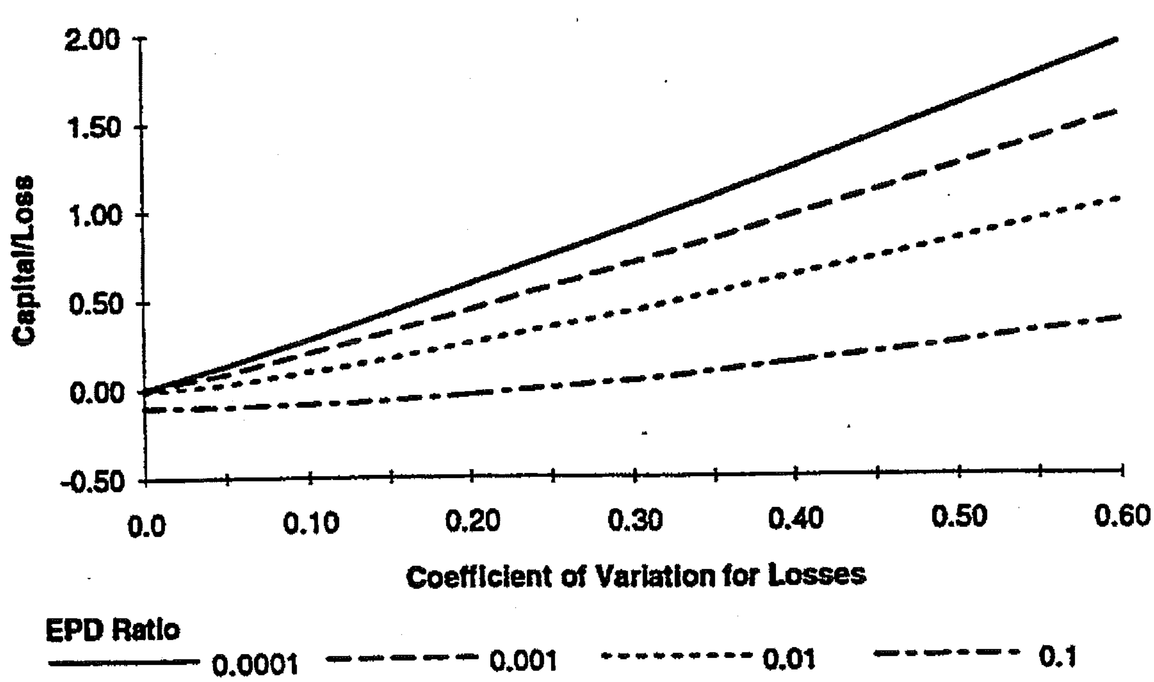
$$d_A = \frac{D_A}{L} = \frac{1}{1-c_A}\left[k_A\phi\left(\frac{-c_A}{k_A}\right) - c_A\Phi\left(\frac{-c_A}{k_A}\right)\right], \tag{4b}$$

where k = the CV of losses,
k_A = the CV of assets,
c_A = the capital/assets ratio,
$\Phi(\bullet)$ = the cumulative standard normal distribution, and
$\phi(\bullet)$ = the standard normal density function.

For the same coefficient of variation, in order for d_A to equal d_L, more capital (per unit of assets) is required for assets than for losses (per unit of losses). This is easily seen by setting $k_A = k$ and $c_A = c$ in equation (4b) and equating with (4a). We get $d_A = d_L/(1-c) > d_L$, for $c > 0$ (negative capital is discussed later). Since the EPD ratio is higher for assets than for losses, the asset capital ratio must be greater than c. Intuitively, asset risk under the normal distribution needs more capital than loss risk because (assuming the same beginning balance sheet), if assets and losses have the same CV, the standard deviation of assets is larger because the value of assets exceeds that of losses. Since the normal probability density is symmetric, the value of capital for both risky assets and losses is distributed with the same mean and with a larger standard deviation under asset risk.

Figure 1 illustrates, for losses as a risk element, capital/loss (c) values for a range of d_L and k values. For a particular EPD ratio, c can be well-approximated by a linear function of k. Notice that the capital ratio is *negative* for high values of d_L and low values of k. When k equals zero, there is *no risk* to policyholders—in order for there to be a positive policyholder deficit, the amount of (riskless) assets must necessarily be less than that of the certain loss. This situation, unlikely in practice, would guarantee a deficit equal to D_L.

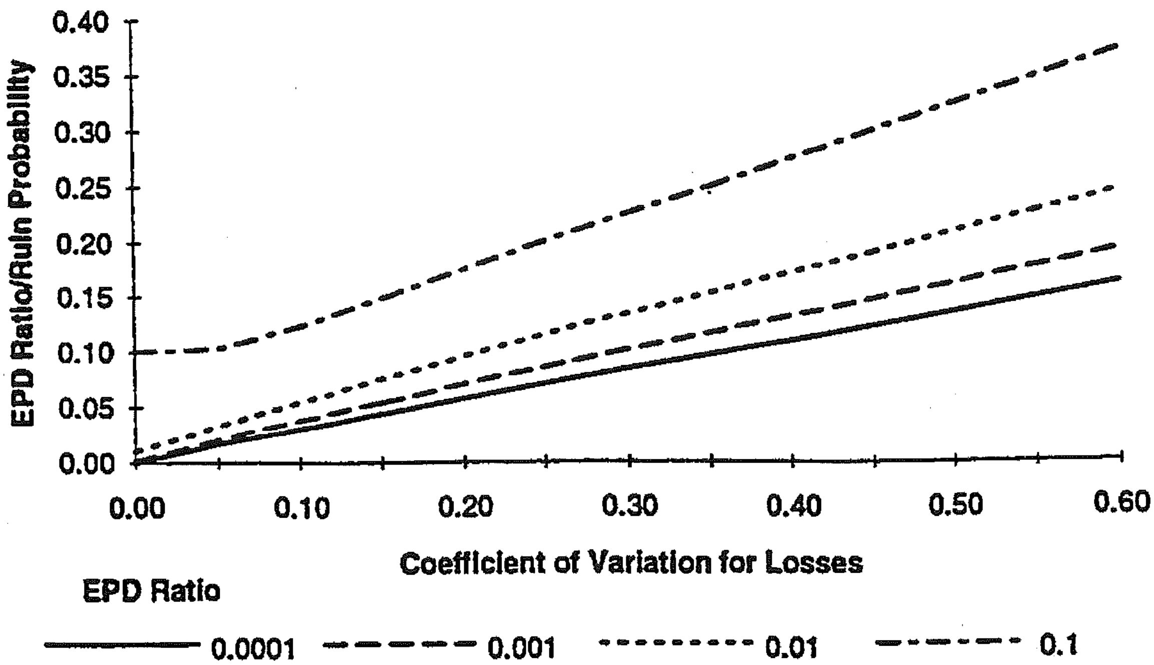
Figure 1
Capital/Loss vs. Expected Policyholder Deficit Ratio and Coefficient of Variation of Losses Under Normal Distribution



It is informative to compare the expected policyholder deficit ratio solvency criterion with the corresponding ruin probability. For loss and asset risk elements, the respective probabilities of ruin are simply $\Phi(-c/k)$ and $\Phi(-c_A/k_A)$. Figure 2 demonstrates that, under the normal distribution, the EPD ratio is an increasing function of the coefficient of variation and the ruin probability.[7] Thus, there is no single ruin probability corresponding to a given EPD ratio.

[7] This relationship follows directly from the comparative statics of the option model, whose analogy with the expected policyholder deficit is discussed below. Specifically, it is well known that an increase in risk of the underlying asset increases the value of a put option on that asset.

Figure 2
Expected Policyholder Deficit Ratio/Ruin Probability vs.
Coefficient of Variation of Losses Under Normal Distribution



The normal distribution might be a reasonable approximation for the variation of aggregate incurred loss amounts arising from a population having a known mean, where individual losses occur independently of each other. An example is noncatastrophe property insurance. For correlated events, and where the mean is unknown, a popular assumption is the *lognormal* distribution (see Aitchison and Brown, 1969, for a thorough explanation of the lognormal distribution and its economic applications). This has the desirable property that negative values are impossible, and the skewness of outcomes appears to accord with observed results. However, the sum of two lognormal variables is only approximately lognormally distributed (the product is a lognormal variable).[8]

For the lognormal distribution, the Appendix also derives analogous formulas for d_L and d_A. Although equation (3) can be used directly, the derivation uses option pricing methods to highlight the option properties of the EPD.[9] The capital ratios c and c_A are determined by solving

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[8] Samuelson (1983, p. 556) proves that the sum of two lognormal variables is in fact lognormal, provided that the sum occurs in a continuous-time framework (as in the Black-Scholes, 1973, option model). However, here we are using a discrete time model.

[9] The standard model for pricing stock options assumes that the stock price exhibits instantaneous geometric Brownian motion, which implies that, at the end of any finite time period, the stock price has a lognormal distribution. The option price can be determined by taking the present value (at riskless interest) of the expected difference between the stock price and the exercise price, where the lognormal distribution has been transformed using a risk-neutral probability measure. This expectation is analogous to the EPD.

$$d_L = \Phi(a) - (1+c)\Phi(a-k) \tag{5a}$$

and

$$d_A = \Phi(b) - \frac{\Phi(b-k_A)}{1-c_A}, \tag{5b}$$

where $a = (k/2) - (\ln(1+c)/k)$,
$b = (k_A/2) + (\ln(1-c_A)/k_A)$, and
$\Phi(\bullet)$ = the cumulative normal distribution.

Unlike the normal distribution, when assets and losses have the same coefficient of variation, under the lognormal model d_A will equal d_L with a *smaller* capital ratio for assets than for losses. Setting $k_A = k$ in equation (5b) and equating with (5a), we get $c_A = c/(1+c) < c$. While the normal distribution is symmetric, the lognormal density is not: the policyholder deficit for losses (actual loss minus fixed assets) can be infinite, but for assets it is limited to the amount of the riskless loss (the minimum asset value is zero). Thus, a relatively smaller capital ratio is required for lognormal asset risk compared to loss risk. For the lognormal distribution, Figure 3 depicts the capital/losses as a function of the coefficient of variation of losses for the same expected policyholder deficit ratios as in Figure 1. Here, the relationship between c and k is not as linear as for the normal case, but the approximation should be reasonable for modest values of the coefficient of variation.

Figure 3
Capital/Loss vs. Expected Policyholder Deficit Ratio and
Coefficient of Variation of Losses Under Lognormal Distribution

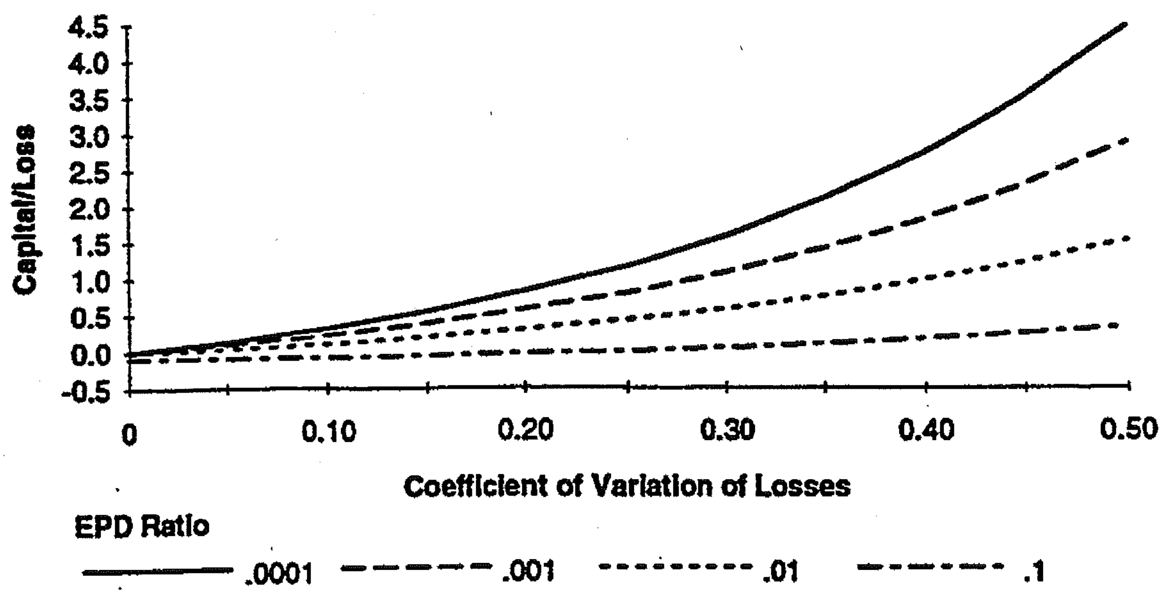
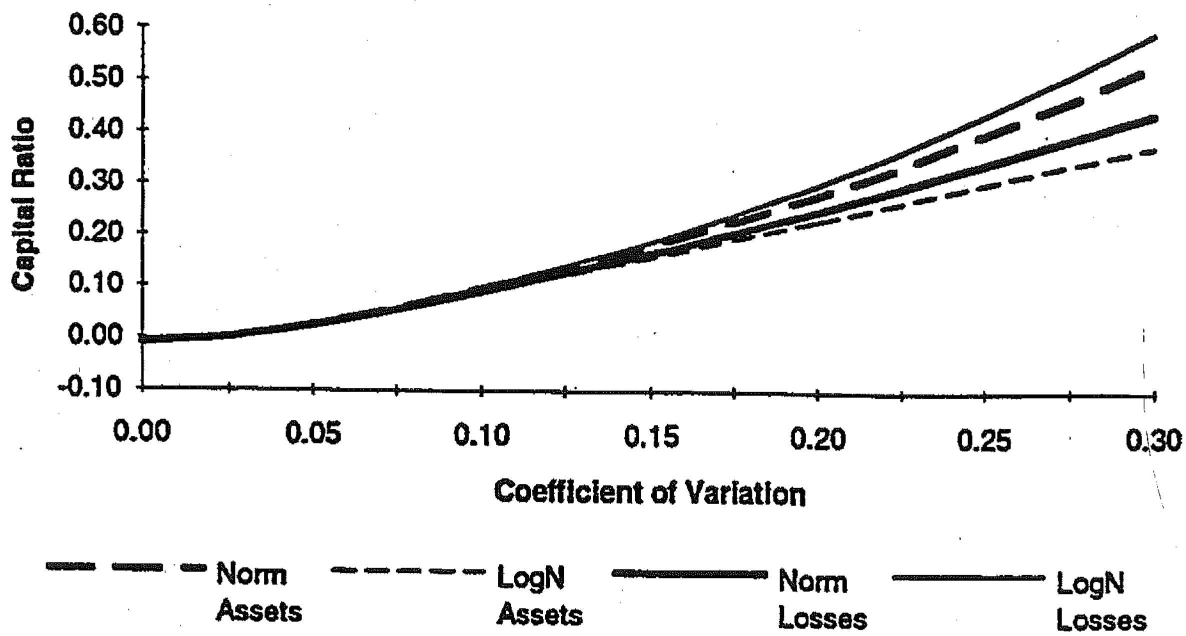


Figure 4 compares the normal and lognormal results directly for d = 0.01. Notice that the capital requirement for losses under the lognormal distribution is greater than for the normal case, with the disparity becoming greater as the coefficient of variation increases. For asset risk, the lognormal distribution requires less capital than the normal case. These relationships are due to the skewness of the lognormal model: the probability of large losses is higher than under the normal distribution and lognormal asset values cannot be negative.

Figure 4
Comparison of Asset vs. Loss Capital Ratios
for Normal and Lognormal Distributions
Expected Policyholder Deficit Ratio = 0.01



The risk-based capital methodology developed so far can be summarized thus: (1) select a particular expected policyholder deficit ratio as the solvency measure, (2) for each risk element, establish a balance sheet with an opposite riskless item (e.g., a risky asset with a riskless liability), keeping the liability amount fixed and (3) using the probability distribution of the risk element amounts, vary the assets to solve for the capital ratio (the amount of capital relative to the amount of the risk element) providing the given EPD ratio.

Risk Measurement and Time

For most risk elements, the probability distribution of realized amounts depends on the length of time until realization. Accounting standards also complicate time-dependent risk measurement.

Accounting Conventions and the Bias Problem

Valuation distortions often appear for financial statement items subject to accounting measurement prior to realization. For example, bonds and real estate will vary in market value, but their statutory or GAAP values, based on purchase price rather than current realizable value, generally remain constant until sold. Conversely, change in an accounting value per se does not connote by the accounting value) that conveys risk. To illustrate, the ultimate value of a discounted unpaid loss may be known with certainty, but although its accounting measure will change (increase) through time, there is no risk present. On the other hand, a risky unpaid loss might carry a constant reserve for several years until the uncertainty is resolved.

For solvency risk measurement, the accounting treatment should directly reveal realizable value variations. Market-value accounting, which sets all balance sheet items at current realizable value, is particularly suitable for solvency assessment, since an insurer's failure usually results in liquidation of the balance sheet or purchase of the company, both in market transactions. Using market valuation, capital is defined as the excess of the market value of assets over the market value of liabilities. Ignoring the value of intangible assets such as goodwill (which would be difficult to incorporate into a practical risk-based capital program), capital is the net liquidation (also called break-up or winding-up) value of the company.

Defining capital as the accounting book value (e.g., statutory surplus or GAAP equity) severely limits the usefulness of a risk-based capital methodology. This is due to accounting bias, which occurs when the current recorded value differs from the current realizable value.

Statutory accounting in particular allows insurers to consistently overvalue or undervalue certain financial statement elements (e.g., loss reserves are not discounted). Further, both SAP and GAAP offer inconsistent measurement between companies by allowing insurers to carry some identical items at different amounts. For example, one insurer may record its loss reserves with a margin for adverse deviation, while another may discount its loss reserves to reflect present value. Also, identical bonds purchased at different times by two insurers may be booked at different amounts.

When bias is present, an insurer's recorded capital will not represent the realizable value of assets over liabilities. For example, if reserves are carried at undiscounted value, the amount of the discount is, in effect, hidden capital. A consistent, unbiased valuation method is essential in equitably assessing insolvency risk among insurers. Consequently, when setting risk-based capital, the financial statement should be adjusted to remove bias.

Time Horizon for Risk Measurement

Under the market valuation standard, it is clear that, in order for the value of capital to change, time must elapse. For example, if the current market value of assets and liabilities are \$1,200 and \$1,000 respectively, the capital will have an unambiguous value of \$200. No matter how risky these items are, market valuation provides a single result when the items are measured concurrently. Since the change in value of capital depends on the passage of time, it follows that insolvency risk must be measured by weighing the possible capital values at a future time. The future capital values are assets minus obligations, both balance sheet quantities. Thus, the relevant insolvency risk elements must be capable of point-in-time estimation, as opposed to a flow-through-time measurement.

In accounting theory, balance sheet items are known as stock quantities, while cash flow and income statement items are called flow quantities. Notice that the commonly-used premium-to-surplus solvency measure is the ratio of a flow to a stock quantity. For consistency in measurement, the ratio of two stock items, such as loss-related reserves and surplus, or two flow items, such as incurred losses and premiums, should be used. The capital ratios (c and c_A, for example) in this model are applied to balance sheet quantities in order to produce the required risk-based capital.

The time span between the current valuation of a financial statement item and a subsequent valuation will greatly affect the measurement of risk. For example, it is more likely for a share of stock to decline 10 percent in value in one year than in one day; similarly, liability reserves to be paid five years from now are more likely to develop adversely by 10 percent than reserves paid in the next six months. Therefore, the degree of risk depends on the time interval between valuations as well as the intrinsic volatility of the item.

The dispersion of future realizable values for many assets, notably stocks, is commonly modeled as a diffusion process.[10] Here the spread of future values becomes greater as time elapses. Similarly, it is known that the variance of unpaid liability losses increases with the time required to pay claims.

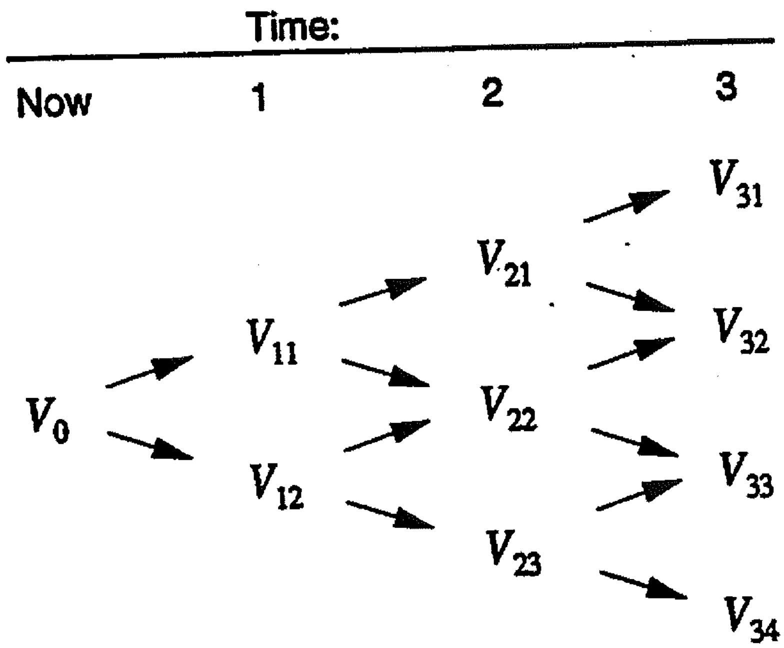
The time-dependent nature of risk is illustrated in Figure 5, where the range of possible risk element values increases with time for three periods. Here the current market value is V_0, and V_{tk} represents the kth possible value at the end of the tth period. The transition of value from one period to the next is governed by a particular probability rule. When the relationship between adjacent nodes is constant (e.g., the probability of moving from, and the rate of change from V_{11} to V_{22} is the same as from V_{23} to V_{34}), the set of possible values at any future point will have the same probability distribution, but with a regularly changing mean and variance.[11] The variance will increase through time (except for trivial cases) even though the mean could remain constant or decrease. A critical notion, therefore, is the variance per unit of time.

Because the variance of realizable values is time-dependent, in order to measure risk consistently, especially for different types of risk elements, it is necessary to establish a common time horizon. Extending the stock vs. reserves example, suppose that the values of the stock at the end of four years and the unpaid loss at the end of one year both have a standard deviation equal to 0.1 times their current value; both risk elements follow a normally distributed

[10] A diffusion process is a type of continuous stochastic process (wherein the probability structure depends on time). The prototype for diffusion processes is Brownian motion, where changes in position are independent increments. It is commonly assumed that infinitesimal stock price changes are normally distributed, producing lognormally distributed stock returns (see Brockett and Witt, 1990, and Cummins, 1988, for additional details).

[11] For example, suppose V_{tk} is a particular node on the lattice and the two subsequent possible values are $V_{tk} + a$ and $V_{tk} - a$ (for all t and k, where a is a constant) with probabilities p and 1 − p. Then the mean will increase by the constant amount $a(2p - 1)$ and the variance by the constant amount $4p(1 - p)a^2$ over each period. If the two subsequent values are *multiples* of V_{tk}, then the mean and the *second moment* each increase by a constant factor each period.

Figure 5
Time Dependence of Risk



diffusion process. Thus, the standard deviation is proportional to the square root of the elapsed time and the reserve standard deviation is twice the stock standard deviation at any particular time. Table 4 illustrates the results for \$1,000 of each item with capital equal to \$100. The two items do not have the same risk: since the reserve standard deviation is higher than that of the stock for a common time horizon, there is a pronounced difference in both the ruin probability and expected insolvency cost.

Table 4
Two Normally Distributed Risk Elements with Different Risk per Time Unit—Beginning Capital = 100

	Value	*Standard Deviation*		*Probability of Ruin (10 Percent Adverse Change)*		*Expected Policyholder Deficit*	
Time	Now	1 Year	4 Years	1 Year	4 Years	1 Year	4 Years
Stock	1,000	50	100	0.023	0.159	0.47	9.26
Unpaid Loss	1,000	100	200	0.159	0.309	8.33	39.56

Note: Equations (4a) and (4b) are used to calculate the expected policyholder deficit values. The capital ratios are 0.10, and the coefficients of variation are equal to (standard deviation)/1,000. Ruin probabilities are determined from the cumulative normal distribution evaluated at [–100/(standard deviation)]. The four-year stock and one-year unpaid loss standard deviations are assumed to be 10 percent of the initial \$1,000 value of each risk element.

Capital Adjustment over a Multiperiod Time Horizon

The parties to the insurance contract are concerned about insolvency of the insurer over a very long time horizon. However, by defining the solvency criterion as the expected policyholder deficit ratio per fixed unit of time, a sufficient and consistent solvency protection can be achieved simply by meet-ing the expected policyholder deficit criterion continually over a short time horizon, equal to the interval between risk-based capital evaluations.

A numerical example will demonstrate this concept. Assume that an insurer has an initial unbiased loss reserve of $1,000. The actual loss payment will occur in three years. Initial assets are $1,100 and remain fixed until the loss is paid; to simplify the example, interest is zero. During each year, information is gathered that enables us to reevaluate our expectation of ultimate loss. The nature of this information is limited, such that the reserve can change, with equal probability, each year up or down by 20 percent of the previous reserve. Thus, the reserve sequence through time is a simple binomial stochastic process.[12] Table 5 shows the progression of possible reserve values, along with the associated probabilities. The EPD when the loss is paid is $98.

Table 5
Time-Dependent Solvency Measurement for
Unpaid Losses Using a Binomial Stochastic Process

Time (t)	*Now (0)*	*1*	*2*	*3*
				1,728 (0.125)
			1,440 (0.25) <	
		1,200 (0.5) <		**1,152 (0.375)**
	1,000 <		960 (0.50) <	
		800 (0.5) <		768 (0.375)
			640 (0.25) <	
				512 (0.125)
Expected Loss	1,000	1,000	1,000	1,000
Assets	1,100	1,100	1,100	1,100
EPD if Liquidated		50	85	98
EPD Ratio		0.050	0.085	0.098

Note: The above values represent either reserve or paid loss amounts, depending on the duration of ultimate loss; the probability of each reserve/payment value is shown in parentheses. The expected policyholder deficit (EPD) is measured now (time 0) for payment at time t. The reserve values in bold represent a policyholder deficit for a three-year horizon if the insurer is liquidated or the claim is paid at that point.

Now assume that the loss follows the same stochastic process but is paid in two years. In other words, the possible paid loss amounts at Year 2 equal the reserve amounts at Year 2 for the three-year case. The two-year EPD is $85 (Table 5). Similarly, under the same stochastic process, a loss paid in one year has a $50 EPD, or a 5 percent EPD ratio.

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[12] By using sufficiently small time intervals, a simple binomial structure can replicate a continuous diffusion process. In particular, if the relationship between adjacent nodes is multiplicative (as in the example here), the result will tend toward a lognormal distribution. Cox, Ross, and Rubinstein (1979) and Trigeorgis (1991) apply the binomial method to evaluate time-dependent contingencies. Some of the numerical results for normal and lognormal distributions in this article were obtained using this technique.

With the amount of assets fixed until settlement, the expected insolvency cost increases with the length of time required to pay the loss, since the spread of possible loss values widens. However, under a typical risk-based capital program, an insurer would be required to adjust its capital (and thus its assets) annually to maintain the minimum requirement. This adjustment process transforms the multiperiod problem into a single-period one.

The annual adjustment procedure is equivalent to liquidating the insurer at the end of each period. When this occurs, the policyholder deficit equals the loss reserve minus the assets. This is true despite the fact that the ultimate loss payment will be unknown at that time. For instance, suppose at Year 1 the reserve is \$1,200 and assets are \$1,100. The insurer is liquidated, releasing its assets to the policyholders. The ultimate losses will be 1,728, 1,152, and 768 with respective probabilities of 0.25, 0.50, and 0.25, yielding respective deficits of 628, 52, and –332. The –332 deficit represents a profit, because the policyholders have, in effect, self-insured for a \$1,100 premium.[13] The expected deficit is \$100, the same as if the \$1,200 reserve were a payment.

Because in this example the probability distribution for the annual percentage reserve change is independent of the elapsed time, a 5 percent one-year forward EPD ratio is achieved when capital is 10 percent of the current expected loss. So, assuming an \$800 reserve outcome at Year 1, the capital will increase to \$300. We can shed most of it, since only \$80 of capital is now needed. As shown in Table 6, the expected deficit over the next period (Year 1 to Year 2) is 0.5(960 – 880) + 0.5(0) = \$40, or 5 percent of the \$800 expected loss.

The capital adjustment can be repeated each period until the loss is finally paid. Thus, although the ultimate expected deficit depends on the duration of the unpaid loss as well as its volatility per unit of time, by measuring the expected policyholder deficit over a fixed time span, we can eliminate the effect of different risk element durations. Notice that without a common time horizon, additional assumptions concerning insurer behavior are required. For example, the holding period for asset sales and capital management strategy must be specified.[14]

In the above simple example, the reserve at any point is equal to the expected paid loss. In a more realistic situation, the market value of the reserve will depend on prevailing interest rates as well as the risk of adverse development. The market value can be obtained by valuing the unpaid losses as in a

[13] To avoid having a profit, the policyholders could reinsure 11/12 of the loss for the \$1,100 premium. In this case the reinsurer's expected loss equals the premium, so the deal is fair. The policyholder deficit remains at \$100.

[14] In the example of Table 5, suppose that the insurer always adjusts capital so that the one-year EPD ratio is 5 percent and defaults only if the loss paid in Year 3 exceeds the Year 2 assets. In this case, the ultimate EPD will be \$50, the same as the EPD over just the first year. But if the insurer always defaults when the reserve or loss payment exceeds the assets and withdraws capital when the reserve becomes smaller (still maintaining the 5 percent one-year EPD ratio), then the ultimate EPD will increase to \$78.

commutation with the policyholder or a loss reserve portfolio transfer to another insurer or reinsurer.[15]

Table 6
Time-Dependent Solvency Measurement for Unpaid Losses
Using a Binomial Stochastic Process Viewed One Year Later
Given that Expected Unpaid Loss is $800

Time (t)	*Now (1)*		*2*			*3*	
						1,152	**(0.25)**
			960	**(0.5)**	<		
	800	<				768	(0.50)
			640	(0.5)	<		
						512	(0.25)
Expected Loss	800		800			800	
Assets	880		880			880	
EPD if Liquidated			40			58	
EPD Ratio			0.050			0.085	

Note: The format is the same as Table 5's. The present time is one year later (time 1). The expected policyholder deficit (EPD) is measured at present for a liquidation at time t. The reserve values in bold represent a policyholder deficit if the insurer is liquidated at that point (or at the third year, when the actual claim must be paid). Capital has been reduced to provide a one-year forward EPD ratio of 5 percent.

The capital adjustment process can be described symbolically by extending the earlier notation. Let $\tilde{A}_t$, $\tilde{L}_t$, and $\tilde{C}_t$ be random variables denoting respective assets, liabilities, and capital at time t. The amounts of these variables are known at present: A_0, L_0, and C_0, with $A_0 = (1+c)L_0$. Further, let $\tilde{A}_1 = (1+\tilde{r})A_0$ and $\tilde{L}_1 = (1+\tilde{g})L_0$, where $\tilde{r}$ and $\tilde{g}$ are random variables denoting the annual return on assets and the annual rate of change in value of the liabilities (i.e., the expected value of $\tilde{g}$ is a risk-adjusted discount rate).

An important variable is $\tilde{C}_1$, the amount of capital at the end of one period. Define $\tilde{c}_1 = \tilde{C}_1/L_0$ as the amount of capital relative to the original expected loss. Then $\tilde{c}_1 = c + (1+c)\tilde{r} - \tilde{g}$, and we have the one-period expected policyholder deficit ratio

$$d_1 = \int_{-\infty}^{0} -zp(z)dz, \tag{6}$$

where $p(\bullet)$ = the density of $\tilde{c}_1$. For a given value of d_1, then, we solve the formula for the beginning capital/loss ratio c. The Appendix applies this formula to derive EPD ratios for the normal distribution when assets and liabilities are

[15] The market value of loss reserves would also implicitly assume that there is no insurer default; otherwise, it would depend on the capital levels of individual insurers and thus could not be a market value. In the absence of a ready market for the trading of loss reserves, market values can be approximated by discounting using a risk-adjusted interest rate (see Butsic, 1988, and D'Arcy, 1988, for details).

both random variables (the negative sign in the integral converts negative capital amounts to positive policyholder deficits).

To summarize, the capital adjustment process guarantees that the policyholders will maintain or exceed the same expected policyholder deficit ratio over each successive year. Therefore, by choosing a common time horizon, a consistent level of policyholder safety can be provided without regard to the actual duration of the risk element. The key requirements are that we know the time-dependent nature of the future realizable values, that the insurer can liquidate its assets and liabilities at each evaluation point (although it does not have to do so), and that insurers will add capital when needed. Notice that it is not necessary for the time-based probability structure to be uniform, as in the preceding example—we only need to know the current market value and the probability distribution of future market values at the *next* valuation date.[16]

The Insurer as a Going Concern

The preceding discussion treated the runoff of an insurer; in other words, it did not consider the risk from policies (both new and renewal) becoming effective in the future. This contingency is considered a major risk element, with rapid growth in business a primary cause of property-liability insolvencies. (A study by A. M. Best, 1991, showed that over the period 1969 through 1990, 21 percent of property-liability insolvencies were caused by rapid premium growth.) Nevertheless, the periodic capital calibration to assure a minimum expected policyholder deficit ratio will also work for an insurer as a going concern, continually writing new business.

To incorporate the future business into the calibration procedure, we still assume the capability of liquidating the insurer at the end of each successive period. However, now the year-end capital $\tilde{C}_1$ will be affected by both the runoff of the initial balance sheet (assets and liabilities A_0, L_0) and by the period-ending value of the business added during the interval. Let P be the premium (net of expenses), assumed to be entirely collected just after the beginning of the period, and let $\tilde{L}_P$ be the loss from the added premium, assumed to be incurred at the end of the period and paid at the end of a subsequent period.[17] Then the end-of-period assets and liabilities are $\tilde{A}_1 = (A_0 + P)(1+\tilde{r})$ and $\tilde{L}_1 = L_0(1+\tilde{g}) + \tilde{L}_P$, where $\tilde{r}$ and $\tilde{g}$ are as previously defined (we assume that the premiums are invested in the same assets as A_0; also the premium is known in advance and thus is not a random variable).[18]

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[16] The current market value will embody the market's knowledge of all possible future values. Thus, knowing (or being able to estimate) the market value will, in effect, provide information concerning the forward probabilities.

[17] The beginning and end-of period assumptions are not necessary, but help to clarify the analysis. A more refined approach would place the premiums and incurred losses in the middle of the period.

[18] This assumption is reasonable, since we can predict the amount of premium for the next year fairly well from the insurer's business plans and knowledge of the current market. However, it would be increasingly difficult to accurately forecast premiums longer into the future. Even

By choosing a constant p we relate the premium to the initial capital: $P = pC_0 = pcL_0$. Also, define a random variable $\tilde{b}$ equal to the incurred loss ratio: $\tilde{L}_P = \tilde{b}P = \tilde{b}pcL_0$. Using the previous definition of the period-ending capital to the initial liabilities, we get

$$\tilde{c}_1 = c(1+p) + [1+c(1+p)]\tilde{r} - pc\tilde{b} - \tilde{g}. \tag{7}$$

Finally, we solve equation (7) for the value of c needed to determine d_1.

Equation (7) is a linear function of three important random variables comprising the bulk of the risk facing the typical property-liability insurer. We have already discussed the role of the asset and loss reserve risk, represented by $\tilde{r}$ and $\tilde{g}$. These can be modeled as diffusion processes. The incurred losses (represented by $\tilde{b}$), as well as a portion of the unearned premium reserve, have components that are qualitatively different from the other risks. In particular, property coverages are subject to catastrophes, which are highly unpredictable, and, being paid quickly, cannot be modeled as diffusion processes (Cummins, 1988, addresses catastrophes by modeling them as a jump process added to a diffusion process). I will return to the three risk categories below in a discussion of correlation of risk elements.

As stated earlier, risk elements are balance sheet quantities. Although the above premiums and incurred losses are income statement items occurring between evaluations, their present value is a balance sheet quantity. A true market valuation would include the present values of future premiums collected and losses and expenses paid arising from business not yet written. But since assessing the worth of an insurer's future business would be a formidable task for a regulator using only public financial statements (except for business added in the upcoming year), this item will be ignored as a risk element for practical risk-based capital applications.

Because the present value of future business is a balance sheet quantity, the capital adjustment process guarantees that the policyholders will maintain at least the minimum EPD ratio each year, even if more exposures occur between evaluation points.

Insolvency Cost as a Financial Option

The model for the expected policyholder deficit ratio as developed so far is nearly complete. However, the *present value* of the policyholder deficit also must be considered: one dollar of forfeited claim a year from now is worth less than one dollar lost now. Since we are evaluating the expected policyholder deficit occurring one year hence, its value is reduced by $1/(1+i)$, where i is a default-free interest rate with a one-year duration (i.e., a one-year principal-only treasury note). For the remainder of this article, it is assumed that the EPD is measured at present value.

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though there is considerable risk to the long-range premium forecast, the insurer does not need extra capital *now* to offset the future uncertainty. The annual risk-based capital calibration process

With the addition of the present value concept, the expected policyholder deficit is now completely analogous to a *financial option* having a one-year maturity. Return to the example with a 50/50 chance of a \$1,200 or \$800 loss reserve value at the end of one year, with \$1,100 in assets at the end of the year. The EPD valued at that point is \$50. With 8 percent interest, the present value of the EPD is \$50/1.08 = \$46.30.

Now suppose that a share of stock has a current price of \$1,000 but will be worth either \$1,200 or \$800 in one year with equal probability. An option to buy (a call option) one share a year from now for \$1,100 (the exercise price) is available.[19] If the stock turns out to be worth \$1,200, the option is worth \$100. If its price is \$800, the option would not be exercised (if so, the holder would lose \$300) and thus its value would be zero. The expected option value at the exercise date is therefore \$50 = 0.5(100) + 0.5(0). Its present value at the same 8 percent interest rate is \$46.30, which is identical to the value of the expected policyholder deficit.

Thus, for a liability risk element paired with riskless assets, the value of the EPD is equivalent to that of a call option on the losses with an exercise price equal to the value of the assets (A_1) at the end of the year (see Table 7). In effect, because liabilities may exceed the insurer's assets, its policyholders have given the insurer's owners the *option* to abandon full payment of claims. The legal concept of corporate limited liability (nonassessment for mutual policyholders/owners) creates this option. Garven (1992) discusses the pricing and incentive implications of the "limited liability" option for stock and mutual insurers.

Table 7
Equivalence Between Call Option on Stock and Policyholder Deficit—Risky Liabilities and Riskless Assets

Stock		*Insurance*
S_0: Current Stock Price	↔	L_0: Current Liability Value
S_1: Stock Price in One Year	↔	L_1: Liability Value in One Year
E_1: Exercise Price	↔	A_1: Asset Value in One Year
E_0: Present Value of Exercise Price	↔	A_0: Current Asset Value
$E_0 - S_0$	↔	C_0: Current Capital Value
$\text{Max}[0, S_1 - E_1]$: Option Value When Exercised	↔	$\text{Max}[0, L_1 - A_1]$: Policyholder Deficit

For an asset risk element (paired with a riskless liability), the EPD is equivalent to the value of a put option on the ending assets implicitly given by the

[19] An option exercisable only at the expiration date, as we have assumed here, is called a *European* option. An option exercisable at any time until the expiration date is called an *American* option. Technically, policyholders have implicitly written American options against their claims, since the insurer's owners can "exercise" during the year, rather than at the evaluation dates. However, the difference in option value (present value of the EPD) would not be significant.

policyholders (see Table 8). Here, if the asset value (stock price) in one year is less than the liability value (exercise price) in one year, the difference is *put* to the policyholders (the option seller). A put option equivalence for both a risky asset and a risky liability is shown in Table 9 (see Brealy and Myers, 1988, and Cox and Rubinstein, 1985, for discussions of option relationships).

Table 8
Equivalence Between Put Option on Stock and Policyholder Deficit—Risky Assets and Riskless Liabilities

Stock		*Insurance*
S_0: Current Stock Price	↔	A_0: Current Asset Value
S_1: Stock Price in One Year	↔	A_1: Asset Value in One Year
E_1: Exercise Price	↔	L_1: Liability Value in One Year
E_0: Present Value of Exercise Price	↔	L_0: Current Liability Value
$S_0 - E_0$	↔	C_0: Current Capital Value
$\text{Max}[0, E_1 - S_1]$: Option Value When Exercised	↔	$\text{Max}[0, L_1 - A_1]$: Policyholder Deficit

Table 9
Equivalence Between Put Option on Stock and Policyholder Deficit—Risky Assets and Risky Liabilities

Stock		*Insurance*
S_0: Current Stock Price	↔	C_0: Current Capital Value
S_1: Stock Price in One Year	↔	C_1: Capital Value in One Year
E_1: Exercise Price	↔	Zero
E_0: Present Value of Exercise Price	↔	Zero
$S_0 - E_0$	↔	C_0: Current Capital Value
$\text{Max}[0, E_1 - S_1]$: Option Value When Exercised	↔	$\text{Max}[0, -C_1]$: Policyholder Deficit

The idea of insurer solvency cost being a financial option is a fairly recent development, trailing the rapid growth of stock option trading in the 1970s. For a more thorough treatment of the topic, see Doherty and Garven (1986), Cummins (1988), Derrig (1989), and Garven (1992). In particular, Cummins shows that the value of the risky asset-liability put option (here, the EPD) is the fair risk-based guaranty fund premium. The Appendix derives EPD ratios for the lognormal distribution by using call and put option equivalents to the respective EPDs for risky losses and assets.

We now have a complete capital-setting model for individual risk elements: determine how much capital per unit of risk element satisfies a standard value of the one-year discounted expected policyholder deficit ratio.[20] For a liability

[20] Due to the annual horizon of a practical risk-based capital program, taking the present value of the EPD will not change the *relative* capital ratios needed for one risk element versus another. This is because the same riskless interest rate should be used for all risk elements. Thus, for example, if the EPD ratio standard is set by requiring a specified percentile of insurers failing to reach the standard, then taking the present value of the EPD is not necessary.

risk element, we assume that the related asset is riskless, with annual return $\tilde{r} = i$. In parallel fashion, a risky asset is paired with a riskless liability, whose market value grows at an annual return of $\tilde{g} = i$. The next section extends the results to the more likely case where both assets and liabilities are risky.

Correlation and Independence of Risk Elements

The preceding sections have demonstrated how risk-based capital for each risk element can be calculated separately by treating each element in the balance sheet of a mini-insurer. As shown below, in order to combine the risk capital for the separate elements, one cannot simply add their required capital amounts together unless the risk elements are perfectly correlated (with the proper sign).

A Numerical Illustration

Suppose that we have a line of business with riskless assets and risky losses, which can have only two possible realizable values. The values and their probabilities are given in Table 10. The desired expected policyholder deficit ratio is 1 percent. The risk-based capital needed for this degree of protection is easily calculated at $2,900.

Assume that another line of business has an identical loss distribution, directly correlated with the first: if a $2,000 loss occurs for the first line, the

Table 10
Insurer with Two Independent Lines of Business with Same Unpaid Loss Distribution—Asset Amount Is Certain

	Asset Amount	*Loss Amount*	*Capital Amount*	*Probability*	*Claim Payment*	*Deficit*
Single Line						
	6,900	2,000		0.6	2,000	0
	6,900	7,000		0.4	6,900	100
Expected Value	6,900	4,000	2,900		3,960	40
Capital/Loss		0.725				
EPD Ratio		0.010				
Two Independent Lines						
	13,800	4,000		0.36	4,000	0
	13,800	9,000		0.48	9,000	0
	13,800	14,000		0.16	13,800	200
Expected Value	13,800	8,000	5,800		7,968	32
Capital/Loss		0.725				
EPD Ratio		0.004				

Note: When capital for the two-line case is reduced to 5,500, the expected policyholder deficit (EPD) ratio becomes 0.16(14,000 – 13,500)/8,000 = 0.010; the capital/loss drops to 0.687.

same amount occurs for the second line; similarly, a \$7,000 amount will occur concurrently for both lines. The effect of combining the two lines is the same as if we now had a single line twice as large as the original single line. The amount of capital per unit of expected loss needed to provide the 1 percent EPD ratio remains the same at 0.725.

Now suppose that the two lines are statistically *independent:* the value of the loss for one line does not depend on the value for the other. Combining the two lines and adding their separate \$2,900 risk-based capital amounts creates the EPD for the composite line (see Table 10). Here the probability of a loss exceeding assets is reduced from 0.40 to 0.16, and the \$32 expected deficit for the combined lines is less than the sum of the individual expected deficits (\$80). This produces a 0.004 protection level, compared to the 0.010 value for the separate lines. To reach the same 1 percent level as before, we need *less* capital than obtained by adding the separate amounts of risk-based capital: only \$5,500 of capital is required, which is \$480 less than the \$5,980 needed when the losses are correlated. The capital ratio to loss drops from 0.725 to 0.687.

For a discrete loss distribution, with assets certain, the EPD for the sum of n independent equal losses is given by $D_L = \sum_{x>A} p_n^*(x)(x-A)$, where $p_n^*(\bullet)$ is the n-fold convolution of the probability density for the losses ($0 \leq x < \infty$).

The reason for the reduced capital requirement through independence of risk elements is the *law of large numbers*. When losses are independent of each other, a small line of business will need a relatively large amount of capital per unit of loss, while a larger one requires a much smaller capital ratio. Practically, however, there is a limit to the risk reduction allowed by the law of large numbers. The mean or other parameters of the loss distribution are rarely known with certainty, introducing systematic or parameter risk affecting all exposures. Thus, even an insurer with a very large homogeneous book of business will remain subject to considerable uncertainty, and consequent capital needs.

Correlation Under the Normal Distribution

Although the preceding numerical example illustrates the capital reduction due to independence of risk elements, one must be careful not to generalize regarding the degree of reduction. For example, using a 0.1 EPD ratio, the capital requirement drops to \$2,000 for the single line of business. The *combined* capital need drops to \$1,000 for the two independent lines—less capital than for a single line. This effect is due to using a discrete probability distribution with a limited range of outcomes. More robust conclusions can be reached by analyzing a continuous probability model, such as the normal distribution.

The normal distribution has the important property that sums of normal random variables are themselves normal random variables with additive means and easily-computed variances. Table 11 provides the mean and variance for

Table 11
Mean and Variance of Capital with Two Normally Distributed Assets or Liabilities

Random Variables	*Mean*	*Variance*
Two Assets	$C = A_1 + A_2 - L$	$\sigma^2 = \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2$
Two Liabilities	$C = A - L_1 - L_2$	$\sigma^2 = \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2$
Asset and Liability	$C = A - L$	$\sigma^2 = \sigma_A^2 + \sigma_L^2 - 2\rho\sigma_A\sigma_L$

Note: σ_1 and σ_2 denote the standard deviations of risk elements 1 and 2 (either assets or liabilities). For the asset and liability combination, σ_A is the total asset standard deviation, and σ_L is the total liability standard deviation. The correlation coefficient between risk elements is ρ.

the composite of two normally distributed assets ($\tilde{A}_1$ and $\tilde{A}_2$), or two liabilities ($\tilde{L}_1$ and $\tilde{L}_2$), or an asset and a liability ($\tilde{A}$ and $\tilde{L}$).

With perfect positive correlation ($\rho = 1$), $\sigma = \sigma_1 + \sigma_2$ for risk elements on the same side of the balance sheet or $\sigma = \sigma_A - \sigma_L$ for assets and liabilities. With perfect negative correlation ($\rho = -1$), $\sigma = \sigma_1 - \sigma_2$ or $\sigma = \sigma_A + \sigma_L$. When the elements are independent, $\rho = 0$, and thus $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$ and $\sigma = \sqrt{\sigma_A^2 + \sigma_L^2}$ for the two cases. As shown in the Appendix, the formula for the EPD ratio with normally distributed combined risk elements is identical to that for individual elements, equations (4a) and (4b).

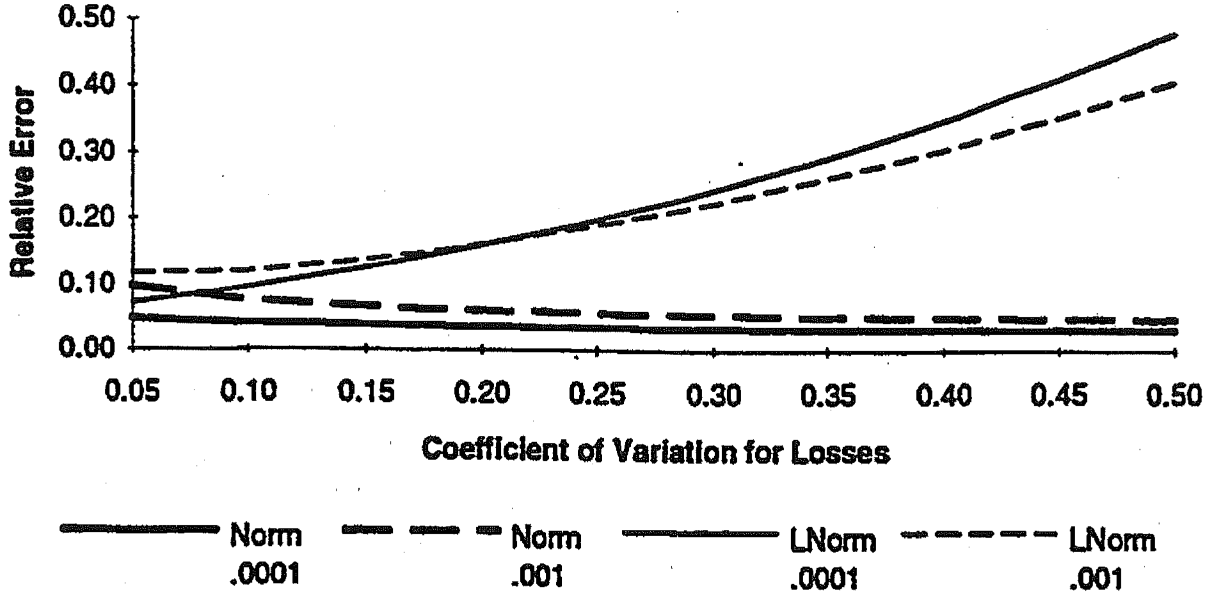
Here c is the capital to loss, k is the total standard deviation divided by the total expected loss L, and D is the total expected policyholder deficit. Similarly, the approximate lognormal (assuming that the sum of two lognormal variables is lognormal) EPD ratio for combined risk elements is identical to equations (5a) and (5b).

As indicated above, for the normal and lognormal distributions, the relationship between c and k is approximately linear for a fixed EPD ratio d and small k. Since $c = -d$ when $k = 0$ (no risk), we have $c \equiv ak-d$ for some constant a. Assuming that a high level of protection is desired (d less than 1 percent or so), we further simplify the relationship to $c \equiv ak$.

Since the total capital C equals cL, and the total standard deviation σ equals kL, it follows that if $c = ak$, then $C = akL = a\sigma$. Therefore, the risk-based capital for the total of separate risk elements is proportional to their combined standard deviation. Risk capital for perfectly correlated items can be added (or subtracted, depending on whether the correlation is positive or negative or whether the items are on the same side of the balance sheet). Risk capital for independent (and partially correlated items) can be combined according to the square root of the sum of the squares of their standard deviations, plus twice the product of their standard deviations and the correlation coefficient. We will refer to this as the *square root rule*.

The relative error in using the square root rule for two independent loss risk elements of the same size and standard deviation is shown in Figure 6. Here the relative error is defined as (approximated minus true capital)/(true capital). For the normal distribution, the error decreases as the EPD ratio decreases and as the risk increases. For a reasonable (i.e., 0.001) protection level, the error is less than 10 percent. For the lognormal model, the error increases with increas-

Figure 6
Relative Error Using Square Root Approximation
for Losses Under Normal and Lognormal Distributions
Expected Policyholder Deficit Ratios = 0.0001 and 0.001



ing risk and tends to be much larger than with the normal distribution. However, for modest values of k, the linear approximation may be acceptable. Notice that for both distributions the relative error is positive, meaning that the square root rule overstates required capital.[21] Thus, the approximation is conservative.

To illustrate, suppose that we have two independent normally-distributed lines of business, each with a \$1,000 expected loss and \$200 standard deviation. For a 0.001 expected policyholder deficit ratio, each requires \$438 of capital in isolation (from equation (4a) with $k = 0.2$ and $c = 0.438$). When the lines are combined, the overall standard deviation increases to \$283, but the capital ratio declines to 0.292, giving \$584 in capital when applied to the \$2,000 total expected losses. The square root rule produces $\$620 = 438\sqrt{2}$, which is about 6 percent more than the exact calculation yields.[22]

A parallel calculation with the lognormal distribution, using equation (5a) shows a 16 percent error: the true required capital is \$700, compared to \$812 from the square root rule.

The square root rule can be extended to incorporate more than two risk elements. The total capital C is a function of the individual element risk capital amounts C_i and the separate correlation coefficients between each pair of n risk

[21] The reason for this is apparent from Figure 3, which shows the capital ratio c to be a *concave upward* function of k. In other words, $d^2c/dk^2 > 0$. Adding an equal, independent loss reduces k to $k' = k/\sqrt{2}$, but the capital ratio drops by even more, to $c' < c/\sqrt{2}$.

[22] Because the error in using the square root for the normal and lognormal distributions overstates the combined amount of capital needed, a closer fit could be had by using a root higher than two. For instance, in the normal example given, using a 2.4th root gives an exact result

elements (notice that the sign of the correlation coefficient depends on which side of the balance sheet the two items reside):

$$C = \left[\sum_{i=1}^{n} C_i^2 + \sum_{i \neq j}^{n} \rho_{ij} C_i C_j \right]^{\frac{1}{2}}. \tag{8}$$

Practical Application of Correlated and Independent Risk Elements

The preceding analysis has shown the effect of correlation between risk elements. Some examples of balance sheet items having varying degrees of correlation are presented in Table 12. In general, reinsurance transactions create a high degree of correlation between ceding and assuming parties. Ownership of insurance subsidiaries (affiliates) or stock also produces highly correlated values. Where it is difficult to determine the numerical correlation between items, a practical approach would be to judgmentally peg the correlation at zero, 1, or –1, whichever is closest to the perceived value.

Table 12
Independent and Correlated Risk Elements

Correlation	*Asset/Asset*	*Liability/Liability*	*Asset/Liability*
Positive	Common Stock/ Preferred Stock, Common Stock/Bonds	Loss Reserve/LAE Reserve	Bonds/Loss Reserve
Zero	Cash/Real Estate	Liability Loss Reserve/ Property Unearned Premium Reserve	Common Stock/ Unearned Premium Reserve
Negative	Common Stock/ Put Options	Loss Reserve/Income Tax Liability, Loss Reserve/ Dividend Reserve	Property-Liability Stock/Loss Reserve, Reinsurance Recoverable/ Loss Reserve

We can demonstrate the effect of independent and correlated risk elements by constructing a numerical example. Table 13 presents risk elements from a hypothetical insurer's balance sheet at market values. The capital ratios assume a 0.005 EPD ratio and are based roughly on empirical data.

All risk elements are assumed to be lognormally distributed, and the EPDs are discounted at an 8 percent riskless interest rate (equations (5a) and (5b), adjusted for the interest rate, are used for this calculation). The loss reserve, equal to the present value of the expected payments, also includes the loss expenses and the liability portion of losses arising from the unearned premiums. The affiliate stock risk is assumed to be the same as for noninsurance stock.

The 20 percent stock capital factor arises from using a 0.168 standard deviation of 1946 to 1989 annual returns from Ibbotson and Associates (1990). Based on the same source, we have used a 0.06 annual standard deviation for bonds (the corporate bond standard deviation is 0.098 for a 20-year maturity;

Table 13
Risk-Based Capital (RBC) Calculation Using the Square Root Rule: Input Assumptions

Risk Element	*Amount*	*Capital Ratio*	*RBC*
Stocks	200	0.20	40
Bonds	1,000	0.05	50
Affiliates	100	0.20	20
Loss Reserve	800	0.40	320
Property UPR	100	0.20	20
Total			450

Correlated Risk Elements		*Correlation Coefficient*
Stocks	Bonds	0.2
Stocks	Affiliates	1.0
Bonds	Affiliates	0.2
Bonds	Loss Reserve	0.3
Affiliates	Loss Reserve	-1.0

adjusting for a more typical property-liability insurer's duration gives a lower value), producing an approximate 5 percent capital ratio.

The loss reserve capital ratio is based on a study of loss ratio variation by Derrig (1989), who used a sample of workers' compensation and private passenger auto loss ratios from 51 insurers over the period 1976 through 1985 (since calendar-year losses were used, the variance should be similar to that for loss reserves). The combined annual variance was 0.059, which was judgmentally reduced to 0.045, reflecting a greater variance in the unpaid loss tail; the variance is lowered when the loss is brought to present value. This produces a capital ratio (to the discounted loss) of about 0.40.

Notice that a further adjustment would be needed to convert the loss capital factor for application to an *undiscounted* loss reserve: using a 16 percent reserve discount (three-year loss duration at a 5 percent risk-adjusted interest rate), the required statutory surplus is $(1 + 0.40)(1 - 0.16) - 1 = 0.176$ times the undiscounted reserve. This illustrates the bias problem inherent in statutory accounting—if in this example the discount is greater than 29 percent, then the required capital is negative! Market-value accounting would avoid this problem.

The sum of the separate risk-based capital amounts is $450. This value assumes that *all* items are fully correlated, ignoring any independence or partial covariance between the items. The pairs of elements that are assumed to be correlated for this example are presented in Table 13.[23]

[23] The correlation coefficient for common stocks and bonds is based on the Ibbotson and Associates (1990) data. The other correlation coefficients are determined judgmentally. In practice, due to limited data, for a particular insurer one would have to rely heavily on industry results (perhaps modified with judgment) to achieve accurate correlation estimates.

The property unearned premium reserve is independent of all other items. Notice that the bonds/reserve correlation coefficient is positive due to the parallel change in value from interest rate movements; because these two items are on opposite sides of the balance sheet, their joint movement will reduce total risk.[24] Similarly, the negative sign of the affiliates/reserve correlation coefficient indicates that these opposing items will increase total risk when combined.

Applying equation (8), the sum of the squares of the separate risk capital amounts is 107,300. The sum of the cross products (each of the above pairs appears twice) of the capital amounts times their correlation coefficients equals 6,000. Thus, the approximate total risk capital is $\$337 = \sqrt{113{,}300}$. If all the risk elements were independent, the total required capital would be only $\$328 = \sqrt{107{,}300}$.

The impact of the bond/reserves covariance can be found by setting the correlation coefficient to zero: here the total risk capital increases to \$351. Thus, the effect of their correlation is to reduce required capital by \$14.

Notice that, for this example, if the correlations between risk elements are not known, the potential error associated with assuming that they are independent when they are fully correlated is about 27 percent of the true risk-based capital. Similarly, the error from assuming that they are fully correlated when they are independent is about 37 percent. This degree of uncertainty can have a greater impact on the insurer's total RBC than the uncertainty in assessing the RBC for a particular risk element. For example, if the risk elements are fully correlated, a 100 percent error in specifying the bond RBC (i.e., \$0 or \$100 instead of the indicated \$50) creates only an 11 percent error in the total risk-based capital. Thus, knowing the degree of correlation between risk elements can be as important as knowing the risk of individual items.

A more sophisticated risk-based capital calculation would divide the risk elements into additional categories and might include a provision for the value of future business.

Summary and Conclusion

This article has addressed insolvency risk measurement for a risk-based capital program, with the following key results:

1. The relevant measure of solvency is the present value of the expected policyholder deficit as a ratio to the expected loss. This value is equivalent to a put option held by the insurer's owners and equals a fair risk-based guaranty fund premium. By requiring sufficient capital to meet or exceed a common

[24] The correlation methodology provides a means of allowing for matching of asset and liability durations. If the durations of fixed maturity assets and loss payments were equal, and the movements in value were due solely to interest rate fluctuations, then a (negative) 100 percent correlation coefficient would be appropriate.

expected policyholder deficit ratio standard for each insurer, policyholders are assured a consistent minimum level of protection.

2. To remove measurement bias caused by accounting conventions and varying insurer practices, the valuation standard for risk-based capital application should determine a market value for each risk element.

3. The major components of insurance risk are time-dependent: the longer the time to realization, the greater the risk. This relationship is particularly important for stocks, bonds, loss reserves, and loss adjustment expense reserves. In order to properly compare risk between these items, a common time horizon must be used.

4. The expected policyholder deficit ratio is based on expected market values at the end of each risk-based capital valuation interval (generally one year). When risk capital levels can be set periodically, with sufficient time for insurers to add capital where necessary, there is no need for additional capital to absorb fluctuations in value beyond the valuation interval. Capital is not required now for distant contingencies.

5. The risk-based capital for an insurer will always be less than the sum of the separate RBC amounts for each risk element, to the extent that all the elements are not fully correlated. By assuming a normal or lognormal distribution, an approximate method for combining risk capital is the square root of the squared individual risk-based capital amounts plus additional terms involving the correlation coefficients. Knowing the degree of correlation between risk elements can be as important as knowing the risk of individual items.

Although this article has viewed the solvency problem from a regulator's perspective, the concepts here could readily be applied to an insurer's in-house capital management. For example, the insurer might want a consistent level of capital higher than the regulatory target RBC. Or, in the absence of a regulatory risk-based capital program, the insurer may wish to set its own standards.

Other applications for the risk measurement concepts presented here include setting risk loadings for reinsurer default, since the relationship between a ceding insurer and an assuming reinsurer is analogous to that of a policyholder and an insurer (the ceding commission for reinsured business should include a provision for the reinsurer's possible insolvency). Another practical use might be establishing solvency ratings for insurers based on the relationship between their recorded capital (adjusted for known bias) and their risk-based capital.

Although some empirical results have been presented in order to explain the application of the methodology, the findings are still rudimentary. It is especially important to determine more accurate assumptions for the distribution of loss reserve risk and to measure the correlation between risk elements.

Appendix

Expected Policyholder Deficit Under the Normal Distribution

The amount of capital is $\tilde{C} = \tilde{A} - \tilde{L}$, where $A = (1+c)L$. $\tilde{A}$ is the value of assets and $\tilde{L}$ is the value of unpaid losses, both random variables. The expected value of $\tilde{C}$ is $cL \equiv \mu$ and the variance of $\tilde{C}$ is $\sigma^2 = \sigma^2_A - 2\rho\sigma_A\sigma_L + \sigma^2_L$, where σ^2_A and σ^2_L are the respective variances of assets and losses, and ρ is the correlation coefficient. The policyholder deficit is $\tilde{L} - \tilde{A} = -\tilde{C}$ for $\tilde{L} > \tilde{A}$ or $\tilde{C} < 0$. The expected policyholder deficit is $D = \int_{-\infty}^{0} -zp(z)dz$, where $p(z) = [1/(\sigma(\sqrt{2\pi}\,)] \exp\{-(z-\mu)^2/2\sigma^2\}$ is the normal probability density function. Let $y = (z - \mu)/\sigma$. Then $dz = \sigma dy$, and we have

$D = \int_{-\infty}^{-\mu/\sigma}\left[-(\mu+y\sigma)/\sigma\sqrt{2\pi}\right] \exp(-y^2/2)dy$, which reduces to

$$D = \sigma\phi\left(\frac{-\mu}{\sigma}\right) - \mu\Phi\left(\frac{-\mu}{\sigma}\right), \qquad \text{(A1)}$$

where $\Phi(\bullet)$ = the cumulative standard normal distribution, and $\phi(\bullet)$ is the standard normal density. Notice that the probability of ruin ($\tilde{C} < 0$) is $\Phi(-\mu/\sigma)$. Define $k_T \equiv \sigma/L$, the ratio of the standard deviation of the capital (total assets minus losses) to the expected loss. Then $\mu/\sigma = (cL)/(k_T L) = c/k_T$. The EPD ratio is

$$d = \frac{D}{L} = k_T\phi\left(\frac{-c}{k_T}\right) - c\Phi\left(\frac{-c}{k_T}\right). \qquad \text{(A2)}$$

Letting the variance of assets be zero, we have the expected policyholder deficit ratio for risky losses:

$$d_L = k\phi\left(\frac{-c}{k}\right) - c\Phi\left(\frac{-c}{k}\right), \qquad \text{(A3)}$$

where $k = \sigma_L/L$, with σ_L being the standard deviation of losses.

Let $c_A = C/A = c/(1+c)$ be the capital/assets ratio; thus, $c = c_A/(1-c_A)$. The asset coefficient of variation is $k_A = \sigma_A/L(1+c)$. Setting the variance of losses to zero, we get $\sigma = \sigma_A$. Then $k_T = \sigma_A/L = k_A(1+c) = k_A/(1-c_A)$, and we have the EPD ratio for risky assets:

$$d_A = \frac{D_A}{L} = \frac{1}{1-c_A}\left[k_A\phi\left(\frac{-c_A}{k_A}\right) - c_A\Phi\left(\frac{-c_A}{k_A}\right)\right]. \qquad \text{(A4)}$$

Expected Policyholder Deficit Under the Lognormal Distribution

To determine the lognormal expected policyholder deficit at the end of one period with no time value ($i = 0$), we use the fact that the EPD for risky losses is a call option with exercise price A and current "stock price" L. Since the well-known Black-Scholes (1973) option pricing model assumes that the future stock price is subject to geometric Brownian motion with instantaneous variance σ^2, at time t the price is lognormally distributed with dispersion parameter $\sigma\sqrt{t}$. The option price is

$$F = S\Phi(a) - Ee^{-it}\Phi(a - \sigma\sqrt{t}), \tag{A5}$$

where $a = \dfrac{\ln(S/E) + (i + \sigma^2/2)t}{\sigma\sqrt{t}}$,

S = the stock price, and
E = the exercise price.

Substituting $i = 0$, $t = 1$, $\sigma = \sigma_L$, $A = (1+c)L = E$ and $L = S$, we get the expected policyholder deficit

$$D_L = L\Phi(a) - (1+c)L\Phi(a - \sigma_L), \tag{A6}$$

where $a = (\sigma_L/2) - (\ln(1+c)/\sigma_L)$, and $\Phi(\bullet)$ = the cumulative standard normal distribution. The coefficient of variation is $k = \sqrt{\exp(\sigma_L^2) - 1} \cong \sigma_L$. Let $k = \sigma_L$ (however, for large values of k, we can use the exact relationship). Dividing equation (A6) by L, the EPD ratio to expected loss is

$$d_L = \Phi(a) - (1+c)\Phi(a - k). \tag{A7}$$

For risky assets with dispersion parameter σ_A, the expected policyholder deficit is equivalent to a put option with exercise price L and stock price A. The corresponding call option value is

$$D'_L = A\Phi(a') - L\Phi(a' - \sigma_A), \tag{A8}$$

where (following the preceding derivation, with $\sigma_A = \sigma_L$) we have $a' = (\sigma_A/2) + (\ln(1+c)/\sigma_A) = \sigma_A - a$. Thus, $D'_L = A\Phi(\sigma_A - a) - L\Phi(-a) = A[1 - \Phi(a - \sigma_A)] - L[1 - \Phi(a)] = D_L + A - L$.

To determine the value of the put option, we use the *put-call parity* relationship $G = F' - S + Ee^{-it}$, where G and F′ are the respective values of put and call options with stock price $S = A$ and exercise price $E = L$. Since $i = 0$, the EPD is $G = D_A = D'_L - A + L = D_L$, and the EPD ratio equals d_L. Since $c = c_A/(1-c_A)$ from the normal EPD derivation and we use the approximation $k_A = \sigma_A$, the EPD ratio in equation (A7) can be cast in terms of the risky asset parameters k_A and c_A:

$$d_A = \Phi(b) - \frac{\Phi(b-k_A)}{1-c_A}, \quad \text{(A9)}$$

where $b = (k_A/2) + (\ln(1-c_A)/k_A)$.

When both $\tilde{A}$ and $\tilde{L}$ are random variables, the EPD can be determined from the associated put option, where assets behave like the stock price and losses like a stochastic exercise price. Cummins (1988) evaluates this option cost under a continuous time framework using the fact that the variable $\tilde{A}/\tilde{L}$ is also lognormal. It has dispersion parameter σ based on the underlying bivariate normal distribution (see the preceding section) where $\sigma^2 = \sigma^2_A - 2\rho\sigma_A\sigma_L + \sigma^2_L$. The value of this option is the same as that for another insurer with risky assets also having expected value A but with dispersion parameter σ instead of σ_A and riskless losses equal to L. Thus, as with the normal distribution, the total risk (measured by σ) determines the EPD, which in this case can be evaluated by equation (A9).

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