2016 CAS Exam 9 Study Manual

Reading Highlights: Key Concepts and Formulas

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Important Note

The *Key Concepts and Formulas* documents in this section represent summaries of the main reading notes. They should be used for review after you have fully read the source readings and/or the main reading notes in the study manual.

Part 1

Portfolio Theory

BKM Chapter 6: Risk and Risk Aversion

1. Utility functions are used to rank investment opportunities with different expected returns and variances, reflecting that more expected return is good and more variance is bad. A typical utility function is of the form:

$$U = E(r) - .5A\sigma^2$$

with A representing the investor's degree of risk aversion (a higher value means they assign a larger penalty for risk).

2. An investor's risk aversion parameter, A, cannot be directly observed. However, it may be possible to infer it based on what they are willing to pay to insure against a loss.

3. We can create a portfolio that contains a combination of a risk-free security (e.g. U.S. Treasury Bill) and a portfolio of risky assets. We care about the expected return and standard deviation of this *complete* portfolio when y percent is invested in the risky portfolio:

$$\begin{aligned} E(r_c) &= yE(r_p) + (1-y)r_f \\ &= r_f + y[E(r_p) - r_f] \\ \sigma_c &= y\sigma_p \end{aligned}$$

4. Combinations of the complete portfolio's expected return and standard deviation can be plotted as the **Capital Allocation Line** (CAL).

5. The slope of the CAL is known as the Sharpe ratio. It is the same as the Sharpe ratio for the risky portfolio.

$$S = \frac{E(r_p) - r_f}{\sigma_p}$$

6. Expected returns for complete portfolios on the CAL are:

$$\begin{aligned} E(r_c) &= r_f + \frac{\sigma_c}{\sigma_p}[E(r_p) - r_f] \\ &= r_f + \sigma_c\left[\frac{E(r_p) - r_f}{\sigma_p}\right] \end{aligned}$$

7. How does an investor choose a complete portfolio? He selects the optimal portion to invest in the risky portfolio, y^*, that maximizes his or her utility.

$$y^* = \frac{[E(r_p) - r_f]}{A\sigma_p^2}$$

8. Passive strategies minimize costs to acquire the information necessary to select an optimal risky portfolio and take advantage of everyone else's efforts to do so. But some argue that passive strategies have limitations:

   a. They're Undiversified

   b. They're Top Heavy

   c. They're Chasing Performance

   d. You Can Do Better

BKM Chapter 7: Optimal Risky Portfolios

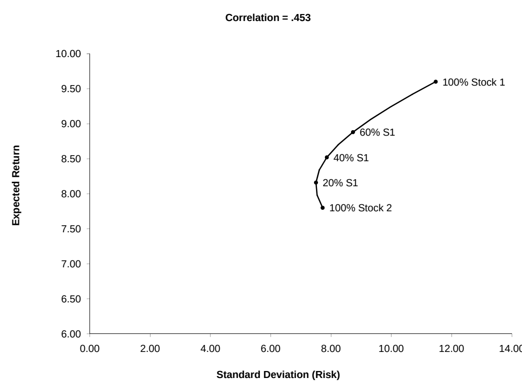
1. There are two broad sources of uncertainty in the returns on risky assets—those related to general economic conditions and those which are firm specific. When multiple risky assets are combined into a single portfolio, the firm specific risks will tend to offset each other (to various degrees), resulting in *diversification*.

2. When two risky assets are combined, the expected return is a weighted average of the expected returns of the two assets and the variance of returns is:

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho$$

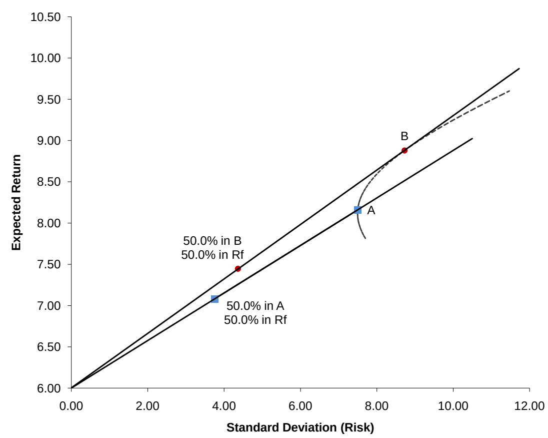
3. When the correlation is not perfect, the expected return and standard deviation combinations for different weights in the two risky assets can be depicted as:

Figure 1. Risk and Return — Two Assets with $\rho = 0.453$



4. When two (or more) risky assets are combined with risk-free borrowing or lending, we can again form a variety of CAL's. But there will be one CAL, formed with a particular combination of risky assets, that results in the maximum Sharpe ratio.

FIGURE 2. Portfolio Opportunity Sets: Two Alternative Risky Assets



With just two risky assets, the weights of the risky assets in this optimal CAL are:

$$w_1 = \frac{[E(r_1) - r_f]\sigma_2^2 - [E(r_2) - r_f]\sigma_{12}}{[E(r_1) - r_f]\sigma_2^2 + [E(r_2) - r_f]\sigma_1^2 - [E(r_1) - r_f + E(r_2) - r_f]\sigma_{12}}$$

5. When there are multiple risky assets, an infinite number of risky portfolios can be formed. Those that have the lowest risk for any given return and the highest return for any level of risk will form an *efficient frontier* that will look just like the risky portfolio choices in the two-asset case. We then select an optimal complete portfolio the same way — by maximizing the Sharpe ratio. This process is referred to as the Markowitz approach and it allows us to separate the two key tasks: finding an optimal combination of risky assets and allocating funds between the risk-free asset and the optimal risky portfolio.

6. Portfolio variance benefits from diversification, since as the number of assets, N, gets large the portfolio variance approaches the average covariance and individual variances become less important.

$$\sigma_p^2 = \frac{1}{N} * \text{Avg. Variance} + \frac{N-1}{N} * \text{Avg. Covariance}$$

7. To really benefit from diversification as a form of *risk reduction* you have to keep the portfolio size constant — as N gets large, the weights on each asset decline to $w_i = 1/N$. This results in a distinction between *Risk Pooling* and *Risk Sharing*.

BKM Chapter 8: Index Models

1. A single factor model of a stock's *actual* return assumes that it equals the expected return, a return component common to all stocks and a company-specific source of variability.

$$r_i = E(r_i) + \beta_i m + e_i$$

2. The factor model isn't too useful because we don't know the expected return, but it does help us understand the variance and covariance for stocks:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma^2(e_i)$$

$$Cov(r_i, r_j) = \beta_i \beta_j \sigma_m^2$$

3. A single index model assumes a particular source of the common factor driving returns, such as the S&P 500 index. It then depicts the actual *excess* returns $R = r - r_f$ as:

$$R_i = \alpha_i + \beta_i R_M + e_i$$

   And then, as before:

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma^2(e_i)$$

$$Cov(R_i, R_j) = \beta_i \beta_j \sigma_M^2$$

4. The single index model helps us solve a practical problem with the Markowitz model — it required too many parameter estimates, including a large number of covariance terms which are very difficult to obtain.

5. An index model allows for the separation of macroeconomic analysis (the analysis of economy-wide issues affecting the overall market) and security analysis (the analysis of the specific issues affecting individual stocks — the alphas).

6. An optimal risky portfolio is still the one that maximizes the Sharpe ratio. But now, we can get there differently by noting that if we can identify assets with positive *alpha* then we would want to tilt our holdings relative to the index portfolio more towards those assets with positive alpha and less towards those with negative alpha. But we also want to ensure diversification, so we hold the passive portfolio and this active portfolio in proportion to the relative return and risk of the index portfolio, with an adjustment downward in the passive holdings the higher the active portfolio beta is.

7. The full-covariance (Markowitz) approach in theory allows us to create a more efficient portfolio but imperfect parameter estimates reduces this benefit. The single index model makes a simplifying assumption that covariances are driven by a single common factor and that the error terms are uncorrelated, which makes it more practical.

8. In practice, measured betas are adjusted to reflect the tendency of betas to regress towards 1.0 over time and to account for estimation error using a Bayesian prior estimate

of 1.0. Two approaches are:

$$\text{Adjusted Beta} = \frac{2}{3}\,(\text{Estimated Beta}) + \frac{1}{3}\,(1.0)$$

$$\text{Forecast Beta} = a + b(\text{Current Beta})$$

Part 2

Equilibrium in Capital Markets

BKM Chapter 9: The Capital Asset Pricing Model

1. Capital Asset Pricing Model - If investors are rational, mean-variance optimizers, use the same assumptions regarding expected returns and standard deviations, and can buy or sell any risky asset and borrow or lend at a risk-free rate of interest, then all investors will own the market portfolio as their "risky asset portfolio".

2. Each investor's Capital Allocation Line (CAL) now goes through the market portfolio and is called the Capital Market Line. The CML now tells us the relationship between the standard deviation and expected return for portfolios investors hold, but what about individual securities?

3. With everyone owning the same market portfolio, the expected return on any risky asset will wind up being a linear function of its beta:

   $$E(r_i) = r_f + \beta[E(r_M) - r_f]$$

   where,

   $$\beta = \frac{Cov(r_i, r_M)}{\sigma_M^2}$$

4. The line depicted by the CAPM equation is known as the Security Market Line.

5. If a stock's expected return doesn't lie along the CAPM line we say it has positive or negative *alpha*. The goal of portfolio management should be to identify assets with positive alpha.

6. CAPM Assumptions

   - All investors follow the methods outlined in previous chapters to select their optimal portfolio, attempting to optimize the trade-off between expected return and standard deviation.

   - All investors make the same assumptions about the inputs to their portfolio optimization problem (they have homogeneous expectations regarding returns, variances and covariances).

   - All assets are publicly traded so that investors can buy or sell risky assets in any amount and can borrow or lend at the risk free rate without limitation.

   - Investors' planning horizons are only one period.

   - All information is publicly available.

   - There are no taxes.

- There are no transaction costs.

7. Extensions of CAPM from relaxing some of these assumptions:

   - Zero-Beta CAPM — CAPM was derived by first assuming that investors can borrow or lend at a purely risk free rate, without restrictions. Without this assumption, we can still depict investors' holdings as a combination of two assets, but instead of it being the market portfolio and a risk-free asset it will be the market portfolio and a portfolio that has a *zero beta* with the market portfolio. Expected returns will then be:
   $$E(r_i) = E(r_z) + \beta[E(r_M) - E(r_z)]$$
   This has a higher intercept and lower slope than the standard CAPM.

   - Labor Income and Non-Traded Assets — CAPM ignores the impact of Labor Income and Non-traded Assets. Betas should reflect whether returns are correlated with human capital and returns on non-traded assets.

   - ICAPM — Other risks might matter besides beta, such as the risk that the betas or market returns actually vary over time and do so in a way that is correlated with other risks. This is reflected in the Intertemporal CAPM.

   - Consumption-Based CAPM — Returns might covary with consumption, so investors will pay less for assets whose returns are bad at particularly bad times for them. This is reflected in the Consumption CAPM.

8. Liquidity and CAPM — The *liquidity* of an asset is the ease and speed at which it can be sold at a fair market value in a timely fashion. It reflects the the cost of engaging in the transaction such as the *bid-ask spread*, the impact that your own selling will have on the price itself, and the immediacy or speed at which the transaction can occur.

   An *illiquidity discount* can be thought of as the discount from fair market value that an investor would have to accept in order to sell an asset quickly, including the cost of trading with more informed investors, and can be thought of as a cost of trading that will impact the price.

   Liquidity betas can be added to CAPM to reflect the following sensitivities to illiquidity:

   a. Sensitivity of the security's illiquidity to market illiquidity

   b. Sensitivity of the stock's return to market illiquidity

   c. Sensitivity of the security illiquidity to the market's rate of return

BKM Chapter 10: APT and Multi-Factor Models

1. Law of One Price — Two assets that are identical in all economically relevant respects should have the same price. If not, then we would say that an arbitrage opportunity exists, as investors would be able to simultaneously buy and sell *identical* assets at two different prices.

2. Efforts of investors to exploit arbitrage opportunities is what causes the Law of One Price to hold. By buying the low price version of an asset and selling the high price version of the asset, investors can earn a net profit up-front with no risk. This puts upward pressure on the low-priced version and downward pressure on the high-priced version, until their prices are equal.

3. A factor model says that the *actual* excess return will reflect an unknown expected return, the effect of a shock to a common factor and the effect of random diversifiable risk.

   $$R_i = E(R_i) + \beta_i F + e_i$$

   It is also possible to expand this to include additional factors.

4. Factor models are not directly useful for determining expected returns (they include expected returns on the right-hand side. But they are useful for measuring the sensitivity of returns to factors and thus for hedging systematic (factor) risks.

5. Arbitrage Pricing Theory (APT) derives a more useful model of the *expected excess return* for a well-diversified portfolio of risky assets, using just two key assumptions: i) a factor model describes returns, ii) investors can buy or sell assets in any quantity such that, in equilibrium, arbitrage opportunities don't exist. These lead to:

   $$E(R_i) = \beta_1 \mathrm{RP}_1 + \beta_2 \mathrm{RP}_2 + \cdots + \beta_N \mathrm{RP}_N$$

   where each β_i term is a factor loading and each RP_i is a factor risk premium indicating the expected return for a portfolio with one unit of sensitivity to that factor. This equation must hold for all well-diversified portfolios, but may not hold for portfolios with large amounts of idiosyncratic (non-systematic) risk because to get from the factor model to this expected excess return equation we used the fact that $\sigma^2(e_i) = 0$.

6. Implications of APT — In the case of a one-factor model, this leads to the result that, in equilibrium, all well diversified portfolios must have the same risk premiums relative to their factor betas. More generally, well diversified portfolios with the same factor betas must have the same expected excess returns.

7. Fama-French 3-Factor Model:

   $$E(r_i) - r_f = b_i[E(r_M) - r_f] + s_i E(\mathrm{SMB}) + h_i E(\mathrm{HML})$$

   where,

a. $E(r_M) - r_f$ is the average excess return for the overall market

b. SMB is a portfolio whose returns reflect the difference in returns for Small stocks and Big stocks

c. HML is a portfolio whose returns reflect the the difference in returns for stocks with High ratios of book value to market value and Low ratios of book value to market value

BKM Chapter 11: Market Efficiency

1. If a market is efficient then prices will reflect all currently available information. In such a market, prices will respond quickly (though not necessarily correctly!) to new information. Ample evidence of this exists, including news of takeovers or analysis of firms featured in CNBC reports.

2. Three forms of market efficiency:

   a. Weak Form — Prices reflect all past price and trading data.

   b. Semi-Strong Form — Prices reflect all publicly available information.

   c. Strong Form — Price reflect all public and private information.

3. *Technical analysis* involves examination of past stock prices in the search for recurring and predictable patterns. *Fundamental analysis* involves forecasting the earnings and dividend prospects for the firm, taking full consideration of all available information, to value the stock from teh ground-up.

4. The EMH simply suggests that you are unlikely to "beat the market" but some degree of active management is important because you still need to select a portfolio that reflects your specific degree of desired risk and which considers factors such as your age, tax bracket and employment.

5. An event study examines the returns on the day of a given event, determines the *abnormal return* relative to a model of expected returns such as CAPM, and then attributes the abnormal return to the specified event. For example, studying abnormal returns around takeover announcements can assess when and how quickly the announcement affects market prices.

6. The EMH suggests that investors cannot earn abnormal returns and instead simply earn returns commensurate with the risk taken. It's difficult to confirm whether this is indeed the case, for several reasons: the magnitude issue, selection bias, and the lucky event issue.

7. There have been numerous tests of the Weak Form of the EMH and the findings can be summarized as follows: there is some intermediate term serial correlation from a momentum effect (past top performers tend to be future top performers), a strong long-term reversal effect (past winners become losers) and some ability to use known variables to predict broad market movement.

8. There have been numerous tests of the Semi-strong Form of the EMH that have uncovered difficult to explain anomolies. Some of these disappear when discovered, some have rational risk-based explanations and some seem to be driven by some degree of irrationality. The key anomolies are:

a. P/E Effect — Low P/E stocks outperform, but maybe because their betas are mismeasured?

b. Small firm in January Effect — Small stocks outperform in January, even after adjusting for beta differences, but perhaps due to tax-loss selling at year-end?

c. Neglected Firm Effect — Firms followed less by analysts seem to earn excess risk adjusted returns, perhaps due to liquidity effects.

d. Highest book-to-market firms tend to outperform lowest book-to-market firms.

e. Post Earnings Announcement Price Drift — Event studies show that positive (negative) earnings surprises are followed by positive (negative) abnormal returns, but the response is sluggish.

9. Strong Form tests show that insiders do earn abnormal returns.

10. They key to the anomolies literature is determining whether we are observing *risk premiums* or *inefficiencies*. All tests of abnormal returns reflect a model of what returns should be, often using CAPM or the Fama-French model. But if there are other risk factors returns would appear to be abnormally high.

11. Neither stock analysts nor mutual fund managers appear able to earn abnormal returns, offering modest confirmation of market efficiency. For the analysts, their recommendations might drive price changes, so it is hard to tell for sure. For managers, most don't beat the market and those that do often don't on an expense-adjusted and risk-adjusted basis, with more sophisticated models indicating negative alpha and little or no persistency.

BKM Chapter 12: Behavioral Finance and Technical Analysis

1. The Behavioral Critique — Conventional financial theory ignores the fact that people commonly make systematic errors assessing risk and making decisions under uncertainty, making it unreasonable to assume that conclusions based on the assumption of super-rational investors capable of doing complex calculations can be right.

2. Errors in information processing can cause investors to misestimate probability distributions. People routinely exhibit the following:

   a. Forecasting Errors — People tend to give too much weight to recent evidence and tend to produce extreme forecasts. This could lead to excessive earnings forecasts and cause high P/E stocks to subsequently under-perform.

   b. Overconfidence — People tend to exhibit extreme overconfidence and overestimate their abilities.

   c. Conservatism — Investors are often slow to update their prior beliefs in the face of new information.

   d. Sample Size Neglect and Representativeness — People tend to infer patterns from limited information.

3. Even with perfect information, investors often make irrational decisions as the result of:

   a. Framing — Decisions tend to be influenced by the way they are framed.

   b. Mental Accounting — People segregate certain decisions arbitrarily (e.g. playing with the "house's money").

   c. Regret Avoidance — People tend to blame themselves less when their choices are more conventional.

   d. Prospect Theory — People are loss averse but focus on *changes* in wealth and react to what they perceive to be gains and losses differently.

4. Several factors limit the ability for investors to arbitrage mispricings, including fundamental risk, implementation costs and model risk. This has lead to some notable apparent arbitrate opportunities that persisted surprisingly long:

   a. Siamese Twin Companies (e.g. Royal Dutch and Shell)

   b. Equity Carve Outs (e.g. 3Com and Palm)

   c. Closed-End Mutual Fund Discounts

5. Fama and French documented return differentials between portfolios (small vs. large firms, high book-to-market vs. low book-to-market firms) that appeared to have nothing to do with differences in their betas. They chose to interpret the results to suggest that the SMB and HML factors proxy for priced risk factors. But another way to interpret the evidence that the HML portfolios help to explain returns on stocks, thus resulting in a *value premium*, is that investors may irrationally prefer certain glamour firms that have had good recent performance, high prices and thus lower book-to-market ratios. This causes their returns to lag behind the value firms.

6. Examination of trading rules based on past price "patterns" generally concludes that these cannot produce abnormal returns. But some past patterns could be driven by *systematic* behavioral biases and so perhaps some insights into investor behavior can be obtained from these patterns.

    a. Exploitable Patterns — The disposition effect may reduce trading of losing stocks and overconfidence may cause a link between trading volume and investor sentiment.

    b. Trends and Corrections — Tools used to uncover trends include the Dow Theory (and variations like the Elliott Wave Theory and Kondrateiff Waves), analysis of moving averages and analysis of whether movement in the index is reflected widely in all of the stocks in the market (breadth indicators).

7. Sentiment Indicators include trading volume stats like the Trin Statistic, Confidence Indices and Put/Call Ratios.

BKM Chapter 13: Empirical Evidence on Security Returns

1. Testing the CAPM Using Two-Stage Regression

    a. First Pass: Estimate betas by regressing historical returns for a large number of stocks or portfolios against the market index (or some other factor), over a specified time period.

    b. Second Pass: Regress the average returns for the stocks or portfolios against their estimated betas and the residual variances from the first pass regression. The regression will be of the form:

    $$\overline{r_i - r_f} = \gamma_0 + \gamma_1 \hat{\beta}_i + \gamma_2 \hat{\sigma}_{ei}^2$$

    c. Use the results of the second-pass regression to test if the intercept equals zero, if the slope equals the average excess market return ($\overline{r_M - r_f}$), and if the coefficient on the residual variance term is zero.

2. Results of Empirical Tests of CAPM (Lintner and Miller & Scholes) – When these analyses was performed, the results were somewhat consistent with CAPM. However, researchers found that $\gamma_0 > 0$, meaning that the intercept of the model was too high. Also, γ_1 was too small, making the slope of the line too flat.

3. Three Problems with the Early CAPM Tests

    a. Didn't use the "true" market portfolio. Tests that reject a positive relationship between average return and beta may simply point to the fact that the market proxy used in the test is not on the efficient frontier. CAPM could be valid and tests using highly diversified, but not mean-variance efficient, portfolios could still fail.

    b. Investors cannot borrow or lend at the risk free rate, which is assumed in the simple versions of the CAPM. The Zero-Beta version of CAPM resolves this issue, resulting in an intercept that is higher than the risk-free rate, consistent with the early empirical findings.

    c. Large measurement errors in the betas (from the first-pass) cause the second-pass regressions to fail, even in simulated data. The problem is reduced when portfolios, rather than individual stocks, are used. Fama & MacBeth used return data from 1935-1968 and found that the relationship between return and beta is indeed linear and that non-systematic risk does not explain excess returns. However, these results are worse with more recent data and worse when a value-weighted market index is used.

4. CAPM Adjusted to Account for Labor Income and Private Businesses – The standard CAPM assumes all sources of cash flow that investors can ultimately use for consumption of

goods and services are tradable. However, the income investors receive from their employment (*labor income* or *human capital*) is not tradable and not reflected in the market index.

Jaganathan and Wang (JW) did an empirical analysis that included aggregate labor income, a proxy for the business cycle and a term to include the size of the company. They found:

a. Standard CAPM betas do not adequately capture the cyclicality of stock returns and thus do not fully capture systematic risk.

b. Human capital is an important component of any model that tries to capture systematic risk.

c. Test of whether firm size was a significant factor showed that it was not once the other JW variables were added to the standard CAPM.

d. when non-traded (private) business wealth is also included (by Heaton and Lucas in a separate study), they also found the market index return did not help to explain the return on individual stocks, rejecting the core implications of the CAPM.

5. Multifactor CAPM & APT (Chen, Roll and Ross Study) – Chen, Roll and Ross (CRR) performed an early empirical analysis wherein they identified several possible variables, in addition to the market index, that might proxy for systematic factors:

   a. growth in industrial production

   b. changes in expected inflation

   c. unexpected inflation

   d. unexpected changes in risk premiums as measured by differences in returns for corporate and government bonds

   e. unexpected changes in the term premium as measured by the difference between long-term and short-term government bond returns

   Their findings: the market index wasn't statistically significant and the only three variables that were significant were industrial production, the risk premium on corporate bonds and unanticipated inflation.

6. Fama-French 3-Factor Model – Fama and French developed a multifactor model that includes the market, a firm size factor (SMB) and a book-to-market ratio factor (HML):

$$E(r_i) - r_f = b_i[E(r_M) - r_f] + s_i E(\text{SMB}) + h_i E(\text{HML})$$

7. Testing the Fama-French 3-Factor Model – The equation above is the Fama-French 3-Factor Model. When testing this model against actual data to see if the three factors (excess market return, SMB and HML) explain the differences in expected returns for different portfolios and if there are any other sources of returns not included in those three factors, we use a slightly modified form:

$$r_i - r_f = a_i + b_i[r_M - r_f] + s_i\text{SMB} + h_i\text{HML} + \epsilon_i$$

This form allows us to test whether there is a non-zero alpha (the a_i term), whether the factor loadings b_i, s_i and h_i are statistically significant, etc. The results of one such test are summarized in the table below:

	B/M	Size	Excess Return	a	b	s	h	$t(a)$	$t(b)$	$t(s)$	$t(h)$	R^2
S/L	0.55	22.39	0.61	−0.42	1.06	1.39	0.09	−4.34	30.78	19.23	1.73	0.91
S/M	1.11	22.15	1.05	−0.01	0.97	1.16	0.37	−0.18	53.55	19.49	9.96	0.96
S/H	2.83	19.05	1.24	−0.03	1.03	1.12	0.77	−0.73	67.32	39.21	26.97	0.98
M/L	0.53	55.85	0.70	−0.06	1.04	0.59	−0.12	−1.29	55.83	18.01	−4.30	0.96
M/M	1.07	55.06	0.95	−0.01	1.05	0.47	0.34	−0.15	32.98	17.50	9.50	0.96
M/H	2.18	53.21	1.13	−0.04	1.08	0.53	0.73	−0.90	47.85	8.99	11.12	0.97
B/L	0.43	94.65	0.58	0.02	1.02	−0.10	−0.23	0.88	148.09	−6.88	−13.52	0.98
B/M	1.04	92.06	0.72	−0.09	1.01	−0.14	0.34	−1.76	61.61	−4.96	13.66	0.95
B/H	1.87	89.53	1.00	−0.09	1.06	−0.07	0.84	−1.40	52.12	−0.86	21.02	0.93

TABLE 13.6

Three-factor regressions for portfolios formed from sorts on size and book-to-market ratio (B/M)

Source: James L. Davis, Eugene F. Fama, and Kenneth R. French, "Characteristics, Covariances, and Average Returns, 1929 to 1997," *Journal of Finance* 55, no. 1 (2000), pp. 396. Reprinted by the permission of the publisher, Blackwell Publishing, Inc.

Size and book-to-market factors appear to be important determinants of average returns - significant t-statistics, high R^2, clear pattern of increasing average returns as the factor loadings increase across the portfolios. But there are alternative interpretations of these statistical results:

a. Risk-Based: Size and value factors are priced factors for risks not captured in CAPM because they capture the risk in the business cycle or time-varying betas and market risk premiums.

b. Behavioral: Investors, perhaps blinded by representativeness errors, may irrationally prefer certain growth or "glamour" firms that have had good recent performance, high prices and thus lower book-to-market ratios. This causes their returns to lag behind the returns for value firms.

8. Liquidity and Asset Pricing – When stocks are sorted into ten portfolios reflecting liquidity beta deciles, we find that both CAPM and FF three-factor alphas are higher for the high liquidity beta portfolios and low for the low liquidity beta portfolios. This supports the notion that liquidity effects play a role in explaining returns for stocks.

9. Equity Premium Puzzle – Mehra and Prescott found that over the period 1889-1978, investors have been excessively rewarded for bearing risk. There have been several attempts to address this puzzle:

   a. Mehra and Prescott used a Consumption CAPM (CCAPM), but their analysis suffered from data issues that have since been improved upon, mitigating some of the perceived puzzle.

   b. Investors just got lucky – they weren't expecting (or demanding) high market risk premiums, but they were fortunate to earn them. In this sense, there really is no puzzle.

   c. The excess returns may have been real, but the measure is biased because it only reflects U.S. market data that suffers from a survivorship bias.

   d. Extensions to the standard CAPM which remove some of its less realistic assumptions also resolves some of the puzzle.

   e. Liquidity risk considerations may also mitigate some of the perceived puzzle.

   f. Behavioral biases could lead to irrational behavior, which in turn causes the equity premium puzzle.

Part 3

Asset-Liability Management

BKM Chapter 15: Term Structure of Interest Rates

1. The **pure yield curve** reflects yields on on zero coupon Treasury bonds. The **on-the-run yield curve** reflects yields on coupon bonds and is based on the most recently issued bonds.

2. *Short rates* represent the interest rate for a specified time interval (such as one year). A cash flow at time n can be valued by stringing together the various short rates:

$$PV = \frac{\$1{,}000}{(1 + r_1)(1 + r_2) \cdots (1 + r_n)}$$

3. The *spot rate* represents the yield on a zero coupon bond with the specified maturity, so it can be found by solving for y in:

$$PV = \$1{,}000(1 + y_n)^n$$

4. A graph of the spot rates for different maturities is the *term structure*.

5. A future, unknown short rate is found from the following:

$$(1 + y_n)^n = (1 + y_{n-1})^{n-1}(1 + f_n)$$

6. There are different theories regarding the relationship among the spot rates (the term structure of interest rates):

   a. Expectations Hypothesis — The forward rates reflect expected future short rates:

   $$(1 + y_n)^n = (1 + y_{n-1})^{n-1}(1 + E(r_n))$$

   This means that investors who want to invest long term have the same expected results from investing in a long term bond at yield y_n as investing in a series of short term bonds at short rates $r_1, r_2 \cdots r_n$. Similarly, they have the same expected results form investing short term at short rate r_1 as investing long term at spot rate y_n and planning to sell the bond at the end of the year at the then-current spot rate.

   b. Liquidity Preference Theory — The expectations hypothesis ignores the risk. The liquidity preference theory argues that the forward rate is equal to the expected spot rate plus a liquidity premium, which could be either positive or negative.

   c. Segmentation Theory — Borrowers and lenders have different preferences for short-, medium- or long-term investments and they do not readily switch from one maturity range to another. The yields in these different markets are determined by supply and demand in each of these markets.

BKM Sections 16.1 and 16.2: Duration and Convexity

1. Six general properties of bond prices:

    a. Bond prices and yields are inversely related.

    b. Increases in yield to maturity has a smaller impact than a comparable decrease in yield.

    c. Long term bonds are more sensitive to changes in yields than short term bonds.

    d. Sensitivity to changes in yields increases at a decreasing rate as maturity increases.

    e. High coupon bonds are less sensitive to changes in yields than low coupon bonds.

    f. The sensitivity of a bond's price to changes in yields is inversely related to the yield at which it is currently selling.

2. *Macaulay Duration, D*, is a bond's average maturity with each cash flow weighted by its present value. For a bond valued with annually compounded rates this is:

$$D = \frac{(1)CF_1(1+y)^{-1} + (2)CF_2(1+y)^{-2} + \cdots + (n)CF_n(1+y)^{-n}}{\text{Bond Price}}$$

3. The percentage change in price of a bond for a given change in its yield is, from its derivative with respect to y,

$$\text{\% Price Change} = \frac{-1}{1+y}\left[\frac{(1)CF_1(1+y)^{-1} + \cdots + (n)CF_n(1+y)^{-n}}{\text{Bond Price}}\right]$$

    This can be written in terms of the Macaulay duration:

$$\frac{dP}{P} = \frac{-1}{1+y} D\, dy$$

4. *Modified Duration* is defined as:

$$D^* = \frac{D}{1+y}$$

    which we can see is just negative one times the percentage change in price for s small change in the yield, dy.

5. With continuous compounding, the Macaulay duration is still defined as the weighted average time to payment, but the formula is now:

$$D = \frac{\sum t_i c_i e^{-y t_i}}{P}$$

And the Modified duration is still minus one times the percentage change in price for a small change in yield, which can be shown to be the same as the Macaulay duration:

$$D^* = \frac{\sum t_i c_i e^{-y t_i}}{P}$$

6. An approximation for Modified Duration is given as:

$$\text{Modified Duration} \approx \frac{P_- - P_+}{2P\Delta y}$$

7. Convexity is a measure of the change in duration as yields change. With annual compounding it is found as:

$$\text{Convextity} = \frac{1}{P(1+y)^2} \sum \left[\frac{CF_t}{(1+y)^t} t(t+1) \right]$$

With continuous compounding it is:

$$C = \frac{1}{P} \frac{\partial^2 P}{\partial y^2} = \frac{\sum t_i^2 c_i e^{-y t_i}}{P}$$

8. Convexity can also be approximated as:

$$\text{Convexity} = \frac{P_- + P_+ - 2P}{P(\Delta y)^2}$$

9. The percentage change in price for a *small* change in yield can be estimated from it's duration, as shown above:

$$\%\ \text{Change in Price} \approx -D^* \Delta y$$

For larger changes, the convexity adjustment is needed:

$$\%\ \text{Change in Price} \approx -D^* \Delta y + \frac{1}{2} \text{Convexity} (\Delta y)^2$$

10. Effective Duration and Convexity are defined similarly but take into account the potential for the cash flows themselves to change as the rates change. Taking first and second derivatives is now harder, so the approximation formulas are used instead:

$$\text{Effective Duration} = \frac{P_- - P_+}{2P\Delta y}$$

$$\text{Effective Convexity} = \frac{P_- + P_+ - 2P}{P(\Delta y)^2}$$

Hull Chapter 4: Interest Rates

1. Bond prices with continuously compounded yields are:

$$P = \sum CF_t e^{-yt}$$

2. The *par yield*, also referred to as the *swap rate*, is the coupon rate on a semi-annual bond whose price is exactly equal to its par value.

3. Zero coupon yields are found using bootstrapping, where prices of bonds with successive maturity dates are used to sequentially solve for the yield at each maturity.

4. Forward rates are found as in BKM, but with continuous compounding the formula for the forward rate between any two points in time T_1 and T_2 simplifies to:

$$R_F = \frac{R_2(T_2) - R_1(T_1)}{T_2 - T_1}$$

5. A *Forward Rate Agreement* allows one party to earn a fixed interest rate of R_K (at a specified compounding frequency) on a specified principal amount, L between time T_1 and T_2.

6. Value of an FRA at inception is usually zero since the rate R_K is set at the forward rate R_F, but at any other point in time its value will be the present value difference between the interest at the agreed rate and the interest at the forward rate:

$$\text{Value of FRA} = L(R_K - R_F)(T_2 - T_1)e^{-R_2 T_2}$$

Using this formula typically involves using a compounding frequency other than continuous for R_K and R_F even though continuous compounding is used for R_2.

7. The FRA value formula above uses a *risk neutral method* to justify using the forward rate, R_F, for the unknown future rate and the risk-free rate, R_2, to discount the amounts.

Hull Chapter 7: Swaps

1. A *plain vanilla* interest rate swap involves one party paying a fixed rate of interest on a notional principal amount to the other party and at the same time receiving from that party a variable rate of interest on the same notional principal amount.

2. Interest rate swaps are used to efficiently manage the interest rate sensitivity of assets or liabilities. If we hold a bond paying a fixed rate of interest and we want to reduce its sensitivity to changes in rates, we can use a swap to exchange the fixed coupon payments to floating rate payments. This will reduce its duration to approximately zero. Similarly, if we have floating rate liability payments we can use a swap to exchange those for fixed payments and eliminate the uncertainty of our liability payment amounts.

3. When two parties face different borrowing costs in both fixed rate and floating rate markets, a swap can be used to lower *both* of their costs by identifying the comparative advantage and negotiating a way to split this amount.

   This example shows ABC has a 70 basis point comparative advantage in the fixed rate market (they pay "more less" in that market):

   Table 1. Determining Comparative Advantage

	ABC	XYZ	ABC's Absolute Advantage
Fixed Rate	7.50%	9.00%	1.50%
Floating Rate	LIBOR + .2%	LIBOR + 1%	0.80%
Comparative Advantage			**0.70%**

   To split these savings equally, we follow these steps:

   a. ABC borrows in the market where it has the comparative advantage (fixed market) and enters into a swap to receive the fixed rate and pay the floating rate.

   b. ABC determines what it would have paid in the floating rate market (LIBOR + .2%) and subtracts the negotiated share of the savings (35 basis points) to determine what it wants to ultimately pay (LIBOR - .15%).

   c. ABC enters into a swap where they receive the full fixed rate payment they have to make (7.5%) and pay the target floating rate payment (LIBOR - .15%).

   d. The terms of the swap can be simplified by noting it is equivalent to receiving 7.65% fixed and paying LIBOR.

   e. If a bank intermediary were to sit between the two companies, the bank's fees would be subtracted from the amount available for the savings and the rest would be the same, except that each side's terms would be determined separately. Or, the bank's fees for each swap could simply be added to each swap party's payments.

4. There are two ways to determine the value of a swap that has already been entered into. One way is to think of a swap as the exchange of a fixed rate bond for a floating rate bond. The other way is to view a swap as a series of Forward Rate Agreements.

5. Value as an Exchange of Bonds — Although the notional amount isn't really paid, if it were then the party receiving the fixed rate just owns a fixed rate bond and is short a floating rate bond. Their position is just worth:

$$V_{swap} = B_{fix} - B_{floating}$$

   Valuing fixed rate bonds should be easy. Floating rate bonds are worth *par* on the rate reset date and so valuing one between reset dates just involves valuing the cash flow due on the next reset date (k^*) and assuming the par value (L) is paid on the reset date too:

$$\text{Value Between Resets} = (L + k^*)e^{-rt}$$

6. Value as Series of FRAs — The swap involves net payments which include unknown future interest rates, so we can't easily value the net payments today without knowing both the expected future rates and the proper risk-adjusted discount rate. But we can use the same trick used to value FRAs, which is to assume that the unknown future floating rate payments will be the *forward rates* and then discount the net payments at the current risk-free rates.

BKM Chapter 16: Managing Bond Portfolios

1. Passive bond portfolio strategies include indexing and immunization.

2. The goal of indexing is to simply mirror the overall results of the broad market, which is tricky because the composition of the bond market changes constantly as bonds mature, default, get called, etc. So indexing is usually done by maturity and sector groupings.

3. Net Worth Immunization — Seeks to set the dollar duration of assets (the duration of the assets multiplied by the dollar value of the assets) equal to the dollar duration of liabilities (the duration of the liabilities multiplied by the dollar value of the liabilities) so changes in rates don't impact the reported net worth.

4. Target Date or Holding Period Immunization — The *future value* of a bond will depend on reinvestment rate of coupons which rises as rates rise and the value of the remaining cash flows at that point which falls as rates rise. These effects are exactly offset when the Macaulay duration is equal to the target date in years.

5. Immunization is harder than it seems. First, durations change continuously as time passes and as rates change. Second, because of convexity duration-matched portfolios are only immune to *small* changes in rates. Immunized portfolios need to be constantly rebalanced.

6. Immunization is also made challenging in practice because rates for all maturities don't always change by the same amount (term structure shifts are not parallel) and inflation concerns mean we may not want to lock in nominal values.

7. Active bond portfolio management includes strategies to replace bonds in the portfolio with different bonds either due to differences in prices for otherwise similar cash flows in risks, differences in spreads in different sectors, anticipation of changes in rates, ability to capture more yield in exchange for assuming more/different risk or ability to minimize taxes.

8. Other forms of active management involve: a) ensuring the portfolio characteristics reflect the investor's time horizon, including perhaps taking advantage of the ability to "ride the yield curve" and hold longer term bonds with the intent to sell as the maturity shortens and yields fall; and b) contingent immunization which combines active and passive strategies whereby active management is used until a trigger level is hit causing a switch to a passive strategy.

Noris: Asset/Liability Management Strategies for P&C Companies

1. Three ways to measure the value of the currently booked surplus: book value surplus uses book values for bonds, market values for stocks and undiscounted liabilities; current value surplus uses market value for all assets and undiscounted liabilities; market value surplus uses market values for assets and discounted liabilities (with municipal bonds yields serving as a proxy for the current after-tax yield).

2. Franchise Equity represents the value of business not yet booked. Because this is so difficult to value, this element of the firm value is ignored in this paper and is not part of the ALM strategy. But it is the focus of the Panning reading.

3. Market value surplus is managed by measuring the Macaulay duration of the market value surplus:
$$D_{\text{MVS}} = \frac{D_{\text{MVA}}\text{MVA} - D_{\text{MVL}}\text{MVL}}{\text{MVS}}$$

4. Duration for stock holdings is measured as:
$$\text{Stock Duration} = \frac{1}{\text{Dividend Yield}}$$

5. Duration Gap of Surplus is defined as the amount by which the duration of the MVS exceeds zero (zero indicating no sensitivity to interest rate changes). With a positive duration gap, MVS would fall as rates rise; a negative duration gap would indicate that MVS rises as rates rise.

6. Three different targets to manage towards are discussed:

   a. Duration Gap of Surplus — One target for the duration gap of surplus (DG_S) is to set it to zero, thus immunizing the surplus from the effects of changes in interest rates. This might be desirable in some contexts but it would result in large fluctuations in earnings and would be unduly restrictive.

   b. Duration Gap of Total Return — Macaulay duration indicates the holding period over which the rate of return is immunized. Setting DG_S equal to a specific holding period, H, would immunize the total return over that holding period. Deviations from this target are then, $DG_{\text{TRS}} = DG_S - H$.

   c. Duration Gap of Leverage — Leverage will stay the same if MVS and MVA change by the same percentage amount. The duration gap of leverage reflects how much the leverage ratio could change and is measured as $DG_{EL} = D_{\text{MVS}} - D_{\text{MVA}}$.

Feldblum: Asset Liability Matching for P/C Insurers

1. Life insurers adopted duration management strategies that tended to focus on "matching" asset and liability durations. To determine if these strategies should be used by P&C insurers, we need to note the following differences between life and P&C insurers:

    a. P&C liabilities are sensitive to inflation so effective duration measures are needed.

    b. The shorter term of P&C liabilities means that matching durations will involve short-term asset profiles and a potentially large cost in terms of lost yield.

    c. Equity cash flows and P&C liabilities share two similarities — both are inflation sensitive and both are subject to considerable risk other than interest rate risk (contagion and legal risk for the liabilities, systematic market risk for equities).

    d. P&C insurers are less susceptible to disintermediation risk.

    e. Long-term bonds held by P&C insurers would lead to a greater risk of having to recognize mark-to-market losses compared to life companies that used to be able to hold bonds at book value.

2. The three drawbacks to cash flow matching: it is cumbersome, inefficient and costly.

3. Duration of P&C loss reserves calculated just like bond duration:

    a. Payment Patterns — The payment patterns used are for the loss reserve payments, not the incurred loss payments.

    b. Discount Rate — The appropriate discount rate to use to calculate the liability duration is the current yield on the asset portfolio, which may require iteration since the duration will dictate the asset portfolio mix.

    c. Yield — The yield we want to use is the yield on new investments.

4. Effective durations of inflation sensitive lines of business are closer to zero. Duration matching in this case will require a very short term asset portfolio or investments in inflation sensitive assets such as common stocks or real estate. But real estate assets are illiquid, require substantial expertise and are limited by regulatory restrictions, so common stock seems to be an appropriate choice.

5. The traditional view of equity duration is:

$$\text{Modified Equity Duration} = \frac{1}{k - g}$$

    Typical measures would be between 10 and 20, but this ignores the inflation sensitivity of the equity cash flows (dividends).

6. How sensitive are equity cash flows to inflation? There are three effects noted:

   a. If interest rates rise because of changes in inflation, the value of real assets should also rise and the inflation adjusted value of the assets should be insensitive to the changes in the rates in the long run.

   b. Inflation could cause cause firm profits to rise or fall, depending on whether the inflation is from increased demand or rising supply costs, some of which cannot be passed on to consumers.

   c. As rates rise, investors may shift their holdings to bonds versus stocks or demand higher stock returns, stock prices to fall.

   The first effect is a long term effect but the latter two are short term, so inflation should cause stock prices to fall at first and then rise.

7. Other considerations besides the reduction in interest rate risk that should be considered are the potential costs in terms of yield that has to be given up, transaction costs and the potential for acceleration of liability payments requiring additional liquidity. Since P&C insurers face relatively less cash flow/liquidity risk (their premiums are heavily front loaded and so new business funds cash flow needs) they can make yield a higher priority.

8. Two recommended strategies: a short (effective) duration common stock portfolio with exposure to systematic market risk or a long-term bond portfolio which exposure to mark to market gains and losses.

Panning: Managing Interest Rate Risk

1. Interest rate risk management should not focus solely on the statutory surplus. We should first be sure to capture the *current economic value* of surplus by using market values of assets and liabilities and we should, more importantly, include the franchise value (value of future business) so that we manage the value of the entire firm from the shareholders' perspective.

2. Aside from being a large component of the value of the firm, franchise value is exposed to interest rate risk.

3. Panning uses the following to determine the fair premium:

$$P = \frac{S(k-y)+L}{1+y} + E$$

   which reflects a target return of k on surplus of S when losses are L, expenses are E and the risk-free rate is y.

4. Current economic value of the firm is the current surplus plus the value of the business just written:

$$\text{Current Economic Value} = \text{Surplus} + \text{Premium} - \text{Expenses} - \text{PV(Losses)}$$

$$C = S + P - E - \frac{L}{1+y}$$

5. Franchise value is calculating assuming future premiums, expenses and claims are simple perpetuity calculations, with all amounts decreasing by the ratio cr each period. So, if we define:

$$d = \frac{cr}{1+y}$$

   then

$$F = \left[P - E - \frac{L}{1+y}\right]\left[\frac{d}{1-d}\right]$$

6. A pricing strategy is defined as a rule to determine how to set k in relation to y by setting parameters a and b:

$$k = a + by$$

   This will be used to quantify the inflation sensitivity of franchise value.

7. With a pricing strategy defined, the franchise value formula is messier:

$$F = \frac{(cr)S(a+(b-1)y)}{(1+y)(1+y-cr)}$$

8. Duration of franchise value then becomes:

$$D = \frac{a - b + 1}{(1 + y)(a + by - y)} + \frac{1}{1 + y - cr}$$

9. The key point is that an insurer could appear to have a low surplus duration and still have its firm value highly sensitive to interest rate changes because the duration of franchise value can be very large.

10. Limits of Using Investment Strategies to Manage Interest Rate Risk — The larger the firm's franchise value, the more problematic it will be to manage the interest rate risk for the total economic value using asset investment strategies.

11. Invisibility of Franchise Value — Because the franchise value is *invisible* to outside parties such as rating agencies and regulators (as well as investors), strategies to manage this risk may actually appear to increase the risk.

12. Using Pricing Strategy to Manage Duration of Total Economic Value — Perhaps a better approach is to use the pricing strategy (a and b parameters in $k = a + by$) to manage the duration of the franchise value. This has a key advantage:

    It avoids the potential rating agency and regulatory risk associated with strategies that focus on managing the duration of the firm's invested assets as a means of managing the risk to its franchise value and total economic value. This key advantage results from the fact that implementing a pricing strategy is nearly as invisible to these external audiences as the franchise value it is intended to protect.

Part 4

Financial Risk Management (a)

Hull Chapter 23: Default Probabilities

1. Three ways to measure default probabilities:

   a. Historical default rates (by rating)

   b. Default rates implied in current prices

   c. Default rates implied by equity prices (Merton Model)

2. Several alternative measures of default probability:

   a. Cumulative Default Probability

   b. Unconditional Default Probability

   c. Hazard Rate/Default Intensity (Conditional Probability of Default)

   d. Average Hazard Rate or Average Default Intensity

3. Approximate average hazard rate based on bond prices, where s is the annual yield spread relative to the risk-free rate and R is the average recovery rate in the event of default:

   $$\bar{\lambda} = \frac{s}{1-R}$$

   Notice that if the spread is measured using continuously compounded rates then $\bar{\lambda}$ will represent an average *instantaneous* default intensity. If instead the spread is measured using annually compounded rates then $\bar{\lambda}$ will represent an average *annual* default intensity.

4. More generally, to estimate the probability of default from bond prices, calculate the risk-free present value of the expected losses from default as a function of the probability of default and then set this equal to the difference between the risk-free value of the bond and the current market price of the bond. Solve for the probability of default or the average hazard rate.

5. When inferring default probability from bond prices, we cannot separate the probability from other risk-adjustments being made by the market. Therefore, the resulting default probability is a *risk neutral probability* and not the *real world probability*. The two may differ for several reasons: liquidity premiums, conservatism among traders, systematic default risks, and skewness.

6. Merton showed that equity can be viewed as a call option on the firm's assets with a strike price equal to the debt. This means that we can use equity prices and equity volatility to infer the (unknown) market value and volatility of the firm's assets and then calculate the default probability as the probability the assets will be worth less than the debt on the

maturity date.

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$

$$d_1 = \frac{\ln(V_0/D) + (r + \frac{1}{2}\sigma_V^2)T}{\sigma_V \sqrt{T}}$$

$$d_2 = d_1 - \sigma_V \sqrt{T}$$

$$\text{Probability of Default} = 1 - N(d_2) = N(-d_2)$$

Hull Chapter 23: Credit Risk in Derivatives Transactions

1. Credit risk in derivatives transactions depends on whether, at the the time of default of your counterparty, the derivative you entered into with them has a positive or negative value to you (i.e. whether the net cash flows are such that they owe you or you owe them).

2. Three methods are used to mitigate exposure to credit risk in derivative contracts:

   a. netting requires default on all transactions in the event of default on any one transaction, which allows you to offset positions with the same counterparty

   b. collateral requirements

   c. downgrade triggers force early termination upon credit deterioration

Background: Mortgage Backed Securities and Related Securitizations

1. Mortgages are essentially bonds issued by the *borrower* and held by the lender. The lender is exposed to default risk, but this is mitigated because the house serves as collateral. The lender also is exposed to prepayment risk, especially since prepayments could occur at the worst possible time for the lender — when interest rates have fallen.

2. Mortgage pools help to manage the prepayment risk in mortgages by selling the mortgages into a pool and then selling interests in the pool to investors. This eliminates the risk for the lender but just transfer it to the investors. Though perhaps not in the same form.

    a. In a *pass-through*, investors share proportionately in all of the cash flows into the pool, but gain the benefits of risk pooling and risk sharing.

    b. In a *collateralized mortgage obligation* (CMO), investors in different tranches have different priority over receiving the cash flows, so some investors take more risk and some take less risk. For instance, the different tranches may receive principal cash flows sequentially and so will be affected very differently by prepayments on some mortgages in the pool.

3. The risk we usually focus on is prepayment risk, in part because in traditional mortgage pools agencies such as Fannie Mae or Freddie Mac provided default protection. This wasn't the case though for CMOs formed with sub-prime mortgages. In those CMOs, the default risk was also passed along to the investors.

4. Stripped MBS (IO & PO) are special forms of mortgage pass throughs where some investors get only the interest payments and some get only the principal payments. These have unusual sensitivity to interest rate changes — IOs have *negative duration* and POs have very high positive duration.

5. Subprime Mortgage Securitizations — Investors' appetite for assuming the prepayment and default risk within subprime mortgage securitization exploded, with $1.2 trillion worth originated in 2005 and 2006 alone. This provided huge flows of cheap financing to subprime borrowers and fueled a massive housing price bubble.

    But unlike traditional MBS, *subprime mortgage securitizations depended heavily on housing prices increasing* because house price increases were virtually the only thing preventing default.

6. Collateralized Debt Obligations (CDOs) are just like CMOs, but rather than contain mortgages they contain any type of "debt obligation". CDOs allowed credit risk to be packaged and sold to investors who had different appetites for such risk. But when tranches of traditional and subprime CMOs began to be used as the debt obligations inside CDOs, they became extremely complex, difficult to rate and difficult to price.

Coval, Jurek & Stafford: The Economics of Structured Finance

1. Two aspects of securitizations contributed significantly to the financial crisis of 2007-2009:

   a. Ratings for senior tranches of securitizations are extremely sensitive to modest imprecision in evaluating the default probabilities and default correlations among the underlying risks.

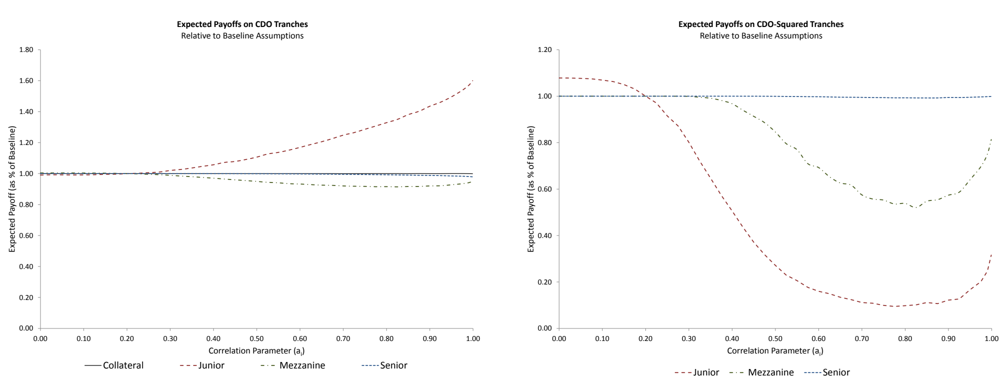
   b. Securitized tranches contain significant exposure to systematic risks.

2. CDOs made it possible to *manufacture* highly rated bonds by pooling lower rated bonds together and then selling low-rated tranches that assumed the bulk of the credit risk and large tranches that were very highly rated because they assumed only the remote risk that substantially all of the bonds would default. Often these top tranches, comprising 80% or more of the pool, would have AAA ratings.

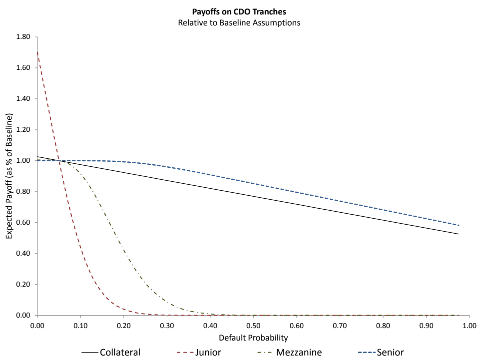
3. Rating CDO tranches is much harder than rating an individual bond because it depends crucially on the assumed correlation of bonds within the pool, rather than characteristics of a particular company. It is also far more sensitive to errors in estimating default rates for the components.

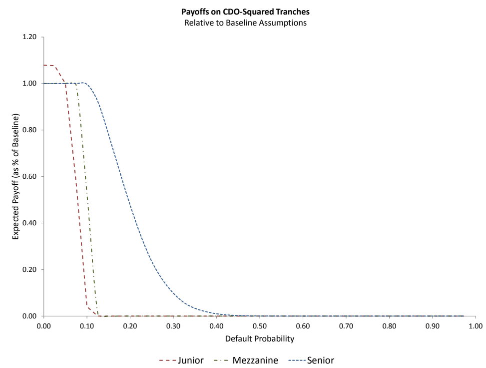
4. The authors evaluated the sensitivity of CDOs and CDO-squareds to both default probabilities and default correlations and their results are shown in the charts below.

Sensitivity to Correlation Parameter



Sensitivity to Default Parameter





5. Aside from the challenges of quantifying the default risk, there are special challenges associated with putting a value on tranches of CDOs. This is because CDO tranches have substantially more *systematic risk* and are essentially "economic catastrophe bonds". Securities with these characteristics need to have very large risk premiums, regardless of the default probability.

6. Reasons why the structured finance market grew so spectacularly:

    a. Seemingly attractive yields due to biased historical default data (especially for subprime mortgages) and lack of appreciation for the systematic risk.

    b. Extreme market optimism

    c. Too little appreciation for fragility of ratings

    d. Perverse incentives for rating agencies

    e. Perverse incentives for banks

Hull Chapter 24: Credit Derivatives

1. Credit default swaps are an efficient way to transfer default risk. The protection buyer (essentially of insurance) pays a periodic swap rate (premium) to the protection seller and the seller pays the difference between the par value of the bond and the market value upon default.

2. Key elements of a CDS are: the reference entity, the reference obligation, the notional principal and the definition of what constitutes "default".

3. The *Credit Default Swap Spread* is the payment rate such that the present values of the buyer and seller payments are equal. The four steps to determine this rate are:

   a. Determine Unconditional Default Probabilities

   b. Determine Protection Buyer's Payments — These will include the periodic swap spread and the accrued swap spread at the time of default.

   c. Determine Protection Seller's Payments — This is the expected default payment, reflecting the probability of default at each point in time and the losses in the event of default at that time. This latter value is easier to determine if the swap payment is a fixed amount or a fixed percent of par, rather than a function of the market value.

   d. Solve for the CDS Spread, s, so that the buyer and seller payments are equal.

4. As long as the same recovery rate in the event of default is used to calculate the risk neutral default probabilities and to value the CDS, the assumed recovery rate doesn't have a significant impact on the estimated CDS spread or the value of an existing CDS.

5. Several indices exist to track the overall CDS market spreads, such as the CDX NA IG, which is a portfolio of CDS on 125 investment grade North American companies, and iTraxx Europe, which is a portfolio of CDS on 125 investment grade names in Europe.

6. Other credit derivatives of interest are CDS forwards and options, basket credit default swaps and total return swaps.

7. When a CDO pool consists of actual bonds it is referred to as a **Cash CDO.** When it consists solely of credit default swaps, it is referred to as a **Synthetic CDO.**

8. A brief outline of the Credit Crunch of 2007 and the role CDO's played in it can be summarized as follows (based on discussion in Hull Chapter 8 in the Eighth Edition):

   a. The housing price bubble distorted the default experience on mortgages.

   b. Lenders began to relax their lending standards (liar loans, NINJA loans, adjustable rate and other exotic payment provisions)

c. Housing bubble bursts in 2006/2007, causing a surge in subprime defaults, losses in junior tranches of CDOs containing subprime mortgages and huge mark-to-market losses on senior tranches.

d. Downgrades of AAA-rated CDO tranches caused urgent needs to post collateral, which was obtained through fire sales of just about everything and massive price declines in many markets with no subprime connection.

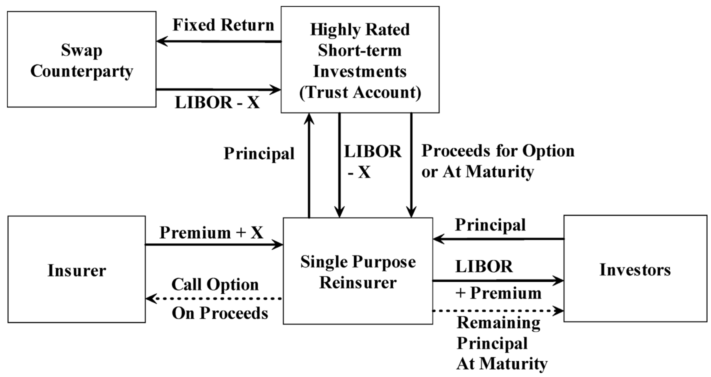
e. Large losses for banks holding securities intended to go into CDOs and for banks that had been dependent on rolling over short-term financing, which was suddenly unavailable because of the inability to post high-quality collateral.

f. With the securitization market dead and liquidity scarce, spreads increased for all forms of debt at all maturities.

Cummins: CAT Bonds and Other Risk-Linked Securities

1. The basic structure of a catastrophe bond:

FIGURE 1. Basic Catastrophe Bond Structure



Investors provide capital (to purchase a bond) to a trust or Single Purpose Reinsurer (SPR), which simultaneously offers a traditional reinsurance contract to the insurer. If a claim is paid on the insurance, investors will not be able to receive all of their principal back, hence catastrophe risk is transferred to the investors.

2. Cat bonds differ from traditional reinsurance in a few ways: the limit is collateralized (with cash), coverage is usually for 3 years, coverage is sold primarily for very risk-remote layers only, coverage is usually on an index basis.

3. Index calculations for the purposes of determining the "claim" payments in index-based bonds can use industry losses, modeled losses based on a particular cat model and a particular exposure database, a parametric calculation based on characteristics of the cat event or a hybrid of these.

4. The choice of loss triggers is a matter of balancing the trade-off between moral hazard, transparency for the investors and basis risk for the insured.

5. Sidecars are structured to provide quota share reinsurance capacity, either for a reinsurer's entire book of business or some subset thereof, as opposed to excess of loss capacity. Otherwise, the structure is similar to a cat bond.

6. Catastrophe Equity Puts (Cat-E-Puts) were used to provide contingent equity capital to a company upon the occurrence of a defined event, at pre-agreed terms.

7. Catastrophe Risk Swaps allow two insurers to "swap" payments tied to different indices — such as one party paying according to industry-wide or company-specific California earthquake claims and the other party paying according to industry-wide or company-specific Florida hurricane claims.

8. Industry Loss Warranties (ILW) are index-based reinsurance agreements, where the payoff is intended to be based on industry-wide claims from defined perils, in defined regions, and for defined classes of business (e.g. personal lines claims from hurricane losses in Florida).

9. Key facts about the Catastrophe Bond Market:

   a. Volume — In 2013, catastrophe bonds represented approximately 8% of the total property-catastrophe reinsurance market (by limits placed). Including ILWs, sidecars, etc. would probably bring this figure to 15%.

   b. Triggers — Index-based triggers dominate the market.

   c. Tenor — Most bonds are issued for 3-year risk periods.

   d. Issuers — Issuance by primary insurers and reinsurers has been nearly equally split, with minimal issuance by non-insurer corporations and governments.

   e. Ratings — Over 90% issued with below-investment grade ratings (BB or B).

   f. Investors — In 2013, dedicated catastrophe bond funds represented 43% of the market and institutional investors (pension funds) represented 41%. The rest captured by mutual funds, hedge funds and (re)insurers.

   g. Loss Experience — Experience has been very good, with only one bond (2005 Kamp Re covering commerical risks in Louisiana) suffering a total loss.

10. Catastrophe Bond Pricing — Similar to reinsurance though it is theoretically possible for non-insurer investors with, for instance, no natural exposure to Florida hurricane risk to price in a lower risk margin for a Florida hurricane catastrophe bond than would an existing reinsurer. But they haven't been cheaper due to higher expenses and the fact that while investors could charge less, they haven't had to.

    a. Multiples of Spreads to Expected Loss Vary by Expected Loss Range

    b. Speads Vary by Peril and Geography

    c. Modeled Expected Losses Have Generally Increased So Spreads Have Been Stable

11. Factors that could facilitate further growth in the market for risk-linked securities:

    a. Improved reporting of insured losses.

b. Regulatory capital requirements should acknowledge the counterparty credit risk associated with reinsurance recoveries, which would give a boost to catastrophe bonds which are fully collateralized.

c. Personal lines insurance rates should be deregulated and more credit should be given to insurers who lock-in multi-year reinsurance coverage.

d. ERISA rules impacting catastrophe bonds should be explored.

Part 5

Financial Risk Management (b)

Stulz: Rethinking Risk Management

1. Firms can maximize their value through the use of derivatives to hedge certain financial risks, thus minimizing the volatility of their cash flows and minimizing the costs of financial distress. It is not the volatility reduction *per se* that impacts value, it is the effect this has on reducing the following costs:

    a. Bankruptcy costs, including direct and indirect costs such as the *under investment problem*

    b. Payments to stakeholders including managers, employees and suppliers who cannot diversify the firm-specific risk

    c. Taxes, which are higher for more volatile earning streams because of increasing marginal tax rates, limits on the use of tax loss carry forwards and the alternative minimum tax

2. The primary objective of risk management is to eliminate costly lower tail outcomes, thereby minimizing the likelihood of financial distress and preserving the financial flexibility to carry out investment objectives.

3. Risk Management in Practice — The above suggests that smaller firms would hedge more often than larger firms and firms that hedge would do so primarily to reduce volatility. In practice though, surveys find that the opposite is true — large firms tend to hedge more than small firms (perhaps due to the investment in personnel, training and computer systems required) and often firms engage in selective hedging. Other survey findings are that most hedging seems to be for executed transactions and near term exposures, rather than company-wide financial risks and firms that hedge want to avoid large losses but not at the expense of giving up large gains.

4. How should a firm's capital ratio impact its hedging decisions?

    a. Highly rated firm with low debt to equity ratio can can afford not to hedge, or can adjust its risk upward by using more debt and use hedging of its financial risks in its business to offset this. Overall risk is the same, but the benefits of debt are achieved: lower taxes, strengthen management incentives, increase concentration of equity ownership.

    b. Firm with low credit rating and significant probability of financial distress should engage in active risk management to reduce the probability of financial distress.

    c. Firm in distress should not hedge its risks because most of the gains will go to the debtholders anyway. In fact, this firm should aggressively take on more risk since downside risk is borne by the debtholders.

5. Manager and shareholder incentives for hedging differ. The greater management's share ownership, the more they hedged. However, compensation in the form of stock options tended to result in less hedging, since the option holder benefits from volatility.

6. Cash flow simulations allow you to measure risk over longer time horizons and study the path of firm value over the entire period and not just at the end of the period as in Value at Risk. It also has the flexibility to allow for non-normal distributions and correlations among variables.

7. Management incentives for managing risk taking can be strengthened by ensuring gains from risk taking activities must be measured on a risk-adjusted basis and ensuring managers are not compensated merely for taking risks, only for earning excess risk adjusted returns.

Culp, Miller & Neves: Value at Risk — Uses and Abuses

1. Value at Risk (VaR) first emerged as a means for trading firms to measure and aggregate risks in their different trading portfolios, using a dollar-based and probability-based measure of risks over a common time horizon for all components in the portfolio.

2. Four derivatives disasters occurred in the 1990's and it is interesting to question whether the use of VaR to measure and manage risk would have prevented them.

   a. Proctor & Gamble — In 1993, P&G entered into a fixed for floating interest rate swap to reduce its financing costs significantly but in exchange they also wrote an interest rate put to its swap counterparty. When rates moved against them, they lost over $100 million.

      If the risk that was taken by writing the put was better understood by senior management, and if senior management was really monitoring individual transactions' VaR, then perhaps the deal would not have been approved. But VaR is not usually measured at the transaction level.

   b. Barings — A young trader made enormous bets on the Japanese stock market using futures contracts and lost over $1 billion. A VaR measure would have told the senior management to shut down this operation much sooner, but they would have needed better IT systems to capture, record and report on all trading activity.

   c. Orange County — This municipality suffered $1.5 billion in losses when its leveraged bets on the slope of the yield curve went sour. VaR would have made it clear what the potential losses were, but if the fund manager was knowingly taking a large risk in order to earn high returns, as he appeared to be doing, then perhaps VaR would not have motivated any change in strategy. For entities whose mission is to take risks, reporting the potential losses alone without also reporting the potential gains renders the information rather meaningless.

   d. Metallgesellschaft (MGRM) — MGRM sold long term fixed price oil contracts to customers and hedged the risk using short term futures contracts. But when prices fell and the futures contracts had huge paper losses (offset for the most part by unrecognized huge paper gains on the long term contracts sold to their customers), senior management forced a liquidation of the futures contracts.

      Aside from the fact that they were taking risks intentionally and apparently with full knowledge of the size of the risk, the real issue is that VaR is a value risk measure and the real issue for MGRM was a cash flow problem not a value problem.

3. Several alternatives to VaR exist including cash flow simulations, risk-based capital allocations to help ensure potential returns are evaluated relative to their risks and shortfall risk measures that focus more on the downside rather than total variability.

Hull Chapter 23: Credit Value at Risk

1. Two ways to capture correlation of default among counterparties:

   a. *Structural models*, such as the Merton model, correlate the stochastic processes the assets of the firms follow.

   b. *Reduced Form models*, such as the Gaussian copula, assume the conditional default probabilities follow a stochastic process and are correlated with macroeconomic variables.

2. Credit Value at Risk attempts to quantify the dollar amount of credit-related (default) losses that will not be exceeded with $X\%$ probability over an N-year period. Two approaches are:

   a. Gaussian Copula Model for Credit VaR — Assume defaults are driven by a (standard normally distributed) common factor, then the percentage of defaults in a portfolio will be less than $V(X,T)$ $X\%$ of the time, where:

   $$V(X,T) = \Phi\left[\frac{\Phi^{-1}[Q(T)] + \sqrt{\rho}\Phi^{-1}(X)}{\sqrt{1-\rho}}\right]$$

   b. CreditMetrics Model for Credit VaR — An alternative is to simulate credit rating changes for each counterparty in the portfolio and quantify the losses that result from default or repricing at higher spreads when credit rating deteriorates.

Butsic: Solvency Measurement for Risk Based Capital Applications

1. Expected Policyholder Deficit (EPD) is defined as the expected value of the difference between the insurer's obligation to pay the claimant and the actual amount paid. It is better than a ruin probability risk measure because it considers the severity of the shortfall and not just its probability.

$$\text{Expected Policyholder Deficit} = D_L = \sum_{x>A} (x - A)p(x)$$

where D_L is the EPD, x is the size of loss, A is the value of the assets (which are assumed constant at this point) and $p(x)$ is the probability density of x. A similar calculation can be done if the assets are random and the liabilities are fixed or, more realistically, if both are random.

2. EPD Ratio is just the EPD divided by the expected loss,

$$\text{EPD Ratio} = d_L = D_L/L$$

where D_L is the EPD and L is the expected loss.

3. Setting Capital Requirements — Set target EPD Ratio (e.g. 5% in example below), solve for the assets needed and set capital equal to assets minus expected loss:

TABLE 1. Capital Needed for 5% EPD Ratio: Fixed Assets

Scenario	Asset	Loss	Capital	Probability	Claim Payment	Deficit
1	10,600	6,900		0.2	6,900	0
2	10,600	10,000		0.6	10,000	0
3	10,600	13,100		0.2	10,600	2,500
Expected Value	10,600	10,000	600		9,500	500
					EPD	500
					Expected Loss	10,000
					EPD Ratio	0.05

4. EPD formulas when asset and/or liability distributions are continuous follow from the definitions above.

   Defining the following variables:

   a. c = ratio of capital to expected losses, such that $A = L + C = (1 + c)L$

   b. $c_A = C/A$ = capital to expected asset ratio

   c. k = coefficient of variation (ratio of standard deviation to mean) for the liabilities

   d. k_A = coefficient of variation for the assets

e. $\phi(\cdot)$ is the standard normal density function

f. $\Phi(\cdot)$ is the standard normal distribution function

we can show the formulas for the EPD ratios depending on whether we assume normal or lognormal distributions.

5. The EPD Ratios in the case of normally distributed assets or liabilities are:

$$d_L = \frac{D_L}{L} = k\phi\left(-\frac{c}{k}\right) - c\Phi\left(-\frac{c}{k}\right)$$

$$d_A = \frac{D_A}{L} = \frac{1}{1-c_A}\left[k_A\phi\left(-\frac{c_A}{k_A}\right) - c_A\Phi\left(-\frac{c_A}{k_A}\right)\right]$$

We use these formulas usually by setting them equal to a target EPD ratio and solving for c, the capital ratio.

6. The EPD Ratios in the case of lognormally distributed assets or liabilities are:

$$d_L = \Phi(a) - (1+c)\Phi(a-k)$$

$$d_A = \Phi(b) - \frac{\Phi(b-k_A)}{1-c_A}$$

where,

$$a = \frac{k}{2} - \frac{\ln(1+c)}{k}$$
$$b = \frac{k_A}{2} + \frac{\ln(1-c_A)}{k_A}$$

7. When *both* assets and liabilities are stochastic, Butsic's analytical formulas cannot be used exactly as shown. See the Cummins reading for how to handle this case.

8. Capital requirements when there are multiple risk elements are determined by using a *square root rule* which sets the required capital equal to the square root of the sum of the squares of the required capital for each risk element on its own plus twice the correlation coefficient times the separate capital amounts.

$$C = \sqrt{\sum C_i^2 + \sum_i \sum_{j\neq i} \rho_{ij} C_i C_j}$$

where C_i is the required capital for each risk element and the correlation coefficients, ρ, are adjusted so that if the risk elements are on opposite sides of the balance sheet their correlations are multiplied by -1.0 (i.e. the sign of the measured correlation is reversed).

Cummins: Allocation of Capital in the Insurance Industry

1. All of the firm's capital stands behind all of the firm's liabilities, so why would we want to allocate capital to individual lines? There are three reasons given:

   a. Pricing, underwriting and other decision making could possibly be enhanced by thinking of capital as being allocated, even though it is not literally allocated.

   b. Allocation of capital could help to tie together certain financial decisions and regulatory risk-based capital rules.

   c. Concepts like risk-adjusted return on capital (RAROC, discussed in the Goldfarb paper) and economic value added (EVA) make use of capital allocation for performance measurement.

2. To maximize shareholder value profits must be weighed against the capital requirement (a risk measure) for the businesses using either Risk-Adjusted Return on Capital (RAROC), which calculates a ratio of the profit to allocated capital, or Economic Value Added (EVA) which penalizes the profit by a "cost" of the allocated capital.

3. Several capital allocation methods are presented:

   a. Regulatory (NAIC) Risk-Based Capital

   b. CAPM

   c. Value at Risk

   d. Insolvency Put Option (Expected Policyholder Deficit)

   e. Merton-Perold

   f. Myers-Read

4. Cummins' Formula for Butsic's Expected Policyholder Deficit — Cummins calls this the Insolvency Put Option and uses it to calculate the capital needed to achieve a 5% EPD ratio.

   He assumes both assets and liabilities are stochastic and so he focuses on the asset to liability *ratio* rather than either the assets or the liabilities. Then he was able to calculate the *EPD Ratio* using the standard Black-Scholes put formula with a strike price of 1.0 and the volatility parameter representing the volatility of the log of the asset to liability ratio.

$$\begin{aligned}
\text{Put} &= \text{Call} + \text{PV(Strike)} - \text{Stock} \\
&= \text{Call} + \text{PV(1.0)} - \text{Asset-to-Liability Ratio} \\
\text{Call} &= (\text{Asset-to-Liability Ratio})N(d_1) - 1.0N(d_2) \\
d_1 &= \frac{\ln(\text{Asset-to-Liability Ratio}/1.0) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \\
d_2 &= d_1 - \sigma\sqrt{T}
\end{aligned}$$

In simpler cases, such as when only the liabilities are stochastic, the formulas from the Butsic reading can be used instead.

5. To calculate the volatility parameter Cummins approximates it as the *coefficient of variation*. In order to get the firmwide σ for a multi-line firm though, he converts the variability measure into standard deviation in dollars by multiplying the σ by the expected losses, derives the overall standard deviation in dollars using the correlation assumptions and then converts back to a coefficient of variation by dividing by the expected aggregate losses.

6. Merton & Perold — The M-P approach looks at the marginal impact of each line of business by including or excluding the line in its entirety and seeing what impact that has on the required capital. The marginals calculated this way will not add to the total though, so some capital is unallocated.

7. Myers-Read — The Myers-Read approach is similar to Merton-Perold conceptually but they measure the marginal capital for a particular line of business by determining the effect of a *small* increase in the size of the line (based on expected loss amount). Using the EPD ratio and using partial derivatives so that each line has the same marginal impact on the overall EPD ratio we get the following surplus to liability ratio for each line:

$$s_i = s - \left(\frac{\partial p}{\partial s}\right)^{-1} \left(\frac{\partial p}{\partial \sigma}\right) [(\sigma_{iL} - \sigma_L^2) - (\sigma_{iV} - \sigma_{LV})] \frac{1}{\sigma}$$

where the σ terms with the V subscripts reflect the covariance of the line i losses with the assets and the covariance of the total losses L with the assets, respectively, σ_L is volatility of the whole firm and $\sigma_i L$ is the covariance of line i losses with the total losses.

8. The Myers-Read method only determines an allocation percentage. Cummins uses a 5% EPD Ratio as the basis for determining the total required capital.

9. Why does capital have a "cost" that has to be allocated? This comes from Agency Costs, Double Taxation and Regulatory Costs.

10. The following are the key conclusions of the paper:

a. EPD is better than VaR, even though both might be useful to calculate. He also thinks it is better to estimate EPD and VaR at different thresholds rather than at just a single point.

b. The option models are better than EPD or VaR because they allow recognition of diversification benefits and he prefers the Myers-Read to the Merton-Perold method.

c. The economic cost of capital (the frictional cost or the spread cost) is what should be allocated to the lines of business.

d. Capital allocation must reflect both the asset and liability risks and in particular the covariance between them.

e. The duration and maturity of the liabilities should be reflected in the capital allocation.

f. The decision making system should dictate the data needs, not the other way around.

g. Capital allocation will lead to better pricing, underwriting and strategy decisions and will lead to shareholder value creation for the winning firms.

Goldfarb: Risk-Adjusted Performance Measures for P&C Insurers

1. RAROC defined:

$$\text{Risk-Adjusted Return on Capital} = \frac{\text{Income}}{\text{Risk-Adjusted Capital}}$$

   where income can be any of GAAP Net Income, Statutory Net Income, IASB Fair Value Basis Net Income, or Economic Profit and risk-adjusted capital is either Regulatory Required Capital, Rating Agency Required Capital or Economic Capital defined to achieve some objective such as a target probability of ruin or target EPD ratio.

2. Two capital measures that are not "risk-adjusted" could arguably be used if allocated to lines of business on a risk-adjusted basis. These include the actual capital (book value) or the market value of the firm's capital (equity).

3. Regardless of the definition of capital, several methods of allocating this to lines of business on a risk-adjusted basis are discussed:

   a. *Proportional Allocation Based on a Risk Measure* — This method simply calculates stand-alone risk measures for each risk source and then allocates the total risk capital in proportion to the separate risk measures.

   b. *Incremental Allocation* — This method determines the impact that each risk source has on the aggregate risk measure and allocates the total risk capital in proportion to these incremental amounts, as in the Merton-Perold method discussed in Cummins.

   c. *Marginal Allocation (Myers-Read Method)* — This method determines the impact of a small change in the risk exposure for each risk source (e.g. amount of assets, amount of reserves, premium volume) and allocates the total risk capital in proportion to these marginal amounts.

   d. *Co-Measures Approach* — This method determines the contribution each risk source has to the aggregate risk measure.

4. The Myers-Read method has become popular because it produces additive capital requirements that sum to the total capital requirement for the firm when the same risk measure is used. But it was developed to allocate frictional costs rather than to allocate capital in a RAROC calculation. Other issues are that it relies on the calculation of the default option and isn't mathematically valid when changes in the business impact the shape of the distribution.

5. Co-Measures Approach — Co-measures such as the Co-CTE are easy to calculate in a simulation setting. For example, 50,000 simulation scenarios are developed, the results are sorted in descending order based on the *total loss* and the worst 1% of the scenarios (the top 500 scenarios) are identified. Then, the average losses for each risk unit in *those*

500 scenarios are calculated and reflect the Co-CTE for each risk unit, which by definition adds to the total CTE.

6. Four applications of RAROC are discussed, including Assessing Capital Adequacy, Setting Risk Management Priorities, Evaluating Alternative Risk Management Strategies, Risk-Adjusted Performance Measurement and Insurance Policy Pricing.

   The latter two are specifically referenced in the learning objectives and appear to be the most relevant for the exam:

   a. RIsk-Adjusted Performance Measurement evaluates actual income relative to the risk that was assumed so that different business units' performance can be compared. By rescaling the profit by the allocated capital we might find that despite higher profits lines of business that require more allocated capital may not be better performers.

      This use of RAROC to better inform the assessment of performance shows that it is possible to take risk into consideration in a relatively simple manner. However, there are a variety of allocation methods that could be used and they may give conflicting results.

   b. Pricing of policies can also use an *ex ante* version of RAROC and set prices so that RAROC is above a specified target rate. The following equation can be used to solve for the additional risk margin, π, that produces the target rate of 15% (under the simplifying assumption that the increased risk margin does not impact expenses).

$$\text{RAROC} = \frac{[P + \pi - E](1 + i) - PV(L)}{\text{Allocated Risk Capital}}$$

      An alternative calculation (the EVA approach) can also produce the same result by requiring the premium to also include the present value of the "cost of capital" applied to the allocated capital.

7. Three specific issues relevant to pricing applications are:

   a. Multi-Period Capital Commitment — If capital is held for more than one period, we need to capture the commitments each period. A simple way to do this is to use the EVA approach and ensure that the target return is earned for each year in which capital will be allocated.

   b. Cost of Risk Capital — The appropriate rate to use should reflect the way in which risk is defined in the RAROC calculations, but this is difficult to do in practice. One compromise is to use CAPM or Fama-French to determine the total shareholder returns required on the firm's total book value or market value and use risk-adjusted capital allocation of one of these actual amounts rather than a theoretical "economic capital" which creates an implicit leverage effect.

c. Investment Income on Allocated Capital — This has to be consistently treated in both the income and the target return calculations.

Part 6

Internal Rate of Return

Felblum: Internal Rate of Return Model

1. Early rating bureau pricing procedures used a fixed profit margin, which didn't consider the time value of money, didn't respond to changes in the competitive market environment and used sales as a rate base, which doesn't take into account the equity provided from the investors (owners).

2. The main reason the shareholders' equity matters is due to the double taxation of investment income on surplus, which creates an inefficiency that must be captured in premiums charged in order to ensure sufficient capital is attracted to the industry.

3. Applying the IRR Model in Insurance — Focus on the investors' equity flows, which typically consist of committed surplus up-front and then eventual return of income and surplus over time. Unlike cash flow models that focus solely on the insurance flows (premium, loss, expense), these cash flows are used only indirectly in the IRR model because they impose constraints on the amount of surplus that can be returned to investors and its timing.

4. At the company level, the initial surplus required is not always clear. Regulatory capital requirements only set *minimum* levels and actual company surplus at any point in time is highly volatile. As a result, it is common to rely on industrywide surplus levels as the basis (e.g. through industry premium to surplus ratios). Using industry levels of surplus implicitly assume that at the industry level the amount of capital invested is appropriate.

5. It is also critical to determine how this surplus can be released. Often in practice it is assumed that the surplus can be released in proportion to paid losses, earned premium or some combination of the two. These choices regarding surplus directly affect how much is allocated to any one line of business at a given time, and can greatly influence the results. This is important because in the end, unlike in other industries where the "initial investment" is known, the committed surplus is a theoretical value.

6. Since surplus exists to handle risks arising from a variety of sources, neither premiums not reserves may be appropriate bases for allocation of surplus. Instead, a risk-based allocation may be more appropriate.

   This consideration is important not just across lines of business but also across policy forms, as the risks in different policies within the same line can vary widely (e.g. claims made vs. occurrence coverage, first dollar coverage vs. high layer excess).

7. Even if IRR is used to price internally it may be wise to use NPV analysis in rate filings because of two factors. One, it isn't always clear how an IRR lower than the company's cost of capital really impacts the company, whereas an NPV calculation clearly shows that this produces a negative NPV. Two, larger losses produce a larger amount of theoretical surplus required, which in turn leads to more investment income and an IRR closer to the investment yield. Both of these factors could be misinterpreted by regulators.

Robbin: IRR

1. IRR on Equity Flows — Track equity flows that would occur had we set up a company to write a *single* policy and set the IRR of equity flows equal to a target IRR.

2. Equity flows reflect statutory income less the change in the required surplus

   Equity Flow = Statutory Income − Change in Statutory Surplus

   $$EF = INC - SCHNG$$

3. IRR defined as the rate such that:

   $$0 = \sum_{j=0}^{n} \frac{\left[INC_j - SCHNG_j\right]}{(1 + IRR)^j}$$

4. Advantages: Simple to interpret, captures impact of accounting rules

5. Disadvantages: Requires surplus assumption (at inception and over time), not clear whether more risk requires more surplus, higher IRR or maybe both

Part 7

Rate of Return and Risk Loads

Ferrari: Total Return on Owners' Equity

1. Using the following notation:

    a. T - Total after-tax return to the insurer

    b. I - Investment gain or loss (after appropriate tax charges)

    c. U - Underwriting profit or loss (after appropriate tax charges)

    d. P - Premium income

    e. A - Total assets

    f. R - Reserves and other liabilities (excluding equity in unearned premium reserves)

    g. S - Stockholders' equity (capital, surplus, and equity in unearned premium reserve)

    the total shareholder return can be written as:

$$\frac{T}{S} = \frac{I}{A}\left(1 + \frac{R}{S}\right) + \frac{U}{P}\frac{P}{S}$$

2. Two leverage factors are embedded in the above formula. In the case of return on assets, we use an *insurance leverage* factor that considers the size of the reserves relative to surplus. In the case of the underwriting profit we use an *insurance exposure* factor that considers the amount of premium that can be written relative to surplus.

3. Three perspectives on profitability are: the regulator or the pricing actuary perspective which focuses on profit relative to premium; society's perspective which focuses on profitability relative to assets dedicated to insurance; and the equity holders' perspective which focuses on profit relative to the value of their equity.

4. An alternative form of the total return equation is:

$$\frac{T}{S} = \frac{I}{A} + \frac{R}{S}\left(\frac{I}{A} + \frac{U}{R}\right)$$

5. Treating the reserves as being "borrowed" from the policyholder, we see that we get to invest those to earn investment income (which does belong to the shareholders). The cost of this borrowing is the underwriting loss relative to reserves. The only catch is that this cost is unknown and variable.

6. A reviewer noted that increasing the T/S ratio may not actually increase shareholder value if doing so makes the returns riskier. More importantly, maximizing the return isn't a matter of just increasing or decreasing the ratios shown because of the dependence between them:

a. Increases in the P/S ratio will likely lead to lower investment return on assets, I/A, as balance sheet items like agents' balances increase or as risk considerations require a more conservative investment policy.

b. P/S ratios will move in the same direction as the underwriting profit, U/P, since the more profitable the business the more insurance risk you can assume without increasing the risk of insolvency.

c. Underwriting profits and investment return on assets will move in the same direction since higher underwriting profits allow for more risk-taking on the investment side.

Roth: Analysis of Rate of Return without Using Leverage Ratios

1. It is not theoretically possible to allocate surplus by line of business and, as a result, no premium to surplus (leverage ratios) exist that can be applied to all insurers.

    a. The appropriate amount of aggregate surplus is unique to each insurer based on all of its risks, so leverage ratios can vary considerably by insurer.

    b. All of the surplus an insurer stands behind all of its risks, so it cannot be allocated to line or state in a realistic fashion.

    c. Actual surplus at any point in time will vary considerably from the theoretical amount needed.

2. The proper way to measure income is to reflect *all* sources of income, thus capturing the full change in surplus during the year (excluding the increases in surplus due to additional paid in capital and the decreases due to the payment of dividends). This, return should be measured as dS/S.

    Whether we use GAAP or Stat surplus may not be a critical issue if we can assume the ratio of GAAP to Stat is roughly constant.

3. The required return for insurers, both stock and mutual, is the amount that will allow them to continue to provide the insurance capacity needed for society, which includes a provision for each of the following — inflation, changes in the valuation of the existing liabilities and changes in demand for insurance.

    Some measurement details:

    a. Expense and claim inflation can be estimated using historical rates of claim inflation, with whatever subjective adjustments seem appropriate to reflect the current economic environment.

    b. Increases in demand for insurance can be measured using premium growth trends on an inflation adjusted basis. Constant premium to surplus ratios mean that premium growth must directly lead to required surplus growth at the same rate.

    c. Surplus growth related to supporting existing business can be measured using the total reserve growth trends (inflation adjusted) and subtracting the premium growth rates.

4. Stock insurers require an additional component of return, which is the dividend that will ensure that their investors earn a reasonable overall rate of return on market value and that additional capital can be attracted as needed.

5. The return stockholders really care about is return on market value not book value, which we've used above. But in equilibrium market to book ratios will adjust so that return on market value will always be "fair" and so the return on surplus will also be "fair".

McClenahan: Insurance Profitability

1. Return on premium ("return on sales") has two very appealing practical benefits compared to return on surplus — it is easy to interpret and it does not require the use of allocated surplus to measure expected profitability. The latter point means that it can be used for all insurers.

2. Only investment income on policyholder supplied funds (reserves net of funds not invested in marketable securities) should be included as the investment income on surplus rightly belongs to the shareholders since they take the risks.

3. From the perspective of the regulator, insurers charging the same premiums should be treated the same, independent of their respective equity bases. Taking into account the equity base might be an important consideration for the investors, but it shouldn't impact the regulation of insurance rates.

4. Rates of return on surplus that are fair and reasonable will be those that lead to desirable market characteristics in terms of the size of the residual market, the degree of competition and the degree of product diversity and innovation.

Robbin: Underwriting Profit Provision

1. Overview of Methods

TABLE 1. Summary of Seven Methods

Approach	Method
Investment Income Offset Methods	CY Investment Income Offset Present Value Offset
Target Total Return Methods	CY ROE IRR on Equity Flows PV of Income to PV of Equity Ratio
Cash Flow Methods	PV Cash Flow Return Risk-Adjusted Discounted Cash Flow

2. Calendar Year Investment Income Offset — Adjust traditional underwriting profit margin U^0 by the investment income from the policyholder supplied funds.

$$\begin{aligned} U &= \text{Traditional U} - \text{After-Tax Inv Income on Policyholder Supplied Funds} \\ &= U_0 - i_{AT}[\text{Policyholder Supplied Funds}] \\ &= U_0 - i_{AT}(PHSF) \end{aligned}$$

where,

$$\begin{aligned} PHSF &= \text{Unearned Premium Balance} + \text{Loss and LAE Reserves} \\ &= [\text{UEPR Net of Prepaid Expenses} - \text{Premium Receivables}] \\ &\quad + [\text{Loss Ratio} * (\text{Loss Reserves/Incurred Loss})] \end{aligned}$$

Advantages: Simple, uses readily available data, stable results

Disadvantages: rapid growth or decline distorts ratios used

3. Present Value Offset — Assume that the traditional underwriting profit margin, U^0 = 5% is appropriate for a *typical* line of business and adjust to reflect investment income differences caused by faster or slower payments than the reference line.

$$\begin{aligned} P &= L[PV(X)] + EXP + U^0(P) + L[1 - PV(X^0)] \\ &= \frac{L + EXP - L[PV(X^0) - PV(X)]}{1 - U^0} \end{aligned}$$

Or in terms of reference line's profit margin:

$$U = U^0 - (PLR)[PV(X^0) - PV(X)]$$

When discounting the losses in this method, current yields on new investments (the *new money rate*) is more theoretically sound. But Robbin argues that in practice existing portfolio yields will tend to be more stable and might be easier to justify to regulators.

Advantages: Simple, isn't affected by growth, doesn't use target return or allocation surplus

4. Calendar Year ROE — This method simply calculates a calendar year ROE and sets the premium so that a target ROE is achieved (prospectively of course, so on an expected basis).

$$ROE = \frac{\text{UPP in Dollars} + \text{Investment Income} - \text{Taxes}}{\text{GAAP Equity}}$$
$$= \frac{U \cdot P + II - FIT}{EQ}$$

Then we solve for U so that the expected ROE equals the target ROE.

Advantages: Inputs readily available, similar to GAAP ROE

Disadvantages: Distorted by growth, requires P:S ratio, uses GAAP equity

5. IRR on Equity Flows — Track equity flows that would occur had we set up a company to write a *single* policy and set the IRR of equity flows equal to a target IRR.

$$\text{Equity Flow} = \text{Statutory Income} - \text{Change in Statutory Surplus}$$
$$EF = INC - SCHNG$$

6. IRR defined as the rate such that: $0 = \sum_{j=0}^{n} \frac{[INC_j - SCHNG_j]}{(1+IRR)^j}$

Advantages: Simple to interpret, captures impact of accounting rules

Disadvantages: Requires surplus assumption (at inception and over time), not clear whether more risk requires more surplus, higher IRR or maybe both

7. PV of Income to PV of Equity Ratio — Set ratio of the PV of income to the PV of average equity balances equal to target return and solves for U.

$$\text{Target Return} = \frac{\text{Present Value of Income Stream}}{\text{Present Value of Avg Equity Balance}}$$
$$r = \frac{PV(INC)}{PV(EQB}$$

The calculation of the income stream, INC, is identical to what we used in the IRR method. But equity calculations reflect *average equity balances*:

$$EQB_i = \frac{EQ_{i-1} + EQ_i}{2}$$

Also note that the discounting of the numerator is done to the *end* of the year and there is an adjustment to the calculation of the PVE in the case of quarterly figures.

Advantages: Not distorted by growth, comparable to GAAP ROE

Disadvantages: Requires selection of discount rates, target ratio and required equity capital in each period.

8. PV Cash Flow Return — Set the PV of equity flows equal to (or less than) the present value of the after-tax cash flows from underwriting and investment of the surplus.

$$PV(\Delta \cdot \text{Equity}; \text{Target Return}) = PV(\text{Total Cash Flow}; \text{Investment Yield})$$
$$PV(\Delta \cdot EQ; r) = PV(TCF; i)$$

Actual tax computations should be used but can can use an approximation — apply tax rate to the present value of cash flows and discount the equity flows at an after-tax investment yield.

Advantages: Focuses on PV of underwriting cash flows

Disadvantages: Not easily reconciled to GAAP ROE

9. Risk-Adjusted Discounted Cash Flow Method — Fair premium equals PV of all loss and expense cash flows, including taxes paid on the present value underwriting income and taxes paid on the investment income on surplus. *Risk-adjusted* discount rates are used for the losses (and if possible for other cash flows, otherwise risk-free rates are used for those).

Risk-free rate of return as i_f, risk-adjusted rate of return is $i_r = i_f + \beta[i_m - i_f]$:

$$PV(\text{Premium}; i_f = PV(\text{Loss}; i_r) + PV(\text{Expenses}; i_f) + PV(\text{Taxes}; i_f)$$
$$PV(P; i_f) = PV(L; i_r) + PV(FX; i_f) + PV(FIT; i_f)$$

Or, an alternative formula is:

$$PV(P; i_f) = PV(L; i_r) + PV(FX; i_f) + \frac{tPV(\text{Inv. Income on Surplus})}{(1-t)}$$

This form of the equation is easier to apply sometimes depending on the information provided.

The critical aspect of this method is the risk-adjusted discount rate used to discount the losses. Robbin essentially uses the CAPM, but reflects the beta for the *liability* and not the equity.

Advantages: Solid theoretical foundation, doesn't require target return or allocation of surplus

Disadvantages: Uncertainty estimating liability betas, especially by line of business.

Mango: Property Catastrophe Risk Load

1. Marginal Surplus Method — Denoting the target return on allocated capital as y, the percentile at which the required capital is set as z, S as the standard deviation of the existing portfolio and S' the standard deviation of the new policy, the marginal surplus formula for a new risk being added to an existing portfolio is:

   $$\text{MS Risk Load} = \frac{yz}{1+y}(S' - S)$$

   The quantity in the brackets is the marginal standard deviation of the new risk and the term $yz/(1+y)$ is a multiplier.

2. Marginal Variance — Using Meyers' paper, the risk load for a new risk n being added to an existing portfolio L is some multiple λ multiplied by the marginal variance:

   $$\text{MV Risk Load} = \lambda[\text{Var}(n) + 2\text{Cov}(L,n)]$$

   In this case the parameter λ is not as clearly defined, but Mango uses the MS multiple, $yz/(1+y)$, scaled by the standard deviation of the combined portfolio:

   $$\lambda = \frac{yz/(1+y)}{\text{Standard Deviation of } L+n}$$

3. The MS and MV methods are easy to apply when we are building up the portfolio from scratch. In that case, the first deal's (X) marginal risk is its total risk and when we add the next deal (Y), its marginal risk is the amount by which the total risk increases. The total risk load will be appropriate for the entire portfolio in both cases.

4. The issue with these methods arises when it comes time to renew the first policy, X. Now the marginal risk is measured assuming Y is already in the portfolio and the effect will be that the sum of the marginal risk loads won't match the risk load we need for the whole portfolio. The MS method, when applied to renewal pricing, results in total risk loads that are too low. The MV method results in total risk loads that are too high. Neither method is *renewal additive*.

5. The Shapley value is a solution to this problem. In the case of the MV method, the lack of renewal additivity is due to the fact that *both* risks get the term $2Cov(X,Y)$ in their marginal variance calculation. The Shapley method says to include only the term $Cov(X,Y)$ in the marginal variance calculation:

   $$\begin{aligned}\text{Risk Load } X &= \lambda[\text{Marginal Variance } X] \\ &= \lambda[\text{Var}(X) + \text{Cov}(X,Y)]\end{aligned}$$

   This eliminates the double counting and each risk shares the covariance contribution to total risk equally.

6. Sharing the covariance equally works to solve the problem, but may not be fair. We can instead weight each risk's allocation of the covariance contributions in proportion to their losses, applying something more like $W_X(2)Cov(X,Y)$ in the marginal variance calculation. This weighted covariance term is called *Covariance Share*.

$$\begin{aligned}\text{Risk Load } X &= \lambda[\text{Marginal Variance } X] \\ &= \lambda[\text{Var}(X) + \text{CovShare}(X)]\end{aligned}$$

7. A detail in the calculation of the CovShare term is that in the case of a property catastrophe application with an event loss table output, the weights are calculated for each event separately.

   If p_i is the probability of occurrence for a given event, then the following formulas are used for the mean, variance, covariance and covariance share calculations:

$$\begin{aligned}\text{Mean} &= \sum p(i)X_i \\ \text{Variance} &= \sum p(i)[1-p(i)]X_i^2 \\ \text{Covariance}(X,Y) &= \sum p(i)[1-p(i)]X_iY_i \\ \text{CovarianceShare}(X) &= \sum \frac{X_i}{X_i+Y_i}(2)p(i)[1-p(i)]X_iY_i\end{aligned}$$

Kreps: Investment-Equivalent Reinsurance Pricing

1. Kreps develops an approach to jointly determining the amount of assets needed to support taking on a new policy and the risk load that must be charged so that the return and risk of writing the policy and investing those assets (in one of two different strategies) is the same as if the same assets are just invested directly in a risky investment.

2. Two strategies can be used to ensure that the assets are invested safely:

    a. Swap Risk-Free Investment for Risky Investment — Invest the assets in risk-free investments rather than risky investments.

    b. Purchase Put Options — Invest in a risky asset yielding y with volatility σ_y but also buy a put option on so that the return cannot be lower than the risk free rate.

3. For each strategy, apply *two* constraints that must be satisfied and use the most stringent:

    a. Safety Constraint — The safety constraint simply requires that the funds available to pay claims at the end of the year are at least equal to a specified loss safety level (a loss amount at some high percentile). This can be viewed as a constraint imposed by the policyholder.

    b. Investment Variance Constraint — The variance constraint simply requires that the combined reinsurance and investment strategy be no more volatile than a direct investment in the risky asset.

4. Formulas — Swap Strategy

    Under this strategy, the premium ($P = \mu_L/(1 + r_f) + R$) and assets contributed by shareholders (A) earn the risk-free rate and then after paying losses the net is returned to shareholders. Their IRR is given as:

    $$(1 + \text{IRR})A = (1 + r_f)(P + A) - L$$

    Requiring this to equal the risky asset yield, y:

    $$\begin{aligned}(1 + y)A &= (1 + r_f)\left[\frac{\mu_L}{1 + r_f} + R + A\right] - \mu_L \\ \Rightarrow R &= \frac{y - r_f}{1 + r_f}A\end{aligned}$$

    a. Safety Constraint — This requires that the premium and assets invested grow to at least s and leads to the constraint on A:

    $$A \geq \frac{s - \mu_L}{1 + y}$$

At the equality, this leads to:

$$R = \frac{y - r_f}{1 + r_f} \frac{s - \mu_L}{1 + y}$$

b. Variance Constraint — Requiring the variance of the risky investment to be the same as the variance of writing the policy and taking no investment risk,

$$A\sigma_y \geq \sigma_L \Rightarrow A \geq \frac{\sigma_L}{\sigma_y}$$

At the equality,

$$R = \frac{y - r_f}{1 + r_f} \frac{\sigma_L}{\sigma_y}$$

5. Formulas — Put Option Strategy

Here, the premium plus the assets contributed by shareholders less the cost of the put option on the total funds invested, at a cost of r as a rate, are invested to now earn an expected return of i, which reflects the yield on the risky assets, y, but with the downside risk eliminated.

Using the same criteria that the IRR for the shareholders must equal the yield if they just invested their assets in the risky investment we get the following risk load as a function of A, which depends on the constraints, r the option cost, i the put-protected return, r_f the risk-free rate and μ_L the mean loss.

$$R = A\left[\frac{(1 + r)(1 + y) - (1 + i)}{1 + i}\right] + \mu_L \left[\frac{1 + r}{1 + i} - \frac{1}{1 + r_f}\right]$$

a. Safety Constraint — Here, the funds grow at least by the rate r_f and if they have to exceed the loss safety constraint then we can derive the value for A at the equality:

$$A = \frac{1}{1 + y}\left[\frac{1 + i}{1 + r_f} s - \mu_L\right]$$

Notice that this formula, unlike the similar formula in the swap case, now contains the variable i which reflects the mean return from the put-protected portfolio. This has to be calculated using very messy formulas, which I will not show here.

The resulting risk load is:

$$R = \frac{1}{(1 + r_f)(1 + y)}\left[s\left((1 + y)(1 + r) - (1 + i)\right) - \mu_L(y - r_f)\right]$$

b. Variance Constraint — Calculating the variance constraint in this case is very messy because the variance of our policy and investment strategy have to both be considered and the latter depends on the variance of the put-protected risky investment.

The result is the following mess for A:

$$A = \frac{b + \sqrt{b^2 + ac}}{a}$$

where

$$a = \sigma_y^2(1+i)^2 - \sigma_i^2(1+y)^2$$

$$b = \mu_L(1+y)\sigma_i^2$$

$$c = \mu_L^2\sigma_i^2 + \sigma_L^2(1+i)^2$$

Then the risk load is just:

$$R = A\left[\frac{(1+r)(1+y) - (1+i)}{1+i}\right] + \mu_L\left[\frac{1+r}{1+i} - \frac{1}{1+r_f}\right]$$

6. For high layer policies there will be an implied minimum risk load as a percent of the limit (ROL):

$$\text{Min ROL Under Swap Strategy/Safety Constraint} = \frac{y - r_f}{(1+r_f)(1+y)}$$

$$\text{Min ROL Under Put/Safety Constraint} = \left[\frac{(1+y)(1+r) - (1+i)}{(1+r_f)(1+y)}\right]$$

7. There are complications when the loss is paid at a single point in time other than $t = 1$ and other complications associated with calculating the parameters of the put-protected return distribution. See the notes and reading for details.