

## Past Exam Questions – BKM6

**2003, Q3** (1.5 points); *Video Explanation Available*

The table below summarizes the probability distribution of potential returns for an investment of \$50,000.

Investment	Probability	Expected Return
Equities	<b>0.1</b>	+ \$20,000
	<b>0.7</b>	+ \$10,000
	<b>0.2</b>	- \$10,000
T-bills	<b>1.0</b>	+ \$2,500

Based on the table, calculate the following:

- (0.5 points)  
The expected rate of return of investing the full \$50,000 in equities.
- (0.5 points)  
The measure of risk (as measured by the standard deviation of the rate of return) of investing fully in equities.
- (0.5 points)  
The risk premium, in dollars, of fully investing in equities versus fully investing in T-bills.

**2003, Q4** (1.0 point); *Video Explanation Available*

Answer the questions below based on the following information about a risky portfolio that you manage, and a risk-free asset:

- $E(r_P) = 11\%$
- $\sigma_P = 15\%$
- $r_f = 5\%$

- (0.5 points)  
Client A wants to invest a proportion of her total investment budget in your risky fund to provide an expected rate of return on her overall or complete portfolio equal to 8%.  
What will be the standard deviation of the rate of return on her portfolio?
- (0.5 points)  
Client B wants the highest return possible subject to the constraint that you limit his standard deviation to be no more than 12%.  
Which client is more risk averse? Explain why.

**2004, Q3 (3.0 point)**

*Note: this question has been modified to reflect the current syllabus*

You are given the following information:

- Expected return of the risky asset,  $E(r) = 0.13$
- Variance of the risky asset ( $\sigma^2$ ) = 0.01
- Risk-free rate = 0.06
- Coefficient of risk aversion ( $A$ ) = 10
- Utility function:  $U = E(r) - 0.5A\sigma^2$

a. (1 point)

Calculate the expected return and standard deviation of a portfolio that is invested 40.0% in the risky asset and 60.0% in a risk-free asset.

b. (2 points)

*Equation is not in current syllabus*

**2005, Q4 (1.75 points); Video Explanation Available**

You are given the following information.

Investment	Expected Return	Standard Deviation
1	12%	20%
2	15%	30%
3	20%	40%

Your utility formula is represented by  $U = E(r) - 0.3A\sigma^2$ .

a. (0.5 point)

Briefly explain which of the three investments a risk-neutral investor would select.

b. (0.5 point)

Identify which investment an investor with the utility function shown above and  $A=2$  would select.

c. (0.75 point)

Calculate the certainty equivalent of the investment selected in part b. above.

**2006, Q4 (2 points)**

Consider the following information about a risky portfolio and a risk-free asset:

- The risk premium of the risky portfolio is 15%.
- The reward-to-variability ratio of the risky portfolio is 0.75.
- The expected return on the risk-free asset is 3.0%.

Assume you can invest in some combination of the risky portfolio and the risk-free asset. Determine the equation for the Capital Allocation Line under these assumptions and graph the Capital Allocation Line. Label all items properly.

**2007, Q1** (1.5 points); *Video Explanation Available*

You are given the following information:

- A risky portfolio has an expected return of 16% and a standard deviation of 25%.
- The T-bill rate is 6%.

a. (0.5 point)

Suppose you invest 60% of your funds in the risky portfolio and 40% in a T-bill money market fund. Calculate the expected value and the standard deviation of the rate of return of the portfolio.

b. (1 point)

Determine the equation of the Capital Allocation Line (CAL) of the risky portfolio and graph the CAL. Plot the position of the overall portfolio on the CAL graph. Label all items properly

**2009, Q1** (1.75 points)

Given the following information regarding a risk-free asset and a portfolio of risky assets:

- The risk-free rate is 3%.
- The expected return on the risky portfolio is 11 %.
- The standard deviation of the risky portfolio's return is 25%.
- An investor has utility function  $U = E(r) - 1/3A\sigma^2$  with risk aversion parameter  $A=2$ .

This investor has a \$50,000 budget for investing.

Calculate how much the investor should invest in the portfolio of risky assets in order to maximize the investor's utility.

**2010, Q2** (1.25 points)

- The return of a risk-free asset is 5%.
- An investment company offers a risky asset, with a Sharpe ratio of 0.2.
- An investor wants to hold a portfolio consisting of the risky asset and the risk-free asset

a. (0.5 point)

Calculate the expected return of the portfolio if the investor wants the standard deviation of the portfolio to be 15%.

b. (0.75 point)

Graph the capital allocation line (CAL) associated with this portfolio.

Plot the position of the overall portfolio on the CAL graph.

Clearly label the axes, the CAL, the risk-free asset, and the overall portfolio.

**2010, Q3 (1.75 points)**

Given the following information regarding a risk-free asset and a risky asset:

- The return of the risk-free asset is 5%.
- An investment company offers a risky asset, with an expected return of 12% and a standard deviation of 15%.
- An investor has the utility function  $U = E(r) - 3A\sigma^2$  with risk aversion parameter  $A = 1.5$ .
- The investor wants to create a portfolio using the risk-free asset and the risky asset

Calculate the proportion of the portfolio comprised of the risky asset that maximizes the investor's utility.

**2011, Q1 (2.25 points); Video Explanation Available**

A portfolio is being constructed for an investor using the following assets and assumptions:

	Asset		
	D	E	F
Expected Return	0.12	0.14	0.04
Standard Deviation	0.15	0.10	0.00

- The utility function is  $U = E(r_c) - 0.5A\sigma_c^2$ 
    - o The subscript c refers to the optimal portfolio.
  - The coefficient of risk aversion, A, is 7.
  - The weight given to asset D in the optimal risky portfolio is 0.16.
  - The reward-to-volatility ratio is 1.03,
  - The investor is allowed to borrow at the risk-free rate.
- a. (1.75 points)  
Calculate the share of the optimal complete portfolio invested in the risk-free asset that would maximize the investor's utility.
- b. (0.5 point)  
Describe the result in part a above in terms of the optimal risky portfolio.

**2012, Q1** (3 points); *Video Explanation Available*

Given the following information:

Asset	Expected Return	Standard Deviation
A	12%	> 0%
B	10%	> 0%
C	6%	> 0%
D	3%	= 0%

- The coefficient of risk aversion is 3.
  - The weight given to Asset A in the optimal risky portfolio is 40%.
  - The standard deviation of the optimal risky portfolio is 16%.
  - The slope of the capital allocation line is 0.35.
- a. (2.5 points)  
Construct an optimal investment plan using some of each asset (A, B, C, and D) and justify your proposed plan. Include brief descriptions of the proportion to be invested in each asset and the overall expected risk and return results.
- b. (0.5 point)  
Explain why the proportion invested in Asset D may be the only difference between plans for investors with the same assets.

**2013, Q1** (1.5 points)

Given the following information about investment options:

- $U = E(r_c) - 0.5A\sigma_c^2$
  - The risk-free rate of return is 3%.
  - The risk premium on the risky portfolio is 5%.
  - The reward-to-volatility ratio of the risky portfolio is 0.25.
  - The risk aversion parameter is 2.
- a. (0.75 point)  
Calculate the certainty equivalent rate of the risky portfolio.
- b. (0.5 point)  
Given that 40% of the available assets are invested in the risky portfolio and the rest in the risk-free asset, calculate the expected return and standard deviation of the complete portfolio.
- c. (0.25 point)  
Calculate the percentage of available assets to invest in the risky portfolio to maximize utility.

**2014, Q1 (1.25 points)**

Given the following about a risky portfolio, P:

- Expected return of P is 16%.
- Standard deviation of expected return of P is 24%.
- Borrowing rate is 10%.
- Lending rate is 6%.

Graph the capital allocation line (CAL). Plot the position of P on the CAL graph. Clearly label the axes, the y-intercept of the CAL, and the borrowing and lending ranges.

**2014, Q2 (1.5 points)**

Contrast active and passive investment strategies regarding:

- i. constructing a portfolio.
- ii. implementation and total costs.
- iii. expected return.

**2016, Q1 (1.25 points)**

An investor has a risk aversion factor of  $A = 2$ . The investor has \$100,000 to invest in a combination of an S&P 500 index fund and risk-free assets. Given the following information:

- The investor's utility is given by  $U = E(r_c) - 0.5A\sigma_c^2$ .
  - The investor's risk-free lending rate is 4%.
  - The investor's risk-free borrowing rate is 6%.
  - The S&P 500 index offers an expected return of 11.5% with a standard deviation of 20%.
- a. (0.5 point)  
The investor would like to achieve the greatest possible return, subject to a maximum standard deviation of 25%. Calculate the amount the investor should allocate to risk-free assets.
  - b. (0.75 point)  
Calculate the expected return and standard deviation of the portfolio in part a. above.

## BKM6 Solutions

### 2003, Q3

The table below summarizes the probability distribution of potential returns for an investment of \$50,000.

Investment	Probability	Expected Return
Equities	<b>0.1</b>	+ \$20,000
	<b>0.7</b>	+ \$10,000
	<b>0.2</b>	- \$10,000
T-bills	<b>1.0</b>	+ \$2,500

Based on the table, calculate the following:

a. (0.5 points)

The expected rate of return of investing the full \$50,000 in equities.

Equities

P	Return
0.1	0.4
0.7	0.2
0.2	-0.2

$$\begin{aligned} E(r) &= 0.1 * 0.4 + 0.7 * 0.2 + 0.2 * -0.2 \\ &= 0.14 \end{aligned}$$

b. (0.5 points)

The measure of risk (as measured by the standard deviation of the rate of return) of investing fully in equities.

$$\begin{aligned} SD &= [0.1 * (0.4 - 0.14)^2 + 0.7 * (0.2 - 0.14)^2 + 0.2 * (-0.2 - 0.14)^2]^{0.5} \\ &= 0.18 \end{aligned}$$

c. (0.5 points)

The risk premium, in dollars, of fully investing in equities versus fully investing in T-bills.

*The risk premium would be the difference in investment income between the two options*

$$\begin{aligned} \text{Prem} &= 0.14 * 50,000 - 2,500 \\ &= 4,500 \end{aligned}$$

### 2003, Q4

Answer the questions below based on the following information about a risky portfolio that you manage, and a risk-free asset:

- $E(r_P) = 11\%$
- $\sigma_P = 15\%$
- $r_f = 5\%$

a. (0.5 points)

Client A wants to invest a proportion of her total investment budget in your risky fund to provide an expected rate of return on her overall or complete portfolio equal to 8%.

What will be the standard deviation of the rate of return on her portfolio?

*In order to calculate the SD, we first need to derive the weights that would produce this return.*

$$0.08 = 0.11w + 0.05(1 - w)$$

$$w = 0.5$$

$$\begin{aligned} SD &= 0.5 * 0.15 \\ &= 0.075 \end{aligned}$$

b. (0.5 points)

Client B wants the highest return possible subject to the constraint that you limit his standard deviation to be no more than 12%.

Which client is more risk averse? Explain why.

**A: the SD that he is willing to accept (0.075) is less than what B is willing (0.12)**

### 2004, Q3

You are given the following information:

- Expected return of the risky asset,  $E(r) = 0.13$
- Variance of the risky asset ( $\sigma^2$ ) = 0.01
- Risk-free rate = 0.06
- Coefficient of risk aversion ( $A$ ) = 10
- Utility function:  $U = E(r) - 0.5A\sigma^2$

a. (1 point)

Calculate the expected return and standard deviation of a portfolio that is invested 40.0% in the risky asset and 60.0% in a risk-free asset.

$$\begin{aligned} E(r) &= 0.4 * 0.13 + 0.6 * 0.06 \\ &= 0.088 \end{aligned}$$

$$\begin{aligned} SD &= 0.4 * 0.01^{0.5} \\ &= 0.04 \end{aligned}$$



**2005, Q4**

You are given the following information.

Investment	Expected Return	Standard Deviation
1	12%	20%
2	15%	30%
3	20%	40%

Your utility formula is represented by  $U = E(r) - 0.3A\sigma^2$ .

a. (0.5 point)

Briefly explain which of the three investments a risk-neutral investor would select.

**Since a risk neutral investor does not account for risk in his decisions, he will choose the investment with the highest expected return: 3.**

b. (0.5 point)

Identify which investment an investor with the utility function shown above and  $A=2$  would select.

Investment	Expected Return	Standard Deviation	Utility
1	12%	20%	0.096
2	15%	30%	0.096
3	20%	40%	0.104

**Select 3, as it provides the highest utility.**

c. (0.75 point)

Calculate the certainty equivalent of the investment selected in part b. above.

***Certainty equivalent rate: the rate that a risk free investment would need to offer to provide the same level of utility as the investment being analyzed.***

$$0.104 = \text{c.e.} - 0.3 * 0$$

$$\text{c.e.} = 10.4\%$$

**2006, Q4**

Consider the following information about a risky portfolio and a risk-free asset:

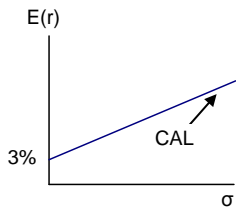
- The risk premium of the risky portfolio is 15%.
- The reward-to-variability ratio of the risky portfolio is 0.75.
- The expected return on the risk-free asset is 3.0%.

Assume you can invest in some combination of the risky portfolio and the risk-free asset. Determine the equation for the Capital Allocation Line under these assumptions and graph the Capital Allocation Line. Label all items properly.

**Y intercept= 0.03**

**Slope = 0.75**

$$E(r_c) = 0.03 + 0.75\sigma_c$$



***You need to include all elements above to get full credit***

**2007, Q1**

You are given the following information:

- A risky portfolio has an expected return of 16% and a standard deviation of 25%.
- The T-bill rate is 6%.

a. (0.5 point)

Suppose you invest 60% of your funds in the risky portfolio and 40% in a T-bill money market fund. Calculate the expected value and the standard deviation of the rate of return of the portfolio.

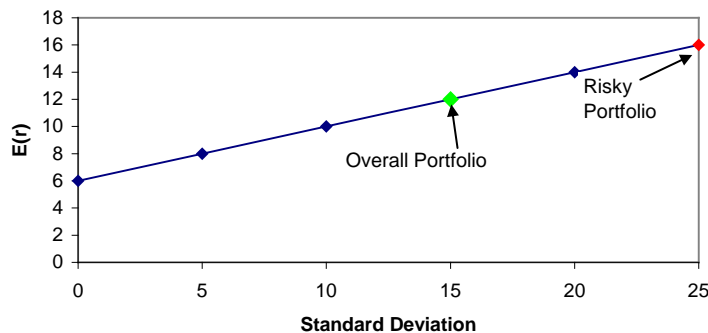
$$\begin{aligned} E(r) &= 0.6 * 0.16 + 0.4 * 0.06 \\ &= 0.12 \end{aligned}$$

$$\begin{aligned} SD &= 0.6 * 0.25 \\ &= 0.15 \end{aligned}$$

b. (1 point)

Determine the equation of the Capital Allocation Line (CAL) of the risky portfolio and graph the CAL. Plot the position of the overall portfolio on the CAL graph. Label all items properly

$$\begin{aligned} E(r) &= 0.06 + (0.12 - 0.06) / 0.15 * \sigma \\ &= 0.06 + 0.4\sigma \end{aligned}$$



### 2009, Q1

Given the following information regarding a risk-free asset and a portfolio of risky assets:

- The risk-free rate is 3%.
- The expected return on the risky portfolio is 11 %.
- The standard deviation of the risky portfolio's return is 25%.
- An investor has utility function  $U = E(r) - 1/3A\sigma^2$  with risk aversion parameter  $A=2$ .

This investor has a \$50,000 budget for investing.

Calculate how much the investor should invest in the portfolio of risky assets in order to maximize the investor's utility.

*First, express the Utility equation in the “standard” format (A should have a coefficient of 0.5)*

$$\begin{aligned}U &= E(r) - 1/3A\sigma^2 \\&= E(r) - 1/2 (2/3 * A)\sigma^2 \\&= E(r) - 1/2 (4/3)\sigma^2\end{aligned}$$

$$\begin{aligned}y^* &= (0.11 - 0.03) / (4/3 * 0.25^2) \\&= 0.96\end{aligned}$$

**Therefore invest \$48K in risky asset, and \$2K in risk free asset**

### 2010, Q2

The return of a risk-free asset is 5%.

- An investment company offers a risky asset, with a Sharpe ratio of 0.2.
- An investor wants to hold a portfolio consisting of the risky asset and the risk-free asset

a. (0.5 point)

Calculate the expected return of the portfolio if the investor wants the standard deviation of the portfolio to be 15%.

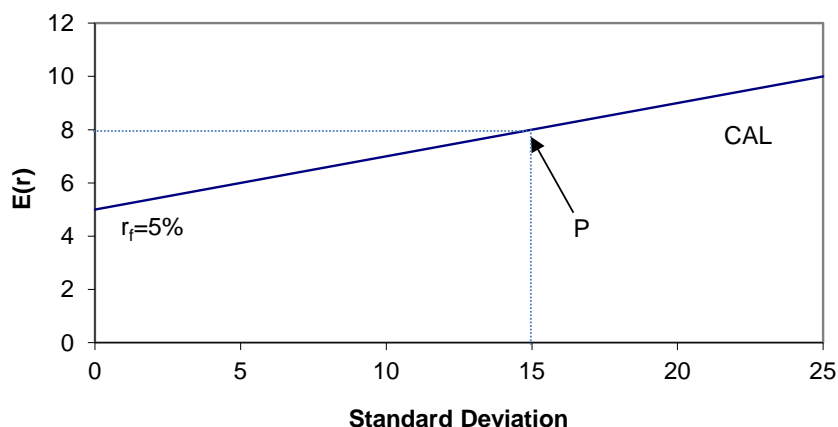
$$\begin{aligned}0.2 &= [E(r_p) - 0.05] / 0.15 \\E(r_p) &= 0.08\end{aligned}$$

b. (0.75 point)

Graph the capital allocation line (CAL) associated with this portfolio.

Plot the position of the overall portfolio on the CAL graph.

Clearly label the axes, the CAL, the risk-free asset, and the overall portfolio.



### 2010, Q3

Given the following information regarding a risk-free asset and a risky asset:

- The return of the risk-free asset is 5%.
- An investment company offers a risky asset, with an expected return of 12% and a standard deviation of 15%.
- An investor has the utility function  $U = E(r) - 3A\sigma^2$  with risk aversion parameter  $A = 1.5$ .
- The investor wants to create a portfolio using the risk-free asset and the risky asset

Calculate the proportion of the portfolio comprised of the risky asset that maximizes the investor's utility.

$$U = E(r) - 3A\sigma^2$$

*We need to restate this equation as it is in a different format to the default Utility equation ( $A\sigma^2$  should have a coefficient of 0.5)*

$$= E(r) - 0.5 (6 * A)\sigma^2$$

$$= E(r) - 0.5 (9)\sigma^2$$

$$y^* = (0.12 - 0.05) / (9 * 0.15^2)$$

$$= 0.346$$

### 2011, Q1

A portfolio is being constructed for an investor using the following assets and assumptions:

	Asset		
	D	E	F
Expected Return	0.12	0.14	0.04
Standard Deviation	0.15	0.10	0.00

- The utility function is  $U = E(r_c) - 0.5A\sigma_c^2$ 
  - o The subscript c refers to the optimal portfolio.
- The coefficient of risk aversion, A, is 7.
- The weight given to asset D in the optimal risky portfolio is 0.16.
- The reward-to-volatility ratio is 1.03,
- The investor is allowed to borrow at the risk-free rate.

a. (1.75 points)

Calculate the share of the optimal complete portfolio invested in the risk-free asset that would maximize the investor's utility.

$$E(r_p) = 0.16 * 0.12 + 0.84 * 0.14 \\ = 0.1368$$

$$1.03 = (0.1368 - 0.04) / \sigma_p \\ \sigma_p = 0.094$$

$$y^* = (0.1368 - 0.04) / (7 * 0.094^2) \\ = 156.5\% \text{ risky portfolio}$$

$$\text{Position in risk-free} = 1 - 1.565 = -56.5\%$$

b. (0.5 point)

Describe the result in part a above in terms of the optimal risky portfolio.

**The optimal complete portfolio requires more invested in the optimal risky portfolio than the investor has, so the investor must borrow funds at the risk-free rate which will then be invested in the optimal risky portfolio.**

## 2012, Q1

Given the following information:

Asset	Expected Return	Standard Deviation
A	12%	> 0%
B	10%	> 0%
C	6%	> 0%
D	3%	= 0%

- The coefficient of risk aversion is 3.
- The weight given to Asset A in the optimal risky portfolio is 40%.
- The standard deviation of the optimal risky portfolio is 16%.
- The slope of the capital allocation line is 0.35.

a. (2.5 points)

Construct an optimal investment plan using some of each asset (A, B, C, and D) and justify your proposed plan. Include brief descriptions of the proportion to be invested in each asset and the overall expected risk and return results.

**Asset D must be the risk free asset, as the  $sd = 0$ .  $\therefore r_f = 3\%$**

**Let the return of the optimal risky portfolio be “x”:**

$$0.35 = (x - 0.03) / 0.16$$

$$x = 0.086$$

**Let the weight of B in the optimal risky portfolio be “w”**

$$0.086 = 0.4 * 0.12 + w * 0.1 + (1 - 0.4 - w) * 0.06$$

$$w = 0.05$$

$$y^* = (0.086 - 0.03) / (3 * 0.16^2) \\ = 0.729$$

$$\text{Risk free \%} = 1 - 0.729 \\ = 0.271$$

**% of portfolio invested in each asset:**

$$A = 0.4 * 0.729$$

$$= 0.292$$

$$B = 0.05 * 0.729$$

$$= 0.036$$

$$C = 0.55 * 0.729$$

$$= 0.401$$

$$D = 0.271$$

$$E(r) = 0.729 * 0.086 + 0.271 * 0.03 \\ = 0.07083$$

$$\sigma = 0.729 * 0.16 \\ = 0.1166$$

b. (0.5 point)

Explain why the proportion invested in Asset D may be the only difference between plans for investors with the same assets.

**Portfolio selection can be separated into two broad categories (“separation property”):**

**1. Selection of the optimal risky portfolio. This is the same for all investors**

**2. Allocation of funds between the optimal risky portfolio & the risk free asset. This will depend on the investor’s risk preferences**

**Asset D is basically the risk free asset & therefore the proportion invested in it will vary based on risk preferences. The proportions in the other assets (forming the optimal risky portfolio) will be the same for all investors**

**2013, Q1 (1.5 points)**

Given the following information about investment options:

- $U = E(r_c) - 0.5A\sigma_c^2$
- The risk-free rate of return is 3%.
- The risk premium on the risky portfolio is 5%.
- The reward-to-volatility ratio of the risky portfolio is 0.25.
- The risk aversion parameter is 2.

a. (0.75 point)

Calculate the certainty equivalent rate of the risky portfolio.

$$\begin{aligned} E(r_p) &= 0.05 + 0.03 \\ &= 0.08 \end{aligned}$$

$$\begin{aligned} 0.25 &= 0.05 / \sigma_p \\ \sigma_p &= 0.2 \end{aligned}$$

$$\begin{aligned} C.E.R &= 0.08 - 0.5 * 2 * 0.2^2 \\ &= 0.04 \end{aligned}$$

b. (0.5 point)

Given that 40% of the available assets are invested in the risky portfolio and the rest in the risk-free asset, calculate the expected return and standard deviation of the complete portfolio.

$$\begin{aligned} E(r_c) &= 0.4 * 0.08 + 0.6 * 0.03 \\ &= 0.05 \end{aligned}$$

$$\begin{aligned} \sigma_c &= 0.4 * 0.2 \\ &= 0.08 \end{aligned}$$

c. (0.25 point)

Calculate the percentage of available assets to invest in the risky portfolio to maximize utility.

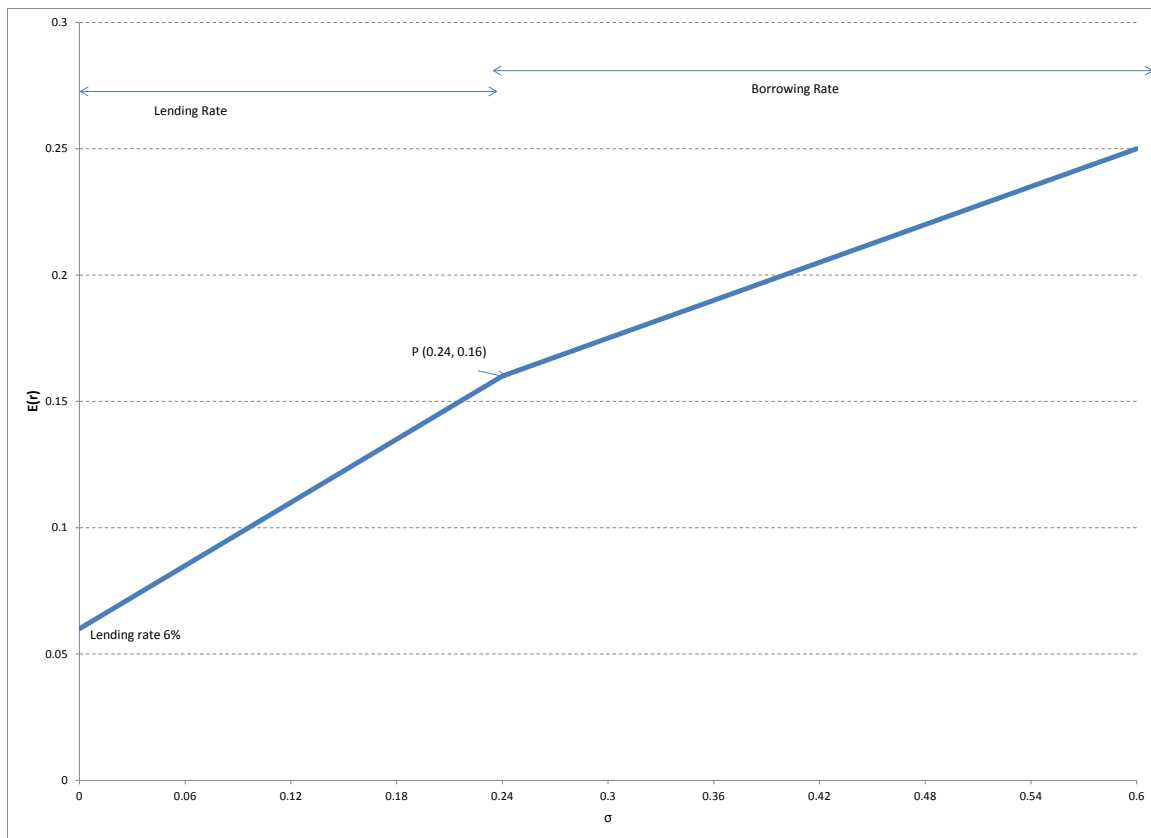
$$\begin{aligned} y^* &= 0.05 / (2 * 0.2^2) \\ &= 0.625 \end{aligned}$$

**2014, Q1 (1.25 points)**

Given the following about a risky portfolio, P:

- Expected return of P is 16%.
- Standard deviation of expected return of P is 24%.
- Borrowing rate is 10%.
- Lending rate is 6%.

Graph the capital allocation line (CAL). Plot the position of P on the CAL graph. Clearly label the axes, the y-intercept of the CAL, and the borrowing and lending ranges.



*See Model Solutions for alternatives*

*Notes from Examiners Report: several candidates spent too much time calculating extra information that was not necessary (eg slopes), and writing explanations*

**2014, Q2 (1.5 points)**

Contrast active and passive investment strategies regarding:

- constructing a portfolio.

*Sample Solutions for both Active and Passive:*



**Active:**

- Active management seeks to discover stocks with positive alphas, so will put more weight on stocks which analysts think are underpriced.
- Active strategies involve hand picking individual investments. The investor will need invest resources to construct that portfolio, or hire someone.

**Passive:**

- Seeks to replicate a well diversified portfolio such as the S&P 500. No effort is made to select stocks; b/c it assumes that it will cost too much for the expected benefits.
- Passive takes prices as given, does not seek to actively analyze securities. Instead focus on well diversified portfolio, often replicating a market index.

ii. implementation and total costs.

**Active:**

- An active portfolio involves lots of on-going trading which incur more fees than passive.
- Active strategy is much more expensive. It requires trading often. So transaction costs are higher. Besides, effort is needed to find the mispriced stocks. This adds additional costs.
- The active strategy will have significantly higher costs.
  - must spend significant time and resources to identify mispriced securities
  - will need to continually analyze portfolio in order to make adjustments if security returns change over time

**Passive:**

- A passive portfolio might incur some cost to set up initially, but then there is minimal additional cost.
- Passive strategy is much cheaper. It doesn't need to trade often so transaction costs are low. Besides, no research is needed

*Note from Examiners Report: In order to receive full credit candidates had to mention transaction costs are higher for active because it involves actively buying and selling securities, while the passive has lower transaction costs because it requires buying and holding.*

*Only partial credit was awarded were if the candidate did not elaborate well enough and we would have to make assumptions, or if only definitions were given without contrasting*

iii. expected return.

**Passive simply earns the market index return, while active strategies will earn an additional profit if they accurately identify non-zero alpha stocks -> but this will be offset by transaction costs and other investment expenses. If markets are indeed efficient (debatable), active strategies will not outperform passive strategies.**

*Note from Examiners Report: In order to receive full credit, the candidate needed to acknowledge that the active strategy does not necessarily outperform the passive strategy because the high transaction costs associated with the active will lower its return.*

**2016, Q1 (1.25 points)**

An investor has a risk aversion factor of  $A = 2$ . The investor has \$100,000 to invest in a combination of an S&P 500 index fund and risk-free assets. Given the following information:

- The investor's utility is given by  $U = E(r_c) - 0.5A\sigma_c^2$ .
- The investor's risk-free lending rate is 4%.
- The investor's risk-free borrowing rate is 6%.
- The S&P 500 index offers an expected return of 11.5% with a standard deviation of 20%.

a. (0.5 point)

The investor would like to achieve the greatest possible return, subject to a maximum standard deviation of 25%. Calculate the amount the investor should allocate to risk-free assets.

**Since the expected return of the S&P 500 index exceeds the risk-free borrowing rate, the investor should allocate as much funds as possible to achieve the greatest expected return.**

$$\begin{aligned} 0.25 &= y * 0.2 \\ y &= 1.25 \end{aligned}$$

$$(1 - 1.25) * 100K = -\$25K$$

**The investor should borrow \$25K at the risk-free rate**

b. (0.75 point)

Calculate the expected return and standard deviation of the portfolio in part a. above.

$$\begin{aligned} E(r) &= 1.25 * 0.115 - 0.25 * 0.06 \\ &= 0.1288 \end{aligned}$$

$$\begin{aligned} \sigma &= 1.25 * 0.2 \\ &= 0.25 \end{aligned}$$

## **Past Exam Questions – BKM7**

**2000, Q8** (2 points); *Video Explanation Available*

Company XYZ's portfolio consists of assets A and B. These assets have the following characteristics:

- Expected return on asset A = 14.5%.
- Expected return on asset B = 12.5%.
- The standard deviation of the return on asset A = 16.5%.
- The standard deviation of the return on asset B = 14.0%.
- Short sales are not allowed.
- No riskless borrowing or lending is allowed.
- XYZ is fully invested in assets A and B.

Calculate the expected portfolio return at the minimum variance level for each of the following correlation coefficients.

a. (0.5 points)  
 $\rho_{AB} = 1.00$

b. (0.5 points)  
 $\rho_{AB} = 0.25$

**2001, Q10** (2 points)

a. (1 point)

Draw a graph that shows the relationship between the capital market line and the efficient frontier for risky assets. Define all items in the graph.

b. (1 point)

State the formula for the capital market line and define all terms.

**2002, Q7** (1.5 points)

Assume that you are determining how to allocate your company's investments between two investment portfolios, aggressive (Portfolio A) and conservative (Portfolio C).

- Portfolio A has a higher expected rate of return and a higher volatility.
- Portfolio C has a lower expected rate of return and a lower volatility.

a. (1 point)

Graph the risk-return relationship for all possible allocations between Portfolio A and Portfolio C, if short sales are not allowed and  $\rho = -1$ . Label both axes and points A and C on the graph.

b. (0.5 point)

What is the standard deviation of the minimum variance portfolio when  $\rho = +1$ ?

**2002, Q13** (1 point)

In *Investments* by Bodie, Kane, and Marcus, the authors discuss the fallacy of time diversification.

Describe how the two main return measures identified by the authors are affected by extending the investment horizon.

**2003, Q5** (4 points); *Video Explanation Available*

The universe of available securities includes two risky stock funds, X and Y, and T-Bills. The expected returns and standard deviations for the universe are as follows:

Asset	E(r)	SD
X	10%	20%
Y	20%	40%
T-Bills	5%	0%

The correlation coefficient between funds X and Y is -0.3.

a. (2.5 points)

Calculate the expected return and standard deviation for the optimal risky portfolio, P.

b. (0.50 points)

Find the slope of the capital allocation line, CAL, supported by T-Bills and optimal portfolio P, from above.

c. (1 point)

If an investor has a coefficient of risk aversion, A, equal to 4, what proportion will he or she invest in fund X, fund Y, and in T-Bills?

**2003, Q7** (1.5 points); *Video Explanation Available*

Identify which one of the following portfolios cannot lie on the efficient frontier and demonstrate why.

Portfolio	Expected Return %	Standard Deviation %
W	15	36
X	12	15
Y	5	7
Z	9	21

**2004, Q4** (3.5 points)

a. (1 point)

Explain the significance of the Separation Property in determining optimal complete portfolios for two clients with different degrees of risk aversion.

b. (1.5 points)

Assuming an investor cannot borrow, graph, in expected return-standard deviation space, the relationship between:

- Capital Allocation Line
- Efficient Frontier of Risky Assets
- Indifference Curve

Be sure to label the axes, and each line/curve.

c. (1 point)

Briefly describe the importance of the following points from the graph in part b. above:

1. The point where the Efficient Frontier and Capital Allocation Line meet
2. The point where the Indifference Curve and Capital Allocation Line meet

**2005, Q5** (3 points)

You are given the following information about two risky assets and a risk-free asset.

Asset	E(r)	SD
A	15%	20%
B	20%	15%
Risk-free	5%	0%

Short sales are not allowed.

a. (2 points)

Calculate the standard deviation of the optimal risky portfolio if the correlation coefficient is 0.1.

b. (0.5 point)

What is the standard deviation of the optimal risky portfolio if the correlation coefficient is -1?

c. (0.5 point)

What is the standard deviation of the optimal risky portfolio if the correlation coefficient is 1?

**2005, Q6 (1 point)**

You are given the following information about stocks and gold.

Asset	E(r)	SD
Stocks	15%	25%
Gold	10%	30%

Demonstrate graphically why an investor would choose to hold gold, in light of the apparent inferiority of gold with respect to expected return and volatility.

Be sure to completely and clearly label all information on the graph.

**2006, Q5 (1.5 points)**

You are considering three portfolios in which to allocate your investment:

- A risk-free asset that yields 4.0%.
- A stock fund with an expected return of 22.0% and a standard deviation of 30.0%.
- A bond fund with an expected return of 8.0% and a standard deviation of 10.0%.

The correlation between the returns of this stock fund and this bond fund is 0.10.

a. (1 point)

Calculate the percentages of the optimal portfolio that would be allocated to the stock fund and to the bond fund.

b. (0.5 point)

Given the answer from part a. above, calculate the expected return of the optimal portfolio.

**2007, Q2 (2.5 points); Video Explanation Available**

Available securities include two risky stock funds, X and Y, and T-bills. Data for the securities follows:

Asset	E(r)	SD
Fund X	8%	22%
Fund Y	27%	70%
T-bills	6%	0%

The correlation coefficient between funds X and Y is -0.15.

a. (1.5 points)

Determine the expected return and the standard deviation of the optimal risky portfolio.

b. (1 point)

"A" is the index of an investor's risk aversion. Calculate how much an investor with a risk aversion index of 4 will invest in each of funds X and Y and in T-bills.

**2007, Q3 (1 point)**

Some financial advisors assert that time diversification reduces risk.

Explain why you either agree or disagree.

**2008, Q1 (3 points)**

A fund manager recently attended a presentation on a new weather catastrophe bond offering and is considering adding the catastrophe bond to his current optimal risky portfolio. Assume the following:

Asset	E(r)	SD
Current optimal risky portfolio	8%	12%
New catastrophe bond	95%	300%
Treasury bills (T-bills)	5%	0%

The fund manager has estimated the correlation coefficient between his current optimal risky portfolio and the new catastrophe bond to be 0.1.

## a. (1.75 points)

Determine the expected return and the standard deviation of the new optimal risky portfolio if the new catastrophe bond is added.

## b. (0.5 point)

Assume the utility function  $U = E(r) - 0.5A\sigma^2$  (note: I modified this from 0.005 to 0.5 to match the text) and risk aversion parameter  $A = 3$ . Calculate the proportion of the new optimal complete portfolio that should be invested in T-bills.

## c. (0.75 point)

Assume that the fund manager invested \$100,000 in the new optimal risky portfolio and T-bills in the proportions determined in part b. above at the beginning of the year. One year later, the investments have grown to \$78,000 in the risky portfolio fund and \$37,800 in T-bills. The investor's risk aversion and the expected returns and standard deviation of the new optimal risky portfolio and T-bills remain unchanged.

The investor wants to rebalance the portfolio by either buying or selling T-bills.

Determine the amount of T-bills required to rebalance the portfolio and state whether the investor must buy or sell these.

**2009, Q2 (2.25 points)**

Given the following information:

	Expected Return	Standard Deviation
T-Bills	4%	0%
Bond Portfolio	8%	15%
Stock Portfolio	20%	40%

The correlation coefficient of the stock and bond portfolios is 10%.

- a. (1.25 points)  
Calculate the weight to invest in each asset to construct the optimal risky portfolio.
- b. (1 point)  
Calculate the Sharpe Ratio for the optimal risky portfolio.

**2009, Q3 (1.75 points); *Video Explanation Available***

Given the following information about the universe of available risky securities:

- Stocks are identically distributed.
  - The expected return for each stock is 10%.
  - The standard deviation of the return for each stock is 50%.
  - The correlation coefficient between any pair of stocks is 0.4.
- a. (0.5 point)  
Calculate the variance of an equally weighted risk/ portfolio of 20 stocks.
  - b. (0.5 point)  
Calculate both the firm-specific risk and systematic risk in this portfolio.
  - c. (0.75 point)  
Calculate the number of stocks necessary for the portfolio's standard deviation to be less than 32%.

**2013, Q2 (1 point)**

Contrast the topics of risk pooling and risk sharing, including their application to insurance companies.



**2014, Q5** (2.75 point); *Video Explanation Available*

Given the following information for constructing an investment portfolio:

Asset	Expected Return	Standard Deviation
A	8%	20%
B	10%	25%

- The risk-free rate is 2%.
- The correlation coefficient between assets A and B is -0.4.
- The investor has a risk aversion coefficient of 5.

Calculate the expected rate of return and standard deviation of the optimal complete portfolio.

**2015, Q1** (2.25 point)

Given the information below describing a three-security financial market, one rational investor's complete portfolio and investors' utility:

Security	Expected Return	Standard Deviation	Portfolio Allocation
X	4%	0%	43%
Y	10%	20%	30%
Z	15%	30%	27%

- Investors assign utility to their portfolios by the formula:  $U = E(r) - 1/2 A\sigma^2$
- (0.5 point)  
Describe the interpretation of the risk aversion index when  $A < 0$ .
  - (0.75 point)  
Calculate the correlation coefficient of the returns of securities Y and Z.
  - (1 point)  
Calculate the value of A for the investor.

**2016, Q3** (4.5 points);

The table below describes the assets available to the investor. All assets are uncorrelated.

Asset	Expected Return	Standard Deviation
A	10%	25%
B	10%	25%
C	10%	25%
D	2%	0%

The table below describes two different portfolio investment options available to the investor:

Asset	Portfolio 1 Weights	Portfolio 2 Weights
A	30%	30%
B	0%	30%
C	0%	30%
D	70%	10%

- a. (1 point)  
Define risk pooling and risk sharing. Provide an example of how each is achieved in the insurance industry.
- b. (1.5 points)  
Calculate each portfolio's expected return, variance and Sharpe ratio.
- c. (0.5 point)  
An investor currently holds Portfolio 1 and is considering moving all investments from Portfolio 1 into Portfolio 2. Explain how this will affect the investor's overall risk.
- d. (1.5 points)  
Construct a portfolio that would give the investor the same expected return as Portfolio 1 and the same Sharpe ratio as Portfolio 2. Briefly explain how this will affect the investor's overall risk.

## BKM7 Solutions

### 2000, Q8

Company XYZ's portfolio consists of assets A and B. These assets have the following characteristics:

- Expected return on asset A = 14.5%.
- Expected return on asset B = 12.5%.
- The standard deviation of the return on asset A = 16.5%.
- The standard deviation of the return on asset B = 14.0%.
- Short sales are not allowed.
- No riskless borrowing or lending is allowed.
- XYZ is fully invested in assets A and B.

Calculate the expected portfolio return at the minimum variance level for each of the following correlation coefficients.

a. (0.5 points)

$$\rho_{AB} = 1.00$$

**Since the assets are perfectly correlated, the total amount should be invested in the asset with lower risk. Therefore, 100% invested in Asset B, and Portfolio Return = 12.5%.**

*TIA forum post(s) discussing this question that may be helpful:*

> <http://www.theinfiniteactuary.com/mb/viewtopic.php?f=2651&t=19090>

b. (0.5 points)

$$\rho_{AB} = 0.25$$

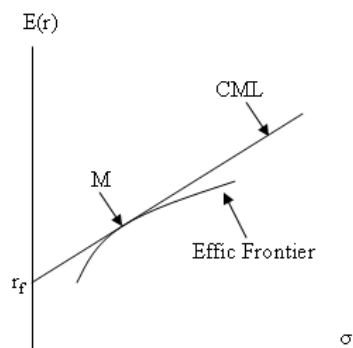
$$w_{\text{MIN}}(D) = \frac{\sigma_E^2 - \text{Cov}(r_D, r_E)}{\sigma_E^2 + \sigma_D^2 - 2\text{Cov}(r_D, r_E)}$$
$$X_A = (0.14^2 - 0.165 * 0.14 * 0.25) / (0.165^2 + 0.14^2 - 2 * 0.165 * 0.14 * 0.25)$$
$$= 0.39$$

$$\text{Return} = 0.39 * 0.145 + 0.61 * 0.125$$
$$= 0.1328$$

### 2001, Q10

a. (1 point)

Draw a graph that shows the relationship between the capital market line and the efficient frontier for risky assets. Define all items in the graph.



b. (1 point)

State the formula for the capital market line and define all terms.

$$E(r) = r_f + [(r_M - r_f)/\sigma_M] * \sigma_E$$

*The above is derived by referring to the graph above*

**Where:**

**$E(r)$  = expected return on the efficient portfolio**

**$r_f$  = risk free rate**

**$r_M$  = expected return on market**

**$\sigma_e$  = SD of portfolio return**

**$\sigma_M$  = SD of market return**

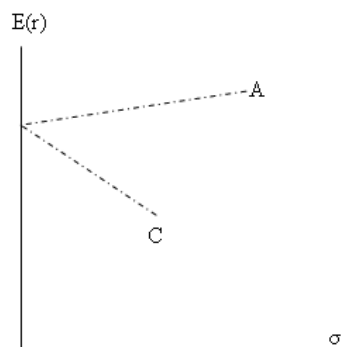
### 2002, Q7

Assume that you are determining how to allocate your company's investments between two investment portfolios, aggressive (Portfolio A) and conservative (Portfolio C).

- Portfolio A has a higher expected rate of return and a higher volatility.
- Portfolio C has a lower expected rate of return and a lower volatility.

a. (1 point)

Graph the risk-return relationship for all possible allocations between Portfolio A and Portfolio C, if short sales are not allowed and  $p = -1$ . Label both axes and points A and C on the graph.



b. (0.5 point)

What is the standard deviation of the minimum variance portfolio when  $p = +1$ ?

**There is no benefit to diversification, so invest all in portfolio C. Thus the standard deviation is the standard deviation of portfolio C.**

**2002, Q13** (1 point)

In Investments by Bodie, Kane, and Marcus, the authors discuss the fallacy of time diversification.

Describe how the two main return measures identified by the authors are affected by extending the investment horizon.

**The volatility of the average annual return decreases with time. This can be easily recognized since the annual return of a one year investment is very volatile but the average annual return of a multi-year investment is less volatile.**

**The volatility of the dollar return increases with the time. A wider range of possible dollar returns become possible as horizon increases**

**2003, Q5**

The universe of available securities includes two risky stock funds, X and Y, and T-Bills. The expected returns and standard deviations for the universe are as follows:

Asset	E(r)	SD
X	10%	20%
Y	20%	40%
T-Bills	5%	0%

The correlation coefficient between funds X and Y is -0.3.

a. (2.5 points)

Calculate the expected return and standard deviation for the optimal risky portfolio, P.

$$w_D = \frac{E(R_D)\sigma_E^2 - E(R_E)\text{Cov}(r_D, r_E)}{E(R_D)\sigma_E^2 + E(R_E)\sigma_D^2 - [E(R_D) + E(R_E)]\text{Cov}(r_D, r_E)}$$

$$\begin{aligned} W_X &= [(0.1-0.05)*0.4^2 - (0.2-0.05)*0.2*0.4*-0.3] / \\ &\quad [(0.1-0.05)*0.4^2 + (0.2-0.05)*0.2^2 - (0.1-0.05+0.2-0.05)*0.2*0.4*-0.3] \\ &= 0.617 \end{aligned}$$

$$W_Y = 0.383$$

$$\begin{aligned} E(r) &= 0.617 * 0.1 + 0.383 * 0.2 \\ &= 0.1383 \end{aligned}$$

$$\begin{aligned} SD &= [0.617^2 * 0.2^2 + 0.383^2 * 0.4^2 + 2 * 0.617 * 0.383 * 0.2 * 0.4 * -0.3]^{0.5} \\ &= 0.1654 \end{aligned}$$

b. (0.50 points)

Find the slope of the capital allocation line, CAL, supported by T-Bills and optimal portfolio P, from above.

$$\begin{aligned}\text{Slope} &= (0.1383 - 0.05) / 0.1654 \\ &= 0.5339\end{aligned}$$

c. (1 point)

If an investor has a coefficient of risk aversion, A, equal to 4, what proportion will he or she invest in fund X, fund Y, and in T-Bills?

$$\begin{aligned}y^* &= (0.1383 - 0.05) / (4 * 0.1654^2) \\ &= 0.8069\end{aligned}$$

$$\begin{aligned}X \% &= 0.8069 * 0.617 \\ &= 0.4979\end{aligned}$$

$$\begin{aligned}Y \% &= 0.8069 * 0.383 \\ &= 0.3090\end{aligned}$$

$$\begin{aligned}\text{Risk free \%} &= (1 - 0.8069) \\ &= 0.1931\end{aligned}$$

### 2003, Q7

Identify which one of the following portfolios cannot lie on the efficient frontier and demonstrate why.

Portfolio	Expected Return %	Standard Deviation %
W	15	36
X	12	15
Y	5	7
Z	9	21

**Z can not lie on the frontier, as it is inferior to X:**

- Z has a lower return than X (9% vs 12%) AND
- Z has a higher risk than X (21% vs 15%)

### 2004, Q4

a. (1 point)

Explain the significance of the Separation Property in determining optimal complete portfolios for two clients with different degrees of risk aversion.

**According to the separation principle, there are 2 steps in portfolio selection:**

**-Selection of the optimal risky portfolio: this will be exactly the same for each investor**

**-Allocation between risk free vs risky assets: this will differ for each investor, depending on their unique risk aversion**

b. (1.5 points)

Assuming an investor cannot borrow, graph, in expected return-standard deviation space, the relationship between:

- Capital Allocation Line
- Efficient Frontier of Risky Assets
- Indifference Curve

Be sure to label the axes, and each line/curve.

*See exhibit from CAS solutions. Note that the CAL shouldn't extend past P, since the investor can't borrow at the risk free*

*(<http://www.casact.org/admissions/studytools/exam8/04-8.pdf>)*

c. (1 point)

Briefly describe the importance of the following points from the graph in part b. above:

1. The point where the Efficient Frontier and Capital Allocation Line meet
2. The point where the Indifference Curve and Capital Allocation Line meet

**1. this is the optimal risky portfolio**

**2. this indicates the proportions of the risky and risk free assets in the complete portfolio**

### 2005, Q5

You are given the following information about two risky assets and a risk-free asset.

Asset	E(r)	SD
A	15%	20%
B	20%	15%
Risk-free	5%	0%

Short sales are not allowed.

a. (2 points)

Calculate the standard deviation of the optimal risky portfolio if the correlation coefficient is 0.1.

$$\begin{aligned} W_a &= [(0.15-0.05)*0.15^2 - (0.2-0.05)*0.15*0.2*0.1] / \\ &\quad [(0.15-0.05)*0.15^2 + (0.2-0.05)*0.2^2 - (0.15-0.05+0.2- \\ &\quad 0.05)*0.15*0.2*0.1] \\ &= 0.24 \end{aligned}$$

$$W_b = 0.76$$

$$\begin{aligned} SD &= [0.24^2 * 0.2^2 + 0.76^2 * 0.15^2 + 2 * 0.24 * 0.76 * 0.2 * 0.15 * 0.1]^{0.5} \\ &= 0.128 \end{aligned}$$

b. (0.5 point)

What is the standard deviation of the optimal risky portfolio if the correlation coefficient is -1?

**Since the correlation equals -1, it is possible to create a portfolio where the SD equals 0. Therefore the optimal portfolio SD = 0.**

*TIA forum post(s) discussing this question that may be helpful:*

> <http://www.theinfiniteactuary.com/mb/viewtopic.php?f=2651&t=4188>

c. (0.5 point)

What is the standard deviation of the optimal risky portfolio if the correlation coefficient is 1?

**Since the assets are perfectly correlated, there is no diversification benefit. Therefore invest in dominant asset, b, which has a higher return and lower risk. SD = 15%**

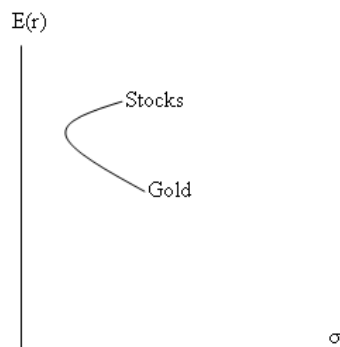
### 2005, Q6

You are given the following information about stocks and gold.

Asset	E(r)	SD
Stocks	15%	25%
Gold	10%	30%

Demonstrate graphically why an investor would choose to hold gold, in light of the apparent inferiority of gold with respect to expected return and volatility.

Be sure to completely and clearly label all information on the graph.



**Even though gold seems to be inferior to stocks, it may provide the investor with diversification benefits if it is not perfectly correlated with stocks: as can be seen above, in this case, it will be possible to construct a portfolio which has a lower level of risk than either stocks or gold alone.**



**2006, Q5**

You are considering three portfolios in which to allocate your investment:

- A risk-free asset that yields 4.0%.
- A stock fund with an expected return of 22.0% and a standard deviation of 30.0%.
- A bond fund with an expected return of 8.0% and a standard deviation of 10.0%.

The correlation between the returns of this stock fund and this bond fund is 0.10.

a. (1 point)

Calculate the percentages of the optimal portfolio that would be allocated to the stock fund and to the bond fund.

*Assume that the question was referring to the optimal RISKY portfolio, as there is no information given about the risk aversion*

$$W_s = \frac{[(0.22-0.04)*0.1^2 - (0.08-0.04)*0.1*0.3*0.1]}{[(0.22-0.04)*0.1^2 + (0.08-0.04)*0.3^2 - (0.22-0.04+0.08-0.04)*0.1*0.3*0.1]}$$

$$= 0.3544$$

$$W_b = 0.6456$$

b. (0.5 point)

Given the answer from part a. above, calculate the expected return of the optimal portfolio.

$$E(r) = 0.3544 * 0.22 + 0.6456 * 0.08$$

$$= 0.1296$$

**2007, Q2**

Available securities include two risky stock funds, X and Y, and T-bills. Data for the securities follows:

Asset	E(r)	SD
Fund X	8%	22%
Fund Y	27%	70%
T-bills	6%	0%

The correlation coefficient between funds X and Y is -0.15.

a. (1.5 points)

Determine the expected return and the standard deviation of the optimal risky portfolio.

$$W_x = \frac{[(0.08-0.06)*0.7^2 - (0.27-0.06)*0.22*0.7*-0.15]}{[(0.08-0.06)*0.7^2 + (0.27-0.06)*0.22^2 - (0.08-0.06+0.27-0.06)*0.22*0.7*-0.15]}$$

$$= 0.58$$

$$W_y = 0.42$$

$$\begin{aligned} E(r) &= 0.58 * 0.08 + 0.42 * 0.27 \\ &= 0.1598 \end{aligned}$$

$$\begin{aligned} SD &= [0.58^2 * 0.22^2 + 0.42^2 * 0.7^2 + 2 * 0.58 * 0.42 * 0.22 * 0.7 * -0.15]^{0.5} \\ &= 0.3024 \end{aligned}$$

b. (1 point)

"A" is the index of an investor's risk aversion. Calculate how much an investor with a risk aversion index of 4 will invest in each of funds X and Y and in T-bills.

$$\begin{aligned} y^* &= (0.1598 - 0.06) / (4 * 0.3024^2) \\ &= 0.27 \text{ in risky portfolio} \end{aligned}$$

$$\begin{aligned} X \% &= 0.27 * 0.58 \\ &= 0.1566 \end{aligned}$$

$$\begin{aligned} Y \% &= 0.27 * 0.42 \\ &= 0.1134 \end{aligned}$$

$$\begin{aligned} T \text{ bills } \% &= 1 - 0.27 \\ &= 0.73 \end{aligned}$$

**2007, Q3** (1 point)

Some financial advisors assert that time diversification reduces risk.

Explain why you either agree or disagree.

**The volatility of the average annual return does decrease with time.**

**However, the volatility of the dollar return increases with the time, which means risk will increase. A wider range of possible dollar returns become possible as horizon increases.**

**I therefore do not agree.**

**2008, Q1**

A fund manager recently attended a presentation on a new weather catastrophe bond offering and is considering adding the catastrophe bond to his current optimal risky portfolio. Assume the following:

Asset	E(r)	SD
Current optimal risky portfolio	8%	12%
New catastrophe bond	95%	300%
Treasury bills (T-bills)	5%	0%

The fund manager has estimated the correlation coefficient between his current optimal risky portfolio and the new catastrophe bond to be 0.1.

a. (1.75 points)

Determine the expected return and the standard deviation of the new optimal risky portfolio if the new catastrophe bond is added.

$$\begin{aligned} W_c &= [(0.95-0.05)*0.12^2 - (0.08-0.05)*0.12*3*0.1] / \\ &\quad [(0.95-0.05)*0.12^2 + (0.08-0.05)*3^2 - (0.95-0.05+0.08- \\ &\quad 0.05)*0.12*3*0.1] \\ &= 0.048 \\ W_c &= 0.952 \end{aligned}$$

$$\begin{aligned} E(r) &= 0.048 * 0.95 + 0.952 * 0.08 \\ &= 0.121 \end{aligned}$$

$$\begin{aligned} SD &= [0.048^2 * 3^2 + 0.952^2 * 0.12^2 + 2 * 0.048 * 0.952 * 0.12 * 2 * 0.1]^{0.5} \\ &= 0.192 \end{aligned}$$

b. (0.5 point)

Assume the utility function  $U = E(r) - 0.5A\sigma^2$  (note: I modified this from 0.005 to 0.5 to match the text) and risk aversion parameter  $A = 3$ . Calculate the proportion of the new optimal complete portfolio that should be invested in T-bills.

$$\begin{aligned} y^* &= (0.121 - 0.05) / (3 * 0.192^2) \\ &= 0.6478 \end{aligned}$$

$$\begin{aligned} \text{T bills} &= 1 - 0.6478 \\ &= 0.3522 \end{aligned}$$

c. (0.75 point)

Assume that the fund manager invested \$100,000 in the new optimal risky portfolio and T-bills in the proportions determined in part b. above at the beginning of the year. One year later, the investments have grown to \$78,000 in the risky portfolio fund and \$37,800 in T-bills. The investor's risk aversion and the expected returns and standard deviation of the new optimal risky portfolio and T-bills remain unchanged.

The investor wants to rebalance the portfolio by either buying or selling T-bills.

Determine the amount of T-bills required to rebalance the portfolio and state whether the investor must buy or sell these.

- **Total portfolio at t=1 is \$115,800 (\$78,000 + \$37,800)**
- **T-bills must be 35.22% of \$115,800 = \$40,785**
- **Need to buy \$2,985 (40,785 - 37,800) of T-bills**

**2009, Q2 (2.25 points)**

Given the following information:

	Expected Return	Standard Deviation
T-Bills	4%	0%
Bond Portfolio	8%	15%
Stock Portfolio	20%	40%

The correlation coefficient of the stock and bond portfolios is 10%.

a. (1.25 points)

Calculate the weight to invest in each asset to construct the optimal risky portfolio.

$$w_S = \frac{[(0.2 - 0.04) * 0.15^2 - (0.08 - 0.04) * 0.1 * 0.15 * 0.4]}{[(0.2 - 0.04) * 0.15^2 + (0.08 - 0.04) * 0.4^2 - (0.08 + 0.2 - 2 * 0.04) * 0.1 * 0.15 * 0.4]} = 0.382$$

$$w_B = 1 - 0.382 = 0.618$$

b. (1 point)

Calculate the Sharpe Ratio for the optimal risky portfolio.

$$E(r_p) = 0.382 * 0.2 + 0.618 * 0.08 = 0.126$$

$$\sigma_p^2 = 0.382^2 * 0.4^2 + 0.618^2 * 0.15^2 + 2 * 0.1 * 0.4 * 0.15 * 0.382 * 0.618 = 0.035$$

$$\sigma_p = 0.187$$

$$\text{Sharpe} = (0.126 - 0.04) / 0.187 = 0.46$$

**2009, Q3**

Given the following information about the universe of available risky securities:

- Stocks are identically distributed.
- The expected return for each stock is 10%.
- The standard deviation of the return for each stock is 50%.
- The correlation coefficient between any pair of stocks is 0.4.

a. (0.5 point)

Calculate the variance of an equally weighted risk/ portfolio of 20 stocks.

$$\sigma_p^2 = (1/n)\bar{\sigma}^2 + [(n-1)/n]Cov$$

$$\begin{aligned}\text{Covar} &= 0.5 * 0.5 * 0.4 \\ &= 0.1 \\ \text{Var} &= (1/20) * 0.5^2 + (19/20) * 0.1 \\ &= 0.1075\end{aligned}$$

b. (0.5 point)

Calculate both the firm-specific risk and systematic risk in this portfolio.

**systematic = 0.1; the variance from the equation above will approach this as the number of stocks approaches infinity**

$$\text{firm specific} = 0.1075 - 0.1 = 0.0075$$

c. (0.75 point)

Calculate the number of stocks necessary for the portfolio's standard deviation to be less than 32%.

$$\begin{aligned}0.32^2 &= (1/n) * 0.5^2 + [(n - 1)/n] * 0.1 \\ n &= 62.5\end{aligned}$$

**63 stocks are required (be sure to round this up to the nearest whole number, as you can not have a fraction of a stock)**

**2013, Q2** (1 point)

Contrast the topics of risk pooling and risk sharing, including their application to insurance companies.

**Risk pooling refers to the practice of adding uncorrelated risks to reduce risk. However, even though the risk per unit reduces and the Sharpe increases, there is also an increase in total risk. Risk pooling is what insurers are in the business of doing**

**Risk sharing refers to selling shares of a risky portfolio (that consists of several risks that are not fully correlated), so that total investment remains constant. Unlike risk pooling, this will actually reduce total risk (while also increasing the Sharpe ratio). Insurance companies can practice this by writing more business, and then ceding business to reinsurers share for example.**

**2014, Q5** (2.75 point)

Given the following information for constructing an investment portfolio:

Asset	Expected Return	Standard Deviation
A	8%	20%
B	10%	25%

- The risk-free rate is 2%.
- The correlation coefficient between assets A and B is -0.4.
- The investor has a risk aversion coefficient of 5.

Calculate the expected rate of return and standard deviation of the optimal complete portfolio.

$$W_A = [(0.08 - 0.02) * 0.25^2 - (0.1 - 0.02) * -0.4 * 0.2 * 0.25] / [(0.08 - 0.02) * 0.25^2 + (0.1 - 0.02) * 0.2^2 - (0.06 + 0.08) * -0.4 * 0.2 * 0.25]$$

$$= 0.5487$$

$$W_B = 0.4513$$

$$E(r_p) = 0.5487 * 0.08 + 0.4513 * 0.1$$

$$= 0.089$$

$$\sigma_p^2 = 0.5487^2 * 0.2^2 + 0.4513^2 * 0.25^2 + 2 * 0.5487 * 0.4513 * -0.4 * 0.2 * 0.25$$

$$= 0.01487$$

$$y^* = (0.089 - 0.02) / (5 * 0.01487)$$

$$= 0.9282$$

$$E(r_c) = 0.9282 * 0.089 + (1 - 0.9282) * 0.02$$

$$= 0.084$$

$$\sigma_c = 0.9282 * 0.01487^{0.5}$$

$$= 0.1132$$

**2015, Q1 (2.25 point)**

Given the information below describing a three-security financial market, one rational investor's complete portfolio and investors' utility:

Security	Expected Return	Standard Deviation	Portfolio Allocation
X	4%	0%	43%
Y	10%	20%	30%
Z	15%	30%	27%

- Investors assign utility to their portfolios by the formula:  $U = E(r) - 1/2 A \sigma^2$

a. (0.5 point)

Describe the interpretation of the risk aversion index when  $A < 0$ .

**A is a measure of investors' risk appetite. When  $A < 0$ , it means this investor is risk seeking. His utility increases by taking on more risk.**

b. (0.75 point)

Calculate the correlation coefficient of the returns of securities Y and Z.

Based on the SDs, Y & Z must be risky assets, whereas X must be risk free

$$\begin{aligned}\text{Weight (risky)} &= 0.3 + 0.27 \\ &= 0.57\end{aligned}$$

$$\begin{aligned}\text{Y's weight in risky} &= 0.3 / 0.57 \\ &= 0.5263\end{aligned}$$

*Plug this into the equation for the optimal risky portfolio:*

$$0.5263 = [0.06 * 0.3^2 - 0.11 * \text{Cov}(Y,Z)] / [0.06 * 0.3^2 + 0.11 * 0.2^2 - (0.11 + 0.06) \text{Cov}(Y,Z)]$$

$$\text{Cov}(Y, Z) = 0.0117$$

$$\begin{aligned}0.0117 &= x * 0.2 * 0.3 \\ x &= 0.195\end{aligned}$$

c. (1 point)

Calculate the value of A for the investor.

$$\begin{aligned}E(r_p) &= 0.526 * 0.1 + 0.474 * 0.15 \\ &= 0.1237\end{aligned}$$

$$\begin{aligned}\sigma_p^2 &= 0.526^2 * 0.2^2 + 0.474^2 * 0.3^2 + 2 * 0.526 * 0.474 * 0.195 * 0.2 * 0.3 \\ &= 0.0371\end{aligned}$$

$$\begin{aligned}0.57 &= (0.1237 - 0.04) / 0.0371A \\ A &= 3.96\end{aligned}$$

**2016, Q3** (4.5 points);

The table below describes the assets available to the investor. All assets are uncorrelated.

Asset	Expected Return	Standard Deviation
A	10%	25%
B	10%	25%
C	10%	25%
D	2%	0%

The table below describes two different portfolio investment options available to the investor:

Asset	Portfolio 1 Weights	Portfolio 2 Weights
A	30%	30%
B	0%	30%
C	0%	30%
D	70%	10%

a. (1 point)

Define risk pooling and risk sharing. Provide an example of how each is achieved in the insurance industry.

**Risk Pooling: increase investment by adding uncorrelated securities to a portfolio**

**Example: selling additional policies to uncorrelated insureds**

**Risk Sharing: keep total investment fixed and allocate amongst additional securities**

**Example (1): selling more insurance policies, but cede some business to reinsurers to keep total net exposure unchanged**

**Example (2): sell more policies, but sell shares of the company to additional investors**

b. (1.5 points)

Calculate each portfolio's expected return, variance and Sharpe ratio.

**Portfolio 1:**

$$\begin{aligned}
 E(\text{Return}) &= 0.3 * 0.1 + 0.7 * 0.02 \\
 &= 0.044 \\
 \text{Variance} &= 0.3^2 * 0.25^2 + 0.7^2 * 0.0^2 \\
 &= 0.005625 \\
 \text{Sharpe} &= (0.044 - 0.02) / (0.005625^{0.5}) \\
 &= 0.32
 \end{aligned}$$

**Portfolio 2:**

$$\begin{aligned}
 E(\text{Return}) &= 0.3 * 0.1 * 3 + 0.1 * 0.02 \\
 &= 0.092 \\
 \text{Variance} &= 0.3^2 * 0.25^2 * 3 + 0.1^2 * 0^2 \\
 &= 0.016875 \\
 \text{Sharpe} &= (0.092 - 0.02) / (0.016875^{0.5}) \\
 &= 0.5543
 \end{aligned}$$

c. (0.5 point)

An investor currently holds Portfolio 1 and is considering moving all investments from Portfolio 1 into Portfolio 2. Explain how this will affect the investor's overall risk.

**This will increase risk (sd) as portfolio 2 has a greater variance than portfolio 1**



d. (1.5 points)

Construct a portfolio that would give the investor the same expected return as Portfolio 1 and the same Sharpe ratio as Portfolio 2. Briefly explain how this will affect the investor's overall risk.

**Invest in a portfolio that consists of Portfolio 2 and the risk free asset (asset D). This would have the same Sharpe ratio as Portfolio 2.**

$$\begin{aligned}0.044 &= w * 0.092 + (1 - w) * 0.02 \\ w &= 0.33\end{aligned}$$

**Therefore invest 33% in Portfolio 2, and 67% in asset D.**

$$\begin{aligned}SD &= 0.33 * 0.016875^{0.5} \\ &= 0.0433\end{aligned}$$

## Past Exam Questions – BKM8

**2003, Q12** (1.5 points); *Video Explanation Available*

Assume that the index model for stocks A and B is estimated with the following results:

- $R_A = 1.0\% + 0.95R_M + e_A$
- $R_B = -2.0\% + 1.2R_M + e_B$
- $\sigma_M = 25\%$
- $\sigma(e_A) = 32\%$
- $\sigma(e_B) = 8\%$

- a. (1 point)  
Find the standard deviation of each stock.
- b. (0.5 points)  
Find the covariance between stocks A and B.

**2004, Q8** (2.25 points); *Video Explanation Available*

You have the following information about two stocks:

Stock	Expected Return	$\beta_i$	Firm-specific SD
A	15.00%	1.2	40.00%
B	10.00%	0.8	30.00%

- The market index has a standard deviation of 25.0%.
  - The risk-free rate is 5.0%.
  - Portfolio P consists of:
    - o 50.0% of Stock A
    - o 30.0% of Stock B
    - o 20.0% of a risk-free security
- a. (0.5 point)  
Calculate the expected return of the portfolio,  $E(r_p)$ .
  - b. (0.5 point)  
Calculate the beta of the portfolio,  $\beta_p$ .
  - c. (0.5 point)  
Calculate the systematic component of the variance of the portfolio.
  - d. (0.5 point)  
Calculate the non-systematic component of the variance of the portfolio.
  - e. (0.25 point)  
Calculate the total variance of the portfolio.

**2013, Q3** (3.25 points); *Video Explanation Available*

Given the following information:

- The expected risk premium of the market index is 8%.
- The standard deviation of the market index is 20%.
- A single-factor model is used to construct an optimal risky portfolio with two components:
  - i. A passive portfolio containing the market index
  - ii. An active portfolio containing some combination of three stocks, with the following estimated parameters:

Stock <sub>i</sub>	$\alpha$	$\sigma^2(e_i)$	$\beta_i$
1	0.06	0.16	0.8
2	0.10	0.20	1.0
3	0.15	0.24	1.2

Calculate the portion of the optimal risky portfolio that is invested in the active portfolio.

**2014, Q4** (1.5 points)

- a. (1 point)  
Discuss two challenges with using a Markowitz model to analyze 200 technology stocks within a broader portfolio.
- b. (0.5 point)  
Briefly discuss how the single-index model does or does not overcome each of the challenges of the Markowitz model identified in part a. above.

**2015, Q3** (1.5 points); *Video Explanation Available*

Given the following information:

Stock	Capitalization (\$)	Alpha	Beta	Expected Return	Standard Deviation
A	2500	1.0%	0.9	7.3%	30%
B	4700	0.0%	0.3	2.1%	10%
C	2800	-0.5%	1.7	11.4%	40%

- An index of stocks A, B and C is used as the market index for a single-index model.
- The standard deviation of the market index portfolio is 20%.

a. (0.25 point)

Calculate the covariance between stock A and the index portfolio.

(Note: this is covered in BKM9)

b. (0.25 point)

Calculate the covariance between stock A and stock B.

c. (0.5 point)

Calculate the systematic and firm-specific components of the variance of stock C.

d. (0.5 point)

Assume 100 stocks are added to this financial market. Briefly describe two disadvantages of using the Markowitz model rather than the single-index model in this case.

**2016, Q4** (1 point);

Given the following information under the single index model:

- Portfolio A's  $\alpha = 2\%$ .
  - Portfolio A's  $\beta = 1.25$ .
- a. (0.5 point)
- Construct a tracking portfolio that matches the systematic component of portfolio A's return.
- b. (0.5 point)
- Demonstrate how to use the tracking portfolio above to earn a positive excess return with no exposure to systematic risk.

## BKM8 Solutions

### 2003, Q12

Assume that the index model for stocks A and B is estimated with the following results:

- $R_A = 1.0\% + 0.95R_M + e_A$
- $R_B = -2.0\% + 1.2R_M + e_B$
- $\sigma_M = 25\%$
- $\sigma(e_A) = 32\%$
- $\sigma(e_B) = 8\%$

a. (1 point)

Find the standard deviation of each stock.

$$\begin{aligned}\sigma_i^2 &= \beta_i^2 * \sigma_M^2 + \sigma(e_i)^2 \\ \sigma_A^2 &= 0.95^2 * 0.25^2 + 0.32^2 \\ \sigma_A &= 0.3985\end{aligned}$$

$$\begin{aligned}\sigma_B^2 &= 1.2^2 * 0.25^2 + 0.08^2 \\ \sigma_B &= 0.3105\end{aligned}$$

b. (0.5 points)

Find the covariance between stocks A and B.

$$\begin{aligned}\text{Cov}[R_A, R_B] &= \beta_A * \beta_B * \sigma_M^2 \\ \text{Cov}[R_A, R_B] &= 0.95 * 1.2 * 0.25^2 \\ &= 0.07125\end{aligned}$$

### 2004, Q8

You have the following information about two stocks:

Stock	Expected Return	$\beta_i$	Firm-specific SD
A	15.00%	1.2	40.00%
B	10.00%	0.8	30.00%

- The market index has a standard deviation of 25.0%.
- The risk-free rate is 5.0%.
- Portfolio P consists of:
  - o 50.0% of Stock A
  - o 30.0% of Stock B
  - o 20.0% of a risk-free security

a. (0.5 point)

Calculate the expected return of the portfolio,  $E(r_p)$ .

$$\begin{aligned}E(r_p) &= 0.5 * 0.15 + 0.3 * 0.10 + 0.2 * 0.05 \\ &= 0.115\end{aligned}$$

b. (0.5 point)

Calculate the beta of the portfolio,  $\beta_p$ .

$$\begin{aligned} B_p &= 0.5 * 1.2 + 0.3 * 0.8 + 0.2 * 0 \\ &= 0.84 \end{aligned}$$

c. (0.5 point)

Calculate the systematic component of the variance of the portfolio.

$$\begin{aligned} \text{Syst.} &= \beta_p^2 \sigma_m^2 \\ &= 0.84^2 * 0.25^2 \\ &= 0.0441 \end{aligned}$$

d. (0.5 point)

Calculate the non-systematic component of the variance of the portfolio.

$$\begin{aligned} \text{Non syst.} &= 0.5^2 * 0.4^2 + 0.3^2 * 0.3^2 + 0.2^2 * 0^2 \\ &= 0.0481 \end{aligned}$$

*TIA forum post(s) discussing this question that may be helpful:*

> <http://www.theinfiniteactuary.com/mb/viewtopic.php?f=2652&t=2933>

e. (0.25 point)

Calculate the total variance of the portfolio

$$\begin{aligned} \text{Total} &= 0.0441 + 0.0481 \\ &= 0.0922 \end{aligned}$$

**2013, Q3** (3.25 points)

Given the following information:

- The expected risk premium of the market index is 8%.
- The standard deviation of the market index is 20%.
- A single-factor model is used to construct an optimal risky portfolio with two components:
  - i. A passive portfolio containing the market index
  - ii. An active portfolio containing some combination of three stocks, with the following estimated parameters:

Stock <sub>i</sub>	$\alpha$	$\sigma^2(e_i)$	$\beta_i$
1	0.06	0.16	0.8
2	0.10	0.20	1.0
3	0.15	0.24	1.2

Calculate the portion of the optimal risky portfolio that is invested in the active portfolio.

**Initial Weights:**

$$w_i^0 = \alpha_i / \sigma^2(e_i)$$

$$\text{Stock}_1 = 0.06 / 0.16 \\ = 0.375$$

$$\text{Stock}_2 = 0.1 / 0.2 \\ = 0.5$$

$$\text{Stock}_3 = 0.15 / 0.24 \\ = 0.625$$

$$\text{Sum} = 0.375 + 0.5 + 0.625 \\ = 1.5$$

**Scaled Weights:**

$$\text{Stock}_1 = 0.375 / 1.5 \\ = 0.25$$

$$\text{Stock}_2 = 0.5 / 1.5 \\ = 0.3333$$

$$\text{Stock}_3 = 0.625 / 1.5 \\ = 0.4167$$

Stock <sub>i</sub>	$\alpha$	$\sigma^2(e_i)$	$\beta_i$
1	0.06	0.16	0.8
2	0.10	0.20	1.0
3	0.15	0.24	1.2
Total	0.1108	0.0739	1.0333

Where  $\alpha_A = \sum w_i \alpha_i$ ,  $\beta_A = \sum w_i \beta_i$ ,  $\sigma^2(e_A) = \sum w_i^2 \sigma^2(e_i)$

$$w_A^0 = \frac{\alpha_A / \sigma^2(e_A)}{E(R_M) / \sigma_M^2}$$

$$w_A^0 = (0.1108 / 0.0739) / (0.08 / 0.2^2) \\ = 0.75$$

$$w_A^* = \frac{w_A^0}{1 + (1 - \beta_A) w_A^0}$$

$$w_A^* = 0.75 / [1 + (1 - 1.0333) * 0.75] \\ = 0.77$$

**2014, Q4 (1.5 points)**

a. (1 point)

Discuss two challenges with using a Markowitz model to analyze 200 technology stocks within a broader portfolio.

**-The number of estimates required to run the Markowitz model on 200 stocks is enormous. The expected return and variance estimates for each stock needs to be estimated, in addition to the covariances between every stock. This is extremely labor intensive.**

**-Having to make so many estimates, increases the chance of producing nonsensical results. In particular, the covariances may not make sense when all the data is rolled up.**

*Note from Examiners Report: one common mistake was not giving information about an issue*

b. (0.5 point)

Briefly discuss how the single-index model does or does not overcome each of the challenges of the Markowitz model identified in part a. above.

**-The number of estimates required in a single index model is significantly fewer: covariances are calculated using variance of the market index and the  $\beta$ 's of each security.**

**-With significantly less estimates, mutually inconsistent results are far less likely.**

**2015, Q3 (1.5 points)**

Given the following information:

Stock	Capitalization (\$)	Alpha	Beta	Expected Return	Standard Deviation
A	2,500	1.0%	0.9	7.3%	30%
B	4,700	0.0%	0.3	2.1%	10%
C	2,800	-0.5%	1.7	11.4%	40%

- An index of stocks A, B and C is used as the market index for a single-index model.
- The standard deviation of the market index portfolio is 20%.



a. (0.25 point)

Calculate the covariance between stock A and the index portfolio.

(Note: this is covered in BKM9)

$$\begin{aligned} 0.9 &= x / 0.2^2 \\ x &= 0.036 \end{aligned}$$

b. (0.25 point)

Calculate the covariance between stock A and stock B.

$$\begin{aligned} \text{Cov}(A,B) &= 0.9 * 0.3 * 0.2^2 \\ &= 0.0108 \end{aligned}$$

c. (0.5 point)

Calculate the systematic and firm-specific components of the variance of stock C.

$$\begin{aligned} \text{Systematic} &= 1.7^2 * 0.2^2 \\ &= 0.1156 \end{aligned}$$

$$\begin{aligned} \text{Total risk} &= 0.4^2 \\ &= 0.16 \end{aligned}$$

$$\begin{aligned} \text{Firm specific} &= 0.16 - 0.1156 \\ &= 0.0444 \end{aligned}$$

d. (0.5 point)

Assume 100 stocks are added to this financial market. Briefly describe two disadvantages of using the Markowitz model rather than the single-index model in this case.

-Due to the large number of parameters, the Markowitz model is subject to estimation errors which can produce nonsensical results. These errors are magnified due to the large number of parameters (as opposed to the small number in index model, this smaller estimation error risk).

-Single factor model uses less parameters; less parameter risk.

-Separating systematic (macroeconomic) and firm-specific risk allows for specialization in securities analysis / decisions.

-Single index model splits out firm specific and systematic risk where Markowitz doesn't. This is important since return is based only on systematic risk and firm specific risk can be diversified away.

**2016, Q4** (1 point);

Given the following information under the single index model:

- Portfolio A's  $\alpha = 2\%$ .
- Portfolio A's  $\beta = 1.25$ .

a. (0.5 point)

Construct a tracking portfolio that matches the systematic component of portfolio A's return.

**Invest 125% of funds in market, and borrow at risk free rate**

*Note from Examiners Report:*

*Candidates who stated that 25% of the portfolio should be borrowed but did not state that this should happen at the risk-free rate were not awarded full credit.*

b. (0.5 point)

Demonstrate how to use the tracking portfolio above to earn a positive excess return with no exposure to systematic risk.

**Short tracking portfolio & use proceeds to invest in A:**

$$\begin{aligned}\alpha &= 1 * 0.02 - (1.25 * 0 - 0.25 * 0) \\ &= 0.02\end{aligned}$$

$$\begin{aligned}\beta &= 1 * 1.25 - (1.25 * 1 - 0.25 * 0) \\ &= 0\end{aligned}$$

*Note from Examiners Report:*

*-This sub-part required candidates to demonstrate (not simply state) that there would be no exposure to systemic risk. They were expected to show how the betas/systematic risk would be offset in the transaction.*

*-Candidates were required to identify that the overall return was the alpha of portfolio A, or 2%.*

## Past Exam Questions – BKM9

**2002, Q11** (1 point)

Argue against the following statement.

"I followed the CAPM theory last year and bought high-Beta stocks. Since then the return on the high-Beta stocks was worse than the average return for low-Beta stocks. The CAPM theory does not work!"

**2003, Q10** (1 point); *Video Explanation Available*

Given the following information, calculate the expected return of portfolio P,  $E(r_P)$ :

- The value of portfolio P is made up of 60% of stock A and 40% of stock B
- Market Price of Risk = Market Risk Premium / Market Variance = 50%
- Expected Market Return,  $E(r_M)$ , is 15%.
- The variance of the market portfolio,  $\sigma_M^2$ , is 20%.
- The covariance of stock A with the market,  $\text{cov}(r_A, r_M)$ , is 25%.
- The covariance of stock B with the market,  $\text{cov}(r_B, r_M)$ , is 10%.

**2003, Q11** (2 points); *Video Explanation Available*

You are given the following information:

- Risk-free rate of return,  $r_f$ , is 5%
- Expected Market Return,  $E(r_M)$ , is 10%
- Stock A has a  $\beta$  of 1.2
- Investor expects a return on A,  $E(r_A)$ , of 12%

a. (0.5 points)

Calculate alpha,  $\alpha$ .

b. (1.5 points)

Graph the security market line (SML) and label the following items:

- x-axis
- y-axis
- coordinate of the risk-free asset
- coordinate of Stock A
- coordinate of the market portfolio, M
- $\alpha$

**2004, Q5** (2 points)

Briefly describe four of the simplifying assumptions that lead to the basic version of the capital asset pricing model (CAPM).

**2004, Q6 (1 point)**

Calculate the expected return for an individual stock using the capital asset pricing model (CAPM) and the following information:

- Risk-free rate is 3.0%.
- Expected market return is 10.0%.
- Standard deviation of the market is 40.0%.
- Standard deviation of the individual stock is 25.0%.
- The correlation of the individual stock with the market is 0.88.

**2004, Q7 (2 points); Video Explanation Available**

The zero-beta model is an extension of the simple capital asset pricing model (CAPM), and is used when one of the major assumptions underlying the model is relaxed.

- (0.5 point)  
Which assumption of the capital asset pricing model is the zero-beta model intended to address?
- (0.5 point)  
Briefly define the zero-beta portfolio.
- (1 point)  
Assume the zero-beta version of the CAPM holds. Calculate the expected return of security A,  $E(r_A)$ , given the following information:
  - Expected market return,  $E(r_M)$ , is 9.0%.
  - Expected return on the zero-beta portfolio,  $E(r_{Z(M)})$  is 5.0%.
  - Risk-free rate,  $R_f$  is 4.0%.
  - Standard deviation of the market portfolio,  $\sigma_M$ , is 10.0%.
  - Beta of security A,  $\beta_A$ , is 0.75.

**2005, Q8 (2 points)**

You are given the following information about quarterly returns.

Quarter Ending	Quarterly Stock X Return	Quarterly Market Return	Quarterly T-Bill Rate
March	-6.0%	0.0%	1.0%
June	1.0%	2.0%	1.0%
September	-4.0%	-1.0%	1.0%
December	5.0%	3.0%	1.0%

Using the single-index model, calculate the estimated beta for Stock X.

**2005, Q12 (1 point)**

You are given the following information.

Stock	Expected Return	Covariance with Market
A	5%	25.00
B	14%	156.25

- Risk-free rate of return ( $r_f$ ) = 3%
- Expected return on market portfolio,  $E(r_M)$  = 6%

Calculate the change in the expected return on Stock B that would bring Stock B into equilibrium with Stock A. Assume the information above with respect to Stock A does not vary with changes in the expected return on Stock B.

**2006, Q6 (2.5 points); Video Explanation Available**

The following table gives a security analyst's opinion about expected returns on two stocks for two different market scenarios:

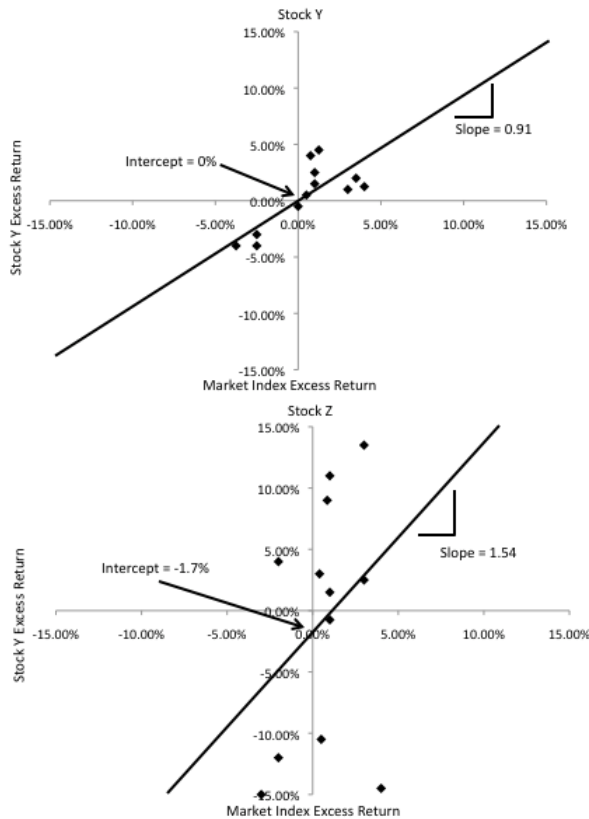
Scenario	Market Return	Stock A	Stock B
#1	6.00%	2.00%	5.00%
#2	22.00%	34.00%	11.00%

The risk-free rate is 4.0% under both scenarios.

- (0.5 point)  
Calculate the beta for Stock A and for Stock B.
- (0.5 point)  
Calculate the alpha for each of Stock A and Stock B if Scenario #1 and Scenario #2 are equally likely.
- (1 point)  
Draw the Security Market Line (SML) for the given economy and plot the two securities on the SML graph. Label all items properly.
- (0.5 point)  
Briefly explain which stock, Stock A or Stock B, would be perceived by the analyst as a better buy.

**2008, Q3 (1.5 points)**

Consider the two regression lines of monthly excess returns for stocks Y and Z in the following graphs.



- (0.25 point)  
Briefly explain which stock has greater systematic risk based on interpretation of the graphs.
- (0.25 point)  
Briefly explain which stock has higher firm-specific risk based on interpretation of the graphs.
- (1 point)  
Briefly explain for each stock whether the intercepts and slopes of the regression lines are consistent with the Capital Asset Pricing Model.

**2010, Q4** (1.25 points)

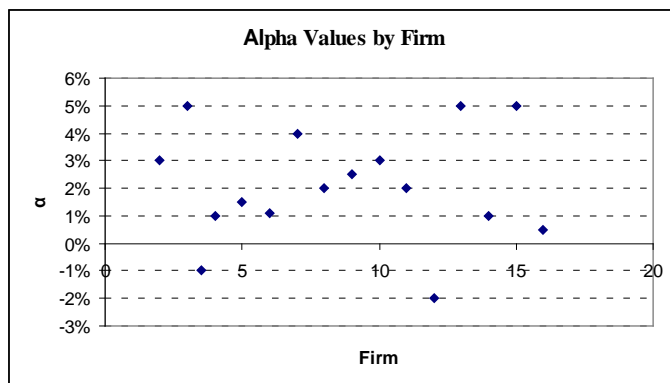
For each of the following, briefly describe the assumption that underlies the basic version of the Capital Asset Pricing Model:

- a. (0.25 point)  
Wealth
- b. (0.25 point)  
Taxes
- c. (0.25 point)  
Human capital
- d. (0.25 point)  
Probability distribution of future cash flows
- e. (0.25 point)  
Portfolio selection models

**2011, Q3** (0.75 points)

Given the following information:

- An index model of the following form is being used:  $R_i = \alpha_i + \beta_i R_M + e_i$ .
  - o  $R_i$  is the return above the risk-free rate for firm  $i$ .
  - o  $R_M$  is the return above the risk-free rate for the market.
- This index model was estimated for several firms using the above equation as a regression equation.
- The residual plot from this regression is shown in the following graph:



Explain whether or not this residual plot is consistent with expectations under the Capital Asset Pricing Model (CAPM).

**2011, Q4 (1.5 points)**

Assume that the simple Capital Asset Pricing Model (CAPM) is valid.

a. (0.75 point)

*Not in current syllabus*

b. (0.75 point)

Evaluate whether Portfolio B below is consistent with CAPM.

Portfolio	Expected Return	Beta
Risk-free	0.07	0
Market	0.16	1.0
B	0.22	1.8

**2013, Q4 (1 point)**

An investment manager currently invests all assets in a market index fund with  $\beta$  equal to 1.0. To reduce the risk, the manager plans to sell the market index fund and invest entirely in a single stock with  $\beta$  equal to 0.75.

Using the Capital Asset Pricing Model (CAPM), fully discuss the impact of the manager's strategy on the overall risk of the investment.

**2014, Q3 (2 points)**

The values in the table below were empirically estimated.

Security	Expected Return	Beta
Stock A	6%	0.5
Stock B	10%	1.5

- The expected market return is 8%.
- The risk-free rate is 2%.

a. (1.5 points)

On a single graph, draw and label:

- The Security Market Line (SML) implied by the expected market return.
- The empirically estimated SML.
- The risk-return point for both securities listed above.

b. (0.5 point)

Explain whether Stock A or Stock B is a better buy according to the Capital Asset Pricing Model (CAPM).



**2015, Q4 (2.75 points)**

Given the following information:

- The variance-covariance matrix:

	Stock A	Stock B	Market Portfolio
Stock A	0.16		
Stock B	-0.48	1.44	
Market Portfolio	-0.0108	0.252	0.09

- Stock A and Stock B are perfectly negatively correlated.
- The risk-free rate is 3%.
- The market portfolio has an expected return of 8%.
- Portfolio P consists of:
  - o 75% invested in Stock A.
  - o 25% invested in Stock B.

a. (1.5 points)

Using CAPM, calculate the expected return of Portfolio P.

b. (1.25 points)

Construct an arbitrage strategy using the risk-free rate and some combination of Stock A and Stock B.

**2015, Q5** (1.5 points); *Video Explanation Available*

Given the following information:

	Market Portfolio	Market's Zero-Beta Portfolio	Minimum-Variance Portfolio
$E(r)$	8%	Not Given	6%
$\sigma$	14%	14%	10%

- The risk-free rate is 3%.
  - The covariance of Security A and the market portfolio is 0.0392.
  - The minimum-variance frontier is symmetric around the minimum-variance portfolio.
- a. (1 point)  
Calculate the expected return for Security A using the zero-beta extension of CAPM.
- b. (0.25 point)  
Identify the assumption that is relaxed under the zero-beta extension of CAPM.
- c. (0.25 point)  
State the correlation between the market portfolio and the market's zero-beta portfolio.

**2016, Q5 (2.75 points)**

Given the following information:

- Risk free rate is 5%.
  - Expected market return is 15%.
  - The market price of risk is 40%.
  - The covariance of stock A with the market is 20%.
  - The current price of stock A is \$90.
- a. (0.75 point)  
Calculate the expected return for stock A using the Capital Asset Pricing Model.
- b. (0.5 point)  
Explain how the current price of stock A would be expected to change if investors forecast the price of stock A to be \$99 in one year.
- c. (0.5 point)  
The current price of stock A remains at \$90 despite the forecasted price of \$99 in one year. Briefly describe an arbitrage opportunity that exists.
- d. (1 point)  
Describe two limitations to being able to take advantage of an arbitrage opportunity.

## **BKM9 Solutions**

### **2002, Q11 (1 point)**

Argue against the following statement.

"I followed the CAPM theory last year and bought high-Beta stocks. Since then the return on the high-Beta stocks was worse than the average return for low-Beta stocks. The CAPM theory does not work!"

**High Beta stocks are riskier and offer higher returns on average, greater than low beta (lower risk) stocks. Thus in some years, returns may be lower than lower Beta stocks, but in the long run, high Beta stocks will have higher returns.**

### **2003, Q10**

Given the following information, calculate the expected return of portfolio P,  $E(r_p)$ :

- The value of portfolio P is made up of 60% of stock A and 40% of stock B
- Market Price of Risk = Market Risk Premium / Market Variance = 50%
- Expected Market Return,  $E(r_M)$ , is 15%.
- The variance of the market portfolio,  $\sigma_M^2$ , is 20%.
- The covariance of stock A with the market,  $\text{cov}(r_A, r_M)$ , is 25%.
- The covariance of stock B with the market,  $\text{cov}(r_B, r_M)$ , is 10%.

$$\beta_i = [\text{Cov}(r_i, r_M) / \sigma_M^2]$$

$$\begin{aligned}\beta_A &= 0.25 / 0.2 \\ &= 1.25\end{aligned}$$

$$\begin{aligned}\beta_B &= 0.1 / 0.2 \\ &= 0.5\end{aligned}$$

$$\begin{aligned}\beta_P &= 0.6 * 1.25 + 0.4 * 0.5 \\ &= 0.95\end{aligned}$$

To calculate the risk free rate,

$$0.5 = (0.15 - r_f) / 0.2$$

$$r_f = 0.05$$

$$\begin{aligned}E(r_p) &= 0.05 + 0.95 * (0.15 - 0.05) \\ &= 0.145\end{aligned}$$

### **2003, Q11**

You are given the following information:

- Risk-free rate of return,  $r_f$ , is 5%
- Expected Market Return,  $E(r_M)$ , is 10%
- Stock A has a  $\beta$  of 1.2
- Investor expects a return on A,  $E(r_A)$ , of 12%

a. (0.5 points)

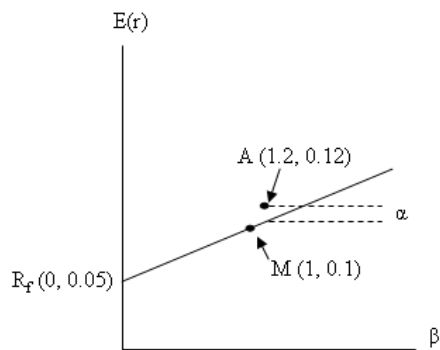
Calculate alpha,  $\alpha$ .

$$\begin{aligned} E(r) - r_f &= \alpha + \beta[E(r_M) - r_f] \\ 0.12 - 0.05 &= \alpha + 1.2 * (0.1 - 0.05) \\ \alpha &= 0.01 \end{aligned}$$

b. (1.5 points)

Graph the security market line (SML) and label the following items:

- x-axis
- y-axis
- coordinate of the risk-free asset
- coordinate of Stock A
- coordinate of the market portfolio, M
- $\alpha$



### 2004, Q5

Briefly describe four of the simplifying assumptions that lead to the basic version of the capital asset pricing model (CAPM).

- Investors are rational mean-variance optimizers: investors are only concerned about mean and variance
- Their planning horizon is a single period
- Investors use identical input lists (homogeneous expectations)
- All assets are publicly traded, and short positions are allowed. Investors can borrow/ lend at a common risk free rate
- All information is publicly available
- No taxes
- No transaction costs

**2004, Q6**

Calculate the expected return for an individual stock using the capital asset pricing model (CAPM) and the following information:

- Risk-free rate is 3.0%.
- Expected market return is 10.0%.
- Standard deviation of the market is 40.0%.
- Standard deviation of the individual stock is 25.0%.
- The correlation of the individual stock with the market is 0.88.

$$\beta_i = [Cov(r_i, r_M) / \sigma_M^2]$$

$$\begin{aligned}\beta &= (0.88 * 0.4 * 0.25) / (0.4^2) \\ &= 0.55\end{aligned}$$

$$\begin{aligned}E(r) &= 0.03 + 0.55 * (0.1 - 0.03) \\ &= 0.0685\end{aligned}$$

**2004, Q7**

The zero-beta model is an extension of the simple capital asset pricing model (CAPM), and is used when one of the major assumptions underlying the model is relaxed.

a. (0.5 point)

Which assumption of the capital asset pricing model is the zero-beta model intended to address?

**Investors can borrow and lend at the risk free rate.**

b. (0.5 point)

Briefly define the zero-beta portfolio.

**A portfolio on the inefficient part of the minimum variance frontier that is uncorrelated with a specific portfolio on the efficient frontier.**

c. (1 point)

Assume the zero-beta version of the CAPM holds. Calculate the expected return of security A,  $E(r_A)$ , given the following information:

- Expected market return,  $E(r_M)$ , is 9.0%.
- Expected return on the zero-beta portfolio,  $E(r_{Z(M)})$  is 5.0%.
- Risk-free rate,  $R_f$  is 4.0%.
- Standard deviation of the market portfolio,  $\sigma_M$ , is 10.0%.
- Beta of security A,  $\beta_A$ , is 0.75.

$$\begin{aligned}E(r) &= 0.05 + 0.75 (0.09 - 0.05) \\ &= 0.08\end{aligned}$$

**2005, Q8**

You are given the following information about quarterly returns.

Quarter Ending	Quarterly Stock X Return	Quarterly Market Return	Quarterly T-Bill Rate
March	-6.0%	0.0%	1.0%
June	1.0%	2.0%	1.0%
September	-4.0%	-1.0%	1.0%
December	5.0%	3.0%	1.0%

Using the single-index model, calculate the estimated beta for Stock X.

Qtr	$r_x$	$r_m$	$(r_m - \text{AVG } r_m)^2$	$(r_x - \text{AVG } r_x) * (r_m - \text{AVG } r_m)$
1	-0.06	0.00	0.0001	0.0005
2	0.01	0.02	0.0001	0.0002
3	-0.04	-0.01	0.0004	0.0006
4	0.05	0.03	0.0004	0.0012
Avg:	-0.01	0.01	0.000250	0.000625

$$\begin{aligned}
 \beta &= \text{cov}(X, M) / \text{var}(M) \\
 &= 0.000625 / 0.000250 \\
 &= 2.5
 \end{aligned}$$

**2005, Q12**

You are given the following information.

Stock	Expected Return	Covariance with Market
A	5%	25.00
B	14%	156.25

- Risk-free rate of return ( $r_f$ ) = 3%
- Expected return on market portfolio,  $E(r_M)$  = 6%

Calculate the change in the expected return on Stock B that would bring Stock B into equilibrium with Stock A. Assume the information above with respect to Stock A does not vary with changes in the expected return on Stock B.

$$5\% = 3\% + \beta_A * (0.06 - 0.03)$$

$$\beta_A = 2/3$$

$$25 / \sigma^2_M = 2/3$$

$$\sigma^2_M = 75/2$$

$$\beta_B = 156.25 / (75/2)$$

$$= 4.167$$

$$E(r_B) = 0.03 + 4.167 * (0.06 - 0.03)$$

$$= 0.155$$

$$\text{Additional} = 0.155 - 0.14$$

$$= 0.015$$

### 2006, Q6

The following table gives a security analyst's opinion about expected returns on two stocks for two different market scenarios:

Scenario	Market Return	Stock A	Stock B
#1	6.00%	2.00%	5.00%
#2	22.00%	34.00%	11.00%

The risk-free rate is 4.0% under both scenarios.

a. (0.5 point)

Calculate the beta for Stock A and for Stock B.

$$0.02 = 0.04 + \beta_A(0.06 - 0.04) + \alpha$$

$$= 0.04 + 0.02\beta_A + \alpha_A$$

$$0.34 = 0.04 + \beta_A(0.22 - 0.04) + \alpha$$

$$= 0.04 + 0.18\beta_A + \alpha_A$$

$$0.32 = 0.16\beta_A$$

$$\beta_A = 2$$

$$0.05 = 0.04 + \beta_B(0.06 - 0.04) + \alpha$$

$$= 0.04 + 0.02\beta_B + \alpha_B$$

$$0.11 = 0.04 + \beta_B(0.22 - 0.04) + \alpha$$

$$= 0.04 + 0.18\beta_B + \alpha_B$$

$$0.06 = 0.16\beta_B$$

$$\beta_B = 0.375$$



b. (0.5 point)

Calculate the alpha for each of Stock A and Stock B if Scenario #1 and Scenario #2 are equally likely.

$$\begin{aligned} 0.34 &= 0.04 + 0.18 * 2 + \alpha_A \\ \alpha_A &= -0.06 \end{aligned}$$

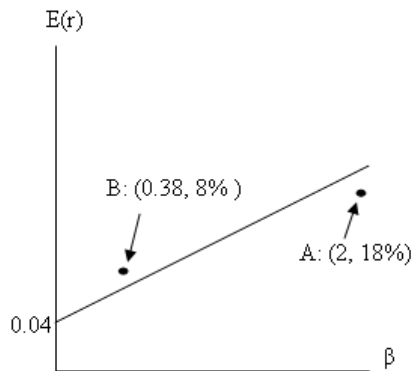
$$\begin{aligned} 0.05 &= 0.04 + 0.02 * 0.375 + \alpha_B \\ \alpha_B &= 0.0025 \end{aligned}$$

c. (1 point)

Draw the Security Market Line (SML) for the given economy and plot the two securities on the SML graph. Label all items properly.

$$\begin{aligned} E(r_M) &= 0.5 * 0.06 + 0.5 * 0.22 \\ &= 0.14 \end{aligned}$$

SML Slope is 0.1 (0.14 – 0.04)



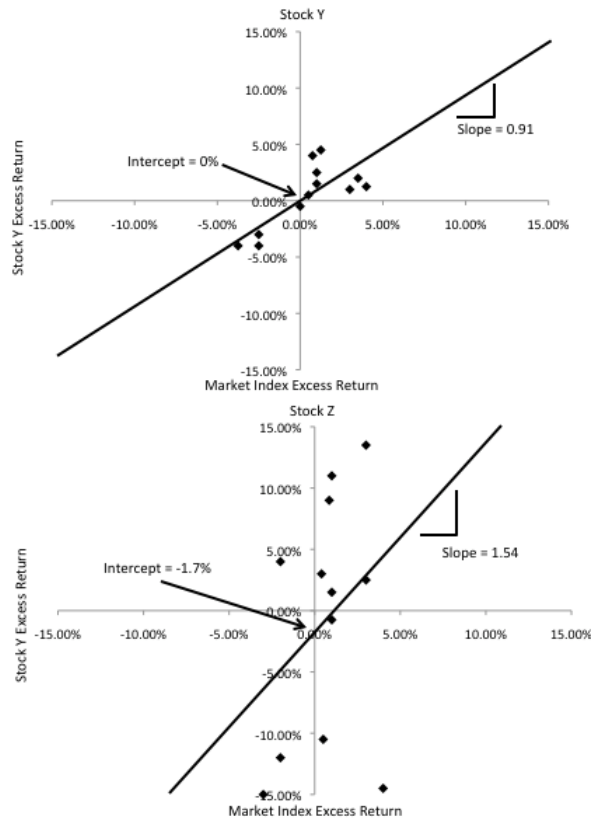
d. (0.5 point)

Briefly explain which stock, Stock A or Stock B, would be perceived by the analyst as a better buy.

**B: it has a positive  $\alpha$ .**

### 2008, Q3

Consider the two regression lines of monthly excess returns for stocks Y and Z in the following graphs.



a. (0.25 point)

Briefly explain which stock has greater systematic risk based on interpretation of the graphs.

*Note that the y axis refers to the “Stock Excess Return” and the x axis refers to the “Market index Excess Return”*

**Z: it has a higher slope (higher beta).**

b. (0.25 point)

Briefly explain which stock has higher firm-specific risk based on interpretation of the graphs.

**Z: its plot points are more spread out from the regression line.**

c. (1 point)

Briefly explain for each stock whether the intercepts and slopes of the regression lines are consistent with the Capital Asset Pricing Model.

**-The slopes are consistent with CAPM: the slope of each graph is constant (which implies a constant beta)**

**-The Y-intercept of CAPM should be 0, so Y is consistent with CAPM while Z is not.**

*TIA forum post(s) discussing this question that may be helpful:*

> <http://www.theinfiniteactuary.com/mb/viewtopic.php?f=2653&t=3788>

#### **2010, Q4**

For each of the following, briefly describe the assumption that underlies the basic version of the Capital Asset Pricing Model:

a. (0.25 point)

Wealth

**There are many investors each with a small amount of wealth compared to the overall wealth, which means that all investors are price takers.**

b. (0.25 point)

Taxes

**No taxes of investment returns.**

c. (0.25 point)

Human capital

**Human capital is ignored since it's not publicly traded.**

d. (0.25 point)

Probability distribution of future cash flows

**All investors have homogenous expectations about future cash flows.**

e. (0.25 point)

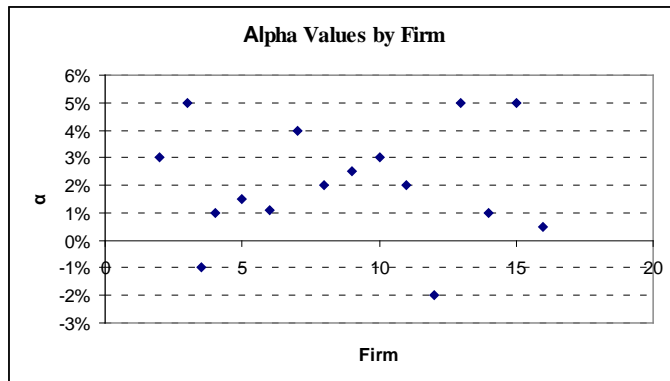
Portfolio selection models

**All investors are mean variance optimizers meaning the use a Markowitz portfolio selection procedure.**

### 2011, Q3

Given the following information:

- An index model of the following form is being used:  $R_i = \alpha_i + \beta_i R_M + e_i$ .
  - $R_i$  is the return above the risk-free rate for firm  $i$ .
  - $R_M$  is the return above the risk-free rate for the market.
- This index model was estimated for several firms using the above equation as a regression equation.
- The residual plot from this regression is shown in the following graph:



Explain whether or not this residual plot is consistent with expectations under the Capital Asset Pricing Model (CAPM).

**This graph is inconsistent with CAPM. Under CAPM,  $E(\alpha) = 0$  for all securities, so the plot of alpha values should center around 0. The graph shows that  $\alpha$  is skewed to be positive.**

**2011, Q4**

Assume that the simple Capital Asset Pricing Model (CAPM) is valid.

b. (0.75 point)

Evaluate whether Portfolio B below is consistent with CAPM.

Portfolio	Expected Return	Beta
Risk-free	0.07	0
Market	0.16	1.0
B	0.22	1.8

**Expected Return of B according to CAPM:**

$$\begin{aligned} E(R_B) &= 0.07 + 1.8 * (0.16 - 0.07) \\ &= 0.232 \end{aligned}$$

**Since this is different to the return of 0.22, we can conclude that it is not consistent with CAPM.**

**2013, Q4 (1 point)**

An investment manager currently invests all assets in a market index fund with  $\beta$  equal to 1.0. To reduce the risk, the manager plans to sell the market index fund and invest entirely in a single stock with  $\beta$  equal to 0.75.

Using the Capital Asset Pricing Model (CAPM), fully discuss the impact of the manager's strategy on the overall risk of the investment.

**The risk of an investment can be calculated using:**

$$\sigma^2 = \beta^2 \sigma_m^2 + \sigma^2(e_i)$$

**The current investment (market index)  $\beta=1$  will only have systematic risk (driven by  $\beta$ ). Non systematic risk  $\approx 0$ .**

**The proposed investment will have  $\beta = 0.75$ . So while the systematic risk will be lower, there will now be significant non systematic risk as the manager holds only one asset (no diversification benefit).**

**This increase in non systematic risk most likely will exceed the systematic risk, thus increasing the total amount of risk.**

**In addition, expected return should fall. CAPM dictates that return only rewards systematic risk. Therefore, the drop in systematic risk will produce lower expected returns.**

**Would not advise- losing diversification benefit of risk sharing!**

**2014, Q3 (2 points)**

The values in the table below were empirically estimated.

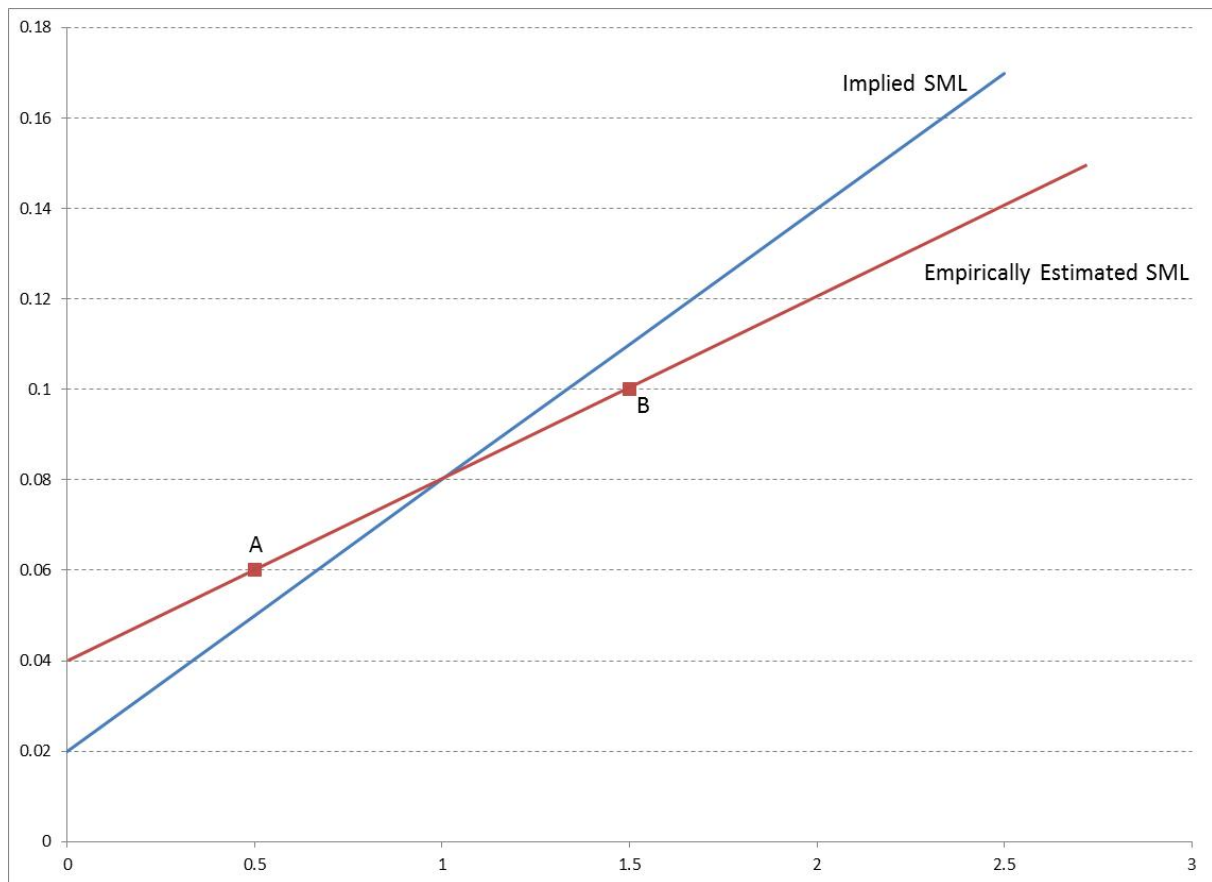
Security	Expected Return	Beta
Stock A	6%	0.5
Stock B	10%	1.5

- The expected market return is 8%.
- The risk-free rate is 2%.

a. (1.5 points)

On a single graph, draw and label:

- a. The Security Market Line (SML) implied by the expected market return.
- b. The empirically estimated SML.
- c. The risk-return point for both securities listed above.



b. (0.5 point)

Explain whether Stock A or Stock B is a better buy according to the Capital Asset Pricing Model (CAPM).

**Stock A is a better buy according to CAPM since it is above the SML (B is below the SML).**

*Note from Examiners Report: candidates who used the positive alpha in the explanation, but did not identify what alpha is, did not receive full credit*

**2015, Q4 (2.75 points)**

Given the following information:

- The variance-covariance matrix:

	Stock A	Stock B	Market Portfolio
Stock A	0.16		
Stock B	-0.48	1.44	
Market Portfolio	-0.0108	0.252	0.09

- Stock A and Stock B are perfectly negatively correlated.
- The risk-free rate is 3%.
- The market portfolio has an expected return of 8%.
- Portfolio P consists of:
  - o 75% invested in Stock A.
  - o 25% invested in Stock B.

a. (1.5 points)

Using CAPM, calculate the expected return of Portfolio P.

$$\begin{aligned}\beta_A &= -0.0108 / 0.09 \\ &= -0.12\end{aligned}$$

$$\begin{aligned}E(r_A) &= 0.03 - 0.12 * (0.08 - 0.03) \\ &= 0.024\end{aligned}$$

$$\begin{aligned}\beta_B &= 0.252 / 0.09 \\ &= 2.8\end{aligned}$$

$$\begin{aligned}E(r_B) &= 0.03 + 0.28 * (0.08 - 0.03) \\ &= 0.17\end{aligned}$$

$$\begin{aligned}E(r_P) &= 0.75 * 0.024 + 0.25 * 0.17 \\ &= 0.0605\end{aligned}$$

b. (1.25 points)

Construct an arbitrage strategy using the risk-free rate and some combination of Stock A and Stock B.

**Since A & B are perfectly negatively correlated, we should be able to construct a portfolio that has a variance of 0.**

$$\begin{aligned}\text{Cov}(A,B) &= -1 * 0.16^{0.5} * 1.44^{0.5} \\ &= -0.48\end{aligned}$$

$$\begin{aligned}W_A &= (1.44 + 0.48) / (1.44 + 0.16 + 2 * 0.48) \\ &= 0.75\end{aligned}$$

$$W_B = 0.25$$

**Check:**

$$\begin{aligned}\sigma^2 &= 0.75^2 * 0.16 + 0.25^2 * 1.44 + 2 * -0.48 * 0.75 * 0.25 \\ &= 0\end{aligned}$$

**The above portfolio (75% in A and 25% in B; which is portfolio P) is therefore risk free. Therefore, borrow at the risk free rate, and use that money to purchase Portfolio P. Use the proceeds to pay back loan.**



**Risk free return of  $6.05\% - 3\% = 3.05\%$**

***Note from Examiners Report:***

***Overall, most candidates had difficulty obtaining full credit on Part b. The majority of candidates seemed to understand the concept of an arbitrage strategy, but struggled to identify the arbitrage opportunity that existed within the problem.***

***Common mistakes included:***

- explaining an arbitrage exploitation strategy that had up-front costs***
- assuming a 50-50 weighting of A and B for the zero variance portfolio***
- constructing a portfolio of A & B with a beta of 0 and outlining an arbitrage strategy between this portfolio and the risk-free security (these portfolios have the same return, so no arbitrage opportunity exists)***
- constructing a portfolio of A & B with beta of 1 and outlining an arbitrage strategy between this portfolio and the market portfolio (these portfolios have the same return, so no arbitrage opportunity exists)***

**2015, Q5 (1.5 points)**

Given the following information:

	Market Portfolio	Market's Zero-Beta Portfolio	Minimum-Variance Portfolio
E(r)	8%	Not Given	6%
$\sigma$	14%	14%	10%

- The risk-free rate is 3%.
- The covariance of Security A and the market portfolio is 0.0392.
- The minimum-variance frontier is symmetric around the minimum-variance portfolio.

a. (1 point)

Calculate the expected return for Security A using the zero-beta extension of CAPM.

*This may be a defective question, as the text did not mention that the zero beta portfolio has to mirror the portfolio on the efficient frontier.*

$$\begin{aligned}\beta_A &= 0.0392 / 0.14^2 \\ &= 2\end{aligned}$$

**Using the fact that the minimum variance frontier is symmetric around the min var portfolio,**

$$\begin{aligned}E(r_Z) &= 0.06 - (0.08 - 0.06) \\ &= 0.04\end{aligned}$$

$$\begin{aligned}E(r_A) &= 0.04 + 2 * (0.08 - 0.04) \\ &= 0.12\end{aligned}$$

TIA forum post(s) discussing this question that may be helpful:

> <http://www.theinfiniteactuary.com/mb/viewtopic.php?f=2653&t=28255>

b. (0.25 point)

Identify the assumption that is relaxed under the zero-beta extension of CAPM.

**Investors can borrow/ lend at the risk free rate**

c. (0.25 point)

State the correlation between the market portfolio and the market's zero-beta portfolio.

**They are not correlated**

**2016, Q5** (2.75 points);

Given the following information:

- Risk free rate is 5%.
- Expected market return is 15%.
- The market price of risk is 40%.
- The covariance of stock A with the market is 20%.
- The current price of stock A is \$90.

a. (0.75 point)

Calculate the expected return for stock A using the Capital Asset Pricing Model.

$$\text{Market Price of Risk} = E(R_M) / \sigma_M^2$$

$$0.4 = (0.15 - 0.05) / \sigma_M^2$$

$$\sigma_M^2 = 0.25$$

$$\beta_A = \text{Cov}(R_A, R_M) / \sigma_M^2$$

$$= 0.2 / 0.25$$

$$= 0.8$$

$$E(r_A) = 0.05 + 0.8 * (0.15 - 0.05)$$

$$= 13\%$$

b. (0.5 point)

Explain how the current price of stock A would be expected to change if investors forecast the price of stock A to be \$99 in one year.

$$\text{Price} = 99 / 1.13$$

$$= 87.61$$

**Therefore price will fall from 90 to 87.61**

c. (0.5 point)

The current price of stock A remains at \$90 despite the forecasted price of \$99 in one year. Briefly describe an arbitrage opportunity that exists.

**Short sell Stock A for 90. Invest  $0.8 * 90 = 72$  in market portfolio and 0.2 in risk-free asset. Expected return =  $72 * 1.15 + 18 * 1.05 = 101.7$ . At the end of the year, buy back the stock to net  $101.7 - 99 = 2.7$ . This creates a risk-free return of 2.7 per share without upfront investment.**

d. (1 point)

Describe two limitations to being able to take advantage of an arbitrage opportunity.

**-Model risk: Maybe the model is not right**

**-Fundamental risks: the market may persist with overpriced securities: The overpricing can get worse.**

**-Fundamental risk: risk that the market doesn't go as expected and arbitrage doesn't pay off.**

**-Transaction costs: Actual cost of implementation may make arbitrage a wash or even cost more money than earned.**

## Past Exam Questions – BKM10

**2000, Q13** (2 points); *Video Explanation Available*

*Note: this question has been modified to reflect the current syllabus.*

The following three portfolios are observed:

Portfolio	Expected Return (%)	$b_{i1}$	$b_{i2}$
1	12	0.50	1.00
2	17	1.25	2.00
3	13	1.00	0.50

Where  $b_{ij}$  is the sensitivity of portfolio  $i$ 's return to the  $j^{\text{th}}$  index.

Assume a new portfolio 4 becomes available, with an expected return of 14%, and with  $b_{i1}$  and  $b_{i2}$  both equal to 0.75.

Briefly describe an arbitrage opportunity that exists with the introduction of this new portfolio.

**2001, Q32** (3 points); *Video Explanation Available*

*Note: this question has been modified to reflect the current syllabus.*

Suppose that the systematic risk of the market can be described by two uncorrelated macro-economic factors,  $F_1$  and  $F_2$ , and that you are to use a two-factor APT to model asset returns.

For stocks A, B, and C you are given the following information:

- The expected excess return (risk premium) for  $F_1$  is 3.25.
- The expected excess return (risk premium) for  $F_2$  is 2.50.
- The sensitivities of the stock returns to the factors  $F_1$  and  $F_2$  are given below.

Stock	$\beta_{i1}$	$\beta_{i2}$
A	1.21	0.8
B	0.68	-0.75
C	0.56	0.5

Answer the following. Show all work.

- a. (1 point)

*Not in current syllabus*

- b. (1 point)

Determine the expected excess returns (risk premiums) for A, B, and C.

- c. (1 point)

Assuming that Portfolio P has \$20 million invested in stock A, \$70 million invested in stock B, and \$30 million invested in Stock C.

Determine the sensitivities  $\beta_{p1}$  and  $\beta_{p2}$  for this portfolio

**2003, Q13** (1.5 points)

You are given the following information about a two-factor APT model:

Factor Portfolio	Expected Return	$\beta_{1i}$	$\beta_{2i}$
1	10%	1.0	0.0
2	15%	0.0	1.0

The risk-free rate,  $r_f$  is 5%

a. (0.5 points)

Calculate the expected return for a portfolio A,  $E(r_A)$ , assuming  $\beta_{A1} = 0.5$  and  $\beta_{A2} = 0.8$

b. (1 point)

Describe the action of an investor who believed  $E(r_A)$  was really 14%. Include the weights of the assets the investor would use to construct a portfolio that would take advantage of a potentially risk-less arbitrage opportunity.

**2004, Q9** (2.5 points); *Video Explanation Available*

Assume there are two independent economic factors, T and S. All stocks have independent firm-specific components with standard deviations of 40.0%.

You have the following information on well-diversified portfolios:

Portfolio 1	Portfolio 2	Portfolio 3
$\beta_{1T} = 2.0$	$\beta_{2T} = 0.8$	$\beta_{3T} = 1.2$
$\beta_{1S} = -0.5$	$\beta_{2S} = 1.0$	$\beta_{3S} = 1.0$
$E(r_1) = 13.0\%$	$E(r_2) = 10.0\%$	?

The risk-free rate is 4.0%.

Calculate the expected return of Portfolio 3,  $E(r_3)$ .

**2006, Q7** (1.5 points); *Video Explanation Available*

The expected returns on Stocks A and B are defined by a multi-factor model with the following estimates:

Stock	$E(r_i)$	$\beta_{i,GDP}$	$\beta_{i,IR}$
A	5.00%	0.8	1.2
B	10.00%	1.2	0.8

Assume the expected values for Gross Domestic Product (GDP) and Interest Rate (IR) are 7.5% and 2.5%, respectively, and that the actual Gross Domestic Product turns out to be 10.0%.

Calculate the actual Interest Rate needed such that Stock A and Stock B generate the same returns. Assume no firm-specific influences.

**2006, Q9** (1 point)

The Fama-French three-factor Model includes two additional factors not in the single-factor Capital Asset Pricing Model (CAPM).

Identify these two additional factors and explain the rationale for including each of them in the three-factor model.

**2008, Q4** (1.5 points); *Video Explanation Available*

You are given the following information:

- There are three independent economic factors:  $F_1$ ,  $F_2$ , and  $F_3$ .
- The risk-free rate is 3%.
- The risk premium for  $F_3$  is 10%.

You are also given the following information about three well-diversified portfolios: A, B, and C:

Portfolio	Beta on $F_1$	Beta on $F_2$	Beta on $F_3$	Expected Return
A	0.80	0.70	1.65	20%
B	-0.25	1.80	0.30	11.90%
C	1.25	0.70	1.40	16.60%

Calculate the risk premium for  $F_1$ .

**2009, Q4** (1 point)

Suppose there are three independent economic factors:  $F_1$ ,  $F_2$ , and  $F_3$ .

- The risk premium for  $F_1$  is 5%.
- The risk premium for  $F_2$  is 2%.
- The risk-free rate is 3%.

Given the following information about well-diversified portfolios:

Portfolio	Beta on $F_1$	Beta on $F_2$	Beta on $F_3$	Expected Return
A	0.4	-0.5	0.5	7.00%
B	-0.8	1	0.5	not given

Calculate the expected return of Portfolio B.

**2011, Q2** (1.5 points)

- (0.5 point)  
Describe the arbitrage argument for restoring equilibrium prices.
- (0.5 point)  
Describe the risk-return dominance argument for restoring equilibrium prices.
- (0.5 point)  
Describe the main difference between the two arguments in parts a and b above.

**2015, Q6 (2.25 point)**

Given the following information:

Firm	Book Value (\$000,000)	Market Value (\$000,000)	CAPM Beta
A	625	500	1.2
B	350	500	1.2
C	1,250	1,000	1.2

a. (1.25 points)

Use the principles underlying the Fama & French 3-Factor Model to evaluate which firm would be expected to have the highest future rate of return.

b. (1 point)

Assume that the firm identified in part a. above does produce the highest rate of return over a future period.

Describe two behavioral explanations that could explain the superior performance.

## **BKM10 Solutions**

### **2000, Q13**

The following three portfolios are observed:

<u>Portfolio</u>	<u>Expected Return (%)</u>	<u><math>b_{i1}</math></u>	<u><math>b_{i2}</math></u>
1	12	0.50	1.00
2	17	1.25	2.00
3	13	1.00	0.50

Where  $b_{ij}$  is the sensitivity of portfolio  $i$ 's return to the  $j^{\text{th}}$  index.

Assume a new portfolio 4 becomes available, with an expected return of 14%, and with  $b_{i1}$  and  $b_{i2}$  both equal to 0.75.

Briefly describe an arbitrage opportunity that exists with the introduction of this new portfolio.

**Combine 50% of Portfolio 1 with 50% of Portfolio 3 to produce a portfolio with expected return of 12.5% and risk factors  $b_{i1}$  and  $b_{i2}$  both equal to 0.75.**

**This new portfolio has the same risk level as Portfolio 4, but with lower return. Therefore, an arbitrage opportunity exists: sell Portfolio 5 short in order to buy Portfolio 4, which would guarantee a risk free profit with no investment commitment.**

### **2001, Q32**

Suppose that the systematic risk of the market can be described by two uncorrelated macro-economic factors,  $F_1$  and  $F_2$ , and that you are to use a two-factor APT to model asset returns.

For stocks A, B, and C you are given the following information:

- The expected excess return (risk premium) for  $F_1$  is 3.25.
- The expected excess return (risk premium) for  $F_2$  is 2.50.
- The sensitivities of the stock returns to the factors  $F_1$  and  $F_2$  are given below.

Stock	$\beta_{i1}$	$\beta_{i2}$
A	1.21	0.8
B	0.68	-0.75
C	0.56	0.5

Answer the following. Show all work.

b. (1 point)

Determine the expected excess returns (risk premiums) for A, B, and C.



$$\begin{aligned} E(r_A) - r_f &= 1.21 * 3.25 + 0.8 * 2.5 \\ &= 5.93 \end{aligned}$$

$$\begin{aligned} E(r_B) - r_f &= 0.68 * 3.25 + -0.75 * 2.5 \\ &= 0.34 \end{aligned}$$

$$\begin{aligned} E(r_C) - r_f &= 0.56 * 3.25 + 0.5 * 2.5 \\ &= 3.07 \end{aligned}$$

c. (1 point)

Assuming that Portfolio P has \$20 million invested in stock A, \$70 million invested in stock B, and \$30 million invested in Stock C.

Determine the sensitivities  $\beta_{p1}$  and  $\beta_{p2}$  for this portfolio

$$\begin{aligned} \beta_{p1} &= 1.21 * (20/120) + 0.68 * (70/120) + 0.56 * (30/120) \\ &= 0.738 \end{aligned}$$

$$\begin{aligned} \beta_{p2} &= 0.8 * (20/120) + -0.75 * (70/120) + 0.5 * (30/120) \\ &= -0.179 \end{aligned}$$

### 2003, Q13

You are given the following information about a two-factor APT model:

Factor Portfolio	Expected Return	$\beta_{i1}$	$\beta_{i2}$
1	10%	1.0	0.0
2	15%	0.0	1.0

The risk-free rate,  $r_f$  is 5%

a. (0.5 points)

Calculate the expected return for a portfolio A,  $E(r_A)$ , assuming  $\beta_{A1} = 0.5$  and  $\beta_{A2} = 0.8$

$$\begin{aligned} E(r_A) &= 5\% + 0.5(10\% - 5\%) + 0.8(15\% - 5\%) \\ &= 15.5\% \end{aligned}$$

b. (1 point)

Describe the action of an investor who believed  $E(r_A)$  was really 14%. Include the weights of the assets the investor would use to construct a portfolio that would take advantage of a potentially risk-less arbitrage opportunity.

**-sell Portfolio A short**

**-buy Portfolio B made up of:**

- **0.5 of Factor Portfolio 1**
- **0.8 of Factor Portfolio 2**
- **-0.3 of the risk free asset (borrow at the risk free rate)**

**This produces an expected return of 15.5%, or a 1.5% risk free return up front.**

**2004, Q9**

Assume there are two independent economic factors, T and S. All stocks have independent firm-specific components with standard deviations of 40.0%.

You have the following information on well-diversified portfolios:

Portfolio 1	Portfolio 2	Portfolio 3
$\beta_{1T} = 2.0$	$\beta_{2T} = 0.8$	$\beta_{3T} = 1.2$
$\beta_{1S} = -0.5$	$\beta_{2S} = 1.0$	$\beta_{3S} = 1.0$
$E(r_1) = 13.0\%$	$E(r_2) = 10.0\%$	?

The risk-free rate is 4.0%.

Calculate the expected return of Portfolio 3,  $E(r_3)$ .

**Portfolio 1:**

$$0.13 = 0.04 + 2.0 * (T - 0.04) - 0.5 * (S - 0.04)$$

$$0.15 = 2T - 0.5S$$

**Portfolio 2:**

$$0.10 = 0.04 + 0.8 * (T - 0.04) + 1 * (S - 0.04)$$

$$0.132 = 0.8T + S$$

**From Portfolio 1:**

$$0.3 = 4T - S$$

**Adding Portfolio 2:**

$$0.432 = 4.8T$$

$$T = 0.09$$

$$S = 0.06$$

$$E(r_3) = 0.04 + 1.2 * (0.09 - 0.04) + 1 * (0.06 - 0.04) = 0.12$$

**2006, Q7**

The expected returns on Stocks A and B are defined by a multi-factor model with the following estimates:

Stock	$E(r_i)$	$\beta_{i,GDP}$	$\beta_{i,IR}$
A	5.00%	0.8	1.2
B	10.00%	1.2	0.8

Assume the expected values for Gross Domestic Product (GDP) and Interest Rate (IR) are 7.5% and 2.5%, respectively, and that the actual Gross Domestic Product turns out to be 10.0%.

Calculate the actual Interest Rate needed such that Stock A and Stock B generate the same returns. Assume no firm-specific influences.

$$\begin{aligned} R(r_A) &= 5 + 0.8 * (10 - 7.5) + 1.2 * (IR - 2.5) \\ &= 4 + 1.2IR \\ R(r_B) &= 10 + 1.2 * (10 - 7.5) + 0.8 * (IR - 2.5) \\ &= 11 + 0.8IR \end{aligned}$$

$$\begin{aligned} 4 + 1.2IR &= 11 + 0.8IR \\ IR &= 17.5\% \end{aligned}$$

**2006, Q9** (1 point)

The Fama-French three-factor Model includes two additional factors not in the single-factor Capital Asset Pricing Model (CAPM).

Identify these two additional factors and explain the rationale for including each of them in the three-factor model.

**1 – Firm size**

**2 – Book to market value ratio.**

**The rationale for including these was that it appeared to measure some of the systematic risk not captured by using only the market portfolio factor used in CAPM. Note that it was not because F-F necessarily thought small firms or high BV to MV ratio firms had better returns.**

**2008, Q4**

You are given the following information:

- There are three independent economic factors:  $F_1$ ,  $F_2$ , and  $F_3$ .
- The risk-free rate is 3%.
- The risk premium for  $F_3$  is 10%.

You are also given the following information about three well-diversified portfolios: A, B, and C:

Portfolio	Beta on $F_1$	Beta on $F_2$	Beta on $F_3$	Expected Return
A	0.80	0.70	1.65	20%
B	-0.25	1.80	0.30	11.90%
C	1.25	0.70	1.40	16.60%

Calculate the risk premium for  $F_1$ .

*Since portfolios A & C have the same beta to factor  $F_2$ , use those to calculate the risk premium:*

**A:**

$$0.20 = 0.03 + 0.80 * F_1 + 0.70 * F_2 + 1.65 * 0.10$$

$$0.005 = 0.80 * F_1 + 0.70 * F_2$$

**C:**

$$0.166 = 0.03 + 1.25 * F_1 + 0.70 * F_2 + 1.40 * 0.10$$

$$-0.004 = 1.25 * F_1 + 0.70 * F_2$$

**Combining:**

$$0.009 = -0.45 F_1$$

$$F_1 = -0.02$$

#### **2009, Q4**

Suppose there are three independent economic factors:  $F_1$ ,  $F_2$ , and  $F_3$ .

- The risk premium for  $F_1$  is 5%.
- The risk premium for  $F_2$  is 2%.
- The risk-free rate is 3%.

Given the following information about well-diversified portfolios:

Portfolio	Beta on $F_1$	Beta on $F_2$	Beta on $F_3$	Expected Return
A	0.4	-0.5	0.5	7.00%
B	-0.8	1	0.5	not given

Calculate the expected return of Portfolio B.

$$7\% = 3\% + 0.4 * 5\% - 0.5 * 2\% + 0.5 * RP(F_3)$$

$$RP(F_3) = 6\%$$

$$E(r_B) = 3\% - 0.8 * 5\% + 1 * 2\% + 0.5 * 6\% = 4\%$$

#### **2011, Q2**

a. (0.5 point)

Describe the arbitrage argument for restoring equilibrium prices.

**When an arbitrage opportunity exists, a few investors will make large changes to their portfolio to take advantage of the mispricing. This will very quickly restore equilibrium.**

b. (0.5 point)

Describe the risk-return dominance argument for restoring equilibrium prices.

**When a security is mispriced many investors will make small changes to tilt the portfolio toward the mispriced security. The cumulative results of all these investors' actions will restore the equilibrium.**

c. (0.5 point)

Describe the main difference between the two arguments in parts a and b above.

**Part a: it is the large actions of just a few investors that restore equilibrium**

**Part b: based on the small actions of many investors.**

**Since the arbitrage argument relies on changes by fewer investors, we can conclude that it is stronger.**

**2015, Q6 (2.25 point)**

Given the following information:

Firm	Book Value (\$000,000)	Market Value (\$000,000)	CAPM Beta
A	625	500	1.2
B	350	500	1.2
C	1,250	1,000	1.2

a. (1.25 points)

Use the principles underlying the Fama & French 3-Factor Model to evaluate which firm would be expected to have the highest future rate of return.

Firm	Book / Market	Market Value (\$000,000)	CAPM Beta
A	1.25	500	1.2
B	0.70	500	1.2
C	1.25	1,000	1.2

**All 3 firms have the same beta, so we can ignore that. A & C should have the highest ratios of book to market, so they should have higher returns than B. A is better than C, as it has a lower market value (small cap firms should outperform large cap). So Firm A should have the highest return.**

b. (1 point)

Assume that the firm identified in part a. above does produce the highest rate of return over a future period.

Describe two behavioral explanations that could explain the superior performance.

**-Small firm tends to be neglected by large investors, because of regret avoidance. Investors would require higher return due to less information and its unconventional choice, so smaller firms had higher return.**

**-Forecast Error – investor place more weight in recent performance that glamour firm got bid up and overpriced → lead to low B/M ratio, on the other hand, other firms with high B/M ratio are undervalued, later on, investor correct their errors, so high B/M value return till go back up.**

**-Sample size neglect and representativeness → investors tend to infer too quickly on pricing pattern/trend and extrapolating price of low book to market ratio firms too high → higher price for these low book to market ratio firms end up generating lower returns than high B to M ratio firms → excessive price represents excessive investor optimism**

*Note from Examiners Report:*

*We gave partial credit for all Behavioral Finance explanations mentioned in the text – Forecast error, Regret Avoidance, Overconfidence, Conservatism, Affect, Sample Size, Framing/Mental Framing, Mental Accounting and Disposition. Most of these were described in chapter 12 of the text. Please note that we did NOT allow credit for Prospect Theory even though it is described in the chapter 12 list of Behavioral Finance explanations. This is because Prospect Theory is an alternative to the standard indifference curve rather than a “Behavioral” – characterized by investor failure to process information correctly or failure to act on information leading to suboptimal decisions.*

## **Past Exam Questions – BKM11**

### **2003, Q14 (1.5 points)**

Briefly describe what is meant by a passive investment strategy and why proponents of the Efficient Market Hypothesis believe that passive investment strategies are superior to active investment strategies.

### **2003, Q15 (1.5 points)**

Fundamental analysis has revealed a number of so-called anomalies, that is, evidence that seems inconsistent with the Efficient Market Hypothesis. Two such anomalies are the Small-Firm-in-January Effect and the Book-to-Market ratios.

Explain one of these two anomalies identified above and how it is inconsistent with the Efficient Market Hypothesis.

### **2004, Q10 (2.5 points)**

a. (1.5 points)

Briefly describe the three different versions of the efficient market hypothesis.

b. (0.25 point)

Briefly describe the concept of technical analysis.

c. (0.25 point)

Briefly explain what the efficient market hypothesis implies about the merit of technical analysis.

d. (0.5 point)

Provide two reasons portfolio management still has a role in an efficient market

### **2006, Q8 (2 points)**

The analyses of easily accessible stock market data seem to predict risk-adjusted returns that are difficult to reconcile with the Efficient Market Hypothesis. These analyses have led to what are known as efficient market anomalies.

Identify and briefly describe two of these anomalies.

### **2007, Q5 (2 points)**

a. (0.5 point)

Briefly describe the semi-strong version of the efficient market hypothesis.

b. (1.5 points)

Fundamental analysis has revealed patterns of stock returns that appear to contradict the semi-strong version of the efficient market hypothesis.

Identify and briefly describe three of these anomalies.

**2008, Q5** (2.25points); *Video Explanation Available*

- a. (1.5 points)  
Identify and briefly describe the three versions of the Efficient Market Hypothesis.
- b. (0.75 point)  
Briefly state whether price drift subsequent to an earnings announcement would tend to invalidate the hypothesis for each version identified in part a. above.

**2008, Q7** (1.5points)

You read a report about 1,000 randomly selected, currently active mutual funds that are independently managed. Three hundred funds that were in the top half of the distribution of returns during one year were also in the top half during the following year.

- a. (1 point)  
Explain how this observation might lead an investor to question the concept of market efficiency.
- b. (0.5 point)  
*No longer on syllabus*

**2010, Q1** (1.5 points)

- a. (1 point)  
In a perfectly efficient market, one might conclude that randomly choosing stocks is as effective as rationally choosing a stock portfolio.  
  
Describe two reasons for active portfolio management.
- b. (0.5 point)  
Describe the expected impact on market efficiency if every investor were to employ a passive investment strategy.



**2012, Q3 (3 points)**

An insurance company's current investment manager has been using fundamental analysis to select stocks for the company's equity portfolio since 2009. The company has recently hired a new CFO who is a firm believer in market efficiency.

a. (1.5 points)

Briefly describe three forms of the efficient-market hypothesis and assess the relevance of each one to the CFO's evaluation of the current investment manager's strategy.

b. (1.5 points)

The following table compares the rates of return on the company's portfolio to the returns on the S&P 500:

Year	Portfolio	S&P 500
2009	+26.7%	+26.5%
2010	+15.5%	+15.1%
2011	-8.5%	-9.0%

The investment manager argues that the portfolio performance is strong evidence against the efficient-market hypothesis. Present a rebuttal that includes three reasons why that position may not be valid.

**2014, Q6 (2 points)**

Given the following information:

- $\beta_{\text{Market}} = 1.0$
- $\beta_{\text{SMB}} = 0.8$
- $\beta_{\text{HML}} = 1.1$
- $\alpha_{\text{Stock A}} = 2\%$

Portfolio characteristic	Expected Return
Small capitalization stocks	9%
Medium capitalization stocks	7%
Big capitalization stocks	5%
High book-to-market ratio stocks	7%
Medium book-to-market ratio stocks	5%
Low book-to-market ratio stocks	4%
Market	6%

- The risk free rate is 1%
  - Assume no firm specific influences
- a. (0.5 point)  
Briefly describe two market anomalies exhibited by the returns of the portfolios that seem to contradict the semi-strong form of the efficient market hypothesis.
- b. (1 point)  
Calculate the expected return for Stock A using the Fama-French three-factor model.
- c. (0.5 point)  
Describe the impact of liquidity on the Fama-French three-factor model.

**2016, Q6** (2 points)

Given the following information:

Portfolio	Total Market Value	$\beta$	Average Annual Return
A	\$ 1 million	0.9	7.10%
B	\$ 58 million	1.5	7.80%
C	\$120 million	1.1	6.70%

- The risk-free rate is 2%.
  - The expected market return is 6%.
- a. (1 point)  
Calculate the alpha of each portfolio.
- b. (0.5 point)  
Describe a market anomaly associated with these three portfolios that appears to contradict the semi-strong form of the efficient market hypothesis.
- c. (0.5 point)  
Describe an extension of CAPM that addresses the cause of the anomaly identified in part b. above.

**2016, Q7** (1.75 points)

- a. (0.75 point)  
Briefly describe the three different versions of the efficient market hypothesis.
- b. (1 point)  
Describe two reasons why it is difficult to determine conclusively that the market is efficient.

## **BKM11 Solutions**

### **2003, Q14**

Briefly describe what is meant by a passive investment strategy and why proponents of the Efficient Market Hypothesis believe that passive investment strategies are superior to active investment strategies.

**A passive investment strategy seeks to replicate a market index by holding a well-diversified portfolio of stocks. There is no attempt to actively look for mispricing.**

**Proponents of the EMH believe that stock prices reflect all available information, and therefore active strategies should not perform better than passive. However, passive strategies are cheaper to implement.**

### **2003, Q15**

Fundamental analysis has revealed a number of so-called anomalies, that is, evidence that seems inconsistent with the Efficient Market Hypothesis. Two such anomalies are the Small-Firm-in-January Effect and the Book-to-Market ratios.

Explain one of these two anomalies identified above and how it is inconsistent with the Efficient Market Hypothesis.

**The Book-to-Market ratio anomaly is that firms with high ratios of book value to market value seem to outperform those stocks with lower ratios.**

**This is inconsistent with EMH because both book and market value are publicly available information. EMH believes that all such information is already reflected in the stock price.**

### **2004, Q10**

a. (1.5 points)

Briefly describe the three different versions of the efficient market hypothesis.

**Weak form: stock prices reflect all information that can be gathered by studying past trading data**

**Semi-strong form: stock prices reflect all publicly available fundamental information about a company**

**Strong form: all relevant information is reflected in a stock's price, including information that is only available to company insiders.**

b. (0.25 point)

Briefly describe the concept of technical analysis.

**Analysis of the history of stock prices in the attempt to identify trends that can be exploited to make profits**

c. (0.25 point)

Briefly explain what the efficient market hypothesis implies about the merit of technical analysis.

**EMH indicates that there is no value to technical analysis, as stock prices already incorporate any relevant information from the path of past stock prices.**

d. (0.5 point)

Provide two reasons portfolio management still has a role in an efficient market

**-it can help an investor achieve appropriate diversification**

**-it can help tailor the portfolio to reflect the time horizon of investment.**

#### **2006, Q8**

The analyses of easily accessible stock market data seem to predict risk-adjusted returns that are difficult to reconcile with the Efficient Market Hypothesis. These analyses have led to what are known as efficient market anomalies.

Identify and briefly describe two of these anomalies.

**P/E Ratio – firms with lower P/E ratios earn higher risk adjusted returns than firms with higher P/E ratios.**

**Small Firm Effect – smaller cap companies seem to enjoy higher return than large companies even when risk adjusted.**

#### **2007, Q5**

a. (0.5 point)

Briefly describe the semi-strong version of the efficient market hypothesis.

**Stock prices reflect all publicly available fundamental information about a company**

b. (1.5 points)

Fundamental analysis has revealed patterns of stock returns that appear to contradict the semi-strong version of the efficient market hypothesis.

Identify and briefly describe three of these anomalies.

**P/E Ratio – firms with lower P/E ratios earn higher risk adjusted returns than firms with higher P/E ratios.**

**Small Firm Effect – smaller cap companies seem to enjoy higher return than large companies even when risk adjusted.**

**Book to Market Ratio – firms with high ratios of book value to market value seem to outperform those stocks with lower ratios**

**2008, Q5**

a. (1.5 points)

Identify and briefly describe the three versions of the Efficient Market Hypothesis.

*See 2004, Q10a*

b. (0.75 point)

Briefly state whether price drift subsequent to an earnings announcement would tend to invalidate the hypothesis for each version identified in part a. above.

- **not violate weak form, because earnings is not trading data**
- **violates semistrong form and strong form**

**2008, Q7**

You read a report about 1,000 randomly selected, currently active mutual funds that are independently managed. Three hundred funds that were in the top half of the distribution of returns during one year were also in the top half during the following year.

a. (1 point)

Explain how this observation might lead an investor to question the concept of market efficiency.

**According to EMH, the market is efficient at pricing stocks and therefore there is little value in actively managing a fund. Of the 50% of funds that are in the top half of performers one year, 50% (250) should remain in the top the following year. The fact that 300 remain in the top half leads an investor to question the market efficiency concept since it is higher than 250.**

### 2010, Q1

a. (1 point)

In a perfectly efficient market, one might conclude that randomly choosing stocks is as effective as rationally choosing a stock portfolio.

Describe two reasons for active portfolio management.

*See 2004, Q10d*

b. (0.5 point)

Describe the expected impact on market efficiency if every investor were to employ a passive investment strategy.

**If every investor were to employ a passive investment strategy, no effects are put to exploit arbitrage opportunities. Therefore, even if there are inefficiencies in the markets, there are no pressures to correct them. Eventually, the market will become inefficient.**

### 2012, Q3

An insurance company's current investment manager has been using fundamental analysis to select stocks for the company's equity portfolio since 2009. The company has recently hired a new CFO who is a firm believer in market efficiency.

a. (1.5 points)

Briefly describe three forms of the efficient-market hypothesis and assess the relevance of each one to the CFO's evaluation of the current investment manager's strategy.

**1. Weak form – prices already reflect all information available on past market data. This tells the CFO that you can't earn excess returns by analyzing past stock prices (technical analysis). Not relevant here since fund manager uses fundamental analysis.**

*Notes from Examiners Report: it is incorrect to state that the market price reflects "all historical data". Instead, it reflects just the data from market trading*

**2. Semi strong form – prices already reflect all publicly available information. This tells the CFO that fundamental analysis won't earn excess returns.**

**3. Strong form – all information is already reflected, even non public information. It is unlikely that strong form efficiency holds, so fundamental analysis could work if you have non public information.**

b. (1.5 points)

The following table compares the rates of return on the company's portfolio to the returns on the S&P 500:

Year	Portfolio	S&P 500
2009	+26.7%	+26.5%
2010	+15.5%	+15.1%
2011	-8.5%	-9.0%

The investment manager argues that the portfolio performance is strong evidence against the efficient-market hypothesis. Present a rebuttal that includes three reasons why that position may not be valid.

**-Lucky Event Issue: Perhaps the investment manager has just gotten lucky. Assuming the chance of beating the market in a given year is 50% (if markets are efficient), there is a non zero probability of beating the market each year over 3 years.**

**-Because of the wide variance in both returns compared to the relatively small difference in returns, it's hard to say whether the difference is due to randomness or not.**

**-The company may have been taking on more risk to achieve higher returns.**

**2014, Q6 (2 points)**

Given the following information:

- $\beta_{\text{Market}} = 1.0$
- $\beta_{\text{SMB}} = 0.8$
- $\beta_{\text{HML}} = 1.1$
- $\alpha_{\text{Stock A}} = 2\%$

Portfolio characteristic	Expected Return
Small capitalization stocks	9%
Medium capitalization stocks	7%
Big capitalization stocks	5%
High book-to-market ratio stocks	7%
Medium book-to-market ratio stocks	5%
Low book-to-market ratio stocks	4%
Market	6%

- The risk free rate is 1%
- Assume no firm specific influences



a. (0.5 point)

Briefly describe two market anomalies exhibited by the returns of the portfolios that seem to contradict the semi-strong form of the efficient market hypothesis.

**-Small capitalization stocks ( $E(r) = 9\%$ ) outperform big capitalization stocks ( $E(r) = 5\%$ )**

**-High book to market ratio stocks ( $E(r) = 7\%$ ) outperform low book to market ratio stocks ( $E(r) = 5\%$ )**

b. (1 point)

Calculate the expected return for Stock A using the Fama-French three-factor model.

$$E(R_A) = 0.02 + 1 * (0.06 - 0.01) + 0.8 * (0.09 - 0.05) + 1.1 * (0.07 - 0.04) \\ = 0.135$$

$$E(r_A) = 0.145$$

*Alternate Solution:*

$$E(R_A) = 1 * (0.06 - 0.01) + 0.8 * (0.09 - 0.05) + 1.1 * (0.07 - 0.04) \\ = 0.115$$

$$E(r_A) = 0.125$$

c. (0.5 point)

Describe the impact of liquidity on the Fama-French three-factor model.

**-Liquidity is not reflected by the Fama-French three-factor model. Therefore stocks with liquidity risk will have a higher  $\alpha$  value than a similar stock without liquidity risk.**

**-Investors prefer liquidity; they will require a premium for illiquidity in the form of  $\alpha$ .**

**2016, Q6 (2 points)**

Given the following information:

Portfolio	Total Market Value	$\beta$	Average Annual Return
A	\$ 1 million	0.9	7.10%
B	\$ 58 million	1.5	7.80%
C	\$120 million	1.1	6.70%

- The risk-free rate is 2%.
- The expected market return is 6%.

a. (1 point)

Calculate the alpha of each portfolio.

$$\begin{aligned} 0.071 &= \alpha_A + 0.02 + 0.9 * (0.06 - 0.02) \\ \alpha_A &= 0.015 \end{aligned}$$

$$\begin{aligned} 0.078 &= \alpha_B + 0.02 + 1.5 * 0.04 \\ \alpha_B &= -0.002 \end{aligned}$$

$$\begin{aligned} 0.067 &= \alpha_C + 0.02 + 1.1 * 0.04 \\ \alpha_C &= 0.003 \end{aligned}$$

b. (0.5 point)

Describe a market anomaly associated with these three portfolios that appears to contradict the semi-strong form of the efficient market hypothesis.

**-Small firm effect:** the lowest capitalization firm has earned an excess return ( $\alpha$ ) of 1.5% which is higher than all other firms/portfolios. Small firms tend to be researched less/have lower investor confidence. Being “lesser known” may cause these firms to be less liquid, resulting in an excess return that is actually demanded due to illiquid nature

**-Small/Neglected Firm:** smaller/neglected firms have less information published about them so investors require higher abnormal returns to bear that additional risk

c. (0.5 point)

Describe an extension of CAPM that addresses the cause of the anomaly identified in part b. above.

**-Liquidity Adjusted CAPM:** CAPM adjusted to account for illiquidity of firm due to lack of analysis/trading. Thus, liquidity adjusted CAPM will produce a higher expected return, reducing the  $\alpha$ .

**-Fama French 3 factor model** includes a small minus big factor in the equation to adjust for the excess returns that small firms experience over larger firms

**2016, Q7** (1.75 points)

a. (0.75 point)

Briefly describe the three different versions of the efficient market hypothesis.

**Weak Form:** stock prices reflect all information regarding past trading/market data

**Semi-Strong Form:** stock prices reflect all publically available fundamental information about the firm's prospects

**Strong Form:** stock prices reflect all available information about a firm, including insider information

b. (1 point)

Describe two reasons why it is difficult to determine conclusively that the market is efficient.

**-Selection Bias:** investors don't share strategies that can beat the market. If they became publically known, they would no longer work since all investors would use it, so we only know about failed strategies.

**-Lucky Event Issue:** Given the large number of investors, it is likely to see some of them consistently outperform the market. However, it is not easy to determine if this is due to luck or skill.

**-Magnitude Issue:** Due to the large volatility of market returns, it is difficult to identify excessive returns since they lie within the standard deviation of market returns.

**-Unobservable Market:** The true market portfolio is unobservable due to non-traded assets and other factors. Therefore, even if we test a market proxy, we can only conclusively determine if the proxy is efficient; it is not truly indicative of the market.

**-Actual vs. Expected Returns:** The efficient market hypothesis is built on expected returns; however we can only observe actual returns. It is therefore difficult to directly test the EMH.

**-Behavioral Bias:** Some investors are not always rational and these irrational behaviors introduce inefficiencies into the market. Therefore, we cannot determine it is completely efficient.

**-Anomalies:** There are a number of known anomalies (such as small firm in January effect) that have been identified. The existence of these anomalies makes it difficult to determine that the market is completely efficient.

**-Insider Trading:** It is difficult to know whether some investors are using insider information. If they are and other investors are not, the prices wouldn't fully reflect this information and hence the market can't be strong form efficient.

## **Past Exam Questions – BKM12**

**2007, Q7** (2.25 points)

a. (1.5 points)

Briefly describe three types of information processing biases that can lead investors to make forecast errors in the financial markets.

b. (0.75 point)

Identify a market anomaly associated with each.

**2008, Q6** (1.5 points)

a. (0.5 point)

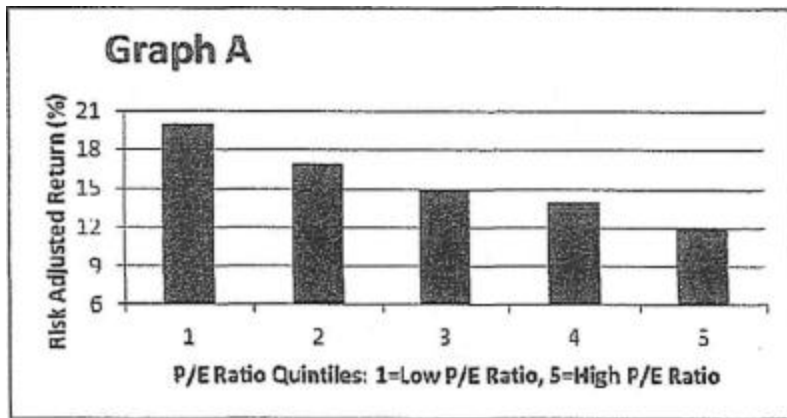
Define the Law of One Price.

b. (1 point)

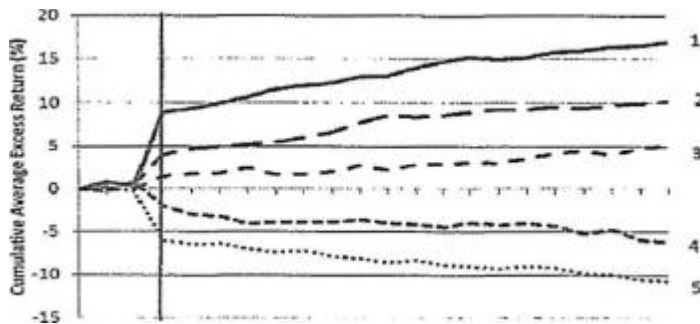
Describe two examples that violate the Law of One Price.

**2013, Q5 (2.5 points)**

Consider the following graphs in the context of the semi-strong form of the efficient market hypothesis:



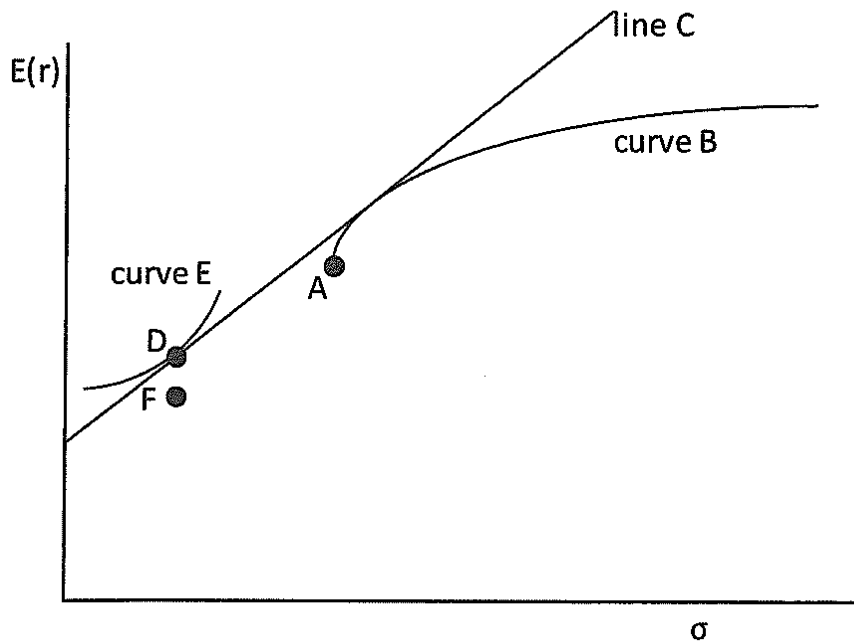
Graph B (Earnings Surprise Quintiles; 1 is Positive surprise. 5 is Negative surprise)



- (0.5 point)  
Describe the market anomaly associated with Graph A.
- (0.5 point)  
Describe the market anomaly associated with Graph B.
- (1 point)  
For each anomaly described in part a. and part b. above, describe an information-processing error that would cause the anomaly.
- (0.5 point)  
For each graph describe, either in words or graphically, what it would look like without the anomaly identified in part a. and in part b. above.

**2016, Q2 (1.75 points)**

The following graph represents the Markowitz portfolio selection model.



A rational investor decides to invest in portfolio D.

a. (0.5 point)

Identify what each of the following represents:

- point A
- curve E

b. (0.25 point)

Briefly explain where the optimal risky portfolio would be on the graph above.

c. (0.5 point)

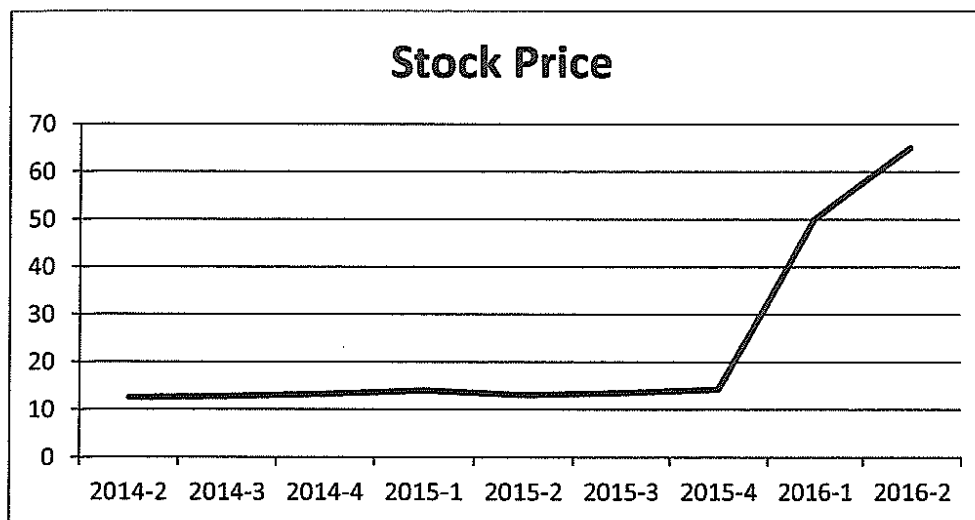
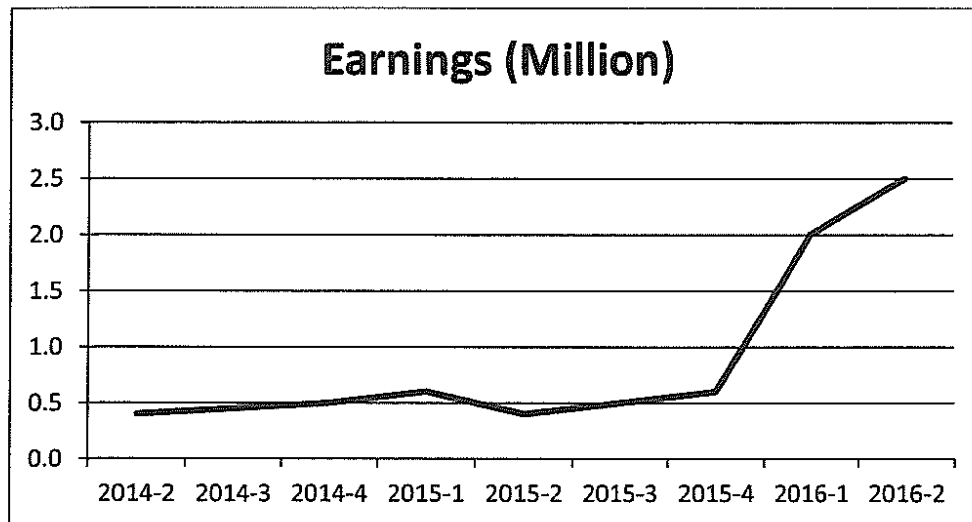
Explain why another rational investor may choose a portfolio other than portfolio D, with reference to the graph above.

d. (0.5 point)

Identify and describe one reason from behavioral finance that may lead an investor to choose portfolio F instead of portfolio D.

**2016, Q8 (1.5 points)**

Given the following quarterly historical information for a firm:



- The market forecasted earnings for 2016-3 are \$3.5 million.
  - The market forecasted stock price for 2016-3 is \$90.
- a. (1 point)
- Identify and briefly describe two potential information processing errors in the scenario above.
- b. (0.5 point)
- An individual investor believes the forecast contains information processing errors as described in part a. above. Briefly explain how the investor expects the stock price to change and identify an investment strategy this investor should pursue.

## **BKM12 Solutions**

### **2007, Q7**

a. (1.5 points)

Briefly describe three types of information processing biases that can lead investors to make forecast errors in the financial markets.

b. (0.75 point)

Identify a market anomaly associated with each.

**Overconfidence:** people tend to overestimate the precision of their beliefs and they tend to overestimate their abilities. An example is the prevalence of active versus passive investment management, since high trading activity is highly predictive of poor investment performance.

**Conservatism:** investors are too slow to update their beliefs in response to recent evidence. An example is post-earnings-announcement drift which gives rise to momentum in stock market returns.

**Sample Size Neglect and Representativeness:** people tend to infer a pattern too quickly based on a small sample. An example is evidence that stocks with the best recent performance show reversals in the few days surrounding earnings announcements suggesting that the correction occurs just as investors learn that their initial beliefs were too extreme.

### **2008, Q6**

a. (0.5 point)

Define the Law of One Price.

**Effectively identical assets should have the same price.**

b. (1 point)

Describe two examples that violate the Law of One Price.

**1) Siamese twins:** Royal Dutch & Shell merged, agreeing to split profits on a 60/40 basis: stock prices of Royal Dutch should have therefore sold for 150% of Shell, but they did deviate from this ratio for periods.

**2) Equity carve-outs:** 3Com announced its decision to spin off Palm, and gave each shareholder 1.5 Palm shares. In theory, 3com stock price should have been higher than Palm's, but it was not.



### 2013, Q5

Consider the following graphs in the context of the semi-strong form of the efficient market hypothesis:

a. (0.5 point)

Describe the market anomaly associated with Graph A.

**P/E effect: lower P/E assets earn higher risk adjusted returns on average**

b. (0.5 point)

Describe the market anomaly associated with Graph B.

**Post Earnings Announcement Drift: prices take time to react to new information given in the earnings announcements.**

c. (1 point)

For each anomaly described in part a. and part b. above, describe an information-processing error that would cause the anomaly.

**a: Forecasting error – give too much weight to the recent data, which will result in deriving an overstated price. This will lead to lower risk adjusted returns**

**b: Conservatism – too slow to update the portfolio to reflect the updated information. It will therefore take time for the stock to reach the equilibrium level**

d. (0.5 point)

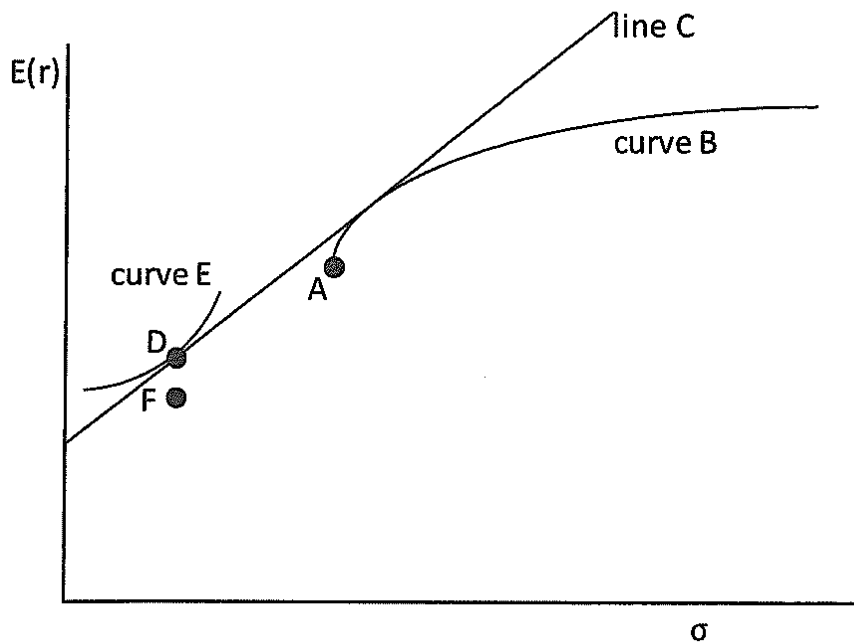
For each graph describe, either in words or graphically, what it would look like without the anomaly identified in part a. and in part b. above.

**a: Each quintile will have the same risk adjusted return**

**b: The excess return would IMMEDIATELY reflect the new earnings data. Rather than being a slope up, there will be a vertical increase, followed by no change (flat horizontal curve).**

### 2016, Q2 (1.75 points)

The following graph represents the Markowitz portfolio selection model.



A rational investor decides to invest in portfolio D.

a. (0.5 point)

Identify what each of the following represents:

point A

- Minimum Variance Portfolio
- A point on the efficient frontier

curve E

- Indifference Curve
- Graphs the risk + return relationships an investor is indifferent about given their risk aversion level
- Curve of equal utility for investor

b. (0.25 point)

Briefly explain where the optimal risky portfolio would be on the graph above.

- Where curve c tangent with curve b is the optimal risky portfolio
- The point at the intersection of line c and curve b

c. (0.5 point)

Explain why another rational investor may choose a portfolio other than portfolio D, with reference to the graph above.

- Different level of risk aversion will result in varying allocation of risk free asset and optimal risky portfolio along line c.
- Different investors will have different indifference curves with different tangency points.

*Note from Examiners Report:*

*A common omission was not explaining how being more or less risk averse related to the nature of the optimal complete portfolio. Some candidates gave explanations from irrelevant sections of the syllabus, such as behavioral finance. Some candidates attempted to explain using reasons relating to maximizing expected return or minimizing variance.*

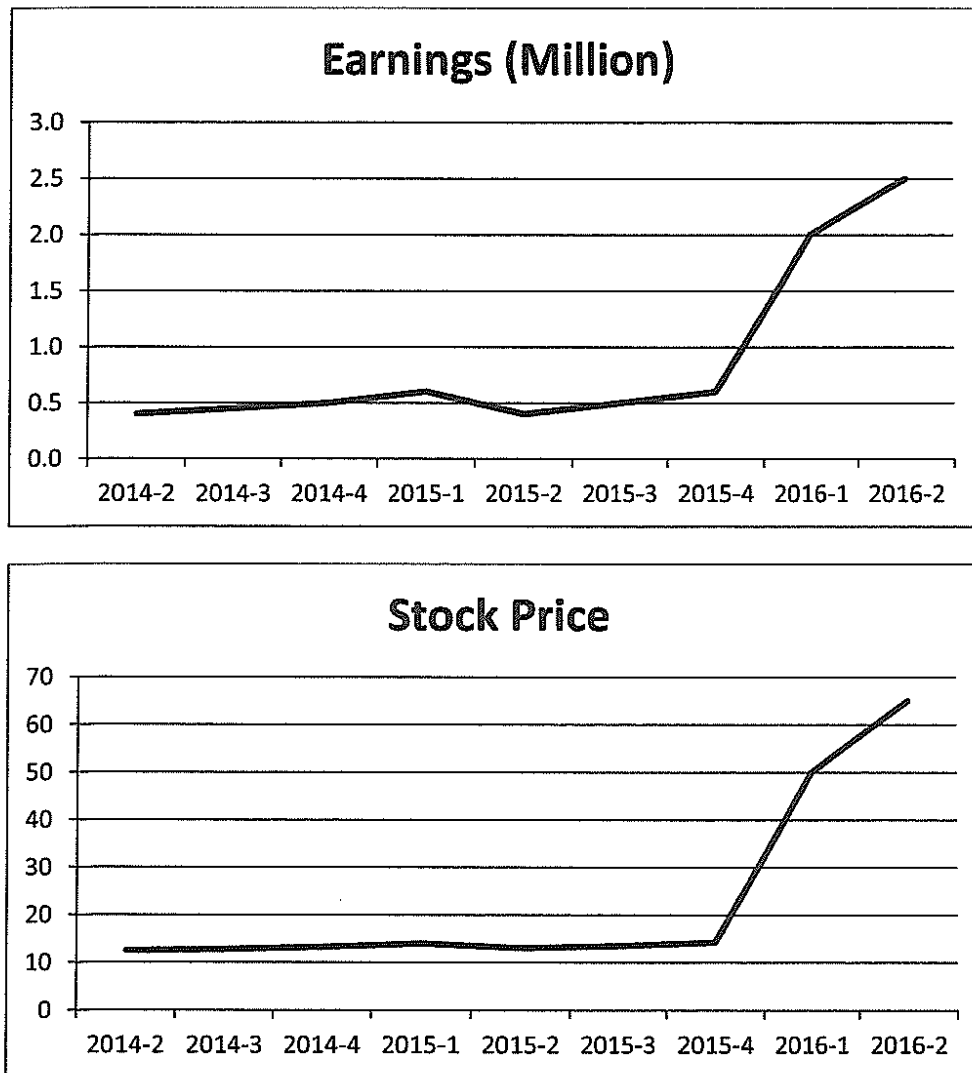
d. (0.5 point)

Identify and describe one reason from behavioral finance that may lead an investor to choose portfolio F instead of portfolio D.

- Forecasting error – F may have had recent good experience and if the investor gives too much weight to it, they may believe high return will continue in the future.
- Conservatism – The Investor could be slow to react to new info that would push their portfolio to a new CAL where portfolio D lies.
- Overconfidence – An investor may exhibit overconfidence in her ability to beat the market by active investing, resulting a suboptimal allocation at F, rather than adopting a passive approach and selecting D.
- Sample Size Neglect and Representativeness – An investor might identify a pattern too quickly extrapolate trends too far out into the future.
- Framing – An investor may act differently where a decision is framed in different ways. They could be risk seeking when it comes to losses but risk averse when it comes to gains.
- Mental Accounting – Investors tend to make decisions in isolated buckets rather than for the portfolio as a whole, this may lead to suboptimal securities within the portfolio.
- Regret Avoidance – If portfolio F has more conventional assets while D is less conventional, then investors with regret avoidance may feel worse if he loses money in portfolio D than portfolio F.
- Prospect Theory – alters the traditional indifference curves. For example, an investor may be more concerned with changes in wealth than levels of wealth.
- Affect – F has negative alpha suggesting below market returns but if F contains investments that serve a social purpose, investors may be willing to accept lower returns to “feel good”.
- Loss Avoidance – An investor is hesitant or unwilling to realize losses on a bad investment, so he holds on to an inefficient portfolio.
- Information Processing Error- like forecasting errors may misestimate probabilities and/or parameters which may lead to a different capital allocation line.

**2016, Q8 (1.5 points)**

Given the following quarterly historical information for a firm:



- The market forecasted earnings for 2016-3 are \$3.5 million.
- The market forecasted stock price for 2016-3 is \$90.

a. (1 point)

Identify and briefly describe two potential information processing errors in the scenario above.

**-Forecasting error: the investors may be forecasting the recently good performance too far into the future**

**-Overconfidence: people sometimes place too much confidence in the precision of their estimates. The steep increasing continuing is not a very conservative**

assumption, so predictors must be pretty confident in their skills to predict continued drastic increase.

**-Sample Size Neglect and representativeness:** The investors may be identifying a growth pattern from limited data i.e. growth from 2015-4 to 2016-2. Such a short period is not adequate to credibly identify a pattern

*Note from Examiners Report:*

*“Conservatism” is not correct. While it is a possible information processing error as described in the exam material, it is not applicable to the specific stock forecast presented in the question. The forecast of a further jump in both earnings and price is indicative of an overreaction by analysts to the recent dramatic growth in earnings and stock price for this firm, whereas conservatism would lead investors to react more slowly to new information. A number of candidates suggested that the fact that the price chart didn't perfectly follow the earnings chart in some earlier quarters prior to 2016 was indicative of the presence of conservatism, but this was more representative of some minor random fluctuations rather than a delayed reaction to a trend.*

b. (0.5 point)

An individual investor believes the forecast contains information processing errors as described in part a. above. Briefly explain how the investor expects the stock price to change and identify an investment strategy this investor should pursue.

**-Stock should come down so short at current prices.**

**-The investor expects the stock price to increase but not by as much as the forecast. Should buy the stock though since he still believes it will increase.**

**-Investor expects stock price to decline in the future as markets realize the irrational excitement built into price. An investment strategy could be to short the security. This is speculative so to reduce the risk could also long a portfolio w/same  $\beta$ .**