# Package 'AVGAS'

May 2, 2023

Title A Variable Selection using Genetic Algorithms

**Version** 0.0.0.9000

ABC	Evaluating ABC for each fitted model	
Index		17
н юр	ABC	1 4 6 7 9 10 13
Roxyge	nNote 7.2.3	
	<pre>iner Leiyue Li &lt;11i289.git@gmail.com&gt; n list(markdown = TRUE)</pre>	
C	Leiyue Li [aut, cre], henglong Ye[aut]	
-	ge en-US	
	s utils, Matrix, pracma, stats, VariableScreening, dplyr, ggplot2	
	GPL (>= 3) ng UTF-8	
fo ac	orm fast interaction selection in high-dimensional linear regression models with two-way interaction effects under strong, weak, or no heredity condition.	er-

ABC

#### **Description**

This function evaluates ABC score for fitted model, one model at a time. For a model I, the ABC is defined as

 $ABC(I) = \sum_{i=1}^{n} \left( Y_i - \hat{Y}_i^I \right)^2 + 2r_I \sigma^2 + \lambda \sigma^2 C_I.$ 

When comparing ABC of fitted models to the same data set, the smaller the ABC, the better fit.

## Usage

```
ABC(

X,

y,

heredity = "Strong",

nmain.p,

sigma = NULL,

extract = "No",

varind = NULL,

interaction.ind = NULL,

pi1 = 0.32,

pi2 = 0.32,

pi3 = 0.32,

lambda = 10
```

#### **Arguments**

Input data. An optional data frame, or numeric matrix of dimension n by nmain.p. Note that the two-way interaction effects should not be included in X because this function automatically generates the corresponding two-way interaction effects if needed

II lieeded

y Response variable. A n-dimensional vector, where n is the number of observa-

tions in X.

heredity Whether to enforce Strong, Weak, or No heredity. Default is "Strong".

nmain.p A numeric value that represents the total number of main effects in X.

sigma The standard deviation of the noise term. In practice, sigma is usually unknown.

In such case, this function automatically estimate sigma using root mean square error (RMSE). Default is NULL. Otherwise, users need to enter a numeric value.

extract A either "Yes" or "No" logical vector that represents whether or not to extract

specific columns from X. Default is "No".

varind Only used when extract = "Yes". A numeric vector of class c() that speci-

fies the indices of variables to be extracted from X. If varind contains indices of two-way interaction effects, then this function automatically generates corre-

sponding two-way interaction effects from X.

interaction.ind

Only used when extract = "Yes". A two-column numeric matrix containing all possible two-way interaction effects. It must be generated outside of this function using t(utils::combn()). See Example section for details.

A numeric value between 0 and 1, defined by users. Default is 0.32. For guidance on selecting an appropriate value, please refer to the Details section.

pi1

ABC 3

pi2	A numeric value between 0 and 1, defined by users. Default is 0.32. For guidance on selecting an appropriate value, please refer to the Details section.
pi3	A numeric value between 0 and 1, defined by users. Default is 0.32. For guidance on selecting an appropriate value, please refer to the Details section.
lambda	A numeric value defined by users. Default is 10. For guidance on selecting an appropriate value, please refer to the Details section.

## **Details**

- For inputs pi1, pi2, and pi3, the number needs to satisfy the condition:  $\pi_1 + \pi_2 + \pi_3 = 1 \pi_0$  where  $\pi_0$  is a numeric value between 0 and 1, the smaller the better.
- For input lambda, the number needs to satisfy the condition:  $\lambda \geq 5.1/log(2)$ .

#### Value

A numeric value is returned. It represents the ABC score of the fitted model.

#### References

Ye, C. and Yang, Y., 2019. High-dimensional adaptive minimax sparse estimation with interactions.

#### See Also

```
Extract, initial.
```

## **Examples**

```
# sigma is unknown
set.seed(0)
nmain.p <- 4
interaction.ind <- t(combn(4,2))</pre>
X \leftarrow matrix(rnorm(50*4,1,0.1), 50, 4)
epl <- rnorm(50,0,0.01)
y<-1+X[,1]+X[,2]+X[,3]+X[,4]+epl
ABC(X, y, nmain.p = 4)
ABC(X, y, nmain.p = 4, extract = "Yes",
    varind = c(1,2,5), interaction.ind = interaction.ind)
# users want to enter a suggested value for sigma
ABC(X, y, nmain.p = 4, sigma = 0.01)
ABC(X, y, nmain.p = 4, sigma = 0.01, extract = "Yes",
    varind = c(1,2,5), interaction.ind = interaction.ind)
# model with only one predictor
## Not run:
ABC(X, y, nmain.p = 4, extract = "Yes",
  varind = 1, interaction.ind = interaction.ind) # warning message
## End(Not run)
```

4 AVGAS

AVGAS

Title

Description

# Title

# Usage

```
AVGAS(
  Χ,
  у,
  heredity = "Strong",
  nmain.p,
  r1,
  r2,
  sigma = NULL,
  interaction.ind = NULL,
  lambda = 10,
  q = 40,
  allout = "No",
  interonly = "No",
  pi1 = 0.32,
  pi2 = 0.32,
  pi3 = 0.32,
  aprob = 0.9,
  dprob = 0.9,
  aprobm = 0.1,
  aprobi = 0.9,
  dprobm = 0.9,
  dprobi = 0.1,
  take = 3
)
```

## **Arguments**

r2

X	Note that the two-way interaction effects should not be included in X because this function automatically generates the corresponding two-way interaction effects if needed.
У	Response variable. A n-dimensional vector, where n is the number of observations in $\boldsymbol{X}$ .
heredity	Whether to enforce Strong, Weak, or No heredity. Default is "Strong".
nmain.p	A numeric value that represents the total number of main effects in X.
r1	A numeric value indicating the maximum number of main effects. This number can be different from the $r1$ defined in detect.

number can be different from the r1 defined in detect.

A numeric value indicating the maximum number of interaction effects. This

AVGAS 5

sigma	The standard deviation of the noise term. In practice, sigma is usually unknown. In such case, this function automatically estimate sigma using root mean square error (RMSE). Default is NULL. Otherwise, users need to enter a numeric value.
interaction.in	d
	A two-column numeric matrix containing all possible two-way interaction effects. It must be generated outside of this function using t(utils::combn()). See Example section for details.
lambda	A numeric value defined by users. Default is 10. For guidance on selecting an appropriate value, please refer to the Details section.
q	A numeric value indicating the number of models in each generation (e.g., the population size). Default is 40.
allout	Whether to print all outputs from this function. A "Yes" or "No" logical vector. Default is "No". See Value section for details.
interonly	Whether or not to consider fitted models with only two-way interaction effects. A "Yes" or "No" logical vector. Default is "No".
pi1	A numeric value between 0 and 1, defined by users. Default is 0.32. For guidance on selecting an appropriate value, please refer to ABC.
pi2	A numeric value between 0 and 1, defined by users. Default is 0.32. For guidance on selecting an appropriate value, please refer to ABC.
pi3	A numeric value between 0 and 1, defined by users. Default is 0.32. For guidance on selecting an appropriate value, please refer to ABC.
aprob	A numeric value between 0 and 1, defined by users. The addition probability during mutation. Default is 0.9.
dprob	A numeric value between 0 and 1, defined by users. The deletion probability during mutation. Default is 0.9.
aprobm	A numeric value between 0 and 1, defined by users. The main effect addition probability during addition. Default is 0.1.
aprobi	A numeric value between 0 and 1, defined by users. The interaction effect addition probability during addition. Default is 0.9.
dprobm	A numeric value between 0 and 1, defined by users. The main effect deletion probability during deletion. Default is 0.9.
dprobi	A numeric value between 0 and 1, defined by users. The interaction effect deletion probability during deletion. Default is 0.1.
take	Number of top candidate models to display. Default is 3.

## Value

A list of output. If allout = "Yes", then the components are:

- final\_modelThe final selected model.
- cleaned\_candidate\_modelAll candidate models where each row corresponding to a fitted model; the first 1 to r1 + r2 columns representing the predictor indices in that model, and the last column is a numeric value representing the ABC score of that fitted model. Duplicated models are not allowed.
- InterRankRank of all candidate interaction effects. A two-column numeric matrix. The first column contains indices of ranked two-way interaction effects, and the second column contains its corresponding ABC score.

6 cross

#### See Also

initial, cross, mut, ABC, Genone, and Extract.

### **Examples**

```
# allout = "No"

set.seed(0)
nmain.p <- 4
interaction.ind <- t(combn(4,2))
X <- matrix(rnorm(50*4,1,0.1), 50, 4)
epl <- rnorm(50,0,0.01)
y <- 1+X[,1]+X[,2]+X[,1]*X[,2]+epl

a1 <- AVGAS(X, y, nmain.p=4, r1=3, r2=3,
    interaction.ind = interaction.ind, q=5)

# allout = "Yes"
a2 <- AVGAS(X, y, nmain.p=4, r1=3, r2=3,
    interaction.ind = interaction.ind, q=5, allout = "Yes")</pre>
```

cross

Performing crossover

## **Description**

This function performs crossover which only stores all fitted models without making any comparison. The selected indices in each fitted model will be automatically re-ordered so that main effects comes first, followed by two-way interaction effects, and zero reservation spaces.

## Usage

```
cross(parents, heredity = "Strong", nmain.p, r1, r2, interaction.ind = NULL)
```

# **Arguments**

A numeric matrix of dimension q by r1+r2, obtained from initial or previous generation where each row corresponding a fitted model and each column representing the predictor index in the fitted model.

heredity Whether to enforce Strong, Weak, or No heredity. Default is "Strong".

nmain.p A numeric value that represents the total number of main effects in X.

r1 A numeric value indicating the maximum number of main effects.

r2 A numeric value indicating the maximum number of interaction effects.

interaction.ind

A two-column numeric matrix containing all possible two-way interaction effects. It must be generated outside of this function using t(utils::combn()). See Example section for details.

detect 7

#### Value

A numeric matrix single.child.bit is returned. Each row representing a fitted model, and each column corresponding to the predictor index in the fitted model. Duplicated models are allowed.

## See Also

```
initial.
```

## **Examples**

```
# Under Strong heredity
set.seed(0)
nmain.p <- 4
interaction.ind <- t(combn(4,2))
X <- matrix(rnorm(50*4,1,0.1), 50, 4)
epl <- rnorm(50,0,0.01)
y<- 1+X[,1]+X[,2]+X[,1]*X[,2]+epl
p1 <- initial(X, y, nmain.p = 4, r1 = 3, r2 = 3,
        interaction.ind = interaction.ind, q = 5)
c1 <- cross(p1, nmain.p=4, r1 = 3, r2 = 3,
        interaction.ind = interaction.ind)</pre>
```

detect

Suggesting values for r2

# **Description**

This function suggests the values for r2.

# Usage

```
detect(
    X,
    y,
    heredity = "Strong",
    nmain.p,
    sigma = NULL,
    r1,
    r2,
    interaction.ind = NULL,
    pi1 = 0.32,
    pi2 = 0.32,
    pi3 = 0.32,
    lambda = 10,
    q = 40
)
```

8 detect

#### **Arguments**

Χ Input data. An optional data frame, or numeric matrix of dimension n by nmain.p. Note that the two-way interaction effects should not be included in X because this function automatically generates the corresponding two-way interaction effects if needed. Response variable. A n-dimensional vector, where n is the number of observa-У tions in X. Whether to enforce Strong, Weak, or No heredity. Default is "Strong". heredity nmain.p A numeric value that represents the total number of main effects in X. The standard deviation of the noise term. In practice, sigma is usually unknown. sigma In such case, this function automatically estimate sigma using root mean square error (RMSE). Default is NULL. Otherwise, users need to enter a numeric value. r1 A numeric value indicating the maximum number of main effects. r2 A numeric value indicating the maximum number of interaction effects. interaction.ind A two-column numeric matrix containing all possible two-way interaction effects. It must be generated outside of this function using t(utils::combn()). See Example section for details. A numeric value between 0 and 1, defined by users. Default is 0.32. For guidpi1 ance on selecting an appropriate value, please refer to ABC. A numeric value between 0 and 1, defined by users. Default is 0.32. For guidpi2 ance on selecting an appropriate value, please refer to ABC. pi3 A numeric value between 0 and 1, defined by users. Default is 0.32. For guidance on selecting an appropriate value, please refer to ABC. lambda A numeric value defined by users. Default is 10. For guidance on selecting an appropriate value, please refer to ABC. A numeric value indicating the number of models in each generation (e.g., the q population size). Default is 40.

#### Value

A list of output. The components are:

InterRank Rank of all candidate interaction effects. A two-column numeric matrix. The

first column contains indices of ranked two-way interaction effects, and the sec-

ond column contains its corresponding ABC score.

mainind.sel Selected main effects. A r1-dimensional vector.

mainpool Ranked main effects in X.

plot Plot of potential interaction effects and their corresponding ABC scores.

#### See Also

initial.

Extract 9

#### **Examples**

```
# under Strong heredity
set.seed(0)
nmain.p <- 4
interaction.ind <- t(combn(4,2))
X <- matrix(rnorm(50*4,1,0.1), 50, 4)
epl <- rnorm(50,0,0.01)
y<- 1+X[,1]+X[,2]+X[,1]*X[,2]+epl
d1 <- detect(X, y, nmain.p = 4, r1 = 3, r2 = 3,
        interaction.ind = interaction.ind, q = 5)
# under No heredity
d2 <- detect(X, y, heredity = "No", nmain.p = 4, r1 = 3, r2 = 3,
        interaction.ind = interaction.ind, q = 5)</pre>
```

Extract

Extracting specific columns from a data

#### **Description**

This function extracts specific columns from X based on varind. It provides an efficient procedure for conducting ABC evaluation, especially when working with high-dimensional data.

# Usage

```
Extract(X, varind, interaction.ind = NULL)
```

## **Arguments**

Χ

Input data. An optional data frame, or numeric matrix of dimension n by nmain.p. Note that the two-way interaction effects should not be included in X because this function automatically generates the corresponding two-way interaction effects if needed.

varind

A numeric vector of class c() that specifies the indices of variables to be extracted from X. Duplicated values are not allowed. See Example section for details.

interaction.ind

A two-column numeric matrix containing all possible two-way interaction effects. It must be generated outside of this function using t(utils::combn()). See Example section for details.

## **Details**

Please be aware that this function automatically renames column names into a designated format (e.g., X.1, X.2 for main effects, and X.1X.2 for interaction effect, etc), regardless of the original column names in X.

Under no heredity condition, this function can be applied in the context of interaction only linear regression models. See Example section for details.

Genone Genone

#### Value

A numeric matrix is returned.

#### See Also

```
ABC, initial.
```

### **Examples**

```
# Extract main effect X1 and X2 from X1,...X4
set.seed(0)
X1 <- matrix(rnorm(20), ncol = 4)
y1 <- X1[, 2] + rnorm(5)
interaction.ind <- t(combn(4,2))

# Extract main effect X1 and interaction effect X1X2 from X1,..X4
Extract(X1, varind = c(1,5), interaction.ind)

# Extract interaction effect X1X2 from X1,...X4
Extract(X1, varind = 5, interaction.ind)

# Extract using duplicated values in varind.
## Not run:
Extract(X1, varind = c(1,1), interaction.ind) # this will not run
## End(Not run)</pre>
```

Genone

Gathering useful information for first generation

# **Description**

This function automatically ranks all candidate interaction effects under Strong, Weak, or No heredity condition, compare and obtain first generation candidate models. The selected models will be re-ordered so that main effects come first, followed by interaction effects. Only two-way interaction effects will be considered.

## Usage

```
Genone(
   X,
   y,
   heredity = "Strong",
   nmain.p,
   r1,
   r2,
   sigma = NULL,
   interaction.ind = NULL,
   lambda = 10,
   q = 40,
   allout = "No",
   interonly = "No",
```

Genone 11

```
pi1 = 0.32,
pi2 = 0.32,
pi3 = 0.32,
aprob = 0.9,
dprob = 0.9,
aprobm = 0.1,
aprobi = 0.9,
dprobm = 0.9,
dprobi = 0.1)
```

#### **Arguments**

X Input data. An optional data frame, or numeric matrix of dimension n by nmain.p. Note that the two-way interaction effects should not be included in X because this function automatically generates the corresponding two-way interaction effects if needed.

y Response variable. A n-dimensional vector, where n is the number of observations in X.

heredity Whether to enforce Strong, Weak, or No heredity. Default is "Strong".

nmain.p A numeric value that represents the total number of main effects in X.

A numeric value indicating the maximum number of main effects.

A numeric value indicating the maximum number of interaction effects.

The standard deviation of the noise term. In practice, sigma is usually unknown. In such case, this function automatically estimate sigma using root mean square error (RMSE). Default is NULL. Otherwise, users need to enter a numeric value.

interaction.ind

aprob

A two-column numeric matrix containing all possible two-way interaction effects. It must be generated outside of this function using t(utils::combn()). See Example section for details.

lambda A numeric value defined by users. Default is 10. For guidance on selecting an appropriate value, please refer to the Details section.

q A numeric value indicating the number of models in each generation (e.g., the population size). Default is 40.

allout Whether to print all outputs from this function. A "Yes" or "No" logical vector.

Default is "No". See Value section for details.

interonly Whether or not to consider fitted models with only two-way interaction effects. A "Yes" or "No" logical vector. Default is "No".

A numeric value between 0 and 1, defined by users. Default is 0.32. For guidance on selecting an appropriate value, please refer to ABC.

pi2 A numeric value between 0 and 1, defined by users. Default is 0.32. For guid-

ance on selecting an appropriate value, please refer to ABC.

A numeric value between 0 and 1, defined by users. Default is 0.32. For guidance on selecting an appropriate value, please refer to ABC.

A numeric value between 0 and 1, defined by users. The addition probability

during mutation. Default is 0.9.

dprob A numeric value between 0 and 1, defined by users. The deletion probability

during mutation. Default is 0.9.

12 Genone

aprobm	A numeric value between 0 and 1, defined by users. The main effect addition probability during addition. Default is 0.1.
aprobi	A numeric value between 0 and 1, defined by users. The interaction effect addition probability during addition. Default is 0.9.
dprobm	A numeric value between 0 and 1, defined by users. The main effect deletion probability during deletion. Default is 0.9.
dprobi	A numeric value between 0 and 1, defined by users. The interaction effect deletion probability during deletion. Default is 0.1.

#### Value

A list of output. If allout = "Yes", then the components are:

- newparents New parents models used for t+1-th generation. A numeric matrix of dimension q by r1+r2 where each row represents a fitted model. Duplicated models are allowed.
- parents\_models A numeric matrix containing all fitted models from initial, cross, and mut where each row corresponding to a fitted model and each column representing the predictor index in that model. Duplicated models are allowed.
- parents\_models\_cleaned A numeric matrix containing fitted models from initial, cross, and mut with ABC scores. Each row corresponding to a fitted model; the first 1 to r1 + r2 columns representing the predictor indices in that model, and the last column is a numeric value representing the ABC score of that fitted model. Duplicated models are not allowed.
- InterRank Rank of all candidate interaction effects. A two-column numeric matrix. The first column contains indices of ranked two-way interaction effects, and the second column contains its corresponding ABC score.

Otherwise, only newparents and InterRank will be returned.

#### See Also

```
initial, cross, mut, ABC, and Extract.
```

# **Examples**

```
# allout = "No"
set.seed(0)
nmain.p <- 4
interaction.ind <- t(combn(4,2))
X <- matrix(rnorm(50*4,1,0.1), 50, 4)
epl <- rnorm(50,0,0.01)
y <- 1+X[,1]+X[,2]+X[,1]*X[,2]+epl
g1 <- Genone(X, y, nmain.p = 4, r1= 3, r2=3,
        interaction.ind = interaction.ind, q = 5)
# allout = "Yes"
g2 <- Genone(X, y, nmain.p = 4, r1= 3, r2=3,
        interaction.ind = interaction.ind, q = 5, allout = "Yes")</pre>
```

initial 13

initial

Setting up initial candidate models

# Description

This function automatically ranks all candidate interaction effects under Strong, Weak, or No heredity condition and obtains initial candidate models.

# Usage

```
initial(
   X,
   y,
   heredity = "Strong",
   nmain.p,
   sigma = NULL,
   r1,
   r2,
   interaction.ind = NULL,
   pi1 = 0.32,
   pi2 = 0.32,
   pi3 = 0.32,
   lambda = 10,
   q = 40
)
```

# **Arguments**

X	Input data. An optional data frame, or numeric matrix of dimension n by nmain.p. Note that the two-way interaction effects should not be included in X because this function automatically generates the corresponding two-way interaction effects if needed.
У	Response variable. A n-dimensional vector, where n is the number of observations in X.
heredity	Whether to enforce Strong, Weak, or No heredity. Default is "Strong".
nmain.p	A numeric value that represents the total number of main effects in X.
sigma	The standard deviation of the noise term. In practice, sigma is usually unknown. In such case, this function automatically estimate sigma using root mean square error (RMSE). Default is NULL. Otherwise, users need to enter a numeric value.
r1	A numeric value indicating the maximum number of main effects. This number

A numeric value indicating the maximum number of main effects. This number can be different from the r1 defined in detect.

A numeric value indicating the maximum number of interaction effects. This number can be different from the r2 defined in detect.

interaction.ind

r2

A two-column numeric matrix containing all possible two-way interaction effects. It must be generated outside of this function using t(utils::combn()). See Example section for details.

14 mut

pi1	A numeric value between 0 and 1, defined by users. Default is 0.32. For guidance on selecting an appropriate value, please refer to ABC.
pi2	A numeric value between 0 and 1, defined by users. Default is 0.32. For guidance on selecting an appropriate value, please refer to ABC.
pi3	A numeric value between 0 and 1, defined by users. Default is 0.32. For guidance on selecting an appropriate value, please refer to ABC.
lambda	A numeric value defined by users. Default is 10. For guidance on selecting an appropriate value, please refer to ABC.
q	A numeric value indicating the number of models in each generation (e.g., the population size). Default is 40.

#### Value

A list of output. The components are:

initialize Initial candidate models. A numeric matrix of dimension q by r1+r2 where each

row represents a fitted model. Duplicated models are allowed.

InterRank Rank of all candidate interaction effects. A two-column numeric matrix. The

first column contains indices of ranked two-way interaction effects, and the sec-

ond column contains its corresponding ABC score.

mainind.sel Selected main effects. A r1-dimensional vector.

mainpool Ranked main effects in X.

#### See Also

ABC, Extract.

# **Examples**

mut

Performing mutation

# Description

This function performs mutation which only stores all fitted models without making any comparison. The selected indices in each fitted model will be automatically re-ordered so that main effects comes first, followed by two-way interaction effects, and zero reservation spaces.

mut 15

## Usage

```
mut(
  parents,
  heredity = "Strong",
  nmain.p,
  r1,
  r2,
  interaction.ind = NULL,
  interonly = "No",
  aprob = 0.9,
  dprob = 0.9,
  aprobm = 0.1,
  aprobi = 0.9,
  dprobm = 0.9,
  dprobm = 0.9,
  dprobi = 0.1
)
```

# Arguments

parents	A numeric matrix of dimension q by r1+r2, obtained from initial or previous generation where each row corresponding a fitted model and each column representing the predictor index in the fitted model.
heredity	Whether to enforce Strong, Weak, or No heredity. Default is "Strong".
nmain.p	A numeric value that represents the total number of main effects in X.
r1	A numeric value indicating the maximum number of main effects.
r2	A numeric value indicating the maximum number of interaction effects.
interaction.in	d d
	A two-column numeric matrix containing all possible two-way interaction effects. It must be generated outside of this function using t(utils::combn()). See Example section for details.
interonly	Whether or not to consider fitted models with only two-way interaction effects. A "Yes" or "No" logical vector. Default is "No".
aprob	A numeric value between 0 and 1, defined by users. The addition probability during mutation. Default is 0.9.
dprob	A numeric value between 0 and 1, defined by users. The deletion probability during mutation. Default is 0.9.
aprobm	A numeric value between 0 and 1, defined by users. The main effect addition probability during addition. Default is 0.1.
aprobi	A numeric value between 0 and 1, defined by users. The interaction effect addition probability during addition. Default is 0.9.
dprobm	A numeric value between 0 and 1, defined by users. The main effect deletion probability during deletion. Default is 0.9.
dprobi	A numeric value between 0 and 1, defined by users. The interaction effect deletion probability during deletion. Default is 0.1.

# Value

A numeric matrix single.child.mutated is returned. Each row representing a fitted model, and each column corresponding to the predictor index in the fitted model. Duplicated models are allowed.

16 mut

## See Also

initial.

## **Examples**

# Index

```
ABC, 1, 5, 6, 8, 10–12, 14

AVGAS, 4

cross, 6, 6, 12

detect, 4, 7, 13

Extract, 3, 6, 9, 12, 14

Genone, 6, 10

initial, 3, 6–8, 10, 12, 13, 16

mut, 6, 12, 14
```