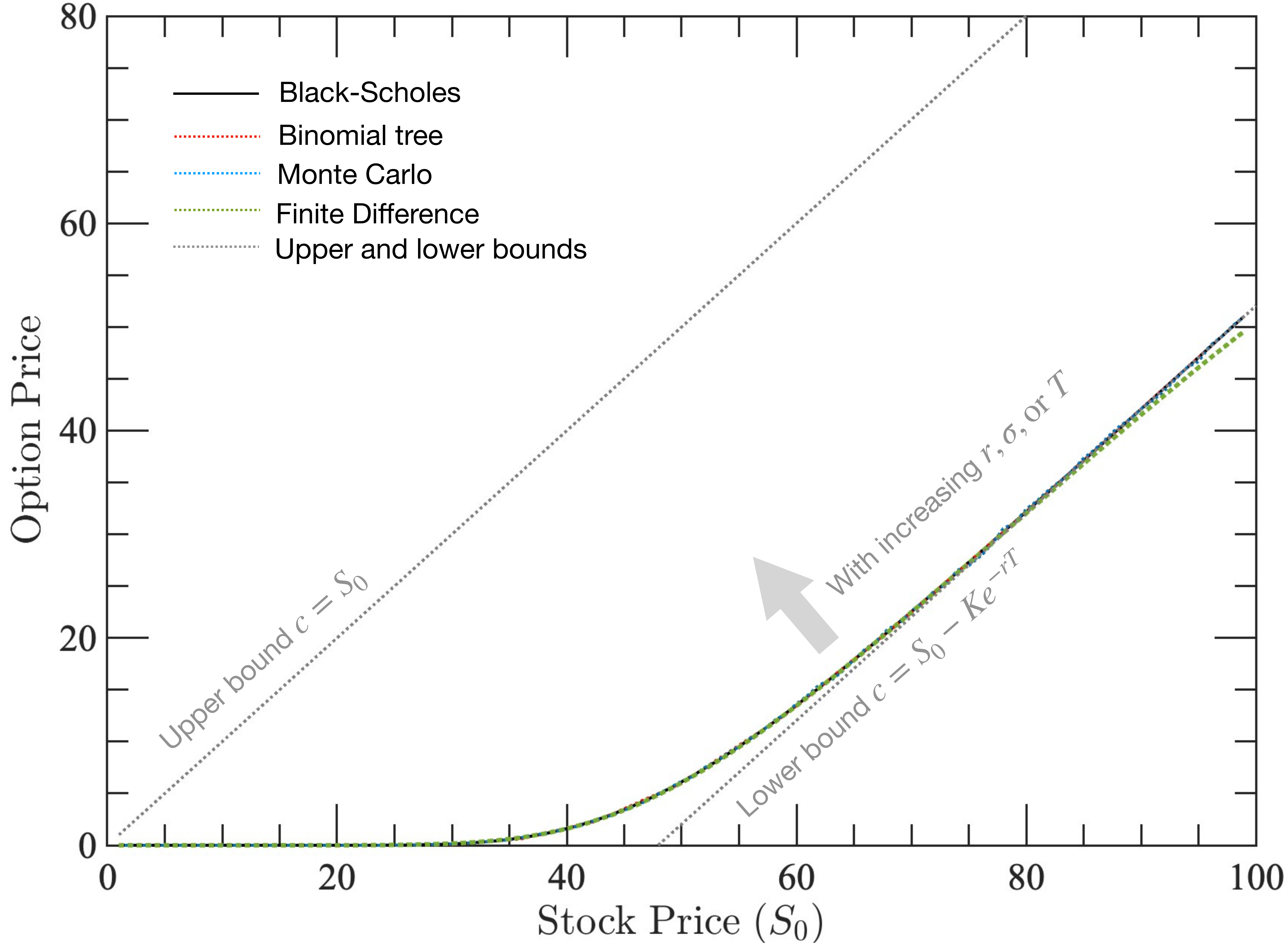
European Call Option

$$c = e^{-rT}\hat{E}[\max(S_T - K, 0)]$$



- Example:
  $K = 50, r = 10\,\%, \sigma = 40\,\%, T = 0.416$ (5 Months)

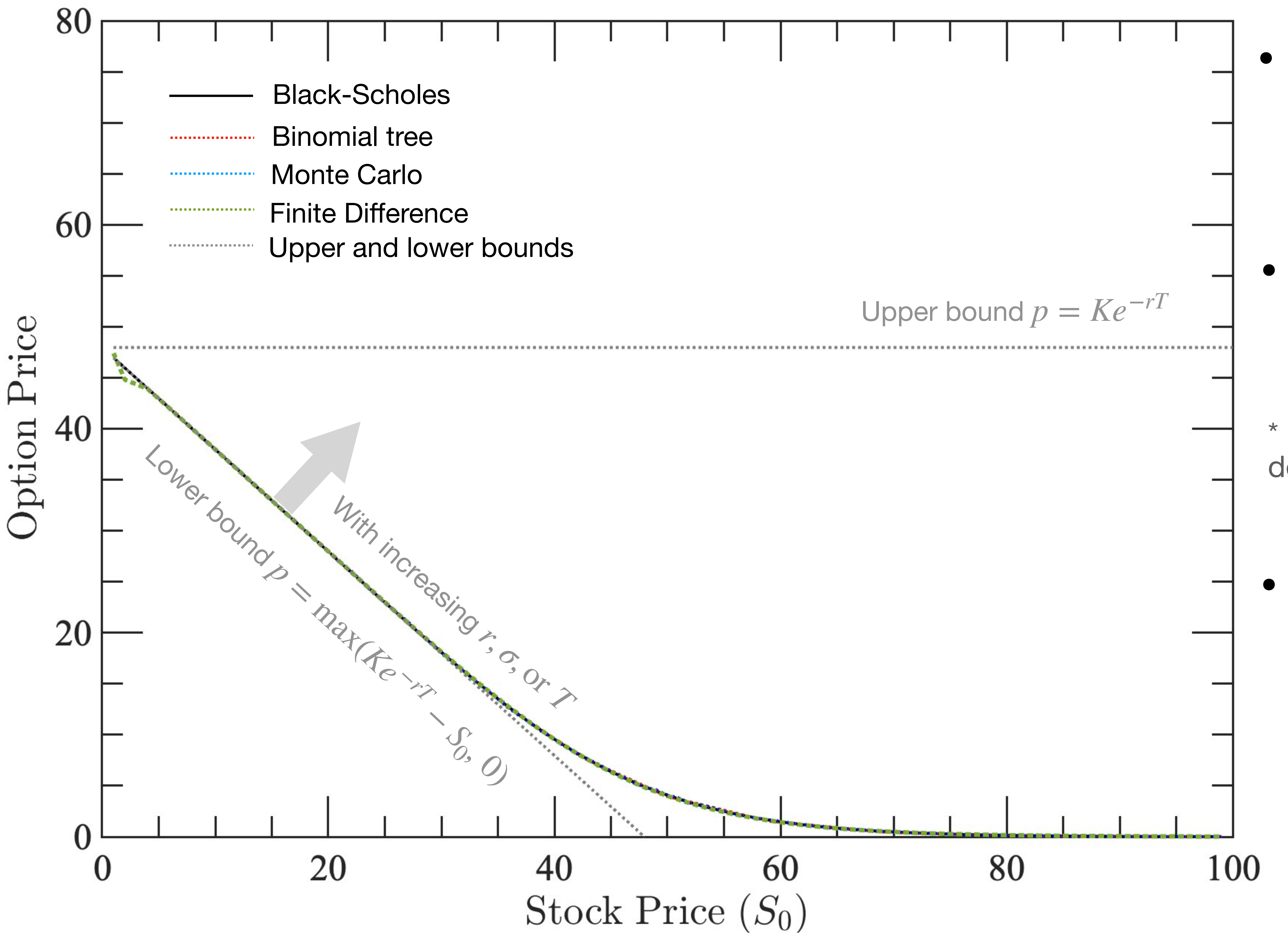
- Black-Scholes formula:
  $c = S_0 N(d_1) + Ke^{-rT} N(d_2)$

* European call and put options have analytic solutions derived from the Black-Scholes-Merton Model.

- Numerical results are consistent with the BS prediction

European Put Option

$$p = e^{-rT}\hat{E}[\max(K - S_T, 0)]$$



- Example:
  $K = 50, r = 10\,\%, \sigma = 40\,\%, T = 0.416$ (5 Months)

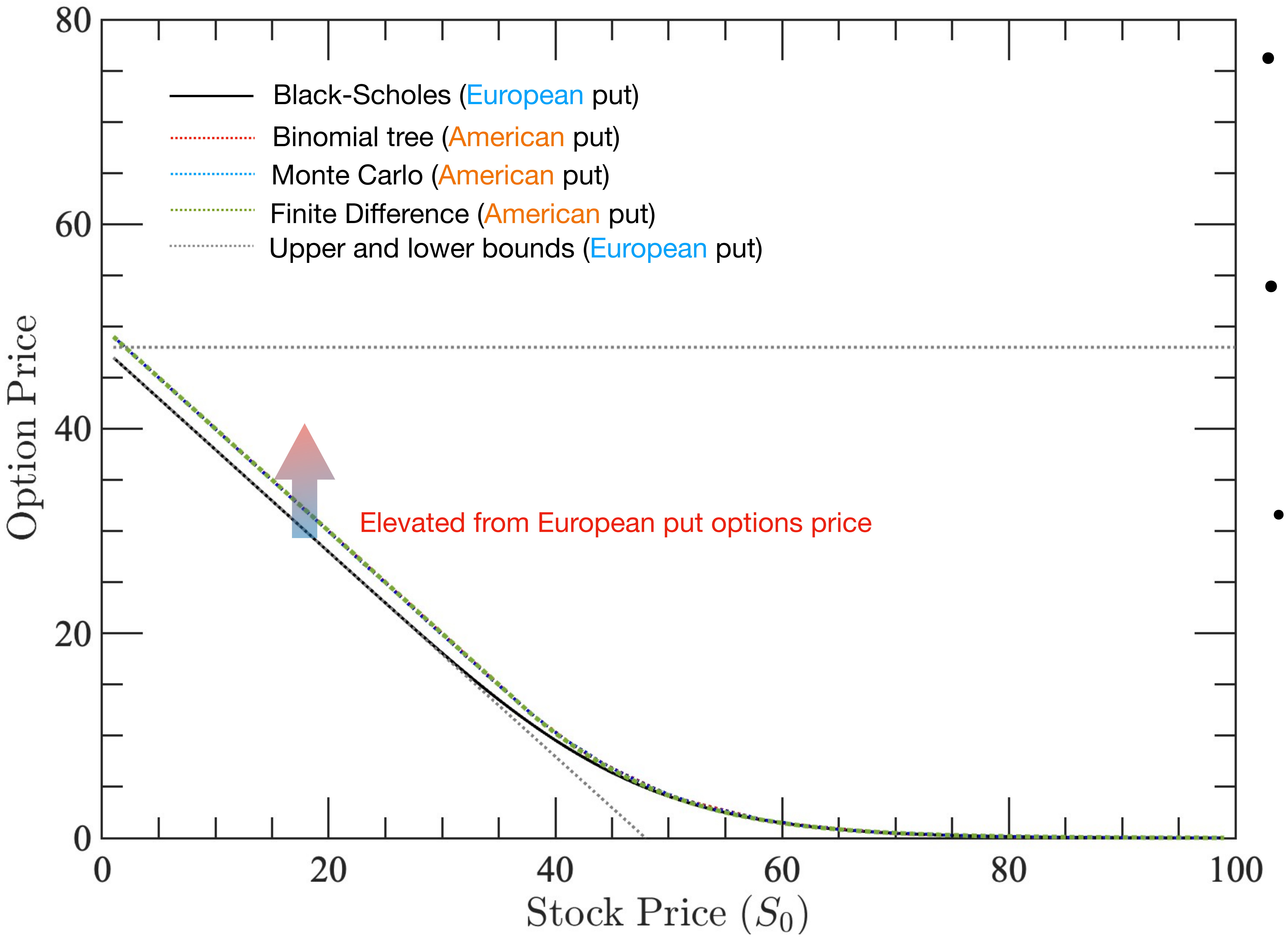
- Black-Scholes formula:
  $p = Ke^{-rT}N(-d_2) - S_0N(-d_1)$

* European call and put options have analytic solutions derived from the Black-Scholes-Merton Model.

- Numerical results are consistent with the BS prediction

American Options

$$c = C,\ P > p$$



- Example:
  $K = 50, r = 10\,\%, \sigma = 40\,\%, T = 0.416$ (5 Months)

- No analytic solutions from Black-Scholes-Merton Model

- Numerical results indicate that American put option is more expensive than European put option given the same conditions (as expected).

Asian Options

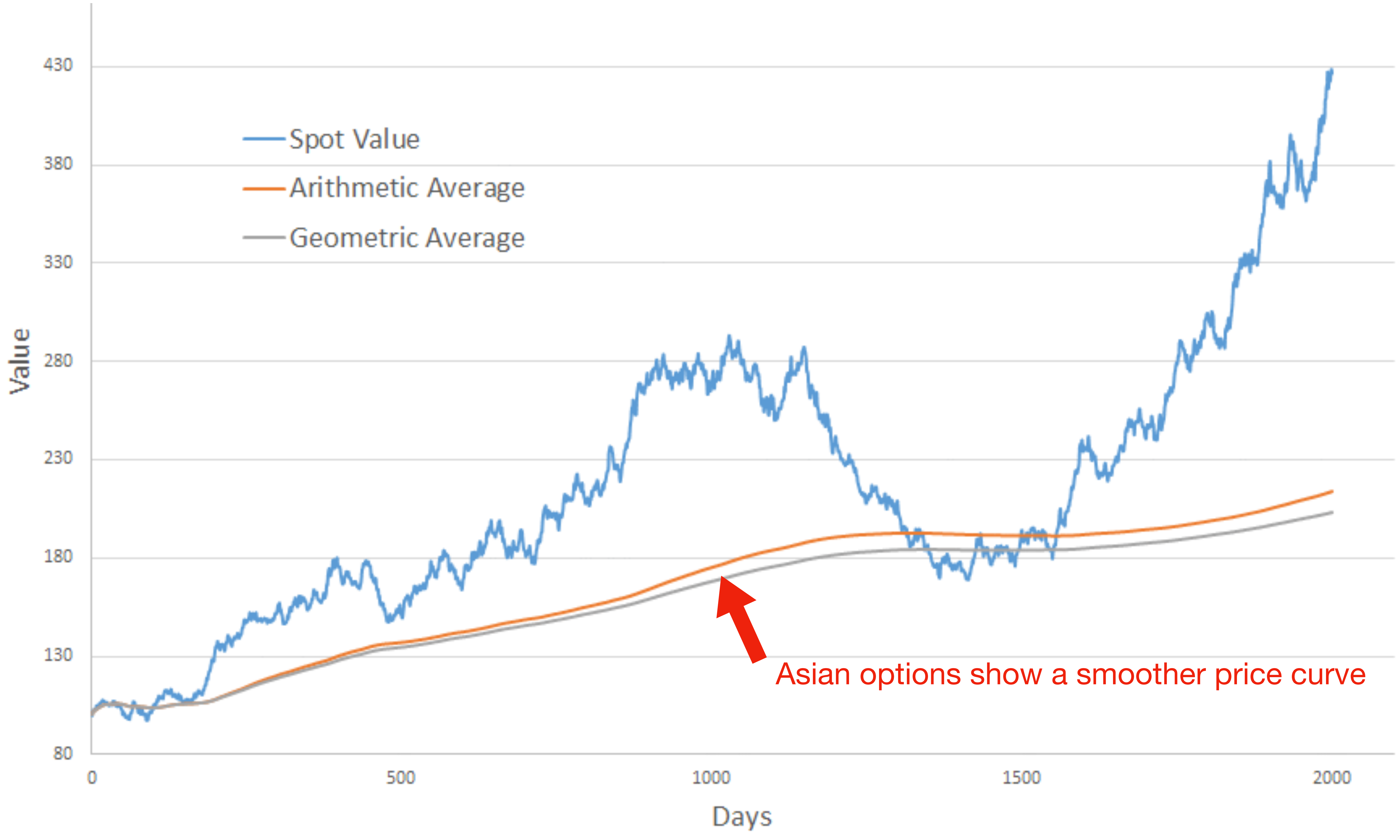


Image Credit: Ioannis Rigopoulos
https://blog.deriscope.com/index.php/en/excel-quantlib-asian-option

- Geometric average vs. Arithmetic average

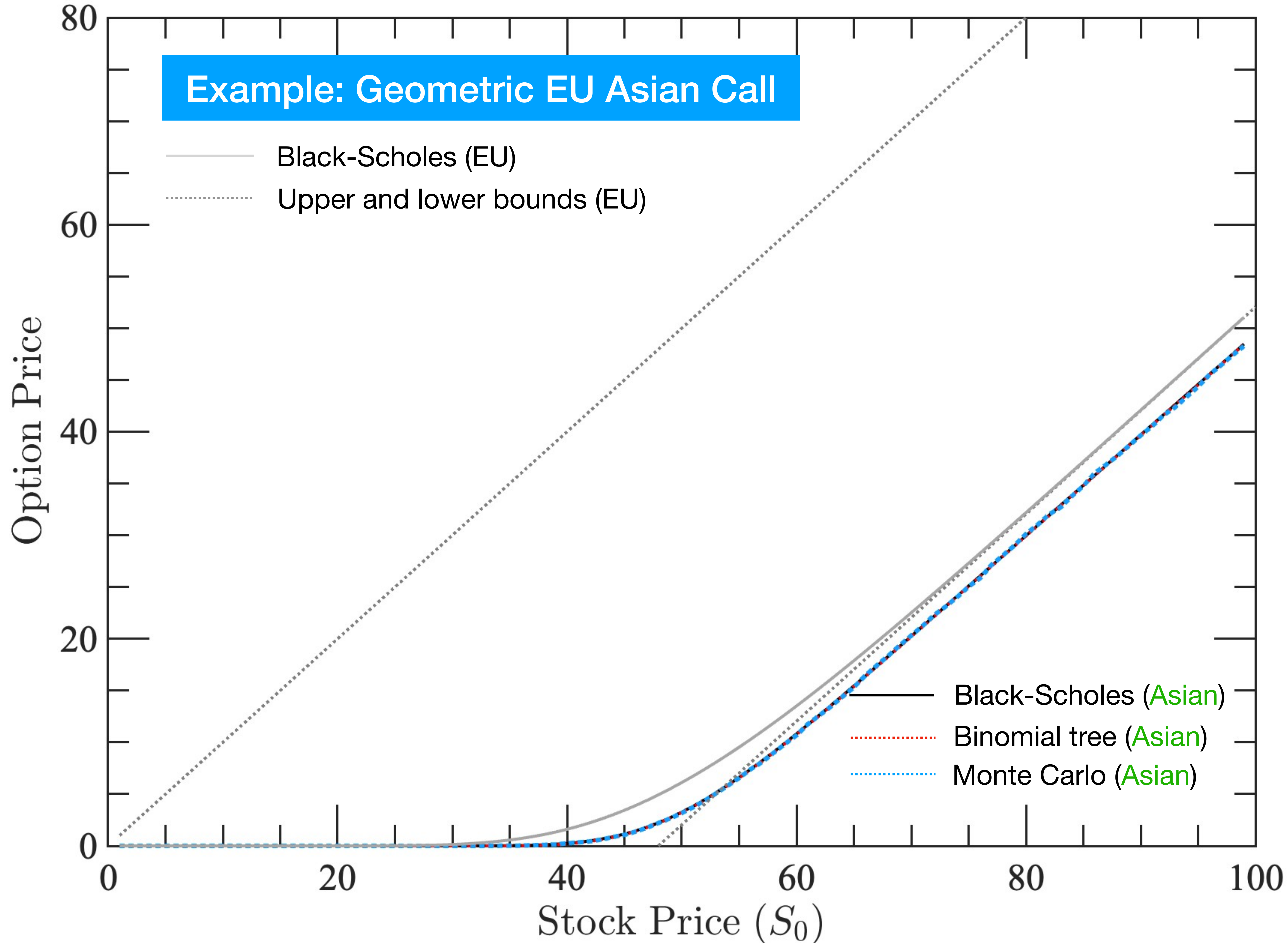
$$S_{\text{ave, G}} = e^{\frac{1}{T}\int_0^T \ln S_{\text{u}} \mathrm{d}u}$$

$$S_{\text{ave, A}} = \frac{1}{T}\int_0^T S_{\text{u}} \mathrm{d}u$$

- American Asian vs. European Asian

* Asian options should be cheaper than vanilla American/European options given the same conditions

Asian Options



- Analytic solution for EU-G Asian options

$$C_{\mathrm{AE,g}}(S_0, T) = e^{(\rho - r)T}\tilde{c}_{\mathrm{E}}(S_0, T)$$

$$P_{\mathrm{AE,g}}(S_0, T) = e^{(\rho - r)T}\tilde{p}_{\mathrm{E}}(S_0, T)$$

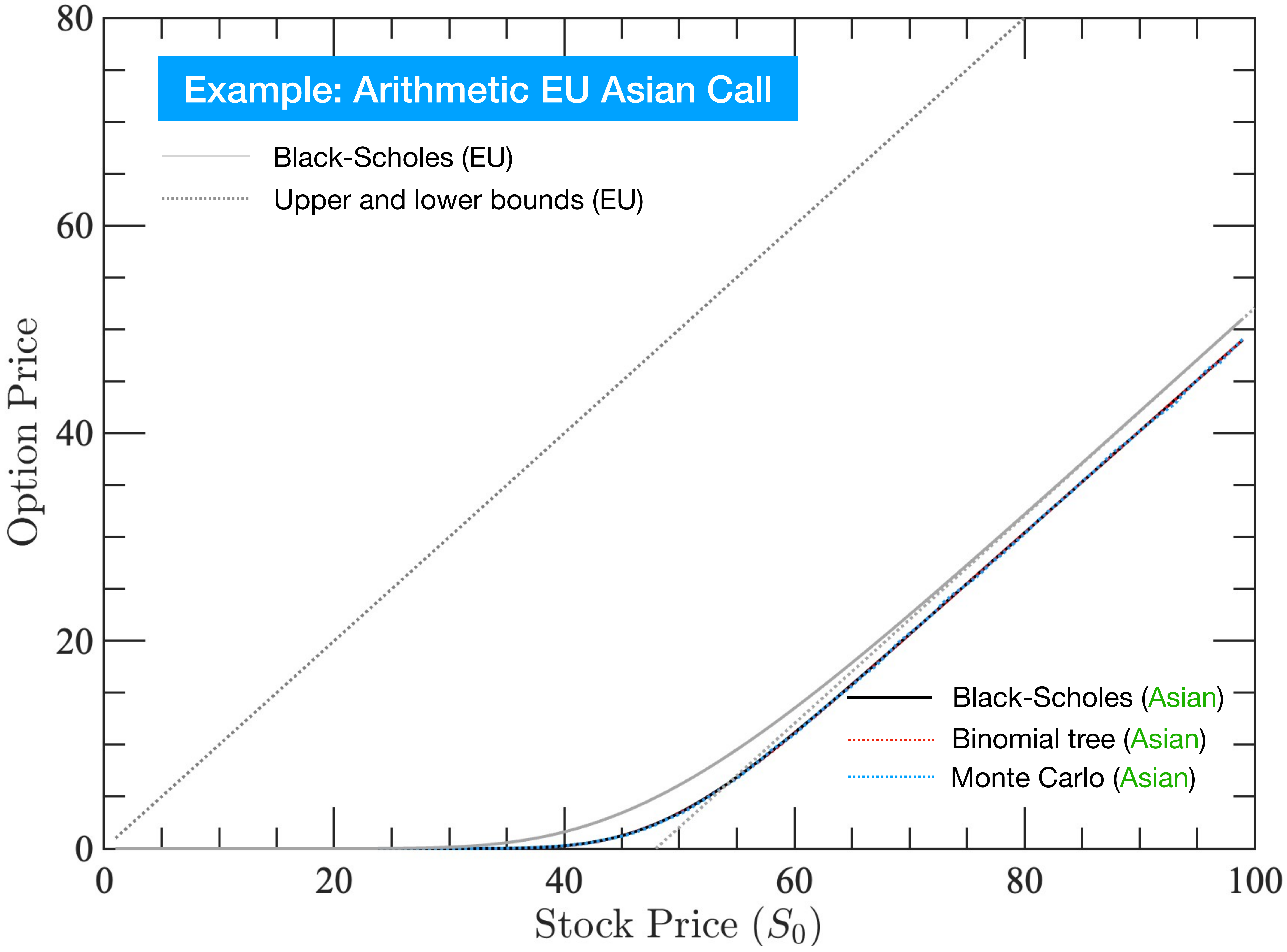
Where

$$\rho = \frac{(r - \sigma^2/6)}{2}$$

and the corrected diffusion term in $\tilde{c}_{\mathrm{E}}$ and $\tilde{p}_{\mathrm{E}}$

$$\sigma_z = \frac{\sigma}{\sqrt{3}}$$

Asian Options



- Quasi-analytic solutions exist for EU-A Asian options through moment-matching approximations

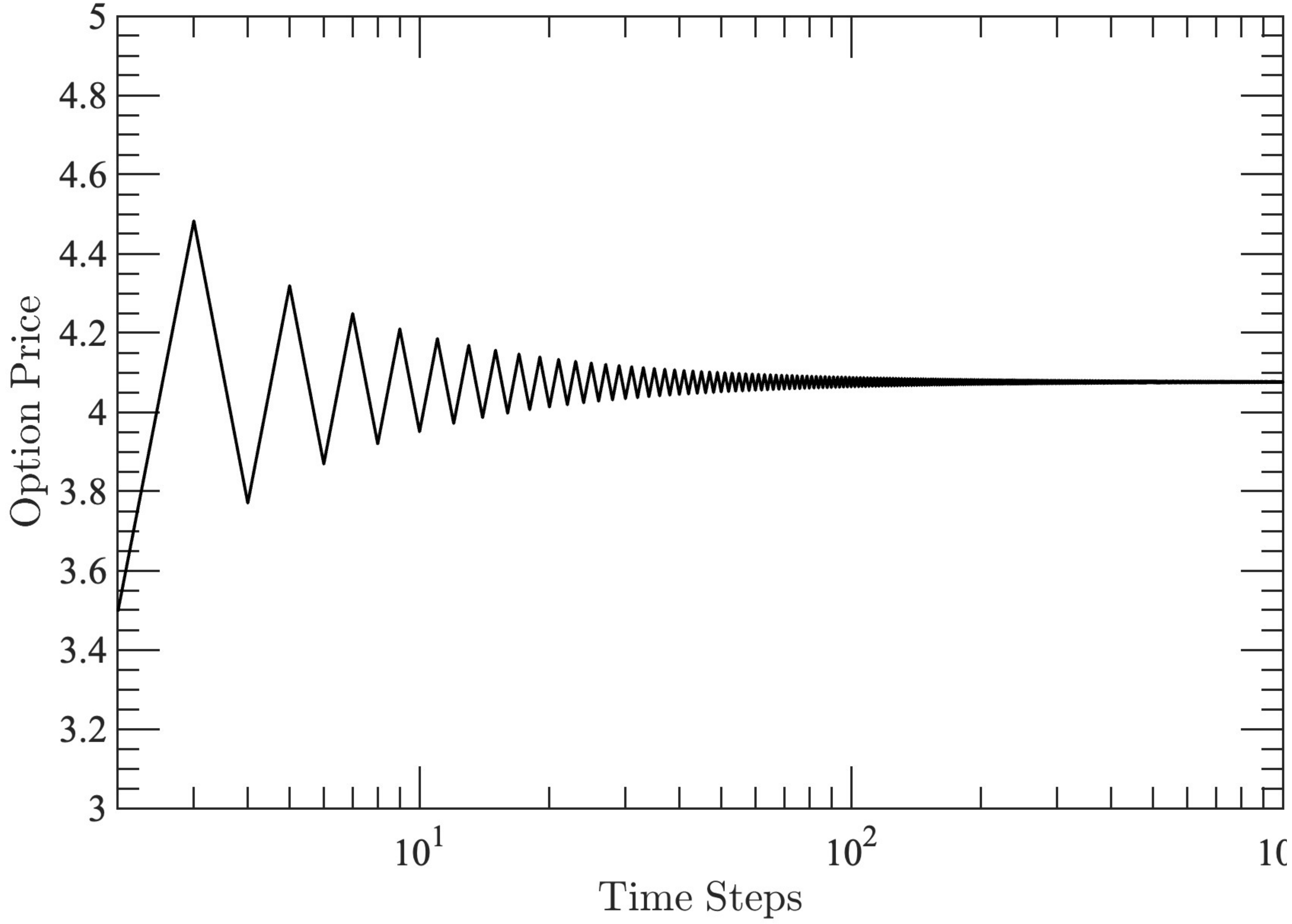
*assuming $S_{\rm ave}$ follows a log-normal distribution

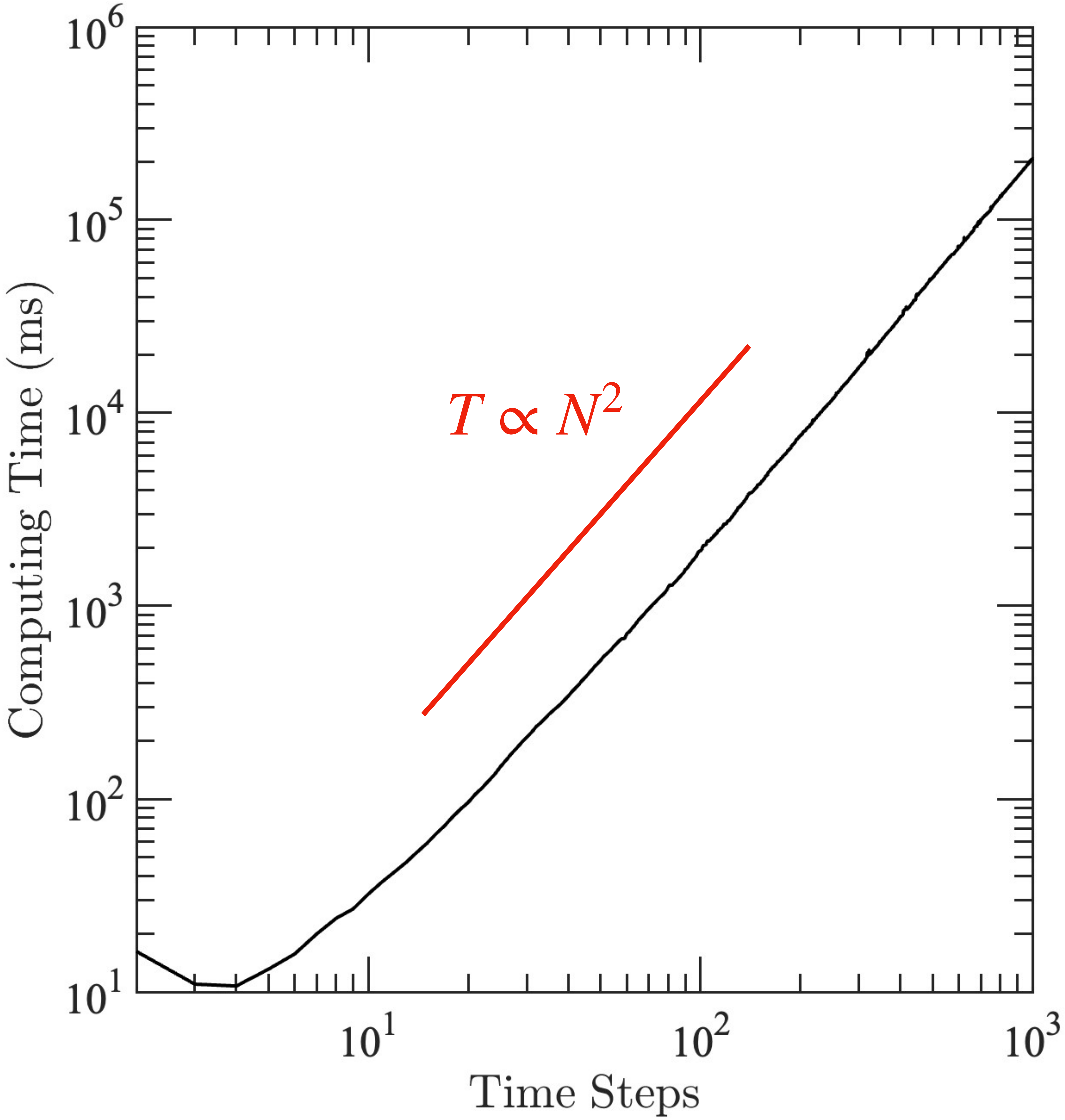
$$F_0 = M_1 \equiv \hat{E}(\int_0^T S_{\rm u} du)$$

$$\sigma^2 = \frac{1}{T}\ln(\frac{M_2}{M_1})$$

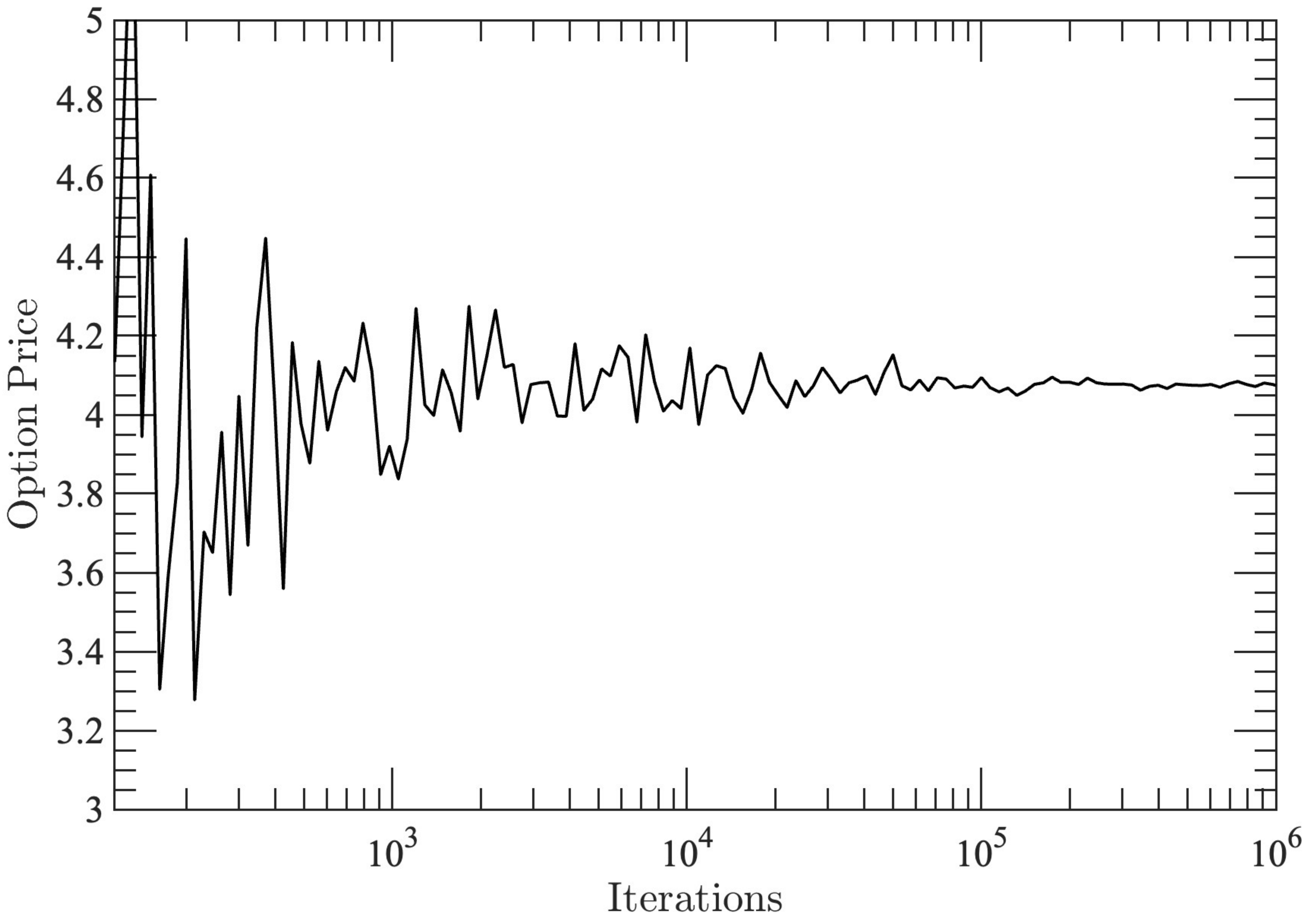
Where $$M_2 \equiv \hat{E}[(\int_0^T S_{\rm u} du)^2]$$

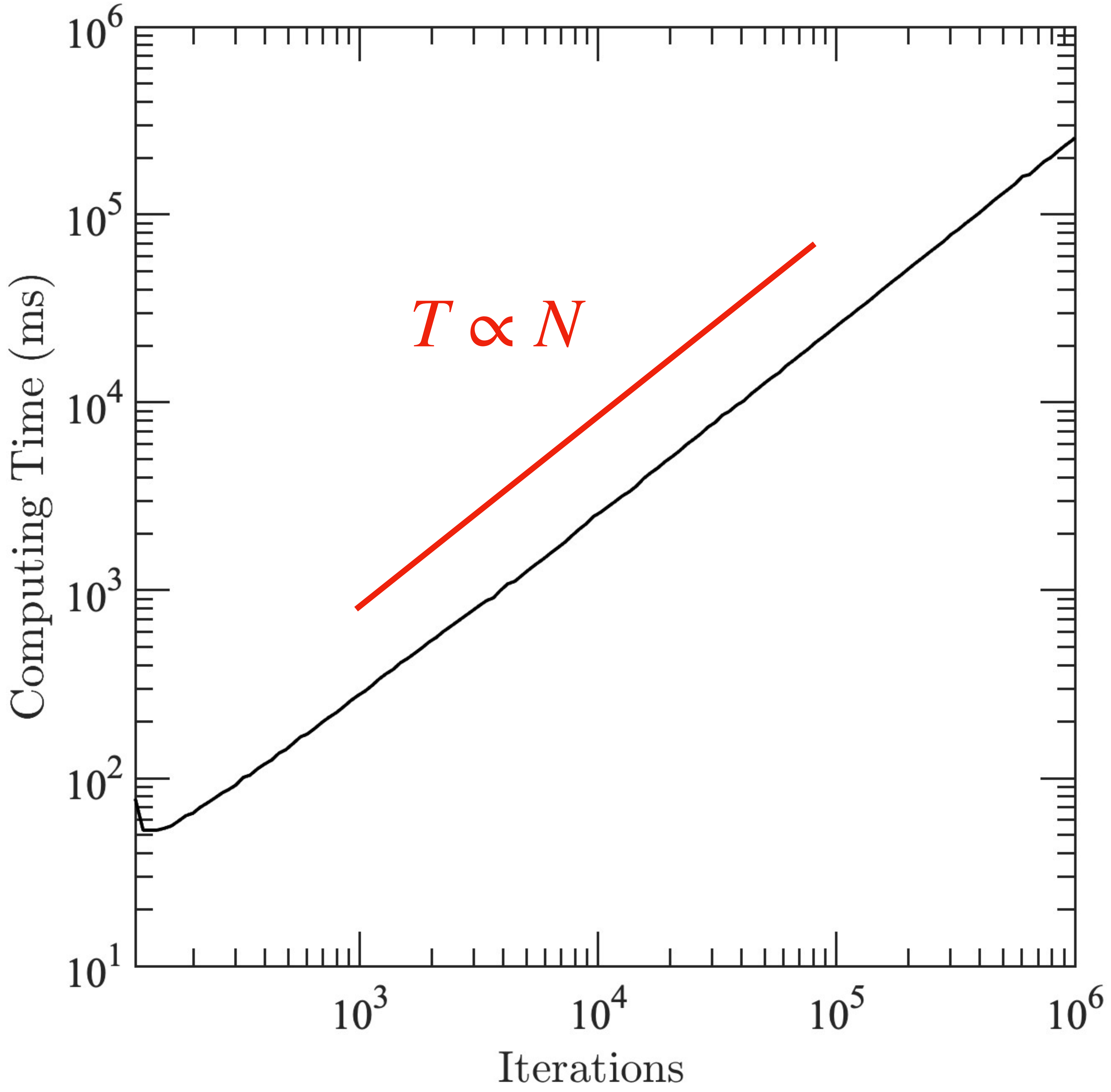
Numerical convergence - binomial tree





Numerical convergence - Monte Carlo





Numerical convergence - Finite difference