Homework 2

Due not later than *Thursday, August* 3

Problem 1. Men, women and yupi live on the planet Alphaomega. They have highly developed technology and in particular, they have stations for teleportation not only in space but also in time. In the city Aleph on this planet, there are two teleportation stations. One of these stations allows travelling both in time and in space, while the other one allows travelling only in time.

One day, there are n requests for teleportation and m of then demand travelling only in time. Each request has the starting time and the ending time for the station utilization. The goal is to satisfy, i.e., to schedule, as many requests as possible without overlapping.

a) (10 pts). Prove that there is always a maximal scheduling.

b) (10 pts). Design an algorithm for building a maximal scheduling and prove that it gives a maximal scheduling.

c) (40 pts). Design an efficient algorithm for building a maximal scheduling and prove that it gives a maximal scheduling.

Problem 2. Men, women and gupi live on the planet Alphaomega. Imagine you are travelling by car on this planet and you need to pass the city Beth. The road goes through this city and you cannot bypass it. There are only two entrances to the city – one from the side you are coming and another from the opposite side. Besides, you know that an evil gupi is sitting at one of the intersections in Beth. When a human being comes close to him, he kidnaps this person and makes the human being a slave. Fortunately, this gupi is lazy and he kidnaps only those who come close to him. The distance in one block is safe for you and can feel the presence of the gupi one block ahead of you. At the same time, other inhabitants of Beth cannot help you because you don't know their language and they don't know your language.

a) (10 pts). Is it always possible to go through the city? Prove that your answer is correct.

b) (10 pts). If it is always possible to go through the city, design an algorithm for doing this.

c) (30 pts). If it is not always possible to go through the city, design an algorithm that will allow you to go through the city when it is possible or to return back when it is impossible.

Problem 3. In a universe W, there are n planets P_i (i = 1, 2, ... , n). Between some pairs of these planets, wormholes exist, which allow travelling from one of these planets to the other in one day. Each group of planets, in which any two planets are directly or indirectly connected by wormholes, decided to form one state.

(30 pts). If it is possible to do this, build an algorithm that finds how many states will be created.

(30 pts). If it is impossible to do this, prove it.