

CS 180 Homework 1

Due Thursday, July 20, 2017

Problem 1. Decide whether the following statement is true or false.

If it is true, give a short explanation.

If it is false, give a counterexample.

In every instance of the Stable Matching Problem, there is a stable matching containing a pair (m, w) such that one of them is ranked first on the preference list of the other and the other is ranked either first or second on the preference list of the first one.

Problem 2. Take the following functions and arrange them in ascending order of growth rate.

$$\log_2^2 n$$

$$\log_{10} 2^n$$

$$\log_2 n^3$$

$$\log_2^2 n^n$$

$$\log_2^2 n + 2n$$

$$\log_{10}(10n)$$

$$a \log_{10}^2 n + b \log_{10} n + c$$

$$\log_2^2(n + 2n)$$

Problem 3. Men, women and yupi live on the planet Alphaomega. Their family pattern is a triple that consists of a man, a woman and a yupi. Three sets are given: M includes n men, W includes n women and Y includes n yupi. A matching is a set H of ordered triples of the form (m, w, y) with the property that each member of M , each member of W and each member of Y appears in at most one triple from H . A matching H is called *perfect* if each member of M , each member of W and each member of Y appears exactly in one triple from H .

Assume that each man ranks all women and all yupi, each woman ranks all men and all yupi, and each yupi ranks all women and all men.

Two triples (m, w, y) and (m', w', y') form an instability in a matching H if one of the following conditions is true:

- (1) m prefers w' to w and w' prefers m to m'
- (2) m prefers y' to y and y' prefers m to m'
- (3) y prefers w' to w and w' prefers y to y'

A matching H is called *stable* if it does not have instabilities.

Decide whether the following statement is true or false.

There is an algorithm that solves the Stable Matching Problem for every instance of this problem.

If it is true, design an algorithm for building a stable perfect matching. Note that when you design an algorithm, you have to prove that it solves the necessary problem

If it is false, give a counterexample.