Optimal Control and Optimization in Robotics

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Introduction

My internship is co-supervised by Justin Carpentier (Willow, Inria Paris) and Nicolas Mansard (Gepetto, LAAS/CNRS) in the Willow research group at INRIA Paris in France.

Figure 1: Justin Carpentier

Figure 2: Nicolas Mansard

Outline

Optimal Control Problem

Optimal Control Problem

Goal 1: Controllability

$$
\begin{aligned}
\text{find} \quad & u \\
\text{subject to} \quad & x(0) = x_0,\ x(T) = p, \\
& \dot{x}(t) = f(x(t), u(t)).
\end{aligned} \tag{1}
$$

Goal 2: Optimal Control

$$
\begin{aligned}
\underset{u}{\text{minimize}} \quad & \int_0^T l(x(t), u(t))dt \\
\text{subject to} \quad & x(0) = x_0,\ x(T) = p, \\
& \dot{x}(t) = f(x(t), u(t)).
\end{aligned} \tag{2}
$$

Transformation of the problem

Adding penalty to the terminal lost:

$$
\begin{aligned}
\underset{u}{\text{minimize}} \quad & \int_{[0,T[} l(x(t),u(t))dt + l_T(x(T)) \\
\text{subject to} \quad & x(0) = x_0, \\
& \dot{x}(t) = f(x(t),u(t)).
\end{aligned} \tag{3}
$$

Discretization of functions and variables:

$$
\begin{aligned}
\underset{x\in\ell^\infty_{N+1},u\in\ell^\infty_N}{\text{minimize}} \quad & J(x,u) = \sum_{i=0}^{N-1} L(x_i,u_i) + L_T(x_N) \\
\text{subject to} \quad & x(0) = x_0, \\
& x_{i+1} = F(x_i,u_i)\ \forall i \in [0..N-1]
\end{aligned} \tag{4}
$$

Differential Dynamic Programming

Dynamic Programming

Optimize one by one:

$$\min_{U} J(U) = \min_{u_0} \min_{u_1} \dots \min_{u_{N-1}} J(U) \tag{5}$$

Definitions of Value Function and Q-functions:

$$\begin{aligned} V_i(x_i) &= \min_{u_i} L(x_i, u_i) + V_{i+1}(x_{i+1}) \\ V_N(x_N) &= L_T(x_N) \end{aligned} \tag{6}$$

$$\begin{aligned} Q_i(x_i, u_i) &= L(x_i, u_i) + V_{i+1}(x_{i+1}) \\ &= L(x_i, u_i) + V_{i+1}(f(x_i, u_i)) \\ &= L(x_i, u_i) + \min_{u_{i+1}} Q_{i+1}(f(x_i, u_i), u_{i+1}), \ \ i \leq N-2 \end{aligned} \tag{7}$$

$$V_i(x_i) = \min_{u_i} Q_i(x_i, u_i) \tag{8}$$

Linear Quadratic Regulator (LQR)

The LQR is an algorithm that solves the problem 4 in one iteration in case L, L_T are quadratic and F is linear.