Chapter 3

Augmented Lagrangian DDP

By the method augmented Lagrangian, for solving the problem 3.1 with equality constraints, we solve sequentially 3.3 by updating Lagrangian multiplier λ and penalization parameter μ until the KKT conditions for the original problem are satisfied:

$$\begin{aligned} \underset{x\in\mathbb{X}^{N+1},u\in\mathbb{U}^N}{\text{minimize}} \quad & J(x,u) = \sum_{i=0}^{N-1} L_i(x_i,u_i) + L_T(x_N) \\ \text{subject to} \quad & x(0) = x_0, \\ & x_{i+1} = F(x_i,u_i) \qquad \forall i \in [0\mathinner{..}N-1] \\ & c_j(x,u) = 0 \qquad \forall j \in [0\mathinner{..}M-1] \end{aligned} \tag{3.1}$$

Let $c : \mathbb{X}^{N+1} \times \mathbb{U}^N \to \mathbb{R}^M$ be the function for the set of constraints. However, one constraint c_j usually does not apply on all the states x or controls u but one x_i, u_i (or a few) of them. So, in practice, I define $c_j^i : \mathbb{X} \times \mathbb{U} \to \mathbb{R}$ for each x_i, u_i and $c_j^T : \mathbb{X} \to \mathbb{R}$ if the constraint c_j applies on them i.e. $j \sim i$. Then I "augment" the loss (and terminal loss) with related constraint(s):

$$\boxed{\mathcal{L}_i^A(x_i,u_i,\lambda,\mu) := L_i(x_i,u_i) - \sum_{j\sim i} \lambda_j c_j(x_i,u_i) + \frac{\mu}{2}\sum_{j\sim i} c_j^2(x_i,u_i) \quad \forall i \in [0\mathinner{..}N-1]} \tag{3.2}$$

$$\begin{aligned} \underset{x\in\mathbb{X}^{N+1},u\in\mathbb{U}^N}{\text{minimize}} \quad & J(x,u) = \sum_{i=0}^{N-1} \mathcal{L}_i^A(x_i,u_i,\lambda,\mu) + \mathcal{L}_T^A(x_N,\lambda,\mu) \\ \text{subject to} \quad & x(0) = x_0, \\ & x_{i+1} = F(x_i,u_i) \qquad \forall i \in [0\mathinner{..}N-1] \end{aligned} \tag{3.3}$$

For all $\lambda \in \mathbb{R}^M, \mu \in \mathbb{R}^{+*}$, the problem3.3 can be solved by a DDP (or FDDP) solver (ref crocoddyl)