Logic in Computer Science - Assignment 1

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Prove the following Theorems with natural deduction:

(1) $p \to q, r \to s \vdash p \wedge r \to q \wedge s$

1	$p \to q$	premise
2	$r \to s$	premise
3	$p \wedge r$	assumption
4	p	$\wedge e_1$ 3
5	r	$\wedge e_2$ 3
6	q	$\to$ e 1, 4
7	s	$\to$ e 2, 5
8	$q \wedge s$	$\wedge$i 4, 5
9	$p \wedge r \to q \wedge s$	$\to$ i 3−8

(2) $\neg(p \wedge q) \vdash \neg q \vee \neg p$

1	$\neg(p \wedge q)$	premise
2	$q \wedge \neg q$	LEM
3	$\neg q$	assumption
4	$\neg q \vee \neg p$	$\vee i_1$ 3
5	q	assumption
6	p	assumption
7	$p \wedge q$	$\wedge$i 5, 6
8	$\bot$	$\neg$e 1, 7
9	$\neg p$	$\neg$i 6−8
10	$\neg q \vee \neg p$	$\vee i_2$ 9
11	$\neg q \vee \neg p$	$\vee$e 2, 3−4, 5−10

(3) $p \to (q \vee r), q \to s, r \to s \vdash p \to s$

1	$p \to (q \vee r)$	premise
2	$q \to s$	premise
3	$r \to s$	premise
4	p	assumption
5	$q \vee r$	$\to$ e $1, 4$
6	q	assumption
7	s	$\to$ e $2, 6$
8	r	assumption
9	s	$\to$ e $3, 8$
10	s	$\vee$e $5, 6-7, 8-9$
11	$p \to s$	$\to$ i $4-10$