Our interest rate model is a two-additive-factor Gaussian model, also called a G++ model. It models the instantaneous-short-rate as the sum of two unobservable, correlated Gaussian factors plus a deterministic function, θ_t, which is chosen so as to exactly fit the current term structure of discount factors. It is described in depth in *Interest Rate Models – Theory and Practice*, Second Edition by Brigo and Mercurio. It is essentially just a slightly more general expression of a 2-Factor Hull-White model.

In addition to θ_t there are five parameters, a, b, σ_1, σ_2, and ρ. Each Gaussian factor follows a Ornstein-Uhlenbeck process where a and b are their mean reversion parameters, σ_1 and σ_2 are their volatility parameters and ρ is the correlation between the Gaussian factors' random terms. Starting in Q4 2008 when swaption volatilities became severely dislocated, leading to highly unstable parameters, we started researching alternative ways to estimate the parameters. This effort converged nicely with our implementation of an alternative equity volatility model for SPX Index options. Concurrent with the implementation of the new equity volatility model we implemented a new process for estimating these parameters that has the same fundamental premise: fit historically observed volatility then add a load that represents the cost of hedging this volatility. In this case it is the cost of convexity hedging.

It is not possible to estimate the parameters directly from historical interest rates if for no other reason that the two stochastic processes are inherently unobservable. Additionally, the two mean reversion parameters would be difficult to estimate even for observable interest rates. The process we have put in place estimates the two mean reversion parameters from the swaptions market using market calibration procedures. We then use a running trimmed mean of the parameters in order to derive estimates that are consistent with the swaptions market but not subject to short-term instability due to dislocations in the market. Once we have the two mean reversion parameters we calibrate σ_1, σ_2, and ρ to be consistent with the historically observed correlation and volatilities of 3-month LIBOR and the 10-year swap rate over a slightly longer than 20-year period (from 11/30/88).

The convexity hedging load that we add is calculated in a similar manner to the equity volatility load. We calculate the difference of the realized volatility of a 10-year swap (Bloomberg Ticker USSWAP10 Index) over a 3-month interval that corresponds to the 3-month interval of the In-3-Month-For-10 year swaption volatility (Bloomberg Ticker USSC0C10 Index) from Bloomberg. This time series is calculated for all of the available data, which is back to January 1997. Due to the complexity of trying to estimate a term structure component and the fact that we are currently convexity hedging with shorter dated instruments due to their liquidity, we did not estimate a term structure component and we also added the same additive factor load to both σ_1 and σ_2.