INTEREST RATES

Understanding Treasury Futures

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Thirty-year Treasury bond futures were originally introduced on the Chicago Board of Trade in 1977. The product line was augmented over the years by the introduction of 10-year, 5-year, 2-year Treasury note and 30-year "Ultra" Treasury bond futures.[1]

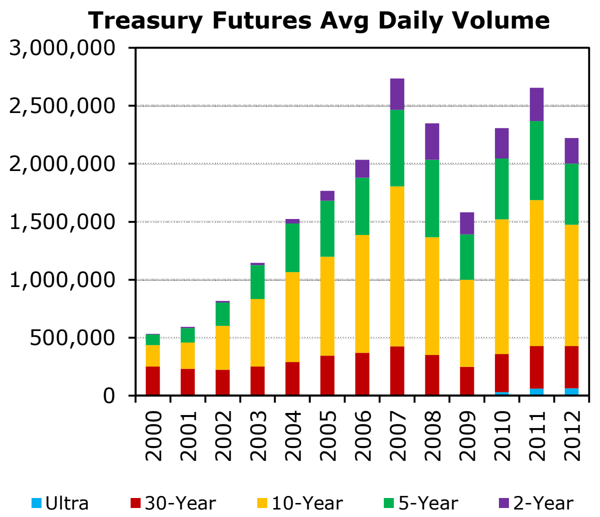
This product line has experienced tremendous success as the scale and global significance of U.S. Treasury investment has grown over the years. Today, these products are utilized on an international basis by institutional and individual investors for purposes of both abating and assuming risk exposures.

This document is intended to provide an overview of the fundamentals of trading U.S. Treasury bond and note futures. We assume only a cursory knowledge of coupon-bearing Treasury securities. Thus, we begin with a primer on the operation of cash Treasury markets before moving on to provide some detail regarding the features of the U.S. Treasury futures contracts as well as a discussion of risk management applications with U.S. Treasury futures.

Coupon-Bearing Treasury Securities

U.S. Treasury bonds and notes represent a loan to the U.S. government. Bondholders are creditors rather than equity- or share-holders. The U.S. government agrees to repay the face or principal or par amount of the security at maturity, plus coupon interest at semi-annual intervals.[2] Treasury securities are often considered "riskless" investments given that the "full faith and credit" of the U.S. government backs these securities. [3]

The security buyer can either hold the bond or note until maturity, at which time the face value becomes due; or, the bond or note may be sold in the secondary markets prior to maturity. In the latter case, the investor recovers the market value of the bond or note, which may be more or less than its face value, depending upon prevailing yields. In the meantime, the investor receives semi-annual coupon payments every six months.



E.g., you purchase $1 million face value of the 3-5/8% note maturing in February 2021. This security pays half its stated coupon or 1-13/16% of par on each six-month anniversary of its issue. Thus, you receive $36,250 (= 3-5/8% of $1 million) annually, paid out in semi-annual installments of $18,125 in February and August. Upon maturity in February 2021, the $1 million face value is re-paid and the note expires.

Price/Yield Relationship

A key factor governing the performance of bonds in the market is the relationship of yield and price movement. In general, as yields increase, bond prices will decline; as yields decline, prices rise. In a rising rate environment, bondholders will witness their principal value erode; in a decline rate

[1] These contracts were originally introduced on the Chicago Board of Trade (CBOT). CBOT was merged with Chicago Mercantile Exchange (CME) in July 2007 and now operates as a unit under the CME Group Holding company umbrella.

[2] Inflation Indexed Treasury Securities (TIPS) were introduced in 1997. These securities are offered with maturities of 30 years; 10 years; and, five years. They are sold with a stated coupon but promise the return of the original principal adjusted to reflect inflation as measured by the Consumer Price Index (CPI) over the period until maturity. Thus, their coupons are typically established at levels that reflect the premium of long- or intermediate-term interest rates relative to inflation. These securities offer investment appeal to those concerned about the long-term prospects for inflation.

[3] This characterization is called in question noting Standard & Poor's downgrade of long-term U.S. sovereign debt from AAA to AA+ status in August 2011.

environment, the market value of their bonds will increase.

IF Yields Rise ↑ THEN Prices Fall ↓

IF Yields Fall ↓ THEN Prices Rise ↑

This inverse relationship may be understood when one looks at the marketplace as a true auction. Assume an investor purchases a 10-year note with a 6% coupon when yields are at 6%. Thus, the investor pays 100% of the face or par value of the security. Subsequently, rates rise to 7%. The investor decides to sell the original bond with the 6% yield, but no one will pay par as notes are now quoted at 7%. Now he must sell the bond at a discount to par in order to move the bond. *I.e.,* rising rates are accompanied by declining prices.

Falling rates produce the reverse situation. If rates fall to 5%, our investment yields more than market rates. Now the seller can offer it at a premium to par. Thus, declining rates are accompanied by rising prices. Should you hold the note until maturity, you would receive the par or face value. In the meantime, of course, one receives semi-annual coupon payments.

Quotation Practices

Unlike money market instruments (including bills and Eurodollars) that are quoted on a yield basis in the cash market; coupon-bearing securities are frequently quoted in percent of par to the nearest 1/32nd of 1% of par.

E.g., one may quote a bond or note at 97-18. This equates to a value of 97% of par plus 18/32nds. The decimal equivalent of this value is 97.5625. Thus, a one million-dollar face value security might be priced at $975,625. If the price moves by 1/32nd from 97-18 to 97-19, this equates to a movement of $312.50 (per million-dollar face value).

But often, these securities, particularly those of shorter maturities, are quoted in finer increments than 1/32nd. For example, one may quote the security to the nearest 1/64th. If the value of our bond or note in the example above were to rally from 97-18/32nds by 1/64th, it may be quoted at 97-18+. The trailing "+" may be read as +1/64th.

Or, you may quote to the nearest 1/128th. If our bond were to rally from 97-18/32^{nds} by 1/128th, it might be quoted on a cash screen as 97-182. The trailing "2" may be read as +2/8^{ths} of 1/32nd; or, 1/128th. If the security rallies from 97-18/32^{nds} by 3/128^{ths}, it may be quoted as 97-186. The trailing "6" may be read as +6/8^{ths} of 1/32nd or 3/128^{ths}.

Sometimes, quotation systems use an alternate fractional reference. *E.g.,* the value of 97-182 might be displayed as 97-18¼. Or a value of 97-18+ might be displayed as 97-18½. A value of 97-186 might be displayed as 97-18¾.

Futures quotation practices are similar but not entirely identical. A quote of 97-182 is the same no matter whether you are looking at a cash or a futures quote. It means 97% of par plus 18/32nds plus 1/128th.

Quotation Practices

Cash Price	Means	Decimal Equivalent (% of Par)	Futures Quote
97-18	97-18/32^{nds}	97.5625000	97-18
97-182 or 97-18¼	97-18/32^{nds} + 1/128th	97.5703125	97-182
97-18+ or 97-18½	97-18/32^{nds} + 1/64th	97.5781250	97-185
97-186 or 97-18¾	97-18/32^{nds} + 3/128^{ths}	97.5859375	97-187

But in the case of the cash markets, that trailing "2" means 2/8ths of 1/32nd = 1/128th. In the case of the futures markets that trailing "2" represents the truncated value of 0.25 x 1/32nd or 1/128th. A quote of 97-18+ in the cash markets is equivalent to 97-185 in the futures market. That trailing "5" represents 0.5 x 1/32nd or 1/64th. A quote of 97-186 in the cash markets is equivalent to 97-187 in the futures market. The trailing "7" represents the truncated value of 0.75 x 1/32nd = 3/128^{ths}.

The normal commercial "round-lot" in the cash markets is $1 million face value. Anything less might be considered an "odd-lot." However, you can purchase Treasuries in units as small as $1,000 face value. Of course, a dealer's inclination to quote competitive prices may dissipate as size diminishes. 30-year Treasury bond, 10-year Treasury note and 5-year Treasury note futures, however, are traded in units of $100,000 face value. 3-year and 2-year

Treasury note futures are traded in units of $200,000 face value.

Accrued Interest and Settlement Practices

In addition to paying the (negotiated) price of the coupon-bearing security, the buyer also typically compensates the seller for any interest accrued between the last semi-annual coupon payment date and the settlement date of the security.

E.g., it is January 10, 2013. You purchase $1 million face value of the 1-5/8% Treasury security maturing in November 2022 (a ten-year note) for a price of 97-18+ ($975,781.25) to yield 1.894%, for settlement on the next day, January 11, 2013.

In addition to the price of the security, you must further compensate the seller for interest of $2,558.70 accrued during the 57 days between the original issue date of November 15, 2012 and the settlement date of January 11, 2013.

This interest is calculated relative to the 57 days between issue date of November 15, 2012 and the next coupon payment date of May 15, 2013 or $2,558.70 [= (57/181) x ($16,250/2)]. The total purchase price is $978,339.95.

Price of Note	$975,781.25
Accrued Interest	$2,558.70
Total	$978,339.95

Typically, securities are transferred through the Fed wire system from the bank account of the seller to that of the buyer vs. cash payment. That transaction is concluded on the settlement date which may be different from the transaction date.

Unlike the futures market where trades are settled on the same day they are transacted, it is customary to settle a cash transaction on the business day subsequent to the actual transaction. Thus, if you purchase the security on a Thursday, you typically settle it on Friday. If purchased on a Friday, settlement will generally be concluded on the following Monday.

Sometimes, however, a "skip date" settlement is specified. *E.g.*, one may purchase a security on Monday for skip date settlement on Wednesday. Or, "skip-skip date" settlement on Thursday; "skip-skip-skip date" settlement on the Friday, etc. Skip or forward date settlements may be useful in order to match Treasury transaction payments with one's anticipated future cash flows at current market prices. Theoretically, there is no effective limitation on the number of days over which one may defer settlement. Thus, these cash securities may effectively be traded as forward contracts.

Treasury Auction Cycle

Treasury securities are auctioned on a regular basis by the U.S. Treasury which accepts bids on a yield basis from security dealers. A certain amount of each auction is set aside, to be placed on a non-competitive basis at the average yield filled.

Prior to the actual issuance of specific Treasuries, they may be bought or sold on a "WI" or "When Issued" basis. When traded on a WI basis, bids and offers are quoted as a yield rather than as a price.

U.S. Treasury Auction Schedule

	Maturity	**Auctioned**
Cash Mgt Bills	Usually 1-7 Days	As Needed
Treasury Bills	4-, 13- and 26-Week	Weekly
Treasury Bills	52-Week	Monthly
Treasury Notes	2-, 3-, 5- and 7-Year	Monthly
	10-Year	February, May, August & November with re-openings in other 8 months
Treasury Bonds	30-Year	February, May, August & November with re-openings in other 8 months
Treasury Inflation Protected Securities (TIPS)	5-Year	April with re-openings in August & December
	10-Year	January & July with re-openings in March, May, September and November
	3-Year	February with re-openings in June & October

After a security is auctioned and the results announced, the Treasury affixes a particular coupon to bonds and notes that is near prevailing yields. At that time, coupon bearing bonds and notes may be quoted on a price rather than a yield basis. However, bills continue to be quoted and traded on a yield basis. Trades previously concluded on a yield basis are settled against a price on the actual issue date of the security, calculated per standard price-yield formulae.

Security dealers purchase these securities and subsequently market them to their customers including pension funds, insurance companies, banks, corporations and retail investors. The most recently issued securities of a particular maturity are referred to as "on-the-run" securities. On-the-runs are typically the most liquid and actively traded of Treasury securities and, therefore, are often referenced as pricing benchmarks. Less recently issued securities are known to as "off-the-run" securities and tend to be less liquid.

The Treasury currently issues 4-week, 13-week, 26-week and 52-week bills; 2-year, 3-year, 5-year, 7-year and 10-year notes; and, 30-year bonds on a regular schedule. In the past, the Treasury had also issued securities with a 4-year and 20-year maturity. Further, the Treasury may issue very short term cash management bills along with Treasury Inflation Protected Securities or "TIPS."

The "Run"

If you were to ask a cash dealer for a quotation of "the run," he would quote yields associated with the on-the-run securities from the current on-the-run Treasury bills, through notes, all the way to the 30-year bond, sometimes referred to as the "long-bond" because it is the longest maturity Treasury available.

Quoting 'the Run'

(As of January 10, 2013)

	Coupon	Maturity	Price	Yield
4-Wk Bill		02/07/13		0.036%
13-Wk Bill		04/11/13		0.051%
26-Wk Bill		07/11/13		0.086%
52-Wk Bill		01/09/14		0.127%
2-Yr Note	1/8%	12/31/14	99-24 1/4	0.244%
3-Yr Note	3/8%	01/15/16	100-00	0.372%
5-Yr Note	3/4%	12/31/17	99-25 3/4	0.788%
7-Yr Note	1-1/8%	12/31/19	99-27+	1.294%
10-Yr Note	1-5/8%	11/15/22	97-18 3/4	1.895%
30-Yr Bond	2-3/4%	11/15/42	93-15	3.085%

The most recently issued security of any tenor may be referred to as the "new" security. Thus, the second most recently issued security of a particular original tenor may be referred to as the "old" security, the third most recently issued security is the "old-old" security, the fourth most recently issued security is the "old-old-old" security.

As of January 11, 2013, the most recently issued 10-year note was identified as the 1-5/8% note maturing in November 2022; the old note was the 1-5/8% note of August 2022; the old-old note was the 1-3/4% of May 2022; the old-old-old note was the 2% of February 2022.

Beyond that, one is expected to identify the security of interest by coupon and maturity. For example, the "2s of '21" refers to the note with a coupon of 2% maturing on November 15, 2021. As of January 10, 2013, there were not any "WI" or "when issued" 10-year notes. WIs typically quoted and traded on a yield basis in anticipation of the establishment of the coupon subsequent to the original auction.

10-Year Treasury Notes

(As of January 10, 2013)

	Coupon	Maturity	Price	Yield
WI				
On-the-Run	1-5/8%	11/15/22	97-18 3/4	1.895%
Old Note	1-5/8%	8/15/22	98-01 3/4	1.847%
Old-Old	1-3/4%	5/15/22	99-18 3/4	1.798%
Old-Old-Old	2%	2/15/22	102-04 3/4	1.743%
	2%	11/15/21	102-17 3/4	1.688%
	2-1/8%	8/15/21	103-28 3/4	1.637%
	3-1/8%	5/15/21	112-05 3/4	1.562%
	3-5/8%	2/15/21	116-04 1/4	1.501%
	2-5/8%	11/15/20	108-18	1.465%
	2-5/8%	8/15/20	108-22	1.414%
	3-1/2%	5/15/20	115-01+	1.341%
	3-5/8%	2/15/20	115-25+	1.288%
	1-1/8%	12/31/19	98-27 3/4	1.295%
	1%	11/30/19	98-05 3/4	1.277%
	3-3/8%	11/15/19	114-00 3/4	1.232%
	1-1/4%	10/31/19	99-31 3/4	1.251%
	1%	3/30/19	98-16 1/4	1.232%

One important provision is whether or not the security is subject to call. A "callable" security is one where the issuer has the option of redeeming the bond at a stated price, usually 100% of par, prior to maturity. If a bond is callable, it may be identified by its coupon, call and maturity date. *I.e.,* the 11-3/4% of November 2009-14 is callable beginning in November 2009 and matures in 2014.

Prior to the February 1986 auction, the U.S. Treasury typically issued 30-year bonds with a 25-

year call feature. That practice was discontinued at that time, however, as the Treasury instituted its "Separate Trading of Registered Interest and Principal on Securities" or STRIPS program with respect to all newly issued 10-year notes and 30-year bonds. [4]

The Roll and Liquidity

Clearly, traders who frequently buy and sell are interested in maintaining positions in the most liquid securities possible. As such, they tend to prefer on-the-run as opposed to off-the-run securities.

It is intuitive that on-the-runs will offer superior liquidity when one considers the "life-cycle" of Treasury securities. Treasuries are auctioned, largely to broker-dealers, who subsequently attempt to place the securities with their customers. Often these securities are purchased by investors who may hold the security until maturity. At some point, securities are "put-away" in an investment portfolio until their maturity. Or, they may become the subjects of a strip transaction per the STRIPS program.

As these securities find a home, supplies may become scare. As a result, bid/offer spreads may inflate and the security becomes somewhat illiquid. Liquidity is a valuable commodity to many. Thus, you may notice that the price of on-the-runs may be bid up, resulting in reduced yields, relative to other similar maturity securities. This tends to be most noticeable with respect to the 30-year bond.

Traders may be interested in conducting a "roll" transaction where one sells the old security in favor of the new security, in order to maintain a position in the on-the-run and most liquid security. Thus, dealers will quote a bid/offer spread in the roll, offering the opportunity to sell the old note/buy the new note; or, buy the old note/sell the new note, in a single transaction.

The "old note" in our table above was quoted at a yield of 1.847% while the "new note" was seen at 1.895%. In this case, the roll is quoted at approximately negative 5 basis points (-0.048% = 1.847% - 1.895%).

This circumstance runs contrary to our typical assumption that traders will be willing to forfeit a small amount of yield for the privilege of holding the most recently issued and presumably most liquid security, underscoring the point that liquidity normally has some observable value.

But circumstances as of January 10, 2013 were a bit unusual to the extent that the yield associated with the on-the-run 10-year note was higher than that associated with the old note. We suggest that this is indicative of heavy supplies and the rather steep shape of the yield curve in the 10-year segment of the curve.

Repo Financing

Leverage is a familiar concept to futures traders. Just as one may margin a futures position and thereby effectively extend one's capital, the Treasury markets likewise permit traders to utilize "repo" financing agreements to leverage Treasury holdings.

A repurchase agreement, repo or simply RP represents a facile method by which one may borrow funds, typically on a very short-term basis, collateralized by Treasury securities. In a repo agreement, the lender will wire transfer same-day funds to the borrower; the borrower wire transfers the Treasury security to the lender with the

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[4] The STRIPS program was created to facilitate the trade of zero-coupon Treasury securities. Prior to 1986, a variety of broker dealers including Merrill Lynch and Salomon Bros. issued zero-coupon securities collateralized by Treasuries under acronyms such as TIGeRs and CATS. For example, if you buy a 10-year Treasury, you can create zero coupon securities of a variety of maturities by marketing the component cash flows. By selling a zero collateralized by a coupon payment due in five years, one creates a five-year zero; or, one may create a ten-year zero by selling a zero collateralized by the principal payment. They engaged in this practice because the market valued the components of the security more dearly than the coupon payments and principal payment bundled together. Today, one might notice that the yield on a Treasury STRIP is usually less than a comparable maturity coupon-bearing Treasury. Beginning with 10s and 30s issued in February 1986, the Treasury began assigning separate CUSIP numbers to the principal value and to tranches of coupon payments associated with these securities. A CUSIP number is a code unique to each security and is necessary to wire-transfer and, therefore, market a security. Thus, the Treasury STRIPS market was created. As a result of their long duration, these securities are most popular when rates are declining and prices rising.

provision that the transactions are reversed at term with the lender wiring back the original principal plus interest.

The borrower is said to have executed a repurchase agreement; the lender is said to have executed a reverse repurchase agreement. Many banks and security dealers will offer this service, once the customer applies and passes a requisite credit check.

The key to the transaction, however, is the safety provided the lender by virtue of the receipt of the (highly-marketable) Treasury security. These repo transactions are typically done on an overnight basis but may be negotiated for a term of one-week, two-weeks, one month. A third party custodian is frequently used to add an additional layer of safety between the lender and borrower, *i.e.,* a tri-party repo agreement. Overnight repo rates are typically quite low in the vicinity of Fed Funds.

Any Treasury security may be considered "good" or "general" collateral. Sometimes when particular Treasuries are in short supply, dealers will announce that the security is "on special" and offer below-market financing rates in an effort to attract borrowers.

Treasury Futures Delivery Practices

While some traders refer to original or "classic" Treasury bond futures as "30-year bond futures," that reference is actually quite misleading. Treasury bond futures permit the delivery in satisfaction of a maturing contract of *any* U.S. Treasury security provided it matures within a range of 15 to 25 years from the date of delivery. That delivery window once extended from 15 to 30 years and, thus, the characterization of the Treasury bond contract as a "30-year bond futures" was apt.

Note that the Ultra T-bond futures contract calls for the delivery of any bond that does not mature for a period of at least 25 years from the date of delivery. Subsequent to the development of the Ultra bond contract, the delivery window of the original T-bond futures contract was amended from 15-30 years to 15-25 years. As such, the Ultra T-bond futures contract currently is most aptly referred to as the 30-year bond contract while the original bond futures contract, as amended, is referred to as the "classic" bond futures contract.

It is likewise tempting to refer to U.S. Treasury bond and note futures as "6% contracts." This too may be somewhat misleading. T-bond and T-note futures are based *nominally* upon a 6% coupon security.

But in point of fact, the contract permits the delivery of *any* coupon security, again provided that it meets the maturity specification mentioned above. *I.e.,* shorts are not necessarily required to deliver 6% coupon bonds. In fact, there may be no eligible for delivery securities that actually carry a coupon of precisely 6% at any given time.

Because of the rather broadly defined delivery specifications, a significant number of securities, ranging widely in terms of coupon and maturity, may be eligible for delivery. This applies with equal effect to 2-, 3-, 5- and 10-year Treasury note futures; as well as the classic and Ultra T-bond futures contracts.

Table 1 included below provides a complete description of the contract specifications of CME Group Treasury futures products.

Conversion Factor Invoicing System

Securities with varying characteristics, such as coupon and maturity, will of course be more or less valued by the investment community. High-coupon securities, for example, will naturally command a greater price than comparable low-coupon securities.

These differences must be reflected in the futures contract. In particular, when a short makes delivery of securities in satisfaction of a maturing futures contract, the long will pay a specified invoice price to the short.

As discussed above, the futures contract permits the delivery of a wide range of securities at the discretion of the short. That invoice value must be adjusted to reflect the specific pricing characteristics of the security that is tendered.

Accordingly, Treasury futures utilize a "conversion factor" invoicing system to reflect the value of the security that is tendered by reference to the 6% futures contract standard. The "Principal Invoice Amount" paid from long to short upon delivery may be identified as the Futures Settlement Price multiplied by the Conversion Factor (CF) multiplied by $1,000.

That $1,000 constant reflects the $100,000 face value futures contract size associated with most T-note and T-bond futures. Note that the 2-year T-note contract is based on a $200,000 face value amount. Thus, this constant must be reset at $2,000 for 2-year Treasury futures.

$$\begin{aligned}&\mathit{Principal\ Invoice\ Price}\\&= \mathit{Futures\ Settlement\ x\ Conversion\ Factor}\ (\mathit{CF})\ x\ \$1{,}000\end{aligned}$$

Any interest accrued since the last semi-annual interest payment date is added to the principal invoice amount to equal the "total invoice amount."

$$\begin{aligned}\mathit{Total\ Invoice\ Amount}&\\&= \mathit{Principal\ Invoice\ Amount}\\&+ \mathit{Accrued\ Interest}\end{aligned}$$

A conversion factor may be thought of as the price of the delivered security as if it were yielding 6%. Clearly, high-coupon securities will tend to have high CFs while low-coupon securities will tend to have low CFs. In particular, bonds with coupons less than the 6% contract standard will have CFs that are less than 1.0; bonds with coupons greater than 6% have CFs greater than 1.0.

E.g., the conversion factor for delivery of the 3-3/8% T-note of 2019 vs. March 2013 10-year T-note futures is 0.8604. This suggests that a 3-3/8% security is approximately valued at 86% as much as a 6% security. Assuming a futures price of 131-23+/32nds (or 131.734375 expressed in decimal format), the principal invoice amount may be calculated as follows.

$$\begin{aligned}\mathit{Principal\ Invoice\ Price} &= 131.734375\ x\ 0.8604\ x\ \$1{,}000\\&= \$113{,}344.26\end{aligned}$$

E.g., the conversion factor for delivery of the 1-3/4% T-note of 2022 vs. March 2013 10-year T-note futures is 0.7077. This suggests that a 1-3/4% security is approximately valued at 71% as much as a 6% security. Assuming a futures price of 131-23+ (131.735375), the principal invoice amount may be calculated as follows.

$$\begin{aligned}\mathit{Principal\ Invoice\ Price} &= 131.734375\ x\ 0.7077\ x\ \$1{,}000\\&= \$93{,}228.42\end{aligned}$$

In order to arrive at the total invoice amount, one must of course further add any accrued interest since the last semi-annual interest payment date to the principal invoice amount.

Cheapest-to-Deliver

The intent of the conversion factor invoicing system is to render equally economic the delivery of any eligible-for-delivery securities. Theoretically, the short who has the option of delivering any eligible security should be indifferent as to his selection.

However, the CF system is imperfect in practice as we find that a particular security will tend to emerge as "cheapest-to-deliver" (CTD) after studying the relationship between cash security prices and principal invoice amounts.

E.g., on January 10, 2013, one might have been able to purchase the 3-3/8%-19 at 114-00¾ ($114,023.44 per $100,000 face value unit). The 1-¾%-22 was valued at 99-18¾ ($99,585.94 per $100,000 face value unit). Compare these cash values to the principal invoice amounts as follows.

	3-3/8%-19	**1-3/4%-22**
Futures Price	131-23+	131-23+
x CF	0.8604	0.7077
x $1,000	$1,000	$1,000
Principal Invoice	$113,344.26	93,228.42
Cash Price	($114,023.44)	($99,585.94)
Delivery Gain/Loss	**($679.18)**	**($6,357.52)**

Our analysis suggests that a loss of $679.18 may be associated with the delivery of the 3-3/8%-19 while an even larger loss of $6,357.52 might be associated with the delivery of the 1-3/4%-22. Thus, we might conclude that the 3-3/8%-19 note is cheaper or more economic to deliver than the 1-3/4%-22.

The Basis

Typically we expect to find a single security, or perhaps a handful of similar securities, will emerge as CTD. This identification has important implications for basis traders who arbitrage cash and futures markets. A basis trader will seek out arbitrage opportunities or situations where they might be able to capitalize on relatively small pricing discrepancies between cash securities and Treasury futures by buying "cheap" and selling "rich" items.

Arbitrageurs will track these relationships by studying the "basis." The basis describes the relationship between cash and futures prices and may be defined as the cash price less the "adjusted futures price" or the futures price multiplied by the conversion factor.

Thus, the basis is analogous to the gain or loss that might be realized upon delivery. Unlike that gain or loss, however, the basis is typically expressed in terms of 32nds. *E.g.*, 1-1/4 points might be shown as 40/32nds. It is also "inverted" in the sense that we are comparing cash less adjusted futures prices - rather than futures invoice price less cash prices.

$$Basis = Cash\ Price - Adjusted\ Futures\ Price$$

$$Adjusted\ Futures\ Price = Futures\ Price\ x\ Conversion\ Factor$$

E.g., a comparison of cash and adjusted futures prices provides us with a quote for the basis associated with the 3-3/8%-19 and 1-3/4%-22 Treasury securities.

	3-3/8%-19	1-3/4%-22
Cash Price	114-00¾	99-18¾
Futures Price	131-23+	131-23+
x CF	0.8604	0.7077
Adjusted Futures	(113-11)	(93-072)
Basis (32nds)	**21.734**	**203.441**

The basis of 21.734/32nds associated with the 3-3/8%-19 corresponds to a loss on delivery of $679.18 as shown above. Similarly, the basis of 203.441/32nds associated with the 1-3/4%-22 corresponds to a loss on delivery of $6,357.52.

As suggested above, and as a general rule, the security with the lowest basis, *i.e.,* the largest gain or smallest loss on delivery, may be considered CTD. Clearly, the 3-3/8%-19 is cheaper-to-deliver than the 1-3/4%-22.

Table 2 included below depicting the basis for all eligible-for-delivery securities vs. the March 2013 10-year T-note futures contract as of January 10, 2013. Referring to Table 2, one may confirm that the 3-3/8%-19 exhibited the lowest basis and, therefore, may be considered *the* CTD security.

Note, however, that there are quite a few securities, with similar coupons and maturities, which are near CTD. In fact, the entire universe of eligible-for-delivery securities features reasonably similar coupons and maturities.

It is important to identify the CTD security to the extent that Treasury futures will tend to price or track or correlate most closely with the CTD. This has interesting implications from the standpoint of a "basis trader" or a hedger as discussed in more detail below.

Suffice it to say at this point that basis trading is a frequent practice in the Treasury futures markets. Certain terminology has been developed to identify basis positions. One may "buy the basis" by buying cash securities and selling futures. One may "sell the basis" by selling cash securities and buying futures.

Basis transactions are typically transacted in a ratio that reflects the conversion factor of the security involved in the trade.

"Buy the Basis" = Buy cash securities & sell futures

"Sell the Basis" = Sell cash securities & buy futures

E.g., if one were to buy the basis by buying $10 million face value of the 3-3/8%-19 note, one might sell 86 March 2013 futures by reference to the conversion factor of 0.8604.

E.g., if one were to sell the basis by selling $10 million face value of the 1-3/4%-22 note, one might buy 71 March 2013 futures by reference to the conversion factor of 0.7077.

By transacting the basis in a ratio identified by reference to the CF, one may roughly balance the movement or volatility on both legs of the spread. This is intuitive to the extent that the conversion factor generally reflects the value of the cash position relative to that of the futures contract. If the CF reflects relative value then presumably it will reflect relative volatility or price movement as well.

Why Is One Issue CTD?

If the conversion factor invoicing system performed flawlessly, all eligible-for-delivery securities would have a similar basis and be equally economic to deliver. As suggested above, however, a single security or several similar securities tend to emerge as CTD.

The CF invoicing system is imperfect because it is implicitly based on the assumption that - (1) all eligible-for-delivery securities have the same yield; and (2) that yield is 6%. But there are any number of "cash market biases" that impact upon the yield of a Treasury security.

Further mathematical biases in the conversion factor calculation will tilt the field towards securities of particular coupons and maturities when yields are greater than or less than the 6% contract standard. Hence, we may further speak of "conversion factor biases."

Cash Market Biases

Cash market bias may be used as a catch-all phrase for anything that impacts upon the relative yields of bonds. Perhaps "supply-demand considerations" is an equally appropriate term. A key concept is that shorts will elect to deliver securities that are cheaper relative to other securities.

Some specific reasons why securities, even those with similar coupons and maturities, may carry somewhat different yields include the shape of the yield curve, reinvestment risks, liquidity preferences, tax considerations, etc.

In an upwardly sloping or "normal" yield curve environment, longer-term securities may carry somewhat higher yields than comparable shorter-term securities. Per an inverted yield curve, shorter-term securities may offer higher yields. Thus, a steep yield curve may bias towards the delivery of lower-yielding securities of longer maturities. While an inverted yield curve may bias towards the delivery of shorter maturity securities.

Prior to the subprime mortgage crisis that erupted in 2008, the yield curve has been rather flat out past 15 years. Thus, this factor has, as an historical matter, had little impact on the delivery of bonds into the 30-year T-bond contract. More recently, however, we see that the yield curve has generally steepened such that the different between 10- and 30-year yields has expanded to greater than 1%. Still, this factor may not be terribly overt as it tends to be obscured by conversion factor biases as discussed below.

Low or generally falling yields may prove problematic to the security investor to the extent that a significant component of one's return is attributable to reinvestment income. Coupon payments, once received, will be reinvested, presumably at prevailing short-term rates. When reinvestment risks become noticeable, investors will prefer low-coupon securities, generating small coupons carrying limited reinvestment risks, over high-coupon securities. Thus, those high-coupon securities may become CTD.

As discussed above, recently issued or "on-the-run" securities generally offer enhanced liquidity relative to "off-the-run" securities. Consequently, on-the-run bond prices may be bid up, their yields pushed down and may, therefore, be unlikely candidates to become CTD. Likewise, tax considerations have the potential to tilt deliveries towards high coupon as opposed to low coupon securities.

Conversion Factor Biases

Perhaps more important that these cash market factors, there are observable biases associated with the mathematics of the conversion factor system or conversion factor biases.

E.g., it is clear that long duration, *i.e.,* low-coupon, long-maturity securities, will become CTD when yields are significantly greater than the 6% contract standard. When yields fall below the 6% contract standard, these factors will bias towards the delivery of short-duration, *i.e.,* high-coupon, short-maturity securities.

If yields > 6%	➔	**Bias to long duration (*i.e.,* low-coupon, long-maturity) securities**
If yields < 6%	➔	**Bias to short duration (*i.e.,* high-coupon, short-maturity) securities**

Duration is explained more thoroughly below but think of duration as a measure of risk. When yields are rising and prices are declining, investors will gravitate towards less risky or short-duration securities. They will want to liquidate riskier long duration securities, creating a delivery bias in favor of those long duration bonds. *I.e.,* while the conversion factor is fixed, the relative price movement of long vs. short duration securities may be very different, thereby creating delivery biases.

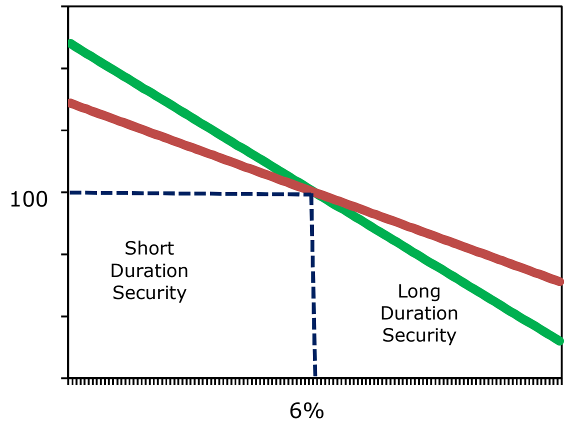
On the other hand, when yields are declining and prices rising, investors will prefer those riskier long duration securities. Thus, they may wish to liquidate less aggressive short duration securities, creating a delivery bias in favor of those short duration securities.

As indicated above, the 3-3/8%-19 was cheapest-to-deliver as of January 10, 2013 vs. the March 2013 Ten-year T-note futures contract. This security, by virtue of its relatively high coupon and short maturity, stood out as the security with the lowest duration of 6.153 years amongst the field of eligible-for-delivery securities. Because yields of 10-year notes were in the range of 1.2% to 1.9%, conversion factor biases severely tilted deliveries towards short duration securities such as the 3-3/8%-19.

Many analysts who consider Treasury basis relationships describe the transaction as a form of arbitrage. Arbitrage, in turn, is often characterized as a riskless or near-riskless transaction. But arbitrage transactions are defined by a "riskless" nature, then Treasury basis trading is certainly not an arbitrage because, of course, the basis may fluctuate to a considerable extent.

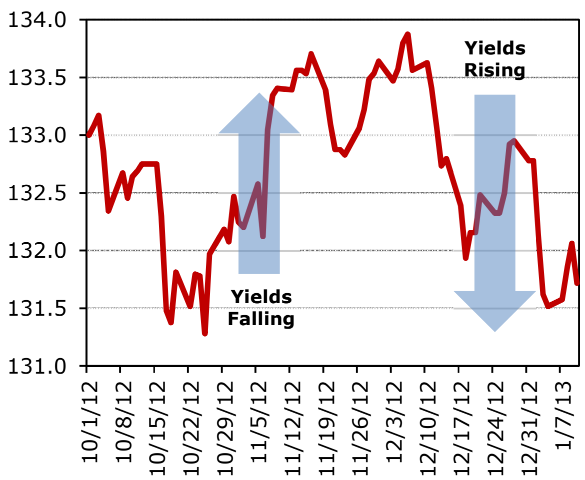
Other analysts suggest that arbitrage transactions are not necessarily riskless. Rather, it may be the case that the value of the transaction is dictated by considerations apart from simple price movement. But if independence from directional price movement is a defining feature of an arbitrage, Treasury basis transactions likewise cannot be considered an arbitrage *per se*. Frequently the most overt factor that dictates the movement of a basis trade is simple directional price movement.

CTD Driven by Yields



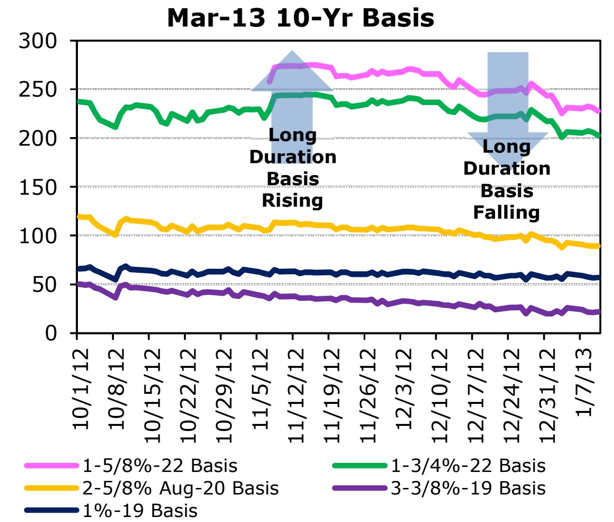
Consider the period between October 2012 and early January 2013 as depicted in our graphic. During this period, the price of the March 2013 Ten-year T-note futures experienced a price advance from approximately 131% to 134% of par. Subsequently, the market reversed downwards along a similar scale.

Mar-13 10-Yr Note Futures



In other words, prices advanced as yields fell, only to be followed by a period where prices declined on rising yields. During the entirety of this period, prices were well above par while yields were well below the 6% futures contract standard. Still, conversion factor biases were diminished or weakened as prices declined only to strengthen once again as the market rallied back.

The impact of these weakening and subsequently strengthening conversion factor biases may be observed by examining the basis for several eligible-for-delivery securities. Actually, the simple and graudual convergence of cash and futures prices may be the feature that is most apparent from an examintion of this graphic.



As prices advanced and yields fell in late October into early November, notice that the basis for long duration securities such as the 1-¾%-22 was buoyed upwards to the extent that its price rose faster than futures price which traced a shorter duration CTD. Again, as yields fall below or further below the 6% futures contract standard, long duration securities tend to become less economic to deliver.

Yields Rising above 6%	Yields Falling Under 6%
Sell long duration basis, *i.e.,* sell long duration securities & buy futures	Buy long duration basis, *i.e.,* buy long duration securities & sell futures
Buy short duration basis, *i.e.,* buy short duration securities & sell futures	Sell short duration basis, *i.e.,* sell short duration securities & buy futures

As prices declined and yields rose in December and into January, the basis for long duration securities such as the 1-5/8%-22 or the 1-3/4%-22 tended to decline more sharply than the basis for short duration securities such as the CTD 3-3/8%-19. This is consistent with our observation above that, as yields rise, long duration securities tend to become more economic to deliver.

Finally, note that 3-3/8%-19 remained cheapest to deliver throughout the period in question, during which time its basis converged rather steadily down towards zero.

It is clear that the performance of the basis is strongly driven by directional price movement in the Treasury markets. Thus, "buying the basis" or "selling the basis" may be motivated by expectations regarding rising or falling yields. The key is to get a sense of market direction and then identify the long or short duration securities whose basis values will be impacted by any sizable price (or yield) movement.

Implied Repo Rate

We often suggest that the security with the lowest basis is cheapest-to-deliver. But to be perfectly correct, we may point out that the structure of coupon receipts and reinvestment of such coupon income plays some (generally small) part in establishing a particular security as cheapest-to-deliver as well. Hence, traders often calculate the "implied repo rate" (IRR) associated with eligible for delivery securities to account for such factors.

The IRR is calculated as the annualized rate of return associated with the purchase of a security, sale of futures and delivery of the same in satisfaction of the maturing futures contract. This calculation takes into account all the cash flows associated with the security. The assumption that the basis for any particular security may completely converge to zero is implicit in the IRR calculation.

E.g., if one were to buy the 3-3/8%-19 basis by buying the cash securities, selling futures in a ratio dictated by the conversion factor and making delivery, or at least witnessing full cash-futures convergence, one would lock-in a return of 0.121%.

E.g., if one were to buy the 1-3/4%-22 basis by buying cash securities and selling futures in a ratio indicated by reference to the conversion factor and making delivery, or at least witnessing full cash-futures convergence, one would lock-in a rate of return of -28.414%.

Clearly, it would be preferable to lock-in a return of 0.121% rather than a return of -28.414%. Thus, the 3-3/8%-19 is cheaper to deliver relative to the 1-3/4%-22. In fact, if we scan the IRRs associated with all securities eligible to be delivered into the March 2013 contract in Table 2 below, we find that the IRR of 0.121% associated with the 3-3/8%-19 is superior to all other IRRs.

Thus, the 3-3/8%-19 Treasury security is associated with the lowest basis and the highest IRR as of January 10, 2013. As a general rule, the security with the lowest basis will likewise exhibit the highest implied repo rate. It is possible that a security with the lowest basis may not quite have the highest IRR because of cash flow considerations. But this statement is generally true. In any event, this observation confirms the CTD status of the 3-3/8%-19 as of January 10, 2013.

By buying the basis of a Treasury security, or buying cash and selling futures, one becomes obligated to make delivery of the Treasury in satisfaction of the maturing futures contract. [5] Thus, buying the basis of the cheapest-to-deliver 3-3/8%-19 vs. a futures contract that matures two or three months hence, may be considered analogous to other short-term investment alternatives.

E.g., we might compare the IRR = 0.121% associated with the CTD security to the prevailing 13-week T-bill yield of 0.051%; or to the effective Fed Funds rate of 0.160%; or, to a 3-month LIBOR rate at 0.300%.

In this example, the IRR associated with the CTD security was essentially equivalent to other short-term investment opportunities. As a general rule, however, the IRR even for the CTD security tends to run at a level that is a bit inferior to the returns associated with comparable short-term investment alternatives. The IRRs associated with all other non CTD securities are even lower.

This begs the question - why would anyone ever want to buy the basis if the returns do not appear to be competitive? The answer lies in the fact that the basis conveys other opportunities apart simply from the opportunity to use the futures contract as a delivery conveyance.

Consider any discrepancy with respect to the CTD to represent a risk premium of sorts. If one buys the CTD security and sells futures with the intention of making delivery, the worst case scenario has the basis converging fully to zero and the hedger essentially locking in a return equal to the IRR, in this case 0.121%.

But if market conditions should change such that another security becomes CTD, this implies that the basis may advance, or at least fail to completely converge to zero. As a result, the trader may realize a rate of return that is in fact greater than the currently calculated IRR.

Basis Optionality

In other words, there is a certain degree of "optionality" associated with the purchase or sale of the basis. Buying the basis is analogous to buying an option which, of course, implies limited risk. Buying the basis implies limited risk to the extent that, even under the worst of circumstances, you make delivery of the security which is effectively equivalent to the possibility that the basis fully converges to zero.

But "crossovers" or "switch" may occur such that the basis converges at a slower rate than otherwise anticipated or actually advances. As a result, this short-term investment may generate a return which is (at least theoretically) unbounded on the upside. Limited risk accompanied by unbounded upside potential is reminiscent of the risk/reward profile of a long option position, thus the analogy between a long basis position and a long option.

The best one may hope by selling the basis, or selling securities and buying futures with the possibility of effectively replacing the sold security by standing long in the delivery process, is that the basis fully converges to zero. This implies limited profit potential.

But in the event of significant changes in market conditions, the basis may advance sharply, exposing the seller of the basis to (theoretically) unbounded risks. Limited profit potential accompanied by unbounded risk is reminiscent of the risk/reward

[5] One may, of course, opt to offset the short futures contract prior to the delivery period and effectively abrogate such obligation.

profile of a short option position, thus the analogy between a short basis position and a short option.

As discussed above, the basis even for the CTD security tends to be in excess of cost of carry considerations. This is manifest in the fact that the IRR even for the CTD is typically a bit below prevailing short-term rates. This premium in the basis essentially reflects the uncertainties associated with which security may become CTD in the future.

Thus, the basis performs much akin to an option. Like any other option, the basis will be affected by considerations including term, volatility and strike price. The relevant term in this case is the term remaining until the presumed delivery date vs. the futures contract. Market volatility affects the probability that a crossover may occur. Rather than speak of a strike or exercise price, it is more appropriate to assess the market's proximity to a "crossover point" or a price/yield at which one might expect an alternate security to become CTD.

Consider the purchase or sale of the CTD basis. The degree to which this basis performs like a call or a put option is contingent upon the relationship between market prices and the 6% futures contract standard.

If yields are below the 6% futures contract standard, the CTD basis may be expected to advance if prices decline (rates rise) towards 6%; or, decline if prices advance (rates fall). Thus, buying the CTD basis when rates are below 6% is akin to the purchase of a put option. Conversely, the sale of the CTD basis when rates are less than 6% is akin to the sale of a put option where the value of transaction is capped if prices should advance while losses may be unbounded if prices should decline.

If yields are above the 6% futures contract standard, the CTD basis may be expected to advance if prices rise (rates fall) towards 6%; or, decline if prices fall (rates rise). Thus, buying the CTD basis when rates are above 6% is akin to the purchase of a call option. Conversely, the sale of the CTD basis when rates are above 6% is akin to the sale of a call option where the value of transaction is capped if prices should decline while losses may be unbounded if prices should advance.

Finally, if rates are close to the 6% futures contract standard, the basis for what is currently CTD may be dictated by considerations apart from conversion factor biases.

Thus, there may be significant crossovers regardless of whether rates rise or fall. Buying the CTD basis under these considerations may be considered akin to the purchase of an option straddle (*i.e.,* the simultaneous purchase of call and put options).

Under these circumstances the basis buyer may be indifferent between advancing or declining prices but has an interest in seeing prices move significantly in either direction. Selling the CTD basis when rates are near the 6% contract standard is akin to selling a straddle (*i.e.*, the simultaneous sale of both call and put options). The basis is sold under these circumstances because the trader anticipates an essentially neutral market.

	Buy CTD Basis	**Sell CTD Basis**
Yields < 6%	Buy Put Option	Sell Put Option
Yields = 6%	Buy Straddle	Sell Straddle
Yields > 6%	Buy Call Option	Sell Call Option

Of course, the basis premium over carry should accrue to the short basis trader under circumstances of continued price stability. But the short basis trader is exposed to the risk of dramatic price movements in either direction.

As of January 10, 2013, the IRR of the CTD 3-3/8%-19 security at 0.121% fell squarely within the range of other short-term investment alternatives. This suggests negligible optionality, *i.e.,* the probability of a crossover or switch is negligible. This is driven by the fact that yields are well below the 6% futures contract standard. Further, the duration of the 3-3/8%-19, with its high coupon and short maturity, was the shortest by some margin relative to other eligible for delivery securities. Thus, the market assessed a negligible probably that this security would not remain CTD by the time we enter the March 2013 delivery period.

Measuring Risk

"You can't manage what you can't measure" is an old saying with universal application. In the fixed income markets, it is paramount to assess the volatility of one's holdings in order reasonably to manage them. The particular characteristics of a coupon-bearing security will clearly impact upon its volatility.

Two readily identifiable ways to define coupon-bearing securities is in terms of their maturity and coupon. Defining volatility as the price reaction of the security in response to changes in yield we might draw conclusions as follows.

Longer Maturity ↑ Greater Volatility ↑
Higher Coupon ↑ Lower Volatility ↓

All else held equal, the longer the maturity of a fixed income security, the greater its price reaction to a change in yield. This may be understood when one considers that the implications of yield movements are felt over longer periods, the longer the maturity.

On the other hand, high coupon securities will be less impacted, on a percentage basis, by changing yields than low coupon securities. This may be understood when one considers that high coupon securities return a greater portion of one's original investment sooner than low coupon securities. Your risks are reduced to the extent that you hold the cash!

There are a couple of popular ways to measure the risks associated with coupon-bearing (and money-market) instruments including basis point value (BPV) and duration.

Basis Point Value (BPV)

BPV represents the absolute price change of a security given a one basis point (0.01%) change in yield. These figures may be referenced using any number of commercially available quotation services or software packages. BPV is normally quoted in dollars based on a $1 million (round-lot) unit of cash securities. The following table depicts the BPVs of various on-the-run Treasuries as of January 10, 2013.

E.g., this suggests that if the yield on the 30-year bond were to rise by a single basis point (0.01%), the price should decline by some $1,858 per $1 million face value unit.

Duration

If BPV measures the absolute change in the value of a security given a yield fluctuation; duration may be thought of as a measure of relative or percentage change. The duration (typically quoted in years) measures the expected percentage change in the value of a security given a one-hundred basis point (1%) change in yield.

Duration is calculated as the average weighted maturity of all the cash flows associated with the bond, *i.e.,* repayment of "corpus" or face value at maturity plus coupon payments, all discounted to their present value.

Measuring Volatility

(As of January 10, 2013)

	Coupon	Mat-urity	Dur-ation (Yrs)	BPV (per mil)
2-Yr Note	1/8%	12/31/14	1.965	$196
3-Yr Note	3/8%	01/15/16	2.980	$298
5-Yr Note	3/4%	12/31/17	4.867	$486
7-Yr Note	1-1/8%	12/31/19	6.676	$660
10-Yr Note	1-5/8%	11/15/22	9.016	$882
30-Yr Bond	2-3/4%	11/15/42	19.788	$1,858

E.g., the 30-year bond is associated with duration of 19.788 years. This implies that if its yield advances by 100 basis points (1.00%), we expect a 19.788% decline in the value of the bond.

In years past, it was commonplace to evaluate the volatility of coupon-bearing securities simply by reference to maturity. But this is quite misleading. If one simply examines the maturities of the current 2-year note and 10-year note, one might conclude that the 10-year is 5 times as volatile as the 2-year.

But by examining durations, we reach a far different conclusion. The 10-year note (duration of 9.016 years) is only about 4-½ times as volatile as the 2-year note (duration of 1.965 years). The availability of cheap computing power has made duration analysis as easy as it is illuminating.

Risk Management

Treasury futures are intended to provide risk averse fixed income investors with the opportunity to hedge or manage the risks inherent in their investment activities. Effective use of these contracts, however, requires a certain grounding in hedge techniques.

Most pointedly, one may attempt to assess the relative volatility of the cash item to be hedged relative to the futures contract price. This relationship is often identified as the futures "Hedge Ratio" (HR). Hedge ratios reflect the expected relative movement of cash and futures and provide risk managers with an indication as to how many futures to use to offset a cash exposure.

Face Value Weighted Hedge

The most superficial way to approach identification of the appropriate hedge ratio is simply to match the face value of the item to be hedged with the face value of the futures contract.

E.g., if one owned $10 million face value of a particular security, the natural inclination is to sell or short one-hundred (100) $100,000 face value futures contracts for a total of $10 million face value. Thus, the face value of hedged security matches the face value held in futures.

While this method has the advantage of extreme simplicity, it ignores the fact that securities of varying coupons and maturities have different risk characteristics.

CF Weighted Hedge

Treasury futures contract specifications conveniently provide a facile means by which to assess the relative risks associated with cash and futures. As discussed above, the conversion factor (CF) represents the price of a particular bond as if it were to yield 6%.
Thus, the CF reflects the *relative value* and, by implication, the *relative volatility* between cash and futures prices. Most basis trades are in fact concluded in a ratio identified by reference to the CF.

E.g., if one held $10 million face value of the 3-3/8%-19 note, one might sell 86 March 2013 futures by reference to the conversion factor of 0.8604 to execute a hedge.

E.g., if one held $10 million face value of the 1-3/4%-22 note, one might sell 71 March 2013 futures by reference to the conversion factor of 0.7077 to execute a hedge.

A conversion factor weighted hedge is likely to be quite effective if you are hedging the cheapest-to-deliver security. Treasury futures will tend to price or track or correlate most closely with the CTD security.

But other securities with different coupons and maturities may react to changing market conditions differently. Thus, one might question if you can or should do better than a CF weighted hedge?

BPV Weighted Hedge

In order to understand the most effective techniques with which to apply a hedge, consider the fundamental objective associated with a hedge. An "ideal" hedge is intended to balance any loss (profit) in the cash markets with an equal and opposite profit (loss) in futures.

Our goal, therefore, is to find a hedge ratio (HR) that allows one to balance the change in the value of the cash instrument to be hedged (Δ_{hedge}) with any change in the value of the futures contract ($\Delta_{futures}$). Note that we use the Greek letter delta or Δ to denote the abstract concept of change in value.

$$\Delta_{hedge} = HR \ x \ \Delta_{futures}$$

We solve for the hedge ratio (HR) as follows.

$$HR = \Delta_{hedge} \div \Delta_{futures}$$

Because we have not defined what we mean by "change in value," the equation above is of an abstract nature and cannot be directly applied. Thus, let's backtrack to discuss the relationship between Treasury futures and cash prices.

Per our discussion above, principal invoice amount paid from long to short upon deliver will be equal to the price of the cash security multiplied by its conversion factor. Rational shorts will, of course, elect to tender the cheapest-to-deliver security. Thus, we might designate the futures price and the conversion factor of the cheapest-to-deliver as $P_{futures}$ and CF_{ctd}, respectively.

$$Principal\ Invoice\ Price = P_{futures} \ x \ CF_{ctd}$$

Because the basis of the CTD is generally closest to zero, relative to all other eligible securities, we might assume that the futures price level and, by implication, any changes in the futures price level ($\Delta_{futures}$) will be a reflection of any changes in the value of the CTD (Δ_{ctd}) adjusted by its conversion factor (CF_{ctd}) as follows.

$$\Delta_{futures} = \frac{\Delta_{ctd}}{CF_{ctd}}$$

Substituting this quantity into our equation specified above, we arrive at the following formula.

$$HR = \Delta_{hedge} \div \left(\frac{\Delta_{ctd}}{CF_{ctd}}\right)$$

We might further rearrange the equation as follows.

$$HR = CF_{ctd}\ x \left(\frac{\Delta_{hedge}}{\Delta_{ctd}}\right)$$

Unfortunately, this concept of "change in value" remains abstract. Let us "operationalize" the concept by substituting the basis point value of the hedged security (BPV_{hedge}) and the basis point value of the cheapest-to-deliver (BPV_{ctd}) for that abstract concept.

Recall from our discussion above that a basis point value represents the expected change in the value of a security, expressed in dollars per $1 million face value, given a one basis point (0.01%) change in yield. Thus, we identify the basis point value hedge ratio (or "BPV HR") as follows.

$$BPV\ HR = CF_{ctd}\ x \left(\frac{BPV_{hedge}}{BPV_{ctd}}\right)$$

Our analysis implicitly assumes that any changes in the yield of the hedged security and that of the cheapest-to-deliver security will be identical. *I.e.,* that we will experience "parallel" shifts in the yield curve. This analysis further presumes that you are able to identify the cheapest-to-deliver security and that it will remain cheapest-to-deliver. The latter assumption is, of course, questionable in a dynamic market.

E.g., let us find the basis point value hedge ratio (HR) required to hedge $10 million face value of the 1-3/4%-22 note security. This security carried a BPV = $8,550 per $10 million. The CTD security was the 3-3/8%-19 with a BPV = $70.50 per $100,000 face value and a conversion factor of 0.8604 vs. March 2013 Ten-year T-note futures. The hedge ratio may be identified as 104 contracts per $10 million face value of the 1-3/4%-12.

$$BPV\ HR = 0.8604\ x \left(\frac{\$8{,}550}{\$70.50}\right) = 104.3\ or\ 104\ contracts$$

Note that the HR = 104 is significantly greater than the 71 contracts suggested by reference to the conversion factor of the 1-3/4%-22 security. This is due to the fact that the CTD security carries a relatively short duration of 6.153 years compared to the duration associated with the hedged security of 8.588 years.

It is no coincidence that the ratio of durations is roughly equal to the ratio between the BPV and CF hedge ratios or (6.153 ÷ 8.558) ~ (71 ÷ 104). *I.e.,* the futures contract is pricing or tracking or correlating most closely with a shorter duration security. Consequently, futures prices will react rather mildly to fluctuating yields. Therefore one requires more futures to enact an effective hedge.

E.g., what would our hedge ratio be if the CTD security was the on-the-run 1-5/8%-22 with a rather longer duration of 9.016 years? This security has a BPV of $88.20 per $100,000 face value and a conversion factor for delivery vs. March 2013 Ten-year T-note futures of 0.6867. Our analysis suggests that one might hedge with 77 contracts per $10 million face value of the 2-5/8%-20.

$$BPV\ HR = 0.6867\ x \left(\frac{\$8{,}550}{\$88.20}\right) = 66.6\ or\ 67\ contracts$$

Note that this hedge ratio of 67 contracts is significantly less than the 104 contracts suggested by our analysis above and reasonably similar to the 71 contracts suggested by the CF hedge ratio. This can be explained by the fact that the 1-5/8%-22 has pricing characteristics that are quite similar to 1-3/4%-22 security which is the subject of the hedge. In particular, the 1-5/8%-22 had a duration of 9.016 years which is reasonably close to the 8.558 duration of the 1-3/4%-22. Because of the similar risk characteristics of the CTD and hedged security, the CF may do a reasonable job of identifying an appropriate hedge ratio.

Crossover Risks

This further suggests that, if there is a crossover in the CTD from a short duration security to a longer duration security, the number of futures needed to hedge against the risk of declining prices is decreased. This may be a favorable circumstance for the hedger who is long cash Treasuries and short futures in a ratio prescribed by the BPV technique.

Consider that as prices decline and longer duration securities become CTD, one is essentially over-hedged in a declining market. If on the other hand, prices advance and even shorter duration securities become CTD, the appropriate hedge ratio will tend to increase. Thus, the long hedger becomes under-hedged in a rising market.

Another way of saying this is that there is a certain degree of "convexity" inherent in the relationship that favors the long hedger or long basis trader (long cash and short futures). Conversely, this convexity tends to work to the disadvantage of the short hedger or short basis trader (short cash and long futures).

Once again, we may liken the basis to an option to the extent that option premiums are also affected by convexity. Further, because the long basis trader effectively owns the option, he pays an implicit premium in the difference between prevailing short-term yields and the return on the basis trade as might be simulated in the absence of any CTD crossovers.

The short basis trader is effectively short an option and receives this implicit premium. This implicit premium is reflected in a comparison of the Implied Rate of Return (IRR) relative to prevailing short-term rates.

Note that the BPV of a debt security is dynamic and subject to change given fluctuating yields. As a general rule, BPV declines as a function of maturity; and, as yields increase (decrease), BPVs decline (advance). This implies that the hedge ratio is likewise dynamic. Over a limited period of time, however, HRs may be reasonably stable, provided there is no crossover in the cheapest-to-deliver. As a general rule in practice, it would be commonplace for hedgers to re-valuate and readjust the hedge if rates were to move by perhaps 20-25 basis points.

Portfolio Hedging

Thus far, our discussion has centered about comparisons between a single security and a Treasury futures contract, a "micro" hedge if you will. But it is far more commonplace for an investor to become concerned about the value of a portfolio of securities rather than focus on a single item within a presumably diversified set of holdings.

How might one address the risks associated with a portfolio of securities, *i.e.*, how to execute a "macro" hedge? The same principles apply whether hedging a single security or a portfolio of securities. Thus, we need to evaluate the risk characteristics of the portfolio in terms of its BPV and duration just as we would examine an individual security. Then we may simply apply the BPV hedge ratio for these purposes.

$$HR = BPV_{portfolio} \div \left(\frac{BPV_{ctd}}{CF_{ctd}}\right)$$

E.g., assume that you held a $100 million fixed income portfolio with a BPV = $80,000 and a duration of 8 years. This duration is similar to the duration associated with securities deliverable against the 10-year T-note futures contract, suggesting use of the 10-year as a hedge vehicle. As of January 10, 2013, the CTD security was the 3-3/8%-19 with a BPV = $70.50 per $100,000 face value unit and a CF = 0.8604. Our analysis suggests that one might sell 976 futures to hedge the portfolio.

$$HR = \$80{,}000 \div \left(\frac{\$70.50}{0.8604}\right) = 976.3 \textit{ or } 976 \textit{ contracts}$$

Thus far, our examples illustrated situations where we had effectively hedged individual securities or portfolios in their entirety. In the process, we might effectively push the risk exposure down to near $0 as measured by BPV or 0 years as measured by duration. But it would actually be uncommon to see an asset manager adjust an actual fixed income risk exposure all the way down to zero.

Asset managers generally measure their performance by reference to a designated "benchmark" or "bogey." The benchmark is often identified as an index of fixed income securities such as the Barcap U.S. Aggregate Bond Index or some other commonly available measure.

The returns on this benchmark may be identified as the "core" or "beta" returns associated with the portfolio. In addition, the asset manager may exercise some limited degree of latitude in an attempt to outperform the benchmark, or to capture some excess return known as "alpha" in current investment parlance.

Asset manager may be authorized to adjust the duration of the portfolio upwards by a limited amount in anticipation of rate declines and price advances. Or, to adjust duration downwards by a limited amount in anticipation of rate advances and price declines. The following formula provides the appropriate hedge ratio for these operations.

$$HR = \left(\frac{D_{target} - D_{current}}{D_{current}}\right) x \left[BPV_{portfolio} \div \left(\frac{BPV_{ctd}}{CF_{ctd}}\right)\right]$$

Where D_{target} is the target duration; $D_{current}$ is the current duration.

E.g., let's return to our example of a $100 million fixed income portfolio. Assume that the portfolio duration of 8 years was designed to coordinate with the duration of the designated benchmark. Thus, the portfolio manager may be authorized to adjust portfolio duration between 6 and 10 years in pursuit of "alpha." The asset manager is now concerned about the prospects for rate advances and wishes downwardly to adjust duration from 8 to 6 years. Our analysis suggests that this may be accomplished by selling 244 futures.

$$HR = \left(\frac{6-8}{8}\right) x \left[\$80{,}000 \div \left(\frac{\$70.50}{0.8604}\right)\right]$$
$$= -244.1 \ or \ sell \ 244 \ contracts$$

The application of this formula provides asset managers with a great deal of flexibility to adjust the portfolio duration – either upward or downward – to meet the demands of the moment.

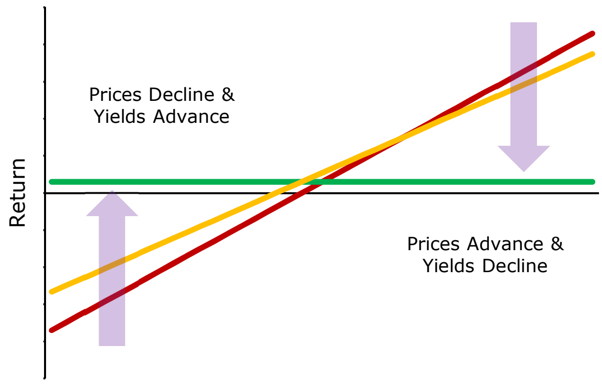
Bullets and Barbells

Typically one looks to hedge a Treasury portfolio with the use of Treasury futures which correspond most closely in terms of duration to the average weighted portfolio duration.

E.g., if one held a portfolio with an average weighted duration of 4 years, it would be natural to look to 5-year Treasury note futures as a suitable risk layoff vehicle. If the portfolio had an average weighted duration of 8 years, it would be natural to look to 10-year Treasury note futures.

This analysis would tend to work well when the portfolio is constructed predominantly of securities which were close in terms of their durations to the average portfolio duration. Certainly if the entire portfolio were populated with a variety of recently issued 5-year T-notes, it would behoove the hedger to utilize 5-year Treasury note futures as a hedge, minimizing basis risk and the need for any subsequent hedge management.

Hedged with Short Futures



A portfolio constructed in such a manner might be labeled a "bullet" portfolio to the extent that it contains reasonably homogeneous securities in terms of maturity and presumably coupon. Under these circumstances, one might simply "stack" the entire hedge in a single Treasury futures contract which most closely conforms to the duration of the portfolio constituents.

Of course, one may attempt to introduce a certain speculative element into the hedge by using longer- or shorter-term futures contracts as the focus of the hedge.

If the yield curve were expected to steepen, a hedge using longer-term futures, *e.g.,* 10- or 30-year Treasury futures rather than 5-year futures, would allow one to capitalize on movement in the curve beyond simply immunizing the portfolio from risk.
If the yield curve is expected to flatten or invert, a hedge using shorter-term futures, *e.g.,* 2-year or 3-

year Treasury futures rather than 5-year futures, could likewise provide yield enhancement.

But a portfolio need not necessarily be constructed per the "bullet" approach. Consider a portfolio with a duration of 4 years that is constructed using a combination of 2- and 10-year notes and no 5-year notes whatsoever.

A portfolio constructed in such a manner may be labeled a "barbell" portfolio to the extent that it is "weighted" with two extreme duration securities with no intermediate duration securities at all. If one were to simply stack the hedge into 5-year Treasury note futures, the investor becomes exposed to the risk that the shape of the yield curve becomes distorted such that 5-year yields sag below yields in the 2- and 10-year sectors of the curve.

The holder of a barbell portfolio might instead attempt to utilize a combination of various tenured Treasury futures which is weighted with an eye to the proportion of the portfolio devoted to each sector of the yield curve. As such, the hedger may insulate from the risks that the shape of the yield curve will shift.

Thus, an asset manager might categorize his holdings into various sectors of the curve corresponding to available Treasury futures "buckets," *i.e.,* 2-, 5-, 10- and 30-year securities. Then, the asset manager may calculate the BPV HRs applicable to each of those bucketed portfolios and essentially hedge each element separately.

If, however, the investor wished to introduce a speculative element into the hedge, the use of longer- or shorter-maturity Treasuries driven by an expectation of a steepening or flattening yield curve, respectively, may be in order.

Table 1: Treasury Contracts Summary

	2-Year T-Note Futures	3-Year T-Note Futures	5-Year T-Note Futures	10-Year T-Note Futures	Classic T-Bond Futures	Ultra T-Bond Futures
Contract Size	$200,000 face-value U.S. Treasury notes		$100,000 face-value U.S. Treasury notes		$100,000 face-value U.S. Treasury bonds	
Delivery Grade	T-notes with original maturity of not more than 5 years and 3 months and remaining maturity of not less than 1 year and 9 months from 1st day of delivery month but not more than 2 years from last day of delivery month	T-Notes with original maturity of not more than 5-1/4 years and a remaining maturity of not more than 3 years but not less than 2 years, 9 months from last day of delivery month	T-notes with original maturity of not more than 5 years and 3 months and remaining maturity of not less than 4 years and 2 months as of 1st day of delivery month.	T-notes maturing at least 6-½ years but not more than 10 years, from 1st day of delivery month.	T-bonds with remaining maturity of at least 15 years but no more than 25 years.	T-bonds with remaining maturity of at least 25 years but no more than 30 years
Invoice Price	Invoice price = settlement price x conversion factor (CF) + accrued interest, CF = price to yield 6%					
Delivery Method	Via Federal Reserve book-entry wire-transfer					
Contract Months	March quarterly cycle – March, June, September, December					
Trading Hours	Open Auction: 7:20 am-2:00 pm, Monday-Friday; Electronic: 6:00 pm - 4:00 pm, Sunday-Friday (Central Times)					
Last Trading & Delivery Day	Last business day of contract month; delivery may occur on any day of contract month up to and including last business day of month			Day prior to last seven (7) business days of contract month; delivery may occur on any day of contract month up to and including last business day of month		
Price Quote	In percent of par to one-quarter of 1/32nd of 1% of par ($15.625 rounded up to nearest cent)		In percent of par to one-quarter of 1/32nd of 1% of par ($7.8125 rounded up to nearest cent)	In percent of par to one-half of 1/32nd of 1% of par ($15.625 rounded up to nearest cent)	In percent of par to 1/32nd of 1% of par ($31.25)	

Table 2: March 2013 Ten-Year T-Note Futures Basis

(As of January 10, 2013)

Coupon	Maturity	Price	Yield	CF	Basis	IRR	Duration
1-5/8%	11/15/22	97-18¾	1.895%	0.6867	227.966	-32.838%	9.016
1-5/8%	8/15/22	98-01¾	1.847%	0.6928	217.252	-31.092%	8.775
1-3/4%	5/15/22	99-18¾	1.798%	0.7077	203.441	-28.414%	8.558
2%	2/15/22	102-04¾	1.743%	0.7307	118.484	-25.314%	8.234
2%	11/15/21	102-17¾	1.688%	0.7367	176.191	-23.420%	8.067
2-1/8%	8/15/21	103-28¾	1.637%	0.7507	160.174	-20.744%	7.789
3-1/8%	5/15/21	112-05¾	1.562%	0.8194	135.569	-15.053%	7.382
3-5/8%	2/15/21	116-04¼	1.501%	0.8544	14.527	-11.469%	7.034
2-5/8%	11/15/20	108-18	1.465%	0.7985	107.923	-12.264%	7.095
2-5/8%	8/15/20	108-22	1.414%	0.8039	89.160	-9.727%	6.853
3-1/2%	5/15/20	115-01+	1.341%	0.8588	61.229	-4.829%	6.530
3-5/8%	2/15/20	115-25+	1.288%	0.8697	39.280	-1.923%	6.266
1-1/8%	12/31/19	98-27¾	1.295%	0.7326	75.475	-10.165%	6.676
1%	11/30/19	98-05¾	1.277%	0.7341	47.151	-6.095%	6.585
3-3/8%	11/15/19	114-00¾	1.232%	0.8604	21.734	0.121%	6.153
1-1/4%	10/31/19	99-31¾	1.251%	0.7474	49.085	-6.008%	6.485
1%	3/30/19	98-16¼	1.232%	0.7341	57.651	-7.637%	6.453

NOTES

March 2013 futures were priced at 131-23+/32nds
Securities highlighted in red represent least economic-to-deliver;
highlighted in green represent most economic-to-deliver.