Theorem 1 (Brouwer fixed point theorem) Let $f : B^n \to B^n$ be a continuous function on the unit ball $B^n := \{x \in R^n : \|x\| \leq 1\}$. Then f has at least one fixed point, thus there exists $x \in B^n$ with $f(x) = x$.

Theorem 2 (Brouwer invariance of domain theorem) Let U be an open subset of R^n and let $f : U \to R^n$ be a continuous injective map. Then $f(U)$ is also open.

Corollary 3 (Topological invariance of dimension) If $n > m$ and U is a non-empty open subset of R^n, then there is no continuous injective mapping from U to R^m. In particular, R^n and R^m are not homeomorphic.