

非真实感绘制

计算机系 张松海

什么是非真实感绘制？

Non-Photorealistic Rendering
Artistic Rendering
Stylization

...

非真实感绘制

- 利用计算机生成不具有照片般真实感,而具有手绘风格的图形的技术
- 目标不在于图形的真实性,而主要在于表现图形的艺术特质、模拟艺术作品(甚至包括作品中的缺陷)



非真实感绘制

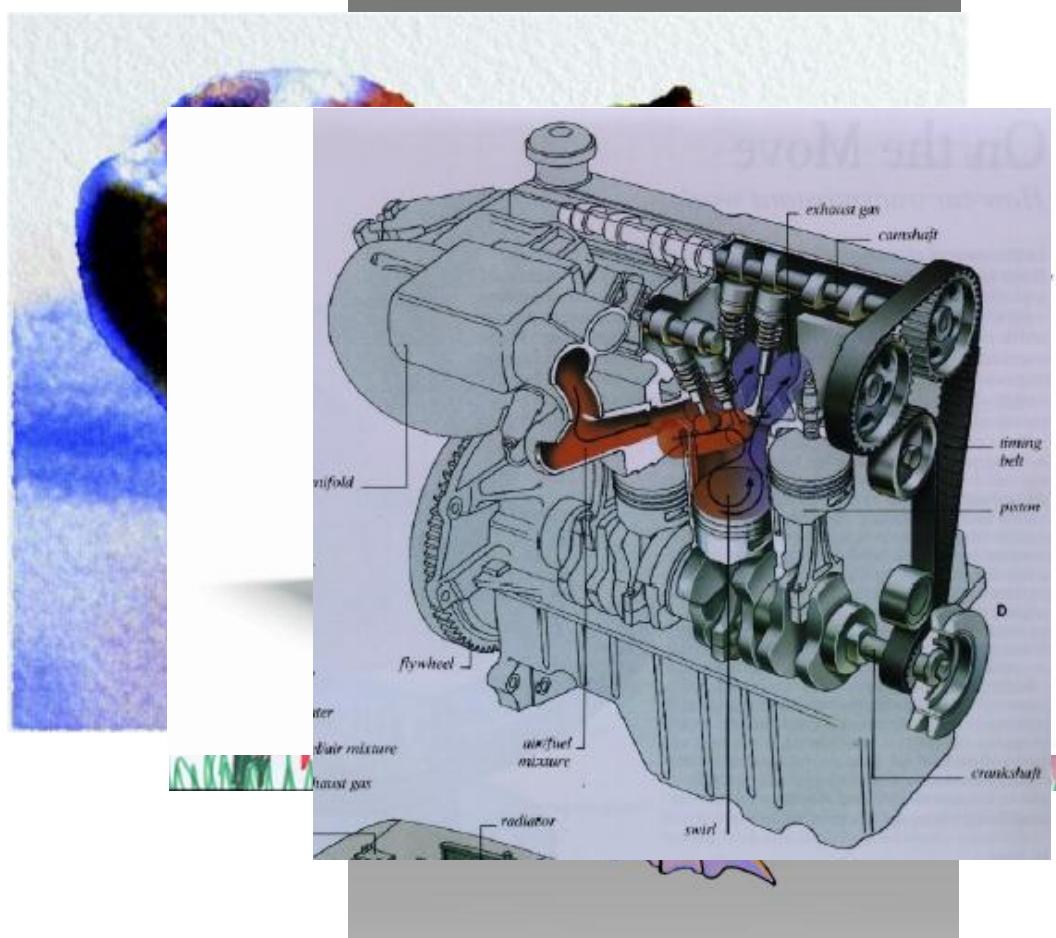
- 利用计算机生成不具有照片般真实感,而具有手绘风格的图形的技术
- 目标不在于图形的真实性,而主要在于表现图形的艺术特质、模拟艺术作品(甚至包括作品中的缺陷)
- 主要模拟画种：油画，水彩画，钢笔画，铅笔画，水墨画和卡通动画

非真实感绘制的特征

- 部分源于真实，超越真实
- 使用抽象的模型传达信息
- 重要信息被加强，其他无关信息可能被去除

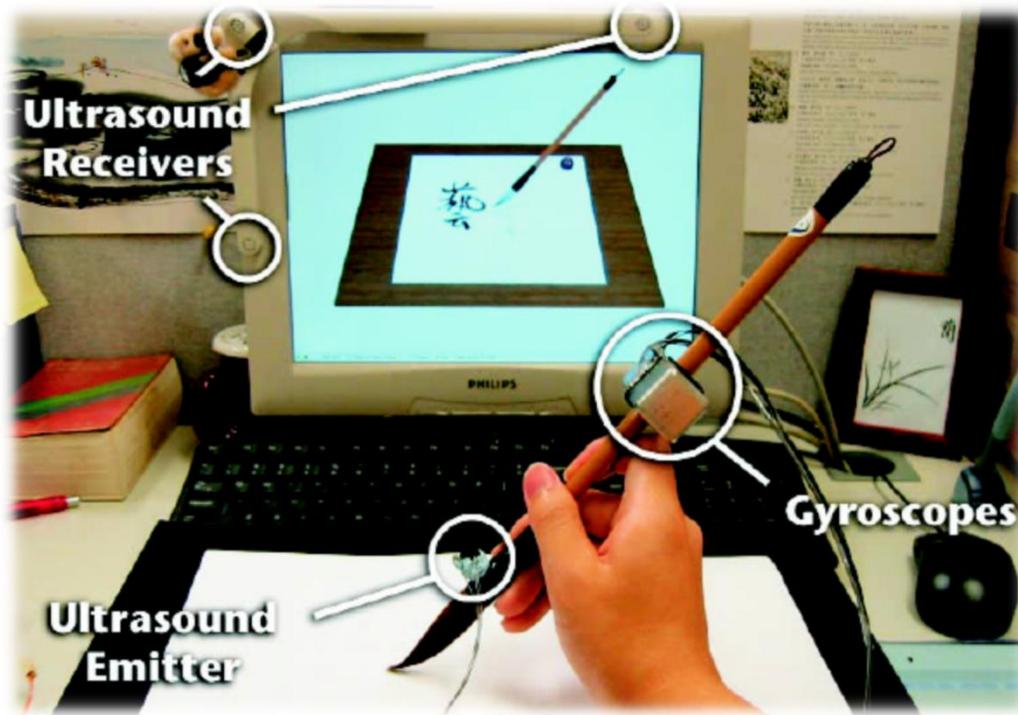
非真实感绘制的应用

- 艺术
- 科学成果可视化
- 技术产品图示
- 教育
- 娱乐



非真实感绘制研究分类

- 创作媒介的计算机模拟



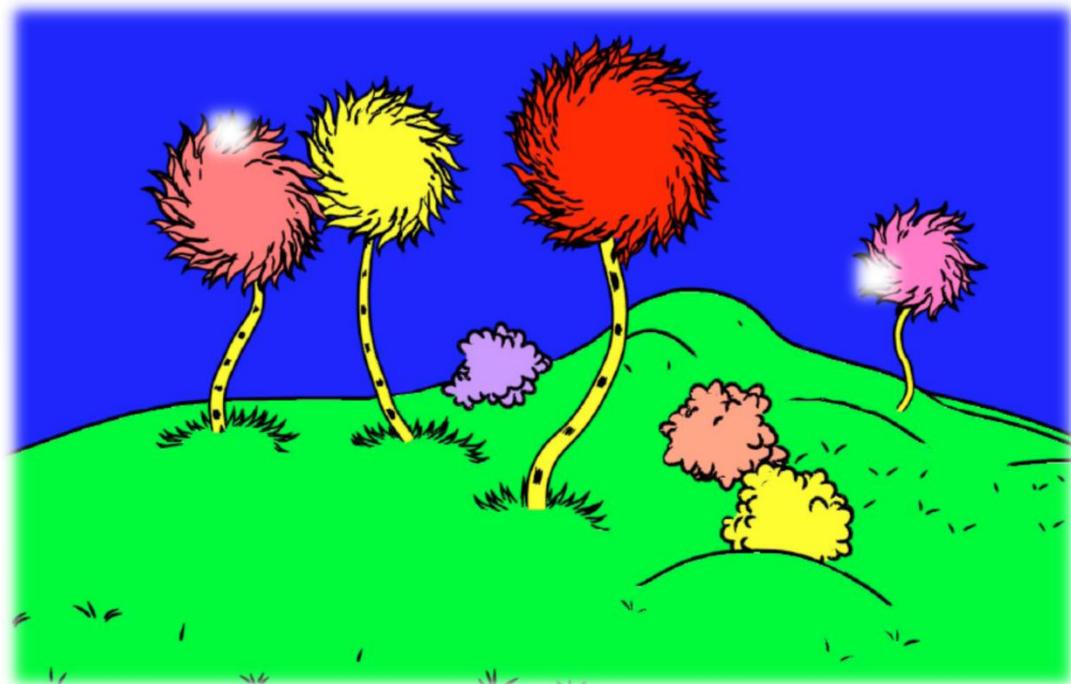
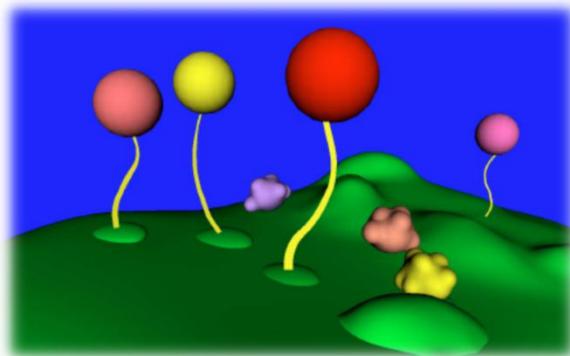
非真实感绘制研究分类

- 创作媒介的计算机模拟
- 创作方法和风格的计算机模拟
 - 图像空间



非真实感绘制研究分类

- 创作媒介的计算机模拟
- 创作方法和风格的计算机模拟
 - 图像空间
 - 物体空间



非真实感绘制研究分类

- 如何画出一条笔划
- 如何设计布局及笔画布置
 - 图像空间
 - 物体空间
- 如何让作品动起来

创作媒介的计算机模拟

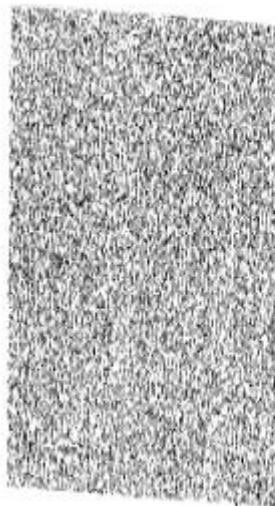
- 主要研究对象：笔、墨、纸
- 主要研究方法：
 - 模拟媒介的微观几何结构
 - 模拟和化简微观物理规律，构建方程或函数

创作媒介的计算机模拟

- 铅笔
 - Observational models of graphite pencil materials@CGF2000
- 水彩
 - Computer-Generated Watercolor@Siggraph1997
- 水墨
 - MoXi: Real-Time Ink Dispersion in Absorbent Paper@Siggraph2005

纸张&铅笔

- 不同硬度的铅笔在不同压力下，在纸面上留下的痕迹



4B pencil
Medium pressure



4B pencil
Heavy pressure



6B pencil
Medium pressure



6H pencil
Light pressure

铅笔建模

• 硬度

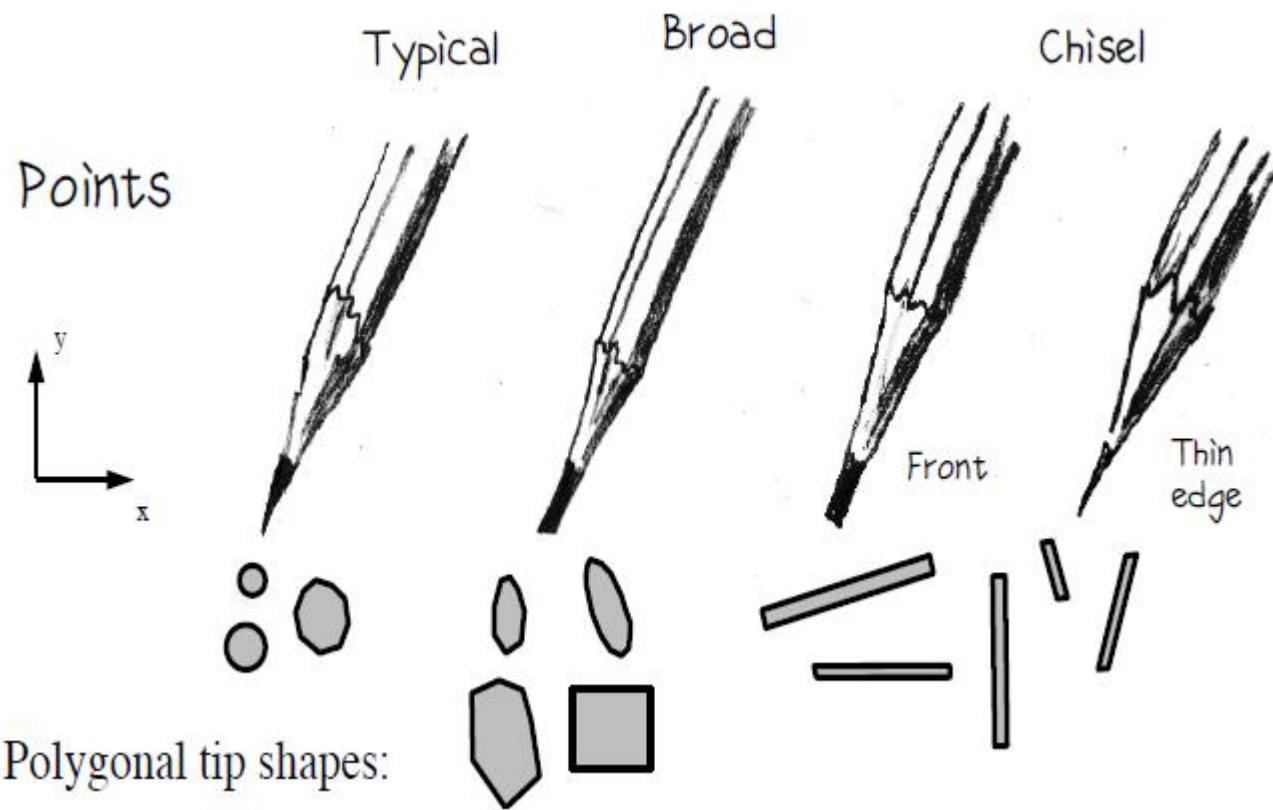
• [9H -8B]

Pencil Number	Graphite	Clay	Wax
9H	0.41	0.53	0.05
8H	0.44	0.50	0.05
7H	0.47	0.47	0.05
6H	0.50	0.45	0.05
5H	0.52	0.42	0.05
4H	0.55	0.39	0.05
3H	0.58	0.36	0.05
2H	0.60	0.34	0.05
H	0.63	0.31	0.05
F	0.66	0.28	0.05
HB	0.68	0.26	0.05
B	0.71	0.23	0.05
2B	0.74	0.20	0.05
3B	0.76	0.18	0.05
4B	0.79	0.15	0.05
5B	0.82	0.12	0.05
6B	0.84	0.10	0.05
7B	0.87	0.73	0.05
8B	0.90	0.04	0.05

铅笔建模

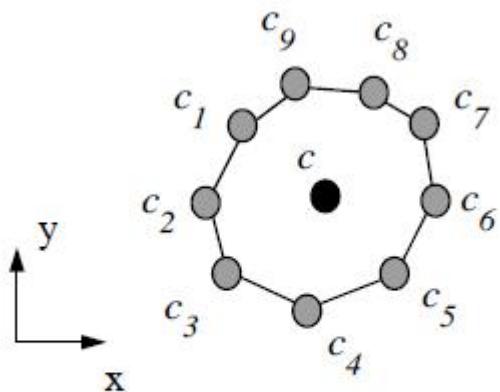
- 笔头形状

- 多边形形状和笔芯粗细共同决定

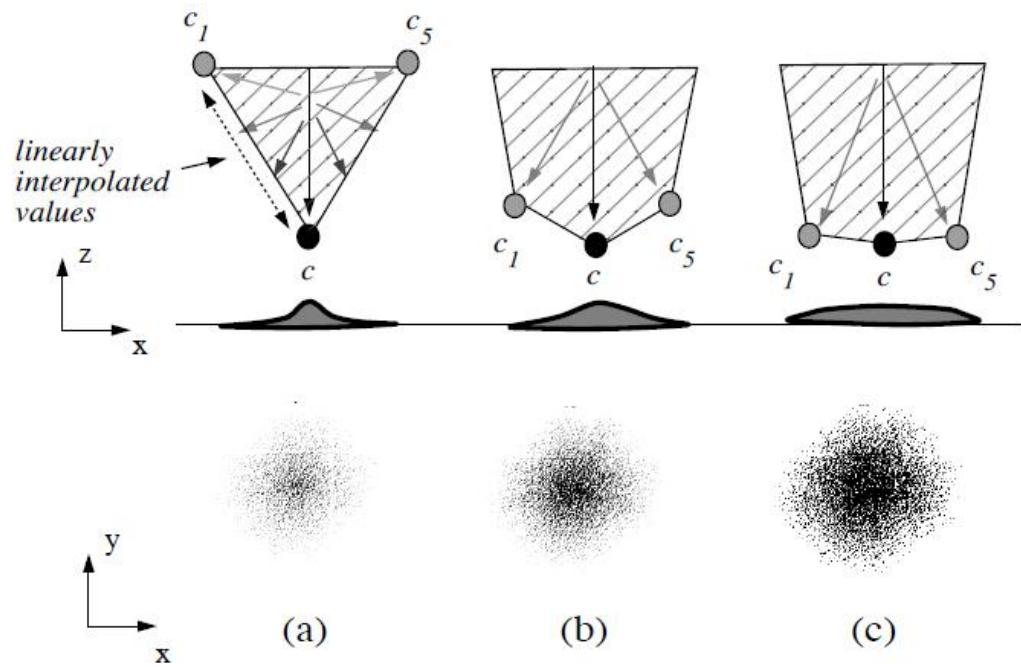


铅笔建模

- 笔头形状
 - 压力分布

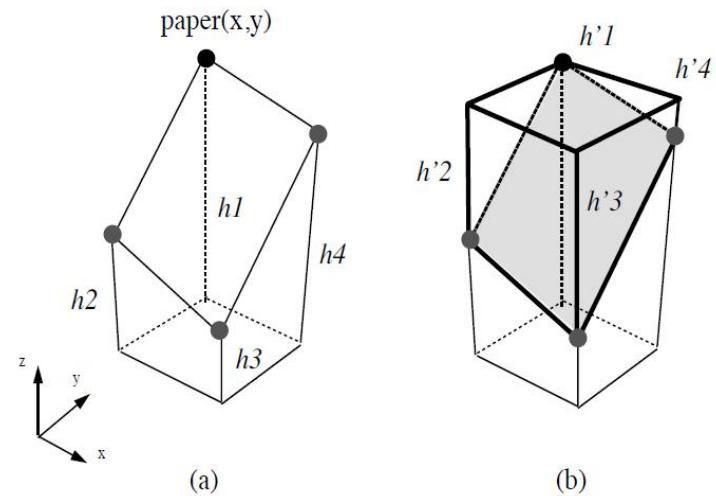
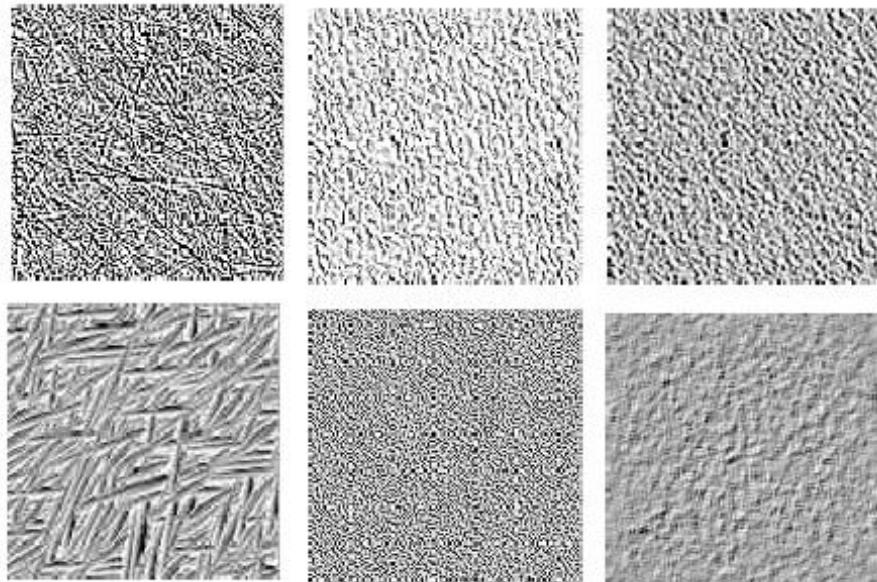


Cross-sections



纸张建模

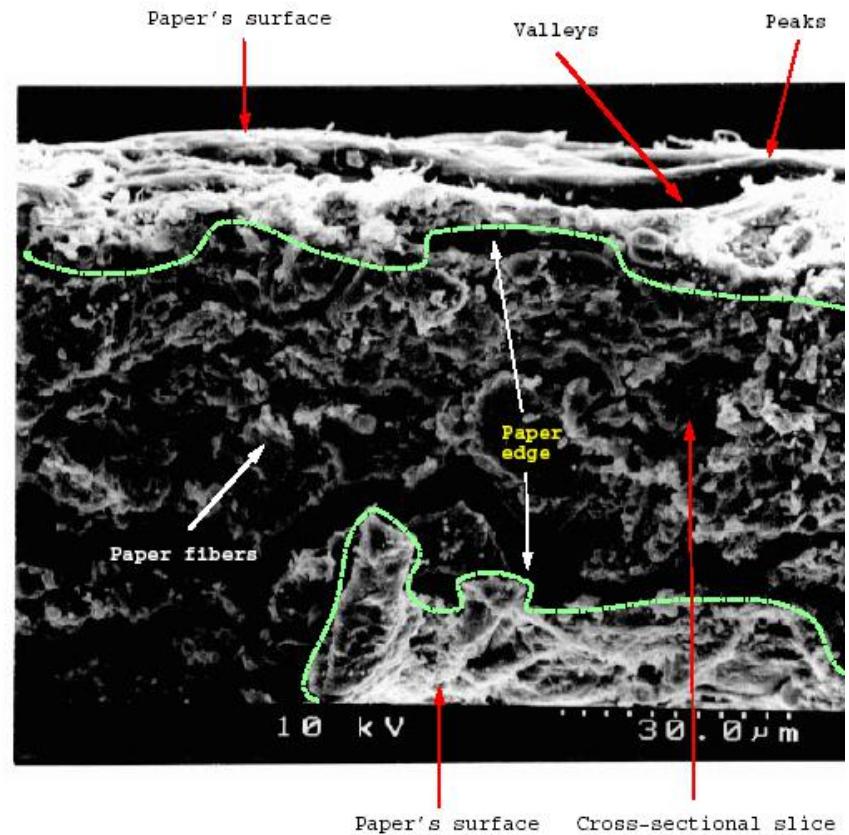
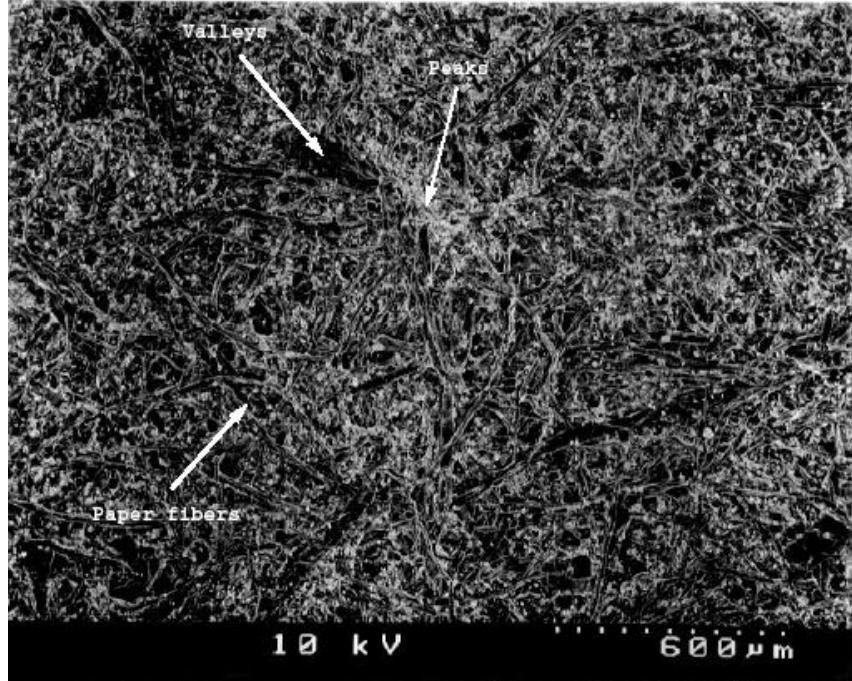
- 影响因素：重量（厚度）、纹理
 - 重量：48-300克/平米
 - 纹理：平滑、半粗糙、粗糙
- 基于高度场 (height field) 构造纹理 [Curtis97]
 - 基于纸张模型的假随机方法



Grain：纸张微观单元建模

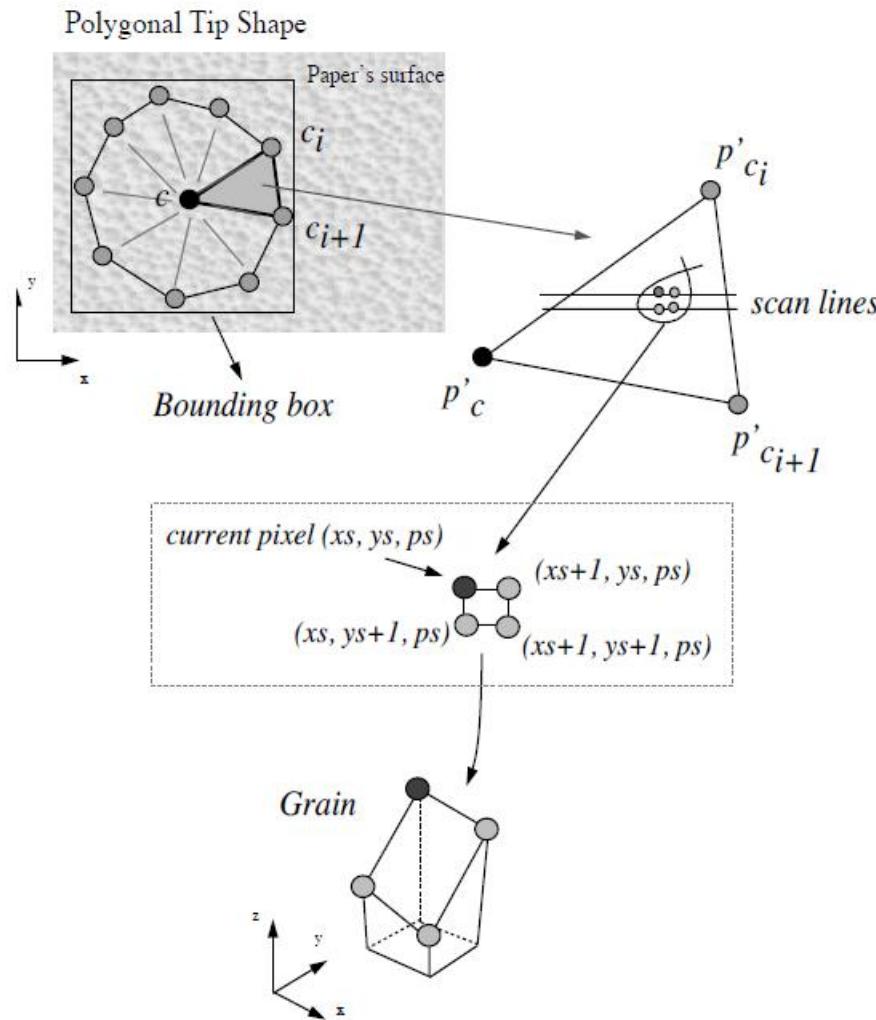
纸张建模

- 扫描电子显微镜下的纸张(scanning electron microscope, SEM)



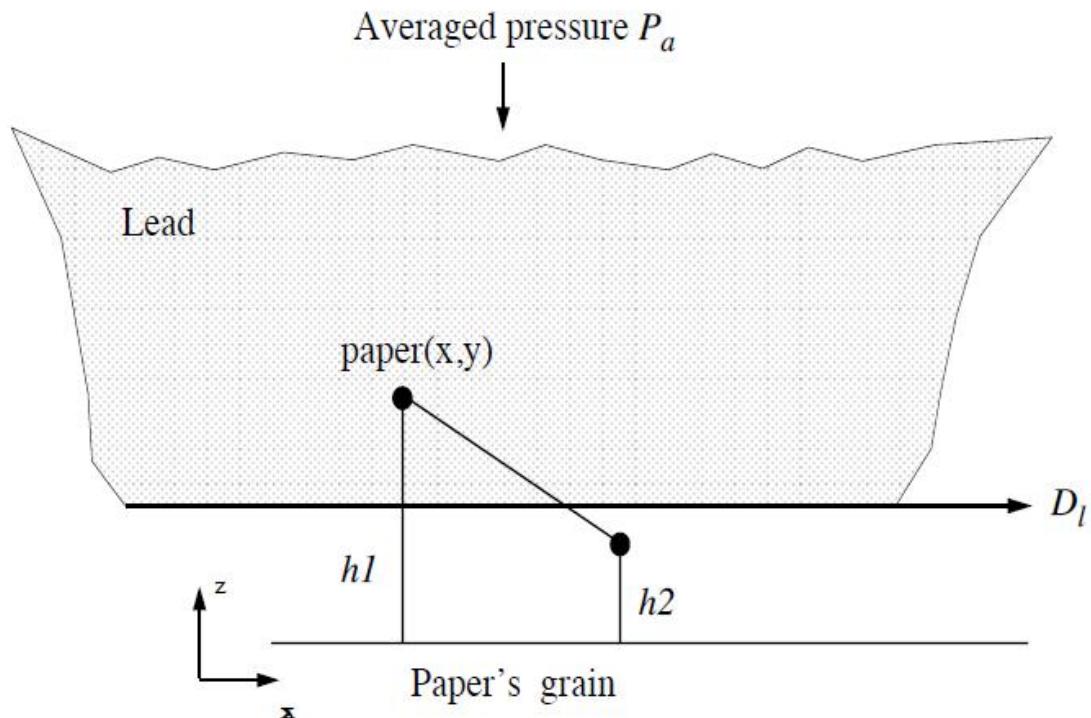
铅笔/纸张相互作用

- 笔头施加到颗粒上的压力分布



铅笔/纸张相互作用

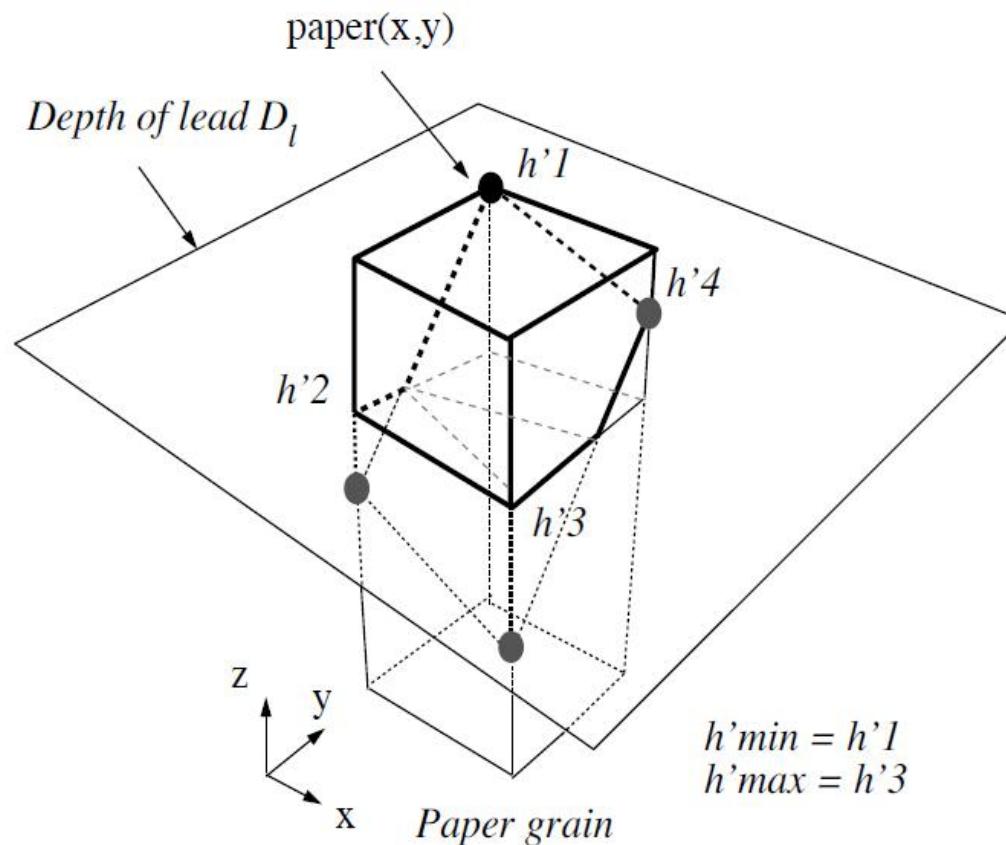
- 铅沉积



$$D_l \leftarrow h_{max} - (h_{max} \times P_a)$$

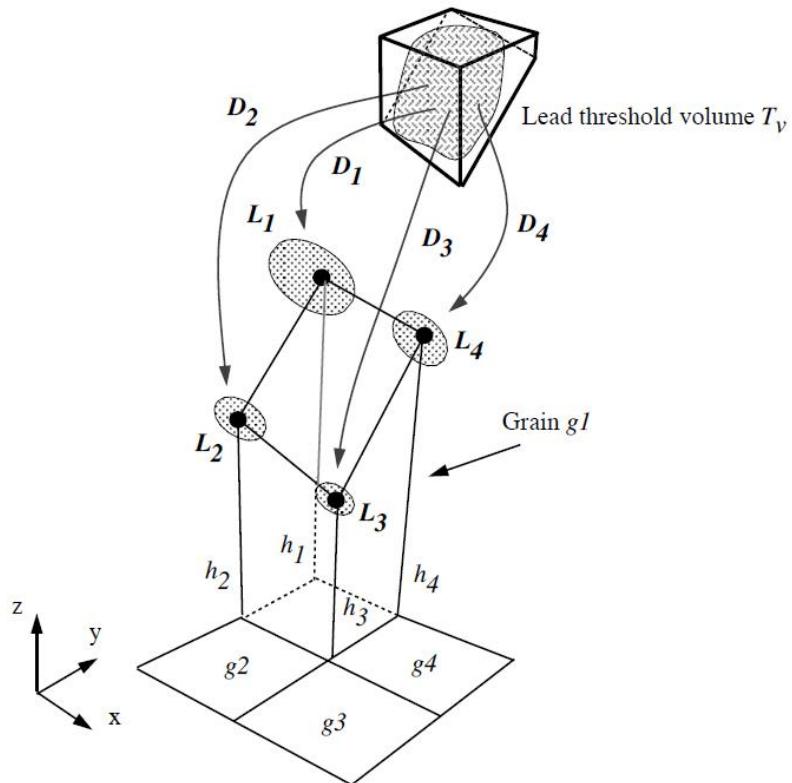
铅笔/纸张相互作用

- ## • 留在颗粒上的铅量



铅笔/纸张相互作用

- 铅量在颗粒中的分布



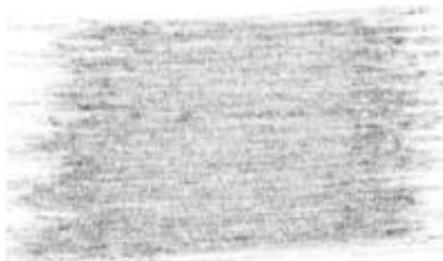
$$A_k \leftarrow \frac{G_k}{F_t}$$

$$I_k \leftarrow 1.0 - A_k$$

铅笔/纸张相互作用

- 破坏纸张颗粒(grains)

- 破坏程度与硬度相关



8H pencil, High pressure

HB pencil, light pressure

Very rough, medium-weight paper

铅笔/纸张相互作用

- 破坏纸张颗粒

- 压力 ↑

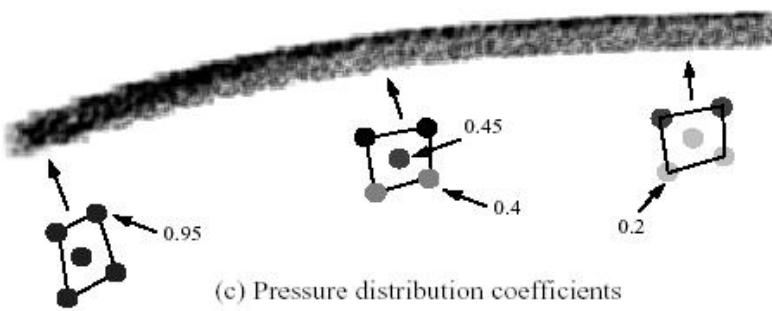
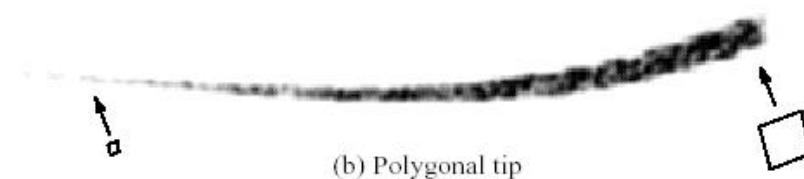
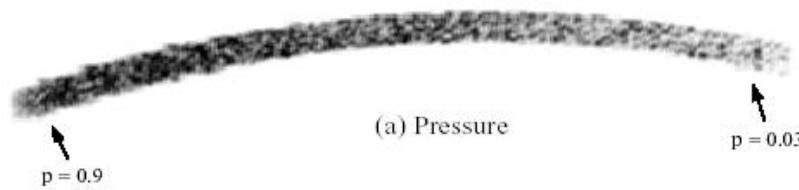
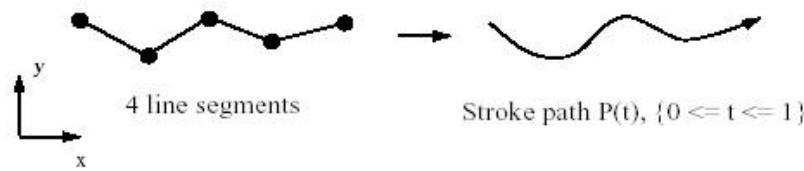
- 硬度 ↑

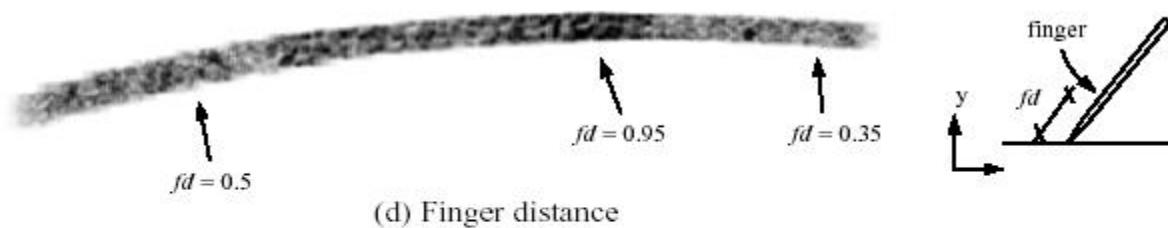
- 纸张刚性 ↓

$$E_k \leftarrow D_l \times da(P_d) \times w$$

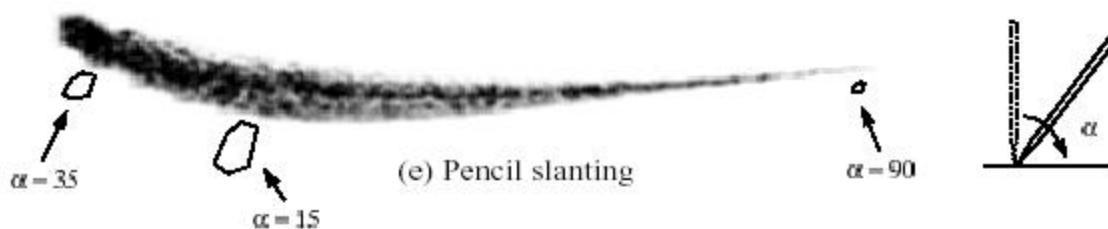
$$h_k \leftarrow h_k - E_k$$

笔触

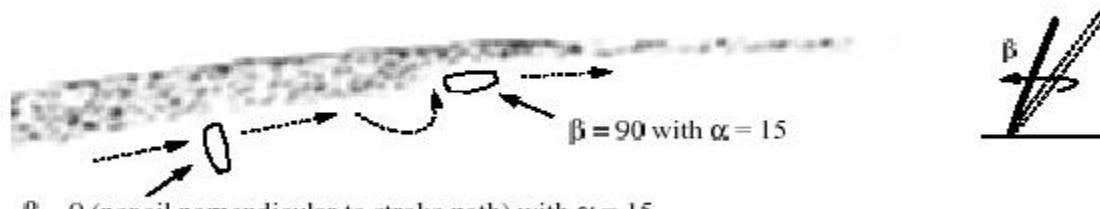




(d) Finger distance



(e) Pencil slanting



(f) Wrist and arm movement

结果



(a)

真实铅笔画



(b)

使用3H铅笔仿真的结果

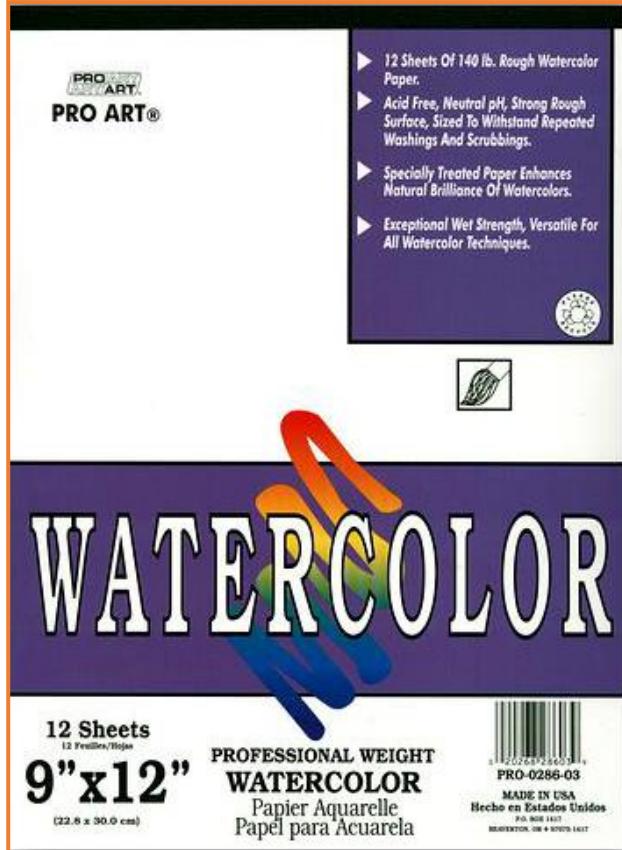
创作媒介的计算机模拟

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水彩的物理性质

- 水彩画材料
 - 画纸
 - 颜料
 - 黏合物
 - 活性剂
- 水彩艺术效果

画纸



- 一般不用木浆制作
- 亚麻布/棉布捣碎而成的纤维
 - 极易吸收液体
- 掺入纤维制作的上胶
 - 延缓液体吸收和扩散的速度

Source: misterart.com

颜料



- 颜料是大量细小粒子组成的固体物质 (大多在0.05 to 0.5 微米之间)
- 颜料密度往往有明显的差别

黏合物和活性剂

黏合物

- 吸收
 - 促使颜料颗粒黏在纸张上面

活性剂

- 促使液体浸透纸张



Binder

Source: Jerry's ARTARAMA

水彩的物理性质

- 水彩画材料
- 水彩艺术效果
 - 干刷效果
 - 边缘暗化
 - 收笔
 - 造粒分离
 - 流动特征
 - 上光

干刷效果 (Dry-brush)



Source: Computer Generated Watercolor

- 技术

- 几乎干燥的笔刷
- 一定的角度

- 效果

- 不规则的缺口
- 参差的边缘

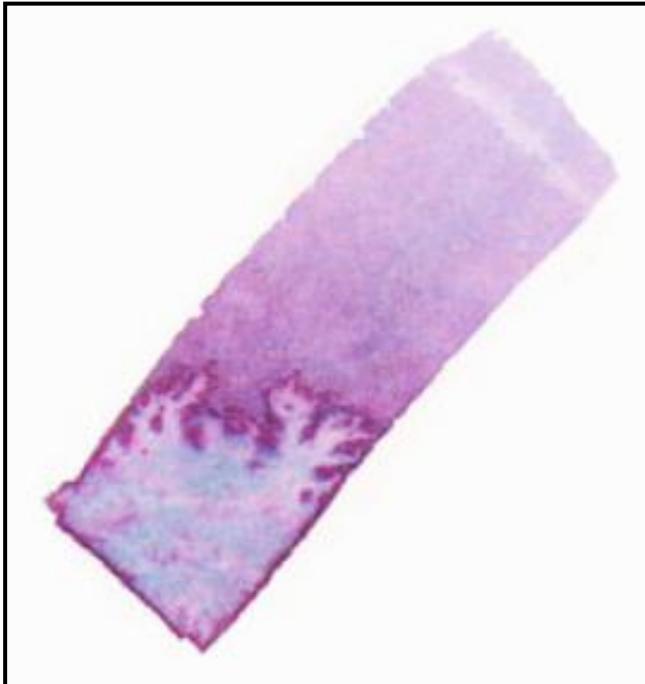
边缘暗化(Edge darkening)



- 技术
 - 湿笔触画于干燥纸面
- 效果
 - 暗化的边缘

Source: Computer Generated Watercolor

收笔(backruns)



- 何时出现?
 - 水蔓延回潮湿区域
 - 笔尖浸染颜料不均匀
 - 颜料随水的蔓延而传播
- 效果
 - 复杂的分叉形状
 - 强烈的边缘暗化效果

Source:

Computer Generated Watercolor

造粒分离(Granulation and separation)

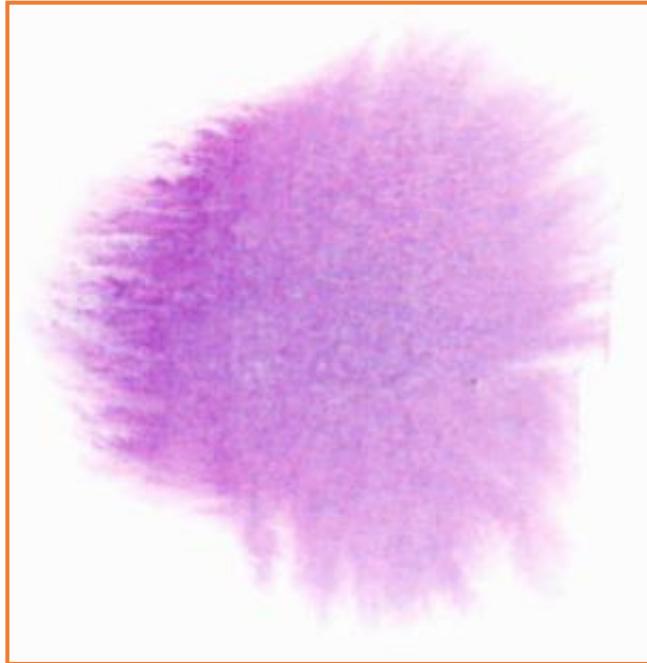


- 颜料造粒
 - 产生颗粒状纹理
 - 不同颜料区域之间效果不同
 - 纸张越干燥，效果越明显
- 颜料分离
 - 即颜料的分离化效果
 - 颜料密度大的区域率先沉淀而成

Source:

Computer Generated Watercolor

流动特征(Flow patterns)



- 湿笔尖画于湿纸之上
 - 潮湿的表面上，笔触传播的更快、更加没有规则
- 效果
 - 柔软、羽毛状的形状

Source:

Computer Generated Watercolor

上光(color glazing)

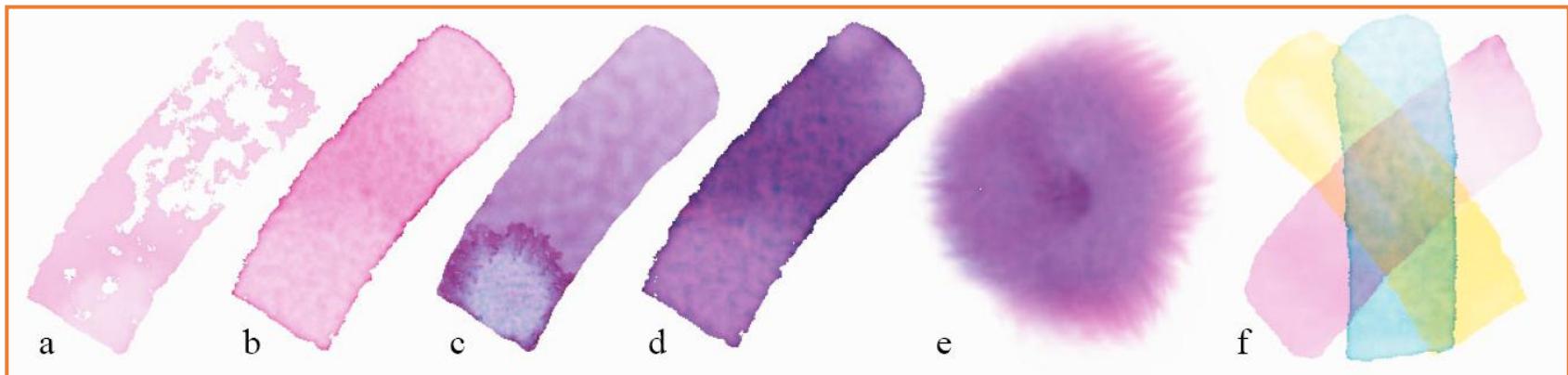
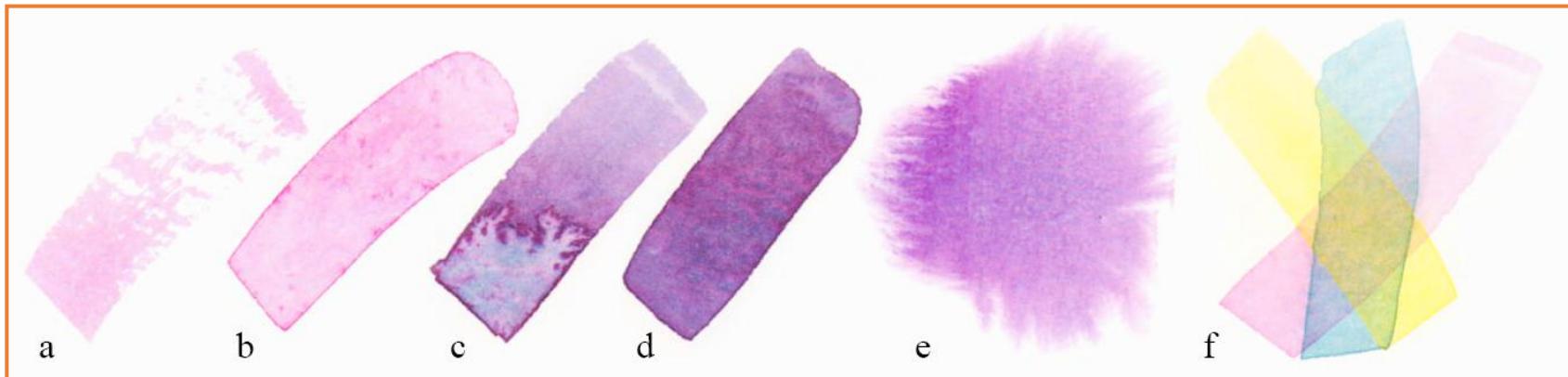


- 技术
 - 于一层水彩之上增加一层薄而浅的笔画
 - 不同颜料从视觉层次上混合
- 效果
 - 不同的颜色
 - 混合内部产生新的色彩

Source:

Computer Generated Watercolor

计算机生成水彩画



计算机生成水彩画

- 纸张生成
- 数据结构
- 流体模拟
- 视觉合成

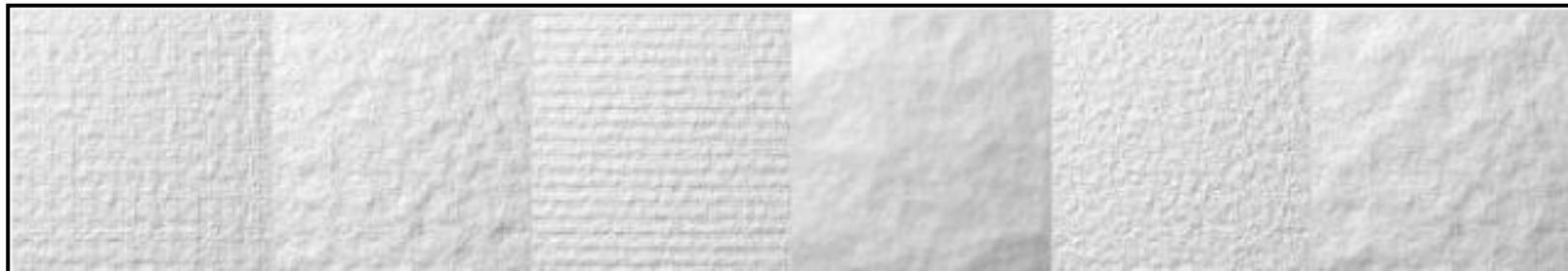
纸张生成

- 一种简单的模型

- 纸张的纹理由高度场 h 和流体容纳场 c (fluid capacity field) 生成

h : 伪随机生成, $0 < h < 1$

$$c = h * (c_{max} - c_{min}) + c_{min}$$



纸张纹理

数据结构

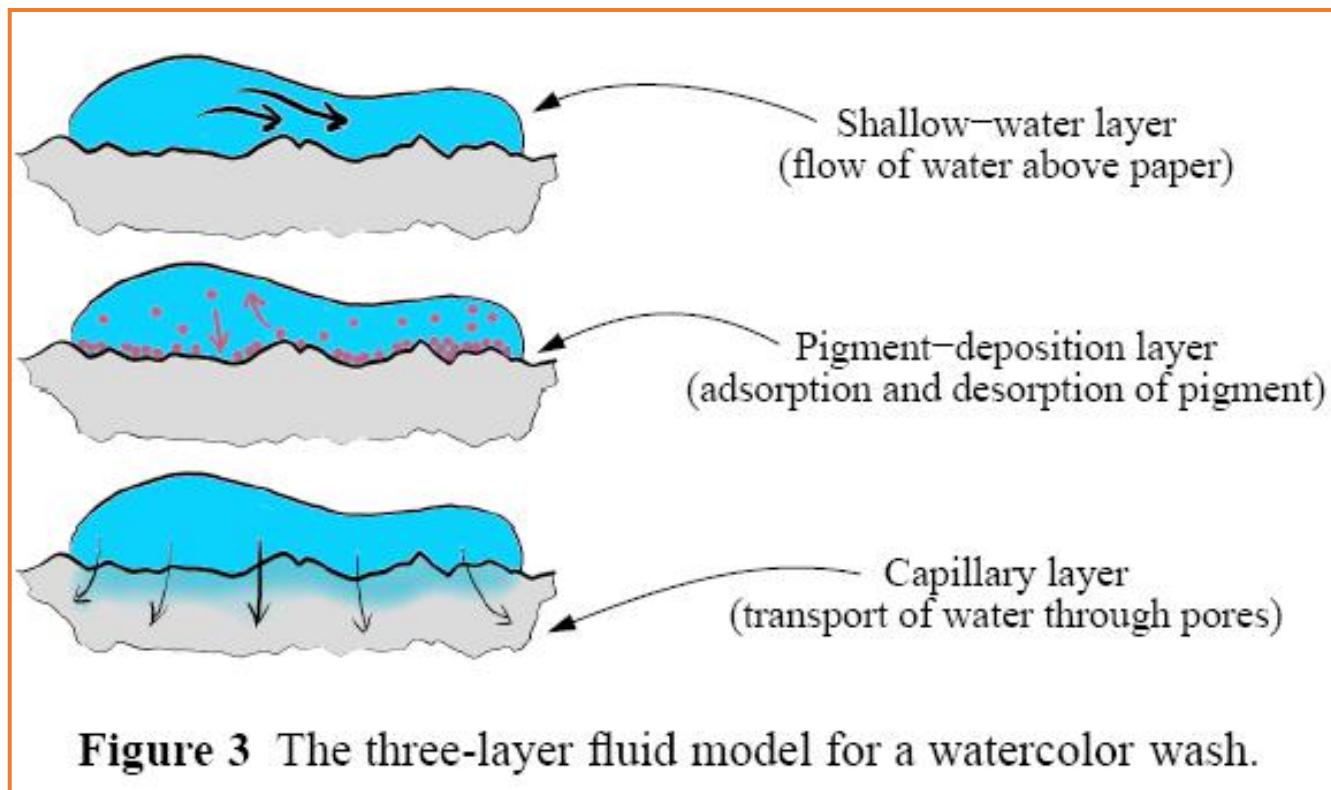
- 一幅完整的作品由一组笔触构成
- 每一个笔触含有不同量的颜料
- 每一个笔触采用一个叫做“glaze”的数据结构来存储

Glaze

- 每一个Glaze通过运行一次流体模拟来生成
- Inputs:
 - 纸张、颜料等性质
 - 潮湿区域(wet-area mask)
- 计算完毕后
 - 通过Kubelka-Munk (KM) 色彩模型合成

流体模拟

- 每个笔触模拟分为三个层次：



Source: Computer Generated Watercolor

流体模拟

Main loop

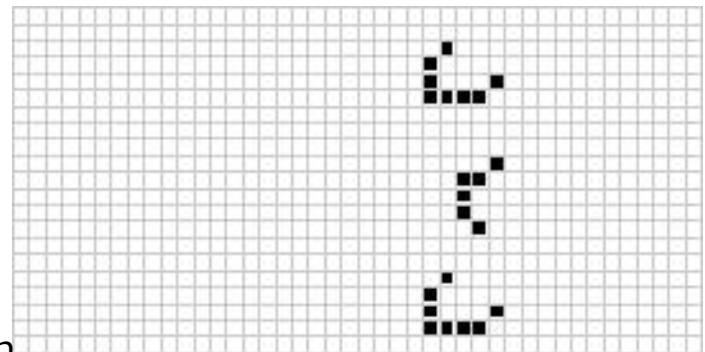
```
proc MainLoop
    for each time step do:
        MoveWater
        MovePigment
        TransferPigment
        SimulateCapillaryFlow
    end for
end proc
```

流体模拟

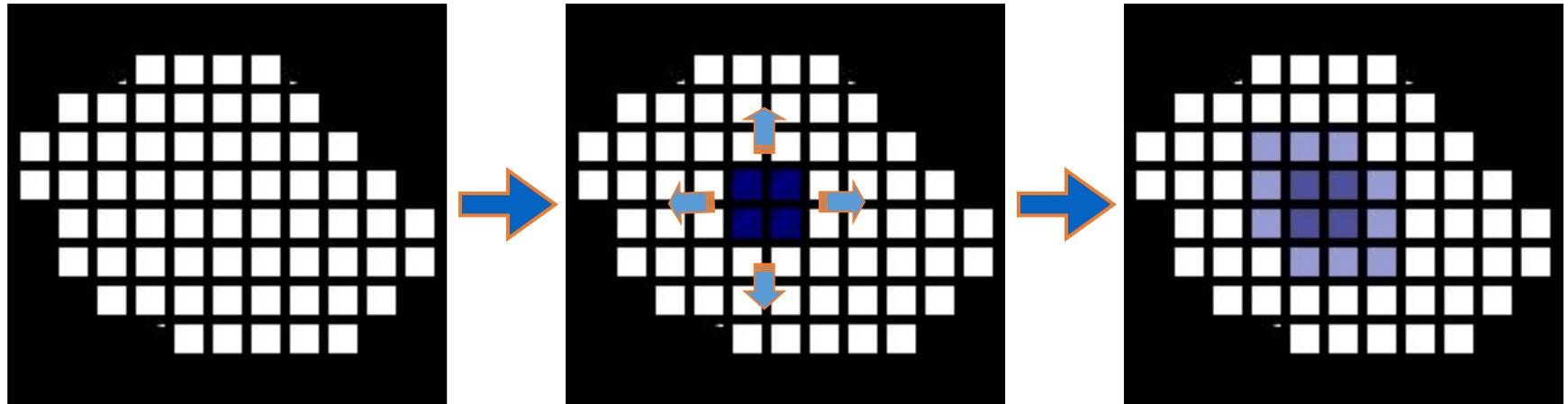
- 细胞自动机 (Cellular Automaton)
- 定义 [Mathworld.com]
 - 细胞自动机是一组带有色彩的细胞，他们在离散的时间点内根据一些固定的规则和邻近细胞的状态而不断进化形成的形状。

Game of Life

Source: Mathworld.com



流动液体



```
proc MoveWater( $M$ ,  $u$ ,  $v$ ,  $p$ ):  
    UpdateVelocities( $M$ ,  $u$ ,  $v$ ,  $p$ )  
    RelaxDivergence( $M$ ,  $u$ ,  $v$ ,  $p$ )  
    FlowOutward( $M$ ,  $p$ )  
end proc
```

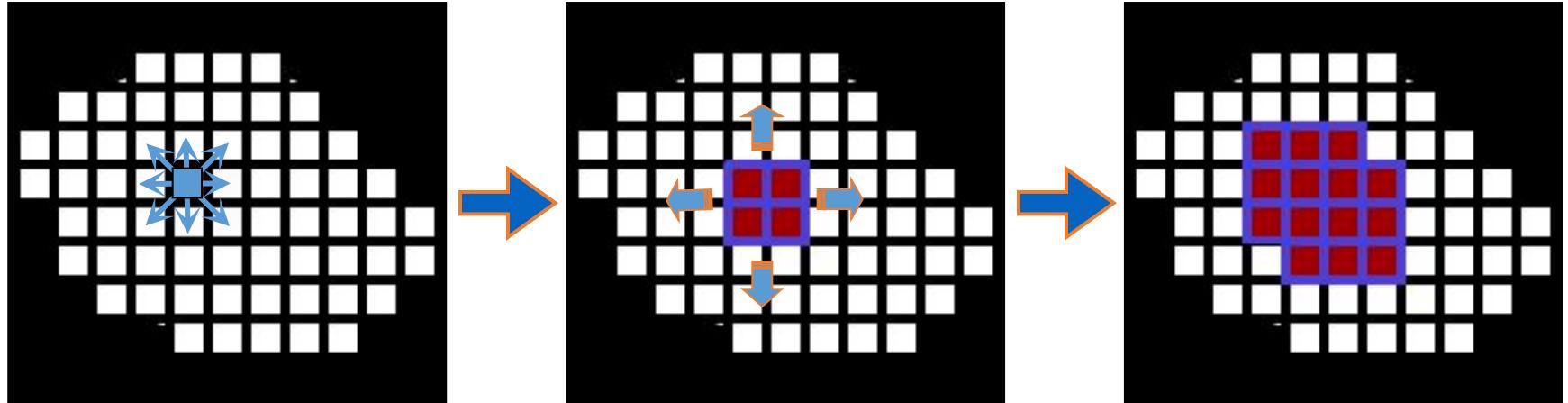
M : 潮湿区域

u, v : 速度

p : 水压

边缘暗化

流动颜料



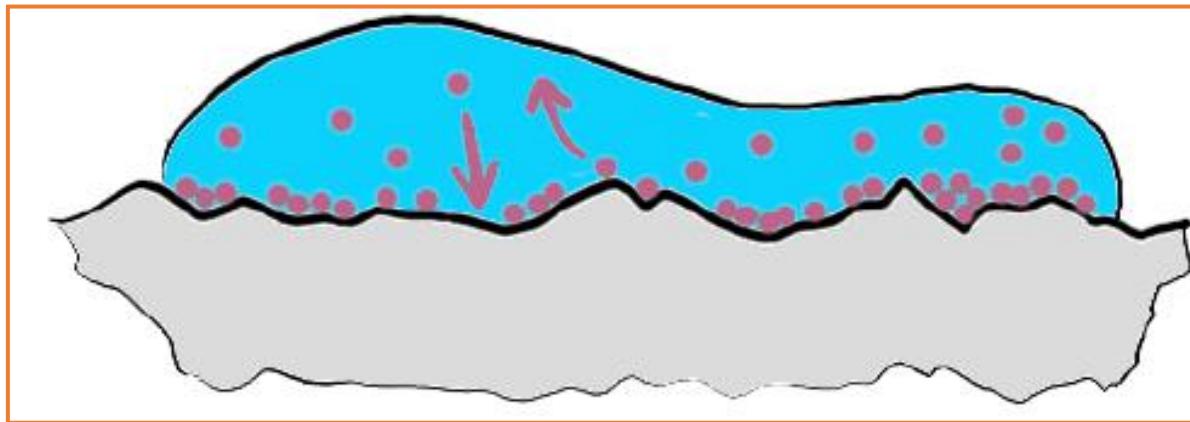
- 颜料依据速度场 u 、 v ，在shallow-water 层中流动
- 每一个细胞中的颜料都根据相应细胞外液体流动的速度向它的邻近区域扩散

颜料传递

- 颜料的吸收和脱附

```
proc TransferPigment( $g^1, \dots, g^n, d^1, \dots, d^n$ ):  
    for each pigment  $k$  do  
        for all cells  $(i, j)$  do g, d : 颜料浓聚物
```

...



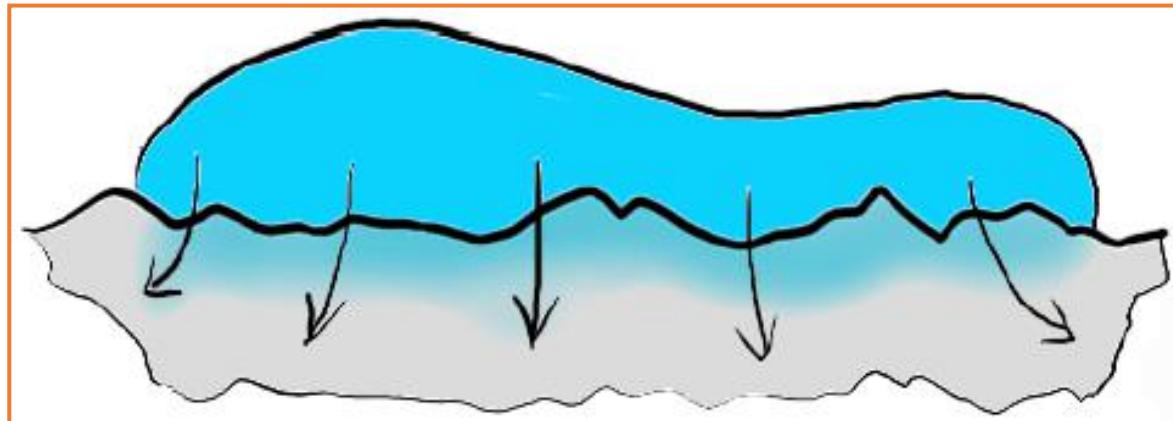
Source: Computer Generated Watercolor

模拟毛细流动

- 在capillary层中扩散的水流

```
proc SimulateCapillaryFlow( $s, M$ ):     $s$ : 纸张饱和度  
    for each pigment  $k$  do  
        for all cells  $(i, j)$  do
```

...



干刷效果
收笔

Source: Computer Generated Watercolor

流体模拟

Main loop

 初始速度
 初始潮湿区域
proc MainLoop($M, u, v, p, g^1, \dots, g^n, d^1, \dots, d^n, s$):

 for each time step do:

 MoveWater(M, u, v, p) 初始颜料浓聚物

 MovePigment(M, u, v, g^1, \dots, g^n)

 TransferPigment($g^1, \dots, g^n, d^1, \dots, d^n$)

 SimulateCapillaryFlow(M, s)

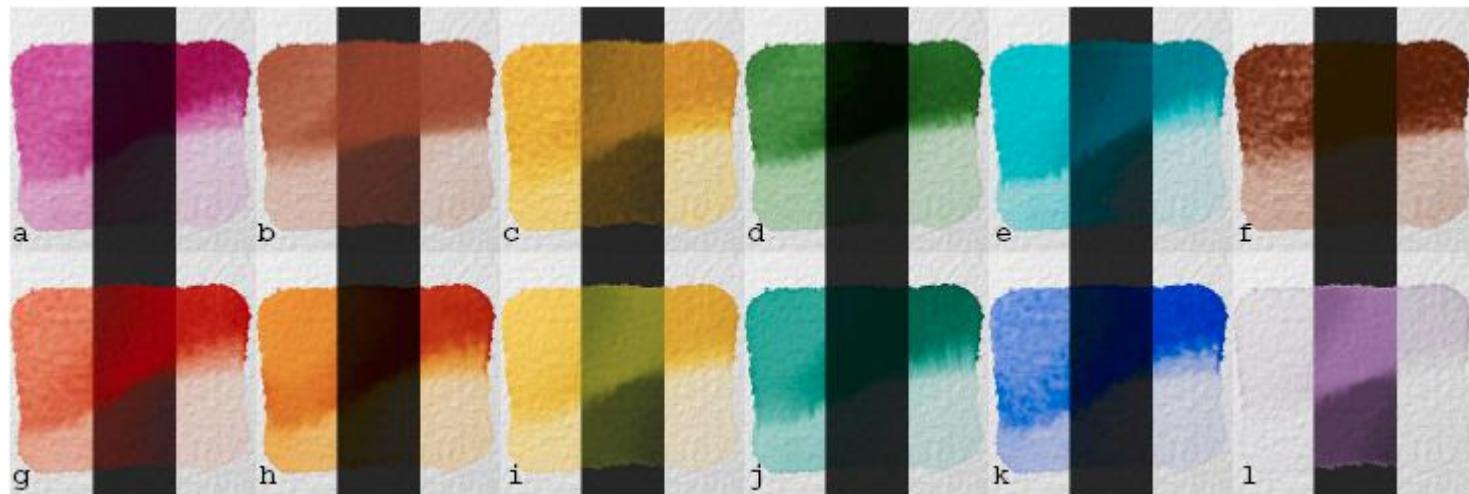
 end for

end proc

视觉合成

- 颜料层次渲染

- 利用Kubelka-Munk (KM) 来合成不同颜料层次



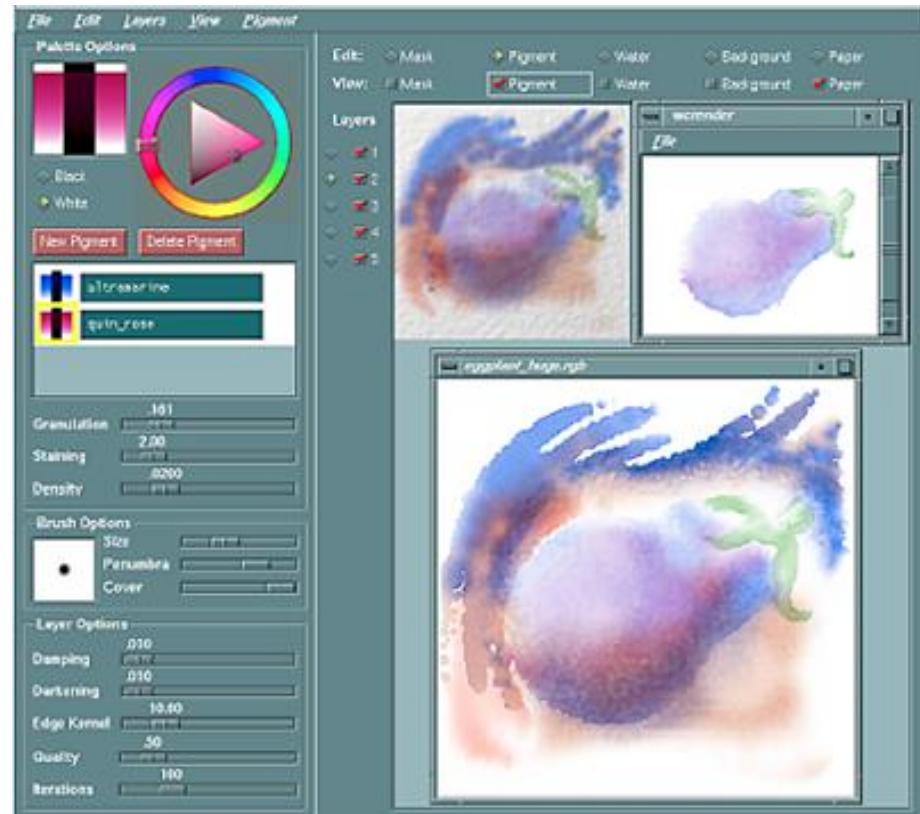
Source: Computer Generated Watercolor

应用

- 交互水彩作画
- 图像自动水彩风格化
- 3D模型的非真实感绘制

交互水彩作画

- 用户创建
 - Glazes
- 用户调整
 - 笔刷大小
 - 颜料
 - 潮湿区域
 - 物理参数

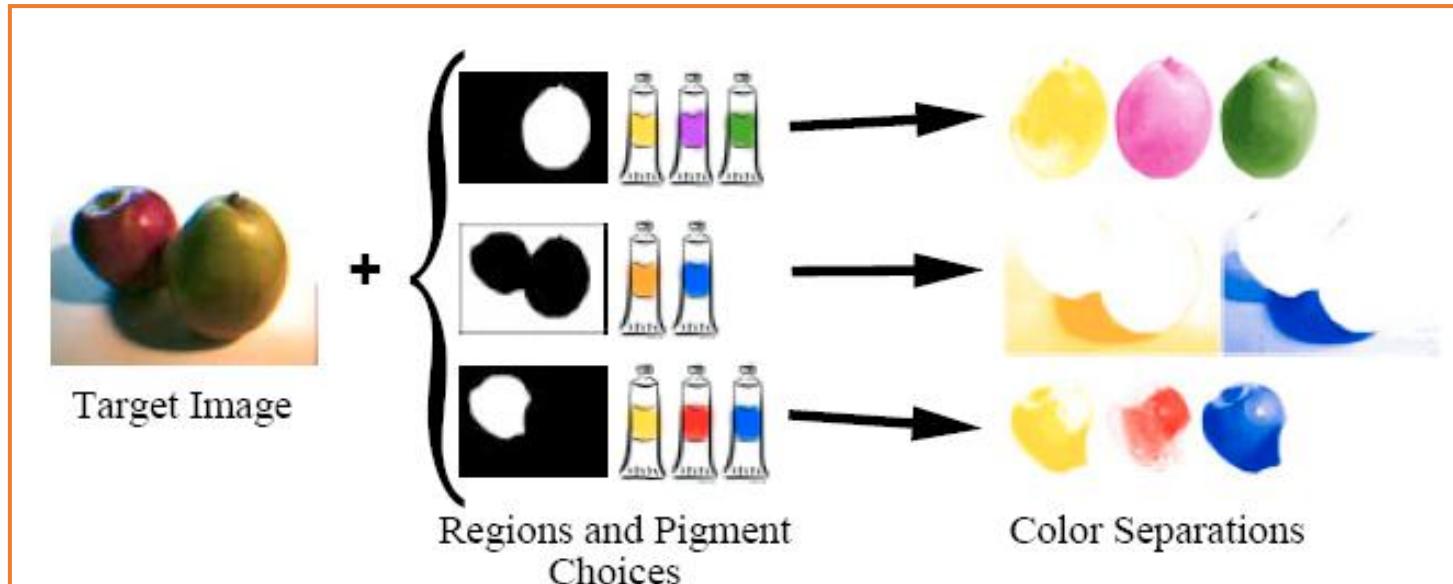


Source: Computer Generated Watercolor

图像自动水彩风格化

- 自动的把图像转换为水彩风格
- 步骤
 - 颜色分割
 - 笔触规划

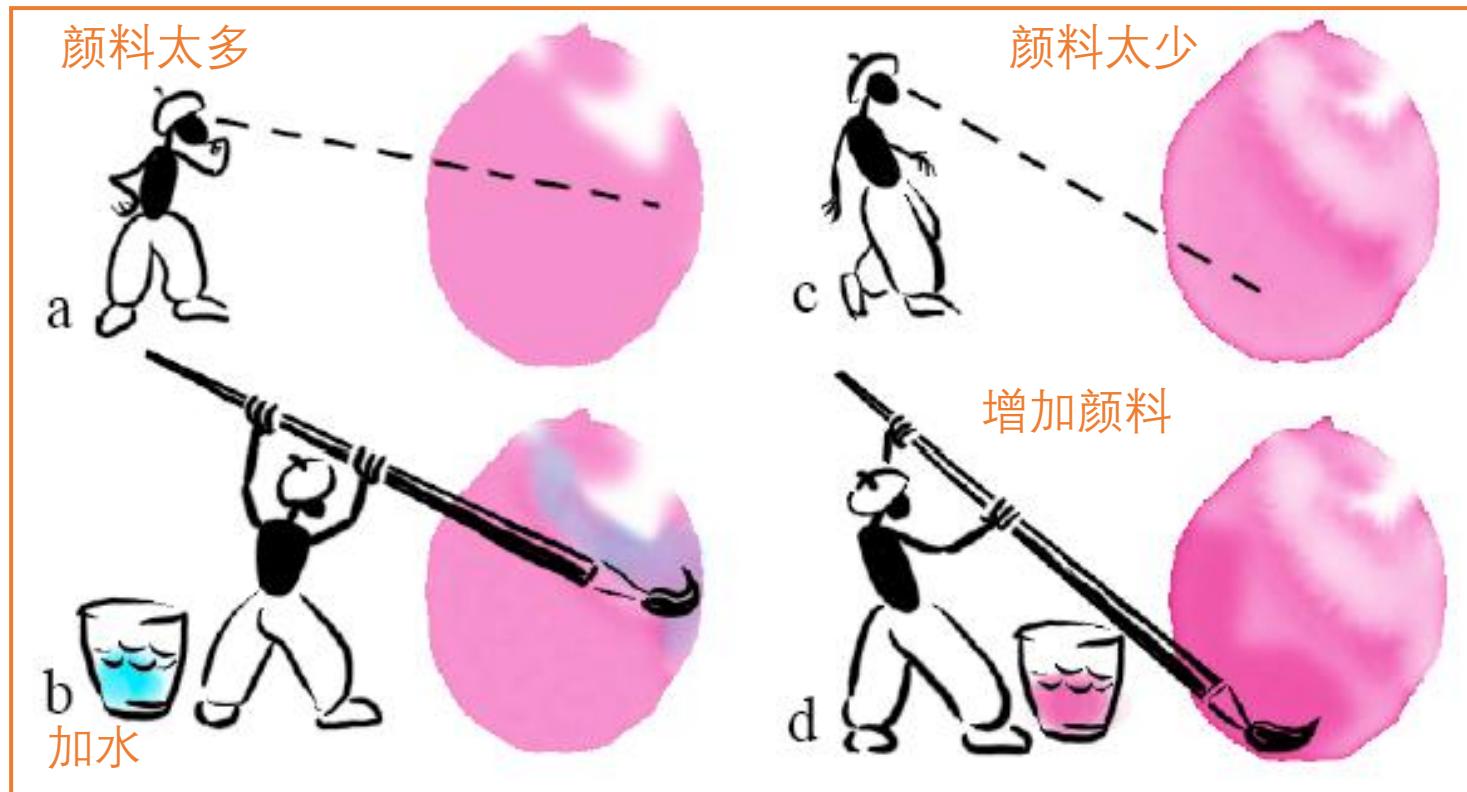
颜色分割



颜色分割过程

笔触规划

- 画家控制笔触的颜料和流动



Brushstroke Planning

Source: Computer Generated Watercolor

图像自动水彩风格化

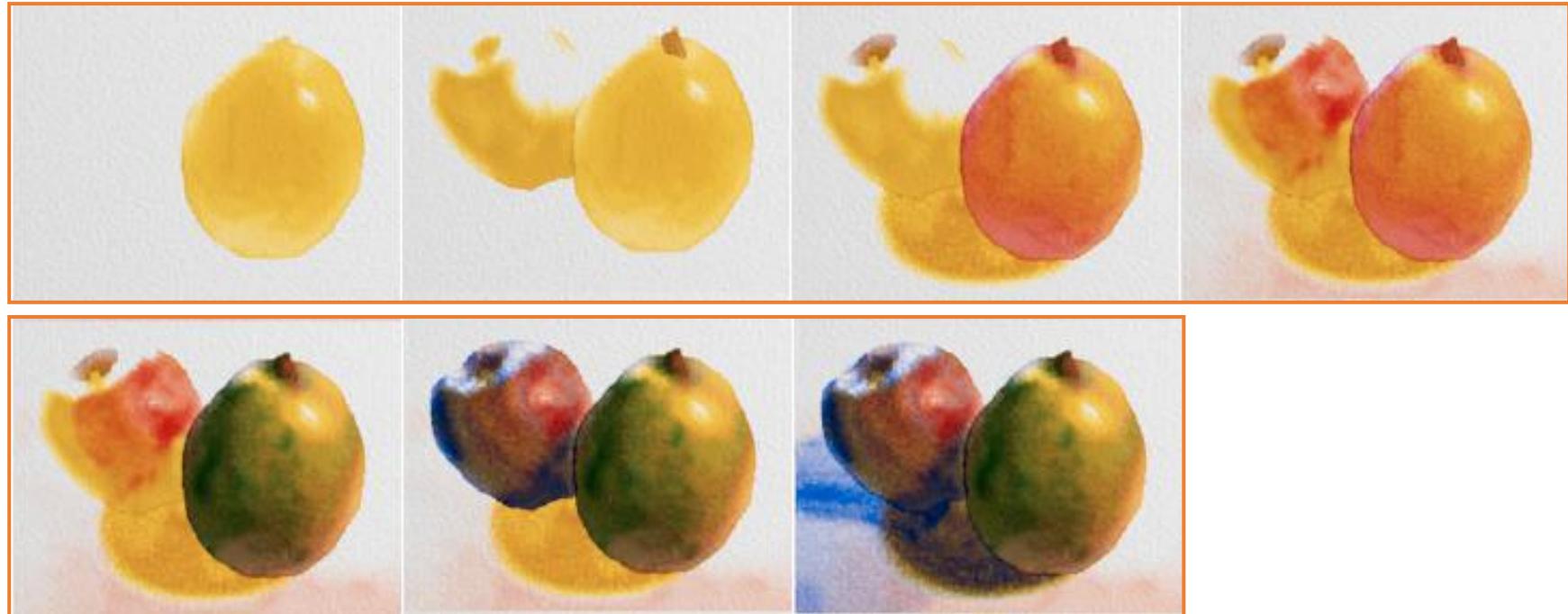


自动生成水彩画



原图

渲染步骤



Source: Computer Generated Watercolor

创作媒介的计算机模拟

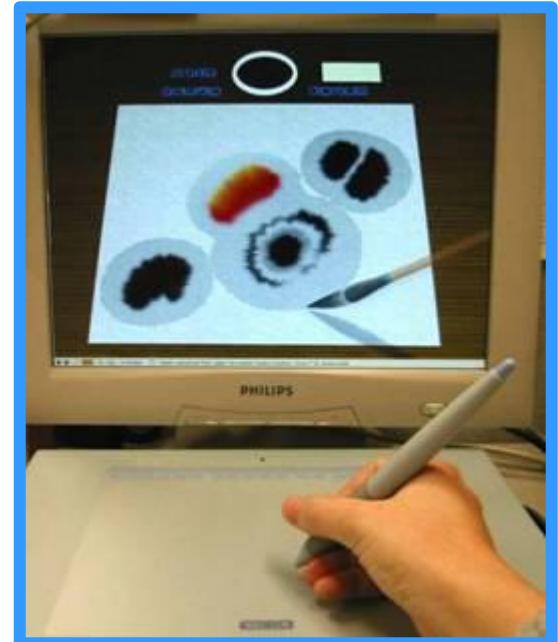
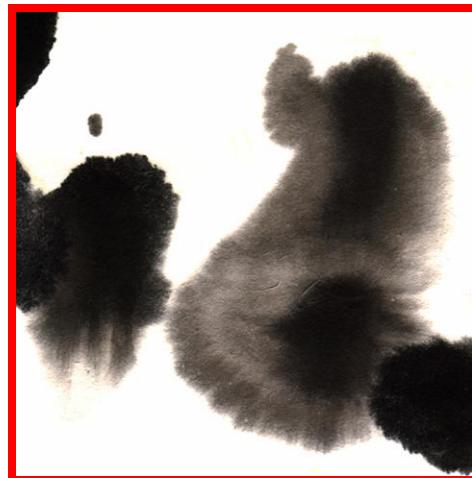
- 铅笔
 - Observational models of graphite pencil materials@CGF2000
- 水彩
 - Computer-Generated Watercolor@Siggraph1997
- 水墨
 - MoXi: Real-Time Ink Dispersion in Absorbent Paper@Siggraph2005

水墨绘图

- 水墨画特点

- 画笔(Brush) → 表达线条和形状

- 水墨(Iink) → 色彩的扩散



The MoXi system

水墨绘图

- 新的电脑绘图工具
 - 数字绘画
 - 美术字体
 - 动画/特效
- 超越真实的画笔和水墨(Brush & Ink)
 - 保留传统，只有发展才能保留，不发展就不可能保留。 — 吴冠中
- 回复传统艺术形式
 - 推广普及大众

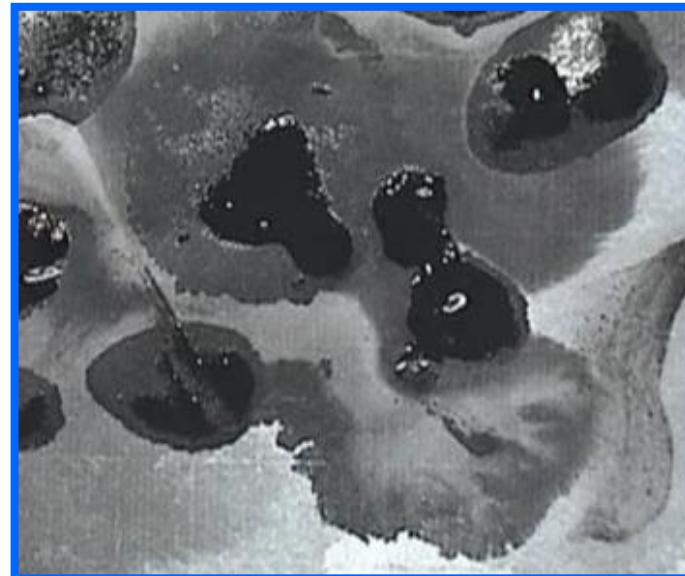


真实水墨扩散



水墨扩散模拟(Water & Pigment)

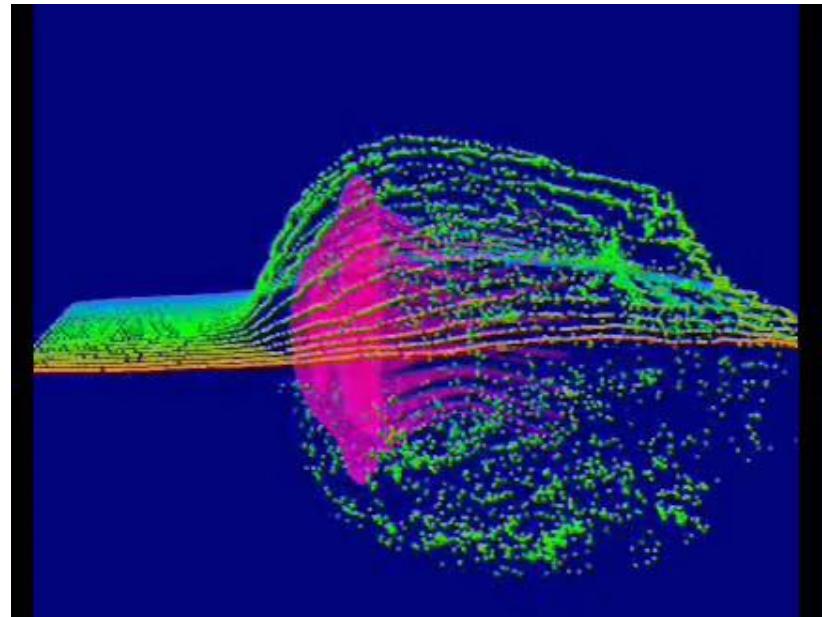
- 液体渗透
 - 2D Lattice Boltzmann (LB) 流体模型
- 颜料颗粒运动
 - 流体扩散



Lattice Boltzmann 流体力学

- 普通流体动态模拟方法
 - 过去二十年间发展迅速
 - 由纽约州立大学石溪分校引入图形学(2002年)

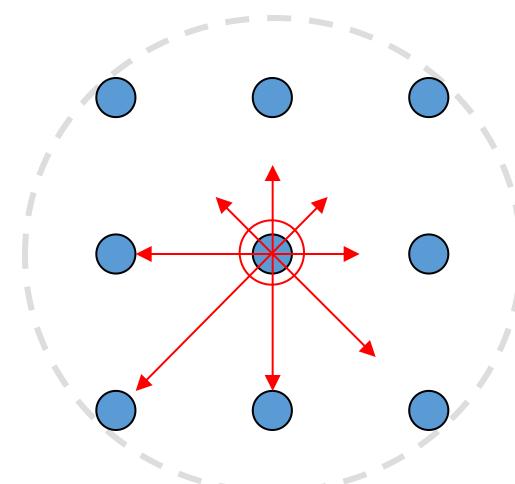
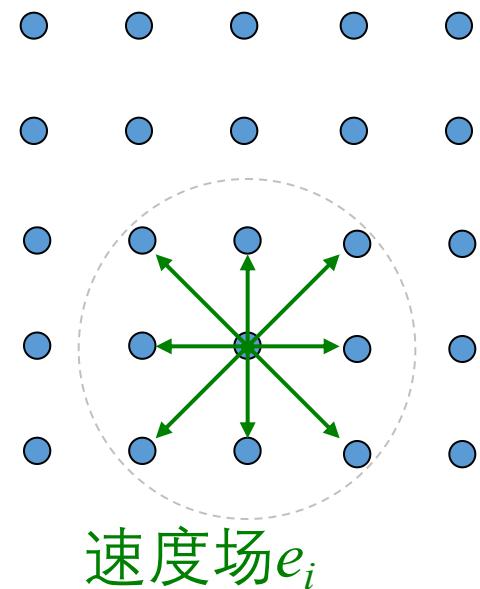
Lattice size = 50^3 . Video
captured in real time on
Geforce FX5900u
[Li et al. 2003]



Lattice Boltzmann 方程

- 离散空间(网格框架)
- 分布函数 $f_i(x, t)$
- 每个时间点(time step)
 - $f_i(x, t)$ 依据速度场 e_i 流动
 - $f_i(x, t)$ 向平衡态松弛

二维9方向网格模型



Lattice Boltzmann 方程

■ 每个时间点(time step)

- $f_i(x, t)$ 依据速度场 e_i 流动

- $f_i(x, t)$ 向平衡态松弛

$$f_i(\mathbf{x} + \mathbf{e}_i \Delta t, t + \Delta t) = (1 - \omega) f_i(\mathbf{x}, t) + \omega f_i^{(eq)}(\mathbf{x}, t)$$

流动

松弛

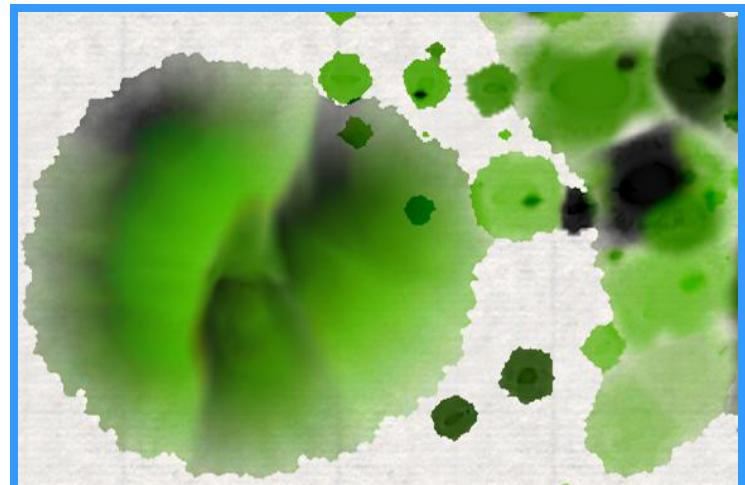
平衡态分布函数

$$f_i^{(eq)} = \rho w_i \left[1 + \frac{3}{c^2} \mathbf{e}_i \cdot \mathbf{u} + \frac{9}{2c^4} (\mathbf{e}_i \cdot \mathbf{u})^2 - \frac{3}{2c^2} \mathbf{u} \cdot \mathbf{u} \right]$$

$$\text{密度 } \rho = \sum_{i=0}^8 f_i \quad \text{速度 } \mathbf{u} = \frac{1}{\rho} \sum_{i=1}^8 \mathbf{e}_i f_i$$

渗透模型

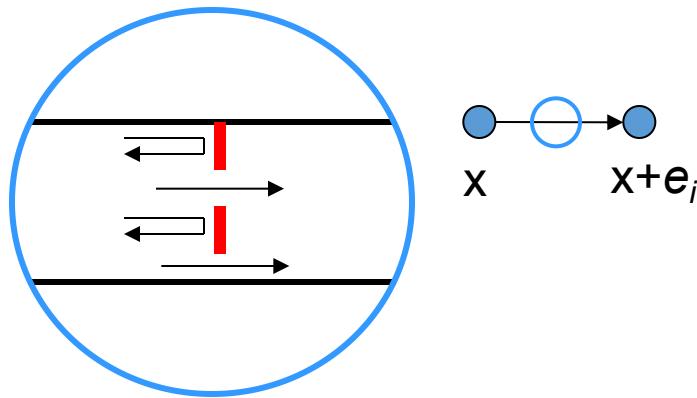
- 基于不可压缩LB模型 (incompressible LB model)
- 增加的特征(features)
 - 半渗透性
 - 对流
 - 边界粗化
 - 蒸发



模拟水墨

半渗透性

- 纸张质地不均、水墨着色力度
- 分支样式
- partially bounce-back $f_i(x, t)$ 模型



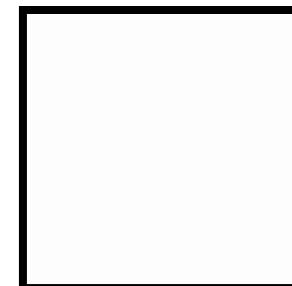
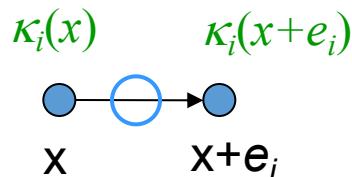
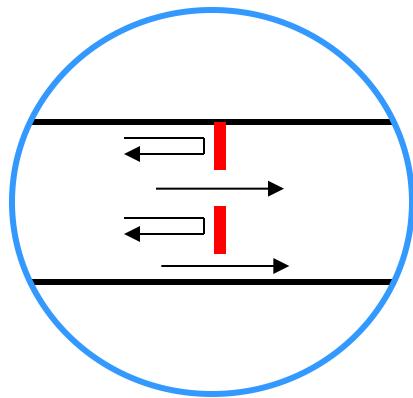
半渗透性

- 每个网格设置系数 K_i

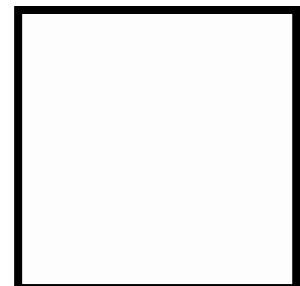
基于Bounce-Back的流动

$$f_i(\mathbf{x}, t+1) = \bar{\kappa}_i(x, e_i) f_k(\mathbf{x}, t) + (1 - \bar{\kappa}_i(x, e_i)) f_i(\mathbf{x} - \mathbf{e}_i, t)$$

$$\bar{\kappa}_i(x, e_i) = \frac{\kappa_i(x) + \kappa_i(x+e_i)}{2}$$



Small K_i



Large K_i

对流

- 单相模型

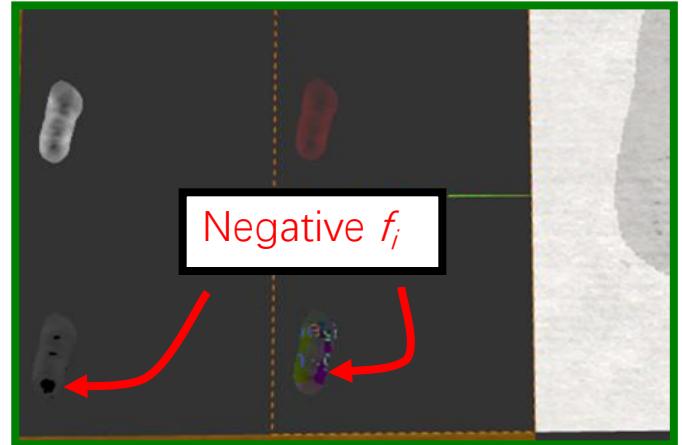
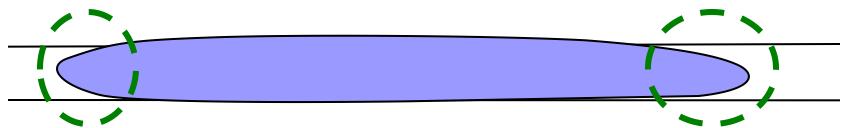
- ρ : 液体量

- 原始LB方程

- ρ 极大, 则 $f_i(x, t) < 0$

- 对流模型

- ρ 较小时对流较低



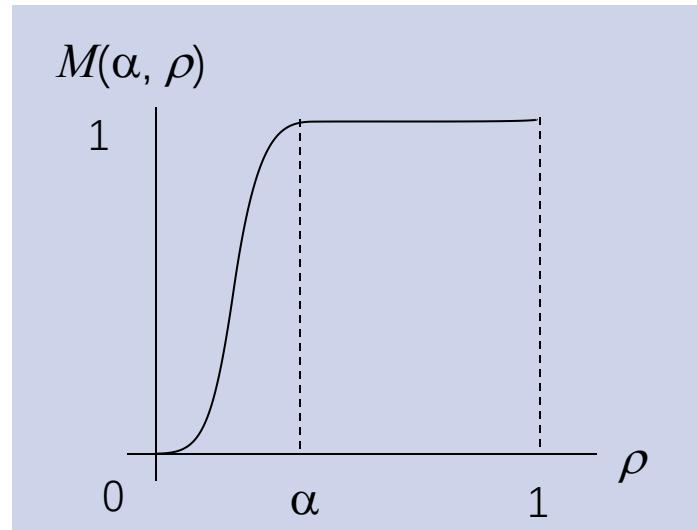
对流

- 我们的模型

- ρ 较小时对流较低

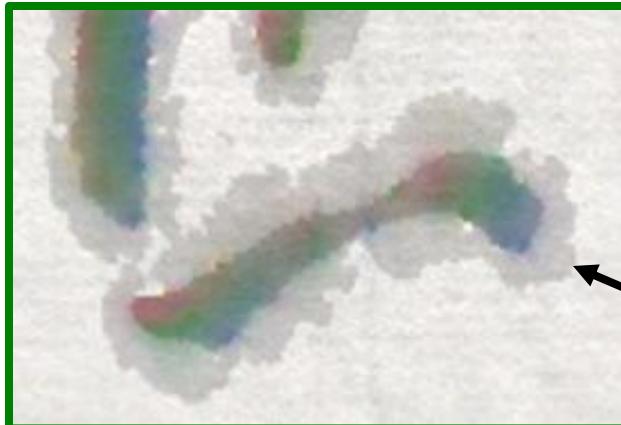
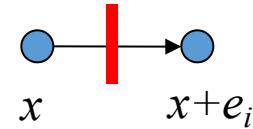
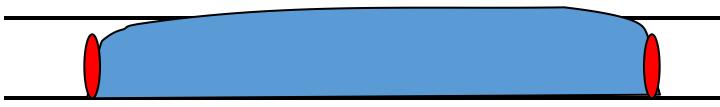
$$f_i^{(eq)} = w_i \left\{ \rho + \rho_0 M \left[\frac{3}{c^2} \mathbf{e}_i \cdot \mathbf{u} + \frac{9}{2c^4} (\mathbf{e}_i \cdot \mathbf{u})^2 - \frac{3}{2c^2} \mathbf{u} \cdot \mathbf{u} \right] \right\}$$

$M(\alpha, \rho) = \text{smoothstep}(0, \alpha, \rho)$

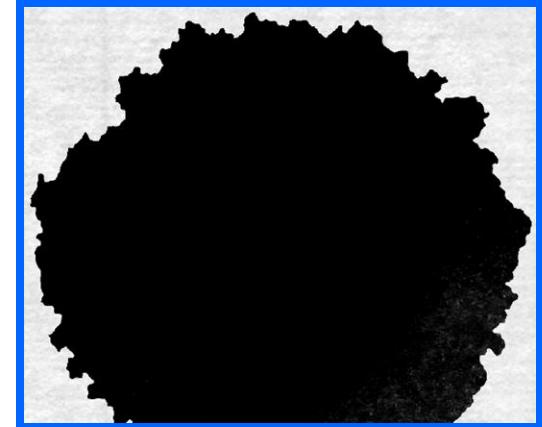


边界粗化

- 利用渗透 (permeability) 和闭合 (pinning) 实现
- 闭合: *Block if (site [$x+e_i$] is dry) & ($f_i(x) < Pin_{thres}$)*



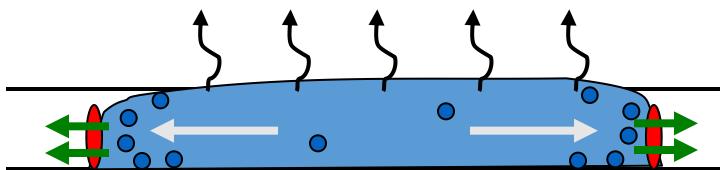
移动边界



边界特性

蒸发

- 闭合部分边界
 - 液体被相邻部分吸收
- 液体流失 \Rightarrow 向外流动减弱 \Rightarrow 边界暗化
- 减少 $f_i(x, t)$ 向边界外的流动



真实边界暗化



模拟边界暗化

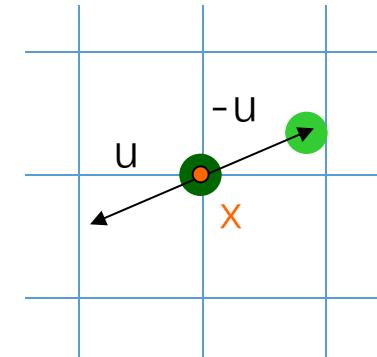
颜料颗粒(pigment)运动

- 特征线法

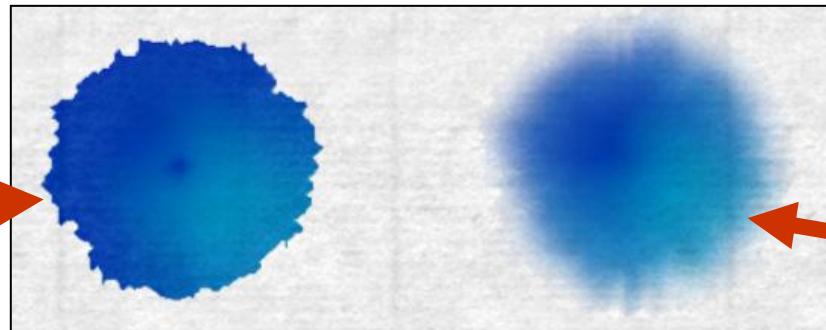
- 追溯每个时间点速度场变化
- $\text{Pigment}'(x) \leftarrow \text{Pigment}(x - u(x))$

- 障碍(hindrance)模型

- 新旧颗粒浓度间插值
- $\text{Pigment}(x) \leftarrow \text{lerp}(\text{Pigment}'(x), \text{Pigment}(x), \text{hinder}(x))$



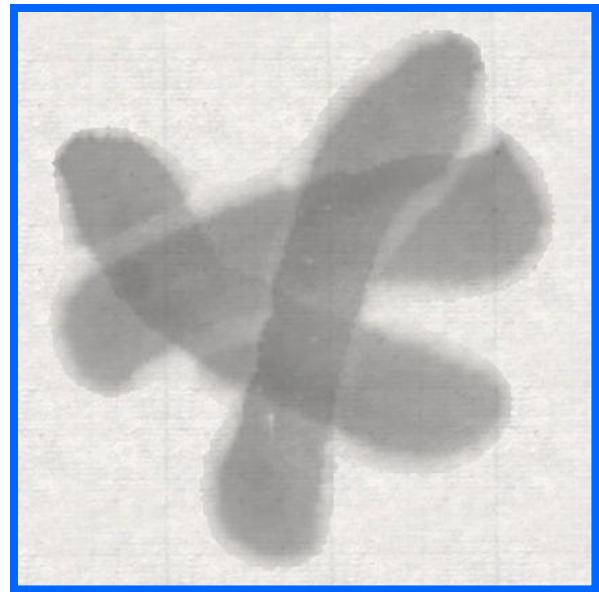
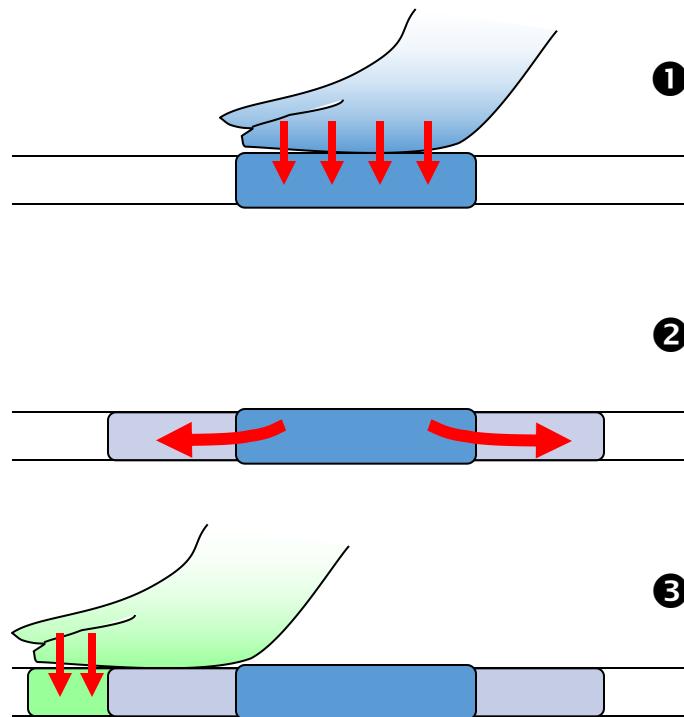
不含障碍模型



加入障碍模型

纸张接受度(Receptivity)

- 如果饱和(saturated)则不会继续吸收



大纲

- 简介
- 水墨模拟 (Ink Simulation)
 - 真实水墨的扩散原理
 - Lattice Boltzmann 方程
 - 我们的水墨扩散模型
 - 液体渗透
 - 颜料颗粒运动
- Demo

用MoXi作画

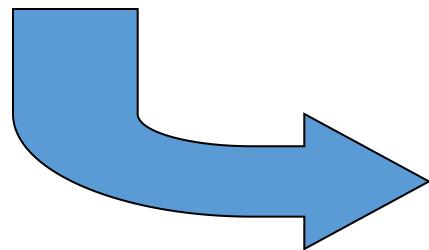


系统性能

- 512² GeForce 6800 Ultra
 - 液体渗透模拟: 100+ fps
 - 整个系统(渗透、颜料颗粒运动、绘制): 48 fps

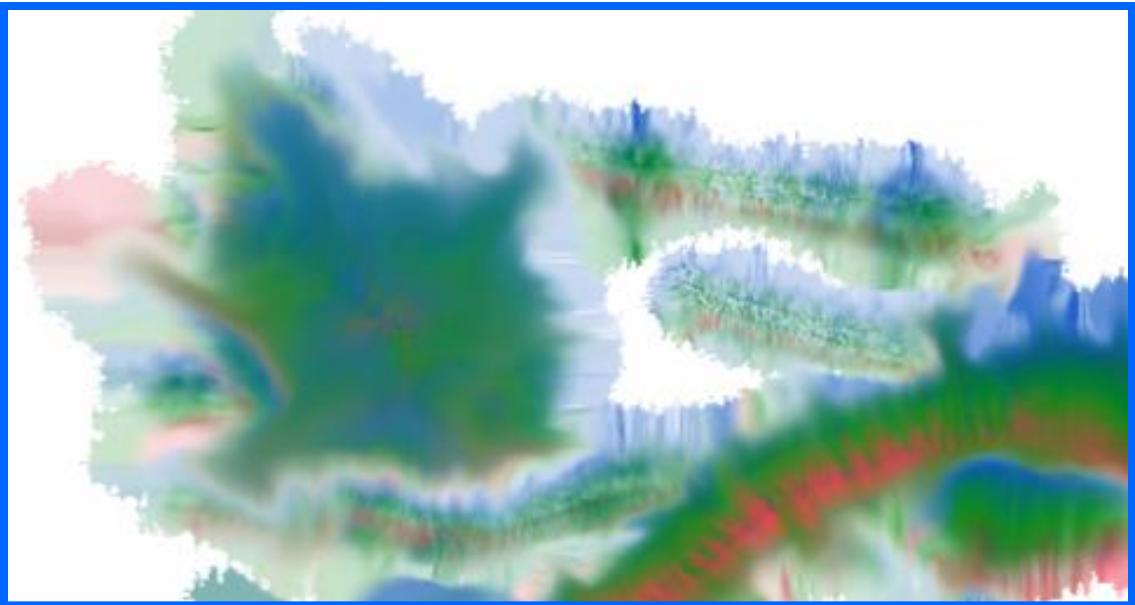
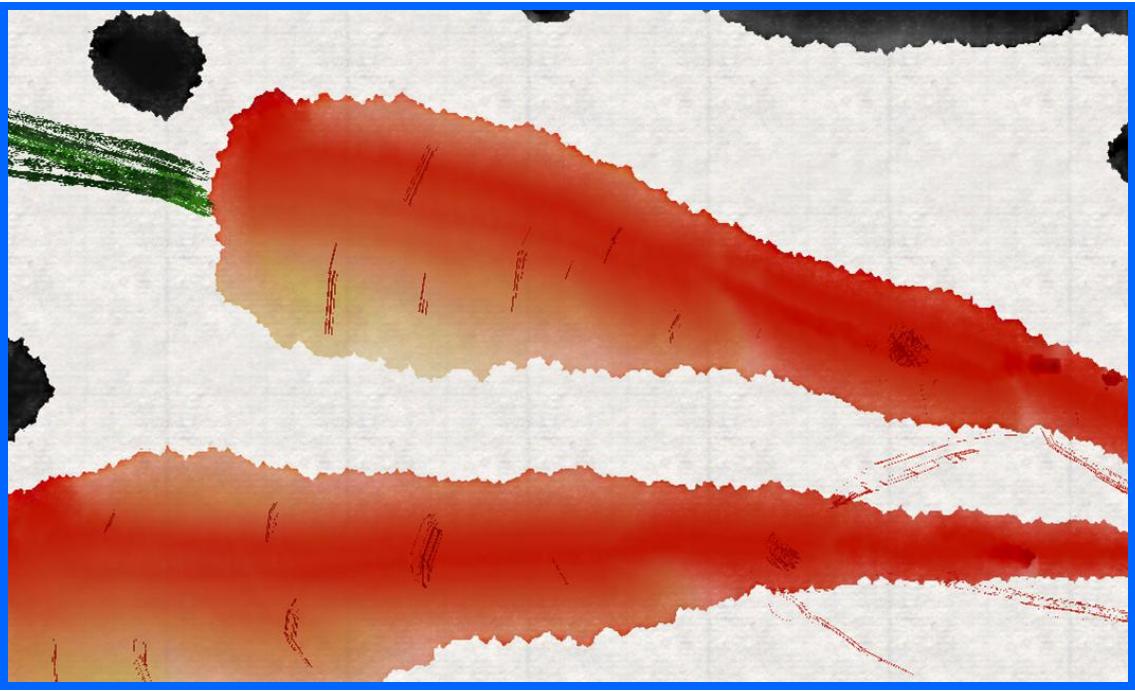
MoXi特效

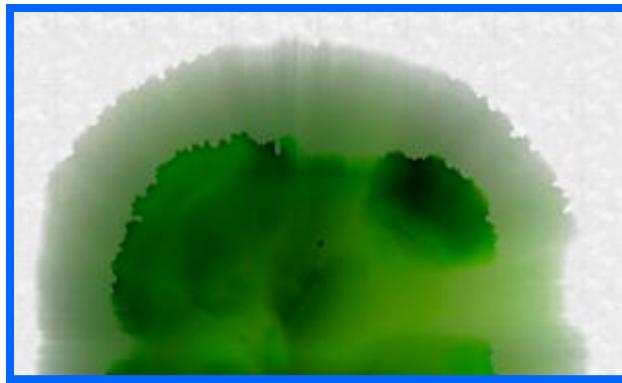
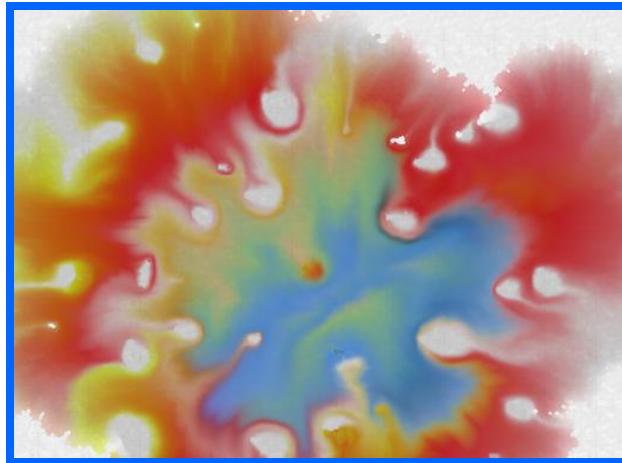
Input Painting

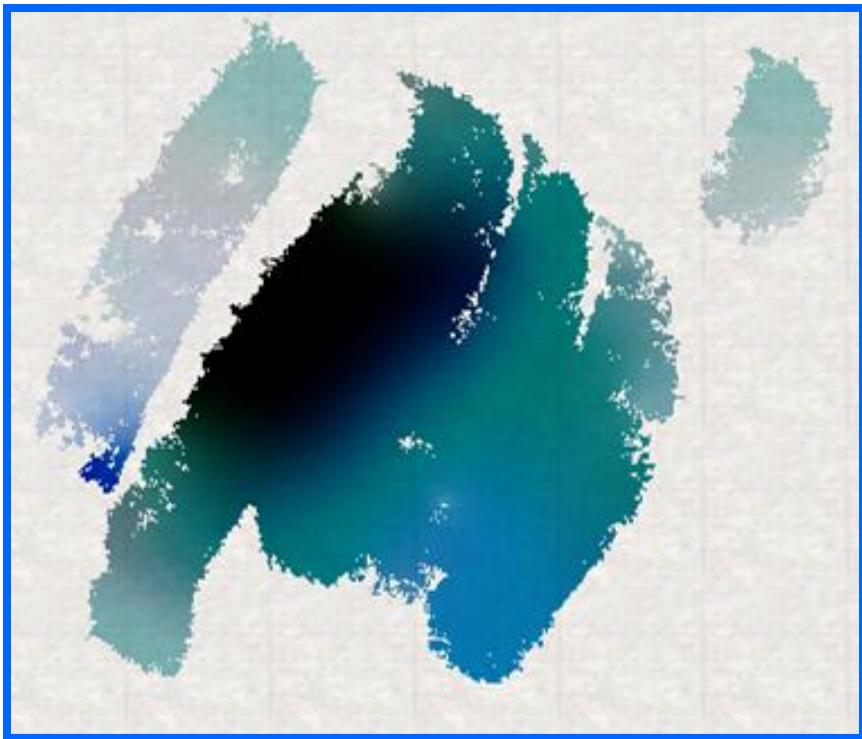


Wash Effect







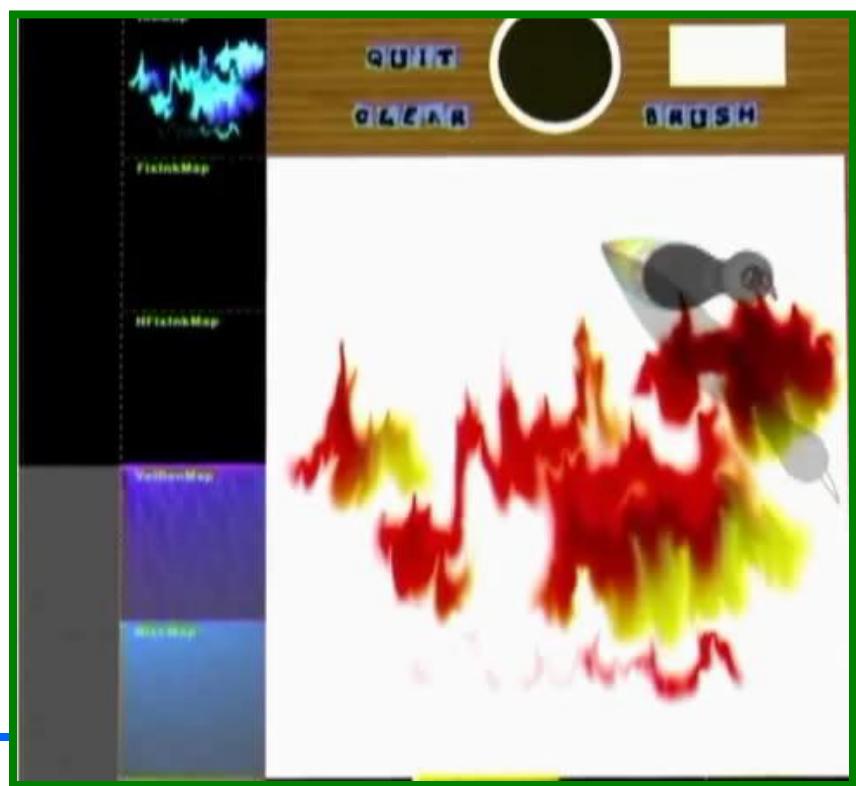


**MoXi
DIGITAL
INK**

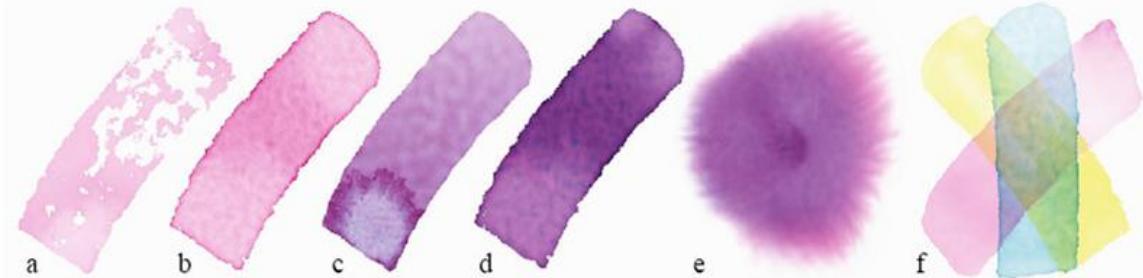
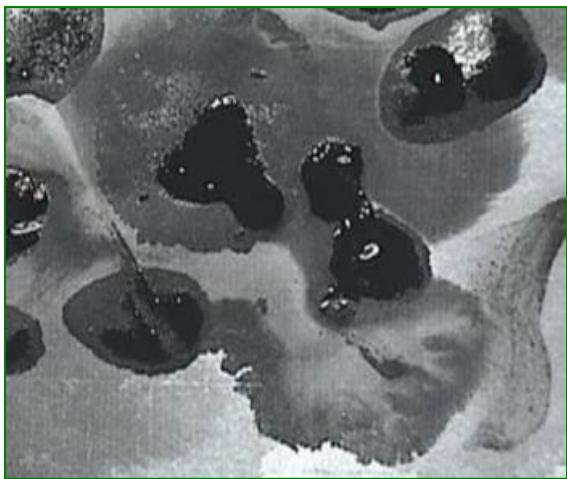
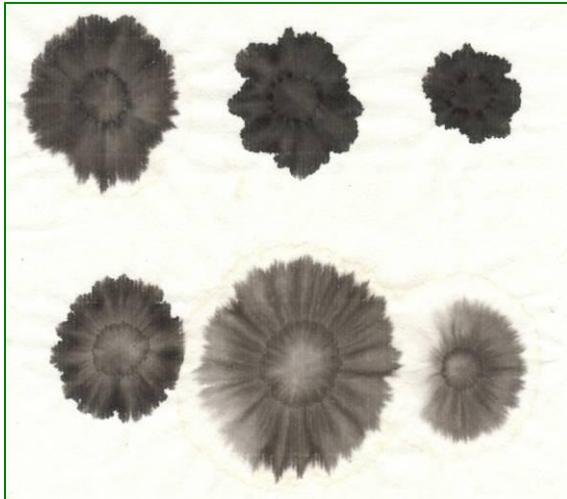




- My Imagination Gone Wild -
This is my imagination. He likes to fly high in the sky.
Don't worry. He won't bite you. He rushes towards you
just because he likes to play with you...



东方 VS. 西方



Curtis et al. 1997

	东方水墨	水彩
流体	附于毛笔纤维之中	附于(平头)画笔表面
	对墨的配料敏感	对纸张纹理敏感
边界	水墨浸入之处即是边界	往往闭合
颜料	传播更远	传播很近

渗透模型很重要！

未来工作

- 高分辨率输出
 - 更好的细节
- 更多流体特征
 - 潺墨 (Splash-Ink)
 - 水拓 (Floating-Ink)
 - 黏性指进 (Viscous fingering)
 - 等等.....



创作媒介的物理模拟

- 小结

- 模拟自然创作媒介主要基于物理建模
- 多使用真实参数，虚拟参数可产生增强效果
- 效率非常重要：实时
- 创作过程需人来完成

- 可能的发展方向

- 提高常用创作媒介的仿真程度
- 尝试新的创作媒介模拟

非真实感绘制研究分类

- 如何画出一条笔划
- 如何设计布局及笔画布置
 - 图像空间
 - 物体空间
- 如何让作品动起来

基于笔划的布置

- 笔划布置的贪婪算法
- 基于Voronoi图的全局优化

Motivation

- Creating a painting from a photograph



Observation : real painting



Observation

- From a rough sketch to fine details
 - Painting a series of layers
- A stroke
 - A line or a curve
 - A contour with constant color
 - A set of strokes represents a picture

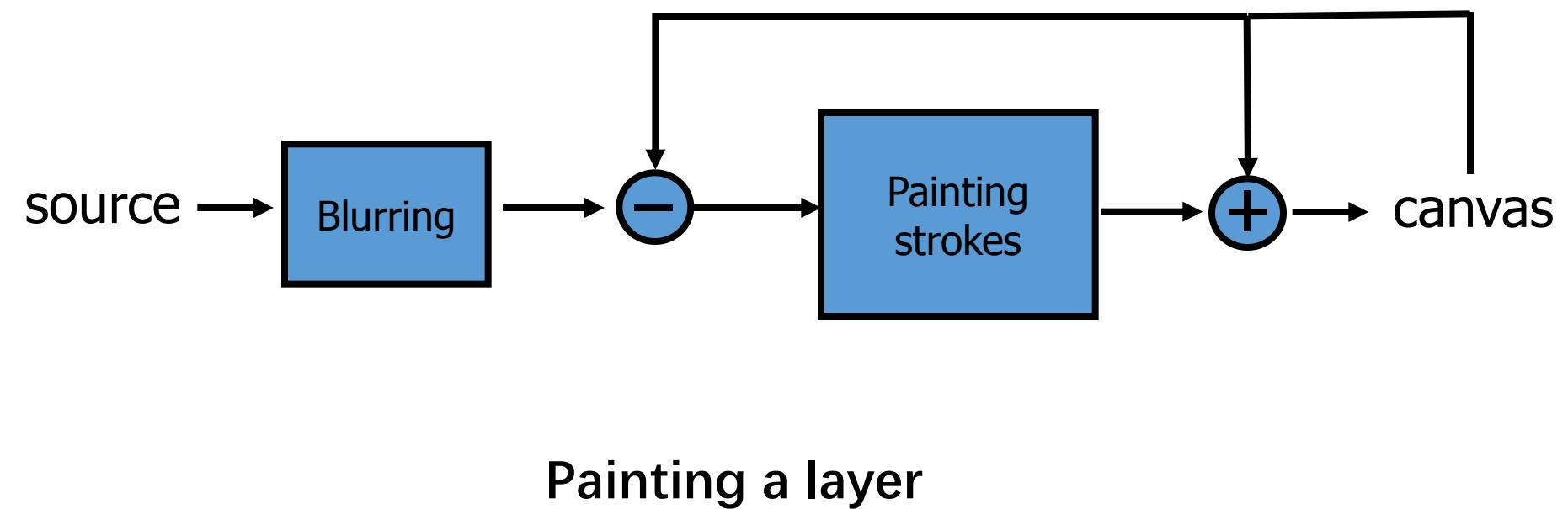
Approach

- Multiple layers from the source image
 - Similar with image pyramid
- Painting with strokes
 - Modeling a stroke : a B-spline curve
 - Creating strokes by analyzing the source image
- Style parameters

Related work

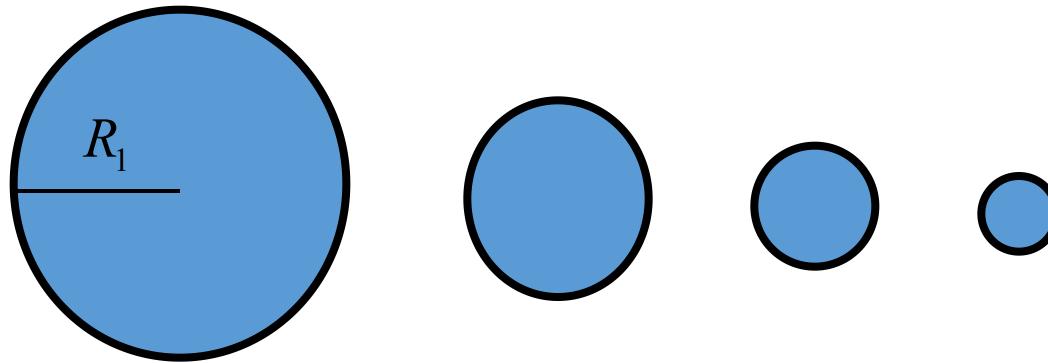
- Physical simulation
 - Curtis et al. 1997, Haeberli 1990
- Automatic painting
 - H. Cohen 1995
- Painting from 3D models
 - Winkenbach et al. 1994, 1996
- Painting from images

Overview



Painting techniques

- Brushes

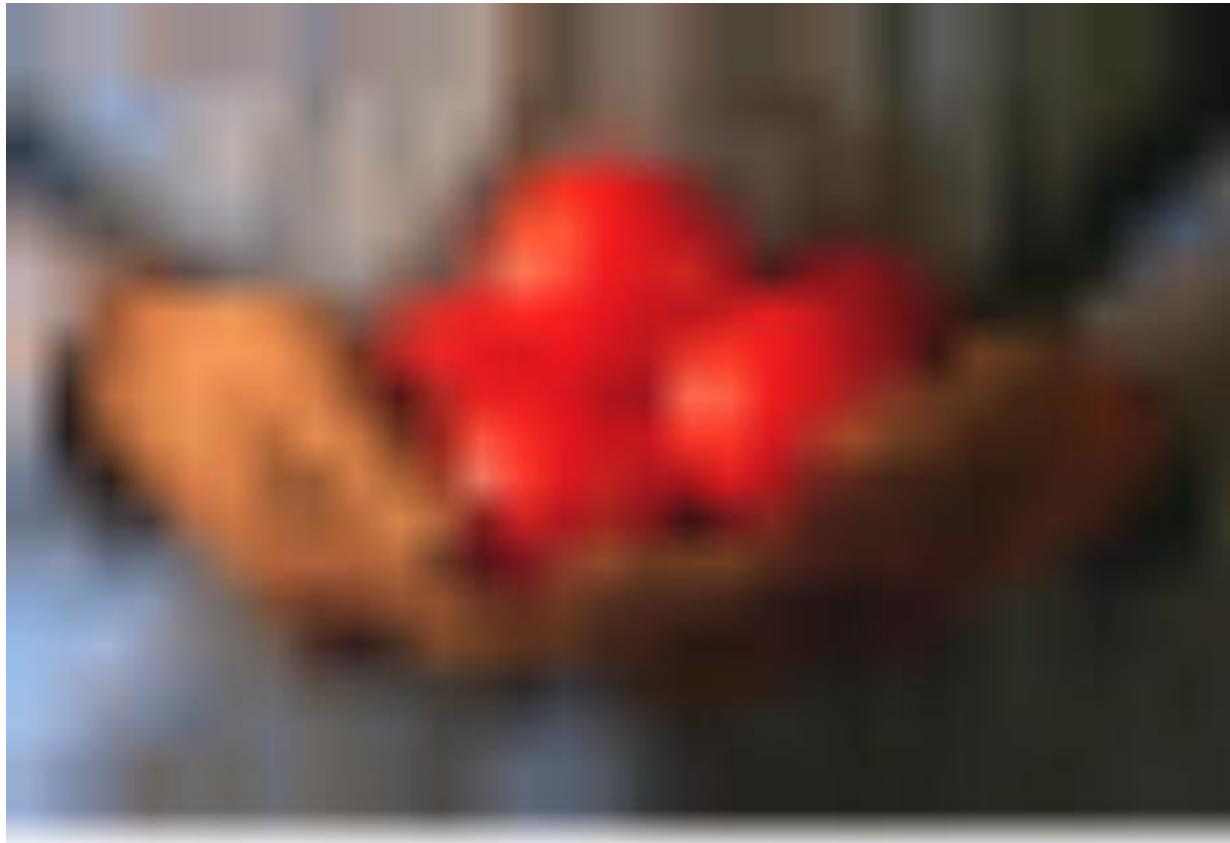


- Creating layers

- Blurring
- A layer corresponds to the radius of a brush $f_\sigma R_i$
 - Gaussian kernel of standard deviation

Painting techniques

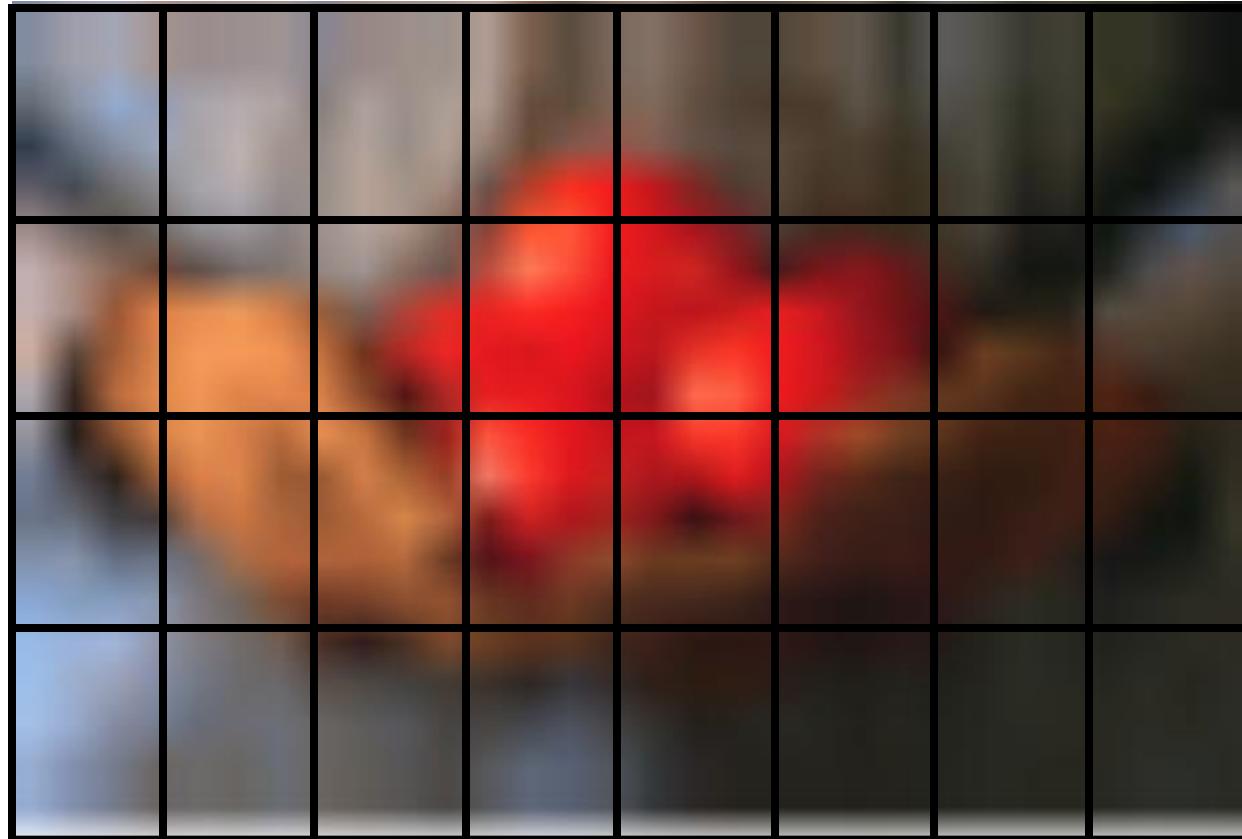
- Placing a stroke



Reference image

Painting techniques

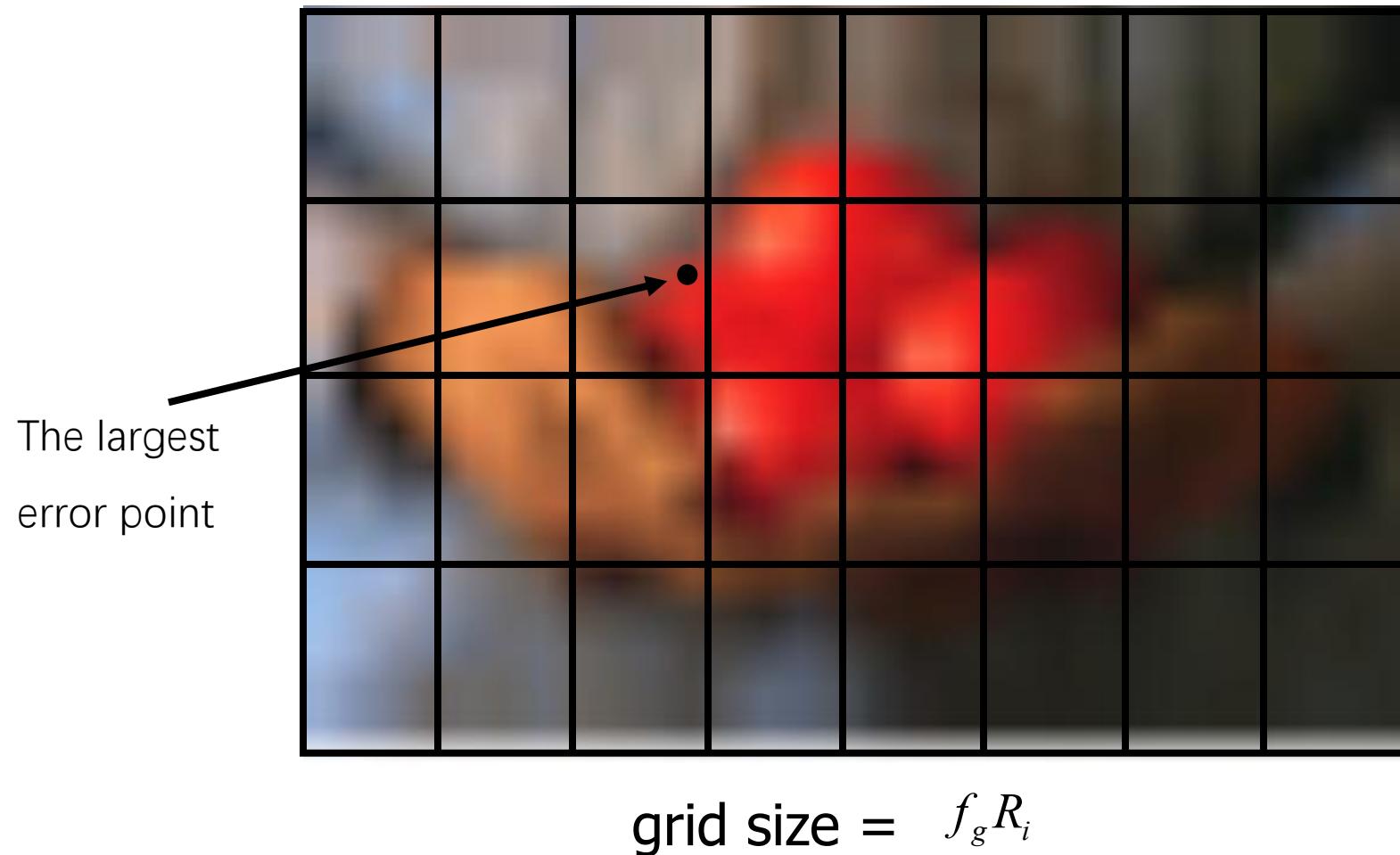
- Placing a stroke



$$\text{grid size} = f_g R_i$$

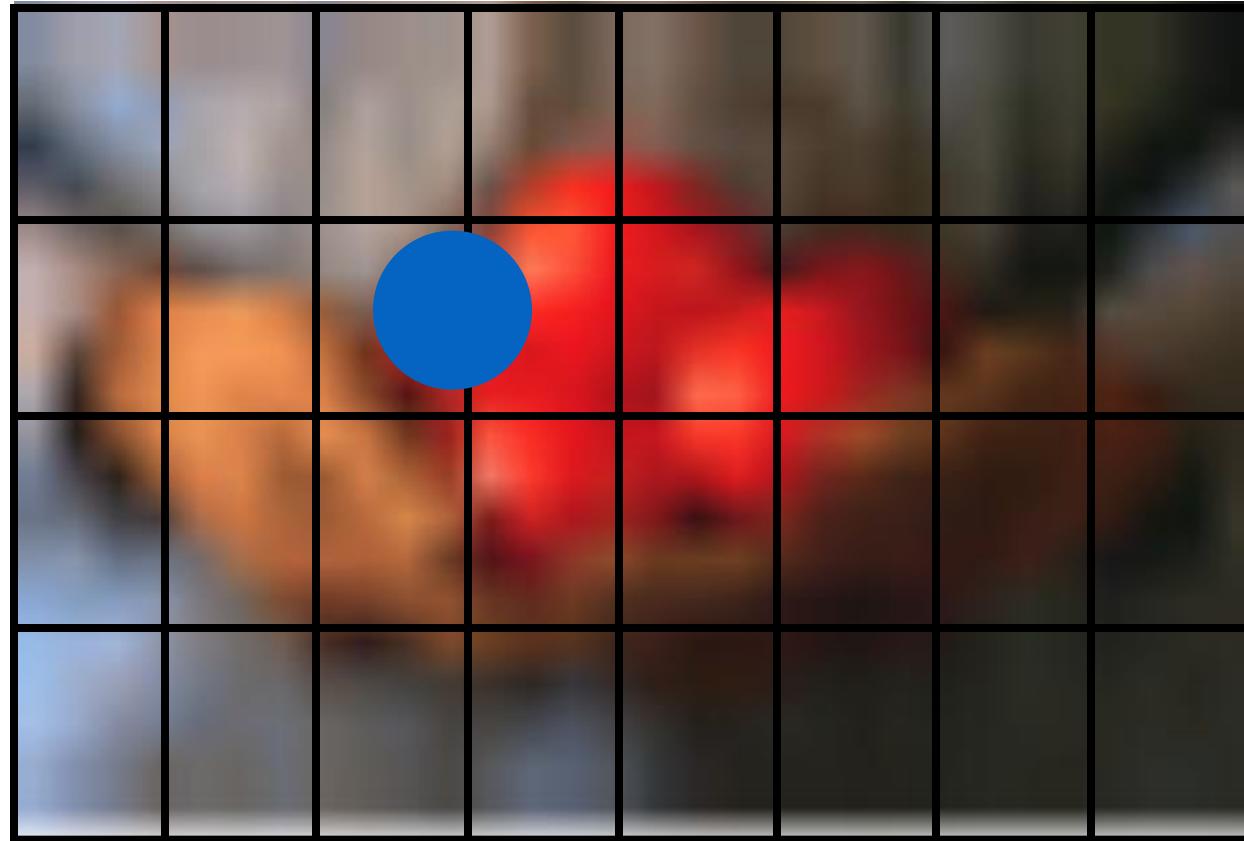
Painting techniques

- Placing a stroke



Painting techniques

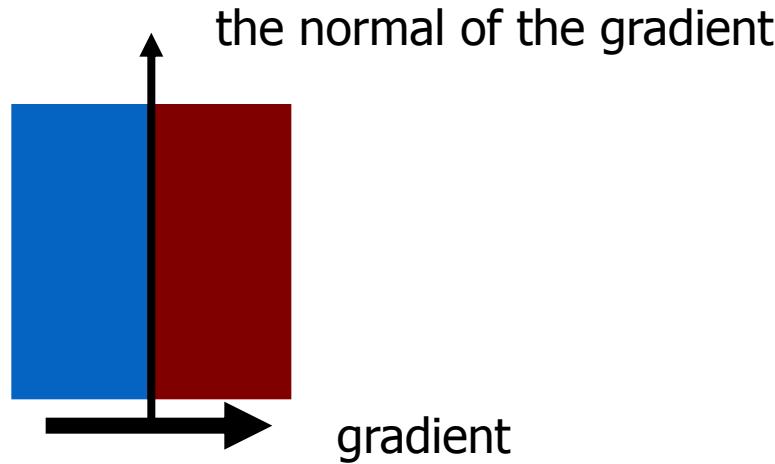
- Placing a stroke



$$\text{grid size} = f_g R_i$$

Creating curved brush strokes

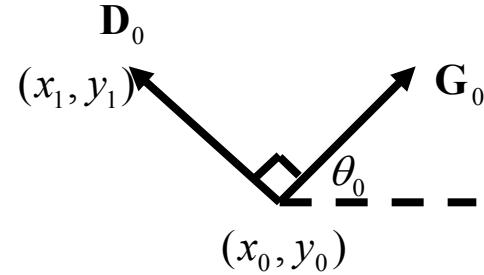
- Basic idea
 - Using image gradients



A stroke follows the normal of the gradient.

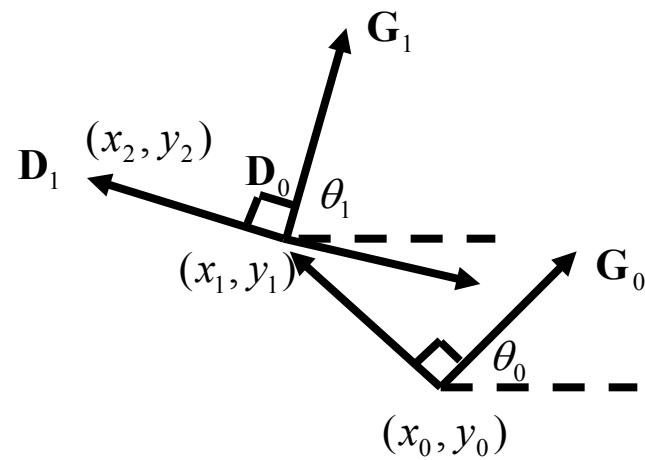
Creating curved brush strokes

Procedure

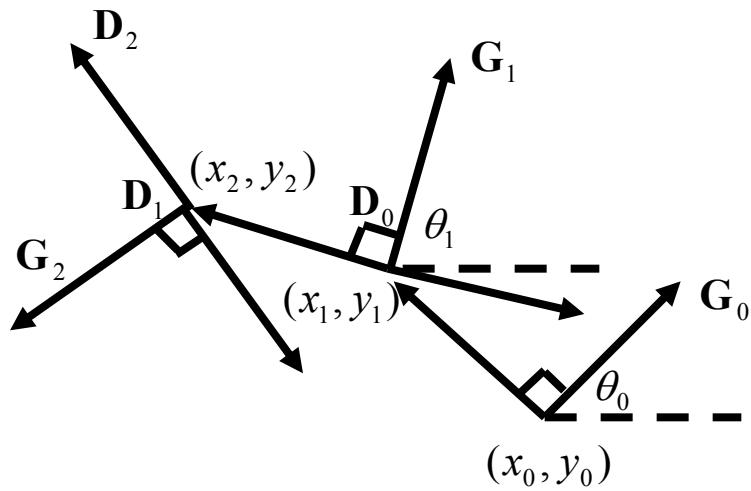


The length of a line segment : R

Creating curved brush strokes



Creating curved brush strokes



Creating curved brush strokes

- Terminating the stroke

- Maximum stroke length
- Color difference

Adjusting the brush stroke

- Curvature filter
 - Curvature filter constant f_c

$$\mathbf{D}'_{i-1} = (dx_{i-1}, dy_{i-1}) \quad \mathbf{D}_i = (dx_i, dy_i)$$

$$\mathbf{D}'_i = f_c \mathbf{D}_i + (1 - f_c) \mathbf{D}'_{i-1}$$

Adjusting the brush stroke

- Minimum & maximum stroke length
 - Maximum length : preventing an infinite loop
 - Minimum length : preventing the speckled appearance
- Brush sizes

Rendering styles

- Style parameters
 - Approximation threshold
 - Brush sizes
 - Curvature filter
 - Blur factor
 - Minimum and maximum stroke lengths
 - Opacity
 - Grid size
 - Color jitter

Rendering styles

- “**Impressionist**” — A normal painting style, with no curvature filter, and no random color. $T = 100$, $\mathbf{R}=(8,4,2)$,
- “**Expressionist**” — Elongated brush strokes. Jitter is added to color value. $T= 50$, $\mathbf{R}=(8,4,2)$, $f_c=.25$, $f_s=.5$, $a=.7$, $f_g=1$, $\text{minLength}=10$, $\text{maxLength}=16$, $j_v=.5$

Rendering styles

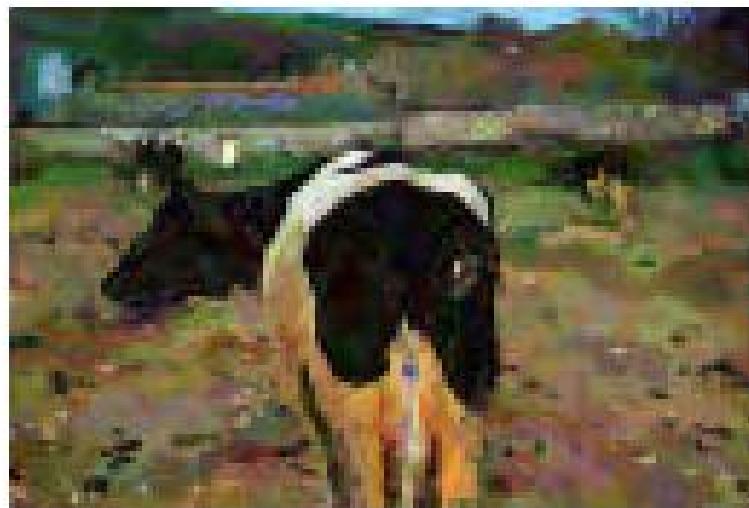
- “**Colorist Wash**” — Loose, semi-transparent brush strokes. Random jitter is added to R, G, and B color components. $T = 200$, $\mathbf{R}=(8,4,2)$, $f_c=1$, $f_s=.5$, $a=.5$, $f_g=1$, $\text{minLength}=4$, $\text{maxLength}=16$, $j_r=j_g=j_b=.3$
- “**Pointillist**” — Densely-placed circles with random hue and saturation. $T = 100$, $\mathbf{R}=(4,2)$, $f_c=1$, $f_s=.5$, $a=1$, $f_g=.5$, $\text{minLength}=0$, $\text{maxLength}=0$, $j_v=1$, $j_h=.3$. (This is

Results

Impressionism

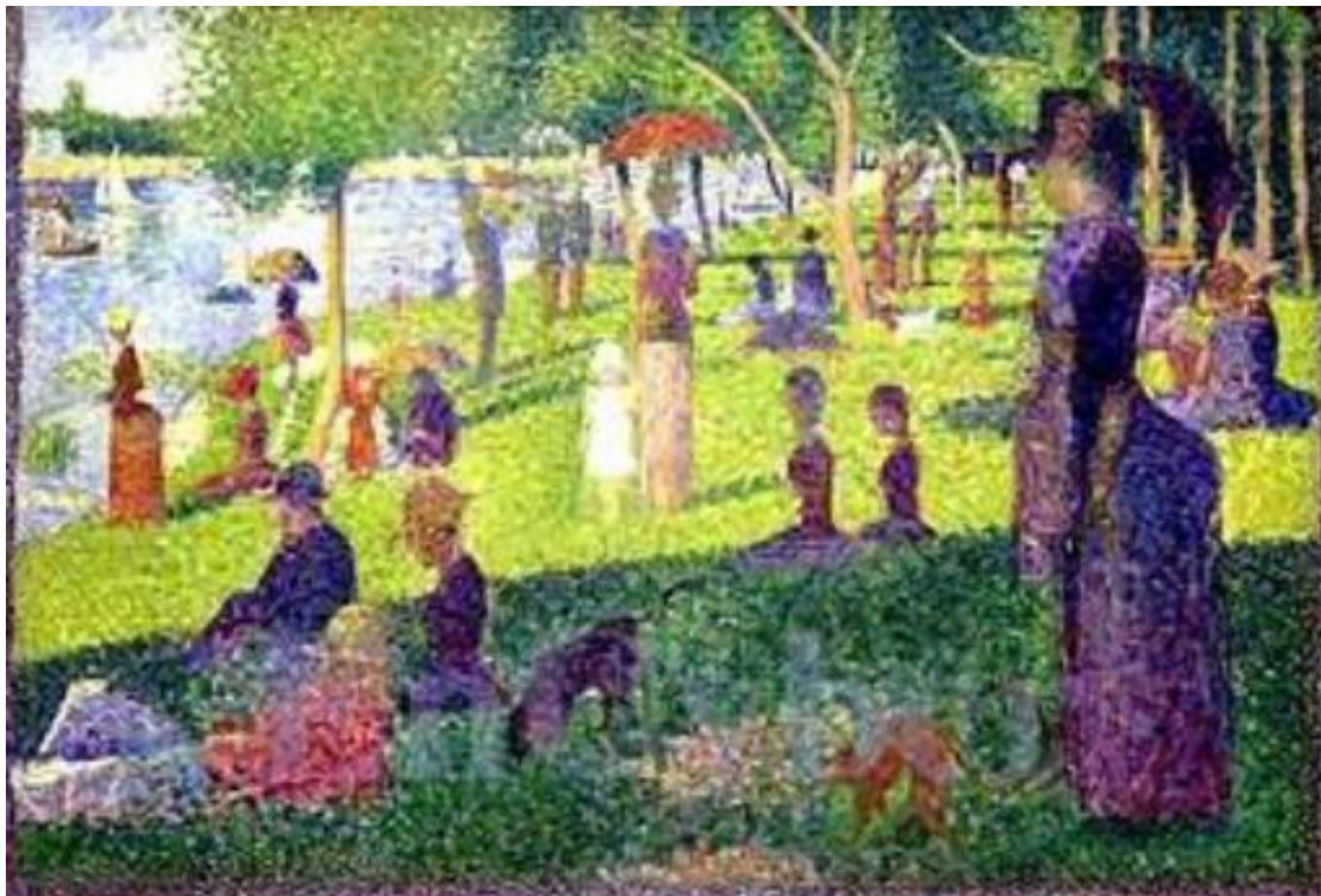


Results

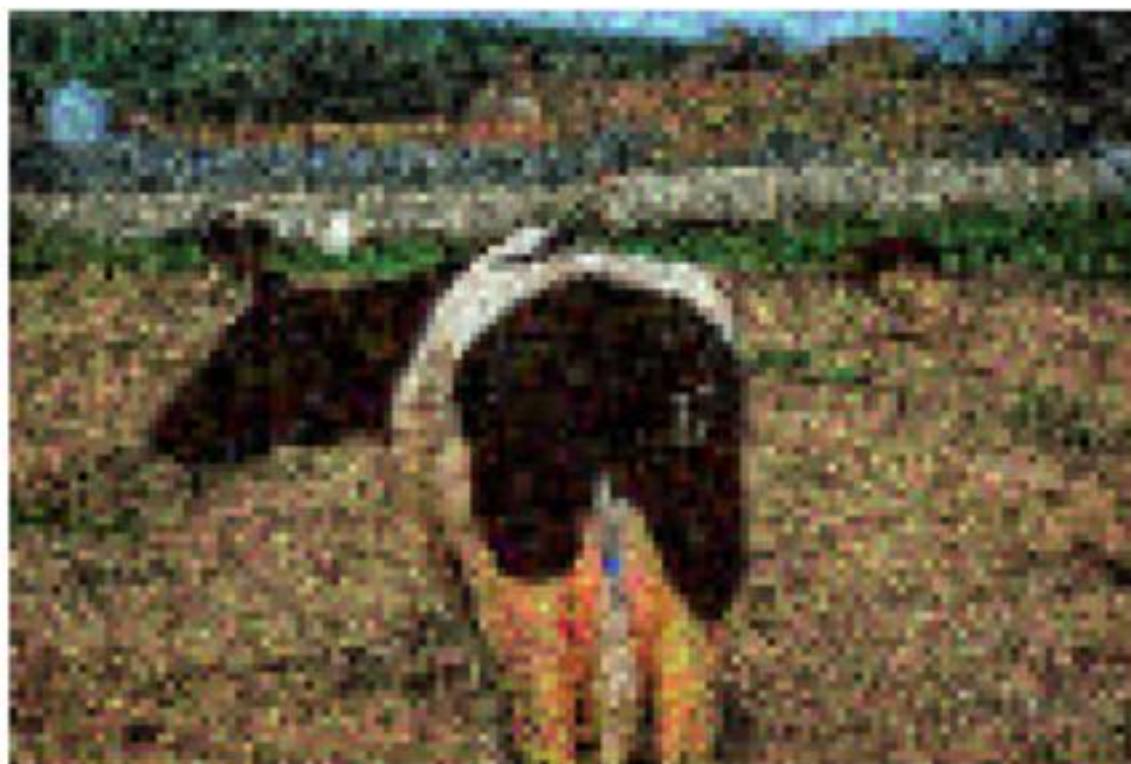


Results

Pointillism



Results

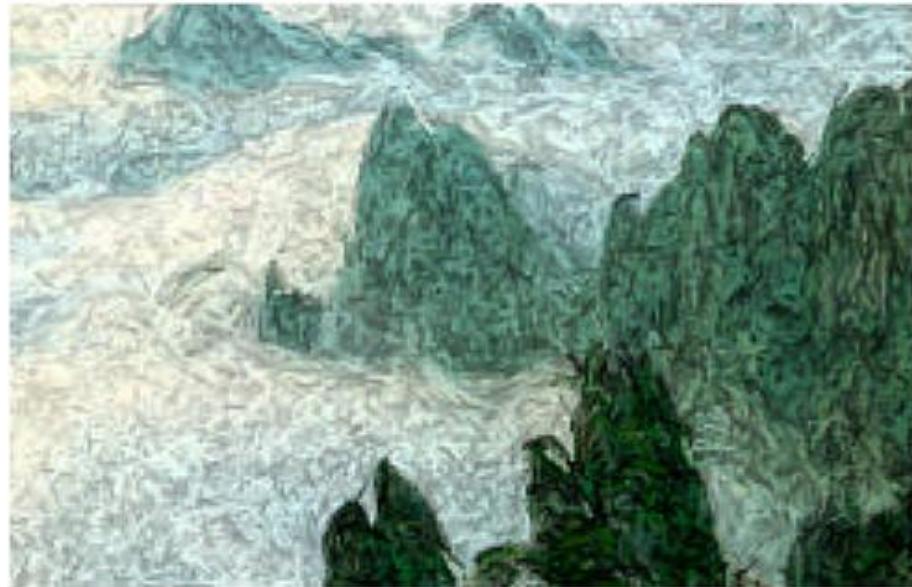


Results

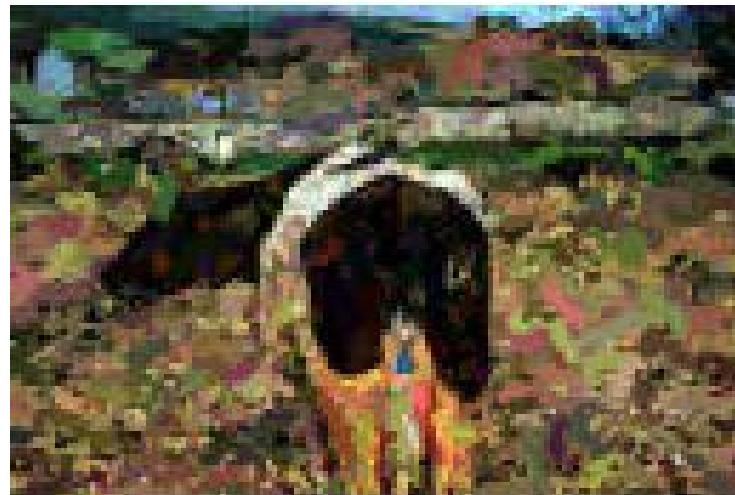
Expressionism



Results



(a)



基于笔划的布置

- 笔划布置的贪婪算法
- 基于Voronoi图的全局优化

基于Voronoi图的全局优化

• LLOYD'S METHOD

function LLOYDSMETHOD(n, I):

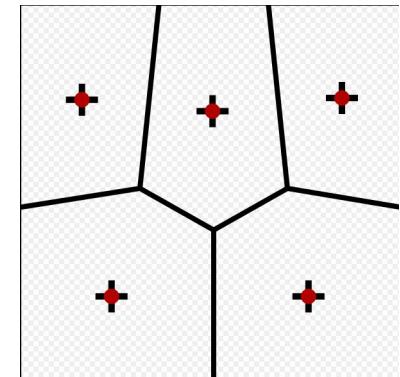
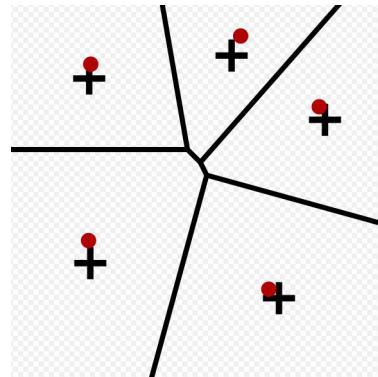
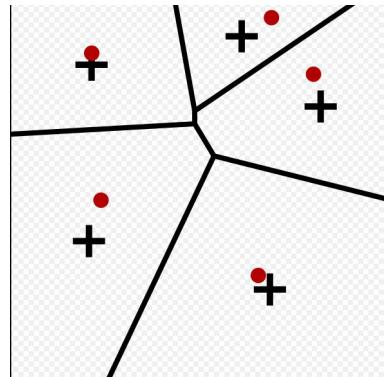
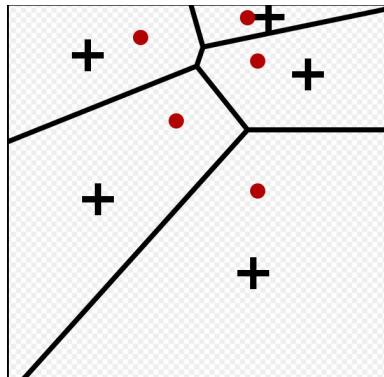
 initialize the centroids \mathbf{C}_i by randomly sampling n points uniformly in the image I

while the algorithm has not converged

 reestimate the labeling by $L_p^i \leftarrow \begin{cases} 1 & i = \arg \min_i \|\mathbf{p} - \mathbf{C}_i\|^2 \\ 0 & \text{otherwise} \end{cases}$

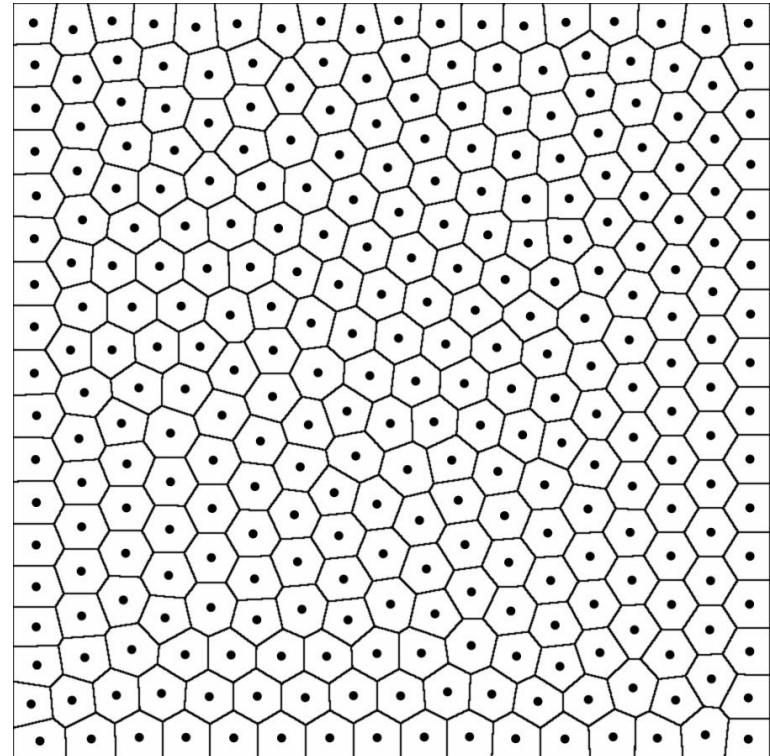
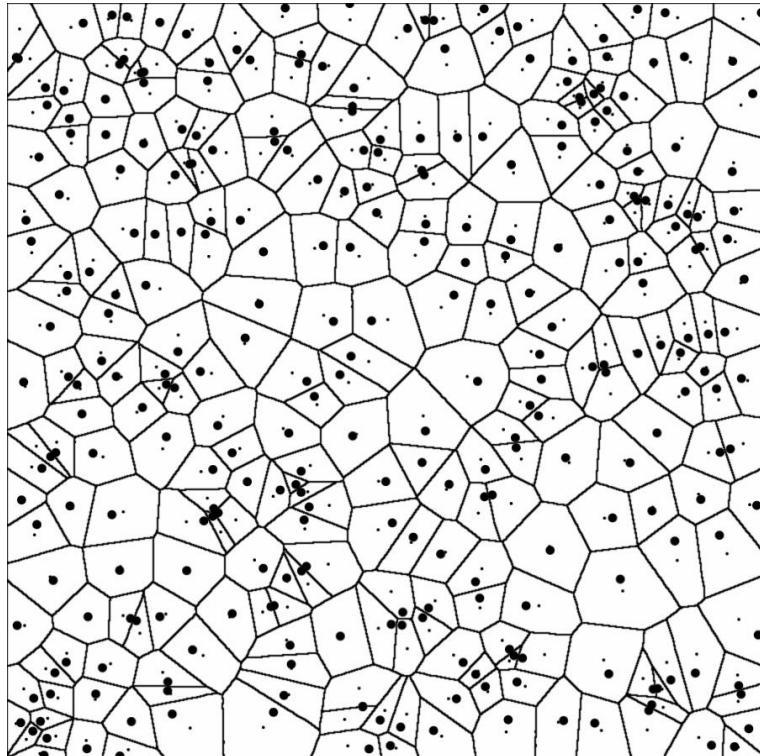
 reestimate the centroids by $\mathbf{C}_i \leftarrow \frac{\sum_p L_p^i \mathbf{p}}{\sum_p L_p^i}$

return the centroids \mathbf{C}_i



基于Voronoi图的全局优化

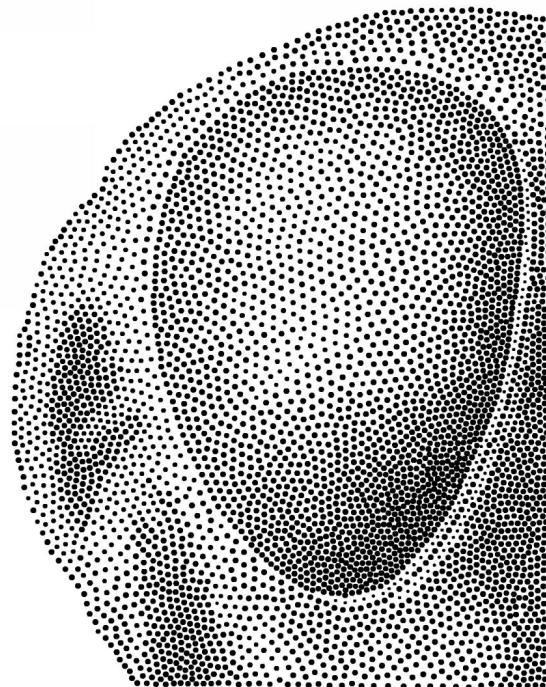
- LLOYD' S METHOD
 - 产生均匀但是随机的结果



基于Voronoi图的全局优化

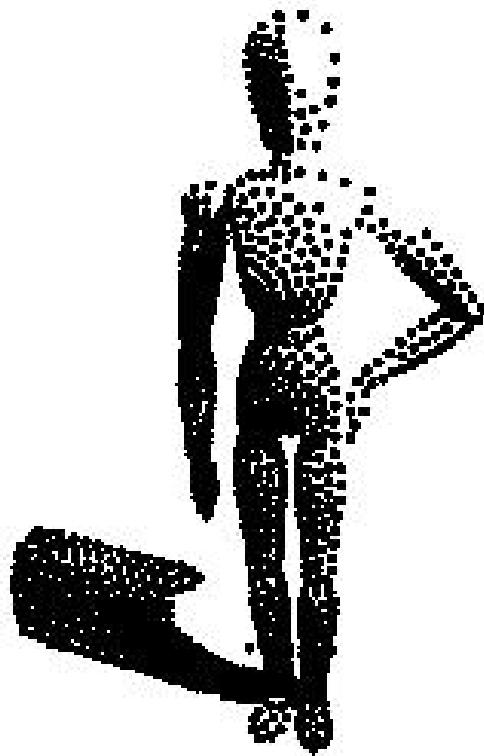
- 实现步骤

- 以色调为概率，随机布点
- 应用LLOYD'S METHOD优化调整位置



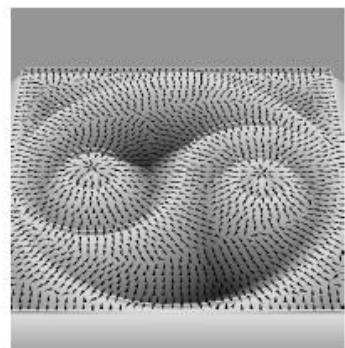
基于Voronoi图的全局优化

- LLOYD'S METHOD的变种（改变点的大小）

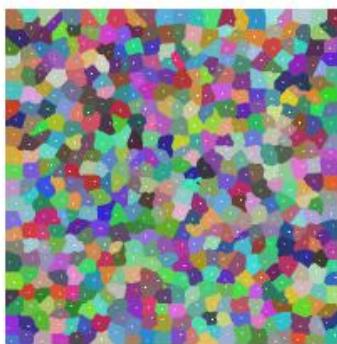


基于Voronoi图的全局优化

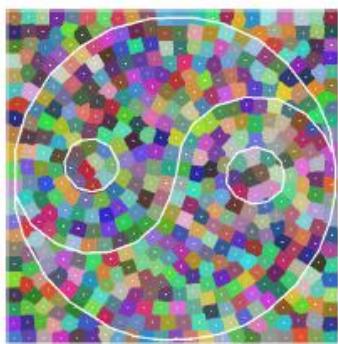
- 应用LLOYD'S METHOD实现方砖铺设 (Tile mosaic)



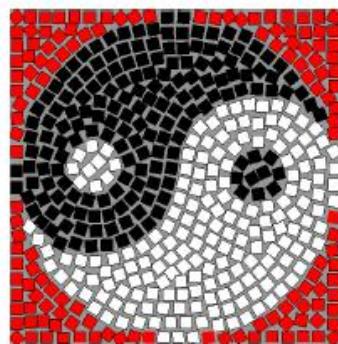
(a)



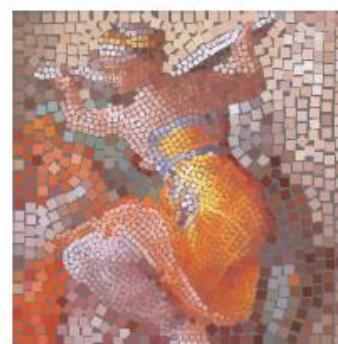
(b)



(c)



(d)



(e)

谢谢！