1: Define $\sim$ on $\mathbb{R}\times\mathbb{R}$ by $(x,y)\sim(z,w)$ iff $x^2-z^2=w^2-y^2$

$x^2-z^2=w^2-y^2 = x^2+y^2 = z^2+w^2$

a: Reflexive: $\forall a\in\mathbb{R}$, $(a,a)\in\sim$ $\quad x^2+y^2=x^2+y^2$

Proof. Let $(x,y)\in\mathbb{R}\times\mathbb{R}$. By the definition of $\sim$, we can have $x^2+y^2=x^2+y^2$ since this is a true statement. This statement indicates that $(x,y)\sim(x,y)$, thus $\sim$ is reflexive. $\blacksquare$

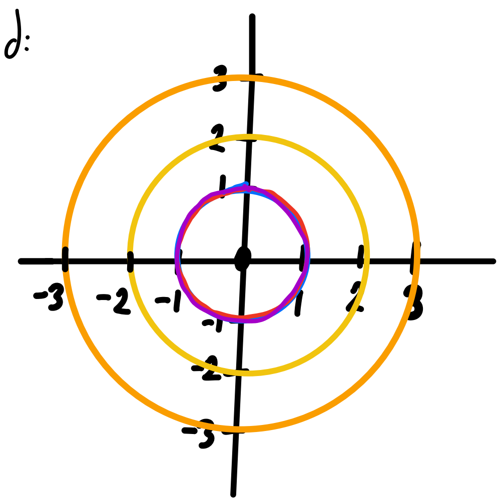
b: Symmetric $(a,b)\in\sim \rightarrow (b,a)\in\sim$

Proof. Let $(x,y)\in\mathbb{R}\times\mathbb{R}$, let $(c,d)\in\mathbb{R}\times\mathbb{R}$, and let $(x,y)\sim(c,d)$. By the definition of $\sim$, we have $x^2+y^2=c^2+d^2$. This statement will still be equivalent when the equation is flipped to $c^2+d^2=x^2+y^2$. This statement indicates that if $(x,y)\sim(c,d)$, then $(c,d)\sim(x,y)$, which makes $\sim$ symmetric. $\blacksquare$

c: Transitive $((a,b)\in\sim$ and $(b,c)\in\sim) \rightarrow (a,c)\in\sim$

Proof. Let $(x,y)\in\mathbb{R}\times\mathbb{R}$, $(c,d)\in\mathbb{R}\times\mathbb{R}$ and $(e,f)\in\mathbb{R}\times\mathbb{R}$ where $(x,y)\sim(c,d)$ and $(c,d)\sim(e,f)$. By the definition of $\sim$, this would give the equations $x^2+y^2=c^2+d^2$ and $c^2+d^2=e^2+f^2$. Because of these statements, we would be able to state that $x^2+y^2=e^2+f^2$, meaning that $(x,y)\sim(e,f)$. Thus if $(x,y)\sim(c,d)$ and $(c,d)\sim(e,f)$, then $(x,y)\sim(e,f)$ and $\sim$ is transitive $\blacksquare$.

d:



Find all pts such that $(x,y)\sim(z,w)$

i: $(x,y)=(0,0)$ $\quad 0^2+0^2=z^2+w^2$

$\boxed{(0,0)\rightarrow(z,w)}$ $\quad z^2+w^2=0$

ii: $(x,y)=(0,1)$ $\quad 0^2+1^2=z^2+w^2$

$z^2+w^2=1$

circle at origin with radius 1

iii: $(x,y)=(1,0)$ $\quad 1^2+0^2=z^2+w^2$

$z^2+w^2=1$

circle at origin with radius 1

iv: $(x,y)=\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ $\quad \left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2=z^2+w^2$

$\frac{1}{2}+\frac{1}{2}=z^2+w^2$

$1=z^2+w^2$

$z^2+w^2=1$

circle at origin with radius 1

v: $(x,y)=(2,0)$ $\quad 2^2+0^2=z^2+w^2$

$4=z^2+w^2$

$z^2+w^2=4$

circle at origin with radius 2

VI: $(x,y)=(3,0)$ $\quad 3^2+0^2=z^2+w^2$

$9=z^2+w^2$

$z^2+w^2=9$

circle at origin with radius 3

2: Define $\sim$ on $\mathcal{P}(\mathbb{N})$ defined by $A \sim B$ iff every element in A is an element of B.

a: Proof. Let $A \in \mathcal{P}(\mathbb{N})$. By the definition of $\sim$ along with the definition of subset, Every element in A is also an element in A. Thus $A \sim A$ and $\sim$ is reflexive $\square$.

b: Counter ex:

$A = \{1,2,3\}$

$B = \{1,2,3,4\}$

Although $A \sim B$ since every element of A is in B, the same cannot be said for $B \sim A$ since not every element in B is in A. Thus, $\sim$ is not symmetric.

c: $A = \{1,2,3\}$

$B = \{1,2,3,4\}$

$C = \{1,2,3,4,5\}$

Proof. Let $A, B, C \in \mathcal{P}(\mathbb{N})$ and let $A \sim B$ and $B \sim C$. By the definition of $\sim$, every element in A is an element of B and every element in B is an element of C. By the definition of subset, every element in A is an element of C. Thus if $A \sim B$ and $B \sim C$, then $A \sim C$ and $\sim$ is transitive $\square$.

d: $A = \{1,2,3\}$ $\quad A \sim B \quad B \sim A$

$B = \{1,2,3\}$

Proof. Let $A, B \in \mathcal{P}(\mathbb{N})$ and $A \sim B$ and $B \sim A$. By the definition of $\sim$, every element of A is an element of B and every element of B is an element of A. By the definition of set equivalence, $A = B$. Thus if $A \sim B$ and $B \sim A$, $A = B$ and $\sim$ is antisymmetric $\square$.