Math 215-800 **PS #4**

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Writing Exercises. Type up your solutions to #1 using LaTeX. Start by going to the course webpage and downloading the .tex file for this assignment; pop that .tex file into your favorite latex editor (probably Overleaf.com) and type your responses just after the corresponding problem in the .tex file.

1. Let X and Y be (unspecified) sets. Write a careful explanation for why $X \cap Y \subseteq X \cup Y$, just like proof explanations we prepared together in class.

   Some of your sentences may explain what needs to be done. Other sentences will make conclusions, and each conclusion needs a justification. Try to format each "conclusion-justification" sentence you use in one of the following ways. These types of formats will be most helpful in writing formal proofs.

   - "We conclude <CONCLUSION>, because <DEFINITION>."
   - "This <DEFINITION> means <CONCLUSION>."
   - "By <DEFINITION>, we know <CONCLUSION>."

Proof. To show that $X \cap Y \subseteq X \cup Y$, we need to show that every element of $X \cap Y$ is also an element of $X \cup Y$. Suppose $a \in X \cap Y$. By the definition of intersection, $a \in X$ and $a \in Y$. This demonstrates that $a \in X \cup Y$ because of the definition of union. Since we now know that $a \in X \cup Y$, this demonstrates that $X \cap Y \subseteq X \cup Y$ by the definition of subset.

$\blacksquare$

Additional Exercises. Complete the next problems, #2-#3. You need not typeset your answers, unless you want to. Staple your answers to your write-up for the Writing Exercise and turn in one homework with your name on the front, at the top.

2. Let A, B, and C be sets. Write an element-chasing proof for the fact that if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

3. The notation for the following Cartesian products involves the familiar interval notation from Calculus classes – no ordered pair is given. Sketch a graph of each Cartesian product in the real plane, $\mathbb{R}^2$. You may wish to use graph paper.

   (a) $[-2, 3] \times [1, 3]$
   (b) $(-2, 3) \times [-4, -1]$
   (c) $[-2, 3) \times \{4\}$
   (d) $(0, 2) \times \mathbb{R}$
   (e) $\{4\} \times \mathbb{R}$
   (f) $\mathbb{R} \times \{2\}$
   (g) $(0, \infty) \times \{2\}$
   (h) $\{2\} \times (-\infty, 0)$

There are a few additional problems on the reverse that are for practice **only.**

Let $A = \{\gamma\}$, $B = \{\square\}$, and $C = \{3\}$.

1. Write out the elements of $A \times B$ using correct set notation.
2. Write out the elements of $B \times A$ using correct set notation.
3. Directly address why these $A \times B$ and $B \times A$ fail to satisfy the *definition* of set equality.
4. Determine $(A \times B) \times C$, using correct set notation to write out the elements.
5. Determine $A \times (B \times C)$ by listing the set's elements using correct set notation.
6. Are $(A \times B) \times C$ and $A \times (B \times C)$ equal? Why or why not? Your explanation needs to address why these sets satisfy or fail to satisfy the *definition* of set equality.

Let $D = \{(x, y) \mid \sqrt{x^2 + y^2} \leq 2\}$ and $H = \{(x, y) \mid y \geq 0\}$. Sketch $D \cup H$, $D \cap H$, $\mathbb{R}^2 - D$, $\mathbb{R}^2 - H$, $D - H$, $\mathbb{R}^2 - (D \cap H)$, $(\mathbb{R}^2 - D) \cup (\mathbb{R}^2 - H)$, $\mathbb{R}^2 - (D \cup H)$, and $(\mathbb{R}^2 - D) \cap (\mathbb{R}^2 - H)$. Are any of these sets equal?

Are the following statements true or false? Justify your answers with proofs (for true) or counterexamples (for false).

1. For all A and B sets, $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.
2. For all A and B sets, $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.
3. For all sets A, the products $A \times \emptyset$ and $\emptyset \times A$ are equal.
4. For all sets A and B, we have $\mathcal{P}(A) \times \mathcal{P}(B) = \mathcal{P}(A \times B)$.
5. Let X be a set such that $A \subseteq X$ and $B \subseteq X$. Then $X - (A \cup B) = (X - A) \cap (X - B)$.