Math 215-800 **PS #3**
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Writing Exercises. Type up your solutions to #1 using LaTeX. Start by going to the course webpage and downloading the .tex file for this assignment; pop that .tex file into your favorite latex editor (probably Overleaf.com) and type your responses just after the corresponding problem in the .tex file.

1. Let X and Y be (unspecified) sets. Write a careful explanation for why $X \cap Y \subseteq X \cup Y$, just like proof explanations we prepared together in class.

   Some of your sentences may explain what needs to be done. Other sentences will make conclusions, and each conclusion needs a justification. Try to format each "conclusion-justification" sentence you use in one of the following ways. These types of formats will be most helpful in writing formal proofs.

   - "We conclude <CONCLUSION>, because <DEFINITION>."
   - "This <DEFINITION> means <CONCLUSION>."
   - "By <DEFINITION>, we know <CONCLUSION>."

Proof. We are able to conclude that $X \cap Y \subseteq X \cup Y$, because all elements in $X \cap Y$, also known as the set of elements found in both X and Y, are in $X \cup Y$, also known as the set of elements in X or Y. By the definition of a subset in this case where every element of $X \cap Y$ must also be an element of $X \cup Y$, we know that $X \cap Y \subseteq X \cup Y$ is a valid statement. ∎

Additional Exercises. Complete the next problems, #2-#3. You need not typeset your answers, unless you want to. Staple your answers to your write-up for the Writing Exercise and turn in one homework with your name on the front, at the top.

2. Let $A = \{\alpha, \beta, \gamma\}$, $B = \{\alpha, a, \star, \oplus\}$, and $C = \{\gamma, \beta, \text{read}, \oplus\}$. Use proper set notation to write out each of the following sets. (The first way to describe a set, as discussed in class.)

   (a) $A \cap B$ (b) $A \cap C$ (c) $A \cup B$ (d) $A \cup C$ (e) $\emptyset \cup A$ (f) $\emptyset \cap C$
   (g) $C - A$ (h) $B \cap C$ (i) $A \cup (B \cap C)$ (j) $B \cap (A \cup C)$

3. (a) Describe *two* infinite sets A and B so that $A \cup B = \mathbb{N}$ and $A \cap B = \emptyset$. For each set equality, write a sentence explaining why your choice of A and B work but limit yourself to one sentence. (i.e., *Two* sentences in addition to the definition of A and B.)

   (b) Describe *three* infinite sets D, E, and F so that $D \cup E \cup F = \mathbb{N}$ and $\{D, E, F\}$ is disjoint. Same deal with a justification, though you may need a few more sentences.

   (c) Describe an infinite set $\mathcal{S}$ of infinite sets so that $\bigcup_{A \in \mathcal{S}} A = \mathbb{N}$ and $\mathcal{S}$ is disjoint. Justify in a few sentences.

   The goal here is to create a pair, A and B, that can be "built up" to the triplet, C, D, and E, and then "built up" again to the collection $\mathcal{S}$. Try to settle on a theme for how to construct these sets that will let you generalize to more, more, and more sets in the collection as you go.