Outline of Surjective Proof

$f: X \to Y$ is surjective iff $\forall y \in Y\ \exists x \in X,\ f(x) = y$

Proof. $\boxed{\text{(Conditions Stated)}}$. Let $b \in \boxed{\text{cod } f}$.

$\boxed{\rightsquigarrow \text{ Go to scrap paper and solve } f(a) = b \text{ for a formula for } a \text{ in terms of } b}$

Then, let $a = \boxed{\text{(formula with } b\text{'s!)}}$, Note that

$\boxed{f(a) = \ldots \text{ Do ALGEBRA} \ldots = b}$

Because $\boxed{\_\_\_\_}$, $a \in \boxed{\text{dom} f}$

Then $\boxed{f}$ is surjective $\blacksquare$.

Scrap work

1: $g((x,y)) = xy$

$\hookrightarrow g(a_1, a_2) = b$

$a_1 a_2 = b$

$a_1 = b$

$\frac{b}{a_2} = b$

$b = b$

$\boxed{\text{Let } a_1 = \frac{b}{a_2} \text{ and } a_2 = 1}$

Scrap work

2: $k(x,y) = (2x+y,\ x+2y)$

$\hookrightarrow k(a,b) = k(c,d)$

$(2a+b,\ a+2b) = (2c+d,\ c+2d)$

$2a+b = 2c+d \qquad a+\cancel{2b} = c+2d$
$\qquad\qquad\qquad\quad\ \cancel{-2b}\quad -2b$

$a = -2b + c + 2d$

$2(-2b+c+2d) + b = 2c+d$

$-4b+2c+4d+b = 2c+d$

$-3b+\cancel{2c}+\cancel{4d} = \cancel{2c}+d$
$\quad\ \cancel{-2c}\ \cancel{-4d}\ \ \cancel{-2c}-4d$

$\frac{\cancel{-3}b}{\cancel{-3}} = \frac{\cancel{-3}d}{\cancel{-3}}$

$\boxed{b=d} \longrightarrow a = \cancel{-2d} + c + \cancel{2d}$

$\boxed{a=c}$

Proof. Suppose $k(a,b) = k(c,d)$. By definition of k, we have $(2a+b, a+2b) = (2c+d, c+2d)$. By using the definition of equality of coordinate pairs, we have $2a+b = 2c+d$ and $a+2b = c+2d$. First, by subtracting the second equation by both sides by $2b$, we get $a = -2b+c+2d$. Then, after plugging this expression for a in the first equation, we get $2(-2b+c+2d)+b = 2c+d$. After distributing the 2, it then becomes $-4b+2c+4d+b = 2c+d$. Next, combining like terms will produce $-3b+2c+4d = 2c+d$. Subtracting $2c$ and $4d$ on both sides creates $-3b = -3d$. Finally, dividing by -3 will indicate $b=d$. With this information, we can plug in d for b in the second equation to obtain $a = -2d+c+2d$. After combining like terms, we see $a=c$. Since we showed that $a=c$ and $b=d$, $(a,b) = (c,d)$ by the definition of equality of coordinate pairs. Thus k is injective. $\blacksquare$

3: a: $M = \mathbb{R} \times \{6\}$

$(0,6)$, $(0.1,6)$, $(\pi,6)$

$g(M) = \{g(n,6) \mid n \in \mathbb{R}\} = \{6n \mid n \in \mathbb{R}\}$, because $g(n,6) = 6n$ by the definition of g. Also, since n can be any value in $\mathbb{R}$, the product of n and 6 will be in $\mathbb{R}$ for all values of n. Thus $g(M) = \mathbb{R}$ where M is any coordinate pair $(n,6)$ where $n \in \mathbb{R}$.

$= \mathbb{R}$

b: $M = \{-1\} \times \mathbb{R}$

$(-1,0)$, $(-1,0.1)$, $(-1,\pi)$

$g(M) = \{g(-1,n) \mid n \in \mathbb{R}\} = \{-n \mid n \in \mathbb{R}\}$, because $g(-1,n) = -n$ by the definition of g. Also, since n can be any value in $\mathbb{R}$, the product of -1 and n will be in $\mathbb{R}$ for all values of n. Thus $g(M) = \mathbb{R}$ where M is any coordinate pair $(-1,n)$ where $n \in \mathbb{R}$.

$= \mathbb{R}$

c: $M = \mathbb{Z} \times \{7\}$

$(0,7)$, $(1,7)$, $(-1,7)$

$g(M) = \{g(n,7) \mid n \in \mathbb{Z}\} = \{7n \mid n \in \mathbb{Z}\}$, because $g(n,7) = 7n$ by the definition of g.

d: $M = \mathbb{Z} \times \mathbb{Z}$

$(0,0)$, $(0,1)$, $(1,1)$, $(8,-4)$

$g(M) = \{g(n,m) \mid n \in \mathbb{Z}, m \in \mathbb{Z}\} = \{nm \mid n \in \mathbb{Z}, m \in \mathbb{Z}\}$, because $g(n,m) = nm$ by the definition of g. Also, since n and m can both be any value in $\mathbb{Z}$, the product of n and m will be in $\mathbb{Z}$ for all values of n and m. Thus $g(M) = \mathbb{Z}$ where M is any coordinate pair (n,m) where $n \in \mathbb{Z}$ and $m \in \mathbb{Z}$.

$= \mathbb{Z}$

e: $M = \mathbb{N} \times \mathbb{N}$

$g(M) = \{g(n,m) \mid n \in \mathbb{N}, m \in \mathbb{N}\} = \{nm \mid n \in \mathbb{N}, m \in \mathbb{N}\}$, because $g(n,m) = nm$ by the definition of g. Also, since n and m can both be any value in $\mathbb{N}$, the product of n and m will be in $\mathbb{N}$ for all values of n and m. Thus $g(M) = \mathbb{N}$ where M is any coordinate pair (n,m) where $n \in \mathbb{N}$ and $m \in \mathbb{N}$.

$= \mathbb{N}$