Math 215-800 **PS #7**

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Writing Exercises. Type up your solutions to #1 using LaTeX. Start by going to the course webpage and downloading the .tex file for this assignment; pop that .tex file into your favorite latex editor (probably Overleaf.com) and type your responses just after the corresponding problem in the .tex file.

1. Let $g : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be defined by $g((x, y)) = xy$ for all $(x, y) \in \mathbb{R} \times \mathbb{R}$. Prove that g is surjective.

   Proof. Suppose $b \in \mathbb{R}$. Then, let $a_2 = 1$ and $a_1 = \frac{b}{a_2}$.
   By definition of g:

   $$g((a_1, a_2)) = a_1a_2$$

   By plugging in for a_1:

   $$= \frac{b}{a_2}a_2$$

   By plugging in for a_2:

   $$= a_1$$

   $$= b.$$

   Since $b \in \mathbb{R}$ and $b = a_1a_2$, $a_1a_2 \in \mathbb{R}$, and since a_2 is a real number and a_1 is calculated using real numbers b and a_2 without dividing by zero, $a_1 \in \mathbb{R}$ and $a_2 \in \mathbb{R}$, making $(a_1, a_2) \in \mathbb{R}^2$ or $\mathbb{R} \times \mathbb{R}$. Therefore, g is surjective.

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Additional Exercises. Complete the next problems, #2-#3. You need not typeset your answers, unless you want to. Staple your answers to your write-up for the Writing Exercise and turn in one homework with your name on the front, at the top.

2. Let $k : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $k(x, y) = (2x + y, x + 2y)$. Prove that k is injective.

3. Let $g : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be defined by $g((x, y)) = xy$ for all $(x, y) \in \mathbb{R} \times \mathbb{R}$ (this is the same function as in #1). You can apply a function to a subset of its domain; for example, if $f : A \to B$ and $M \subseteq A$ then:

   $$f(M) = \{b \in B \mid \exists m \in M, f(m) = b\}.$$

   Using this general definition of how to apply a function to a set, please determine $g(M)$ for each of the following choices for M. When possible use set builder notation to summarize $g(M)$ succinctly and then justify your response in 1 - 2 sentences.

   (a) $M = \mathbb{R} \times \{6\}$
   (b) $M = \{-1\} \times \mathbb{R}$

(c) $M = \mathbb{Z} \times \{7\}$

(d) $M = \mathbb{Z} \times \mathbb{Z}$

(e) $M = \mathbb{N} \times \mathbb{N}$

Examples:

- $g(M) = \{8, 12\}$ when $M = \{2, 3\} \times \{4\}$. This is because another way to write M is $M = \{(2, 4), (3, 4)\}$, and so $g(M) = \{g(2, 4), g(3, 4)\} = \{8, 12\}$.
- If $M = \{5\} \times \mathbb{N}$ then $g(M) = \{g(5, n) | n \in \mathbb{N}\} = \{5n \mid n \in \mathbb{N}\}$, because $g(5, n) = 5n$ by the definition of g.