Math 215-800 **PS #12**
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Writing Exercises. Type up your solutions to #1 using LaTeX. Start by going to the course webpage and downloading the .tex file for this assignment; pop that .tex file into your favorite latex editor (probably Overleaf.com) and type your responses just after the corresponding problem in the .tex file.

1. Prove using induction that $\sum_{i=1}^{n}(i!)i = (n+1)! - 1$ for all $n \geq 1$.

Proof. Let $s(n)$ be the statement:

$$\sum_{i=1}^{n}(i!)i = (n+1)! - 1$$

for all $n \geq 1$.

Base Case: When $n = 1:$

$$LHS : (1!)1 = 1,$$

$$RHS : (1+1)! - 1 = 1,$$

$$LHS = RHS.$$

Thus $s(1)$ is true. Now, suppose $s(k)$ is true for some k. To show that $s(k+1)$ is true, let's look at the LHS:

$$LHS = \sum_{i=1}^{k+1}(i!)i = (1!)1 + (2!)2 + ... + (k!)(k) + ((k+1)!)(k+1),$$

$$LHS = (k+1)! - 1 + ((k+1)!)(k+1),$$

by the hypothesis that $s(k)$ is true.

$$LHS = (k+1+1)(k+1)! - 1,$$

$$LHS = (k+2)(k+1)! - 1,$$

$$LHS = (k+2)! - 1 = RHS.$$

Then, putting these together, we have:

$$LHS = RHS.$$

Thus if $s(k)$ is true then $s(k+1)$ is true. By PMI, $s(n)$ is true for all $n \geq 1$.

$\blacksquare$

Additional Exercises. Complete the rest of these problems. You need not typeset your answers, unless you want to. Staple your answers to your write-up for the Writing Exercise and turn in one homework with your name on the front, at the top. **You definitely should read the next few paragraphs so that you can do #2-4.**

2. Read *Appendix A. Guidelines for Writing Mathematical Proofs* from "Mathematical Reasoning: Writing and Proof" (Version 2.1) by Ted Sundstrom. A handout has been provided and an electronic version is on the webpage. Give an example when you have not followed one of these guidelines in your problem sets for this class, and explain how that could have introduced confusion about your meaning. Include a quote of your original work and clearly state what problem set and question number you are discussing.

3. Let $U = [0,1] \times [0,1]$. We define $R \subseteq U \times U$ as follows: $((a,b),(c,d)) \in R$ if and only if at least one of the following is true:

   - $(a,b) = (c,d)$, or
   - $a = c$ and $\{b,d\} = \{0,1\}$.

   Draw a graph of U. Be careful to indicate whether or not boundary line segments and corner points are included or excluded. For each point (a,b) of U provided, use the indicated color to mark all points (c,d) of U that $((a,b),(c,d)) \in R$.

   (a) $\left(\frac{1}{2}, \frac{1}{3}\right)$; red (b) $(0,0)$; blue (c) $\left(\frac{1}{3}, 0\right)$; yellow (d) $(1,1)$; green

4. Let $U = [0,1] \times [0,1]$. We define $S \subseteq U \times U$ as follows: $((a,b),(c,d)) \in S$ if and only if at least one of the following is true:

   - $(a,b) = (c,d)$, or
   - $a = c$ and $\{b,d\} = \{0,1\}$, or
   - $b = d$ and $\{a,c\} = \{0,1\}$, or
   - $\{a,c\} = \{0,1\}$ and $\{b,d\} = \{0,1\}$.

   Draw a (new) graph of U. Be careful to indicate whether or not boundary line segments and corner points are included or excluded. For each point (a,b) of U provided, use the indicated color to mark all points (c,d) of U that $((a,b),(c,d)) \in S$.

   (a) $\left(\frac{1}{2}, \frac{1}{3}\right)$; red (b) $(0,0)$; blue (c) $\left(\frac{1}{3}, 0\right)$; yellow (d) $(1,1)$; green