$h(x_1) = h(x_2)$

$$mx_1 + \cancel{b} = mx_2 + \cancel{b}$$
$$\quad -b \qquad\quad -b$$

$$\frac{\cancel{m}x_1}{\cancel{m}} = \frac{\cancel{m}x_2}{\cancel{m}}$$

$$\boxed{x_1 = x_2}$$

2: $\mathcal{P}(A \cup B) \neq \mathcal{P}(A) \cup \mathcal{P}(B)$

Let $A = \{1,2,3\}$ $\quad \mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$

$B = \{2,3,5\}$ $\quad \mathcal{P}(B) = \{\emptyset, \{2\}, \{3\}, \{5\}, \{2,3\}, \{3,5\}, \{2,5\}, \{2,3,5\}\}$

$A \cup B = \{1,2,3,5\}$

$\mathcal{P}(A \cup B) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{5\}, \{1,2\}, \{1,3\}, \{1,5\}, \{2,3\}, \{2,5\}, \{3,5\}, \{1,2,3\}, \{2,3,5\}, \{1,2,5\}, \{1,3,5\}, \{1,2,3,5\}\}$

$\mathcal{P}(A) \cup \mathcal{P}(B) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}, \{5\}, \{3,5\}, \{2,5\}, \{2,3,5\}\}$

$\mathcal{P}(A \cup B) \neq \mathcal{P}(A) \cup \mathcal{P}(B)$ since in this case where $A = \{1,2,3\}$, and $B = \{2,3,5\}$, $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$, but $\mathcal{P}(A \cup B) \not\subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$ because not all of the elements in $\mathcal{P}(A \cup B)$ are in $\mathcal{P}(A) \cup \mathcal{P}(B)$.

3: Proof. Suppose $f(a,b) = f(c,d)$. By definition of f, we have $(2a+b, a-b) = (2c+d, c-d)$. By using the definition of equality of coordinate pairs, we have $2a+b = 2c+d$ and $a-b = c-d$. First, by using the second equation to solve for a, we get $a = b+c-d$. Then, substituting that expression for a in the first equation gives the equation $2(b+c-d)+b = 2c+d$, which then by distributing the 2 gives $2b+2c-2d+b = 2c+d$, and then after combining like terms gives $3b+2c-2d = 2c+d$. Subtracting $2c$ on both sides will cancel out $2c$ on both sides and adding $2d$ on both sides gives the new equation $3b = 3d$. Finally, dividing by 3 will indicate $b = d$. With this information, we can plug in d for b in the second equation to get $a = d+c-d$ and after combining like terms, we also get $a = c$. Since we showed that $a = c$ and $b = d$, $(a,b) = (c,d)$ by the definition of equality of coordinate pairs. Thus, f is injective. $\blacksquare$

Scrap work:

$2a+b = 2c+d$ and $a-b = c-d$

$\quad a = b+c-d$

$2(b+c-d)+b = 2c+d$

$\quad a = d+c-d$

$2b+2c-2d+b = 2c+d$

$\quad \boxed{a=c}$

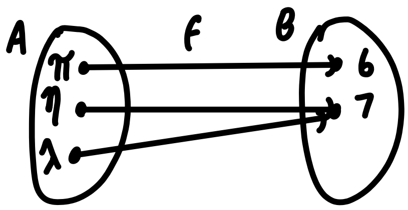
$$3b + \cancel{2c} - \cancel{2d} = \cancel{2c} + d$$
$$\quad -\cancel{2c} + \cancel{2d} \quad -\cancel{2c} + 2d$$

$$\frac{\cancel{3}b}{\cancel{3}} = \frac{\cancel{3}d}{\cancel{3}}$$

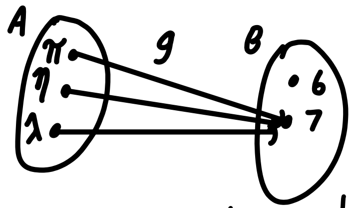
$$\boxed{b=d}$$

4:

a: $f: A \to B$ defined by:

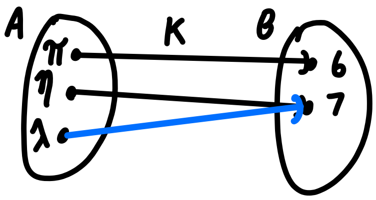


b: $g: A \to B$ defined by:



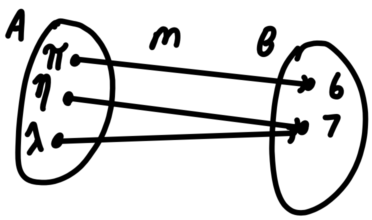
c: It is not possible to define a function $h: B \to A$ so that h is surjective because there are more elements that are in A, the codomain, than in B, the domain. Therefore, it's impossible to assign every element in the codomain to an element in the domain while assigning each element in the domain to at most one element in the codomain, in which breaking this rule would not allow h to be a function.

d: $K: A \to B$ defined by



It is not possible to define a function $K: A \to B$ so that K is injective because there are more elements in A, the domain, than in B, the codomain. Since all elements in the domain must be assigned to an element in the codomain, it is certain in this case that a collision will occur since there will be more than one element input from A that will result in the same element output from B, thus making K unable to be injective.

e: $m: A \to B$ defined by



f: $n: B \to A$ defined by