Outline of injectivity Proof:

Proof. (conditions stated). Let $a, b \in \text{dom } f$ and suppose $f(a) = f(b)$ (Evaluate $f(a)$ and $f(b)$ using the dfn of f!)

Algebra, etc.

Then, $a = b$, thus f is injective ☑

Outline of Surjective Proof

$f: X \to Y$ is surjective iff $\forall y \in Y\ \exists x \in X,\ f(x) = y$

Proof. (Conditions Stated). Let $b \in \text{cod } f$.

Go to scrap paper and solve $f(a) = b$ for a formula for a in terms of b

Then, let $a =$ (formula with b's!), Note that $f(a) = \dots$ Do ALGEBRA ... $= b$

Because ____, $a \in \text{dom} f$

Then f is surjective ☑.

1: $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$

$f(x) = \tan(x)$

$f(a) = \tan(a)$

$f(b) = \tan(b)$

$f(a) = b$

$\tan(a) = b$

Let $b \in \mathbb{R}$

Let $a = \frac{\pi}{4}$

$f(\frac{\pi}{4}) = \tan(\frac{\pi}{4})$

$b = \tan(\frac{\pi}{4}) = 1$

2: Let $A = \{\pi, \beta, -8.76, ♡, □, a, q\}$

a: $Y = \{\beta, -8.76, □\}$

$\boxed{f_Y = 0110100}$

b: $Y = \emptyset$

$\boxed{f_Y = 0000000}$

c: $k \in F$, $k = 1111111$

$\boxed{Y = \{\pi, \beta, -8.76, ♡, □, a, q\}}$

d: $g \in F$, $g = 1011010$

$\boxed{Y = \{\pi, -8.76, ♡, a\}}$

3: $A = \{a_1, a_2, a_3, \dots\}$

a: $Y = \{a_1, a_3\}$

$\boxed{f_Y = 101000\dots}$

b: $Y = \{a_i \mid i \text{ is odd}\} \leftarrow \{a_1, a_3, a_5, \dots\}$

$\boxed{f_Y = 101010\dots}$

c: $Y = \{a_i \mid \exists k \in \mathbb{Z} \text{ with } k \geq 0 \text{ and } i = 3k+2\} \leftarrow$ There is a $k \in \mathbb{Z}$ with $k \geq 0$ and $i = 3k+2$ $\to$

$k=0 \to i=2$
$k=1 \to i=5$
$k=2 \to i=8$
$k=3 \to i=11$
... ...

$Y = \{a_2, a_5, a_8, a_{11}, \dots\}$

$\boxed{f_Y = 010010010010\dots}$

4: $A = \{a_1, a_2, a_3, \ldots\}$

$f_Y = 101001000100001\cdots$

$Y = \{a_1, a_3, a_6, a_{10}, a_{15}, \ldots\}$

$\frac{k(k+1)}{2} =$

$\boxed{Y = \{a_i \mid \exists k \in \mathbb{Z} \text{ with } k \geq 1 \text{ and } i = \frac{k(k+1)}{2}\}}$ ← There is a $k \in \mathbb{Z}$ with $k \geq 1$ and $i = \frac{k(k+1)}{2}$

$k = 1 \rightarrow i = 1$

$k = 2 \rightarrow i = 3$

$k = 3 \rightarrow i = 6$

$k = 4 \rightarrow i = 10$

$k = 5 \rightarrow i = 15$

$\ldots \quad \ldots$