1: Proof. Let $S(n)$ be the statement

$\sum_{i=1}^{n} (i!)i = (n+1)! - 1$ for all $n \geq 1$

Base case: When $n=1$

LHS: $(1!)1 = 1$

RHS: $(1+1)! - 1 = 2! - 1 = 2 - 1 = 1$

LHS = RHS, thus $S(1)$ is true.

$(1!)1 + (2!)2 = 5$

$(2+1)! - 1 = 5$

Now, suppose $S(K)$ is true for some K. To show that $S(K+1)$ is true, let's look at the LHS

$S(K)$: $\sum_{i=1}^{K} (K!)K = (K+1)! - 1$ for all $K \geq 1$

$\sum_{i=1}^{K+1} ((K+1)!)(K+1)$

$((K+1)+1)! - 1$

$\rightarrow (K+2)! - 1$ (Goal)

$= (1!)\cdot 1 + (2!)\cdot 2 + \dots$
$+ (K!)(K) + ((K+1)!)(K+1)$

$= (K+1)! - 1 + ((K+1)!)(K+1)$ by the hypothesis that $S(K)$ is true.

$= (K+1+1)(K+1)! - 1$

$= (K+2)(K+1)! - 1$

$= (K+2)! - 1 = \text{RHS}$

$(1\cdot 2\cdot 3\cdot \dots \cdot K\cdot K+1) - 1 + (1\cdot 2\cdot 3\cdot \dots \cdot K\cdot K+1)(K+1)$

2. I did not always follow guideline #10 when writing proofs, which discusses numbering equations when they are referenced later in the same proof. This may lead to confusion in some proofs, like the one written for PS#6 problem 3. This problem involves proving a function is an injection using coordinate pairs. Although not many equations are used, it is still helpful for the reader to know where the referenced equations came from. Specifically, it would be beneficial to numerically assign the equations "$2a+b=2c+d$," and "$a-b=c-d$." Referencing these equations as the "first equation" and "second equation" may cause the proof to be more difficult to read than just labelling them (1) and (2) and then referencing them by mentioning equation (1) or equation (2).

3: $U = [0,1] \times [0,1]$. $R \subseteq U \times U$ is defined as $((a,b),(c,d)) \in R$ iff $(a,b) = (c,d)$ and/or $a=c$ and $\{b,d\} = \{0,1\}$

$\{0,1\} = \{1,0\}$

(a) $(\frac{1}{2}, \frac{1}{3}) \rightarrow (a,b)$

$\boxed{(\frac{1}{2}, \frac{1}{3}) \rightarrow (c,d)}$

(b) $(0,0) \rightarrow (a,b)$

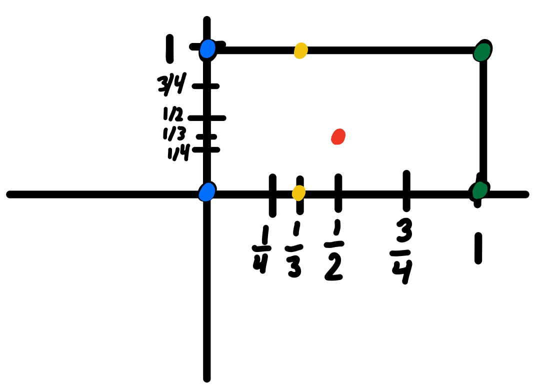
$\boxed{\begin{array}{l}(0,0) \rightarrow (c,d) \\ (0,1) \rightarrow (c,d)\end{array}}$

(c) $(\frac{1}{3}, 0) \rightarrow (a,b)$

$\boxed{\begin{array}{l}(\frac{1}{3},0) \rightarrow (c,d) \\ (\frac{1}{3},1) \rightarrow (c,d)\end{array}}$

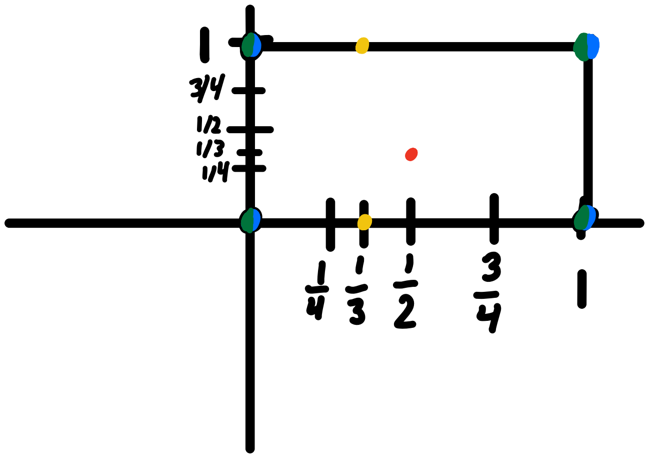
(d) $(1,1) \rightarrow (a,b)$

$\boxed{\begin{array}{l}(1,1) \rightarrow (c,d) \\ (1,0) \rightarrow (c,d)\end{array}}$



4: $U = [0,1] \times [0,1]$. $S \subseteq U \times U$ $((a,b),(c,d)) \in S$ iff at least one is true

- $(a,b) = (c,d)$, or
- $a = c$ and $\{b,d\} = \{0,1\}$, or
- $b = d$ and $\{a,c\} = \{0,1\}$, or
- $\{a,c\} = \{0,1\}$ and $\{b,d\} = \{0,1\}$



(a) $(\frac{1}{2}, \frac{1}{3}) \to (a,b)$

$(\frac{1}{2}, \frac{1}{3}) \to (c,d)$

(b) $(0,0) \to (a,b)$

$(0,0) \to (c,d)$
$(0,1) \to (c,d)$
$(1,0) \to (c,d)$
$(1,1) \to (c,d)$

(c) $(\frac{1}{3}, 0) \to (a,b)$

$(\frac{1}{3}, 0) \to (c,d)$
$(\frac{1}{3}, 1) \to (c,d)$

(d) $(1,1) \to (a,b)$

$(1,1) \to (c,d)$
$(1,0) \to (c,d)$
$(0,1) \to (c,d)$
$(0,0) \to (c,d)$