

$$2: a: A \cap B = \{\alpha\}$$

$$b: A \cap C = \{\beta, \gamma\}$$

$$c: A \cup B = \{\alpha, \beta, \gamma, \alpha, \star, \emptyset\}$$

$$d: A \cup C = \{\alpha, \beta, \gamma, \text{read}, \emptyset\}$$

$$e: \emptyset \cup A = \{\alpha, \beta, \gamma\}$$

$$f: \emptyset \cap C = \emptyset$$

$$g: C - A = \{\text{read}, \emptyset\}$$

$$h: B \cap C = \{\emptyset\}$$

$$i: A \cup (B \cap C) = \{\alpha, \beta, \gamma, \emptyset\}$$

$$j: B \cap (A \cup C) = \{\alpha, \emptyset\}$$

$$3: a: A \cup B = \mathbb{N} \quad A \cap B = \emptyset$$

$$A = \{a \mid a = 2x, x \in \mathbb{N}\} = \{0, 2, 4, 6, \dots\}$$

$$B = \{b \mid b = 2x+1, x \in \mathbb{N}\} = \{1, 3, 5, 7, \dots\}$$

The definition of  $A$  represents the set of all even natural numbers while the definition of  $B$  represents the set of all odd natural numbers. Because all elements in the set of natural numbers are either even or odd, this means  $A \cup B = \mathbb{N}$  and  $A \cap B = \emptyset$ .

b:

$$D = \{d \mid d = 3x, x \in \mathbb{N}\} = \{0, 3, 6, 9, 12, \dots\}$$

$$E = \{e \mid e = 2x, x \in \mathbb{N} \text{ and } x \text{ is not divisible by } 3 \text{ and } x \neq 0\} = \{2, 4, 8, 10, 14, 16, \dots\}$$

$$F = \{f \mid f = x, x \in \mathbb{N} \text{ and } x \text{ is not divisible by } 3 \text{ and } x \text{ is not divisible by } 2 \text{ and } x \neq 0\} = \{1, 5, 7, 11, 13, 17, \dots\}$$

The definition of  $D$  represents the set of all natural numbers in which 0 is an element followed by all natural numbers that are a multiple of three.

The definition of  $E$  represents the set of all natural numbers that are a multiple of 2 and are not a multiple of 3.

Lastly, the definition of  $F$  represents the set of all natural numbers that are not a multiple of 2 and not a multiple of 3. Because  $D$  considers 0 and all elements in the set of natural numbers that are a multiple of 3 and  $E$  considers all elements in the set of natural numbers that are multiples of 2, excluding elements that are also multiples of 3, and  $F$  considers the remaining elements that are not a multiple of 3 or a multiple of 2, this information indicates  $D \cup E \cup F = \mathbb{N}$  and  $\{D, E, F\}$  is disjoint.

$$C: S = \left\{ \begin{array}{l} \{0, 1, 2, 4, 6, 8, 10, 12, 14, \dots\} \\ \{3, 9, 15, 21, 27, 33, \dots\} \\ \{5, 25, 35, 55, 65, 85, 95, \dots\} \\ \{7, 49, 77, 91, 119, 132, \dots\} \\ \{11, 121, 143, \dots\} \\ \vdots \end{array} \right\}$$

The definition of  $S$  represents an infinite set of infinite sets where the union of all of the infinite sets of  $S$  represent the set of natural numbers and the intersection of all the infinite sets of  $S$  represent the empty set. The first infinite set has 0 and 1 as elements followed by all natural numbers that are a multiple of two. The second infinite set of  $S$  covers all natural numbers that are multiples of three, excluding those that are also multiples of two. The third infinite set of  $S$  contains all natural numbers that are multiples of five, excluding those that are also multiples of two or three. This pattern continues where the infinite sets of  $S$  will have elements that are multiples of specific values while excluding those that were already an element of a prior infinite set due to also being a multiple of another value.

This information indicates that  $\bigcup_{A \in S} A = \mathbb{N}$  and  $S$  is disjoint.