Math 215-800 **PS #6**
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Writing Exercises. Type up your solutions to #1 using LaTeX. Start by going to the course webpage and downloading the .tex file for this assignment; pop that .tex file into your favorite latex editor (probably Overleaf.com) and type your responses just after the corresponding problem in the .tex file.

1. Let $m, b \in \mathbb{R}$ with $m \neq 0$. Prove that $h : \mathbb{R} \to \mathbb{R}$ defined by $h(x) = mx + b$ is injective.

   Proof. Suppose $h(x_1) = h(x_2)$. By definition of h, we have $mx_1 + b = mx_2 + b$. Then, subtracting b from both sides gives $mx_1 = mx_2$. Finally, dividing by m, where $m \neq 0$ gives $x_1 = x_2$. Therefore, h is injective. ∎

Additional Exercises. Complete the next problems, #2-#3. You need not typeset your answers, unless you want to. Staple your answers to your write-up for the Writing Exercise and turn in one homework with your name on the front, at the top.

2. Find sets A and B that demonstrate $\mathcal{P}(A \cup B) \neq \mathcal{P}(A) \cup \mathcal{P}(B)$. You must calculate the unions and power sets, and explain why the sets you found actually have different $\mathcal{P}(A \cup B)$ and $\mathcal{P}(A) \cup \mathcal{P}(B)$.

3. Fix this proof. Justifications and steps are missing all over the place. Some things are just wrong. Please, before I die from the heart palpitations induced by reading the assignment, *fix this proof.*

   Proposition 1. The function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ defined by $f(x, y) = (2x + y, x - y)$ is an injection.

   Proof. For each (a, b) and (c, d) in $\mathbb{R} \times \mathbb{R}$, if $(a, b) = (c, d)$ then

   $$(2a + b, a - b) = (2c + d, c - d).$$

   We will use systems of equations to prove that $a = c$ and $b = d$.

   $$\begin{aligned} 2a + b &= 2c + d \\ a - b &= c - d \\ 3a &= 3c && \text{by adding the two equations together} \\ a &= c \end{aligned}$$

   Since $a = c$, we see that $(2c + b, c - b) = (2c + d, c - d)$. So $b = d$. Therefore we have proved that the function f is one-to-one. ∎

4. Let $A = \{\pi, \eta, \lambda\}$ and $B = \{6, 7\}$.

   (a) Define, if possible, a function $f : A \to B$ so that f is surjective.

   (b) Define, if possible, a function $g : A \to B$ so that g is *not* surjective.

   (c) Define, if possible, a function $h : B \to A$ so that h is surjective.

(d) Define, if possible, a function $k : A \to B$ so that k is injective.

(e) Define, if possible, a function $m : A \to B$ so that m is *not* injective.

(f) Define, is possible, a function $n : B \to A$ so that n is injective.

If any of the above are not possible, use 1-2 sentences to explain why.

Pure mathematics is, in its way, the poetry of logical ideas. – Albert Einstein