MAT 373 Homework 1 (Revised)

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Problem 1: Let a, b be integers such that $a|b$ and $b|a$, prove $a = \pm b$.

Proof. Since it is known that $a|b$ and $b|a$, there are integers c, d, such that:

$$ac = b \tag{1}$$

$$bd = a \tag{2}$$

where $a, b \neq 0$.

Equation (2) allows us to substitute bd for a, which turns equation (1) into:

$$bdc = b \tag{3}$$

Note that equation (3) is only true when:

$$dc = 1 \tag{4}$$

Since we know that c, d are integers, the only two cases in which equation (4) is true is when either,

$$c = d = 1 \tag{5}$$

or,

$$c = d = -1 \tag{6}$$

By applying equations (5) and (6) to equations (1) and (2), we end up with either,

$$a = b$$

or,

$$a = -b$$

Thus, if we know $a|b$ and $b|a$, we can say that $a = \pm b$.

$\square$

Problem 2: Let a, b, m be integers, $m \neq 0$. Prove $a|b \iff ma|mb$

Proof. Suppose $a|b$, then there must be an integer k such that:

$$ak = b \tag{7}$$

where $a \neq 0$.

We can multiply both sides of equation (7) by m to give us:

$$mak = mb \tag{8}$$

Since we know that m, a are integers, ma is also an integer. This knowledge, along with equation (7) allows us to state that $ma|mb$.

Moreover, now suppose $ma|mb$, then there is some integer l such that:

$$mal = mb \tag{9}$$

where $ma \neq 0$.

Since we assumed $m \neq 0$, we can divide m on both sides of equation (9) to then give us:

$$al = b \tag{10}$$

Thus, it is now possible to conclude that if $ma|mb$, then $a|b$. Combining both parts of this proof can now indicate that $a|b \iff ma|mb$.

$\square$