Let's do some scientific stuff

Randomized matrix multiplication

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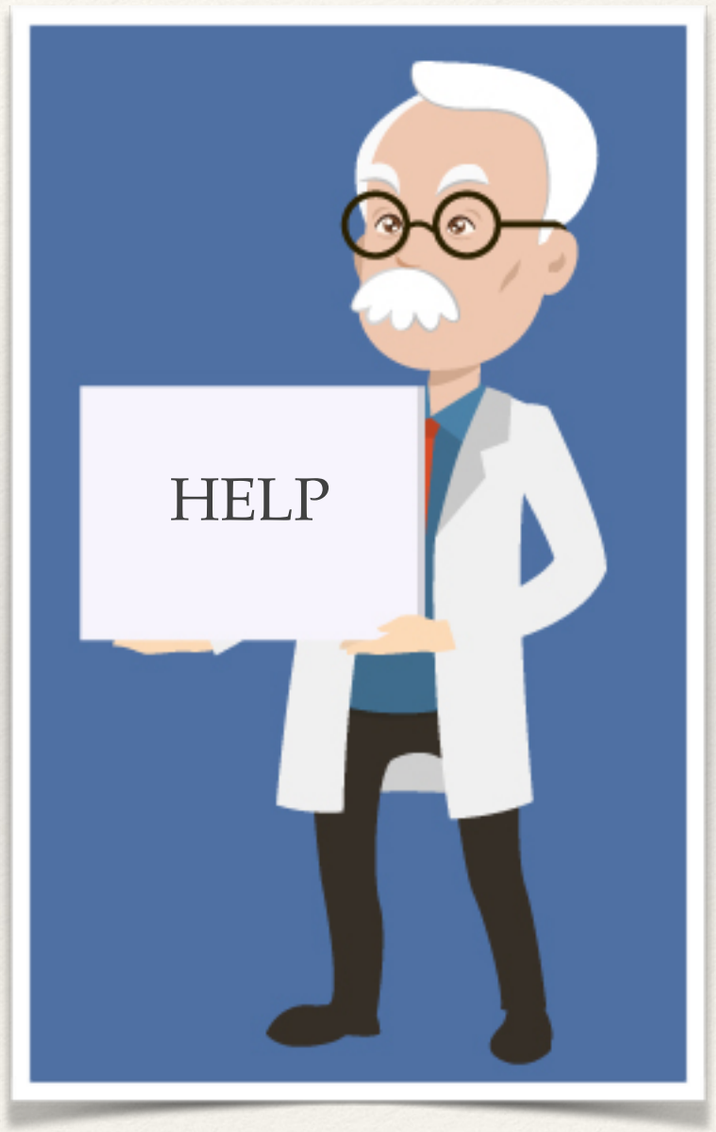
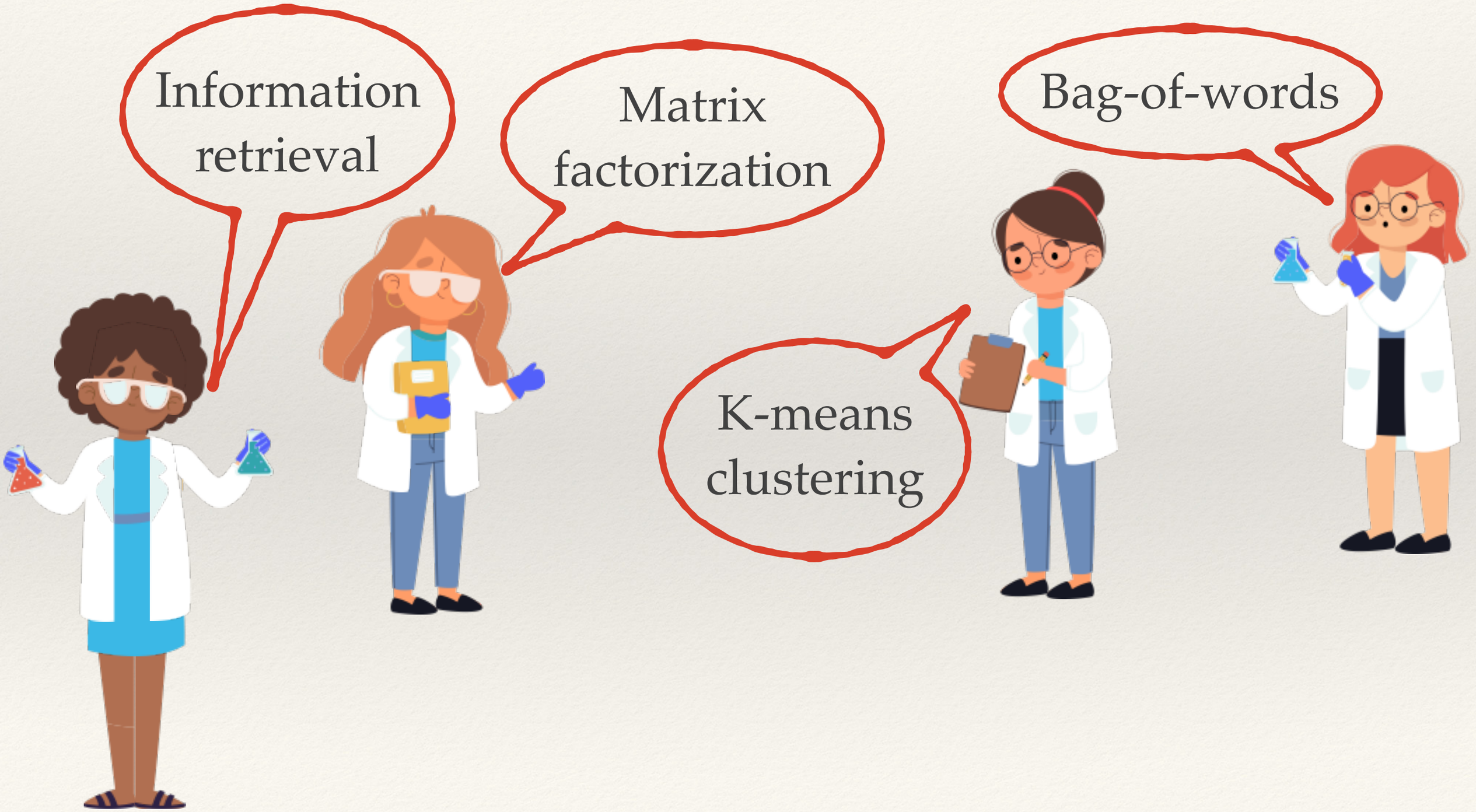
NLA problems

- Matrix multiplication
- Computation of the SVD
- Low-rank approximation
- Least squares



Application



General idea

$$\boldsymbol{BC} = \sum_{j=1}^{N} \boldsymbol{b}_{:j}\boldsymbol{c}_{j:}.$$

$$p_j = \frac{\|\boldsymbol{b}_{:j}\|^2 + \|\boldsymbol{c}_{j:}\|^2}{\|\boldsymbol{B}\|_{\mathrm{F}}^2 + \|\boldsymbol{C}\|_{\mathrm{F}}^2} \quad \text{for } j = 1, 2, 3, \ldots, N.$$

General idea

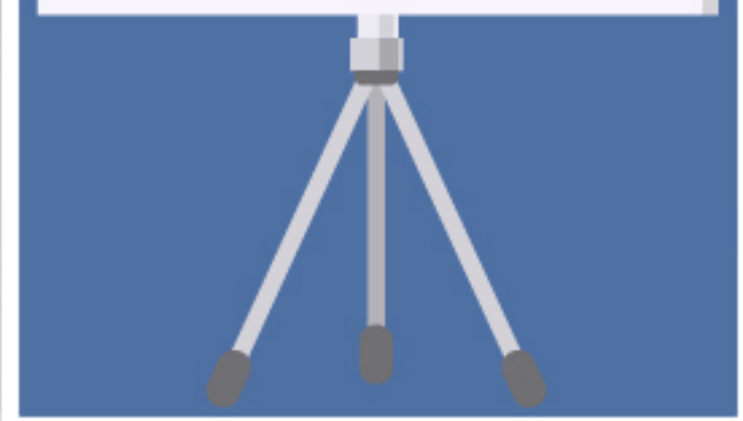
$$\boldsymbol{R} = \frac{1}{p_j} \cdot \boldsymbol{b}_{:j} \boldsymbol{c}_{j:}$$

$$\mathbb{E}\,\boldsymbol{R} = \sum_{j=1}^{N} \frac{1}{p_j} \cdot \boldsymbol{b}_{:j} \boldsymbol{c}_{j:} \cdot p_j = \sum_{j=1}^{N} \boldsymbol{b}_{:j} \boldsymbol{c}_{j:} = \boldsymbol{BC}.$$

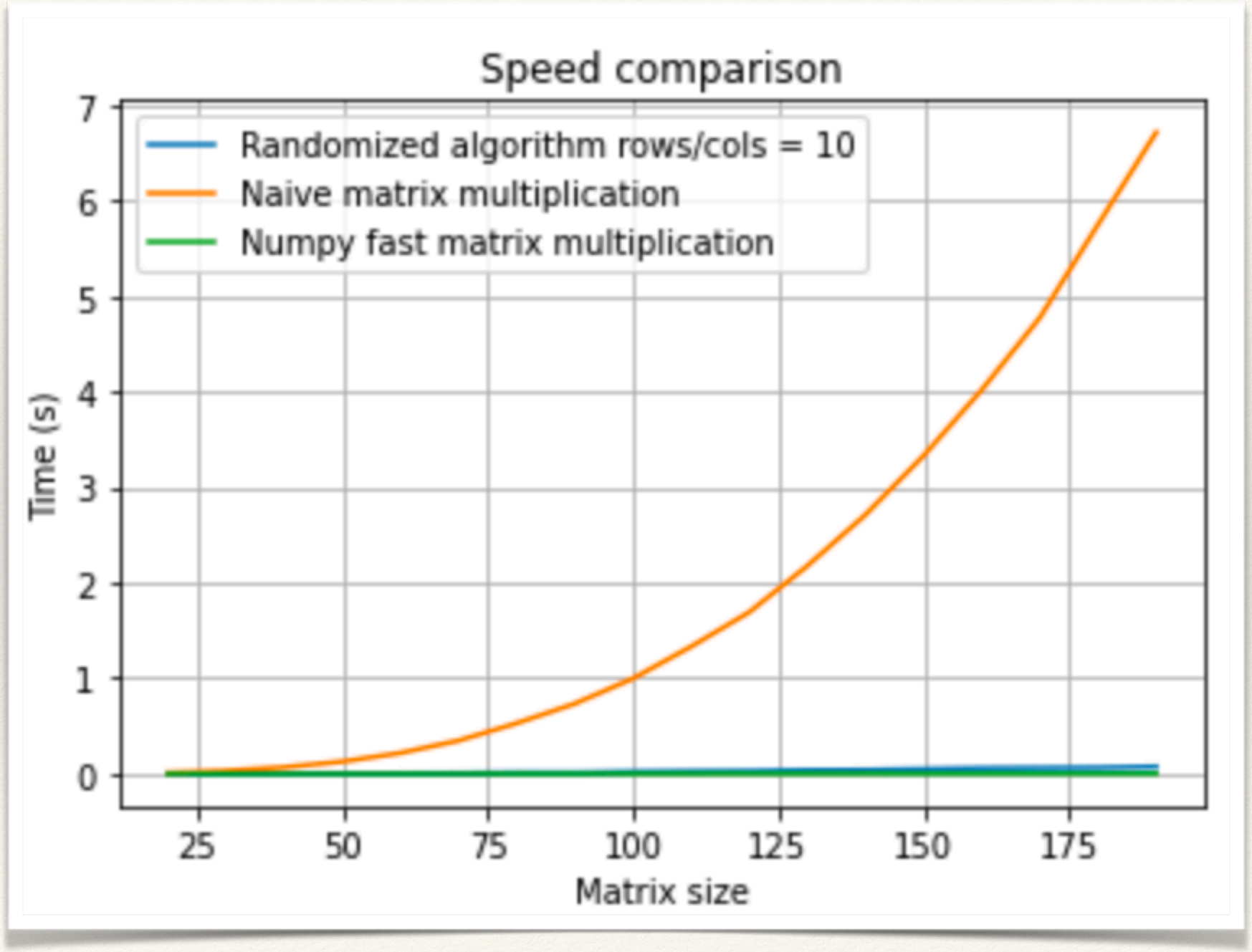
$$\bar{\boldsymbol{R}}_n = \frac{1}{n} \sum_{k=1}^{n} \boldsymbol{R}_k$$

R is random matrix with probability p_j

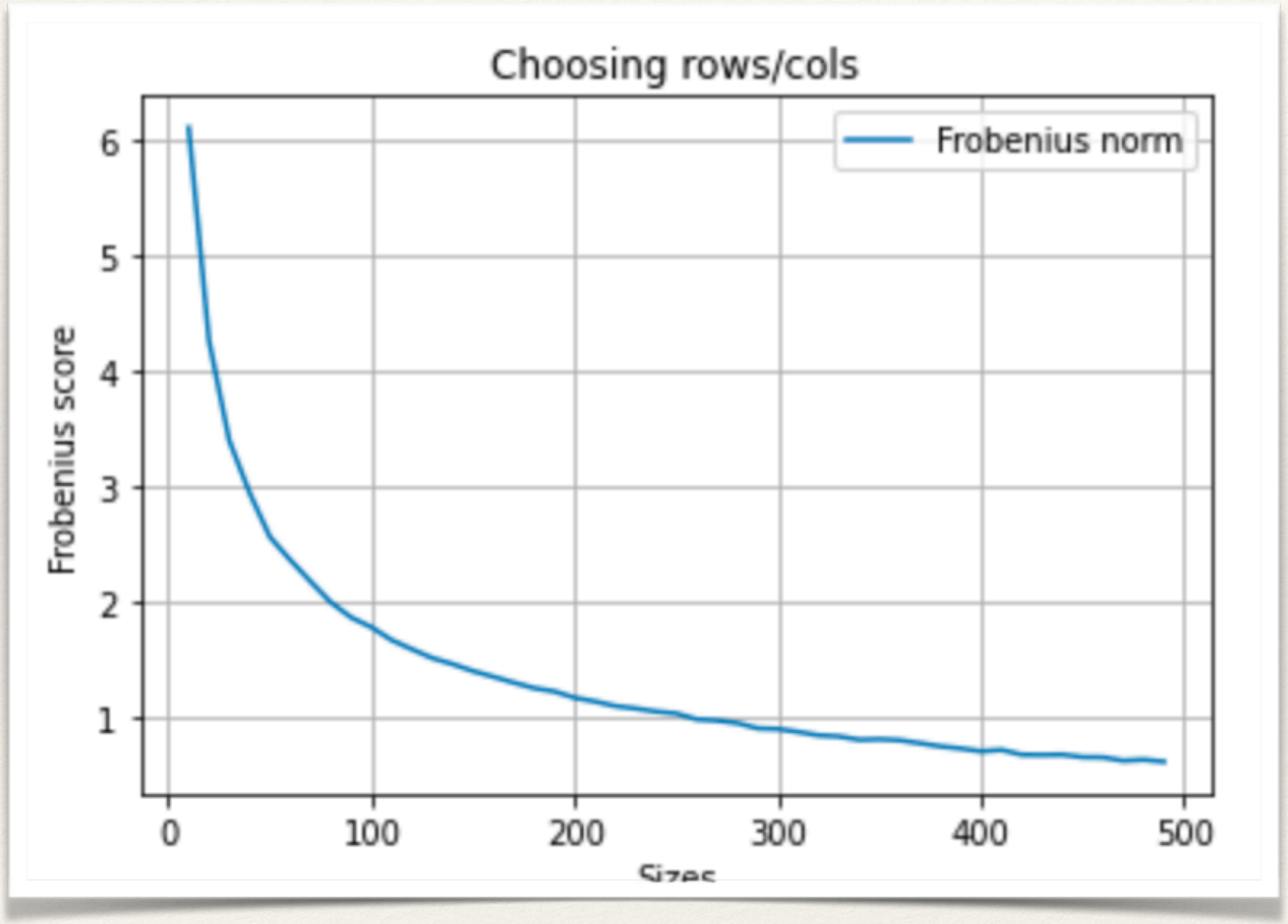
R_k is independent copy of R



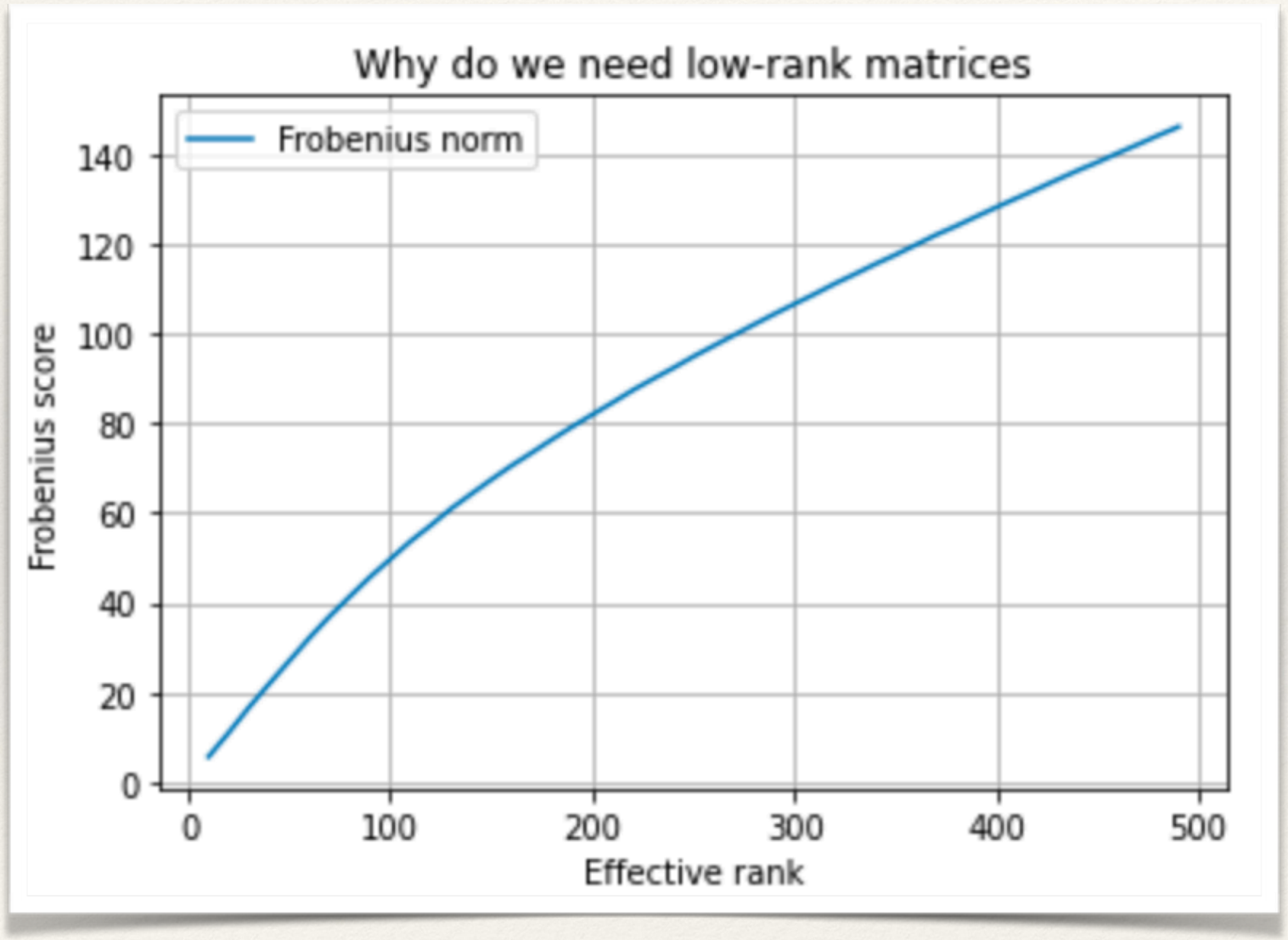
Speed results



Accuracy results



Accuracy results



Streaming application

- No need to store the entire matrix in memory

$$p_j = \frac{\|\boldsymbol{b}_{:j}\|^2 + \|\boldsymbol{c}_{j:}\|^2}{\|\boldsymbol{B}\|_{\mathrm{F}}^2 + \|\boldsymbol{C}\|_{\mathrm{F}}^2} \quad \text{for } j = 1, 2, 3, \ldots, N.$$

$$\boldsymbol{R} = \frac{1}{p_j} \cdot \boldsymbol{b}_{:j} \boldsymbol{c}_{j:}$$

Complexity

- Naive algorithm - $O(N^3)$

- Andrew Stothers (Best) - $O(N^{2.374})$

- Randomized - $O(N \times (d_1 + d_2))$

Questions?

Literature

- A practical streaming approximate matrix multiplication algorithmAuthor links open overlay panel. Deena P.Francis Kumudha Raimond
- An Introduction toMatrix Concentration Inequalities. Joel A. Tropp
- Frequent Direction Algorithms for Approximate Matrix Multiplication with Applications in CCA. Qiaomin Ye, Luo Luo, Zhihua Zhang