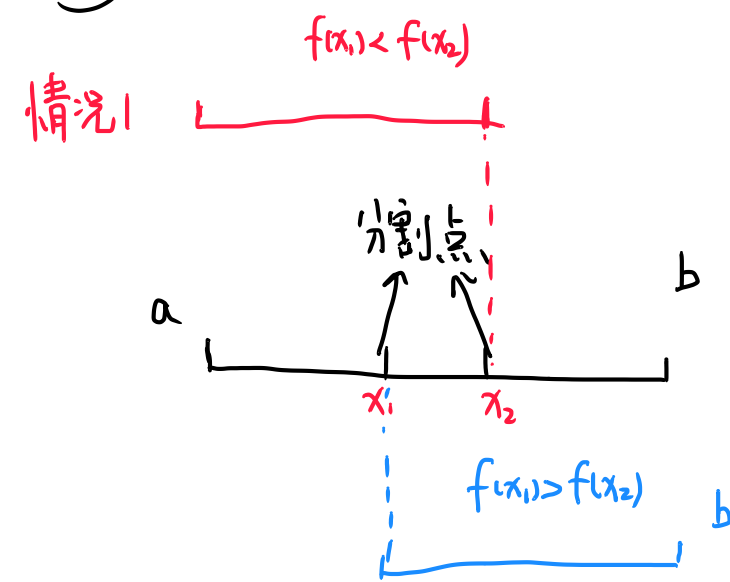


§ 1.1 黄金分割法



$$\overline{ax_1} = \overline{x_2b}$$

$$\frac{x_1x_2}{ax_1} = \frac{x_2b}{x_2b} = \frac{ax_2}{ab} = 0.618$$

$$\Rightarrow \begin{cases} x_1 = a + 0.382(b-a) \\ \quad = 0.382b + 0.618a \\ x_2 = a + 0.618(b-a) \\ \quad = 0.382a + 0.618b \end{cases}$$

例题. 作业题: $\min f(x) = \sin x + \cos x$.
 $x_0 = 3.0$, $h = 0.2$, $\varepsilon = 0.1$

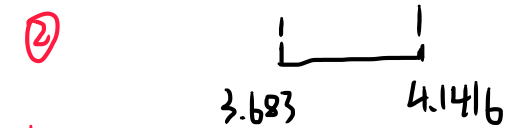
Step 1: 找初始区间:

$$f(x_0) = 0.1411, \quad f(x_0+h) = -1.0567, \quad f(x_0+2h) = -1.2223$$

$$\therefore f(x_0+4h) = -1.4028, \quad f(x_0+8h) = f(4.6) = -1.1058$$

$$\therefore [x_0+2h, x_0+8h] \Rightarrow [3.4, 4.6]$$

Step 2: 黄金分割法:



$$\textcircled{1} \begin{cases} x_1 = 3.4 + 0.382(4.6-3.4) = 3.8584 \\ x_2 = 3.4 + 0.618 \times (4.6-3.4) = 4.1416 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} f(x_1) = -1.4109 \\ f(x_2) = -1.3818 \end{cases} \quad f(x_2) > f(x_1)$$

$$\therefore \text{区间更新: } [3.4, 4.1416]$$

$$\textcircled{2} \begin{cases} x_1 = 3.4 + 0.382 \times (4.1416-3.4) = 3.683 \\ x_2 = 3.4 + 0.618 \times (4.1416-3.4) = 3.858 \end{cases}$$

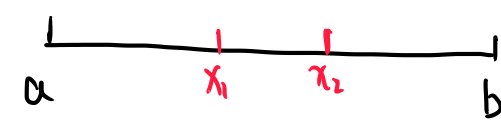
$$\Downarrow$$

$$\begin{cases} f(x_1) = -1.3723 \\ f(x_2) = -1.4108 \end{cases} \quad f(x_1) > f(x_2)$$

$$\therefore \text{区间更新: } [3.683, 4.1416]$$

§ 斐波那契法

$$F_n \geq \frac{b-a}{L}$$



1	2	3	4	5	6	7	8
1	1	2	3	5	8	13	21

区间缩小的方式同 0.618 法

我们看一下, x_1, x_2 怎么求. 类似地: $0.382 + 0.618 = 1$

$$\Rightarrow \begin{cases} x_1^k = a + \frac{F_{n-k}}{F_n}(b-a) \\ x_2^k = a + \frac{F_{n-1-k}}{F_n}(b-a) \end{cases}$$

第1次迭代 ($k=0$)
 第2次迭代用上次 (F_{n-k})

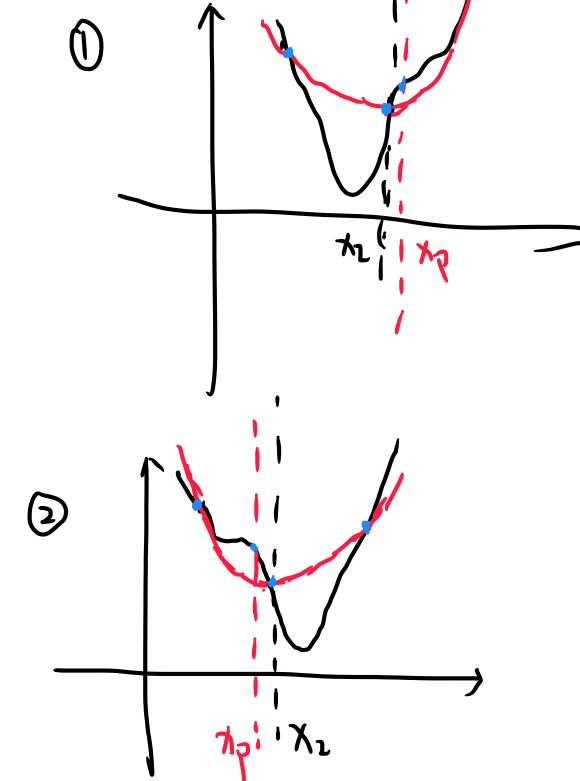
例: $\min f(x) = 3x^3 - 4x + 2$, $x_0 = 0$, $h = 1$, $\varepsilon = 0.2$
 $[0, 2]$, $F_n \geq \frac{2}{0.2} = 10$, $F_n = 13$

$$\therefore \textcircled{1} \begin{cases} x_1 = 0 + \frac{F_{n-2}}{F_n} \Delta x = 2 \times \frac{5}{13} = \frac{10}{13} \\ x_2 = 0 + 2 \times \frac{8}{13} = \frac{16}{13} \end{cases} \Rightarrow [0, \frac{16}{13}]$$

$$\textcircled{2} \begin{cases} x_1' = 0 + \frac{F_{n-2-1}}{F_{n-1}} \Delta x = 0 + \frac{16}{13} \times \frac{3}{8} = \frac{6}{13} \\ x_2' = 0 + \frac{F_{n-1-1}}{F_{n-1}} \Delta x = 0 + \frac{16}{13} \times \frac{5}{8} = \frac{10}{13} \end{cases} \Rightarrow [\frac{6}{13}, \frac{16}{13}]$$

...

§ 二次插值法



区间选择

$$\text{if: } f_p \geq f_z$$

$$\text{if: } x_p \geq x_z: \textcircled{1}$$

$[x_1, x_p]$ 中间点不变

$$\text{if: } f_p < f_z$$

$$\text{if: } x_p \geq x_z: \textcircled{3}$$

$[x_2, x_3]$ 且中间点取 x_p

$$\text{if: } x_p < x_z: \textcircled{2}$$

$[x_p, x_3]$ 中间点不变

$$\text{if: } x_p < x_z: \textcircled{4}$$

$[x_1, x_z]$ 且中间点取 x_p



§ 三次插值法