Q1 1. Proof by Truth table enumeration

2. Proof by Resolution

Q2 Convert KB, $\neg G$ to CNF:

(1) $F_1 \wedge (\neg F_4 \Leftrightarrow \neg F_3)$:

$F_1 \wedge (\neg F_4 \Rightarrow \neg F_3) \wedge (\neg F_3 \Rightarrow \neg F_4)$

$F_1 \wedge (F_4 \vee \neg F_3) \wedge (F_3 \vee \neg F_4)$

(2) $F_4 \Rightarrow \neg F_2$:

$\neg F_4 \vee \neg F_2$

(3) $F_3 \Rightarrow F_5$:

$\neg F_3 \vee F_5$

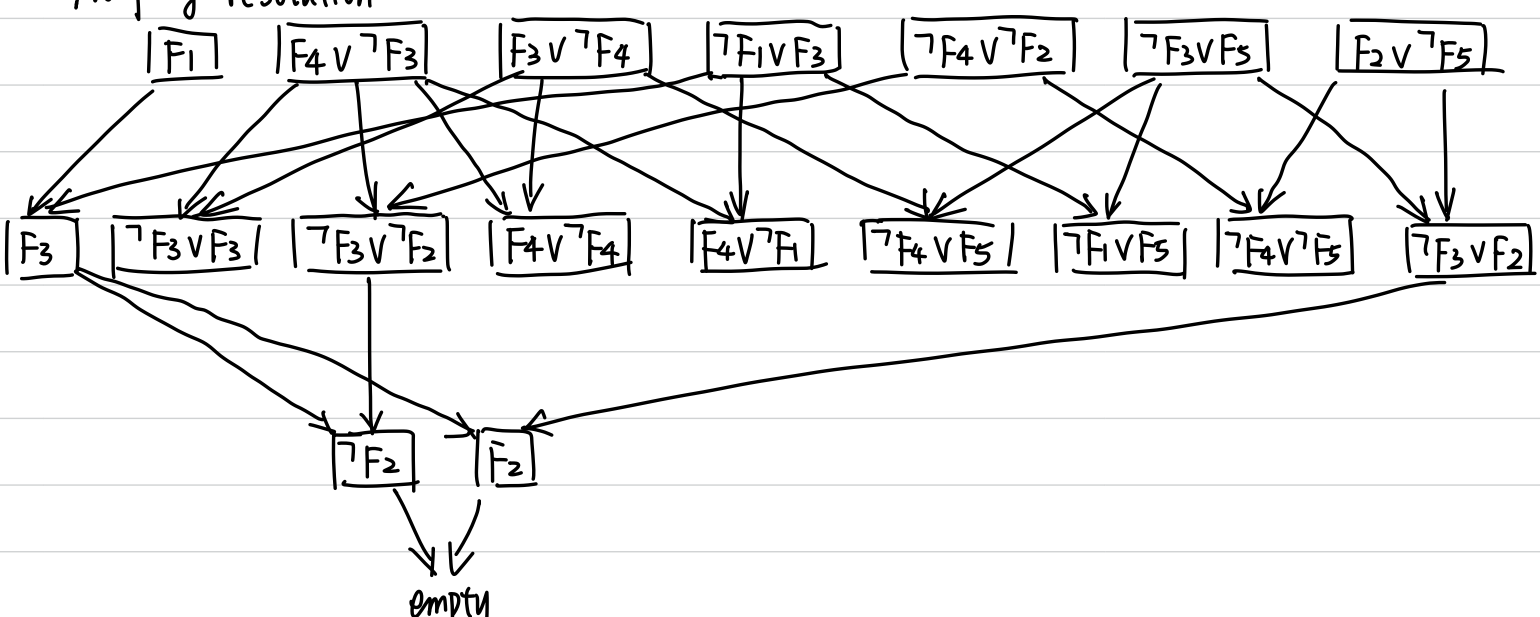
(4) $\neg(\neg F_2 \wedge F_5)$:

$F_2 \vee \neg F_5$

Convert CNF into clauses:

F_1, $(F_4 \vee \neg F_3)$, $(F_3 \vee \neg F_4)$, $(\neg F_1 \vee F_3)$, $(\neg F_4 \vee \neg F_2)$, $(\neg F_3 \vee F_5)$, $(F_2 \vee \neg F_5)$

Proof by resolution:



So there exists contradiction, $KB \wedge \neg G$ is unsatisfiable. So $KB \models G$ is proven.

Q3 $B\left(\frac{p}{p+n}\right) = B\left(\frac{1}{2}\right) = -\left(\frac{1}{2}\log_2\frac{1}{2} + \frac{1}{2}\log_2\frac{1}{2}\right) = 1$

$\text{Remainder}(\text{Hun}) = \frac{3}{8}B\left(\frac{1}{3}\right) + \frac{5}{8}B\left(\frac{3}{5}\right)$

$= -\frac{3}{8}\left(\frac{1}{3}\log_2\frac{1}{3} + \frac{2}{3}\log_2\frac{2}{3}\right) \approx 0.951$

$\text{Gain}(\text{Hun}) = B\left(\frac{1}{2}\right) - \text{Remainder}(\text{Hun}) \approx 0.049$

$\text{Remainder}(\text{Res}) = \frac{2}{8}B\left(\frac{0}{2}\right) + \frac{6}{8}B\left(\frac{4}{6}\right)$

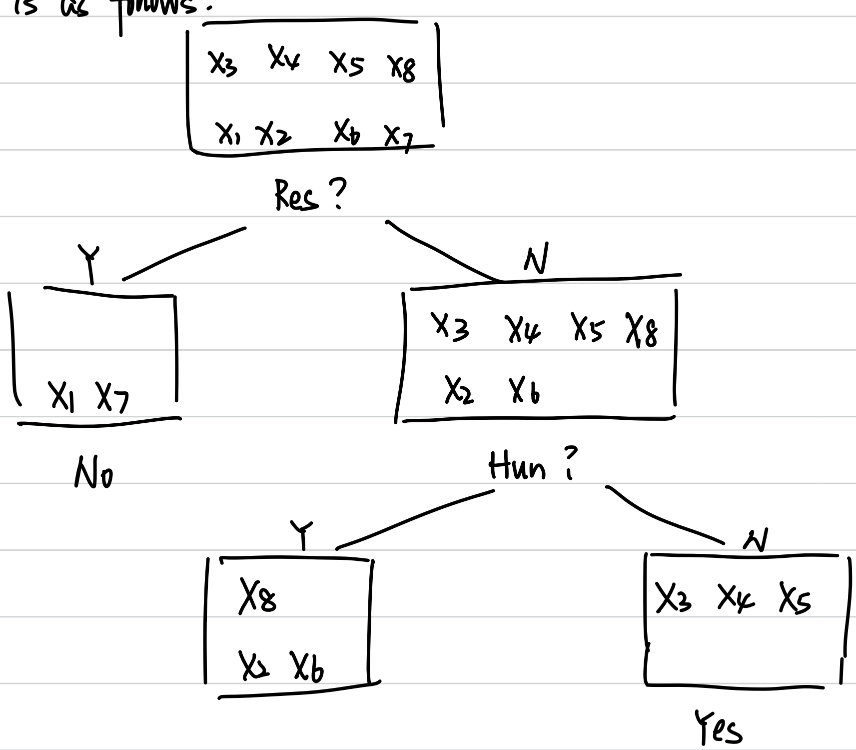
$= -\frac{3}{4}\left(\frac{2}{3}\log_2\frac{2}{3} + \frac{1}{3}\log_2\frac{1}{3}\right) \approx 0.689$

$\text{Gain}(\text{Res}) = B\left(\frac{1}{2}\right) - \text{Remainder}(\text{Res}) \approx 0.311$

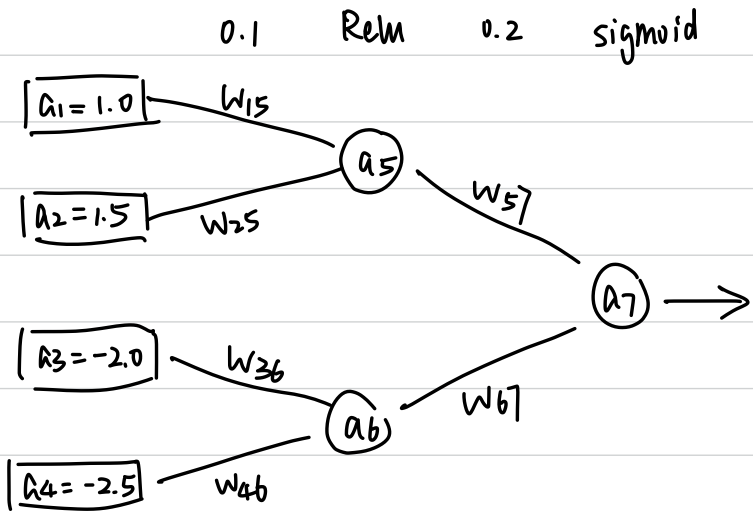
$\text{Gain}(\text{Res}) > \text{Gain}(\text{Hun})$

"Res" has the largest Gain and therefore chosen as the attribute

The tree is as follows:



Q4



$a_5 = Relu(in_5) = Relu(W_{15}a_1 + W_{25}a_2) = Relu(0.25) = 0.25$

$a_6 = Relu(in_6) = Relu(W_{36}a_3 + W_{46}a_4) = Relu(-0.45) \simeq 0$

$a_7 = \delta(in_7) = \delta(W_{57}a_5 + W_{67}a_6) = \delta(0.05) = 0.512$

Q5 $Err_7 = y_7 - h_7(W, x) = 1 - 0.512 = 0.488$

$\delta'(x) = \left(\frac{1}{1+e^{-x}}\right)' = \frac{e^{-x}}{(1+e^{-x})^2}$

$\Delta_7 = Err_7 \times g'(in_7) = Err_7 \times \delta'(in_7) = 0.448 \times \frac{e^{-0.05}}{(1+e^{-0.05})^2} \approx 0.112$

$W_{57} = W_{57} + \alpha \times \Delta_7 \times a_5 = 0.2 + 0.1 \times 0.112 \times 0.25 \approx 0.2028$

$W_{67} = W_{67} + \alpha \times \Delta_7 \times a_6 = 0.2 + 0.1 \times 0.112 \times 0 = 0.2$

$Relu'(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$

$\Delta_5 = g'(in_5) W_{57} \Delta_7 = 1 \times 0.2 \times 0.112 = 0.0224$

$W_{15} = W_{15} + \alpha \times \Delta_5 \times a_1 = 0.1 + 0.1 \times 0.0224 \times 1.0 \approx 0.102$

$W_{25} = W_{25} + \alpha \times \Delta_5 \times a_2 = 0.1 + 0.1 \times 0.0224 \times 1.5 \approx 0.103$

$\Delta_6 = g'(in_6) W_{67} \Delta_7 = 0$

$W_{36} = W_{36} + \alpha \times \Delta_6 \times a_3 = 0.1$

$W_{46} = W_{46} + \alpha \times \Delta_6 \times a_4 = 0.1$

Q6 (a) $5 \times 5 = 25$

(b) $124 \times 124 \times 5 \times 5 = 384400$

(c) The size of layer 4 is 60×60

(d) $32 \times 32 \times 32 \times 32 = 1048576$