Problem Chosen	2021	Team Control Number
A	MCM/ICM Summary Sheet	2106187

Optimization of the future transportation: Lightweight Suspension Transportation System

Summary

Due to current large consumption of ground space, future transportation system needs to take advantage of higher space. Thus, basic structure design of lightweight suspension transportation system at a span of 100m has been proposed, similar to suspension bridge and cable way. This paper aims at making such kind of design into reality by using mathematical modeling to ensure its feasibility through fields including safety concerns, cost and materials, speed and driving rules, vibration and custom experience, as well as rescue scheme for accidents.

In order to reduce cost while ensure safety, the **elastic deformation** of all contents (beams, rods, cables) for bearing the trailer's weight is considered. The force exerted to the track is modeled to have a **distribution of its deflection curve**, using the simulation of Solidworks. Then the force is distributed onto each rod and cable by force analysis to compare with their maximum bearing force. Variables that can be optimized include the number of rods, the length of the cable and allowable factor of safety. After applying **Multiple Attribute Decision Making method**, the best score leads to the final structure design: **10 rods placed uniformly** along the 100-meter beam and a cable of **291.6343 meters** for one single track, which cost **20053 dollars** buying cables and rods.

The factor of safety ensures that extra load may occur due to the trailer's movement. Assuming the trailer moves with uniform acceleration and consider both **impact and dynamic coefficient**, both the cable and rods are pushed to failure condition to test the maximum allowed speed. After modifying the exact force from the moving trailer, the maximum speed is **33.6 km/h** for the design proposed before. At the same time, another plan is provided to achieve a maximum speed of 60 km/h required by the task: **changing the radius** of cable from 3 cm **to 3.02cm** with **a cost of 20320**. While adding rods also works, it costs more.

Hanging trailers also have a problem of vibration when moving. To make sure such suspension structure won't allow the trailer to vibrate with unacceptable amplitude causing discomfort, the environmental factor: wind load is analyzed. Since the wind force is related to the angle between the wind direction and trailer's velocity, **the maximum effect** appears to be **at 136^o**. At this direction, the amplitude is linearly related to the length of the hanging beam, with a maximum of **0.29 mm for one-meter beam**, which is already acceptable. Possible improvement is optimizing the shape of the trailer to reduce air resistance coefficient C. After **changing C from 1 to 0.5**, the **amplitude decrease to 0.14 mm.**

Finally, such traffic should also have a rescue scheme. **Adding truss structure** brings lots of advantages including reducing time for clearing up the track as well as reducing rescue distance and etc. The design of the truss structure only has one parameter, the distance between two parallel truss c. Due the graphic feature of the bridge, the possible number for c that is reasonable are 10,20,25 and 50 meters. In order to find the best mode, the average rescuing time, the maximum rescuing time as well as the total length of truss that need to be added are evaluated. The final result of the best choice is **c=20**, with **a maximum rescuing time of 24.74 s, average time of 13.65 s and total truss length of 133.08 m.**

Sensitivity analysis guarantees the effectiveness of all the modeling. While there's still some aspects that are not perfect to model real conditions, lots of achievements have been made for the lightweight suspension system much closer to real-life application after evaluating all the fields above.

Keywords: keywords1,keywords2,keywords3

Contents

1 Introduction

1.1 Problem

Due to the rapid development of city population and the ground transportation system, the land space has already been overcrowded. Thus, the idea of lightweight suspended traffic system that can take advantage of higher city space becomes reasonable and doable. According to its name, lightweight suspended traffic system refers to railway tracks supported in the air by pile foundation with lightweight trailers suspended on the track to run. Such design generates lots of advantages such as avoiding road surface occupation, adapting all-terrain environments, operating point-to-point as a network system as well as bringing economic advantages.

However, the new system is still under testing. A small-scale testbed is designed to simulate the original system as a suspension bridge structure. While the rough design is already given, detailed design including the number of rods, the length of cable, as well as traffic rules for the running cars above. People call for a rigorous plan to achieve least cost as well as system safety. And a new regulations for the lightweight system to handle accidents is also important.

1.2 Analysis & Problem Restatement

The priority of any structure is to ensure safety. For the simplest case, such structure should be strong enough to carry the weight of a stable trailer. The upper limited load of a trailer is required to be 1t. The beam which directly supports the movement of the cable car needs to be pulled by enough rods above to avoid large and dangerous deformation. The total number and positions of all rods along the bar are important to make sure none of them exceeds its maximum bearing load. The suspension cable connects all rods, which in turn restricts its shape. Among all the possible design choices, price cost is the last parameter to finalize the decision. The least cost, which means using the least cable and rod materials in total gives the best design.

Furthermore, the movement of car may bring extra damage to the system. Things need to be considered includes: the velocity and acceleration of the car, the environmental effects (wind), the vibration and etc. To simplify the model, we assume that the car is moving under "uniform acceleration - uniform velocity - uniform deceleration" condition with the restriction that the running speed should be less than 60km/h. The movement above the ground then brings in the factor of wind and environment which leads to small vibrations. Vibrations affect not only the structure safety, but also the ride comfort of the occupant. Thus, the design model can be further modified.

Finally, the new transportation system calls for new regulations such as rescue scheme. Due to the specialty of the new traffic system, accidents should be handled in a relatively short time to avoid tie-up since one single car getting stuck may affect a whole line. Designers also need to provide effective solutions for efficient rescue procedure based on the specialty of our own design.

Thus, the problems that will be addressed in this project is further specified as follows:

1. How to decide the number of rods and arrange them in a most effective way with the beam and suspension cable to avoid deformation? Then what's the limit status of the

suspension cable, under which the cost of material can also be minimized.

2. Based on previous model, let the car move and accelerate. What's the speed limit of the car that is safe driving in such system? Is there any way to improve the suspension model after taking the factor of car movement into consideration?

3. Environmental factors such as wind affect the transportation a lot above the ground. How will the trailer vibrate moving under wind load? How to improve the design to reduce vibration which brings discomfort to occupants?

4. How should the rescue scheme be regulated under such special transport system to achieve a best effectiveness?

2 Model Design

2.1 Assumptions

In order to extract an effective model for solving such engineering problem, assumptions below are made. Some assumptions are throughout the text which is for simplifying the problem. Others may be modified or improved during the procedure to make it closer to reality.

1. The only failure mode for all elements (beams, rods, cables) considered under such condition is elastic deformation. And the elastic deformation in such system includes only: longitudinal deformation for cable and rods, bending and laterial deformation for beams

2. The motion of the trailer is assumed to be "uniform acceleration - uniform velocity - uniform deceleration".

3. The requirement of two station structure at each end is neglected in the model.

4. The tracks are reduced to one in model 1 & 2 and two in model 3&4

3 Model 1: Static Forces Model of the System

3.1 Model Overview

In order to estimate the minimum amount of material we need for the cable and the rods, we firstly need to find out the loading conditions and loading limitations of them. When the speed of the trailer and the environmental factors are not considered, the system is in a static state. The processes to analysis and calculate the axial forces on the rods(f_i), the shear forces on the beam(f_b) and the tension on the cable($f_{i,1}$ or $f_{i,2}$) are divided into two main steps:

1. Calculate the forces on pull rods based on the equilibrium condition and the distribution of forces on the beam. (We denote the force on the ith rod as f_i)

2. Based on the force f_i on each rod, the tension on the cable can be calculated and we denote them as $f_{i,1}$ and $f_{i,2}$

Then, we mainly change two parameters which are function coefficientλ and rod number n to calculate the result of the three kinds of forces and the corresponding cost for the whole system.

Finally, a weight ω is introduced to measure the importance of safety comparing with the importance of cost. We use the weight to generate a score for different structures and the best one will be find.

To make it clearer, Table 2 lists out all notations used in Model 1.

Table 1: Notations

Symbol	Definition
f_i	Force on ith rod
$f_{i,1}, f_{i,2}$	Tention on the cable
F	Load force
n	rod number
$d(x)$	Rod position function
$D(x)$	Force distribution function
E	Modulus of Elasticity of the Beam (Steel Alloy: A-36)
I_B	Second Moment of Inertia for the cross section of the beam
h_0	The height difference between the beam and the top
λ	Quadratic equation constant
P_n	Cost for applying n rods
m_i	The ith rod mass

3.2 Axial Force on Each Pull Rod

As shown in the following picture, in Model 1, the truss won't be considered and we just analyze one side of the transpotation system to simplify the model. For the forces are not uniformly distributed on these rods, we will get a family of forces which are f_i where $0 < i < n$.

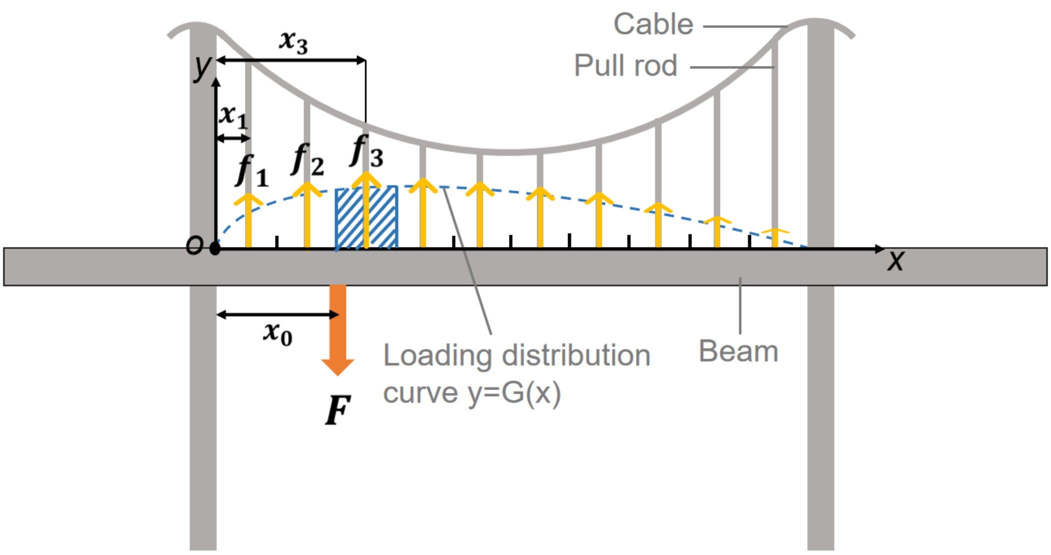

Figure 1: Equivalent axial forces on pull rods.

When there are no weight pass by, the task of the rod is supporting the beam, in this situation the forces are equaly distributed and we denote it as f_{ini} and we can get the following equations.

$$M_B g = n f_{ini} \tag{1}$$

where M_B is the mass of the beam.

When we apply a force F on the beam which is x_0 meter to the left, the situation becomes different, the corresponding force on the rods which is $f_{F,i}$ are not equally distributed and we have:

$$f_i = f_{ini} + f_{F,i} \tag{2}$$

Suppose that the force distribution function for the beam is $D(x)$, $f_{F,i}$ can be calculated as

$$f_{F,i} = \frac{\int_{d(i-1)}^{d(i)} D(x)dx}{\int_0^L D(x)dx} F \tag{3}$$

$$d(x) = \frac{x}{n} L \tag{4}$$

Where the function $d(x)$ is the position function for the ith middle point of the two rods. So, use Equation(1) (2) and (3), we can get the axial force on each pull rod. The only task for us is to find the function $D(x)$.

3.3 Find the Distribution Function

When a force is applied to the beam, a small deformation occurs, we use Solidworks to construct a beam and use FEA (Finite Element Analysis) to simulate this kind of deformation.

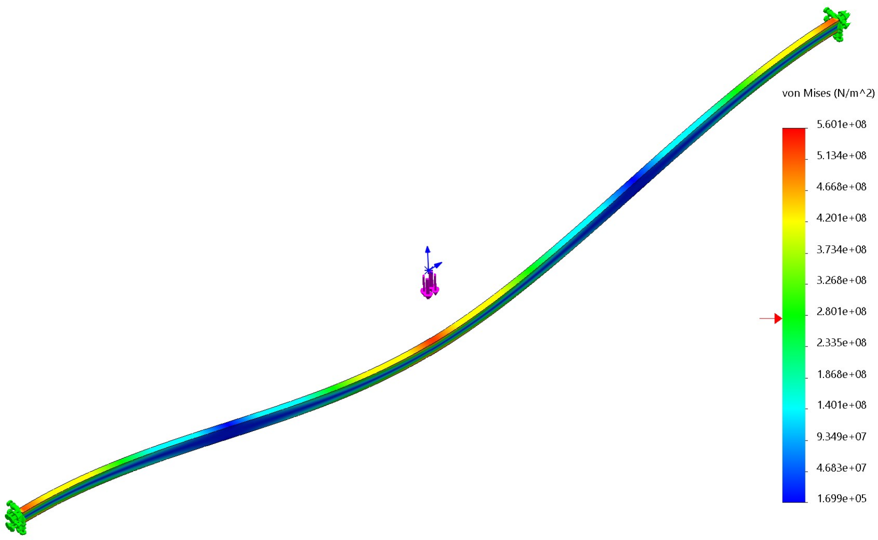

Figure 2: Deformation Simulation.

We can find that the simulation result matches the theoretical equation of the deflection curve of beams very well which is

$$V(x) = \begin{cases} -\dfrac{Fx}{6EI_BL}(L-x_0)[L^2-(L-x_0)^2-x^2] & x \le x_0 \\ L+\dfrac{Fx}{6EI_BL}(L-x_0)[L^2-(L-x_0)^2-x^2] & x > x_0 \end{cases} \quad [1]$$

Where I_B is the second moment of inertia for the cross section of the beam, E is the modulus of Elasticity of the beam and x_0 is the position that we apply the force on.

For $D(x)$ is directly proportional to $V(x)$, we have

$$D(x) = k_dV(x) \tag{5}$$

So that Equation(3) becomes

$$f_{F,i} = \frac{k_d \int_{d(i-1)}^{d(i)} V(x)dx}{k_d \int_0^L V(x)dx}F \tag{6}$$

$$f_{F,i} = \frac{\int_{d(i-1)}^{d(i)} V(x)dx}{\int_0^L V(x)dx}F \tag{7}$$

3.4 Tensions on the Cable

As shown in the following picture, we will then calculate the tensions on the cable.

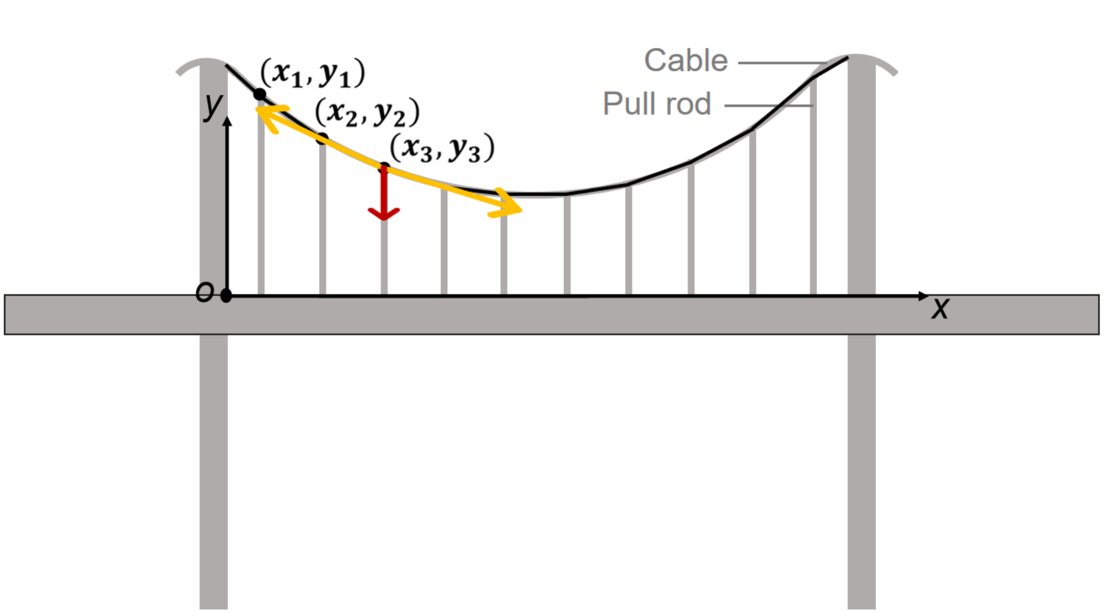

Figure 3: Tensions on the Cable.

Previously, we have calculated the force on the ith rod which is f_i, here we should do a little update on the value of f_i to further consider the mass of the rod which is m_i.

$$f_{i'} = f_i + m_ig \tag{8}$$

Then, the position function of the rod and the quadratic function of the cable will give us three point and we can use the three point and the force on the cable to calculate the tentions on the cable which are $f_{i,1}$ and $f_{i,2}$.

The position function of the rod and the quadratic function are given by

$$x_i = p(n,i) = \frac{2i-n-1}{2n}L \tag{9}$$

$$y_i = \lambda x_i^2 \tag{10}$$

So we can calculate the two tensions by

$$(f_{i,1}, f_{i,2}) = T[f_{i'}, (x1, y1), (x2, y2), (x3, y3)] \tag{11}$$

The tension function can be written as

$$T[.] = \begin{cases} f_{i,1} = \dfrac{\cos\beta}{\cos\alpha} f_{i,2} \\ f_{i'} = sgn(y_0 - y_1)\sin\alpha f_{i,1} + sgn(y_2 - y_1)\sin\beta f_2 \\ \alpha = \arctan\dfrac{|y_0 - y_1|}{|x_0 - x_1|} \\ \beta = \arctan\dfrac{|y_2 - y_1|}{|x_2 - x_1|} \end{cases}$$

3.5 Results

We will use Matlab and apply the following algorithm to find out the best structure.

Algorithm 1: Optimal Architecture

Input: λ_0, λ_{max}, ω, $x_{0,ini}$, δ, n_{max},
Result: ΣP, $f_{i,1}$, $f_{i,2}$, $score$
while *there are other combinations of* (λ, n, x_0) **do**
  apply the distribution of the force $D(x)$
  get the position of the rod $p(i, n, L, \lambda)$
  get the force on each rod $f_i(i, n, F, X, x0)$
  calculate tention $[f_{i,1}, f_{i,2}] = T_i([x0, y0], [x1, y1], [x2, y2], f_i)$
  next (λ, n, x_0)
end
Data normalization for $(f_{i,1}, f_{i,2}, (\Sigma P)_i)$
Find the informationentropy and use it to calculate the weight of each variable.
Use TOPSIS to Enlarge the gap and calculate the optimal solution.

The following two pictures show the two curve family. The first one is the relationship between λ and tension $f_{t,n}$ and the second one is about the relationship between $ambda$ and cost P_n.

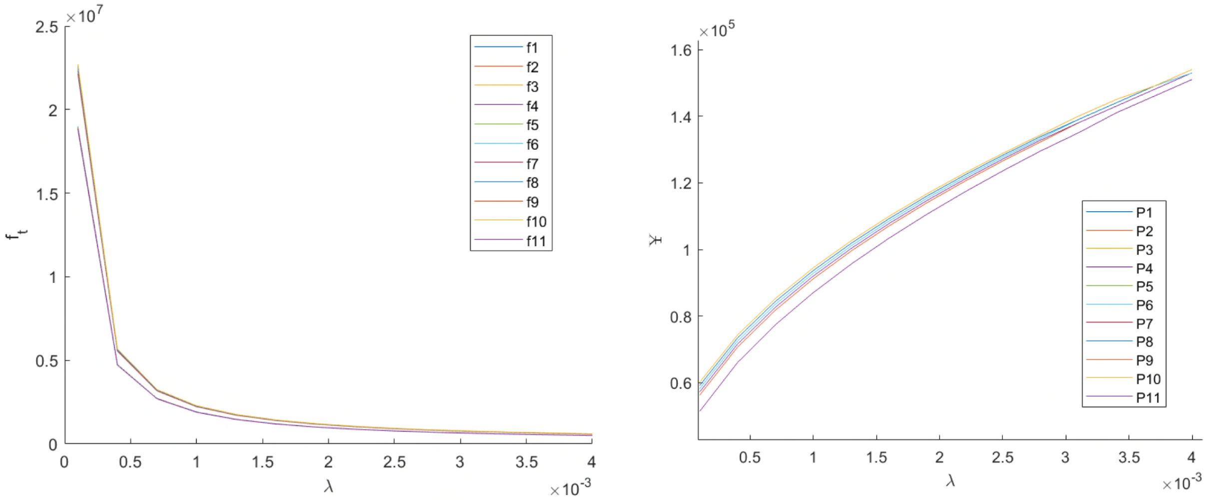

Figure 4: Two curve families

And we can apply MADM(Multiple Attribute Decision Making), during this process, we use informationentropy to estimate the weight objectively and then use TOPSIS to calculate the optimal solution. The following are the Feature scaling formula and TOPSIS estimation criteria for data

$$x'_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{12}$$

$$score = \frac{l_i^-}{l_i^- + l_i^+} \tag{13}$$

And the result is shown on the following picture.

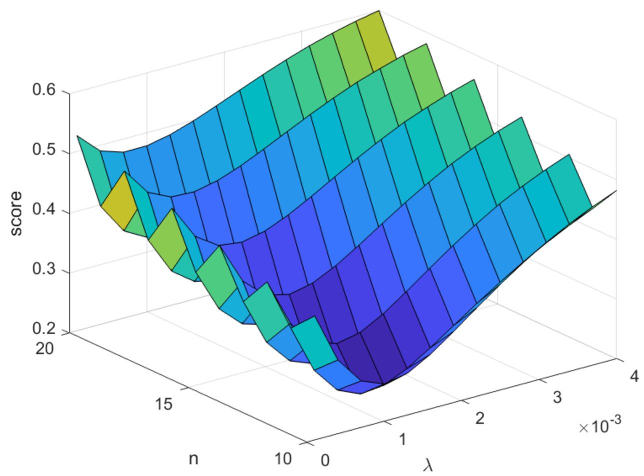

Figure 5: MAMD results for Model1.

Consider the limitation of the cable and rod, we can find that the maximal force they can bear separately is 706850N ($r_c = 0.03m$) and 78539N ($r_r = 0.01m$). The optimal structure is $n = 10$, $\lambda = 0.28$ and the cost is 20053 dollars.

4 Model 2: Forces Model of the System in a Dynamic Process

4.1 Total Forces on the Beam

When a trailer is moving on the track, the total force F' acting on the beam is greater than that in the static state. Although the motion of the trailer is in horizontal direction, the trailer will still exert extra vertical force onto the beam, counted in F'. The extra part of force is due to the following reason: As shown in Fig. 6, when the trailer is moving on the track, a large downward force is exerted on the point under the wheel, which is point A of the beam. At point B (near point A), there is no downward force at the moment. In a very short period of time, the position of the wheel will change to point B, and point A and B will exchange their force condition. This process continues as the trailer moving along the beam. As a result, vibration is caused in the beam, and the total forces bearing by the rods increase.

Equation (14) is the expression of the total external force F'. F' is proportional to the static force, namely the weight of the trailer M. And it is determined by a groups of coefficient μ, u, and D, which are related to the feature of the track as well as the speed and acceleration of the trailer. In Equation (14), μ is called the impact coefficient, which has the expression of Equation (15). This expression is derived from data in bridge construction practice, according to literature [4]. L is the span of one section of track, which indicates that the grater the span of one section is, the larger the impact force will be.

$$F' = (1+\mu)(1+u)DM \tag{14}$$

$$\mu = \frac{15}{37.5+L} \quad [4] \tag{15}$$

$$D = 1 + \frac{\pi}{fL}v \quad [5] \tag{16}$$

The parameter D in Equation (14) is called the dynamic coefficient. According to previous researches, when the car velocity is relatively slow, which means under 60 km/h, the relationship between dynamic coefficient and velocity can be approximated to be linear [5].

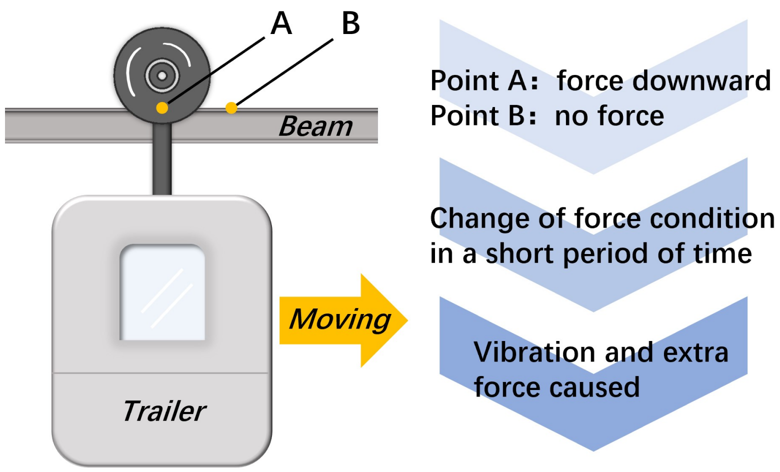

Figure 6: Reason for the extra force.

As shown in Equation (16), this coefficient is determined by L, the fundamental frequency f of the system, and the velocity of the trailer v [5]. The faster the trailer moves, the larger the force is. Equation (17) shows the expression of f. In the equation, I is the second moment of area of the beam cross section, calculated as in Equation (18). H is the axial force of the cable. m_b, m_c, and m_r are masses per meter of the beam, cable, and rod, respectively. After plugging the data of the most effective case obtained in Model 1, the fundamental frequency f turns out to be 0.31Hz.

$$f = \frac{1}{L}\sqrt{\frac{EI(2\pi/L)^2 + H}{m_b + m_c + nm_r/L}} \quad [3] \tag{17}$$

$$I = \frac{1}{12} \times (0.5m)^4 - \frac{1}{12} \times (0.4m)^4 = 0.0031m^4 \tag{18}$$

Another factor influencing F' is the acceleration (or deceleration) of the trailer. When the trailer is accelerating, the force condition of different points on the beam will change faster. Therefore, we add the coefficient $1+\mu$ to describe F', where $u = 0.001a$, and a is the absolute value of the acceleration/ deceleration.

4.2 Forces on Rods and Cable and the Speed limit of the Trailer

With F', we are able to recalculate the axial forces in the rods and cable. Inversely, with the parameters of rods and cable fixed according to Section 1, we can calculate the maximum F', so that the maximum allowed speed of the trailer is obtained. The motion of the trailer is simplified as "uniform acceleration-uniform velocity-uniform deceleration". We assume that when the maximum speed is reached, the acceleration of the trailer must be 0, and the deceleration of the trailer should not exceed a certain value. According to literature, the suitable acceleration/ deceleration of vehicles (under which people do not feel uncomfortable) is between $0.11g$ and $0.15g$ ($1.08m/s^2$ to $1.47m/s^2$) [10]. Hence, the value of a should not exceed $1.47m/s^2$. Then the expression of v_{max} is:

$$v_{max} = [\frac{F'}{(1+\mu)(1+u)M} - 1]\frac{fL}{\pi}$$

After calculating the result using Matlab, we find out that the maximum speed of the trailer is 33.6km/h. At this speed, the cable nearly reach its elastic limit, and the total force exerted on the beam is 21597N. 33.6km/h is a reasonable speed limit in comparison with 60km/h (the maximum speed can be reached by the trailer).

4.3 Controlling the Maximum Speed: Re-analysis of the Cost of Suspension System

In practice, if the trailer is required to reach a higher speed, such as 60km/h, then we need to re-analyze how to construct the suspension system and its cost. To allow the system to bear a greater F', we can adjust the parameter of the cable, as well as the number of rods. By operating the program, we find out that the axial force on the cable becomes 716491N, if the parameters of the rods are not changed. Then there are two ways to re-design the system:

1. If the number of rods is kept unchanged, the radius of the cable should become 0.0302m, and the new design costs 20320 dollars.
2. If the radius of the cable is kept unchanged, the number of rods should become 17, and the new design costs 20720 dollars.

Therefore, the first plan is more economical.

5 Model 3: The Vibration of the Trailer Under Wind Effect

5.1 Modeling External Wind Load

The environmental factor that is taken into consideration refers to the force caused by wind resistance. To simplify the question, assume that the wind speed remains unchanged and force exerted by the wind to the trailer remains unchanged. The wind force, we suppose, is related to factors including air density, windward area as well as the wind speed. After referring to the internet, we obtained the formula for wind resistance that is accord with our assumption:

$$F = \frac{1}{2} C \rho S v^2 \qquad [2] \tag{19}$$

where C is for the coefficient of wind resistance, ρ for air density, S for the windward area and v for the relative velocity of the wind. For coefficient C, it is simply one for regular shapes such as cuboids [8]. But it can be decreased by improving the shape of the trailer. For example, the coefficient C is reduced to 0.5 for spheres, and is only 0.28 to 0.4 for normal cars [8]. Figure 16 provides a possibility that can be analyzed.

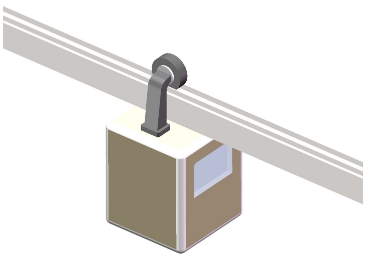

Figure 7: C=1

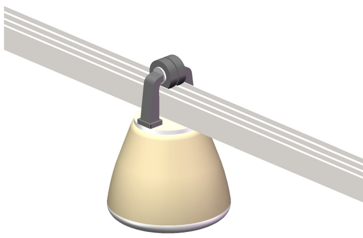

Figure 8: C<0.5

However, since we've already consider the additional impact of the moving trailer's velocity, the wind speed here should also be modified. To be more accurate, the velocity should be interpreted as "the relative velocity" of the wind to the car, which means taking it as a vector and consider it as the addition of both car velocity vector and wind velocity vector. This allow us to follow the discussion about the trailer's speed in task 2.

$$\overrightarrow{v}_{relative} = \overrightarrow{v}_{trailer} + \overrightarrow{v}_{wind}$$

Thus, among all parameters we need for calculating wind force, both S and $v_{relative}$ are related with the direction of the wind. Assume the direction of the velocity of the trailer is unchanged, one more variable α should be inserted which refers to the angle between wind direction and trailer's moving direction, using counterclockwise. Let $\alpha \in [0, \pi]$. Two related formulas are rewritten as follows:

$$v_{relative} = \sqrt{v_{wind}^2 + v_{car}^2 - 2 v_{wind} v_{car} cos(\alpha)} \tag{20}$$

$$S = \frac{A_{trailer} + A_{hangingbeam}}{max\{cos(\alpha), |sin(\alpha)|\}} \tag{21}$$

In order to ensure that the structure is completely safe, we need to select the condition under which the effect of wind is maximized regarding the moving trailer. Since the relation between α and wind force follows a rough trend that won't be affected by changing other parameters, it's reasonable to first choose a set of parameters and find out the angle at which the wind force is maximized to simplify following calculations. Let the cross section of the hanging beam be 0.3 $\times$ 0.3 = 0.09 m^2, and the length be 0.5 m. Choose the material to be cast iron alloy with E=6.7 $\times 10^7$. C =1 for cuboid shapes and ρ = 1.293 for dry air.Inserting all the exact numbers, the result of wind load related to its direction against car velocity can be obtained as the polar diagram below. It's clear that the effect of wind under same wind speed is maximized when the angle between trailer velocity and wind velocity is $\alpha = 136^o$. Thus, we keep $\alpha = 136^o$ unchanged from now on.

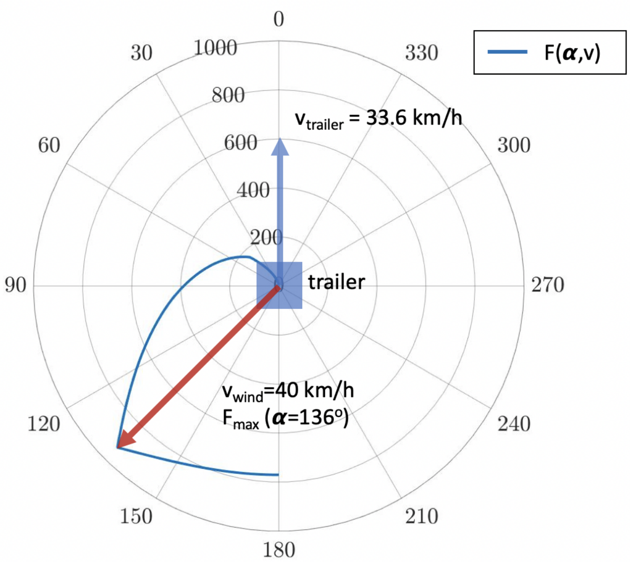

Figure 9: Maximum effect of the wind

5.2 Obtain parameters for vibration equation

The equations of motion of such kind of vehicle–bridge system can be derived from equilibrium considerations applied to the free-body diagrams of the vehicle components and their interaction forces with the bridge element. According to the vibration formula:

$$MX^{''} + CX^{'} + KX = F_{wind} \quad [6] \tag{22}$$

In order to describe the vibration of trailer, there're still some basic physical parameters need to be determined: the mass of the trailer, the air resistance, the stiffness of the hanging beam as well as the wind resistance force affect the vibration condition of the trailer. Among the three parameters on the left side, the stiffness K can be calculated using the formula of "elastic deformation of an axial loaded member" [7]:

$$\delta = \int_0^L \frac{P(x)dx}{A(x)E(x)} \quad = \frac{PL}{AE} \quad \text{for Constant Load and Cross-Sectional Area} \quad [7] \tag{23}$$

where δ = displacement of one point on the bar relative to the other point, L = original length of bar, P = internal axial force at the section,A = cross-sectional area of the bar and

E = modulus of elasticity for the material. Refer to the basic Hook's Law F=Kx, we can consider

$$K = \frac{AE}{L} \tag{24}$$

Considering that the trailer ask for a relatively easier movement, the hanging beam shouldn't be designed to be too heavy or stiff. Also, in order to make the modeling closer to reality, the mass of the hanging beam should also be added into the total mass. To make the calculation easier, we choose cast iron alloy as the material, but change its basic size by altering the ratio of cross area A divided by length L.

Then to make a summary, table 2 lists out all the exact numbers we choose to determine the parameters in Equ. (22), including those involved in the wind force.

Table 2: Parameters for Equ. (22)

Parameter	Physical Meaning	Numerical Value
M	The weight of the trailer	9.8 $\times 10^3$N provided by the question
C	Coefficient of air resistance	$\begin{cases} 1 & \text{for cuboid} \\ 0.5 & \text{for cuboid} \end{cases}$ [8]
A	Cross section area of the hanging beam	0.3$\times$0.3 m^2
E	Modulus of Elasticity of the hanging beam	6.7 $\times 10^7$ (for Cast Iron Alloy)
ρ	Air density	1.293 g/l for dry air
v_{wind}	Wind speed	40 km/h for level 6 [9]

5.3 Vibration amplitude & Improvement

After writing out the vibration equation, it's then easy to solve for the vibration amplitude. Since under such condition the hanging beam can be analogous to an extremely stiff spring, the length of the beam may have an close effect to the final amplitude. Thus, the length is changed form 0.1 to 1 m, which also reflects the ratio between cross area and length that is mentioned in previous section.

After calculation, the amplitude shows a linearly increasing relationship with the length with the maximum result reaching 0.3 mm. Thus, an improved plan is put forward, that is, decreasing the number of coefficient C. According to literature review, cars can be designed into a much better shape to reach quite small C in order to reduce vibration caused wind effect. Thus, referring again to Figure 16, we suggest changing the shape of the car to obtain a smaller wind coefficient C. Thus, the same procedure is repeated with C=0.5, and an obvious decrease in maximum vibration amplitude can be observed. The two lines shown in Figure 10 reflects the amplitude change and we've seen that the decrease of amplitude almost changed the same with the change in coefficient C.

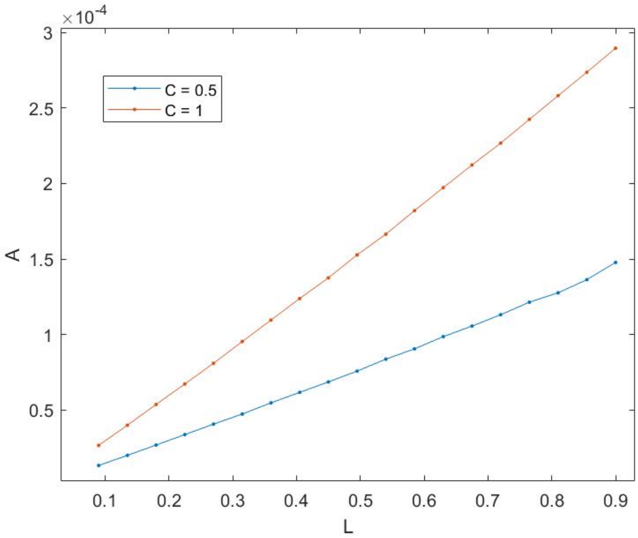

Figure 10: Vibration Amplitude

C	A
1	0.29 mm
0.5	0.14 mm

Table 3: Amplitude under different C

Thus the final result and improvement is shown in Table 3 by optimizing the shape of the trailer.

6 Model 4: The rescue scheme

6.1 Defining Rescue Scheme & Rescue Effectiveness

Rescue work on the suspended traffic system is much harder than the work on the ground since the tracks are limited. Thus, in order to come up with a best rescue system, the effectiveness can be measured by the time needed for rescuing, the number and position of rescue trailers it requires, as well as the number of tracks it may impact.

6.2 Completing the structure by adding truss

To simplify the modeling, assume that the rescue team consists of trailers the same with normal cars. Although it may need extra equipment, we assume that the additional mass can be neglected. The advantage of having truss is listed as follows:

1. Provide area for rescue trailers to rest between two terminal station so that it can get into work quickly when accidents happens.
2. Provide possible shortcut for rescue trailers to reach the broken car when there's other normal trailers in front of or behind it.
3. Eliminate the time for pulling the broken trailer away and let the track operate as normal since as long as the broken trailer is pushed onto the truss structure, the original track is cleared up for other trailer to pass by.
4. Allow one rescue trailer to reach two adjacent tracks, thus the shortest route can also be reduced from all the possible choices

Before designing the exact truss structure, it's still important to make sure that the above structure is still within safety concerns after this tiny change. While the rods are now inclined and the cable needs to support two adjacent tracks, the safety can still be ensured simply because: firstly, the force in the rods is increased by csc($\alpha_{inclined}$) while their length is also increased by csc($\alpha_{inclined}$), thus making sure that the maximum bearing force is increased by csc($\alpha_{inclined}$); secondly, using two parallel cables as shown in the original design provided by the question can disperse the load on the cable, at the same time not increasing the cost.

Then, for the design of the truss, according to the original design, it is constructed into triangular shapes which is the most stable structure. The only two parameters that needs to be decided is the distance b between two adjacent tracks and the distance c between two parallel truss beams. Considering the width of a single trailer which is 1.5m, and the extreme possibility is that there maybe two trailers on both adjacent tracks and one rescue trailer in the middle. Thus it's reasonable to set b=5m, leaving enough space for possible condition. Since we consider 100m as the total length of one section of track, the number of c is quite limited since it needs to be a factor of 100. While keeping c too small may cause the trailer to move extra distance to reach middle point of the bridge, and keeping it too big may cause the truss to have fewer advantage of reducing the rescue distance, the reasonable number choice is finally reduces to c=10,20,25,and 50 m.

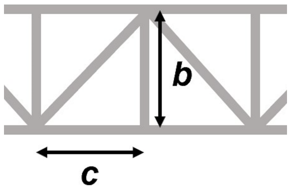

Figure 11: Truss structure

6.3 Calculation Results for the Best Route

Considering the maximum speed of the trailer is 33.6 km/h, we can calculation a safety distance between two trailers on the track: 55 meters. Thus, when accidents occur, it's possible that there maybe other normal moving trailers in front of and behind the stuck one. According to Fig. 12, we need to choose a best route which can take advantage of the truss structure design.

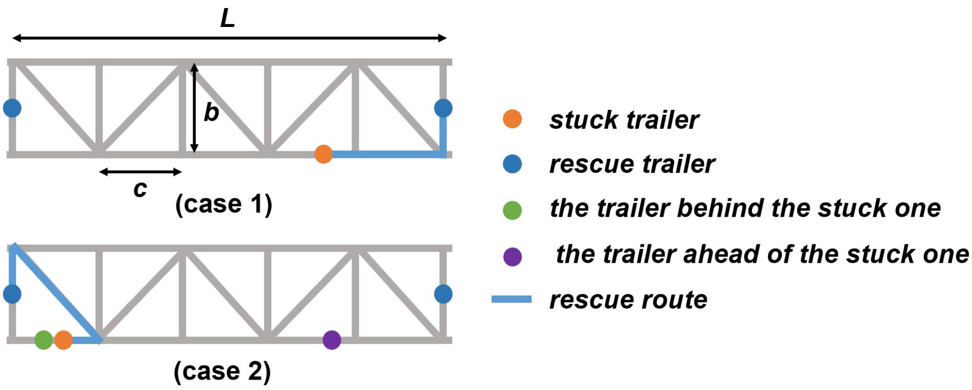

Figure 12: Truss structure

We denote the distance of the stuck trailer to the left end of the track as x_0. For $c = 50m$, 25m, 20m, and 10m, the shortest rescue time t are all piecewise functions of x_0, denoted as $t_{50}(x_0)$, $t_{25}(x_0)$, $t_{20}(x_0)$, and $t_{10}(x_0)$ (unit: s). The results are as follows (Note: "left" means that the rescue trailer on the left can reach the stuck trailer more quickly, and the best route is from the left. The meaning of "right" is similar.):

$$t_{50}(x_0) = \begin{cases} 38.79 - 0.36x_0 x \le 27.36 \quad (left) \\ 41.72 - 0.47x_0 27.36 < x \le 45 \quad (right) \\ 36.9 - 0.36x_0 45 < x_0 \le 100 \quad (right) \end{cases} \quad t_{25}(x_0) = \begin{cases} 19.98 - 0.36x_0 x \le 25 \quad (left) \\ 41.72 - 0.47x_0 25 < x \le 45 \quad (right) \\ 36.9 - 0.36x_0 45 < x_0 \le 100 \quad (right) \end{cases}$$

$$t_{20}(x_0) = \begin{cases} 15.52 - 0.36x_0 x \le 20 \quad (left) \\ 31.94 - 0.36x_0 20 < x_0 \le 40 (left) \\ 41.72 - 0.47x_0 40 < x \le 45 \quad (right) \\ 36.9 - 0.36x_0 45 < x_0 \le 100 \quad (right) \end{cases} \quad t_{10}(x_0) = \begin{cases} 8.52 - 0.36x_0 x \le 10 \quad (left) \\ 17.95 - 0.36x_0 10 < x_0 \le 20 \quad (left) \\ 22.87 - 0.36x_0 20 < x_0 \le 30 \quad (left) \\ 33. - 0.36x_0 30 < x_0 \le 40 \quad (left) \\ 41.72 - 0.47x_0 40 < x \le 45 \quad (right) \\ 36.9 - 0.36x_0 45 < x_0 \le 100 \quad (right) \end{cases}$$

According to the expression of rescue time, we can plot out the graphs of the functions, as shown in Fig. 16. With the functions, we then calculate the maximum and average rescue time, as well as the total length of trusses for each value of c. The results are listed in Table 4.

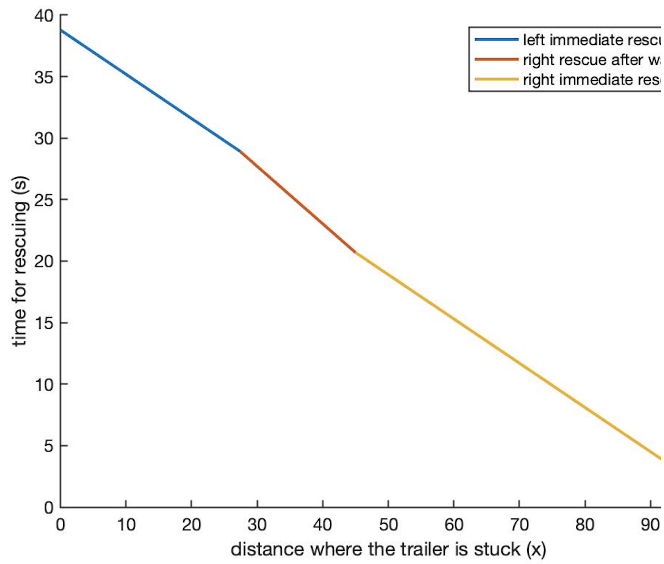

Figure 13: c=50

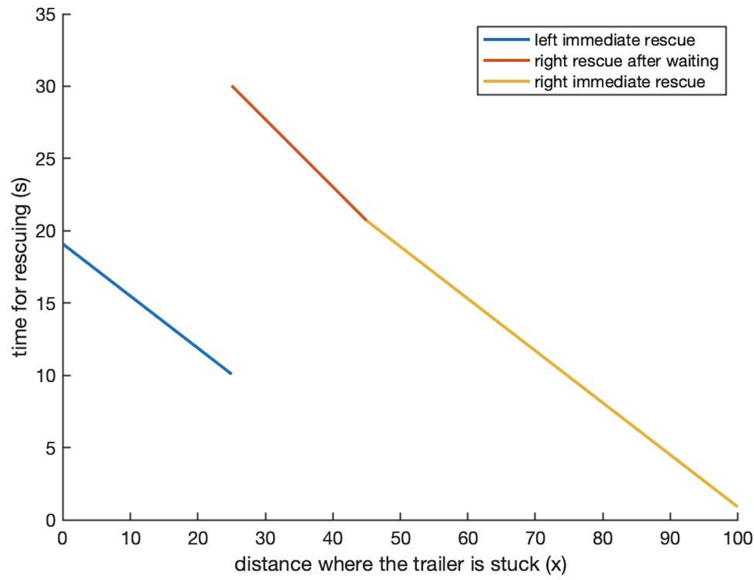

Figure 14: c=25

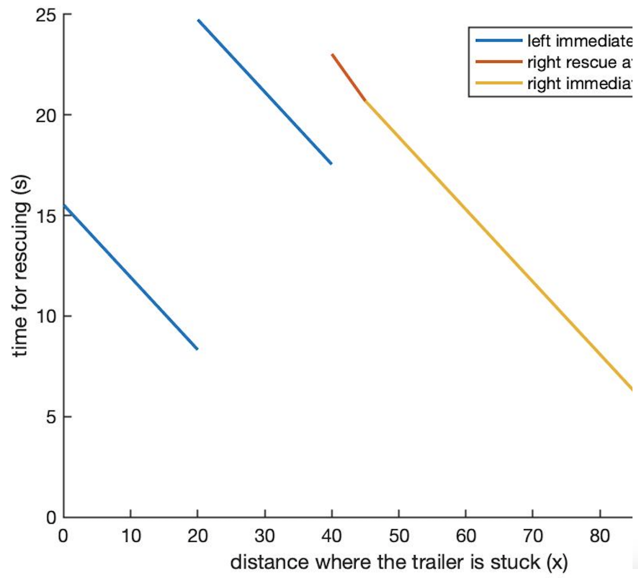

Figure 15: c=20

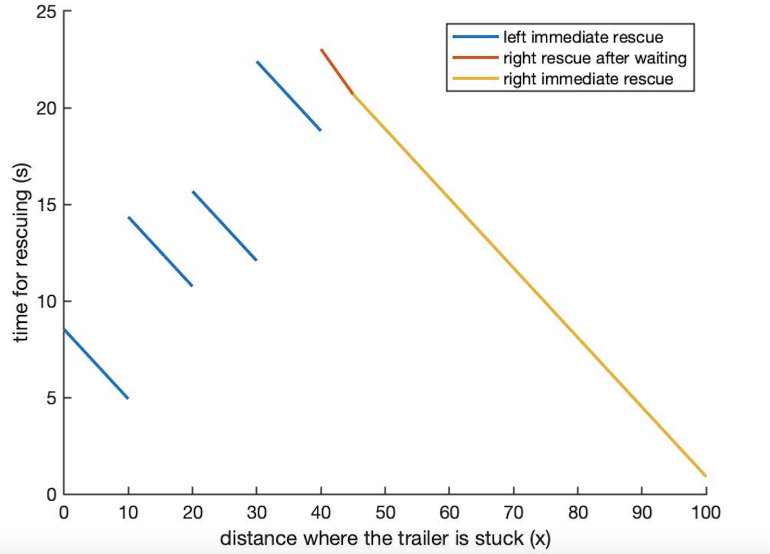

Figure 16: C=10

Figure 17: Rescue time with respect to position of the stuck trailer.

Table 4: Calculation results for the choice of c

	$c = 50m$	$c = 25m$	$c = 20m$	$c = 10m$
maximum time needed (s)	38.79 ($x_0 = 0$)	30.04 ($x_0 = 25$)	24.74 ($x_0 = 20$)	22.22 ($x_0 = 30$)
average time needed (s)	19.58	14.88	13.65	12.41
total length of truss (m)	115.50	126.98	133.08	166.80

From Table 4 we can see, for smaller c, both maximum and average rescue time is smaller, which means the rescue efficiency is higher. However, the total length of truss is increasing as c decreasing (especially when $c = 10m$), which means it will cost people more money. Therefore, in order to get an efficient and economical rescue scheme, we take $c = 20m$ as the best choice.

7 Sensitivity Analysis

7.1 Changing Values of L and h_0

In the previous sections, to reduce the complexity of calculations, we assumed that the span L of the track is $100m$, and the height h_0 from the beam to the top of the system is $10m$. However, the value of L can be taken from $50m$ to $100m$, and h_0 is set by ourselves. Therefore, we need to take different values of L and h_0 to see whether the results are still reasonable.

We will first apply a $\pm5\%$ change for L and h_0, and check their influence on the maximum tension of the cable $max\{\frac{f_{i,1}+f_{i,2}}{2}\}$, maximum force on the rod $max\{f_i\}$ and the maximum speed v_{max}.

Table 5: Sensitivity Analysis for L

L \ result	$max\{\frac{f_{i,1}+f_{i,2}}{2}\}$	$max\{f_i\}$	v_{max}
-5%	3.284%	-0.266%	3.281%
-4%	2.613%	-0.214%	2.610%
-3%	1.949%	-0.161%	1.947%
-2%	1.291%	-0.108%	1.291%
-1%	0.642%	-0.054%	0.642%
+1%	-0.635%	0.054%	-0.635%
+2%	-1.264%	0.107%	-1.263%
+3%	-1.885%	0.160%	-1.884%
+4%	-2.501%	0.211%	-2.499%
+5%	-3.109%	0.262%	-3.107%

Table 6: Sensitivity Analysis for h_0

h_0 \ result	$max\{\frac{f_{i,1}+f_{i,2}}{2}\}$	$max\{f_i\}$	v_{max}
-5%	-0.0002%	-0.0213%	-0.0002%
-4%	-0.0002%	-0.0171%	-0.0002%
-3%	-0.0001%	-0.0128%	-0.0001%
-2%	-0.0001%	-0.0085%	-0.0001%
-1%	-0.0000%	-0.0043%	-0.0000%
+1%	+0.0000%	+0.0000%	-0.0000%
+2%	+0.0000%	0.0043%	+0.0000%
+3%	0.0001%	0.0086%	0.0001%
+4%	0.0001%	0.0128%	0.0001%
+5%	0.0002%	0.0171%	0.0002%

From Table.5 and Table.6 we can see that the influence of h_o on our model is negligible, and the influence of L on the model is acceptable which means that our model is stable and the process of fixing the value of h_0 and L is reasonable.

7.2 Changing the Initial Value of the Vibration Equation

The vibration equation is a partial differential equation, and the initial values will effect the solution. In the real world, the wind keeps changing its speed and direction, leaving different initial states of the trailer's vibration in a period of time. Therefore, we change the initial values of the vibration equation to check how the solutions varies.

The cross section area of the hanging beam A, the air coefficient C, the air density ρ, the wind speed v_w and the maximum speed of the vehicle v_c all have influence on the coefficient of the vibration equation, the following table shows the influence of these variables on the maximum amplitude of the vibration.

Table 7: Sensitivity Analysis for Vabration Equation

	A	C	ρ	v_w	v_c
-5%	-0.71%	-4.97%	-5.52%	-6.32%	-5.16%
-4%	-0.89%	-4.00%	-4.00%	-4.18%	-4.00%
-3%	-0.98%	-3.02%	-3.38%	-3.47%	-2.58%
-2%	-0.18%	-2.04%	-1.87%	-2.22%	-1.78%
-1%	-0.71%	-1.07%	-1.25%	-1.16%	-1.16%
1%	0.09%	0.80%	1.07%	1.25%	0.98%
2%	0.09%	1.33%	2.05%	2.22%	1.16%
3%	0.11%	2.22%	2.76%	2.14%	2.58%
4%	0.27%	4.00%	4.00%	6.41%	3.20%
5%	0.36%	4.62%	5.16%	5.34%	4.00%

From Table.7 we can see that the influence of these five variables on the amplitude of the vibration in Model 3 is acceptable which peoves the reliable of our model.

7.3 Changing the value of b

In Model 4: The rescue scheme, we fix the value of the distance between two beams to $b = 5m$ and analyze the shortest rescue route and its corresponding time. However, the constrain of the value of b is $b \geq 4.5m$ so that we can apply a $\pm 5\%$ change of b to see whether it will influence the result a lot or not.

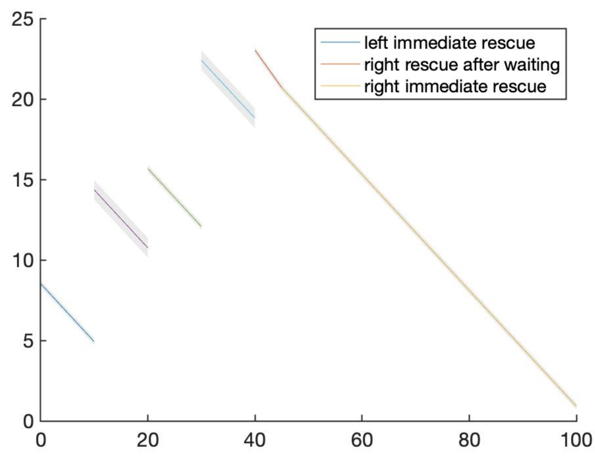

Figure 18: Sensitivity Analysis of b

The shadow on Figure.18 around the lines shows the influence of b and we can conclude that the influence of b is small which means that our results are credible.

8 Conclusion

8.1 Static Force Model and Results for Task 1

To analyze the limit status of the cable and to get the optimal cost, we build Model 1 for static state of the system. Following representations and approximations are made for the model:

1. We build axes along the track and get the distribution formula $V(x)$ of force F along the x-axis.
2. Axial forces of each rods are calculated as the integral of $V(x)$ around it.
3. The shape of the cable is considered as a parabola. To get the tension inside it, the cable is simplified as a series of straight lines.

By running the program with a groups of changeable parameters, and applying MADM, we finally get the optimal result:

Maximum force the cable can bear	706850N (r_c=0.03m)
Maximum force a rod can bear	78539N (r_r=0.01m)
Number of rods n	10
λ	0.28
Total length of the cable	291.63m
Optimal cost	20053$ per 100m

8.2 Speed Limit of the Trailer

Due to the speed of the trailer, there will be an extra impact force on the beam, which increases the total force F' the system needs to bear. Based on literature, we derived the formula of F', which is related to parameter of the suspension system, speed of the trailer, as well as the acceleration. According to the results of task1, we do calculations in two cases:

1. The parameter of the suspension system is kept as the optimal value. The maximum speed when the tension in the cable reaches the elastic limit is 33.6km/h.
2. The maximum speed is set as 60km/h. And the parameter of the suspension system is re-analyzed. We should either change the radius of the cable to 0.0302m, or increase the number of rods to 17. The former choice is more economical, with the cost of 20320 dollars.

8.3 Vibration Analysis Results

We quantified the wind force upon the trailer as a function of the air density, windward area, and the relative speed. By solving the second ordered ordinary differential equation of the vibration, we find out that: the amplitude of vibration is nearly proportional to the length of the hanging arm, for both $C = 1$ and $C = 0.5$.

In comparison with the cubic shaped trailer hanging on the track with one arm, the one with a smoother shape and hanging with two symmetrical arms is more stable. Hence, adjusting the shape of the trailer (namely changing C) and the way of hanging are two methods to reduce the vibration. It turns out that the maximum amplitude for $C = 0.5$ is 0.14mm, which is only a half of the amplitude 0.29mm when $C = 1$.

8.4 Rescue Scheme and Discussion of the Effectiveness

For the rescue scheme, we make the trailers able to move on the truss, and take other trailers on the stuck track into consideration. Under the assumption we made, we find out that the results of rescue time are piecewise functions of stuck position x_0. Comparing the maximum and average rescue time, as well as the total length of truss, the best choice of c turns out to be 20m. In this case, the maximum rescue time is 24.74s, and the average rescue time is 13.65s, which are very short. Hence, we are convinced that the rescue scheme is very efficient.

9 Strength and Weakness

9.1 Strength

- When analyzing the optimal cost of the material for task 1, we applied Multiple Attribute Decision Making, so that all the changeable parameters are taken into consideration.
- During the modeling and calculation process, the parameters of the rods, cable and beam has experienced several modifications to make the result closer to reality. Although the choice of exact numbers are relatively less important, there's still consideration behind.
- Based on solid knowledge foundation of Solid Mechanics, the evaluation of deformation and the analysis of the materials and structures are relatively accurate.

9.2 Possible Improvements

- In task 1, since there are different parameters taking lots of different values, it takes a long time to run the program in Matlab. If we have more time, we may improve the model or the algorism, so that the calculations can take less time in practice.
- In task 3, the final result shows a linearly increasing trend between length and amplitude. However in reality, there must be other situations or restrictions in actual construction for engineers to make the best decision. Thus, the model can be improved by considering fields such as cost or possible material deformation.

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Appendices

Appendix A Codes

A.1 Codes for Calculating the Forces in Model 1

```
clear, clc
h0 = 10; X = 100; lambda = 0.0001; %function constant %%%%%
maxlambda = 0.0013; intevallambda = 0.0003; omega = 0.8; F = 1000 *9.8; n = 10; x0ini = 0;
rodMax = 20;  prod = 788.768/16; pbeam = 0; pcable = 443.682; RESULT = [1,2,3,4,5,6,7,8,9]
while lambda <= maxlambda
    disp(lambda); n = 10;
    while n <= rodMax
        disp(n); %test use
        x0 = x0ini;
        while x0 <= 50
            for i = 1:2:n
                if i == 1
                    if n == 1
                        [a1,a2] = calPosition(i, n, X, lambda);
                        [f1,f2,fr] = tension(0, a1, 100, X^2*lambda/4 ,a2, X^2*lambda/4, r
                    else
                        [a1,a2] = calPosition(i, n, X, lambda);
                        [b1,b2] = calPosition(i+1, n, X, lambda);
                        [f1,f2,fr] = tension(0, a1, b1, X^2*lambda/4 ,a2, b2, rodForce(i,
                    end
                elseif i == n
                    if n == 1
                        [a1,a2] = calPosition(i, n, X, lambda);
                        [f1,f2,fr] = tension(0, a1, 100, X^2*lambda/4 ,a2, X^2*lambda/4, r
                    else
                        [a1,a2] = calPosition(i-1, n, X, lambda);
                        [b1,b2] = calPosition(i, n, X, lambda);
                        [f1,f2,fr] = tension(a1, b1, 100, a2 ,b2, X^2*lambda/4, rodForce(i
                    end
                else
                    [a1,a2] = calPosition(i-1, n, X, lambda);
```

```
                    [b1,b2] = calPosition(i, n, X, lambda);
                    [c1,c2] = calPosition(i+1, n, X, lambda);
                    [f1,f2,fr] = tension(a1, b1, c1, a2 ,b2, c2, rodForce(i, n, F, X, x0),
                end
                syms d;
                q(d) = sqrt(1 + lambda*4*d^2); price = prod*calculateRodL(i, lambda, X, h0
                res = [lambda, i, n, x0, f1, f2, price, fr, rodForce(i, n, F, X, x0)]; RES
            end
            x0 = x0 + inteval;
        end
        n = n + 1;
    end
    lambda = lambda + intevallambda;
end
xlswrite('C:\Users\liche\Desktop\rawData1.xlsx',RESULT);
function [length] = singleRodL(i, n, lambda, X, h0)
    [p1,p2] = calPosition(i, n, X, lambda); delta = 0;
    if i==1 && i==n
        delta = 0.001/lambda;
    end
    length =  p2 + h0 - X^2/4*lambda - delta;
end
function [length] = calculateRodL(n, lambda, X, h0)
    sum = 0;
    for i = 1:1:n
        [p1,p2] = calPosition(i, n, X, lambda);
        sum = sum + p2 + h0 - X^2/4*lambda;
    end
    length = sum;
end
function [fn] = rodForce(i, n, F, X, x0)
    P = 1000 * 9.8; b = X - x0; E = 200;
    I = 1/12*1*1 - 1/12*0.8*(0.8)^3; L = X;
    syms x;
    f(x) = (P*b*x) / (6*E*I*L) * (L^2 - b^2 - x^2);
    g(x) = (P*b*(-x)) / (6*E*I*L) * (L^2 - b^2 - x^2) + L;
    if ((i-1)/n*X - x0) * (i/n*X - x0) < 0
        intS = int(f(x), x, (i-1)/n*X, x0) + int(g(x), x, x0, i/n*X);
    elseif (i-1)/n*X >= x0
        intS = int(g(x), x, (i-1)/n*X, i/n*X);
    else
        intS = int(f(x), x, (i-1)/n*X, i/n*X);
    end
    intA = int(f(x), x, 0, x0) + int(g(x), x, x0, X);
    fn = (F * intS / intA + 9.8 * 7850 * (0.5*100*0.5 - 0.4*100*0.4) / n) * 0.8;
end
function [x0, y0] = calPosition(i, n, X, lambda)
    x0 = (2*i - n - 1) / (2*n) * X + 50;
    y0 = lambda * (x0 - 50)^2;
end
function [f1, f2, fr] = tension(x0, x1, x2, y0, y1, y2, F, length) %F rodforce
    alpha = atan(abs(y0 - y1)/abs(x0 - x1));
    beta = atan(abs(y2 - y1)/abs(x2 - x1));
    eq = sign(y2 - y1) * sin(beta) + sign(y0 - y1) * sin(alpha) * cos(beta) / cos(alpha);
    fr = (F + 7850 * 3.14 * (1/100)^2 * length *9.8)/2;
    f2 = fr / eq;
    f1 = f2 * cos(beta) / cos(alpha);
end
```

A.2 Codes for Multiple Attribute Decision Making in Model 1

```
clear,clc
R = xlsread('C:\Users\liche\Desktop\(UPDATED)resultData.xlsx');
D = R(:,[3 4 5 6]);D = 1./D;[m,n]=size(D);
for i=1:n
    AA(1,i)=norm(D(:,i));
end
AA=repmat(AA,m,1); DUP=D./AA; E = [1,2,3,4];
for j = 1:4
    sum0 = 0;
    for i = 1:154
        sum0 = sum0 + DUP(i,j)*log(DUP(i,j));
    end
    k = 1/log(154); E(j) = -k*sum0;
end
F = 1 - E; omega = F./sum(F); %weight
for i = 1:154
    for j = 1:4
        DUP(i,j) = DUP(i,j)*omega(j);
    end
```

```
end
%TOPSIS
max1 = max(DUP(:,1));min1 = min(DUP(:,1));
max2 = max(DUP(:,2));min2 = min(DUP(:,2));
max3 = max(DUP(:,3));min3 = min(DUP(:,3));
max4 = max(DUP(:,4));min4 = min(DUP(:,4));
s = [0];
for i = 1:154
    smax0 = sqrt((max1 - DUP(i,1))^2+(max2 - DUP(i,2))^2+(max3 - DUP(i,3))^2+(max4 - DUP(i
    smin0 = sqrt((min1 - DUP(i,1))^2+(min2 - DUP(i,2))^2+(min3 - DUP(i,3))^2+(min4 - DUP(i
    s0 = (smin0)/(smax0 + smin0);
    s = [s;s0];
end
s(1) = []; Dfinal = [R(:,[1 2]) s];
res = find(Dfinal(:,3)==max(Dfinal(:,3)));
disp(res);
y0 = [10,11,12,13,14,15,16,17,18,19,20];
y = [y0 ;y0; y0; y0; y0; y0; y0; y0; y0; y0; y0; y0; y0; y0];
x0 = [0.0001];
for i = 1:13
    usl = 0.0001 + 0.0003*i;
    x0 = [x0;usl];
end
x = [x0 x0 x0 x0 x0 x0 x0 x0 x0 x0 x0];
z = zeros(14,1);
for i = 14:14:154
    z0 = s(i-13:i,:);
    z = [z z0];
end
z(:,1) = [];
surf(x,y,z);
```

A.3 Codes for Vibration Analysis in Model 3

```
clear,clc
rho=1.293; A=0.3*0.3; E= 67000000; vw=40/3.6;vc=33.6/3.6; C = 1; RESULT = [1];
for d=1:0.5:10
    L=d*A; K=A*E/L;
    M = 9800 + L*A*7190;
    for a=135:1:135
        v=sqrt(vc^2+vw^2-2*vc*vw*cos(a*pi/180));
        if a>= 45 && a <= 135
            S= (1.5*1.8+L*0.3)/abs(sin(a*pi/180));
        else
            S= (1.5*1.8+L*0.3)/abs(cos(a*pi/180));
        end
        F=1/2*C*rho*S*v^2;
        syms y(x);
        dy = diff(y);
        y = dsolve(M*diff(y,2) + C*diff(y) + K*y == F, y(0) == 0, dy(0) == 0);
        RESULT = [RESULT;y];
    end
end
```