2021 MCM-SJTU

Problem D Multicast Strategy

In computer networks, source routing is a routing method in which routing information is filled into the header of data packet, and then the router sends the data packet by hopping. Take the simplified topology in Figure.1 as an example. In this figure, A, B, ... G, H represent routers, and X, Y, Z, U, V, W represent the edges connected to the routers. If H wants to send a packet to G, then H node can fill the path information (V, W) into the header of data packet, and then send the data packet. After node F receives the data packet, it will check all the edges connected to it. Then it finds that edge W is in the packet. So the data packet will be transmitted to G. Therefore, after two hops, the data packet will reach G from H.

The example shown above is a simple unicast case. In fact, we can extend the case to a multicast case. Still taking Figure 1 as an example, if H wants to send a data packet to A and B respectively, then H can fill the branch formed from H to A, B into the data packet, namely U, Z, X, Y. When E node obtains this packet, it will recognize that edge Z in the routing information is connecting itself and node C, and then send the data packet to C. C checks the routing information and finds edge X, Y. Through them, package can be sent to A and B respectively, thereby realizing multicast.

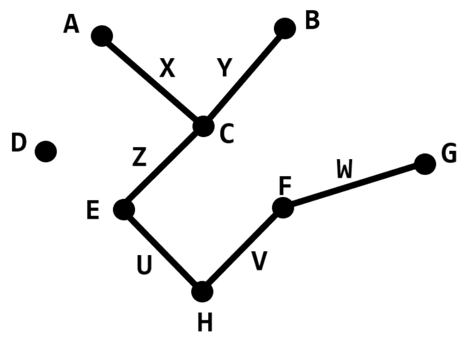

Figure 1

From the two examples mentioned above, we can see that when we want to send data packets to many receiving nodes in the multicast case, we may have to fill many routing information into the package. Imagine if there are 100 receiving nodes, then we need to fill in more than 100 edges, thus effective information will be greatly reduced. Therefore, an optimization problem needs to be solved.

Please complete the following tasks according to the different backgrounds:

Task 1.

Topology is given, the number of branches that can be contained in the header is also given. In the case of Figure 2, the maximum number of branches in the header is 3, and duplicate payloads are not allowed to appear, how many receiving points can be covered at most?

In the case of Figure 3, the maximum number of branches in the header is 3, but the existence of redundant packets is now allowed. What is the minimum number of redundancy on the premise that all receiving points are covered?

Note: In the following situations, redundancy will occur. Take Figure 1 as an example, when the packet capacity is set to 3, and two packets (U, Z, X) and (U, Z, Y) are set. In the transmission process, packets with the same payload pass through U and Z twice, although the path information of the two is different.

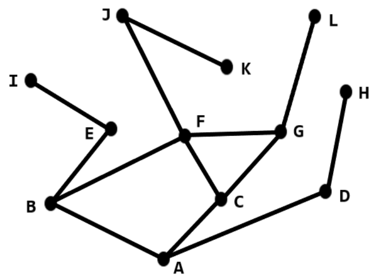

Figure 2

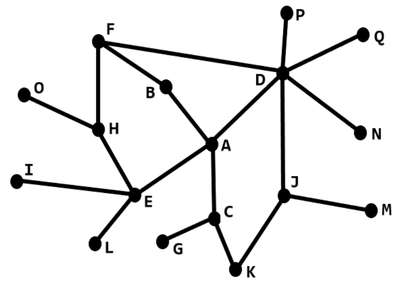

Figure 3

Task 2.

Topology is given, the number of branches that the header can hold is unknown at this time. The existence of duplicate payloads is permitted. As mentioned above, the more branches in a package, the less effective information it can carry. In the case of Figure 4, set the capacity of the packet and the corresponding packaging method, to make the transmission system reach a comprehensive optimum. Because a large capacity may bring difficulty, assume the capacity of the package is considerably small, for example, 15,20 and so on.

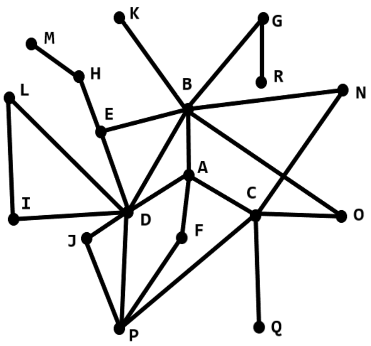

Figure 4

Task 3.

In fact, in the real Internet case, the topology is unknown. Based on the experience of the two tasks above, if the multicast technology needs to be applied in practice, how to design the algorithm for packaging branch information to achieve a relatively optimal state?

Acknowledgement

This problem is designed by Prof. Xiaohua Tian from Department of Electric Engineering at Shanghai Jiao Tong University, and then scribed, modified, and finalized by Prof. Xiaofeng Gao, Chang Liu (frenkiedejong@sjtu.edu.cn), Shijie Wang (wang_sj@sjtu.edu.cn), Hanwen Shi (shihanwen@sjtu.edu.cn) from Team 006 of SJTU MCM Group. Tongxin Ren (rentongxin@sjtu.edu.cn) helped proof reading and provided many suggestions.