①

	S_s	S_d
S_l	u_{ls}	u_{ld}
S_n	u_{ns}	u_{nd}

$$k_n' = k_n(1-\varepsilon) + \varepsilon k_l$$

$$k_l' = k_l(1-\varepsilon) + \varepsilon k_n.$$

$$\pi_l(i, k_l) = (1-\alpha)B + \alpha[k_s u_{ls} + (k-k_s)u_{ld}]$$

$$\pi_n(i, k_l) = (1-\alpha)B + \alpha[k_s u_{ns} + (k-k_s)u_{nd}]$$

②

无看错.

$$\pi_{l1} = (1-\alpha)B + \alpha\left[k_l' u_{ls} + (k-k_l') u_{ld}\right]$$

$$\pi_{n1} = (1-\alpha)B + \alpha\left[k_n' u_{ns} + (k-k_n') u_{nd}\right]$$

看错

$$\pi_{l2} = (1-\alpha)B + \alpha\left[k_n' u_{ls} + (k-k_n') u_{ld}\right]$$ 实际 n

$$\pi_{n2} = (1-\alpha)B + \alpha\left[k_l' u_{ns} + (k-k_l') u_{nd}\right]$$ 实际 l

(1) 自己无看错.

$$P_{1n\to l} = \frac{(k-k_s)\pi_l'}{k_s\pi_n + (k-k_s)\pi_l}$$

$$= \frac{k_l'\pi_l'}{k_n'\pi_n' + k_l'\pi_l'}$$

$$= \frac{k_l(1-\varepsilon)\cdot\pi_{l1} + \varepsilon\cdot k_n\cdot\pi_{n2}}{\left[k_n(1-\varepsilon)\pi_{n1} + \varepsilon k_l\cdot\pi_{l2}\right] + \left[k_l(1-\varepsilon)\cdot\pi_{l1} + \varepsilon k_n\cdot\pi_{n2}\right]}$$

(2) 自己看错

$$P_{2n\to l} = \frac{(k-k_s)\pi_l'}{k_s\pi_n' + (k-k_s)\pi_l'}$$

$$= \frac{k_n'\pi_n'}{k_l'\pi_l' + k_n'\pi_n'}$$

$$= \frac{k_n(1-\varepsilon)\pi_{n1} + \varepsilon k_l\cdot\pi_{l2}}{\left[k_l(1-\varepsilon)\pi_{l1} + k_n\cdot\varepsilon\cdot\pi_{n2}\right] + \left[k_n(1-\varepsilon)\pi_{n1} + \varepsilon k_l\cdot\pi_{l2}\right]}$$

$$\therefore P_{n\to l} = (1-\varepsilon)P_{1n\to l} + \varepsilon P_{2n\to l}$$

$$= \frac{(1-\varepsilon)\left[k_l(1-\varepsilon)\pi_{l1} + \varepsilon\cdot k_n\cdot\pi_{n2}\right] + \varepsilon\left[k_n(1-\varepsilon)\pi_{n1} + \varepsilon k_l\cdot\pi_{l2}\right]}{k_l(1-\varepsilon)\pi_{l1} + k_n\varepsilon\cdot\pi_{n2} + k_n(1-\varepsilon)\pi_{n1} + \varepsilon k_l\cdot\pi_{l2}}$$

(1) 自己无看错 ③

$$P_{1l\to n} = \frac{(k-k_s)\,\pi_n}{k_s \pi_l + (k-k_s)\pi_n}$$

$$= \frac{k_n' \pi_n}{k_l' \pi_l + (k_n' \pi_n}$$

$$= \frac{(1-\varepsilon)\cdot k_n \cdot \pi_{n1} + \varepsilon \cdot k_l \cdot \pi_{l2}}{(1-\varepsilon)\cdot k_n \cdot \pi_{n1} + \varepsilon \cdot k_l \cdot \pi_{l2} + (1-\varepsilon) k_l \cdot \pi_{l1} + \varepsilon k_n \cdot \pi_{n2}}$$

(2) 自己看错

$$P_{2l\to n} = \frac{(k-k_s)\,\pi_l'}{k_s \pi_l' + (k-k_s)\pi_n'}$$

$$= \frac{k_l' \pi_l}{k_l' \pi_l + k_n' \pi_n}$$

$$= \frac{(1-\varepsilon) k_l \cdot \pi_{l1} + \varepsilon \cdot k_n \cdot \pi_{n2}}{(1-\varepsilon)\cdot k_l \cdot \pi_{l1} + \varepsilon \cdot k_n \cdot \pi_{n2} + (1-\varepsilon) k_n \cdot \pi_{n1} + \varepsilon k_l \cdot \pi_{l2}}$$

$$P_{l\to n} = (1-\varepsilon)\, P_{1n\to l} + \varepsilon P_{2n\to l}$$

考虑拜占庭人

K_L的人数为 $K_L + l_2$ 拜占庭人数.

⑤ 推导.

$$P_{1n\to l} = \frac{k_l \pi_{l1} - \varepsilon k_l \cdot \pi_{l1} + \varepsilon (k - k_l)\cdot \pi_{n2}}{总}$$

$$= \frac{(k_l - \varepsilon k_l)\pi_{l1} + (\varepsilon k - \varepsilon k_l)\cdot \pi_{n2}}{(k - \varepsilon k - k_l + \varepsilon k_l)\pi_{n1} + \varepsilon k_l \cdot \pi_{l2} + (k_l - \varepsilon k_l)\pi_{l1} + (\varepsilon k - \varepsilon k_l)\pi_{n2}}$$

令 $\varepsilon = A$

$(1-\varepsilon) = B$

$k_l = C$

$k - k_l = D$

$$\pi_{l1} = \alpha 1 - \alpha + \alpha [\{k_l(1-\varepsilon) + \varepsilon(k-k_l)\} u_{ls} + \{(k-k_l)(1-\varepsilon) + \varepsilon k_l\} u_{l}$$

$$\pi_{n1} = 1-\alpha+\alpha\{[(k-k_l)(1-\varepsilon)+\varepsilon k_l] u_{ns} + [k_l(1-\varepsilon)+\varepsilon(k-k_l)] u_{nd}\}$$

$$\pi_{n2} = 1-\alpha+\alpha\{[k_l(1-\varepsilon)+\varepsilon(k-k_l)] u_{ns} + [(k-k_l)(1-\varepsilon)+\varepsilon k_l] u_{nd}\}$$

$$\pi_{l2} = 1-\alpha+\alpha\{[(k-k_l)(1-\varepsilon)+\varepsilon k_l] u_{ls} + [k_l(1-\varepsilon)+\varepsilon(k-k_l)] u_{ld}\}$$

$M = a+d$ $\quad$ $N = b+c$

$$\pi_{l1} = 1-\alpha+\alpha[(BC + AD)u_{ls} + (BD + AC)u_{ld}]$$

$$\pi_{n1} = 1-\alpha+\alpha[(BD \overset{N}{+} AC)u_{ns} + (BC \overset{M}{+} AD)u_{nd}]$$

$$\pi_{l2} = 1-\alpha+\alpha[(BD \overset{N}{+} AC)u_{ls} + (BC \overset{M}{+} AD)u_{ld}]$$

$$\pi_{n2} = 1-\alpha+\alpha[(BC \overset{M}{+} AD)u_{ns} + (BD \overset{N}{+} AC)u_{nd}]$$

7 $\{\cdot 1 \cdot\}$ $\{\cdot 2 \cdot\}$

$$P_{n\to l} = \frac{(1-\varepsilon)\left[(k_l - \varepsilon k_l)\pi_{l1} + (\varepsilon k - \varepsilon k_l)\cdot\pi_{n2}\right] + \varepsilon\left[(k - \varepsilon k - k_l + \varepsilon k_l)\pi_{n1} + \varepsilon k_l \cdot \pi_{l2}\right]}{(k - \varepsilon k - k_l + \varepsilon k_l)\pi_{n1} + \varepsilon k_l \cdot \pi_{l2} + (k_l - \varepsilon k_l)\pi_{l1} + (\varepsilon k - \varepsilon k_l)\pi_{n2}}$$

$$= \frac{B\cdot\left[B\cdot C\cdot \pi_{l1} + A\cdot D\cdot \pi_{n2}\right] + A\left[B\cdot D\,\pi_{n1} + A\cdot C\,\pi_{l2}\right]}{B\cdot D\cdot \pi_{n1} + A C\cdot \pi_{l2} + BC\,\pi_{l1} + AD\,\pi_{n2}}$$

$$= \frac{B^2 C\,\pi_{l1} + A^2 C\,\pi_{l2} + ABD\,\pi_{n2} + ABD\,\pi_{n1}}{BD\,\pi_{n1} + AD\,\pi_{n2} + BC\,\pi_{l1} + AC\,\pi_{l2}}$$

$$= \frac{(1-\alpha)(B^2C + A^2C + 2ABD) + \alpha\{\cdot 1 \cdot\}}{(1-\alpha)(BD + AD + BC + AC) + \alpha\{\cdot 2 \cdot\}}$$

$AD = a$

$BD = b$

$AC = c$

$BC = d$

$$= \frac{B^2C + A^2C + 2ABD}{a+b+c+d} \cdot \frac{1 + \alpha\cdot\left(\frac{\{\cdot 1 \cdot\}}{B^2C + A^2C + 2ABD} - 1\right)}{1 + \alpha\left(\frac{\{\cdot 2 \cdot\}}{a+b+c+d} - 1\right)}$$

⑦ $\{\cdot\overset{1}{\cdot}\cdot\}\ \{\cdot\overset{2}{\cdot}\cdot\}$

$$\{\cdot\overset{2}{\cdot}\cdot\} = BD(N \cdot U_{ns} + M U_{nd}) + AD(M U_{ns} + N U_{nd}) + BC(M U_{cs} + N U_{cd}) + AC(N U_{cs} + M U_{cd}$$

$$= (BDN + ADM) U_{ns} + (BDM + ADN) U_{nd} + (BCM + ACN) U_{cs} + (BCN + ACM) U_{cd}$$

$$= [a(a+d) + b(b+c)] U_{ns} + [a(b+c) + b(a+d)] U_{nd} + [c(b+c) + d(a+d)] U_{cs} + [c(a+d) + d(b+c)] U_{cd}$$

$$\{\cdot\overset{1}{\cdot}\cdot\}' = B^2C(M U_{cs} + N U_{cd}) + A^2C(N U_{cs} + M U_{cd}) + ABD(N U_{ns} + M U_{nd} + M U_{ns} + N U_{nd})$$

$$= B^2C[(a+d) U_{cs} + (b+c) U_{cd}] + A^2C[(b+c) U_{cs} + (a+d) U_{cd}] + ABD[(a+b+c+d)(U_{ns} + U_{nd})]$$

$$= [B^2C(a+d) + A^2C(b+c)] U_{cs} + [B^2C(b+c) + A^2C(a+d)] U_{cd} + ABD(a+b+c+d) U_{ns}$$
$$+ ABD(a+b+c+d) U_{nd}$$

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$$\frac{1+\alpha\left(\dfrac{\{\cdots 1 \cdots\}}{B^2C + A^2C + 2ABD} - 1\right)}{1+\alpha\left(\dfrac{\{\cdots 2 \cdots\}}{a+b+c+d} - 1\right)} \qquad \frac{1+ax}{1+bx} = 1 + \underline{\underline{(a-b)}}x + O(x^2) \qquad \frac{1+a\alpha}{1+b\alpha} = 1 + (a-b)\alpha + O(\alpha^2)$$

$$a-b = \frac{\{\cdots 1 \cdots\}}{B^2C + A^2C + 2ABD} - \frac{\{\cdots 2 \cdots\}}{a+b+c+d}$$

$$= \frac{(a+b+c+d)\{\cdots 1 \cdots\} - (B^2C + A^2C + 2ABD)\{\cdots 2 \cdots\}}{(B^2C + A^2C + 2ABD)(a+b+c+d)} \quad 上$$

⑨ 上 $= \{(a+b+c+d)[B^2C(a+d) + A^2C(b+C)] - (B^2C + A^2C + 2ABD)[c(b+c) + d(a+d)]\} U_{Ls} +$

$\{(a+b+c+d)[B^2C(b+c) + A^2C(a+d)] - (B^2C + A^2C + 2ABD)[c(a+d) + d(b+c)]\} U_{Ld} +$

$\{(a+b+c+d)[ABD(a+b+c+d)] - (B^2C + A^2C + 2ABD)[a(a+d) + b(b+c)]\} U_{ns} +$

$\{(a+b+c+d)[ABD(a+b+c+d)] - (B^2C + A^2C + 2ABD)[a(b+c) + b(a+d)]\} U_{nd}$

$$a-b = \left\{ \frac{B^2C(a+d) + A^2C(b+c)}{B^2C + A^2C + 2ABD} - \frac{c(b+c) + d(a+d)}{a+b+c+d} \right\} U_{Ls} +$$

$$= \left\{ \frac{B^2C(b+c) + A^2C(a+d)}{B^2C + A^2C + 2ABD} - \frac{c(a+d) + d(b+c)}{a+b+c+d} \right\} U_{cd} +$$

$$\left\{ \frac{ABD(a+b+c+d)}{B^2C + A^2C + 2ABD} - \frac{a(a+d) + b(b+c)}{a+b+c+d} \right\} U_{ns} +$$

$$\left\{ \frac{ABD(a+b+c+d)}{B^2C + A^2C + 2ABD} - \frac{a(b+c) + b(a+d)}{a+b+z+d} \right\} U_{nd}$$

⑩ $\frac{B^2C+A^2C+2ABD}{a+b+c+d} \cdot (a-b) \cdot \partial$

$$= \frac{\partial}{a+b+c+d} \Big\{ \Big[B^2C(a+d)+A^2C(b+c) - \frac{c(\overset{b+c}{\cancel{a+d}})+d(\overset{a+d}{\cancel{b+c}})}{a+b+c+d} \cdot (B^2C+A^2C+2ABD) \Big] U_{cs} +$$

$$\Big[B^2C(b+c)+A^2C(a+d) - \frac{c(a+d)+d(b+c)}{a+b+c+d} \cdot (B^2C+A^2C+2ABD) \Big] U_{cd} +$$

$$\Big[ABD(a+b+c+d) - \frac{a(a+d)+b(b+c)}{a+b+c+d} (B^2C+A^2C+2ABD) \Big] U_{ns} +$$

$$\Big[ABD(a+b+c+d) - \frac{a(b+c)+b(a+d)}{a+b+c+d} \cdot (B^2C+A^2C+2ABD) \Big] U_{nd}$$

$a+b+c+d = k$

⑪ 第一行

$$
\begin{aligned}
&(1-2\varepsilon)^2 k_1\left[k(1-\varepsilon)\varepsilon + (1-2\varepsilon)^2 k_1 - \frac{(1-2\varepsilon)^2 k_1^2}{k}\right] U_{1s} + \\
&(1-2\varepsilon)^2 k_1\left[k(1-\varepsilon+\varepsilon^2) - 2(1-2\varepsilon+2\varepsilon^2)k_1 + \frac{(1-2\varepsilon)^2}{k} k_1^2\right] U_{1d} + \\
&(1-2\varepsilon)^2 (k-k_1)\left[k(-1+\varepsilon)\varepsilon - (1-2\varepsilon)^2 k_1 + \frac{(1-2\varepsilon)^2}{k} k_1^2\right] U_{ns} + \\
&(1-2\varepsilon)^2 (k-k_1)\left[k(1-\varepsilon)\varepsilon - 4\cancel{\,}(1-\varepsilon)\varepsilon k_1 \cancel{+} \frac{(1-2\varepsilon)^2 k_1^2}{k}\right] U_{nd}
\end{aligned}
$$

$$
\begin{aligned}
= K\cdot\Big\{ & -(-1+\varepsilon)\varepsilon\cdot(U_{nd}-U_{ns}) + (1-2\varepsilon)^2\frac{k_1^3}{K^3}(U_{1d}-U_{1s}+U_{nd}-U_{ns}) \\
& + \left(\frac{k_1}{K}\right)^2\left\{[-2-4(-1+\varepsilon)\varepsilon]U_{1d} + U_{1s} - U_{nd} + 2U_{ns} + 4(-1+\varepsilon)\varepsilon(U_{1s} - 2U_{nd} \overset{+2U_{ns}}{}\right\} \\
& + \frac{K_1}{K}\left\{[1+(-1+\varepsilon)\varepsilon]U_{1d} - U_{ns} - (-1+\varepsilon)\varepsilon(U_{1s} - 5U_{nd} + 5U_{ns})\right\}\Big\}
\end{aligned}
$$

$$\text{⑫}\ dk \cdot \Big(\frac{k_l^3}{K^3} (1-2\varepsilon)^2 \Delta + \left(\frac{k_l}{K}\right)^2 \Big[-4(-1+\varepsilon)\varepsilon \Delta_n + [-2+4(-1+\varepsilon)\varepsilon]\Delta + \Delta' \Big]$$

$$+ \frac{k_l}{K} \Big\{ [-1+4(-1+\varepsilon)]\Delta_n + [1+(-1+\varepsilon)\varepsilon]\Delta + \Delta' \Big\} + (1-\varepsilon)\varepsilon \Delta_n \Big)$$