Раздел 2

1. $p_1 \cdot x + p_2 \cdot y = m$
$1.1 p_1 \cdot x + p_2 \cdot y = m + S$ ⊖ $0.1 p_1 \cdot x = S$
$p_1 \cdot x = 10 S$
$p_2 \cdot y = m - 10 S$

~~11 S~~

$$\frac{p_1 x + p_2 y}{1.1 p_1 x + p_2 y} = \frac{m}{m+S}$$

$p_1 x m + p_2 y m + p_1 x S + p_2 y S = 1.1 p_1 x m + p_2 y m$

$(p_1 x + p_2 y) S = 0.1 p_1 x m$

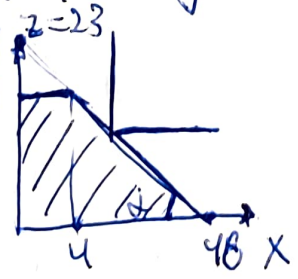
$S = \frac{0.1 p_1 x}{p_1 x + p_2 y} m = 0.1 p_1 x$

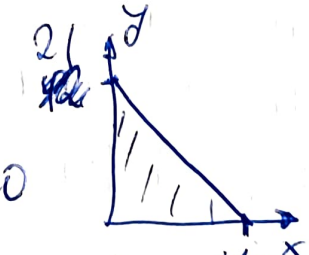
2. Задача устная.

3. $I = 420$ ₽
$p_1 = 10$
$p_2 = 20$

а) $p_1 x + p_2 y \leq I$
$10x + 20y \leq 420$

б)





$tg\alpha = \frac{21}{42} = \frac{z}{46}$

$z = \frac{21 \cdot 46}{42} = 23$

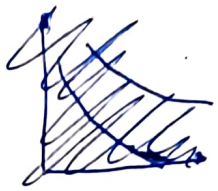
$y = 21 \quad x \in [0; 4]$

в)

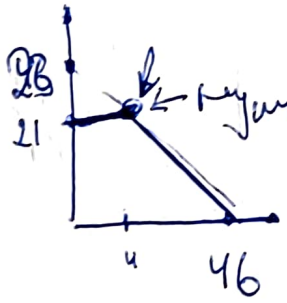
8] $\bar{p}_1 x + \bar{p}_2 y = 420$, где $\bar{p}_1 = \frac{420}{23}$, $\bar{p}_2 = \frac{420}{46}$

$\frac{420}{23} x + \frac{420}{46} y = 420$

$\frac{x}{23} + \frac{y}{46} = 1$



$u(x_1, x_2) = \min\{21x_1, 4x_2\}$



$\Rightarrow$ точное $y = 105$, $x = 20$

$\Rightarrow \frac{x}{y} = \frac{20}{105}$

$\Rightarrow x = \frac{20y}{105}$

$\frac{20y}{105 \cdot 23} + \frac{y}{46} = 1$

$y = \frac{1}{\frac{20}{105 \cdot 23} + \frac{1}{46}} = \frac{1}{0.008 + 0.021}$

$21y = 4x \Rightarrow y = \frac{4x}{21}$

$\frac{x}{23} + \frac{4x}{21 \cdot 46} = 1 \Rightarrow x = \frac{1}{\frac{1}{23} + \frac{4}{21 \cdot 46}} = 21 \rightarrow y = 4.$

№1 $u(x_1, x_2)$

$m > 0$ - доход

$p = (p_1, p_2) > 0$

а) $u(x_1, x_2) = (x_1)^\alpha (x_2)^\beta$

$p_1 x_1 + p_2 x_2 = m$

$p_1 x_1^0 + p_2 x_2^0 = m$

$\frac{\partial u}{\partial x_1} = \alpha x_1^{\alpha-1} x_2^\beta$

$\frac{\partial u}{\partial x_2} = \beta x_1^\alpha x_2^{\beta-1}$

$\frac{\partial u}{\partial x_1} / \frac{\partial u}{\partial x_2} = \frac{p_1}{p_2} = \frac{\alpha x_1^{\alpha-1} x_2^\beta}{\beta x_1^\alpha x_2^{\beta-1}} = \frac{\alpha}{\beta} \frac{x_2}{x_1}$

$\begin{cases} \frac{\alpha}{\beta} \frac{x_2}{x_1} = \frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 = m \end{cases} \Rightarrow p_1 = \frac{\beta}{\alpha} x_2 = \frac{p_1}{p_2} x_1 \frac{\beta}{\alpha}$

$p_1 x_1 + p_1 x_1 \frac{\beta}{\alpha} = m$

$p_1 x_1 (1 + \frac{\beta}{\alpha}) = m \Rightarrow$

$\begin{cases} x_1 = \frac{m}{p_1(1+\frac{\beta}{\alpha})} \\ x_2 = \frac{\beta}{\alpha} \frac{m}{p_2(1+\frac{\beta}{\alpha})} \end{cases}$

б) $u(x_1, x_2) = \alpha_1 x_1 + \alpha_2 x_2$, $\alpha_1, \alpha_2 > 0$

$\frac{\partial u}{\partial x_1} / \frac{\partial u}{\partial x_2} = \begin{cases} \frac{\alpha_1}{\alpha_2} = \frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 = m \end{cases} \Rightarrow$ НЗЧ.

в) $u(x_1, x_2) = \min \left\{ \frac{x_1}{\alpha_1}, \frac{x_2}{\alpha_2} \right\}$

$\begin{cases} \frac{x_1}{\alpha_1} = \frac{x_2}{\alpha_2} \\ p_1 x_1 + p_2 x_2 = m \end{cases} \Rightarrow$ НЗРС.

г) $u(x_1, x_2) = \sqrt{x_1} + x_2$

$\frac{\partial u}{\partial x_1} = \frac{1}{2\sqrt{x_1}} \Rightarrow \frac{\partial u}{\partial x_1} / \frac{\partial u}{\partial x_1} = \begin{cases} \frac{1}{2\sqrt{x_1}} = \frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 = m \end{cases}$

$\frac{\partial u}{\partial x_2} = 1$

№5. $I = 600$

$u(x, y) = xy$

$25x + 40y = 600$

$\frac{y}{x} = \frac{25}{40} \Rightarrow x = \frac{40}{25} y$

$40y + 40y = 600$

$80y = 600$ $\quad y = \frac{600}{80} = \frac{60}{8} = \frac{15}{2}$