2.1) $A = \dfrac{\partial F}{\partial U} - ?$ ~~Введём вектора (γ, E, U), но~~

$$\bar U = \begin{Vmatrix} \rho \\ \rho u \\ E \end{Vmatrix}, \quad \bar F = \begin{Vmatrix} \rho u \\ \rho u^2 + p \\ (E+p)u \end{Vmatrix}, \text{ где } E = \frac{1}{2}\rho u^2 + \rho e, \quad e = \frac{p}{(\gamma-1)\rho}$$

$$\bar U = \begin{Vmatrix} U_1 \\ U_2 \\ U_3 \end{Vmatrix}$$

$$\Rightarrow E = \frac{1}{2}\rho u^2 + \frac{p}{\gamma-1} \Rightarrow \frac{p}{\gamma-1} = \left(E - \frac{1}{2}\rho u^2\right)$$

$$\Rightarrow p = (\gamma-1)\left(\underset{U_3}{\underset{\parallel}{E}} - \frac{1}{2}\rho u^2\right)$$

$$\frac{1}{2}\rho u^2 = \frac{1}{2}\frac{\rho^2 u^2}{\rho} = \frac{1}{2}\frac{U_2^2}{U_1}$$

$$\Rightarrow p = (\gamma-1)\left(U_3 - \frac{1}{2}\frac{U_2^2}{U_1}\right)$$

$$\Rightarrow \bar F = \begin{Vmatrix} U_2 \\ \frac{U_2^2}{U_1} + (\gamma-1)U_3 + \frac{1-\gamma}{2}\frac{U_2^2}{U_1} \\ \left(U_3 + (\gamma-1)\left(U_3 - \frac{1}{2}\frac{U_2^2}{U_1}\right)\right)\frac{U_2}{U_1} \end{Vmatrix} =$$

$$= \begin{Vmatrix} U_2 \\ \frac{3-\gamma}{2}\frac{U_2^2}{U_1} + (\gamma-1)U_3 \\ \left(\gamma U_3 + \frac{1-\gamma}{2}\frac{U_2^2}{U_1}\right)\frac{U_2}{U_1} \end{Vmatrix} = \bar f(U_1, U_2, U_3)$$

$$\Rightarrow \frac{\partial \bar F}{\partial \bar U} = \text{матрица (тензор 2-го ранга)} =$$

$$= \begin{Vmatrix} \frac{\partial F_1}{\partial U_1} & & \\ \frac{\partial F_1}{\partial U_2} & \ddots & \\ \frac{\partial F_1}{\partial U_3} & & \frac{\partial F_3}{\partial U_3} \end{Vmatrix} = \boxed{\begin{Vmatrix} 0 & \frac{\gamma-3}{2}\frac{U_2^2}{U_1^2} & (\gamma-1)\frac{U_2^3}{U_1^3} - \gamma\frac{U_3 U_2}{U_1} \\ 1 & (3-\gamma)\frac{U_2}{U_1} & \gamma\frac{U_3}{U_1} + \frac{3(1-\gamma)}{2}\frac{U_2^2}{U_1^2} \\ 0 & \gamma-1 & \gamma\frac{U_2}{U_1} \end{Vmatrix}}$$

Ответ

$$\frac{\partial F_3}{\partial U_1} = \frac{\gamma-1}{2}\frac{U_2^2}{U_1^2}\cdot\frac{U_2}{U_1} - \left(\gamma U_3 + \frac{1-\gamma}{2}\frac{U_2^2}{U_1}\right)\frac{U_2}{U_1^2} = \frac{\gamma-1}{2}\frac{U_2^3}{U_1^3} - \gamma\frac{U_3 U_2}{U_1} + \frac{\gamma-1}{2}\frac{U_2^3}{U_1^3} = (\gamma-1)\frac{U_2^3}{U_1^3} - \gamma\frac{U_3 U_2}{U_1}$$

$$\frac{\partial F_3}{\partial U_2} = \frac{1}{U_1}(\ldots) + (1-\gamma)\frac{U_2^2}{U_1^2} = \gamma\frac{U_3}{U_1} + \frac{3(1-\gamma)}{2}\frac{U_2^2}{U_1^2}$$

2.2) СВ матрицы системы для примитивных переменных: ?

$$\text{Матрица прим. пер.} - \Pi = \begin{Vmatrix} u & \rho & 0 \\ 0 & u & \frac{1}{\rho} \\ 0 & \gamma p & u \end{Vmatrix}$$

Даны собственные числа:

$\lambda_1 = u - a$

$\lambda_2 = u$ $\qquad a = \sqrt{\dfrac{\gamma p}{\rho}}$ — скорость звука

$\lambda_3 = u + a$

$$\Rightarrow \lambda_1: \begin{Vmatrix} a & \rho & 0 \\ 0 & a & \frac{1}{\rho} \\ 0 & \gamma p & a \end{Vmatrix} \cdot \bar h_1 = \bar 0 \Rightarrow \bar h_1 = \begin{Vmatrix} \frac{\rho}{a} \\ -1 \\ a\rho \end{Vmatrix}$$

Для λ_3, очевидно, изменится только знак второй компоненты $\bar h_1 \Rightarrow$

$$\bar h_3 = \begin{Vmatrix} \rho/a \\ +1 \\ a\rho \end{Vmatrix}$$

$$\lambda_2: \begin{Vmatrix} 0 & \rho & 0 \\ 0 & 0 & \frac{1}{\rho} \\ 0 & \gamma p & 0 \end{Vmatrix} \bar h_2 = \bar 0 \Rightarrow \bar h_2 = \begin{Vmatrix} 1 \\ 0 \\ 0 \end{Vmatrix}$$

$$\Rightarrow \boxed{\bar h_1 = \begin{Vmatrix} \rho/a \\ -1 \\ a\rho \end{Vmatrix} \quad \bar h_2 = \begin{Vmatrix} 1 \\ 0 \\ 0 \end{Vmatrix} \quad \bar h_3 = \begin{Vmatrix} \rho/a \\ 1 \\ a\rho \end{Vmatrix}}$$

Ответ

2.3) $\dfrac{\partial \rho}{\partial t} + \rho_0 \dfrac{\partial u}{\partial x} = 0$

$\dfrac{\partial u}{\partial t} + \dfrac{a^2}{\rho_0}\dfrac{\partial \rho}{\partial x} = 0$

$\dfrac{\partial \rho}{\partial t} + \rho_0 \dfrac{\partial u}{\partial x} + 0 \cdot \dfrac{\partial \rho}{\partial x} = 0$

$\dfrac{\partial u}{\partial t} + 0 \cdot \dfrac{\partial u}{\partial x} + \dfrac{a^2}{\rho_0}\dfrac{\partial \rho}{\partial x} = 0$

$\Updownarrow$

$\Rightarrow \partial_t \begin{Vmatrix} \rho \\ u \end{Vmatrix} + A\,\partial_x \begin{Vmatrix} \rho \\ u \end{Vmatrix} = 0$

$A = \begin{Vmatrix} 0 & \rho_0 \\ \frac{a^2}{\rho_0} & 0 \end{Vmatrix}$

СЧ: $\begin{vmatrix} 0-\lambda & \rho_0 \\ \frac{a^2}{\rho_0} & -\lambda \end{vmatrix} = \lambda^2 - a^2 = 0$

$\Rightarrow \lambda_{1,2} = \pm a$

λ_1: $\begin{Vmatrix} -a & \rho_0 \\ \frac{a^2}{\rho_0} & -a \end{Vmatrix} \cdot \bar{h}_1 = \bar{0} \Rightarrow \boxed{\bar{h}_1 = \begin{Vmatrix} \rho_0 \\ a \end{Vmatrix}}$

λ_2: $\begin{Vmatrix} a & \rho_0 \\ \frac{a^2}{\rho_0} & a \end{Vmatrix} \cdot \bar{h}_2 = \bar{0} \Rightarrow \boxed{\bar{h}_2 = \begin{Vmatrix} -\rho_0 \\ a \end{Vmatrix}}$

Ответ

2.4) Линейно вырожд, истинно нелин.!

$\begin{Vmatrix} \rho \\ u \\ p \end{Vmatrix}_t + \Omega \begin{Vmatrix} \rho \\ u \\ p \end{Vmatrix}_x = 0; \quad \bar{U} = \begin{Vmatrix} \rho \\ u \\ p \end{Vmatrix}$

$\lambda_1 = u - a \qquad \bar{h}_1 = \begin{Vmatrix} \rho/a \\ -1 \\ a\rho \end{Vmatrix}$

$\lambda_2 = u \qquad \bar{h}_2 = \begin{Vmatrix} 1 \\ 0 \\ 0 \end{Vmatrix}$

$\lambda_3 = u + a \qquad \bar{h}_3 = \begin{Vmatrix} \rho/a \\ 1 \\ a\rho \end{Vmatrix}$

$a = \sqrt{\dfrac{\gamma p}{\rho}} \Rightarrow \lambda_1 = u - \sqrt{\dfrac{\gamma p}{\rho}} = U_2 - \sqrt{\gamma}\sqrt{\dfrac{U_3}{U_1}}$

$\Rightarrow \nabla \lambda_1 = \begin{Vmatrix} -\sqrt{\gamma} \cdot \frac{\sqrt{U_3}}{2\sqrt{U_1}} \cdot \left(\frac{-1}{U_1^2}\right) \\ 1 \\ -\sqrt{\gamma} \cdot \frac{1}{U_1 2\sqrt{U_3}} \end{Vmatrix} = \begin{Vmatrix} \frac{1}{2}\sqrt{\gamma}\sqrt{\frac{U_3}{U_1^3}} \\ 1 \\ \frac{1}{2}\sqrt{\frac{\gamma}{U_1 U_3}} \end{Vmatrix} = \begin{Vmatrix} \frac{1}{2}\sqrt{\frac{\gamma p}{\rho^3}} \\ 1 \\ -\frac{1}{2}\sqrt{\frac{\gamma}{\rho p}} \end{Vmatrix}$

$\nabla \lambda_2 = \begin{Vmatrix} 0 \\ 1 \\ 0 \end{Vmatrix}$

$\nabla \lambda_3$ = замена знаков у 1-ой и 3-ей компонент в $\nabla \lambda_1$ =

$= \begin{Vmatrix} -\frac{1}{2}\sqrt{\frac{\gamma p}{\rho^3}} \\ 1 \\ +\frac{1}{2}\sqrt{\frac{\gamma}{\rho p}} \end{Vmatrix}$

$\Rightarrow \nabla\lambda_1 \cdot \bar{h}_1 = \frac{1}{2}\sqrt{\frac{\gamma p}{\rho^3}} \cdot \frac{\rho}{a} - 1 - \frac{1}{2} a \sqrt{\frac{\gamma}{\rho p}} = \frac{1}{2}\sqrt{\frac{\gamma p \rho^2}{\rho^3 \gamma p}} - 1 - \frac{1}{2}\sqrt{\frac{\gamma p}{\rho}\frac{\gamma}{\rho p}} =$

$= \frac{1}{2} - 1 - \frac{1}{2}\sqrt{\frac{\gamma^2}{\rho^2}} = -\frac{1}{2} - \frac{1}{2}\left(1 + \frac{\gamma}{\rho}\right) \quad \forall \bar{U} \in R^m$

$\neq 0$ – истинно нелинейное

$\nabla \lambda_3 \cdot \bar{h}_3 = 1 + 1 + \frac{1}{2}\frac{\gamma}{\rho} \neq 0 \;\; \forall \bar{U} \in R^m$ – истинно нелинейное

$\nabla \lambda_2 \cdot \bar{h}_2 = \begin{Vmatrix} 0 \\ 1 \\ 0 \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ 0 \\ 0 \end{Vmatrix} \equiv 0$ – не линейно вырожд.

$\Rightarrow \forall$ λ_1-поле, λ_3-поле – ~~[illegible]~~ истинно нелинейное

λ_2 – линейно вырожденное

Ответ