Сжимаемая жидкость Вар. 2.

1. $\rho_0 = const$ лин. ур-ие - ?

$a = a_0 = const$

$|\delta\rho| << \rho_0$

$\rho = \rho_0 + \delta\rho$.

1: $\rho_t + \frac{\partial}{\partial x_\alpha}(\rho u_\alpha) = 0$

$\delta\rho_t + (\rho_0 + \cancel{\delta\rho})\operatorname{div}\bar u + \underbrace{u_\alpha \frac{\partial \cancel{\delta\rho}}{\partial x_\alpha}}_{\approx 0} = 0$

$\Rightarrow \delta\rho_t + \rho_0 \operatorname{div}\bar u = 0$. — 1-ое ур-ие $\Rightarrow \delta\rho_{tt} + \rho_0 \frac{\partial}{\partial t}\operatorname{div}\bar u = 0$.

2: $\frac{\partial u_i}{\partial t} + \underbrace{\frac{\partial}{\partial x_\alpha}(u_\alpha u_i)}_{\approx 0} + \frac{a_0^2}{\rho}\frac{\partial \rho}{\partial x_i} = 0$

$\frac{\partial \bar u}{\partial t} + \frac{a_0^2}{\rho_0}\nabla\delta\rho = 0 \Rightarrow \frac{\partial}{\partial t}\operatorname{div}\bar u + \frac{a_0^2}{\rho_0}\Delta\delta\rho = 0$

$\Rightarrow$ имеем $\begin{cases} \delta\rho_{tt} + \rho_0\frac{\partial}{\partial t}\operatorname{div}\bar u = 0 \\ \frac{\partial}{\partial t}\operatorname{div}\bar u + \frac{a_0^2}{\rho_0}\Delta\delta\rho = 0 \quad /\cdot\rho_0 \end{cases} \Rightarrow \begin{cases} \delta\rho_{tt} + \rho_0\frac{\partial}{\partial t}\operatorname{div}\bar u = 0 \\ \rho_0\frac{\partial}{\partial t}\operatorname{div}\bar u + a_0^2\Delta\delta\rho = 0 \end{cases} \ominus$

$\Rightarrow \boxed{\delta\rho_{tt} - a_0^2\Delta\delta\rho = 0}$

2. $\frac{d\rho'}{dt} + \rho_0\frac{dv'}{\partial x} = 0$

$\frac{dv'}{dt} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial x}$

$p' = a^2\rho'$

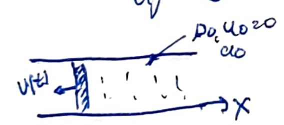
$a^2 = \left(\frac{dp}{d\rho}\right)_0$

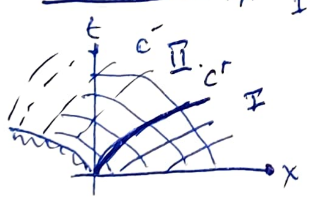
$\frac{d^2 p'}{dt^2} - a^2\frac{d^2 p'}{dx^2} = 0 \Rightarrow p' = f_1(x-at) + f_2(x+at)$

$\Rightarrow \boxed{\begin{aligned} \rho'(x,t) &= \frac{1}{a^2}\left(f_1(x-at) + f_2(x+at)\right) \\ v'(x,t) &= \frac{1}{a\rho_0}\left(f_1(x-at) - f_2(x+at) + C\right) \end{aligned}}$

для медленного вдвижения?!!

3.





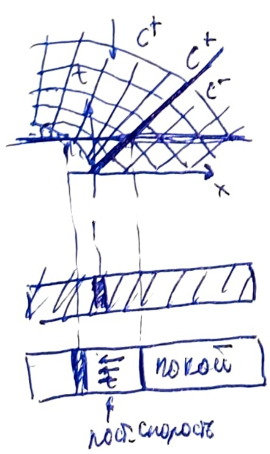
I: C^+: $\frac{dx^+}{dt} = u + a$

$I_+ = u + \frac{2a}{\gamma-1} = \frac{2a_0}{\gamma-1}$

C^-: $\frac{dx^-}{dt} = u - a$

$I_- = u - \frac{2a}{\gamma-1} = -\frac{2a_0}{\gamma-1}$

$\Rightarrow u = 0$, $a = a_0$ $\Rightarrow$ зона покоя. $\frac{dx^+}{dt} = a_0$, $\frac{dx^-}{dt} = -a_0$



II: C^+: $\frac{dx^+}{dt} = u + a$

$I_+ = u + \frac{2a}{\gamma-1}$ $= 2u + \frac{2a_0}{\gamma-1} = const \Rightarrow u = const$ вдоль C^+.

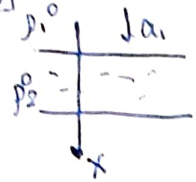
$\Rightarrow \frac{dx^+}{dt} = \ldots$

C^-: $\frac{dx^-}{dt} = u - a$

$I_- = u - \frac{2a}{\gamma-1} = -\frac{2a_0}{\gamma-1}$

$\Rightarrow a = u\frac{\gamma-1}{2} + a_0$

4.



$p_1' = f_0(t-\frac{x}{a_1})$ - пад

$p_1' = f_1(t+\frac{x}{a_1})$ - отраж. $\Rightarrow p_1' = f_0(t-\frac{x}{a_1}) + f_1(t+\frac{x}{a_1})$

$p_2' = \varphi(t-\frac{x}{a_2})$

p_1' - ?

p_2' - ?

$\frac{p_1'}{p_1^0}$ - ?

$\frac{p_2'}{p_1^0}$ - ?

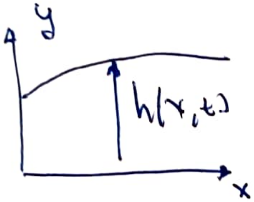
$\Rightarrow V_1(x,t) = \frac{1}{a_1\rho_1}(f_0(t-\frac{x}{a_1}) - f_1(t+\frac{x}{a_1}))$

$V_2(x,t) = \frac{1}{a_2\rho_2}\varphi(t-\frac{x}{a_2})$

$p_1|_{x=0} = p_2|_{x=0} \Leftrightarrow f_0(t) + f_1(t) = \varphi(t)$

$v_1|_{x=0} = v_2|_{x=0} \Leftrightarrow \frac{1}{a_1\rho_1} f_0(t) - \frac{1}{a_1\rho_1} f_1(t) = \frac{1}{a_2\rho_2}\varphi(t)$ $\Rightarrow$ нужные отношения.

5.



$$\begin{cases} \frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) = 0 \\ \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} = 0 \end{cases}$$

$\exists\, a = \sqrt{gh} \Rightarrow h_t = \frac{2}{g}aa_t$

$h_x = \frac{2}{g}aa_x$

$\Rightarrow \frac{2}{g}aa_t + u\frac{2}{g}aa_x + \frac{1}{g}a^2u_x = 0$

$$\Rightarrow \begin{cases} a_t + ua_x + \frac{1}{2}au_x = 0 \\ u_t + uu_x + 2aa_x = 0 \end{cases}$$

Это тоже с газодин.:

$$\begin{cases} a_t + ua_x + \frac{\gamma-1}{2}au_x = 0 \\ u_t + uu_x + \frac{2}{\gamma-1}aa_x = 0 \end{cases}$$

при $\gamma = 2$.

6. Характеристики - кривые, вдоль которых ур-ие в ЧП превращается в обыкновенное диф. ур-ие.

7. ЗСМ и ЗСИ, сохр. анализ энтропии.

Вариант 1.

1.

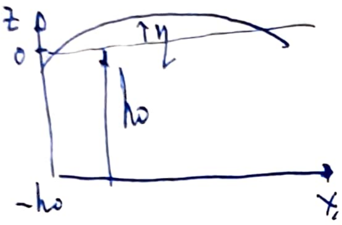
$$\begin{cases} \rho_t + u\rho_x + \rho u_x = 0 \\ u_t + uu_x + \frac{a^2(\rho)}{\rho}\rho_x = 0 \end{cases}$$

$a \; \forall \; a = a(\rho)$

$$\begin{cases} a_t + ua_x + \frac{\gamma-1}{2} a u_x = 0 \\ u_t + uu_x + \frac{2}{\gamma-1} a a_x = 0 \end{cases}$$

$\forall \; a = a_0 \left(\frac{\rho}{\rho_0}\right)^{\frac{\gamma-1}{2}}$

2.



$h_0 \ll L$

$\eta \ll h_0$

$$\begin{cases} \frac{\partial \bar u}{\partial t} + (\bar u \nabla)\bar u = -\frac{1}{\rho}\nabla p + \bar g \\ \operatorname{div} \bar u = 0 \end{cases}$$

$\bar u = (u, v, w)$

w - мало

$$\Rightarrow \begin{cases} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \cancel{w\frac{\partial u}{\partial z}} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \cancel{w\frac{\partial v}{\partial z}} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \\ \cancel{\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \end{cases}$$

$$\Rightarrow -\frac{dp}{dz} - \rho g = 0 \Rightarrow \Delta p = \rho g(\eta - z)$$

$$\begin{cases} \frac{\partial u}{\partial t} + \ldots + g\frac{\partial \eta}{\partial x} = 0 \\ \frac{\partial v}{\partial t} + \ldots + g\frac{\partial \eta}{\partial y} = 0 \end{cases}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \Rightarrow \int_{-h_0}^{\eta} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dz + w\Big|_{-h_0}^{\eta} = 0$$

$(u, v) = (u_1, u_2)$

$(x, y) = (x_1, x_2)$

$$\Rightarrow \int_{-h_0}^{\eta} \frac{\partial}{\partial x_i} u_i dz + w\Big|_{-h_0}^{\eta} = 0 \Rightarrow \frac{\partial}{\partial x_i}\int_{-h_0}^{\eta} u_i dz - u_i\frac{\partial \eta}{\partial x_i}\Big|_{z=\eta} - u_i\frac{\partial h_0}{\partial x_i}\Big|_{z=-h_0} + w\Big|_{-h_0}^{\eta} = 0$$

Кин. усл. пов-ти: $\frac{\partial \eta}{\partial t} + u_i\frac{\partial \eta}{\partial x_i} = w$

$$-u_i\frac{\partial \eta}{\partial x_i}\Big|_{z=\eta} + w\Big|_{z=\eta} = \frac{\partial \eta}{\partial t}$$

дно: $u_i\frac{\partial h_0}{\partial x_i}\Big|_{z=-h_0} + w\Big|_{z=+h_0} = 0$

$$\frac{\partial}{\partial x_i}\int_{-h_0}^{\eta} u_i dz + \frac{\partial \eta}{\partial t} = 0$$

$$\Rightarrow \frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x_i} u_i(\eta + h_0) = 0$$

$$\frac{\partial \eta}{\partial t} + \operatorname{div}(\bar u(\eta + h_0)) = 0$$

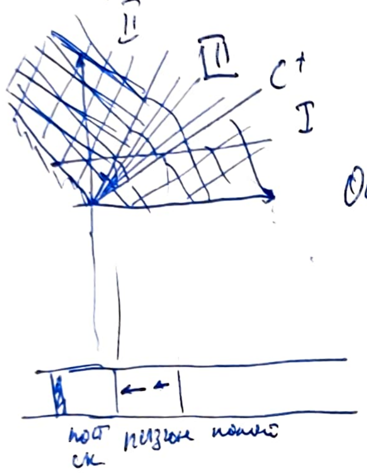
3. Инв. Римана - сохраняющиеся в д/я однородного газа вдоль характеристик комбинации параметров течений газообр. сред.

4. 3СМ, 3С4, 3С7.

5. см. вопр. 2.

6. рассмотрим быстрое (мгновенное)

$$\dot{X} = \begin{cases} 0, & t<0 \\ -v, & t \geq 0 \end{cases}$$



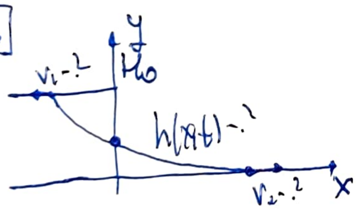
III: $C^+: \frac{dx}{dt} = a_0 + \frac{\gamma+1}{2}u$

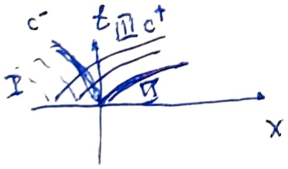
$I_- = \frac{2a}{\gamma-1} - u = \frac{2a_0}{\gamma-1} \Rightarrow a = a_0 + \frac{\gamma-1}{2}u$

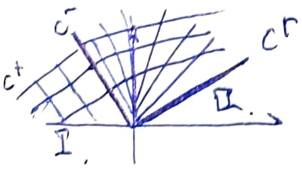
Очень быстрое: II. C^-: $\frac{dx}{dt} = u - a = -v_{кр} - a \Rightarrow v_{кр} = \dots$

Если очень быстро, то вместо пос.м. - вакуум.

7.







I: $u + 2\sqrt{gh} = I_+ = 2\sqrt{gH_0} \Rightarrow h = H_0$

$-u + 2\sqrt{gh} = 2\sqrt{gH_0} = I_- \quad u = 0$

$\frac{dx^+}{dt} = \sqrt{gH_0} \Rightarrow x^+ = t\sqrt{gH_0} + C \quad \left(\frac{dx^+}{dt} = u + \sqrt{gh}\right)$

$\frac{dx^-}{dt} \Rightarrow x^- = -t\sqrt{gH_0} + C \Rightarrow \boxed{V_1 = \sqrt{gH_0}}$

II: $I_+ = \cancel{u+2\sqrt{gh}} = 2\sqrt{gH_0}$ $u + 2\sqrt{gh} = 2\sqrt{gH_0}$

$\frac{dx^+}{dt} = \boxed{2\sqrt{gH_0} = V_2} \Rightarrow x^+ = 2\sqrt{gH_0}$

III: $2\sqrt{gh} + u = 2\sqrt{gH_0} \Rightarrow h = const \Rightarrow u = const$

$2\sqrt{gh} - u = C$

$\frac{dx^+}{dt} = u + \sqrt{gh}$

$\frac{dx^-}{dt} = u - \sqrt{gh} \Rightarrow x^- = C_1 t + C_2$

$\frac{x}{t} = u - \sqrt{gh} \Rightarrow x = ut - t\sqrt{gh} \Rightarrow h = \dots = \dots$

$2\sqrt{gh} + u = 2\sqrt{gH_0} \Rightarrow u = \dots$