REAL OPTION APPROACH FOR JACK UP VALUATION

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Abstract
Successful investing in oil and gas offshore projects is highly dependent on technology and market conditions. A pivotal aim of an investor is to allocate resources to financially rewarding projects. In order to identify value-increasing projects, companies perform the discounted cash flow method or the more advanced decision tree analyses. These methods, however, cannot properly capture the value of the flexibility to re-adjust plans to possible external and internal "shocks". Moreover, traditional valuation tools give no insight about how these future decision contingencies affect the risk of a project during its lifetime. The real option approach may help to solve these problems, and provide an insightful investment decision framework. The objective of this paper is to demonstrate how the real option approach can be practically applied to the investment decision making in jack ups, accounting for flexibility, strategic growth opportunities and idle time. The suitability of real option valuation for investment decision making is also discussed.

Key Words
Jack up, Drilling, Real Options, Investment Analysis, DCF, NPV, Oil and Gas, Project evaluation

Nomenclatures

y	option expiration time in years	u	up movement factor of the underlying asset
N	quantity of tree steps	d	down movement factor of the underlying asset
r	Risk free interest rate per year	NPV	Net Present value
σ	project volatility	DCF	Discounted Cash Flow
b	dividends rate	S	Price
p	up movement probability	X	Exercise price

1. Introduction
The offshore drilling operation is complex, and the cost per day of a drilling rig (the day-rate) varies as a function of market conditions governed by crude oil price and the water depth. Additional factors are cost of rig acquisition, manpower, regulations (safety, environment and certification) as well partnership formation. A drilling company acquires or constructs a unit and, in general, receives day-rate remuneration from the prospector which leases the rig. In some cases receives an incentive bonus for operational performance or high equipments availability. Also, there are turn-key contracts in which a fixed value is received for drilling a well, irrespective of duration; but this practice is not common, as it could lead to priority in operational speed and not in well quality.

The remuneration value is affected by economic uncertainty (motion of market influencing prices) or by technical uncertainty (new technologies, obsolescence, new contract types and performance of newly-installed equipments). The drilling rig is depreciated by use and by maintenance needs; and the technological development may render it less competitive.

The volatility of day-rate is very high, suggesting that economic evaluations based on NPV (net present value) are inadequate. Some companies support periods of negative cash flow in expectation of the situation reversal. In general the rig operating cost is almost constant, as the asset depreciation is a very significant contributing factor. Decisions about drilling rig acquisition or its sale, operation interruption or reactivation, avoiding inopportune attitudes and available time to exercise such options, require considerable managerial flexibility and almost always such decisions are irreversible ones.

For **investment or re-entry** the more significant variables are: acquisition cost, operating cost, volatility and discount rates (net and risk-free) and, at a lower level, the abandonment cost.
For **reactivation and temporary stoppage**, the operational and maintenance costs are the main influences. With a smaller impact, the lump-sum costs for suspension and reactivation and the volatility.

For **abandonment** the more sensible parameters are the exit cost (residual value) and operational cost. Acquisition cost, volatility and discount rates have less influence.

2. Risk Factors

All offshore contracts are associated with considerable risks and responsibilities with regard to technical, operational, commercial, financial and political risks.

2.1 Financial Risk

Interest rate and currency fluctuations: Drilling companies are exposed to risks due to fluctuations in interest and exchange rates. They attempt to minimize these risks by implementing hedging arrangements as appropriate, but they would not be able to avoid these risks. Changes in currency exchange rates relative to the US dollar may affect the US dollar value of some assets and thereby impact upon the total return on such assets.

Borrowing and leverage: Borrowings create leverage to the extent income derived from assets obtained with borrowed funds exceeds the interest and other expenses that a company have to pay; the net income will be greater than if borrowings were not made.

Fluctuating value of the fleet: The value of the rigs owned by a company may fluctuate with market conditions. Any downturn in the market could have a material adverse effect on the company's liquidity and may result in breaches of its financial obligations. In such a case, sales of the rigs could be forced at prices that may represent a potential loss of value.

2.2 Operational Risk

Project risk: It is customary in the drilling service business for contracts to be structured as time charters or bareboat charters. In some instances, market participants may accept fixed prices for certain components of the overall contract work-scope. Such instances include mobilization and demobilization of a unit to/from a worksite, and the conversion/upgrade of units to meet specific requirements as may be required for a specific project. Generally, the policy is to mitigate project risk by assessing and addressing termination risk, breakdown risk, off-hire situations, weather events, force majeure risk, etc.

Rig operation: The fleet will be exposed to operational risks, such as breakdown, bad weather, technical problems, force majeure situations (nationwide strikes etc.), collisions and grounding.

Political instability: Many of areas are characterised by political instability. The rigs may be located in the immediate proximity of platforms, floating production vessels and other offshore oilfield infrastructure which could be subject to hostile attacks. Even though the rigs have insurance coverage against attacks, such an incident could result in a substantial loss of revenue and large policy claims to be handled.

Insurances: Operational risks include personal injury, the loss of a unit, operational disruption, off hire and termination of contract. In order to mitigate these risks, rig owners implement an insurance program in line with market practice, and additional insurance is always considered when a specific project is judged to be of a high risk nature.

Environmental Laws and regulations: Regulation of uncontrolled discharges to the marine environment via international conventions and legislation has become increasingly stringent and thus potentially poses a material risk. In addition oil companies typically attempt to devolve increased risk and liability to the contractor and require indemnification for an increased range of pollution events. Laws and regulations also affect the construction and ongoing operational cost of the rigs as health and safety requirements become progressively stringent over time.

Risk associated with upgrade, refurbishment and repair projects: Fleet requires refurbishment and repair from time to time, including when units are acquired or when repairs or upgrades are required by law, in response to an inspection by a governmental authority or when a unit is damaged. These upgrades, refurbishment and repair projects are subject to risks, including delays and cost overruns, which could have an adverse impact on our available cash resources and results of operations.

Accidents: Some drilling rigs operate in harsh environments. There are several factors that could contribute to an accident, including, but not limited to, human errors, weather conditions and faulty constructions.

2.3 Commercial Risk

Market risks: The demand for rigs will always fluctuate depending on global market drivers such as global oil and gas price levels, political climate, economical climate, availability of capital, operators' willingness to invest, levels of exploration and development for oil and gas, increases in the number of rigs available for contract, etc.. New entrants in the market could also have a negative effect on the contractual prices as well as market value.

Political and regulatory risks: Changes in the legislative and fiscal framework governing the activities of the oil companies could have a material impact on exploration and development activity in general or affect the Group's operations directly. In particular, changes in political regimes will constitute a material risk factor.

Taxation Risk: Generally drilling companies operate in numerous foreign jurisdictions in which it is subject to taxation. As a result, the ultimate tax liability is determined by the application of local tax laws and other regulations as interpreted by authorities in each taxable jurisdiction.

Requisition or arrest: Rigs could be requisitioned by a government in the case of war or other emergencies or become subject to arrest.

Oil prices: Extended periods of low oil prices historically lead to a reduction in exploration drilling as the oil companies scale down their investment budgets. The sharp reduction in production costs on new oil fields may reduce the strong historical correlation between day rates and oil prices.

Service life and technical risks: The service life of a new rig is generally assumed to be more than 30 years, but will ultimately depend on its capability. There will always be some exposure to technical risks, with unforeseen operational problems leading to unexpectedly high operating costs and/or lost earnings.

Credit risk of customers: Lack of payments from customers may significantly and adversely impair the liquidity**.**
Idle Period: It is possible for a unit cannot secure a contact at reasonless prices and hence becomes idle.

3. Evolution of Project Valuation Methods

Often the concept of "net present value" is used to determine whether the profits generated by a project would be worthwhile. Net present value does not take into account the flexibility inherent in certain projects. Net present value calls for to measure the project's eventual revenue—if it's successful—against the cost of investment using cash inflows. You can first invest money to explore the field and determine the best location for drilling, then decide whether or not to drill; you can also decide how much time and money to invest in exploring. Each option—to explore or not, explore quickly or slowly, drill or not—has a value in itself which should be factored into determining the overall value of the project. A real options analysis allows gauging the project's value with all the options taken into account.

The most common project valuation techniques are shown in Figure 1. The first set of valuation techniques is quite well known:

- Return on Investment (ROI)
- Net Present Value (NPV)
- Internal Rate of Return (IRR)

The second set of techniques is usually applied when a new rig is acquired or refurbished to fulfill a perceived marketplace need:

- Sensitivity Analysis
- Monte Carlo Simulation

The theory of real options is an extension of financial option which compliments DCF. A *call option* gives the holder the right, but not the obligation, to buy a security at a specified price in the future. The buyer of the call option is taking an optimistic view of the stocks underlying the call option. Similarly, a capital investment today that gives the investor the future right, but not the obligation, to make a further investment is a real option. A variety of factors can influence the value of the option. For example, as the value of the stock (or the present value of the expected cash flows) increases, so does the value of the call option. Real options are directly analogous to financial options in several ways, as shown in Figure 2.

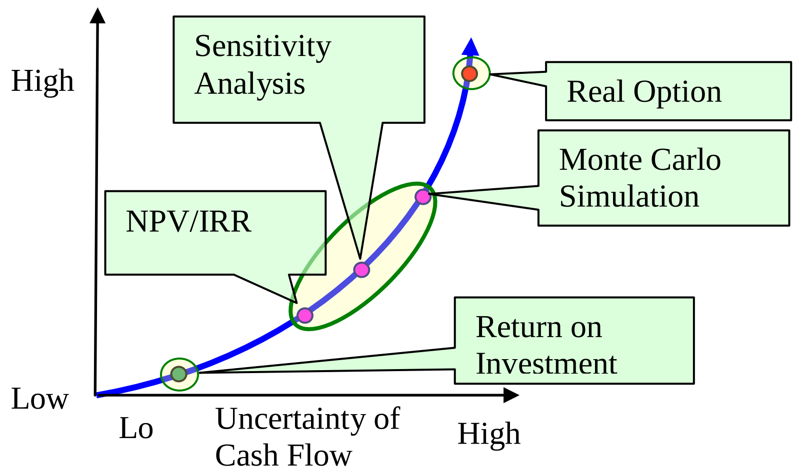

Figure 1: Value measurement techniques are shown along the blue curve from low to high.

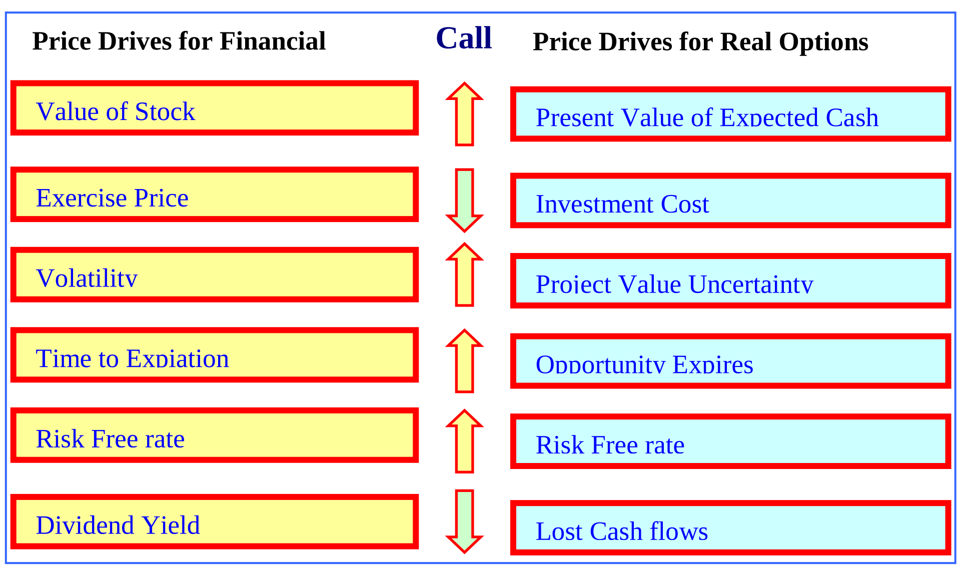

Figure 2: Comparison of real options and financial options

Another critical difference between DCF and real options is the effect of uncertainty (or risk) on value. Uncertainty typically is considered bad for the valuation of traditional cash flows. In contrast, uncertainty *increases* the value of real options. So, in today's uncertain environment, the value of options actually increases. After an investment is made, time passes, uncertainty is resolved and the present value of cash flows (analogous to the future value of a stock) can be calculated more accurately. If the environment is volatile, then the chance that the value of the project in the future will exceed the necessary investment (or, in other words, that the NPV will be positive) is higher.

Figure 3 shows two investments; one with a wide range of possible outcomes, the other with a relatively narrow range. In the former, more volatile scenario, there is a good chance that the project would have a positive NPV in the future;

hence a real option under this set of outcomes would have value. The latter, more stable scenario has no chance of producing a positive NPV. An option using the latter set of outcomes would have no value.

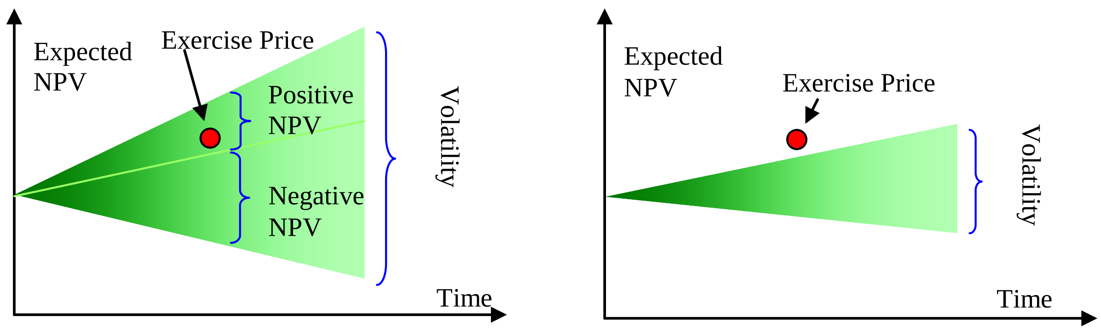

Figure 3: Volatility increases the value of real options

The most common definition of an *option* is an agreement between two parties, the *option seller*(writer) and the *option buyer*, whereby the option buyer is granted a right (but not an obligation), 'written' by the option seller to carry out some operation (or *exercise* the option) at some moment in the future for a price (the option premium). The predetermined price is referred to as *strike price*, and future date is called *expiration date* [2, 3 and 5].

There are several types of options, mostly depending on when the option can be exercised [5]. European options can be exercised only on the expiration date. American-style options are more flexible as they may be exercised at any time up to and including expiration date and as such, they are generally priced at least as high as corresponding European options. Other types of options are path-dependent or have multiple exercise dates (Asian, Bermudian).

4. Definitions of Variables

Value of the underlying asset: This variable is the foundation from which the option derives its value. For a stock option, it is the current stock price. In the case of real options on a project, it is the future cash flow. Given that the call option holder has the right to purchase the share for the fixed exercise price, the higher the current stock price (or the future cash flow for real assets) the more valuable is the call stock option.

Risk-free interest rate*:* This variable is the return on holding the risk-free asset such as government bonds, and remains identical in both financial options and real options framework. An increase in the risk-free interest increases the value of the call option.

Maturity (or expiry) date: This variable is the time remaining until the option expires, and after which the option ceases to exist. In general, a longer term to maturity increases the value of the call option, because the option provides its flexibility over a longer period of time.

Exercise price: For a stock call option, this is the fixed price at which the call option holder can purchase the stock. In the case of real options on projects, this represents the cost to develop the project. It should be noted that the exercise price is typically fixed throughout a financial option's life. However in real option the exercise price (the cost to develop) may vary through time. A lower exercise price increases the value of a call option.

Volatility: The value of a financial (or real) option is influenced by the uncertainty of returns on the underlying stock. Even though many systematic and non-systematic factors influence returns, a reasonable estimate of volatility for the purposes of computing the option value can be calculated by simply measuring the variation in historical returns on the traded stock; however, such information is not available for real assets. Techniques for doing this may vary, but the volatility implied in a competitive financial market are most likely going to reflect some reasonable estimate of future return volatility. On the other hand, there is no market exchange for real option opportunities that can be referenced to produce an implied volatility. There is no widely accepted technique that captures the systematic and non-systematic risks affecting the cash flows of real investments.

Exercise: Financial investors can exercise options almost instantaneously. Real investment opportunities can be much more complex and time consuming to act on. Companies, for many reasons, maintain varying degrees of agility or control; and this will affect their ability to exercise and capture the option payoffs from their project.

5. Binomial option model

The binomial model(Figure 4) represents the price evolution of the option's underlying asset as the binomial tree of all possible prices at equally-spaced time steps from today under the assumption that at each step, the price can only move up and down at fixed rates and with respective pseudo-probabilities P_u and P_d [7]. In other words, the root node is today's price, each column of the tree represents all the possible prices at a given time, and each node of value *S* has two child nodes of values Su and Sd, where *u* and *d* are the factors of upward and downward movements for a single time-step dt. Variables *u* and *d* are derived from volatility σ [7]

$u = e^{-\sigma\sqrt{dt}}$

and (1)

$d = e^{\sigma\sqrt{dt}}$ (2)

P_d is simply equal to $1 - P_u$ and P_u is derived from the assumption that over a period of dt the underlying asset yields the same profit as a riskless investment on average, so that if

it is worth *S* at time *t*, then it is worth Se^{rdt} at time $t + dt$. This leads to the following equation:

$$Se^{rdt} = (P_u uS + (1 - P_u)dS) \quad (3)$$

from which we deduce

$$P_u = \frac{Se^{rdt} - d}{u - d} \quad (4)$$

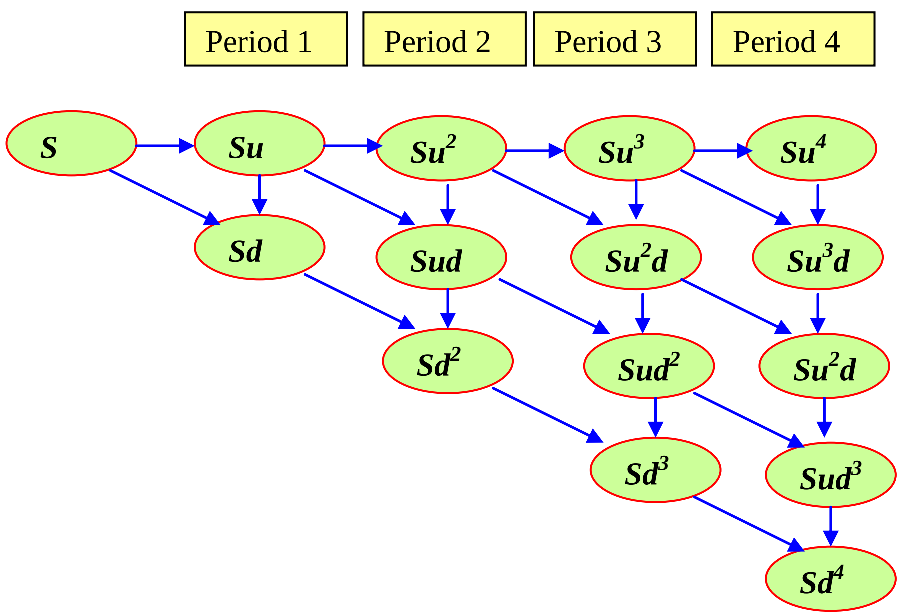

Figure 4: Binomial tree with 4 steps

From the binomial tree representation, we can then iteratively derive the option price for each node of the tree, starting at the leaves. At each leaf of the tree (i.e. at option expiry) deriving call and put option price is simple:

$$V_{call} = \max(X - S, 0) \quad (5)$$

Indeed, if market price S at expiry date is greater than strike price X, a call option returns its holder $S - X$ dollars of profit — for a same-day sale transaction — or zero profit otherwise.

$$V_{put} = \max(X - S, 0) \qquad (6)$$

Similarly, if market price S at expiry date is lower than strike price X, a put option gives its holder $X - S$ dollars of profit, or zero profit otherwise.

Having calculated all possible option prices at expiry date, we start moving back to the root, using the following formula

$$V_t = \left(P_u V_{u,t+1} + P_d V_{d,t+1}\right)e^{-rdt}, \qquad (7)$$

where V_t is the option price for one of the nodes at time t and $V_{u,t+1}$ and $V_{d,t+1}$ are the prices of its two child nodes. This formula is derived from the observation that an option which is worth V_t at time t, is worth $V_t e^{rdt}$ at time, and its expected value on the other hand, which is, $P_u V_{u,t+1} + P_d V_{d,t+1}$, by definition.

The up multiplier u predicts that if the stock price makes an up movement, it will rise by an incremental amount related to time and volatility. The expression $u = e^{-\sigma\sqrt{dt}}$ actually has its roots in physics of "Brownian motion", not statistics or finance. Brownian motion is the motion of gas molecules as they travel randomly through space, having their direction of motion altered by chance as they bang into each other.

Example 1: A stock option is a contract giving the holder the right to purchase a stated number of shares of stock at a fixed price in a specified period of time. Typically, the holder is under no obligation to exercise his/her rights. The value of a stock option consists of two parts: intrinsic value and time value. Intrinsic value is defined as the difference between the stock's value and the exercise price (the price at which the option holder can purchase the stock). The intrinsic value is never less than zero since the contract involves no liability on the part of the option holder but can be higher. For those options where the exercise price is greater than the current stock price ("out of the money"), the intrinsic value is zero, although such options may still have time value. The time value of a stock option is the present value of the expected difference between the value of the stock at the time of exercise and the option's exercise price.

Consider a company giving shares to its mangers. The underlying stock price is $30 and the exercise price is also $30 and hence it is currently at the money. Mangers must hold these shares for 5 years. The volatility of the company's share is 43%. The risk free rate is assumed to be 5%.

The up factor (u) and down factor (d) are calculated using Equations (1) and (2) to be 1.537258 and 0.650509 respectively. We start with the underlying value of $30 and multiply it with u and d to obtain $46.118 and $19.515respectively. Starting with $46.118 and multiplying it by u and d we obtain $70.895 and $30.000. Multiplying $19.515 by u and d we obtain $30.00 and $12.695 respectively. The remaining of the lattice entries are calculated by in this manner.

To build the decision lattice, we start with the terminal nodes. The Node **(Su^5)** shows a value $**227.546M** (=257.546-$30 (exercise price)). The value of all other node in this column is determined by subtracting the exercise price ($30) from the value of the node.

Moving to the intermediate Node **(Su^2d^2)**, the exercise value of scaling up at this node is 19.515-$30=-$10.485; i.e. the stock option has no value. Similarly the option value at Nodes **Sud^4** and **Sd^5** are also zero**.**

The binomial value of continuing the solution at the node **(Su^2d^2)** is calculated with Equation (7), as

$$\left[p(\$78.984) + (1-p)(\$16.118)\right]\exp(-0.05 \times 1) = \$6.929$$

$p = 0.451946$ is calculated by Equation (4). The option is in the money at this node.

Rolling back to the starting point (Node **(S)**) using backward induction method, the value of the scaling up is $13.854, which mangers should gain for each stock at the end of expiration date.

Start	**1**	**2**	**3**	**4**	**5**
S 30.000	**Su** 46.118	**Su^2** 70.895	**Su^3** 108.984	**Su^4** 167.536	**Su^5** 257.546
13.854	**25.716**	**46.559**	**81.838**	**138.999**	**227.546**
	Sd 19.515	**Sud** 30.000	**Su^2d** 46.118	**Su^3d** 70.895	**Su^4d** 108.984
	5.368	**10.934**	**21.822**	**42.358**	**78.984**
		Sd^3 12.695	**Sud^2** 19.515	**Su^2d^2** 30.000	**Su^3d^2** 46.118
		1.281	**2.979**	**6.929**	**16.118**
			Sd^4 8.258	**Sud^3** 12.695	**Su^2d^3** 19.515
			0.000	**0.000**	**0.000**
				Sd^4 5.372	**Sud^4** 8.258
				0.000	**0.000**
					Sd^5 3.495
					0.000

Figure 5: Binomial tree and Option value (shaded area and bold face) for Example 1.

Example2: A drilling firm's assets, currently valued at \$330M, vary with an annual volatility of 35%. If the risk-free rate is 5% and the firm holds \$160M of debt, is bankruptcy likely?

We use a one-year horizon with three-month steps. We also, price the stock as a call option on the firm's assets using the Binomial Lattice. Results are shown in Figure XX. It can be seen that the firm worth \$177.803M. Even at worth state of nature the firm is solvent and worth \$8.864M.

Period	**1**	**2**	**3**	**4**
330.000	390.174	461.320	545.440	644.898
177.803	**236.063**	**305.271**	**387.427**	**484.898**
	279.106	330.000	390.174	461.320
	124.995	**173.950**	**232.161**	**301.320**
		236.062	279.106	330.000
		80.012	**121.094**	**170.000**
			199.655	236.062
			41.643	**76.062**
				8.864

Figure5: Binomial tree and Option value (shaded area and bold face) for Example 2. Values are in \$M

6. The Effect of Idle Time

Repeated short-term leases are a feature of drilling as well as many other capital intensive operations. In lease contracts, operators (lessees) require services that are short term relative to the life of the equipment and may be repeated, possibly at different locations, many times. Leases often include options affecting lease length, particularly extension, termination, and assignment or sublet pitons.

The *idle time* between consecutive lease contracts, i.e., the level of equipment or service utilization is an important factor in viability of these services. Certain options in leasing are specifically designed to make the exercise decision independent of the leasing rate. For example, an extension option on a lease may be specified as floating or at market rate, meaning that if the extension option is exercised, the renewal leasing rate will be the prevailing leasing rate at the time of exercise. However, the owner of the asset may still be affected by the existence of the option because of the uncertainty regarding the time when the asset will be available. The compensation to the owner of the asset for the options in the contract reflects differences in expected cash flows between the leased asset and the market resource -caused by the option presence and exercise- between the start of a lease and the start of the subsequent lease. While the expected discounted value of a lease contract is unique, there may be many payment schemes with the same expected value.

The inclusion of idle time can result in qualitatively different effects on asset utilization rates. The presence of extension options, for example, can lead to *both* a decrease and an increase in the utilization rate of an asset. Consider the case of a tight market, where a large percentage of rigs have contract, and a leasing contract with an extension option with a short notification time that is unlikely to be exercised. This contract leaves the leasor with an under-utilized asset, for which the leasor will demand compensation. On the other hand, in down markets, with only a few rigs have contract, a lease with an extension option with a high probability of exercise would reduce under-utilization. Current business practice indicates that in such situations a discount may be offered, especially when the utilisation is low.

Example 3: Drilling contracts commonly confer flexibility to the prospector by allowing drilling additional wells if market and geological conditions are favourable for drilling. Such contracts consist of a fixed-price component of the contract extending to the option's exercise date and that of a second stage of the contract given the possibility that the option might be exercised. The first part of the contract can be valued through use of simple NPV techniques. The second stage is valued with Real Option.

Consider a contract which has provided for the drilling of 6 wells at a fixed price with the option to extend the contract to two more wells at the same price. If it will take 60 days to drill each well at a contracted day rate of $112,500 then the value of the contract is the present value of 12 months' total revenue of $40,500,000 plus the expectation of the value of the two additional wells being drilled.

The example presented here uses a six-step binomial tree, and hence each time period lasts 2 months. It is assumed that the annual volatility is 40% giving the bimonthly (or local) rate of $\sigma_{local} = \sigma\sqrt{t} = 0.4\sqrt{1/6} = 0.1633$. It is also assumed that the bi-monthly risk free interest rate is 0.008165 giving an annual rate of 0.05. These data leads to the parameters as noted in Table 1. Movement of the day rate is shown in Table Figure 7.

$$\Delta t = 1 \qquad d = e^{-\sigma\sqrt{\Delta t}} = 1/u = 0.8493$$

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.1633\sqrt{1}} = 1.1774 \qquad p_u = \frac{\left(e^{(r-b)\Delta t} - d\right)}{(u-d)} = 0.61556 \quad p_d = 1 - p_u = 0.38444$$

Table 1: Parameters of the binomial tree of Figure 7 (Example 3)

1	2	3	4	5	6
$ 112,500	$ 132,456	$ 155,953	$ 183,617	$ 216,188	$ 254,538
$ 37,496	**$ 50,915**	**$ 67,969**	**$ 88,963**	**$ 113,839**	**$ 142,038**
	$ 95,550	$ 112,500	$ 132,456	$ 155,953	$ 183,617
	$ 16,810	**$ 24,696**	**$ 35,802**	**$ 51,030**	**$ 71,117**
		$ 81,154	$ 95,550	$ 112,500	$ 132,456
		$ 4,542	**$ 7,439**	**$ 12,184**	**$ 19,956**
			$ 68,928	$ 81,154	$ 95,550
			$0	**$0**	**$0**
				$ 58,543	$ 68,928
				$0	**$0**
					$ 49,722
					$0

Figure 7: The Binomial tree for Example 3. Option value is shown in shaded areas.

The market day rate at the beginning is $112500. At the end of period one this either goes up by a u=1.1774 to $132,456 or go down by d=0.8493 to $95,550. In the next period $132,456 can go up to $155,953 or go down to $112,500. Also, $95,550 can go up to $112,500 or go down to $81,154. The rest of entries in Figure 7 are found in this manner.

If day rates fall below the contract prices at the end of 12 month, the option to renew will not be exercised or contract terms might need to be renegotiated. In this situation the value of the option is zero but cannot become negative. If market rates rise, the prospector, by exercising the option, captures the lower rate in the original contract.

Contract without option Minus Option value Total contract value	\$ 54,000,000 \$ 4,499,547 \$ 49,500,453	(\$112500x60 days per well x 8 wells) (\$32398x60 days per well x 2 wells)
Alternatively		
Contract Value for the first 6 wells Contract value for remaining 2 wells Option value of 2 wells Total contract value	\$ 40,500,000 \$ 13,500,000 \$ 4,499,547 \$ 49,500,453	(\$112500x60 days per well x 6 wells) (\$112500x60 days per well x 2 wells) (\$32398x60 days per well x 2 wells)

Table 2: Contact value calculation using a six period model

At Node u^5V the market day rate is \$254,538 giving the proposctor\$142,038= \$254,538-\$11250 and hence the prospector should exercise his option. At a node, where the market rate exceeds the contact rate, prospector exercises his option. At Node d^3u^2V the falls below the contract rate and hence the prospector is better off to re-negotiate the rate. The value at Node u^4V is a dependant on Nodes u^5V *and* du^4V, i.e.

$$V = (\$254{,}535 \times 0.61556 + \$183{,}617 \times 0.38444) \times EXP(-0.0008165 \times 1) = \$113{,}83$$

However, this calculation for d^3uV gives a negative value, thus the value of option is zero. Rolling back to the initial node gives the value of option as \$37,494- See Figure 7.

The total contract value is the present value of drilling 8 wells at \$112,500/day less the value of the extension option - underwritten by the drilling contractor - of \$4,499,547 for the last two wells- Calculations are shown in Table 2. In contrast, NPV analysis produces a higher contract value: \$54 million (8 wells at \$112,500/day).

Early termination- The option to terminate early given sufficient notice is also common in drilling contracts. This is the equivalent of an American put because it can be exercised at any time before the end of the contract period. Consider a fixed-price, 5-month drilling contract includes the option to terminate early- Assume values of all parameters are given above are still hold. During the course of the contract, day rates may move from their original level and thus, if the wells still need to be drilled, the option to terminate early effectively becomes the option to switch into a lower-priced drilling contract. The difference between the two contract values is the intrinsic value of the option at that particular point in time. Figure 8 shows the binomial tree for the day rate of this floating rate contact. The last row of Figure Y3 gives the contact value as product of the initial day rate times the remaining days.

	Start	1st Month	2nd Month	3rd Month	4th Month	5th Month
Binomial movement of the day rate	\$112,500	\$126,270 \$100,232	\$141,726 \$112,500 \$89,301	\$159,073 \$126,270 \$100,232 \$79,563	\$178,544 \$141,726 \$112,500 \$89,301 \$ 70,886	\$200,398 \$159,073 \$126,270 \$100,232 \$79,563 \$63,156
Fixed rate contact	**\$20,250,000**	**\$16,875,000**	**\$13,500,000**	**\$10,125,000**	**\$ 6,750,000**	**\$3,375,000**

Figure 8: Binomial tree for the movement of day rate. Last row is the value of a fixed rate contact. Fixed rate contract at any point in time is the fixed day rate times the number of remaining days of the contact.

Figure 9 shows the binomial value of the contract value as the product of the current day rate times the number of remaining days. At the beginning of each month the prospector has to decide whether to exercise his option or not. If the payoff is greater than the expected value of the contract in the next period then the prospector should exercise his option.

1	2	3	4	5	6
$20,250,000	$18,940,515	$17,007,081	$ 14,316,572	$10,712,623	$ 6,011,929
$ 0	**$ 2,065,515**	**$ 3,507,081**	**$ 4,191,572**	**$ 3,962,623**	**$ 2,636,929**
Wait	**Wait**	**Wait**	**Wait**	**Wait**	**Wait**
	$15,034,735	$13,500,000	$ 11,364,309	$ 8,503,541	$ 4,772,191
	$ -1,840,265	**$0**	**$ 1,239,309**	**$ 1,753,541**	**$ 1,397,191**
	Exercise	**Wait**	**Wait**	**Wait**	**Wait**
		$10,716,125	$ 9,020,841	$ 6,750,000	$ 3,788,103
		$ -2,783,875	**$ -1,104,159**	**$ 0**	**$ 413,103**
		Exercise	**Exercise**	**Wait**	**Wait**
			$ 7,160,626	$ 5,358,062	$ 3,006,947
			$ -2,964,374	**$ -1,391,938**	**$ -368,053**
			Exercise	**Exercise**	**Exercise**
				$ 4,253,160	$ 2,386,875
				$ -2,496,840	**$ -988,125**
				Exercise	**Exercise**
					$ 1,894,671
					$ -1,480,329
					Exercise

Figure 9: Decision tree for option valuation and exercise timing

The shaded rows in Figure 9 show the intrinsic payoff at each node. To determine this, the fixed rate of contract (last row of Figure 8) is subtracted from value of each node. If the value is negative (negative pay off) then the prospector should exercise otherwise wait until the next period. Starting at the last nodes of Figure 9 and applying this rule, the binomial decision tree can be populated as shown in Figure 9 (the shaded areas).

7. Case Study

7.1 The Project and its Net Present Value

Mid 2008 a Gulf of Mexico jack up which was damaged in a hurricane was offered to Zirak Drillers for $43M. The jack up had lost all its legs, but the hull was almost intact. All equipment seemed to be in working order, but requiring overhaul. Zirak negotiated the price and took an option to buy the jack up for $25M (paid end of Q2 2009). The option premium, inspection and towing to the yard were $1M. Zirak Drillers are hoping to extend the legs from 340 foot to 410 feet as well as well as extending the cantilever and results of preliminary analyses (cost included in the initial payment $1m) were quite encouraging. The classifying authority did not reject the idea outright, but had serious reservation. After some enquiry Zirak Drillers got a budget estimate of $4M to cover the cost of design and $81M to cover the cost of renovation (all at Q2-2008 Prices). A newly design spud-can will also be fitted to make suitable for its new location and condition. These compared favorably with the cost of a new design at the time. Zirak Drillers thought that with the intended enhancement of specifications they will get better day rate and lower idle time.

Basic Data	Q2-08 prices ($)		Assumptions					
Number of days chargeable per year			320	% Equity		100.00%	Inflation	2.50%
Day Rate			113,000	% Debt		0.00%	Utilisation	85%
WACC			0.16	Interest Rate Paid		0.00%	Local Tax	0.000
Salvage Value (-) Future price			180,000,000	Period				
				1	2	3	4	5
Capital Expenditures			PV (2008)	Q4 2008	Q2-2009	Q4-2009	Q2-2010	Q4-2010
Option premium +Survey & legal costs			$ 1,000,000	-1,000,000				
Design Contractor's fee & Admin+moring	Budget		$ 4,000,000			-4,997,433		
Price of hull			$ 21,551,724		- 25,000,000			
Fabrication	Budget		$ 80,000,000				-75,353,600	-34,782,133
Working Capital, Debt Reserve	Budget		$ 1,000,000					- 1,449,256
Disposal Expense	Budget		$ -					
Total Capital Expenditures (PV)			$ -107,551,724	-1,000,000	-25,000,000	-4,997,433	-75,353,600	-36,231,389
Net Cash Flow			$ 108,760,093					
NPV			$ 1,208,369					

Figure 10: Summary of assumptions for the case study

Zirak Drillers is a up and coming drilling company with three jack ups which takes high risk ventures in the hope of growing fast. Their cost of capital is 16%. Figure 10 summarizes the assumptions.

At the end of fifth period $1,000,000 (Q2-2008 value) working capital will be injected into the business without increase during the lifetime of the rig. The residual value of the rig is assumed to be 180,000,000 at the end of its 25-year service life.

The present value of the rig (PV), i.e. the exercise price, is calculated using certain assumptions, following traditional valuation techniques, such as the discounted cash flow. Results are presented in Appendix and a summarized in Figure 10. For the assumed data the net present value is $1,208,369and hence this venture can be accepted. This is not the real value of the project as NPV neglects managerial flexibility and hence cannot capture a project's true value. If the price of hull was $1.21M more, then NPV would be negative and hence the project would be rejected.

7.2 Expected Net Present value

Some firms' capital-budgeting decisions are based on the expected net present value (ENPV) model, which is a variation of decision tree models. This was especially developed to capture the effect of ***technical uncertainty*** (represented by probabilities to succeed for each phase) on the value of a project. If the prediction of the future cash flows of the project is uncertain the decision tree model was supplemented with sensitivity and scenario analyses. Using these two types of analyses a firm tries to take into consideration the impact of economic uncertainty on the project value. For instance, sensitivity analysis ranks input variables of the future cash flows (e.g. potential market share, unit price, unit costs, etc.) according to their contribution to the risk of the project present value. Managers attribute to the most risky input parameters their possible minimum and maximum values and compute the expected net present values of the project that correspond to these extremes. However, the calculated set of mutually exclusive outcomes of the project value does not facilitate the undertaking of the right investment decision because the true probability of each outcome is unknown.

Revenues are calculated using the expected day rate, number of day chargeable per year and the expected utilization of the rig. The net income before taxes and bank interests is calculated from the gross income by deducing depreciation and taxes, as well as all operating expenses. Employing a corporate tax rate of 35%, the provision of taxes is subtracted resulting in net income after taxes. Finally, the net cash flows are calculated by adding back depreciation, which is a non-cash charge, subtracting out annual capital expenditures (such as maintenance, repair insurance etc.). Working Capital is assumed to be part of the expenditure and hence the free cash flow is not adjusted. The calculated NPV strongly depends (among other things) on the cost of capital (Figure 11). For a firm with a slightly higher WACC the NPV will be negative. The internal rate of return (IRR) for this data is about 16.14%

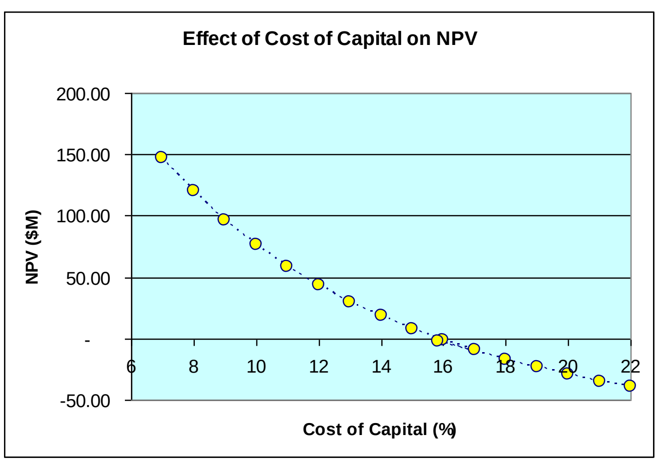

Figure 11: Effect of Weighted Average cost of capital (WACC) on NPV

The annual cash-inflows and outflows are discounted using the weighted average cost of capital (*WACC=16%*). Thus, after discounting the free cash flows, we obtained a total present value of $1,208,369. This is the net present value at the beginning of Q2-2008. The Project could generate this $1,208,369, only if the probability of estimates for all stages to be correct is 100%. The probability of being correct is, however, 100% in phase I, 100% in phase II, 85% in Phase III and 95% for the fabrication phase. Also, the revenue has 85% probability to be correct. Thus, the net present value should be decreased with the corresponding probabilities associated to the different phases, respectively. This adjustment results in a value of $328,230, which is the *expected net present value* at the end of Q2-2008.

Period	**1**	2	3	4	5	Operation
Probability of Success	100%	100%	85%	95%	95%	85%
Cumulative Probability	100.00%	100.00%	85.00%	80.75%	76.71%	
Probability adjusted Cash	$ -1,000,000	$ -21,551,724	$ -3,400,000	$ -46,172,000	$ -19,994,125	$ -92,117,849
PV Cash inflow						$ 92,446,079
ENPV						$ 328,230

Table 3 Expected Net Present value (ENPV) for the case study

The ability of ENPV analysis to account for the managers' flexibility related to the resolution of technical uncertainty is its advantage over traditional NPV. The latter views investments as now-or never opportunities. It means that deciding to invest in the current phase of the project, the company's managers pre-commit to undertake all future phases of the project even if one of them fails. Thus, traditional NPV includes all costs with certainty.

7.3 Project's Volatility

The Crystal Ball® program was used to make the Monte Carlo simulation necessary to estimate the percent variation of standard deviation from project present value along the years, named, the project's volatility [9]. This project return rate is, calculated by

$$Z = \ln\left(\frac{PV_1 + CF_1}{PV_0}\right) \qquad (1)$$

where

$$PV_0 = \sum_{t=1}^{T} \frac{CF_t}{(1+WACC)^t} \qquad (2)$$

and $$PV_1 = \sum_{t=2}^{T} \frac{CF_t}{(1+WACC)^{t-1}} \qquad (3)$$

In the Crystal Ball® program, using the spreadsheet for cash-flow calculation, the uncertainty variable correspondent to the day rate was defined, using a normal distribution, with mean $110000 and standard deviation of 0.43.

A simulation was made with five thousand trials and the standard deviation value of the rate of return on the project is 41% which is the project annual volatility. Drilling companies use values ranging from 0.35 to 0.55 for volatility.

7.3 Real Option Valuation

The project started at the end of second quarter of 2008 by spending $1M to survey, tow and buy the option with a promise to pay $25M at the end of 2008. It is assumed that this process takes 2.5 years from start to finish, divided into five of 6 month long- see Figure 12. It is estimated that it takes $5M to produce design drawing and prepare bid documents and takes 6 month to finish. The fabrication is assumed to take 18 month to complete. The budget estimate of fabrication and all other costs is $81M (Figure 12).

$$\Delta t = \frac{y}{N} = 2.5/5 = 1/2 \qquad p_u = \frac{\left(e^{(r-b)\Delta t} - d\right)}{(u-d)} = 0.42802$$

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.41\sqrt{1/2}} = 1.336312 \qquad p_d = 1 - p_u = 0.57198$$

$$d = e^{-\sigma\sqrt{\Delta t}} = 1/u = 0.748328$$

Table4: Parameters of the event tree

Table 4 show parameters needed to build the event tree.

	Data	First Option	Second Option	Third Option	Fourth Option
Five Time Steps From the end of	Q2-2008	Q4-2008	Q2-2009	Q4-2009	Q4-2010
Option Expiration period (6 months)		1	1	1	2
Project Volatility	41%				
Capital Expenditure (M)-Exercise price	$ **107.552**				
Risk Free Rate	5,00%				
Dividends Rate	0,00%				
Expenditure Timing (PV=$107.552M* Q2 2008)	$1M	$25.00M	$4.997M		$117.390M

* $ **107.552** is the Investment Present Value in the year at the start of Q2-2008 discounted by 16%: 1.0 +25.0/1.16 ^{1} +4.997/1.16 $^{1..5}$+117.390/1.16 $^{2..5}$

Figure 12: Summary of the rig's estimated expenditure requirements

Initially the present value of the asset (**$108.760M**) is introduced in period 0. The other elements of the first line are calculated by multiplying the previous element by u . Each remaining element is obtained by multiplying the element of previous column by d . The constructed event tree is shown in Figure 13. The PV of project is V = $108.760M which can go up by a factor u=1.33632 to $145.337M or go down by a factor of 0.758328 to $81.388M. In the next period, $145.337 can go up to $194.216M or down to $108.760M. Similarly, $60.905M can go up to $108.760M or down to $60.905M. The rest of entries in Figure 13 are calculated in a similar fashion. The event tree of Figure 12 shows the evolution of the rig values at each period until its operation.

Phase	I	II	III		
	Q4 2008	Q2-2009	Q4-2009	Q2-2010	Q4-2010
	***V* 108.760**	***uV*** 145.337	***u²V*** 194.216	***u³V*** 259.534	***u⁴V*** 346.818
		dV 81.388	***duV*** 108.760	***du²V*** 145.337	***du³V*** 194.216
			d² 60.905	***d²uV*** 81.388	***d²u²V*** 108.760
				d³V 45.577	***d³uV*** 60.905
					d⁴V 34.107
Cost at the time	**-1.000**	**-25.000**	**-4.997**		**-117.390**

Figure 13: Event tree of the case study- all figures are in $M.

The valuation begins at the last columns and it is rolled back to the beginning in a process known as backward induction. In the last period the option value is calculated as follows:

$$V = Max[S - X;0] \qquad (8)$$

where V = Real Option Value, S = Event tree PV and X = price to exercise the option.

At node $\boldsymbol{u^4V}$ the project worth S=$346.818 but we need to spend X=$107.390M to get this and hence the value of this node is $346.818M-$117.390M = $229.428. Similarly the value of $\boldsymbol{du^3V}$ is $76.827. The value at Node $\boldsymbol{d^2u^2V}$ ***is***

$$V = Max[\$108.760 - \$117.390M;0] = -\$8.6$$

Since a profit maximizing agent would walk away when the value of the node is negative, a zero will be entered at this node. Similarly all negative nodes have zero value to the investor (Figure 13). That is to say, when the project present value (***S***) calculated in event tree is higher than the invested value (***X***), the option should be exercised and its price will be ***S – X***. Otherwise, the option should not be exercised, and its value is zero.

To calculate the value at Node $\boldsymbol{u^3V}$ requires the future value of the projected cash flows be discounted back to the present at risk free rate, i.e.:

$$S = [S_u p + S_d (1-p)]e^{-rt} = [229.428 \times 0.4282 + 76.827 \times 0.57197]e^{-0.05\times 0.5} = \$138.634M$$

The same approach gives $30.173M for Node ***du²V.***

For Node ***d²uV*** we have

$$S = [S_u p + S_d (1-p)]e^{-rt} = [0 \times 0.4282 + 0 \times 0.57197]e^{-0.05\times 0.5} = 0$$

Figure 14 is completed by this rolling back or backward induction method and a value of $2.050M is obtained as the real option value of the project.

I	II	III		
Q4 2008	Q2-2009	Q4-2009	Q2-2010	Q4-2010
V 2.850	***uV*** 9.224	***u²V*** 70.768	***u³V*** 138.634	***u⁴V*** 229.428
	dV 0.000	***duV*** 8.391	***du²V*** 32.072	***du³V*** 76.827
		d² V 0.000	***d²uV*** 0.000	***d²u²V*** 0.000
			d³V 0.00	***d³uV*** 0.000
				d⁴V 0.000

Figure 14: Rig's decision tree- all figures are in $M.

In calculating entries of Figure 14 we ignored that the rig has some positive market value as conversion to accommodation barge or as a scrap metal. It is assumed that if everything fails the hull worth $10M.

At Q4-2009 nodes when the value of a node is less than $10M, a value of $10M is entered and rolled back again. With this option to sell for $10M the value of real option rises to $2.850 (Figure 14). Note that a value of $10 was entered for Nodes ***duV*** *and* ***d² V*** , but Zirak would walk away at Node ***dV*** and hence the value the Node is zero. However, if Zirak is legally bound to pay $25M, then the value of the Node is $10M and the option value would be $8.804.

I	II	III		
Q4 2008	Q2-2009	Q4-2009	Q2-2010	Q4-2010
V 8.804	***uV*** 10.121	***u²V*** 70.768	***u³V*** 138.634	***u⁴V*** 229.428
	dV 10	***duV*** 10.0	***du²V*** 32.072	***du³V*** 76.827
		d² V 10.0	***d²uV*** 0.000	***d²u²V*** 0.000
			d³V 0.00	***d³uV*** 0.000
				d⁴V 0.000

Figure 15: Option value assuming the hull can be sold for $10.

Remembering that $25M is the hull's price, one can conclude that the total value of renovation for Zirak Drillers is $33.8M. However, for another company with lower cost of capital this hall worth more. This contrasts with the net present value of minus $1,208,369and calculated with no flexibility, using the traditional discounted cash-flow method.

The decision tree (Figure 16) shows the optimal strategies to be applied to the investment forecasting.

I	II	III		
Q4 2008	**Q2-2009**	**Q4-2009**	**Q2-2010**	**Q4-2010**
Invest $1M	***Invest $25M***	***Invest $4.99M***	***Wait***	**Invest $117.390M**
	Transfer Rights	***Transfer Rights***	***Wait***	**Invest $117.390M**
		Transfer Rights	***Wait***	***Don't Invest***
			Wait	***Don't Invest***
				Don't Invest

Figure 16: Decision tree for the case study

7.4 Option to Exit

DAANA Drilling operates is shallower water and has a dozen of rigs in their portfolio. Daana Drilling believes they can use this rig but it is too costly and makes the offer of taking this project for sums noted for each stage in Figure 17, for a premium of $10M paid upfront. This create an exit option for Zirak Drillers and the question is how much this offer worth to Zirak.

At Q4-2010 nodes (Figure 14) when the value of a node is less than $80M, a value of $80M is entered and rolled back to Q2-2010. At this date Daana is willing to pay only $30M but non of the entries fall below $30M and hence Daana's offer has no value. Rolling back to the beginning we note that the amount that Daan's agrees to pay worth less than the node value. As can be seen from Figure 17 the option value for this exit option is $46.789M.

Phase	I	II	III		
	Q4 2008	Q2-2009	Q4-2009	Q2-2010	Q4-2010
	46.789	55.216	97.142	140.405	229.428
		44.345	71.101	78.025	80.000
			71.101	78.025	80.000
				78.025	80.000
					80.000
DAANA's Offer		**10**	**10**	**30**	**80**

Figure 17: Option to exit- all figures are in $M.

Exit Option worth $46.789M-$8.804M-$10M (option premium)=$28M to Zirak.

Maximum loss= -$107,551,724 (PV of expenditure) -$10(Daana's premium)+$80M(Daana pays)=-$37.6M

8. Conclusions

Advances in pricing methods for financial securities have served to benefit investors in the valuation of the strategic options. There is great potential for using such asset pricing theories to the evaluation of real investments, and making capital allocation decisions, value the businesses, and assess performance. Pricing real options arguably involves as much art as science, and the application of traditional models can produce misleading output. This paper has outlined the binomial approach which is designed to bring about heightened understanding of strategic values of real option opportunities.

The specific numerical examples in this paper, demonstrated the value of flexibility in decision making. This result is not surprising because of the asymmetric risk structure in the model that arguably corresponds with the underlying reality. The benefit of flexibility-to-change in an upward scenario may be quite substantial while the benefit in a downward situation may be comparatively small.

Real options recognize that the ability to delay, suspend, expand or abandon a project is valuable when there exists flexibility associated with management decision making. ROA offers a better tool to guide investment decisions in the context of uncertainty and flexibility. In practice, managers realize that the value of timing in making investments is significant considering rapidly evolving market conditions and an uncertain business climate. A key contribution of our analysis involved understanding the complexities of the projects in order to determine their interdependencies, how one project can be leveraged to launch other projects, and its impact on projected business benefits.

We conclude that the practical measurement of the value of flexibility should include real option thinking that can provide valuable information to management.

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