Bike Redistribution Optimization Problem Formulation

Objective

Minimize the total number of bike transfers between stations while balancing the bike distribution across all stations.

Definitions

- ullet Let N be the number of bike stations.
- Let S_i be the current number of bikes at station i (i.e., $S_i = \text{CurNumberOfBikes}[i]$).
- Let M_i be the maximum capacity of bikes at station i (i.e., $M_i = \text{MaxNumberOfBikes}[i]$).
- Let T_{ij} be the number of bikes transferred from station i to station j.
- Let A be the average number of bikes that should be at each station, calculated as:

$$A = \frac{1}{N} \sum_{i=1}^{N} S_i$$

Decision Variables

• T_{ij} : Integer variable representing the number of bikes transferred from station i to station j. This variable must be non-negative:

$$T_{ij} \ge 0 \quad \forall i, j$$

Objective Function

Minimize the total number of bike transfers:

Minimize
$$Z = \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} T_{ij}$$

Constraints

1. **Supply Constraints:** The number of bikes transferred out of each station cannot exceed the current number of bikes available:

$$\sum_{j=1, j \neq i}^{N} T_{ij} \le S_i \quad \forall i$$

2. **Demand Constraints:** The total number of bikes received by each station plus the current number of bikes must be at least equal to the average number of bikes:

$$\sum_{j=1, j\neq i}^{N} T_{ji} + S_i \ge A \quad \forall i$$

3. Non-Negativity: The transfers must be non-negative:

$$T_{ij} \ge 0 \quad \forall i, j$$

4. Integer Constraint: The transfer variables are integers:

$$T_{ij} \in \mathbb{Z} \quad \forall i, j$$

Summary

This problem formulation focuses on redistributing bikes among multiple stations to ensure a balanced supply across all stations while minimizing the total number of bike transfers. The objective function aims to reduce the number of movements, while the constraints ensure that no station transfers more bikes than it has and that each station receives enough bikes to meet the average target.