

**Table 1** Microeconomic Model Calibration

| Calibrated Parameters                 |                 |       |                      |
|---------------------------------------|-----------------|-------|----------------------|
| Description                           | Parameter       | Value | Source               |
| Permanent Income Growth Factor        | $\Gamma$        | 1.03  | PSID: Carroll (1992) |
| Interest Factor                       | $R$             | 1.04  | Conventional         |
| Time Preference Factor                | $\beta$         | 0.96  | Conventional         |
| Coefficient of Relative Risk Aversion | $\rho$          | 2     | Conventional         |
| Probability of Zero Income            | $\wp$           | 0.005 | PSID: Carroll (1992) |
| Std Dev of Log Permanent Shock        | $\sigma_\psi$   | 0.1   | PSID: Carroll (1992) |
| Std Dev of Log Transitory Shock       | $\sigma_\theta$ | 0.1   | PSID: Carroll (1992) |

**Table 2** Model Characteristics Calculated from Parameters

| Description                         | Symbol and Formula   | Approximate<br>Calculated<br>Value |
|-------------------------------------|--|------------------------------------|
| Finite Human Wealth Measure         | $\mathcal{R}^{-1} \equiv \Gamma/R$   | 0.990                              |
| PF Finite Value of Autarky Measure  | $\sqsupset \equiv \beta\Gamma^{1-\rho}$  | 0.932                              |
| Growth Compensated Permanent Shock  | $\underline{\psi} \equiv (\mathbb{E}[\psi^{-1}])^{-1}$   | 0.990                              |
| Uncertainty-Adjusted Growth         | $\underline{\Gamma} \equiv \Gamma\underline{\psi}$   | 1.020                              |
| Utility Compensated Permanent Shock | $\underline{\underline{\psi}} \equiv (\mathbb{E}_t[\psi^{1-\rho}])^{1/(1-\rho)}$                     | 0.990                              |
| Utility Compensated Growth          | $\underline{\underline{\Gamma}} \equiv \Gamma\underline{\underline{\psi}}$                           | 1.020                              |
| Absolute Patience Factor            | $\mathfrak{P} \equiv (R\beta)^{1/\rho}$  | 0.999                              |
| Return Patience Factor              | $\mathfrak{P}_R \equiv \mathfrak{P}/R$   | 0.961                              |
| PF Growth Patience Factor           | $\mathfrak{P}_\Gamma \equiv \mathfrak{P}/\Gamma$   | 0.970                              |
| Growth Patience Factor              | $\mathfrak{P}_{\underline{\Gamma}} \equiv \mathfrak{P}/\underline{\Gamma}$                           | 0.980                              |
| Finite Value of Autarky Measure     | $\underline{\underline{\sqsupset}} \equiv \beta\Gamma^{1-\rho}\underline{\underline{\psi}}^{1-\rho}$ | 0.941                              |

**Table 3** Definitions and Comparisons of Conditions

| Perfect Foresight Versions  | Uncertainty Versions  |
|---|---|
| Finite Human Wealth Condition (FHC)   |   |
| $\Gamma/R < 1$<br>The growth factor for permanent income $\Gamma$ must be smaller than the discounting factor $R$ for human wealth to be finite.  | $\Gamma/R < 1$<br>The model's risks are mean-preserving spreads, so the PDV of future income is unchanged by their introduction.  |
| Absolute Impatience Condition (AIC)   |   |
| $\mathbf{P} < 1$<br>The unconstrained consumer is sufficiently impatient that the level of consumption will be declining over time:<br>$c_{t+1} < c_t$  | $\mathbf{P} < 1$<br><i>If wealth is large enough, the expectation of consumption next period will be smaller than this period's consumption:</i><br>$\lim_{m_t \rightarrow \infty} \mathbb{E}_t[c_{t+1}] < c_t$   |
| Return Impatience Conditions  |   |
| Return Impatience Condition (RIC)   | Weak RIC (WRIC)   |
| $\mathbf{P}/R < 1$<br>The growth factor for consumption $\mathbf{P}$ must be smaller than the discounting factor $R$ , so that the PDV of current and future consumption will be finite:<br>$c'(m) = 1 - \mathbf{P}/R < 1$    | $\wp^{1/\rho} \mathbf{P}/R < 1$<br>If the probability of the zero-income event is $\wp = 1$ then income is always zero and the condition becomes identical to the RIC. Otherwise, weaker.<br>$c'(m) < 1 - \wp^{1/\rho} \mathbf{P}/R < 1$                              |
| Growth Impatience Conditions  |   |
| PF-GIC  | GIC   |
| $\mathbf{P}/\Gamma < 1$<br>Guarantees that for an unconstrained consumer, the ratio of consumption to permanent income will fall over time. For a constrained consumer, guarantees the constraint will eventually be binding. | $\mathbf{P} \mathbb{E}[\psi^{-1}]/\Gamma < 1$<br>By Jensen's inequality, stronger than the PF-GIC.<br>Ensures consumers will not expect to accumulate $m$ unboundedly.<br>$\lim_{m_t \rightarrow \infty} \mathbb{E}_t[m_{t+1}/m_t] = \mathbf{P}_{\underline{\Gamma}}$ |
| Finite Value of Autarky Conditions  |   |
| PF-FVAC   | FVAC  |
| $\beta \Gamma^{1-\rho} < 1$<br>equivalently $\mathbf{P}/\Gamma < (R/\Gamma)^{1/\rho}$<br>The discounted utility of constrained consumers who spend their permanent income each period should be finite.                       | $\beta \Gamma^{1-\rho} \mathbb{E}[\psi^{1-\rho}] < 1$<br>By Jensen's inequality, stronger than the PF-FVAC because for $\rho > 1$ and nondegenerate $\psi$ , $\mathbb{E}[\psi^{1-\rho}] > 1$ .  |

**Table 4** Sufficient Conditions for Nondegenerate<sup>‡</sup> Solution

| Model              | Conditions             | Comments  |
|--------------------|------------------------|---|
| PF Unconstrained   | RIC, FHWC <sup>°</sup> | RIC $\Rightarrow  v(m)  < \infty$ ; FHWC $\Rightarrow 0 <  v(m) $<br>RIC prevents $\bar{c}(m) = 0$<br>FHWC prevents $\bar{c}(m) = \infty$   |
| PF Constrained     | PF-GIC*                | If RIC, $\lim_{m \rightarrow \infty} \dot{c}(m) = \bar{c}(m)$ , $\lim_{m \rightarrow \infty} \dot{\kappa}(m) = \underline{\kappa}$<br>If <del>RIC</del> , $\lim_{m \rightarrow \infty} \dot{\kappa}(m) = 0$   |
| Buffer Stock Model | FVAC, WRIC             | FHWC $\Rightarrow \lim_{m \rightarrow \infty} \dot{c}(m) = \bar{c}(m)$ , $\lim_{m \rightarrow \infty} \dot{\kappa}(m) = \underline{\kappa}$<br><del>FHWC</del> +RIC $\Rightarrow \lim_{m \rightarrow \infty} \dot{\kappa}(m) = \underline{\kappa}$<br><del>FHWC</del> + <del>RIC</del> $\Rightarrow \lim_{m \rightarrow \infty} \dot{\kappa}(m) = 0$<br>GIC guarantees finite target wealth ratio<br>FVAC is stronger than PF-FVAC<br>WRIC is weaker than RIC |

<sup>‡</sup>For feasible  $m$ , the limiting consumption function defines the unique value of  $c$  satisfying  $0 < c(m) < \infty$ . <sup>°</sup>RIC, FHWC are necessary as well as sufficient. \*Solution also exists for ~~PF-GIC~~ and RIC, but is identical to the unconstrained model's solution for feasible  $m \geq 1$ .

**Table 5** Taxonomy of Perfect Foresight Liquidity Constrained Model Outcomes

For constrained  $\dot{c}$  and unconstrained  $\bar{c}$  consumption functions

| Main Condition<br>Subcondition | Math                    | Outcome, Comments or Results  |
|--------------------------------|-------------------------|---|
| <del>PF-GIC</del>              | $1 < \mathbf{P}/\Gamma$ | Constraint never binds for $m \geq 1$   |
| and RIC                        | $\mathbf{P}/R < 1$      | FHWC holds ( $R > \Gamma$ ); $\dot{c}(m) = \bar{c}(m)$ for $m \geq 1$   |
| and <del>RIC</del>             | $1 < \mathbf{P}/R$      | $\dot{c}(m)$ is degenerate: $\dot{c}(m) = 0$  |
| PF-GIC                         | $\mathbf{P}/\Gamma < 1$ | Constraint binds in finite time for any $m$   |
| and RIC                        | $\mathbf{P}/R < 1$      | FHWC may or may not hold<br>$\lim_{m \uparrow \infty} \bar{c}(m) - \dot{c}(m) = 0$<br>$\lim_{m \uparrow \infty} \dot{\kappa}(m) = \underline{\kappa}$ |
| and <del>RIC</del>             | $1 < \mathbf{P}/R$      | <del>FHWC</del><br>$\lim_{m \uparrow \infty} \dot{\kappa}(m) = 0$   |

Conditions are applied from left to right; for example, the second row indicates conclusions in the case where ~~PF-GIC~~ and RIC both hold, while the third row indicates that when the PF-GIC and the RIC both fail, the consumption function is degenerate; the next row indicates that whenever the PF-GIC holds, the constraint will bind in finite time.