

$$\begin{aligned}\mathbf{Y}_{t+1}/\mathbf{Y}_t &= \mathbb{M}[\xi_{t+1}\Gamma\psi_{t+1}\mathbf{p}_t] / \mathbb{M}[\mathbf{p}_t\xi_t] \\ &= \Gamma\end{aligned}$$

because of the independence assumptions we have made about ξ and ψ .

If all consumers were at their target m where $\mathbb{E}_t[m_{t+1}] = m$, we can show that the growth factor for aggregate assets is approximately Γ :

Aggregate assets are:

$$\begin{aligned}\left(\frac{\mathbf{A}_{t+1}}{\mathbf{A}_t}\right) &= \mathbb{M}\left[\frac{a_{t+1,i}\Gamma\mathbf{p}_{t+1,i}}{a_{t,i}\mathbf{p}_{t,i}}\right] \\ &= \Gamma\mathbb{M}\left[\frac{a_{t+1}\psi_{t+1}}{a_t}\right] \\ &= \Gamma\mathbb{M}\left[\frac{(a_t\mathcal{R}_{t+1} + \xi_{t+1} - c(a_t\mathcal{R}_{t+1} + \xi_{t+1}))\psi_{t+1}}{a_t}\right] \\ &\quad \Gamma\mathbb{M}\left[\frac{(a_t\mathbf{R}/\Gamma + \psi_{t+1}\xi_{t+1} - c(a_t\mathcal{R}_{t+1} + \xi_{t+1}))\psi_{t+1}}{a_t}\right] \\ &\quad \Gamma\mathbb{M}\left[\frac{(a_t\mathbf{R}/\Gamma + \psi_{t+1}\xi_{t+1} - (c(\mathbb{E}_t[m_{t+1}]) + (c'(\mathbb{E}_t[m_{t+1}]))(m_{t+1} - \mathbb{E}_t[m_{t+1}]))\psi_{t+1}}{a_t}\right]\end{aligned}$$

but the first term reduces to

$$\Gamma\mathbb{M}\left[\frac{a_t\mathbf{R}/\Gamma + \psi_{t+1}\xi_{t+1}}{a_t}\right] = \mathbf{R} + 1/a_t$$

but defining $\bar{\mathcal{R}}_{t+1} = \mathbb{E}_t[\mathcal{R}_{t+1}]$

$$\begin{aligned}\Gamma\mathbb{M}\left[\frac{-c(a_t\mathcal{R}_{t+1} + \xi_{t+1})\psi_{t+1}}{a_t}\right] &\approx \Gamma\mathbb{M}\left[\frac{-\left(c(a_t\bar{\mathcal{R}}_{t+1} + \xi_{t+1}) + c'(a_t\bar{\mathcal{R}}_{t+1} + \xi_{t+1})(\mathcal{R}_{t+1} - \bar{\mathcal{R}}_{t+1})\right)\psi_{t+1}}{a_t}\right] \\ &\quad (1) \\ &= \Gamma\mathbb{M}\left[\frac{-\left(c(a_t\bar{\mathcal{R}}_{t+1} + \xi_{t+1})\psi_{t+1} + c'(a_t\bar{\mathcal{R}}_{t+1} + \xi_{t+1})(\mathbf{R}/\Gamma - \bar{\mathcal{R}}_{t+1}\psi_{t+1})\right)\psi_{t+1}}{a_t}\right] \\ &\quad (2)\end{aligned}$$

while the remainder of the expression is

where \mathbf{P}_t designates the mean level of permanent income across all individuals, and we are assuming that $a_{t,i}$ was distributed according to the invariant distribution with a mean value of a . Since permanent income grows at mean rate Γ while the distribution of a is invariant, if we normalize \mathbf{P}_t to one we will similarly have for any period $n \geq 1$

$$\mathbf{A}_{t+n} = A\Gamma^n + \text{cov}(a_{t+n,i}, \mathbf{p}_{t+n,i}).$$