

Theoretical Foundations of Buffer Stock Saving

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Drawbacks of Numerical Solutions

A Black Box

- Can Construct Solution to Model Without Really Understanding It
- Hard Even To Be Sure Your Numerical Solution Is *Right*
- Little Intuition for How Results Might Change With
 - Calibration
 - Structure
- *Very Hard To Teach!*

I Am A *Big* Fan Of Numerical Methods

- Have Done A Good Deal Of Work With Them Myself
- But As A Result, Have Felt All These Drawbacks Keenly

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The Gap This Paper Fills

Foundations For Microeconomic Household's Problem With

- Uncertain Labor Income
- No Liquidity Constraints
- CRRA Utility
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Key Result

Restrictions On Parameter Values Such That

- Problem Defines A Contraction Mapping
 - $\Rightarrow \exists$ A Unique Consumption Function $c(m)$
- There Is A 'Target' Ratio Of Assets to Permanent Income
 - Requires A Key 'Impatience' Condition To Hold
 - Good News
 - Condition Is Weaker (Easier To Satisfy) Than Previous Papers Imposed

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Limit as horizon T goes to infinity of

$$\begin{aligned}
 a_t &= m_t - c_t \\
 b_{t+1} &= a_t R \\
 \mathbf{p}_{t+1} &= \mathbf{p}_t \underbrace{\Gamma^{\psi_{t+1}}}_{\equiv \Gamma_{t+1}} \\
 m_{t+1} &= b_{t+1} + \mathbf{p}_{t+1} \xi_{t+1},
 \end{aligned} \tag{1}$$

$$\xi_{t+n} = \begin{cases} 0 & \text{with probability } \wp > 0 \\ \theta_{t+n}/\wp & \text{with probability } \wp \equiv (1 - \wp) \end{cases} \tag{2}$$

- $u(\bullet) = \bullet^{1-\rho}/(1-\rho)$; $\mathbb{E}_t[\psi_{t+n}] = \mathbb{E}_t[\xi_{t+n}] = 1 \ \forall \ n > 0$; $\beta < 1, \rho > 1$

Surely This Problem Has Been Solved?

No.

- Can't Use Stokey et. al. theorems because CRRA utility
- Lit thru ? Can't Handle Permanent Shocks
- Must Use Boyd's 'Weighted' Contraction Mapping Theorem
- Surprisingly Subtle

Fortunately, the Conclusions Are Simple!

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Benchmark: Perfect Foresight Model

Definitions:

Absolute Patience Factor

$$\mathbf{P} = (R\beta)^{1/\rho}$$

Return Patience Factor

$$\mathbf{P}_R = \mathbf{P}/R$$

Perfect Foresight Growth Patience Factor

$$\mathbf{P}_\Gamma = \mathbf{P}/\Gamma$$

Name	Condition	Implication
(AIC) Absolute Impatience Condition	$\mathbf{P} < 1$	$c \downarrow$ over time
(RIC) Return Impatience Condition	$\mathbf{P}_R < 1$	$c/a \downarrow$ over time
(PF-GIC) Growth Impatience Condition	$\mathbf{P}_\Gamma < 1$	$c/\mathbf{p} \downarrow$ over time

When Does A Useful Limiting Solution Exist?

Finite Human Wealth (FHW) condition:

$$\Gamma < R \quad (3)$$

Return Impatience Condition:

$$\mathbb{D}_R < R \quad (4)$$

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What If There Are Liquidity Constraints?

- FHWC is *not* necessary for solution to exist
- Other Key Condition For Useful Solution is
'Perfect Foresight Finite Value of Autarky Condition (PF-FVAC)':

$$\beta \Gamma^{1-\rho} < 1 \quad (5)$$

- Without RIC, Constraints Are Irrelevant
 - Because Wealth Always Wants To Rise, So Constraint Never Binds

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Liquidity Constraints and Uncertainty

- Introduce permanent shocks to income
- Finite Value of Autarky Condition Becomes

$$\overline{\overline{\overline{\beta \Gamma^{1-\rho}}}} < 1 \quad (6)$$

$$\beta < \overline{\overline{\Gamma}}^{\rho-1}$$

Contraction Mapping Requirements

Finite Value of Autarky Condition: Same As In Liq Constr Problem!

$$\overbrace{\beta \Gamma^{1-\rho}}^{\equiv \Gamma} < 1 \quad (7)$$

$$\beta < \underline{\Gamma}^{\rho-1}$$

'Weak Return Impatience Condition' (WRIC)

$$0 \leq \beta^{1/\rho} \underline{\Gamma} < 1 \quad (8)$$

Requirement For Existence Of A Target

Definitions: 'Uncertainty-Adjusted' Growth:

$$\underline{\Gamma} = \Gamma\psi \quad (9)$$

Adjusted Growth Patience Factor:

$$\mathbf{P}_{\hat{r}} = \mathbf{P}/\underline{\Gamma} \quad (10)$$

Growth Impatience Condition:

$$\mathbf{P}_{\hat{r}} < 1 \quad (11)$$

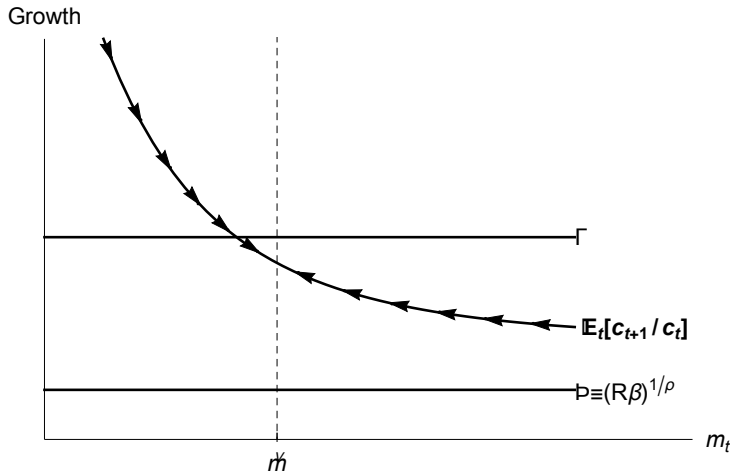
Why? Because it can be shown that

$$\lim_{m_t \rightarrow \infty} \mathbb{E}_t \left[\frac{m_{t+1}}{m_t} \right] = \mathbf{P}_{\hat{r}} \quad (12)$$

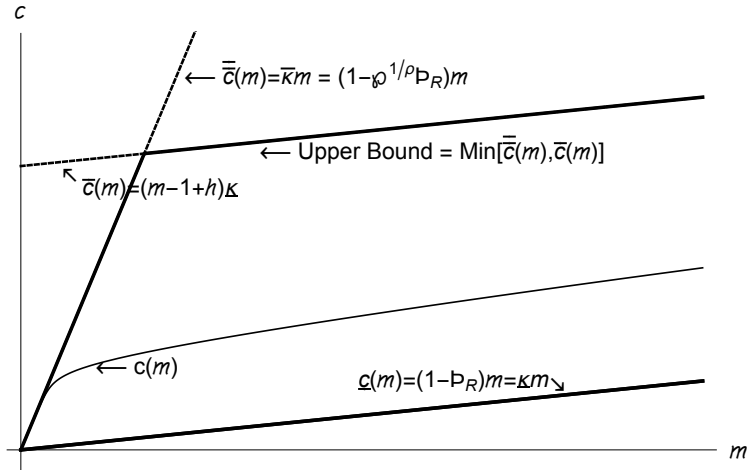
Five Propositions

- ① $\lim_{m_t \rightarrow \infty} \mathbb{E}_t[c_{t+1}/c_t] = \mathbf{P}$
- ② $\lim_{m_t \rightarrow 0} \mathbb{E}_t[c_{t+1}/c_t] = \infty$
- ③ \exists a unique target value of m , called \check{m}
- ④ $\mathbb{E}_t[c_{t+1}/c_t | m_t = \check{m}] = \Gamma - \epsilon$
- ⑤ $\left(\frac{d\mathbb{E}_t[c_{t+1}/c_t]}{dm_t} \right) < 0$

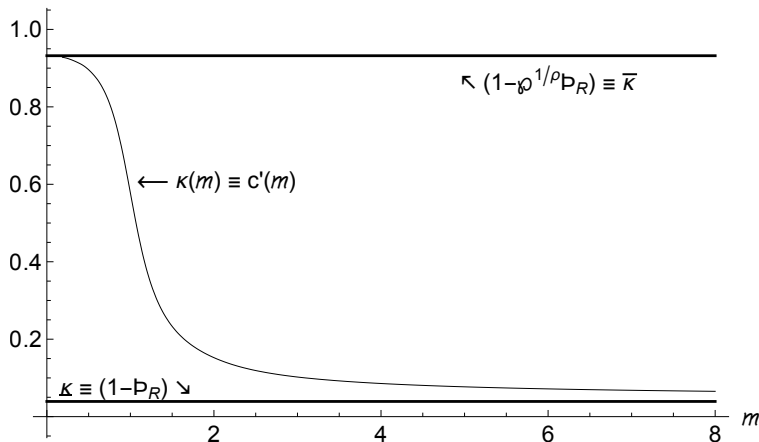
The Target Saving Figure



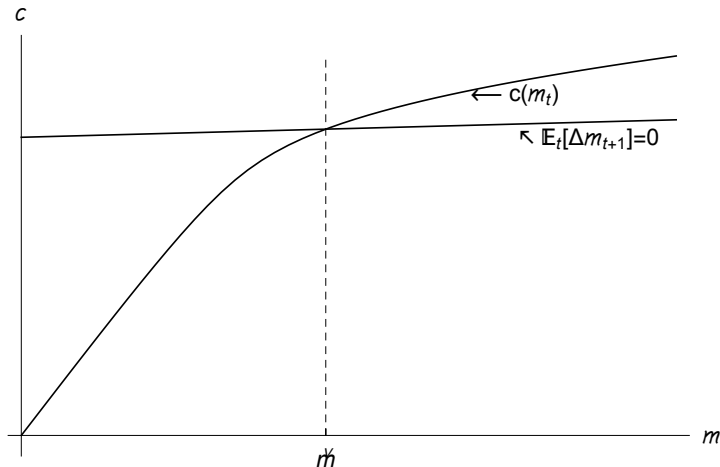
Bounds On the Consumption Function



The Marginal Propensity to Consume

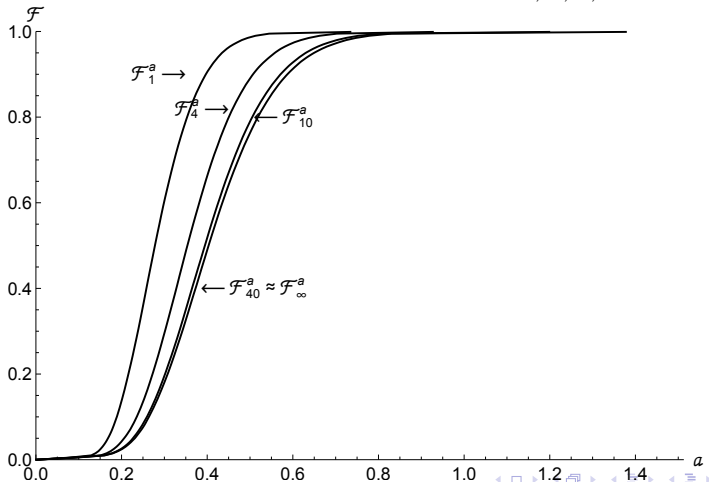


The Consumption Function and Target Wealth



Convergence To The Invariant Distribution

? Proves Existence of an Invariant Distribution of m, c, a , etc.



Balanced Growth Equilibrium

Achieved When Cross Section Distribution Reaches Invariance

$$Y_{t+1}/Y_t = C_{t+1}/C_t = \Gamma \quad (13)$$

Fisherian Separation Fails, Even Without Liquidity Constraints!

Insight:

- Precautionary Saving \approx Liquidity Constraints
- If $\bar{c}(m)$ is solution for constrained consumer,

$$\lim_{\varphi \downarrow 0} c(m; \varphi) = \bar{c}(m) \quad (14)$$

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The MPC Out Of Permanent Shocks

<http://econ.jhu.edu/people/ccarroll/papers/MPCPerm.pdf>

Lots of Recent Papers Trying to Measure the MPCP

Paper Proves:

- $MPCP < 1$
- But not a lot less:
 - 0.75 to 0.95 (annual rate) for wide range of parameter values

- Defined Conditions Under Which Widely Used Problem Has Solution
 - Finite Value of Autarky Condition Guarantees Contraction (with WRIC)
 - Growth Impatience Condition Prevents $m \rightarrow \infty$
- Economy Of Buffer Stock Consumers Exhibits Balanced Growth
 - Even In Absence of General Equilibrium Adj of Interest Rate

