Convergence in Euclidian Space Convergence of ${}_t$vFunc-t
Boyd's theorem shows that defines a contraction mapping in an -bounded space. We now show that also defines a con
Calling * the unique fixed point of the operator , since $^*() = ^*()$,

On the other hand, ${}_T - ^* \in \mathcal{C}\,(A, B)$ and $= \|{}_T - ^*\| < \infty$ because ${}_T$ and * are in $\mathcal{C}\,(A, B)$. It follows that

Then we obtain

Since ${}_T() = \frac{1-}{1-}$, ${}_{T-1}() \leq \frac{()^{1-}}{1-} <_T ()$. On the other hand, ${}_{T-1} \leq_T$ means ${}_{T-1} \leq_T$, in other words, ${}_{T-2}() \leq_{T-1} ()$. Induct
Convergence of ${}_t$cFunc-t
Given the proof that the value functions converge, we now show the pointwise convergence of consumption functions $\{_T$
Consider any convergent subsequence $\left\{{}_{T-n(i)}()\right\}$ of $\{{}_{T-n+1}()\}_{n=1}^{\infty}$ converging to c^*. By the definition of ${}_{T-n}()$, we have
$\geq (c_{T-n(i)}) +_{T-n(i)} [{}^{1-}_{T-n(i)+1} {}_{T-n(i)+1}()]$, for any ${}_{T-n(i)} \in [,]$. Now letting $n(i)$ go to infinity, it follows that the left hand si

Hence, $c^* \in {}_{T-n(i)} \in [,] \arg\max \left\{({}_{T-n(i)}) +_t [{}^{1-}_{t+1}()]\right\}$. By the uniqueness of $()$, $c^* = ()$.