ApndxMTargetIsStable Unique and Stable Target and Steady State Points

This appendix proves Theorems 2-3 and:

Lemma 1 *If ˇ and ˆ both exist, then ˇ ≤ ˆ.*

Lemma 2 *If ˇ and ˜ both exist, then ˇ ≤ ˜.*

Proof of Theorem 2

Theorem 2 *thm:target For the nondegenerate solution to the problem defined in Section 2.1Setupsubsection.2.1 when FVAC*

Moreover, ˆ is a point of 'stability' in the sense that

The elements of the proof of Theorem 2 are:

Existence and continuity of $_t[_{t+1}/_t]$

Existence of a point where $_t[_{t+1}/_t] = 1$

$_t[_{t+1}]-_t$ is monotonically decreasing

Existence and Continuity of $_t[_{t+1}/_t]$Ex-t[mNrm-t+1/mNrm-t] The consumption function exists because we have impose

Section 2.8Concave Consumption Function Characteristicssubsection.2.8 shows that for all t, $_{t-1} = _{t-1} - _{t-1} > 0$. Since

Existence of a point where $_t[_{t+1}/_t] = 1$ Ex-t[mNrm-t+1/mNrm-t]=1.

This follows from:

Existence and continuity of $_t[_{t+1}/_t]$ (just proven)

Existence a point where $_t[_{t+1}/_t] < 1$

Existence a point where $_t[_{t+1}/_t] > 1$

The Intermediate Value Theorem

Existence of m where $_t[_{t+1}/_t] < 1$E[mt+1/mt

If RICRIC holds. Logic exactly parallel to that of Section 3.1Limits as

$_{t+1}$ from the RHS, establishes that $\lim_{t\uparrow\infty} {}_t[_{t+1}/_t] = \lim_{t\uparrow\infty} {}_t \left[\frac{\mathcal{R}_{t+1}(_t-(_t))+_{t+1}}{_t}\right]$

$=_t [(/_{t+1})]$

$=_t [/_{t+1}]$

< 1 where the inequality reflects imposition of the GICModGIC-Mod eq:GICMod.

If RICRIC fails. When the RICRIC fails, the fact that $\lim'_{\uparrow\infty}() = 0$ (see equation eq:MPCminInv) means that the lim

So we have $\lim_{\uparrow\infty} {}_t[_{t+1}/_t] < 1$ whether the RICRIC holds or fails.

Existence of m ¿ 1 where $_t[_{t+1}/_t] > 1$E[mt+1/mt] ¿ 1 Paralleling the logic for in Section 3.2Limits as mm Approaches

Intermediate Value Theorem. If $_t[_{t+1}/_t]$ is continuous, and takes on values above and below 1, there must be at least one po

$_t[_{t+1}]-_t$Delta m is monotonically decreasing.

Now define $\zeta(_t) \equiv_t [_{t+1}]-_t$ and note that $\zeta(_t) < 0 \leftrightarrow_t [_{t+1}/_t] < 1$

$\zeta(_t) = 0 \leftrightarrow_t [_{t+1}/_t] = 1$

$\zeta(_t) > 0 \leftrightarrow_t [_{t+1}/_t] > 1$, so that $\zeta(\check{}) = 0$. Our goal is to prove that $\zeta(\bullet)$ is strictly decreasing on $(0,\infty)$ using the fact that ζ

$=^{-}(1-' (_t)) - 1.$

Now, we show that (given our other assumptions) $\zeta'()$ is decreasing (but for different reasons) whether the RICRIC hol

If RICRIC holds. Equation eq:MPCminDef indicates that if the RICRIC holds, then > 0. We show at the bottom o

$=^{-} - 1$

$=_t \left[\ \right] - 1$

$= \underbrace{_t \left[\ \right]}_{=} -1$ which is negative because the GICModGIC-Mod says < 1.

If RICRIC fails. Under RICRIC, recall that $\lim'_{\uparrow\infty}() = 0$. Concavity of the consumption function means that $'$ is a d

Proof of Theorem 3

Theorem 3 *thm:MSSBalExists For the nondegenerate solution to the problem defined in Section 2.1Setupsubsection.2.1 whe*

Moreover, ˇ is a point of stability in the sense that

The elements of the proof are:

Existence and continuity of $_t[_{t+1 t+1}/_t]$

Existence of a point where $_t[_{t+1 t+1}/_t] = 1$

$_t[_{t+1 t+1}-_t]$ is monotonically decreasing

Existence and Continuity of the Ratio

Since by assumption $0 < \ \leq_{t+1} \leq^{-} < \infty$, our proof in that demonstrated existence and continuity of $_t[_{t+1}/_t]$ implies exis

Existence of a stable point

Since by assumption $0 < \ \leq_{t+1} \leq^{-} < \infty$, our proof in Subsection that the ratio of $_t[_{t+1}]$ to $_t$ is unbounded as $_t \downarrow 0$ impli

The limit of the expected ratio as $_t$ goes to infinity is most easily calculated by modifying the steps for the prior theore

$= \lim \left[(/)(_t)+_{t+1 t+1} \right]$