

Equality of \bar{c} and p Growth with Transitory Shocks
 Section 4.1 Individual Balanced Growth of Income, Consumption, and Wealth subsection.4.1 asserted that in the absence of permanent shocks, the optimal consumption c_t is constant over time. First define \bar{c} as the function that yields optimal end-of-period assets as a function of \bar{c} . Suppose the population starts in period t with an arbitrary value for $cov_t(a_{t+1,i}, c_{t+1,i})$. Then if \bar{c} is the invariant mean level of consumption, we have

where the combination of the bar and the $\bar{\cdot}$ are meant to signify that this is the average value of the derivative over the interval $[t, t+1]$.

so

But since $\bar{c}^{-1}(\bar{c})^{1/\sigma} < \bar{c}'(\bar{c}) < \bar{c}$,

and for the version of the model with no permanent shocks the GIC-Mod says that $\bar{c} < \bar{c}$, while the FHWCFHWC says that $\bar{c} = \bar{c}$.

This means that from any arbitrary starting value, the relative size of the covariance term shrinks to zero over time (converges to zero). This logic unfortunately does not go through when there are permanent shocks, because the $c_{t+1,i}$ terms are not independent. To see the problem clearly, define $\bar{c}_{t+1,i} = c_{t+1,i}$ and consider a first order Taylor expansion of $\bar{c}'(\bar{c}_{t+1,i})$ around $\bar{c}_{t+1,i} = \bar{c}_{t,i} + 1$.