HAFiscal

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Using a heterogeneous agent model calibrated to match the initial MPC and subsequent spending dynamics over four years, we assess the effectiveness of three fiscal stimulus policies employed during recent recessions. Unemployment Insurance (UI) extensions are the clear 'bang for the buck' winner, especially when effectiveness is measured in utility terms. 'Stimulus checks' are second best, and have the advantage (over UI) of being scalable to any desired size. A temporary (two year) cut in the rate of wage taxation is considerably less effective than the other policies, and has negligible benefits in the version of our model without a multiplier.

REMARK:	https://econ-ark.org/materials/bufferstocktheory
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Thanks.

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1 Introduction

Using a heterogeneous agent model calibrated to match the initial MPC and subsequent spending dynamics over four years, we assess the effectiveness of three fiscal stimulus policies employed during recent recessions. Unemployment Insurance (UI) extensions are the clear 'bang for the buck' winner, especially when effectiveness is measured in utility terms. 'Stimulus checks' are second best, and have the advantage (over UI) of being scalable to any desired size. A temporary (two year) cut in the rate of wage taxation is considerably less effective than the other policies, and has negligible benefits in the version of our model without a multiplier.

Fiscal policies that aim to boost consumer spending in recessions have been tried repeatedly in many countries in recent years. The nature of such policies has been quite varied, at least in part because traditional macroeconomic models were unable to provide clear guidance about which policies were likely to be most effective.

But new sources of microeconomic data, such as those from Scandinavian national registries, have recently enabled unprecedentedly fine-grained measurement of the dynamics of different types of consumers' spending patterns in response to income shocks. Simultaneously, advances in Heterogeneous Agent macro modeling have made it possible to construct structural models capable of matching these spending patterns with a reasonably high degree of fidelity. This combination of developments makes it possible, really for the first time, to conduct quantitatively credible structural analyses of the likely effectiveness of such policies.

Because spending dynamics in our model reflects the behavior of utility maximizing consumers, we are able to evaluate the policies not only by their effects on aggregate consumption expenditures, but also directly in terms of the impact on consumers' utility. The principal difference between the two metrics is that is that the utility-metric evaluation further increases the already considerable advantage that the UI extension exhibited in the aggregate-consumption-boosting metric, because the benefits of the UI extension are enhanced because the payments are specifically directed to a set of consumers who have high marginal utility.

- [Briefly describe the model - EC, emphasizing our welfare criterion to nullify the redistributive benefits]

- [Briefly describe the recessions (as we model them) and the treatment of the multiplier - IF: I (IF) have added a short section, mostly based on EC's description of MIT shocks.] Recessions are unexpected and double the unemployment rate and the average length of unemployment spells. The end of the recession occurs as a Poisson process calibrated for an average length of recession of six quarters, leading to a return of the unemployment rate to normal levels over time. In an extension to the model, we allow for aggregate demand effects during the recession. With this extension, any reduction in aggregate consumption below its steady-state level directly reduce labor income. Hence, any policy stimulating consumption will also boost incomes through this aggregate demand multiplier channel.

- [Briefly elaborate on the ways in which we have calibrated the model to match "dynamics of the MPC" (splurge, matching liquid asset distribution, etc). Mention that

we are mixing-matching US and Norwegian data and briefly defend, but say that details and a more extended justification will follow. - HT and/or IF] We parametrize the model in two steps. First, we estimate the extent to which consumers "splurge" when receiving an income shock. We do so using Norwegian data to be consistent with the best available evidence on the time profile of the marginal propensity to consume provided by ?. Second, we move on to the calibration of the full model on US data taking the splurge-factor as given. In the model, agents are ex-ante heterogeneous. The population consists of types that differ according to their level of education and their subjective discount factors. The distribution of subjective discount factors is estimated separately for each education group to match features of the wealth distribution within that group. In addition, agents experince different histories of idiosyncratic income shocks and periods of unemployment, so that within each type there is heterogeneity induced by different shock realizations.

Our results are intuitive.

In the economy with no multiplier during recessions, the benefit of a sustained wage tax cut is small. One reason there is any benefit at all is that, even for people who have not experienced an unemployment spell, the hightened risk of unemployment during a recession increases the marginal value of income because it helps them build the extra precautionary reserves induced by the extra risk. A second benefit is that, by the time a person does become unemployed, the temporary tax reduction will have allowed them to accumulate a larger buffer stock of resources to sustain them during unemployment. Finally, in a recession there are more people who will have experienced a spell of unemployment, and the larger population of beneficiaries means that the consequences of the two prior mechanisms will be greater. But, quantitatively, all of these effects are small.

When a multiplier exists, the tax cut has more benefits, especially if the recession continues long enough that most of the spending induced by the tax cut happens while the economy is still in recession (and therefore the multiplier still is in force). The typical recession, however, ends long before our "sustained" tax cut is reversed, so even in an economy with a multplier that is powerful during recessions, much of the tax cut's effect on consumption occurs when any multiplier that might have existed in a recession is no longer operative.

In contrast to the tax cut, both the UI extension and the stimulus checks concentrate most of the marginal increment to consumption at times when the multiplier (if it exists) is still powerful. Even leaving aside any multiplier effects, the stimulus checks have more value than the wage tax cut, because at least a portion of them go to people who are unemployed and therefore have both high MPC's and high marginal utilities (while wage tax cuts by definition go only to persons who are employed and earning wages). But the greater bang-for-the-buck of the UI extension reflects the fact that *all* of the recipients are in circumstances in which they have a high MPC and a high marginal utility.

We conclude that extended UI benefits should be the first weapon employed from this arsenal. But a disadvantage is that the total amount of stimulus that can be accomphshed with this tool is constrained by the fact that only a limited number of people become unemployed. If more stimulation is called for than can be accomplished

via UI extension, checks have the advantage that their effects scale almost linearly in the size of the stimulus. The wage tax cut is also in principle scalable, but it's effects are smaller than those of checks because its recipients have considerably lower MPCs and u' than check and UI recipients. In the real world, a tax cut is also likely the least flexible of the three tools: UI benefits can be further extended, multiple rounds of checks can be sent; but multiple rounds of changes in wage tax rates would likely be administratively and politically more difficult to achieve.

   - The tools we are using could be reasonably easily modified to evaluate a number of other policies. For example, in the COVID recession, not only was the duration of UI benefits extended, those benefits were supplemented by very substantial extra payments to every UI recipient. We did not calibrate the model to match this particular policy, but the framework could easily accommodate such an analysis.

2 Model

In this section we describe our heterogeneous agent model featuring households that differ according to their level of education and their subjective discount factors. We first describe the problem faced by these households given the income process they face with permanent and transitory shocks as well as shocks to their employment status. Then we describe how we model the arrival of a recession and the policies that we study as potential responses. Finally, we discuss an extension of the model where we include aggregate demand effects that induce a feedback effect from aggregate consumption to income and hence, amplify the impact of a recession when it occurs.

2.1 The Household Problem

A household i is characterized by the level of education $e(i)$ and their subjective discount factor β_i. The household faces a stochastic income stream, $y_{i,t}$, and chooses to consume some of that income when it arrives (the 'splurge') and then to optimize consumption with what is left over. Therefore, consumption each period for household i can be written:

$$c_{i,t} = c_{sp,i,t} + c_{opt,i,t}, \tag{1}$$

where $c_{i,t}$ is total consumption, $c_{sp,i,t}$ is the splurge consumption and $c_{opt,i,t}$ is the household's optimal choice of consumption after splurging. Splurge consumption is simply a fraction of income:

$$c_{sp,i,t} = \varsigma y_{i,t}, \tag{2}$$

while the optimized portion of consumption is chosen to maximize lifetime expected consumption:

$$\sum_{t=0}^{\infty} \beta_i^t (1 - D)^t \mathbb{E}u(c_{opt,i,t}). \tag{3}$$

The optimization is subject to the budget constraint given existing market resources $m_{i,t}$ and income state, and a no-borrowing constraint:

$$a_{i,t} = m_{i,t} - c_{i,t} \tag{4}$$

$$m_{i,t+1} = \tag{5}$$

$$a_{i,t} \geq 0 \tag{6}$$

The Income Process Households face a stochastic income process with permanent and transitory shocks to income, along with unemployment shocks. In normal times, households receive unemployment benefits for two quarters before they run out. Permanent income in the model is described by the following equation:

$$p_{i,t} = \psi_{i,t} \Gamma_{e(i)} p_{i,t-1}, \tag{7}$$

where $\psi_{i,t}$ is the shock to permanent income and $\Gamma_{e(i)}$ is the growth rate of income for education group $e(i)$ of the household. The shock to permanent income is normally distributed with variance σ_ψ^2.

The actual income a household receives will be subject their employment status as well as transitory shocks, $\xi_{i,t}$:

$$y_{i,t} = \begin{cases} \xi_{i,t} p_{i,t}, & \text{if employed} \\ 0.3 p_{i,t}, & \text{if unemployed with benefits} \\ 0.05 p_{i,t}, & \text{if unemployed without benefits} \end{cases} \tag{8}$$

where $\xi_{i,t}$ is normally distributed with variance σ_ξ^2. A Markov transition matrix with four states generates the unemployment dynamics. An employed household can continue being employed, or move to being unemployed with benefits (with one remaining period of benefits). This household can then either become employed or move to being unemployed with benefits (but no remaining periods of benefits). A household in this state can then either become employed or unemployed without benefits, where they will remain until they become employed again. The probability of becoming employed is the same for each unemployed state and the probabilities of transitioning from employment to unemployment and vice-versa are chosen to match the unemployment rate for each education group (in steady state) and an average duration of unemployment of 1.5 quarters.

2.2 MIT Shocks

We model the arrival of a recession, and the government policy response to it, as an unpredictable event—an MIT shock. We have four types of shock representing a recession and the three different policy responses we consider. The policy responses are usually modeled as in addition to the recession, but we also consider a counterfactual in which the policy response occurs without a recession in order to understand the welfare effects of the policy.

Recession   At the onset of a recession, several changes occur. First, the unemployment rate for each education group doubles. Those who would have been unemployed remain so, and an additional number of households move from employment to unemployment. Second, conditional on the recession continuing, the emplyment transtion matrix is adjusted so that unemployment remains at the new high level, and the expected length of time for an unemployment spell increases from two to four quarters. Third, the end of the recession occurs as a Poisson process calibrated for an average length of recession of six quarters. Finally, at the end of a recession, the employment transition matrix switches back to its original probabilities and as a result the unemployment rate tends down over time back to its steady-state level.

Stimulus Check   In this policy response, the government sends money to every household that directly increases their market resources. The checks are means-tested depending on permanent income. A fixed check amount is sent to every household with permanent income less than a threshold and this amount is then linearly reduced to zero for households about a higher permanent income threshold.[IF: Do we want to provide the actual numbers? $ 1200; The check is only paid out fully to individuals with a permanent yearly income smaller than $ 100,000. Individuals with a permanent income greater than $150,000 receive no check.]

Extended Unemployment Benefits   In this policy response, unemployment benefits are extended from 2 quarters to 4 quarters. That is, those who become unemployed at the start of the pandemic, or who were already unemployed, will receive unemployment benefits for up to four quarters (including quarters leading up to the recession). Those who become unemployed one quarter into the recession will receive up to three quarters of unemployment benefits. These extended unemployment benefits will occur regardless of whether the recession ends, and no further extensions are granted if the recession continues.

Payroll Tax Cut   In this policy response, employee-side payroll taxes are reduced for a period of 8 quarters. During this period, which continues irrespective or whether the recession continues or ends, employed households' income is increased by the amount of the tax cut. The income of unemployed households is unchanged by this policy.[IF: Again, do we want to provide the size of the tax cut here (2%)? Also note that there is a 50% chance, that the policy is extended by another 8 quarters if the recession is still ongoing in the 8th quarter of the payroll tax cut (although this add-on does not really affect the results in any significant way.)]

2.3 Aggregate Demand Effects

Our baseline model is a partial equilibrium model that does not include any feedback from aggregate consumption to income. In an extension to the model, we add aggregate demand effects during the recession. With this extension, any changes in consumption

away from the steady state consumption level feed back into labor income. Aggregate demand effects are evaluated as:

$$AD(C_t) = \begin{cases} \left(\frac{C_t}{\tilde{C}}\right)^\kappa, & \text{if in a recession} \\ 1, & \text{otherwise} \end{cases} \tag{9}$$

where $\tilde{C}$ is the level of consumption in steady state. Idiosyncratic income in the aggregate demand extension is multiplied by $AD(C_t)$:

$$y_{AD,i,t} = AD(C_t)y_{i,t} \tag{10}$$

The series $y_{AD,i,t}$ is then used for each household's budget constraint.

3 Parameterizing the model

This section describes how we set the various parameters of the model. First, we estimate the extent to which consumers "splurge" when receiving an income shock. We do so using Norwegian data to be consistent with the best available evidence on the time profile of the marginal propensity to consume provided by **?**. For this exercise we use a version of the model calibrated to the Norwegian economy.

Second, we move on to the calibration of the full model on US data taking the splurge-factor as given. We then have different types of agents that differ according to their level of education and their subjective discount factors. Some parameters are calibrated equally for all of these different types, while some parameters are calibrated separately for each education group. Finally, a distribution of subjective discount factors is estimated separately for each education group to match features of the wealth distribution within that group.

3.1 Estimation of the "splurge" factor

We define splurging as the free spending of available income without concern for intertemporal maximization of utility. As we will show in this section, it is necessary to allow for splurging in order to capture the shorter and longer term response of consumption to income shocks. Specifically, we show that our model can account well for the results of **?**, who study the impact of lottery winnings in Norway on consumption using millions of datapoints from the Norwegian population registry. To do so we calibrate our model to reflect the Norwegian economy and estimate the splurge factor, as well as the distribution of discount factors in the population to match two empirical moments.

First, we take from **?** the marginal propensity to consume out of a one-period income shock. We not only target the contemporaneous response of consumption to the income shock, but also the subsequent impact on consumption in years one through four after the shock. The share of lottery winnings expended at different time horizons, as found in **?**, are plotted in figure 1a.

Second, we match the steady-state distribution of liquid wealth in the model to its empirical counterpart. Due to the lack of data on the liquid wealth distribution in Norway, we resort to the corresponding data from the US - assuming that liquid wealth inequality is comparable across these countries.[1] Specifically, we impose as targets the cumulative liquid wealth share for the entire population at the 20th, 40th, 60th and 80th income percentile, which in data from the Survey of Consumer Finance in 2004 equal 0.03 percent, 0.35 percent, 1.84 percent, and 7.42 percent.[2] Hence, 92.6 percent of the total liquid wealth is held by the top income quintile. The data is plotted in figure 1b.

(a) Share of lottery win spent

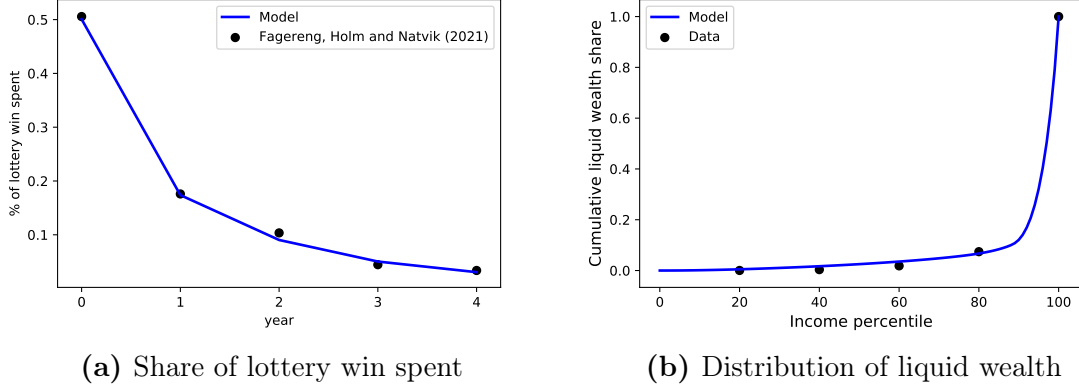**(b)** Distribution of liquid wealth

Figure 1 Targets and model moments from the estimation

For this estimation exercise, the remaining model parameters are calibrated to reflect the Norwegian economy. Specifically, we set the real interest rate to 2 percent annually and the unemployment rate to 4.4 percent, in line with **?**. The quarterly probability to survive is calibrated to $1 - 1/160$, reflecting an expected working life of 40 years. Aggregate productivity growth is set to 1 percent annually following **?**. The unemployment net replacement rate is calibrated to 60 percent following **?**. Finally, we set the real interest rate on liquid debt to 13.6 percent and the borrowing constraint to 80 percent of permanent income following data from the Norwegian debt registry **?**.[3]

Estimates of the standard deviations of the permanent and transitory shocks are taken from **?** who estimate an income process on administrative data for Norwegian males from 1971 to 2014. The estimated annual variances for the permanent and transitory shocks are 0.004 and 0.033, respectively.[4] As in **?**, these are converted to quarterly values by

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[1] Data from the Norwegian tax registry contains information on liquid assets, but not liquid debt. Only total debt is reported, and this is mainly mortgage debt. Therefore, we cannot construct liquid wealth as in for example **?**.

[2] See section 3.2 for details.

[3] Specifically, we determine the average volume-weighted interest rate on liquid debt, which consists of consumer loans, credit and payment card debt and all other unsecured debt. To determine the borrowing limit on liquid debt we determine the ratio between total credit card limit divided by total wage income in Norway. We use data from December 2019. Note that although these data let us pin down aggregate quantities, they do not solve the issue referred to in footnote 1, since we cannot link them to the tax registry at the individual level.

[4] As shown in **?**, an income process of the form that we use here should be estimated using moments in levels not differences. Hence, we take the numbers from column 3 of their Table 4.

multiplying the permanent and transitory shock variances by 1/4 and 4, respectively. Thus, we obtain quarterly standard deviations of $XX = 0.0316$ and $XX = 0.363$.

Using the calibrated model, unexpected lottery winnings are simulated and the share of the lottery spent in each year is calculated. Specifically, each simulated agent receives a lottery win in a random quarter of the first year of the simulation. The size of the lottery win is itself random and spans the range of lottery sizes found in **?**. The estimation procedure minimizes the distance between the target and model moments by selecting the splurge factor and the distribution of discount factors in the population, where the latter are assumed to be uniformly distributed in the range $[\beta - \nabla, \beta + \nabla]$. We approximate the uniform distribution of discount factors with a discrete approximation and let the population consist of 7 different types.

The estimation yields a splurge factor of 0.32 and a distribution of discount factors described by $\beta = 0.986$ and a $\nabla = 0.0174$. Given these estimated parameters and the remaining calibrated ones, the model is able to replicate the time path of consumption in response to a lottery win from **?** and the targeted distribution of liquid wealth very well, see figure 1.

3.2 Data on permanent income, liquid wealth and education

Before we move on to the parameterization of the full model for the US, we describe in detail the data that we use to get measures of permanent income, liquid wealth and the division of households into educational groups. We use data on the distribution of liquid wealth from the 2004 wave of the Survey of Consumer Finance (SCF). We restrict our attention to households where the the head of the household is of working age which we define to be in the range from 25 to 62. The SCF-variable "normal annual income" is our measure of the household's permanent income, and to exclude outliers we drop the observations that make up the bottom 5 percent of the distribution of this variable. The smallest value of permanent income for households in our sample is thus \$16,708.

Liquid wealth is defined as in **?** and consists of cash, money market, checking, savings and call accounts, directly held mutual funds, stocks and bonds. We subtract off liquid debt which is the revolving debt on credit card balances. Note that the SCF does not contain information on cash holdings, so this is imputed with the procedure described in Appendix B.1 of **?** which also describes the credit card balances that are considered part of liquid debt. We drop any households that have negative liquid wealth.

Households are classified into three educational groups. The first group "Dropout" applies to households where the head of household has not obtained a high school diploma, the second group "Highschool" includes heads of households that have a high school diploma and those who in addition have some years of college education without obtaining a bachelor's degree, and the third group "College" consists of heads of households who have obtained a bachelor's degree or higher. With this classification of the education groups, the "Dropout" group makes up 9.3 percent of the population, the "Highschool" group 52.7 percent, and the "College" group 38.0 percent.

With our sample selection criteria we are left with a sample representing about 61.3 million US households.

3.3 Calibrated parameters

With households divided into the three education groups, some parameters, presented in table 1, are calibrated equally across all groups, while other parameters, presented in table 2, are education-specific. Households are also assumed to be ex-ante heterogeneous in their subjective discount factors in addition to their level of education.

Parameter	Notation	Value
Risk aversion		1.0
Splurge		0.32
Survival probability, quarterly		0.994
Risk free interest rate, quarterly		1.01
Standard deviation of transitory shock		0.346
Standard deviation of permanent shock		0.0548
Unemployment benefits replacement rate (share of PI)		0.3
Unemployment income w/o benefits (share of PI)		0.05
Avg. duration of unemp. spell in normal times (quarters)		1.5
Avg. duration of unemp. benefits in normal times (quarters)		2

Table 1 Calibrated parameters that apply to all types. "PI" refers to permanent income.

All households are assumed to have log preferences over consumption, so the coefficient of relative risk aversion is set to XX=1. We also assume that all households have the same propensity to splurge out of transitory income gains and set XX=0.32, the value estimated in section 3.1. However, each education group is divided into types that differ in their subjective discount factors. The distributions of discount factors for each education group are estimated to fit the distribution of liquid wealth within that group, and this is described in detail in section 3.4. Regardless of type, households face a constant survival probability each quarter. This is set to $1-1/160$, reflecting an expected working life of 40 years. The real interest rate on households' savings is set to 1 percent annually.

When households are born, they receive an initial level of permanent income. This initial value is drawn from a log-normal distribution which depends on the education level the household is born with. For each education group, the parameters of the distribution are determined by the mean and standard deviation of log-permanent income for households of age 25 in that education group in the SCF 2004. For the "Dropout" group the mean initial value of quarterly permanent income is $6,200, for the "Highschool" group it is $11,100, and for the "College" group it is $14,500. The standard deviations of the log-normal distributions for each group are respectively 0.32, 0.42, and 0.53.

While households remain employed, their income is subject to both permanent and transitory idiosyncratic shocks. These shocks are distributed equally for the three

Parameters calibrated for each education group			
	Dropout	Highschool	College
Percent of population	9.3	52.7	38.0
Avg. quarterly PI of "newborn" agent ($1000)	6.2	11.1	14.5
Std. dev. of log(PI) of "newborn" agent	0.32	0.42	0.53
Avg. quarterly gross growth rate of PI	1.0036	1.0045	1.0049
Unemployment rate in normal times (percent)	8.5	4.4	2.7

Table 2 Parameters calibrated for each education group. "PI" refers to permanent income.

education groups. The standard deviations of these shocks are taken from **?** who set the standard deviations of the transitory and permanent shocks to $XX = 0.346$ and $XX = 0.0548$, respectively. Permanent income also grows on average with a growth rate XX that depends on the level of education. These average growth rates are based on numbers from **?** who construct age-dependent expected permanent income growth factors using numbers from **?** and fit the age-dependent numbers to their life-cycle model. We construct the quarterly growth rates of permanent income in our perpetual youth model by taking the average of the age-dependent growth rates during a household's working life. The average gross quarterly growth rates that we obtain for the three education groups are then $XX_d = 1.0036$, $XX_h = 1.0045$, and $XX_c = 1.0049$.

Households also face the risk of becoming unemployed, and the parameters describing unemployment in normal times are taken from **?**. The unemployment benefits replacement rate is thus set to XX=0.3 for all households, and when benefits run out, the unemployment income without any benefits is set to XX=0.05. These replacement rates are set as a share of the households' permanent income. The probability of transitioning out of unemployment is also the same for all households, and is set to XX=2/3. This implies that the average duration of an unemployment spell in normal times is 1.5 quarters. The duration of unemployment benefits in normal times is set to 2 quarters. However, the different education groups do differ in the probability of transitioning into unemployment in the first place. These probabilities are set to match the average US unemployment rate by education group in 2004.[5] This average was 8.5 percent for the "Dropout" group, 4.4 percent for the "Highschool" group, and 2.7 percent for the "College" group. This implies that the probability of transitioning into unemployment in normal times are $XX_d = 6.2$ percent, $XX_h = 3.1$ percent and $XX_c = 1.8$ percent.

3.4 Estimating the discount factor distributions

Discount factor distributions are estimated separately for each education group to match the distribution of liquid wealth for households in that group. To do so, we let each

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[5]Source: Statista.com.

education group consist of different types that differ in their subjective discount factor, β. The discount factors within each group $e \in \{d, h, c\}$ are assumed to be uniformly distributed in the range $[\beta_e - \nabla_e, \beta_e + \nabla_e]$. The parameters β_e and ∇_e are chosen for each group separately to match the median liquid wealth to permanent income ratio and the 20th, 40th, 60th, and 80th percentile points of the Lorenz curve for liquid wealth for that group. We approximate the uniform distribution of discount factors with a discrete approximation and let each education group consist of 7 different types.

Panel A of Table 3 shows the estimated values of (β_e, ∇_e) for each education group. The panel also shows the minimum and maximum values of the discount factors we actually use in the model when we use a discrete approximation with 7 values to the uniform distribution of discount factors. As is clear from the maximum values, all types in the model have a discount factor below 1.

The minimum values for the discount factors on the other hand, indicate that some of the household types in the model are very impatient, particularly in the Dropout group. This reflects that liquid wealth is very concentrated within that education group with the top quintile holding 96.4 percent of the liquid wealth for that group. Such low estimates for discount factors are in line with those obtained in the literature on payday lending (see for example XX and XX).

Panel (A) Estimated discount factor distributions

	Dropout	Highschool	College
(β_e, ∇_e)	(0.799, 0.228)	(0.937, 0.066)	(0.985, 0.012)
(Min, max) in approximation	(0.604, 0.995)	(0.881, 0.994)	(0.975, 0.996)

Panel (B) Estimation targets

	Dropout	Highschool	College
Median LW/PI (data)	4.64	30.2	112.8
Median LW/PI (model)	4.64	30.2	112.8
$[20, 40, 60, 80]$ pctiles of Lorenz curve (data)	$[0, 0.01, 0.6, 3.6]$	$[0.06, 0.6, 3.0, 11.6]$	$[0.2, 0.9, 3.3,$
$[20, 40, 60, 80]$ pctiles of Lorenz curve (model)	$[0.0, 0.0, 0.5, 3.6]$	$[0.04, 0.9, 3.7, 11.3]$	$[0.3, 1.5, 4.0,$

Panel (C) Non-targeted moments

	Dropout	Highschool	College	Population
Percent of total wealth (data)	0.8	17.9	81.2	100
Percent of total wealth (model)	1.6	21.2	77.3	100
Average quarterly MPC (model)	0.63	0.38	0.14	0.31
Average annual MPC (model, incl. splurge)	0.88	0.79	0.57	0.72

Table 3 Estimated discount factor distributions, estimation targets and non-targeted moments.

Panel B of Tabel 3 and Figure 2 show the estimation targets and how well the model manages to fit them. The bottom-right quadrant of Figure 2 also shows how well the model fits the liquid wealth distribution for the population as a whole. The fit is quite close, but the model does produce a population where liquid wealth is slightly more concentrated than in the data.

Finally, panel C of Table 3 shows that the estimated model produces a wealth distribution across the three education groups that is fairly close to the one in the data. The panel also reports the average, quarterly and annual MPCs for each education group. The quarterly MPC results from the households' decision problem where they optimally allocate between consumption and savings. The annual MPC takes into account the initial splurge factor when an income shock is first received as well as the decisions to consume out of additional income that remains after splurging for four quarters.
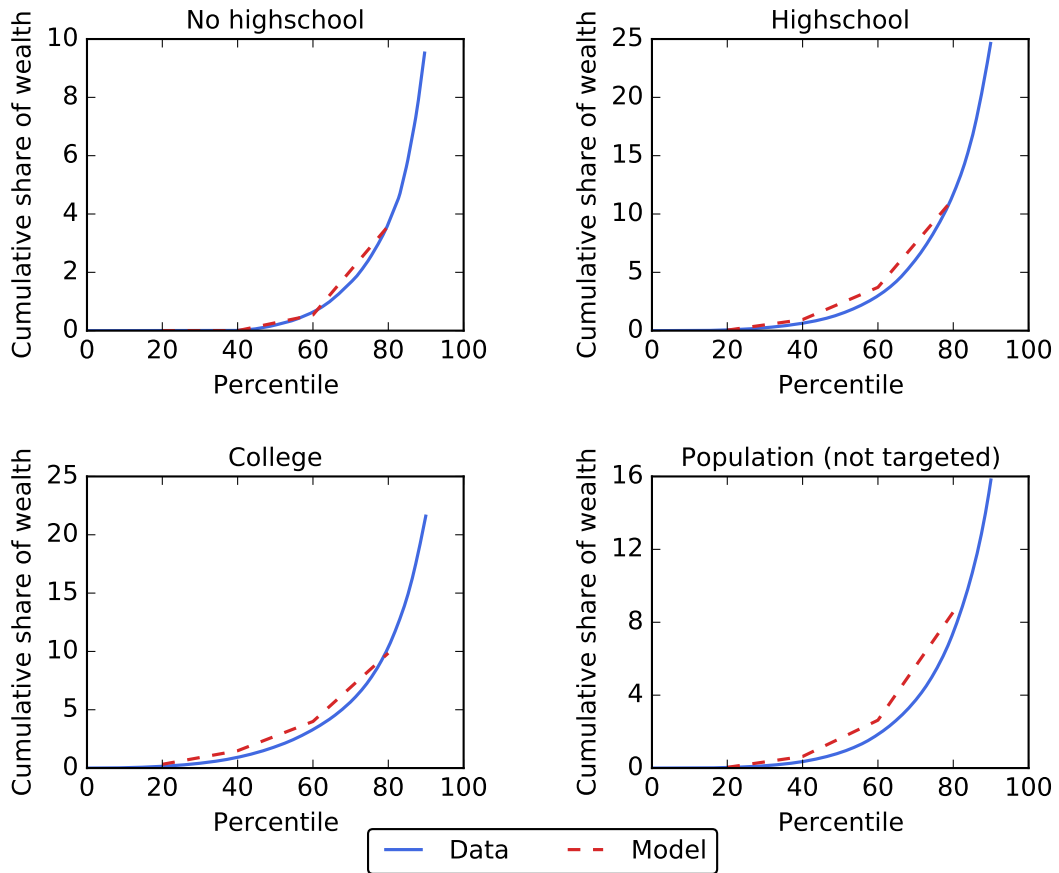

Figure 2 Distributions of liquid wealth within each educational group and for the whole population from the 2004 Survey of Consumer Finance and from the estimated model.
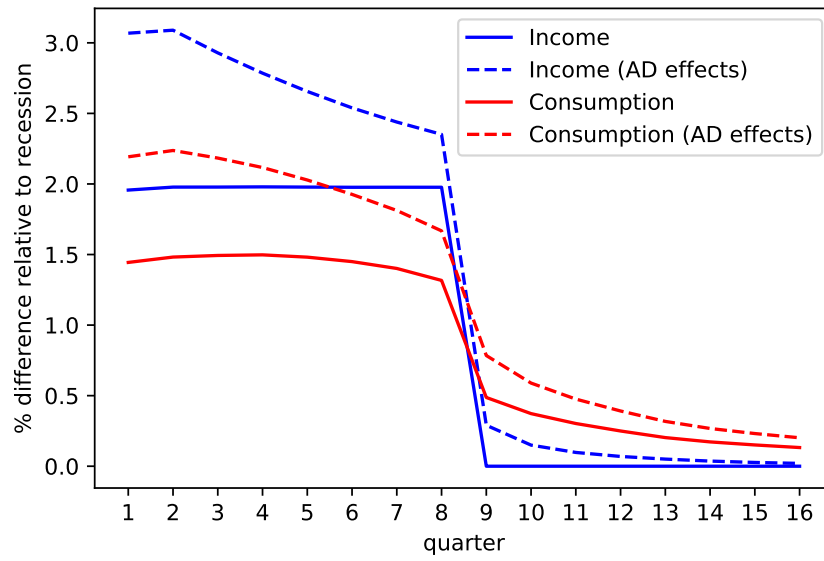
Figure 3 Impulse responses of aggregate income and consumption to a pay roll tax cut during a recesssion lasting eight quarters with and without aggregate demand effects

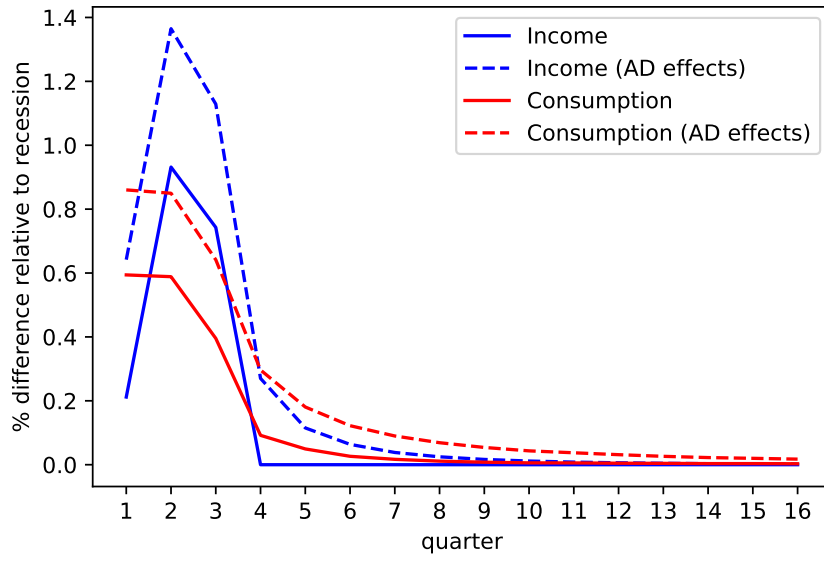# 4 Results

4.1 Impulse responses

Figure 4 Impulse responses of aggregate income and consumption to a UI extension during a recesssion with and without aggregate demand effects
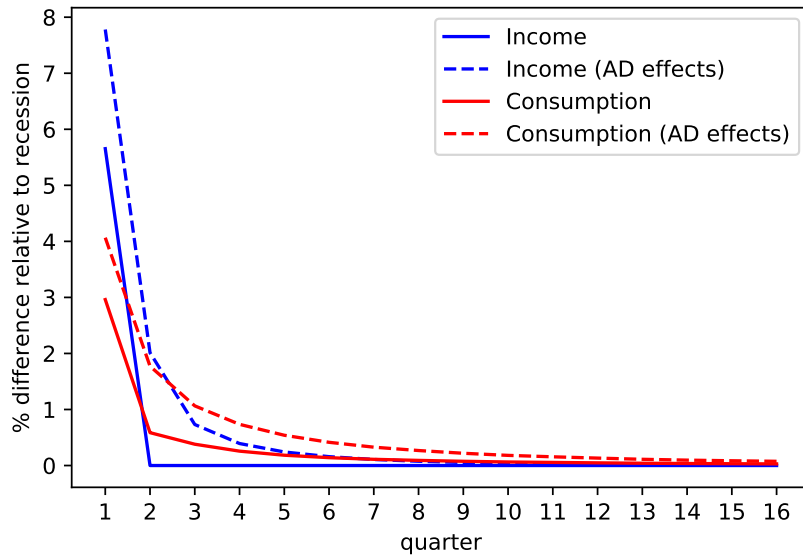

Figure 5 Impulse responses of aggregate income and consumption to a stimulus check during a recesssion with and without aggregate demand effects

4.2 Multipliers

Definitions:

- The *net present value (NPV)* of a variable X at horizon t is given by

$$NPV(t, X) = \sum_{s=0}^{t} \left(\prod_{i=1}^{s} \frac{1}{R_i} \right) X_s \tag{11}$$

- The *cummulative multiplier (CM)* of a policy is given by

$$CM(t) = \frac{NPV(t, \Delta C)}{NPV(T_{max}, \Delta G)} \tag{12}$$

where ΔC is the additional aggregate consumption spending in the policy scenario relative to the baseline and ΔG is the government expenditures caused by the policy.

4.3 Linearity of the check multiplier

Table 6 shows the amount of additional consumption stimulated (as % of baseline consumption) caused by stimulus checks of different sizes as well as their multiplier (i.e. the ratio between stimulated consumption and the cost of the policy, in net present value). The table shows that the impact of the check stimulus experiment scales roughly linearly with the size of the stimulus check, leaving the multiplier largely unaffected by the size of check.

5 Welfare analysis

We want to convert welfare units to consumption units. A proportional increase in every agents' consumption in the baseline by fraction x, in welfare, is equal to:

$$x \frac{1}{N} \sum_{i=1}^{N} \sum_{t=0}^{\infty} D^t c_{it,\text{base}} u'(c_{it,\text{base}}) \tag{13}$$

where c_{it} is consumption (including the splurge) of agent i at time t and D is the social planner's discount rate. N is the number of agents.

	Tax Cut	UI extension	Stimulus check
Multiplier (with AD effects)	1.285	1.795	1.850
Multiplier (with only 1st round AD effects)	1.146	1.480	1.481
Share of policy expenditure during recession	46.4%	71.4%	66.0 %

Table 4 Multipliers as well as the share of the policy ocurring during the recession for the three policies considered

	Tax Cut	UI extension	Stimulus check
Recession lasts 2q	1.096	1.648	1.689
Recession lasts 4q	1.224	1.718	1.842
Recession lasts 8q	1.471	1.864	1.999

Table 5 Multipliers (with AD effects) for different recesssion lengths for the three policies considered
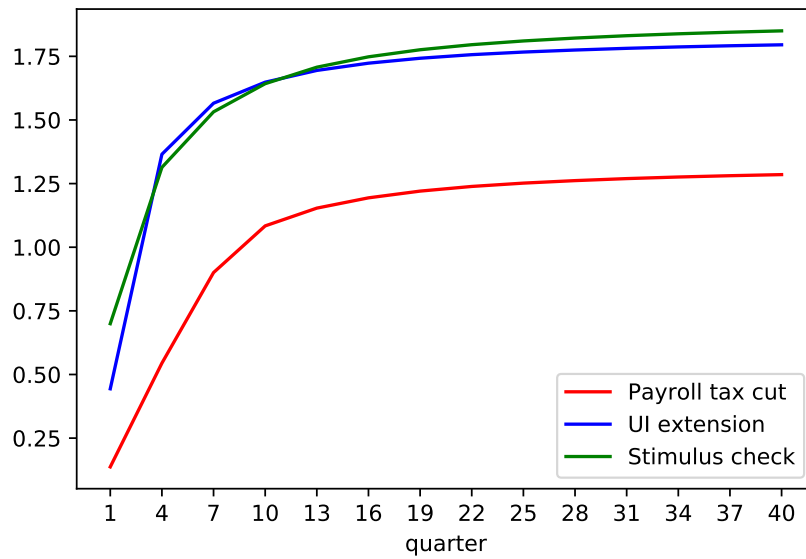

Figure 6 Cummulative Multiplier as a function of the horizon in quarters for the three policies considered. Policies are implemented during a recession with AD effects active

Size of stimulus check	$75	$1200	$5000
Add. cons. as share of baseline cons. (recession, AD)	0.019	0.308	1.294
Multiplier (recession, AD)	1.849	1.850	1.866

Table 6 Multipliers for different sizes of the stimulus check

	Check	UI	Tax Cut
$\mathcal{C}(Rec, \text{policy})$	0.090	3.395	0.004
$\mathcal{C}(Rec, AD, \text{policy})$	0.426	5.005	0.133

Table 7 Consumption Equivalent Welfare Gains in Basis Points

The cost of such an increase is

$$x \frac{1}{N} \sum_{i=1}^{N} \sum_{t=0}^{\infty} R^{-t} c_{it,\text{base}} \tag{14}$$

Define

$$\mathcal{W}^c = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=0}^{\infty} D^t c_{it,\text{base}} u'(c_{it,\text{base}}) \tag{15}$$

$$\mathcal{P}^c = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=0}^{\infty} R^{-t} c_{it,\text{base}} \tag{16}$$

Aside - with log utility, $\mathcal{W}^c = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=0}^{\infty} D^t = \frac{1}{1-D}$

We will assume that a government expenditure of size F with welfare benefit $\mathcal{W}$ will be funded by a proportional consumption tax of size $\frac{F}{\mathcal{P}^c}$ resulting in a welfare loss of $\frac{F}{\mathcal{P}^c} \mathcal{W}^c$. The overall welfare benefit will be equivalent to consumption units:

$$\mathcal{C} = \frac{\mathcal{W}}{\mathcal{W}^c} - \frac{F}{\mathcal{P}^c} \tag{17}$$

There is also an 'unseen' cost to the government policy exactly equal to implementing the policy in normal times.

Define welfare of a policy as:

$$\mathcal{W}(\text{policy}, AD, Rec) = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=0}^{\infty} D^t u(c_{it,\text{policy},AD,Rec}) \tag{18}$$

So the consumption equivalent of a policy implemented in recession is:

$$\mathcal{C}(\text{policy}, AD, Rec) = \left(\frac{\mathcal{W}(\text{policy}, AD, Rec) - \mathcal{W}(AD, Rec)}{\mathcal{W}^c} - \frac{PV(\text{policy}, Rec)}{\mathcal{P}^c} \right) \tag{19}$$
$$- \left(\frac{\mathcal{W}(\text{policy}) - \mathcal{W}(\text{base})}{\mathcal{W}^c} - \frac{PV(\text{policy})}{\mathcal{P}^c} \right)$$

Table 7 shows results for this method. Note that the policy expenditures of each policy have been equalized.

6 Conclusion

Appendices

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