

APPENDIX A: [HTTPS://ECON-ARK.GITHUB.IO/HAFISCAL/#SEC:APPENDIX-HANKDETAILS](https://econ-ark.github.io/HAFiscal/#sec:appendix-hankdetails)
 OF THE HANK AND SAM MODEL

A.1 <https://econ-ark.github.io/HAFiscal/#sec:hank-householdsHouseholds>

The household block follows closely to the main text with a few exceptions. First, the splurge only occurs out of equilibrium—that is, the steady state of the model is calculated without the splurge behavior. Second, the level of permanent income of all newborns is equal to one. Furthermore, all households face the same employment to unemployment and unemployment to employment probabilities. The probabilities are calibrated to the transition probabilities of high school graduates from the main text. Lastly, following the notation of ?, r_t^a will denote the economy wide ex-ante real interest rate.

A.2 <https://econ-ark.github.io/HAFiscal/#sec:goods-marketGoods Market>

A continuum of monopolistically competitive intermediate goods producers, indexed by $j \in [0, 1]$, produces intermediate goods Y_{jt} , which are sold to a final goods producer at price P_{jt} . Each period, these producers fully consume their profits.

A.3 <https://econ-ark.github.io/HAFiscal/#sec:hank-final-goods-producerFinal Goods Producer>

A perfectly competitive final goods producer purchases intermediate goods Y_{jt} from intermediate good producer j at price P_{jt} and produces the final good Y_t using a CES production function:

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\epsilon_p - 1}{\epsilon_p}} dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}},$$

where ϵ_p is the elasticity of substitution.

Given P_{jt} , the price of intermediate good j , the final goods producer maximizes profits by solving:

$$\max_{Y_t} P_t \left(\int_0^1 Y_{jt}^{\frac{\epsilon_p - 1}{\epsilon_p}} dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} - \int_0^1 P_{jt} Y_{jt} dj.$$

The first order condition leads to demand for good j given by

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\epsilon_p} Y_t,$$

and the price index

$$P_t = \left(\int_0^1 P_{jt}^{1-\epsilon_p} dj \right)^{\frac{1}{1-\epsilon_p}}.$$

A.4 <https://econ-ark.github.io/HAFiscal/#sec:hank-intermediate-goods-producers>

Intermediate goods producers produce according to a production function linear in labor L_t :

$$Y_{jt} = ZL_{jt},$$

where Z is total factor productivity.

Each intermediate goods producer hires labor L_t from a labor agency at cost h_t . Given this labor cost, each producer sets P_{jt} to maximize profits while facing price stickiness à la ?. In HANK models with sticky prices, profits tend to be countercyclical, and when households have high MPCs, this can generate countercyclical consumption responses to dividends. To simplify, we assume that intermediate goods producers fully consume their profits rather than distributing them to households, thereby abstracting from consumption responses to firm profits. Each producer maximizes profits by solving:

$$J_t(P_{jt}) = \max_{\{P_{jt}\}} \left\{ \frac{P_{jt}Y_{jt}}{P_t} - h_t L_{jt} - \frac{\varphi}{2} \left(\frac{P_{jt} - P_{jt-1}}{P_{jt-1}} \right)^2 Y_t + J_{t+1}(P_{jt+1}) \right\},$$

where φ determines the cost of adjusting the price and, hence, the degree of price stickiness.

The problem can be rewritten as the standard New Keynesian maximization problem:

$$\max_{\{P_{jt}\}} E_t \left[\sum_{s=0}^{\infty} M_{t,t+s} \left(\left(\frac{P_{jt+s}}{P_{t+s}} - MC_{t+s} \right) Y_{jt+s} - \frac{\varphi}{2} \left(\frac{P_{jt+s}}{P_{jt+s-1}} - 1 \right)^2 Y_{t+s} \right) \right],$$

where $MC_t = \frac{h_t}{Z}$.

Given that all firms face the same adjustment costs, there exists a symmetric equilibrium where all firms choose the same price with $P_{jt} = P_t$ and $Y_{jt} = Y_t$.

The resulting Phillips Curve is

$$\epsilon_p MC_t = \epsilon_p - 1 + \varphi(\Pi_t - 1)\Pi_t - M_{t,t+1}\varphi(\Pi_{t+1} - 1)\Pi_{t+1} \frac{Y_{t+1}}{Y_t}$$

where $\Pi_t = \frac{P_t}{P_{t+1}}$.

A.5 <https://econ-ark.github.io/HAFiscal/#sec:hank-labor-market>

A risk-neutral labor agency supplies labor N_t to intermediate goods producers at cost h_t by hiring households at wage w_t . To hire workers, the agency posts vacancies v_t , which are filled with probability ϕ_t . Household job search is random. Following ?, we assume the labor agency cannot observe individual household productivity. Instead, it only observes the average productivity of all employed workers, which is normalized to one.

Labor agency. The labor agency determines how many vacancies to post and how much labor to sell by solving the following problem:

$$J_t(N_{t-1}) = \max_{N_t, v_t} \left\{ (h_t - w_t)N_t - \kappa v_t + E_t \left[\frac{J_{t+1}(N_t)}{1 + r_t^a} \right] \right\},$$

subject to

$$N_t = (1 - \omega)N_{t-1} + \phi_t v_t.$$

The parameters κ and ω are, respectively, the cost of posting a vacancy, and the job separation rate.

The resulting job creation curve is:

$$\frac{\kappa}{\phi_t} = (h_t - w_t) + (1 - \omega)E_t \left[\frac{\kappa}{(1 + r_t^a)\phi_{t+1}} \right].$$

Matching. The matching process between households and the labor agency follows a Cobb-Douglas matching function:

$$m_t = \chi e_t^\alpha v_t^{1-\alpha},$$

where m_t is the mass of matches, e_t is the mass of job searchers, α is the matching function elasticity, and χ is a matching efficiency parameter.

The vacancy filling probability ϕ_t and the job finding probability η_t evolve as follows:

$$\eta_t = \chi \Theta_{it}^{1-\alpha}$$

$$\phi_t = \chi \Theta_t^{-\alpha}$$

where $\Theta_t = \frac{v_t}{e_t}$ is labor market tightness.

Wage Determination. Following ? and ?, we assume the real wage evolves according to the following rule:

$$\log \left(\frac{w_t}{w_{ss}} \right) = \phi_w \log \left(\frac{w_{t-1}}{w_{ss}} \right) + (1 - \phi_w) \log \left(\frac{N_t}{N_{ss}} \right),$$

where ϕ_w dictates the extent of real wage rigidity.

A.6 <https://econ-ark.github.io/HAFiscal/#sec:hank-fiscal-policyFiscal Policy>

The government issues long term bonds B_t at price q_t^b in period t that pays δ^s in period $t+s+1$ for $s \in \{0, 1, 2, \dots\}$.

The bond price satisfies the no arbitrage condition:

$$q_t^b = \frac{1 + \delta E_t[q_{t+1}^b]}{1 + r_t^a}.$$

The government funds its expenditures through debt and taxes, subject to the following budget constraint:

$$(1 + \delta q_t^b)B_{t-1} + G_t + S_t = \tau_t w_t N_t + q_t^b B_t,$$

where S_t are payments for unemployment insurance and other transfers.

For all stimulus policies, except tax cuts, we follow ? and allow the tax rate to adjust in order to stabilize the debt-to-GDP ratio:

$$\tau_t - \tau_{ss} = \phi_B q_{ss}^b \frac{B_{t-1} - B_{ss}}{Y_{ss}}$$

where ϕ_B governs the speed of adjustment.

For the tax cuts, we assume government expenditures adjust following:

$$G_t - G_{ss} = \phi_G q_{ss}^b \frac{B_{t-1} - B_{ss}}{Y_{ss}}$$

where ϕ_G governs the speed of adjustment of government spending in response to debt.

A.7 <https://econ-ark.github.io/HAFiscal/#sec:hank-monetary-policy> Monetary Policy

The central bank follows a standard Taylor rule that responds solely to inflation:

$$i_t = r^* + \phi_\pi \pi_t,$$

where ϕ_π is the coefficient on inflation. Inflation is given by $\pi_t = P_t / P_{t-1} - 1$, and r^* is the steady state interest rate.

A.8 <https://econ-ark.github.io/HAFiscal/#sec:hank-equilibrium> Equilibrium

An equilibrium in this economy is a sequence of:

- Policy Functions $(c_{it}(m))_{t=0}^\infty$ normalized by permanent income.
- Prices $(r_{t+1}^a, i_t, q_t^b, w_t, h_t, \pi_t, \tau_t)_{t=0}^\infty$.
- Aggregates $(C_t, Y_t, N_t, \Theta_t, B_t, A_t)_{t=0}^\infty$.

Such that:

- $(c_{it}(m))_{t=0}^\infty$ solves the household's maximization problem given $(w_t, \eta_t, r_t^a, \tau_t)_{t=0}^\infty$.
- The final goods producer and intermediate goods producers both maximize their respective objective functions.
- The nominal interest rate is determined by the central bank's Taylor rule.
- The tax rate is set according to the fiscal rule, ensuring that the government budget constraint is satisfied.

TABLE 1. Calibration

Description	Parameter	Value	Source/Target
Elasticity of Substitution	ϵ_p	6	Standard
Price Adjustment Costs	φ	96.9	??
Vacancy Cost	κ	0.056	$\frac{\kappa}{w\phi} = 0.071$
Job Separation Rate	ω	0.092	Match $\pi(eu)$ for Highschool group
Matching Elasticity	α	0.65	??
Job Finding Probability	η_{ss}	0.67	$\pi(ue)$ in section ??
Vacancy Filling Rate	ϕ_{ss}	0.71	den Haan et al. (?)
Real Wage Rigidity parameter	ϕ_w	0.837	Gornemann et al. (?)
Government Spending	G	0.38	Gov. budget constraint
Decay rate of Gov. Coupons	δ	0.95	5 Year Maturity of Debt
Response of Tax Rate to Debt	ϕ_B	0.015	Auclert et al. (?)
Taylor Rule Inflation Coefficient	ϕ_π	1.5	Standard

- The value of assets is equal to the value of government bonds:

$$A_t = q_t^b B_t.$$

- The goods market clears¹:

$$C_t = w_t N_t + G_t,$$

where $C_t \equiv \int_0^1 \mathbf{c}_{it} di$.

- The labor demand of intermediate goods producers equals labor supply of labor agency:

$$L_t = N_t.$$

A.9 <https://econ-ark.github.io/HAFiscal/#sec:hank-calibration> Calibration of Non-Household Blocks

The elasticity of substitution is set to 6, and the price adjustment cost parameter is set to 96.9 as in ?. The vacancy cost is set to 7 percent of the real wage as in ?.² The matching

¹Note if profits were not held by firms then the goods market condition would be $C_t + G_t = Y_t - \kappa v_t - \frac{\varphi}{2} \left(\frac{P_t}{P_{t-1}} - 1 \right)^2 Y_t$. In particular, since firm profits are $D_t = Y_t - w_t N_t - \kappa v_t - \frac{\varphi}{2} \left(\frac{P_t}{P_{t-1}} - 1 \right)^2 Y_t$, then the goods market condition would become $C_t + G_t = w_t N_t + D_t = Y_t - \kappa v_t - \frac{\varphi}{2} \left(\frac{P_t}{P_{t-1}} - 1 \right)^2 Y_t$.

²The range of plausible values lie between 4percent and 14percent ?

elasticity is 0.65 following [??](#). The job separation rate is set to 0.092. As in section [??](#), we set the job finding probability in the steady state for the unemployed η_{ss} to 0.67. Along with the job separation rate, this gives a probability of transitioning from employment to unemployment within a quarter of 3.1 percent which is the value we use for the High-school group in section [??](#). The quarterly vacancy filling rate is 0.71 as in [?](#) (and together with our other choices, this pins down the matching efficiency χ). The degree of wage rigidity ϕ_w is set to 0.837 following [?](#). The tax rate is set to 0.3 and government spending is set to clear the government budget constraint. The parameters that dictate the speed of fiscal adjustment, ϕ_B and ϕ_G , are set to 0.015, the lower bound of the estimates in [?](#).³ Furthermore, the decay rate of government coupons is set to $\delta = 0.95$ to match a maturity of 5 years.⁴ Finally, the Taylor rule coefficient on inflation is set to the standard value of $\phi_\pi = 1.5$.

³The speed of adjustment parameter is set to the lower bound to ensure that the policies evaluated in the HANK and SAM model are almost entirely deficit financed.

⁴The duration of bonds in the model is $\frac{(1+r)^4}{(1+r)^4 - \delta}$