1 The Method of Moderation

moderation}

The endogenous gridpoints method constructs a consumption function $\grave{\mathrm{c}}_t$ by interpolating a finite set of (m, c) pairs. A practical problem arises when the approximation must be evaluated at values of m outside the grid: naïve linear extrapolation can predict consumption so high that precautionary saving turns negative, an economically impossible result.

A solution, developed in **?**, exploits the fact that the true consumption function is bounded between two analytically computable perfect-foresight rules.

- **Optimist.** An agent who is certain that future income shocks will always equal their mean has human wealth $h_{\succ}$ and consumes

AboveBrief}
$$\bar{\mathrm{c}}_t(m) \;=\; (m + h_{\succ})\,\underline{\kappa}_t, \tag{1}$$

  where $\underline{\kappa}_t$ is the minimal marginal propensity to consume of the corresponding perfect-foresight problem.

- **Pessimist.** An agent who assumes the worst possible income realisation in every future period has lower human wealth and a consumption floor at

BelowBrief}
$$\underline{\mathrm{c}}_t(m) \;=\; (m - \underline{m}_t)\,\underline{\kappa}_t, \tag{2}$$

  where $\underline{m}_t$ is the natural borrowing constraint.

- **Realist.** The true solution satisfies $\underline{\mathrm{c}}_t(m) < \mathrm{c}_t(m) < \bar{\mathrm{c}}_t(m)$ for all m.

Because the realist's consumption always lies between these bounds, one can define a *moderation ratio*

odRteBrief}
$$\omega_t(m) \;\equiv\; \frac{\mathrm{c}_t(m) - \underline{\mathrm{c}}_t(m)}{\bar{\mathrm{c}}_t(m) - \underline{\mathrm{c}}_t(m)} \;\in\; (0, 1), \tag{3}$$

which measures how close the realist is to the optimist ($\omega = 1$) versus the pessimist ($\omega = 0$). The key insight is that the logit of this ratio, $\log\bigl(\omega/(1-\omega)\bigr)$, is nearly linear in m and therefore easy to interpolate accurately. Consumption is recovered via the inverse transformation:

omChiBrief}
$$\mathrm{c}_t(m) \;=\; \underline{\mathrm{c}}_t(m) \;+\; \frac{1}{1 + e^{-\chi_t(m)}}\bigl(\bar{\mathrm{c}}_t(m) - \underline{\mathrm{c}}_t(m)\bigr), \tag{4}$$

where $\chi_t \;\equiv\; \log\bigl(\omega_t/(1 - \omega_t)\bigr)$ is the logit-transformed moderation ratio. Since the recovered c is automatically sandwiched between the bounds, the extrapolation problem vanishes.

Figure **??** illustrates the idea: the realist consumption function (middle curve) is bounded above by the optimist and below by the pessimist.

The full treatment in **?** extends the method with tighter upper bounds, applies an analogous transformation to the value function, and shows how to handle stochastic returns and Hermite interpolation refinements.

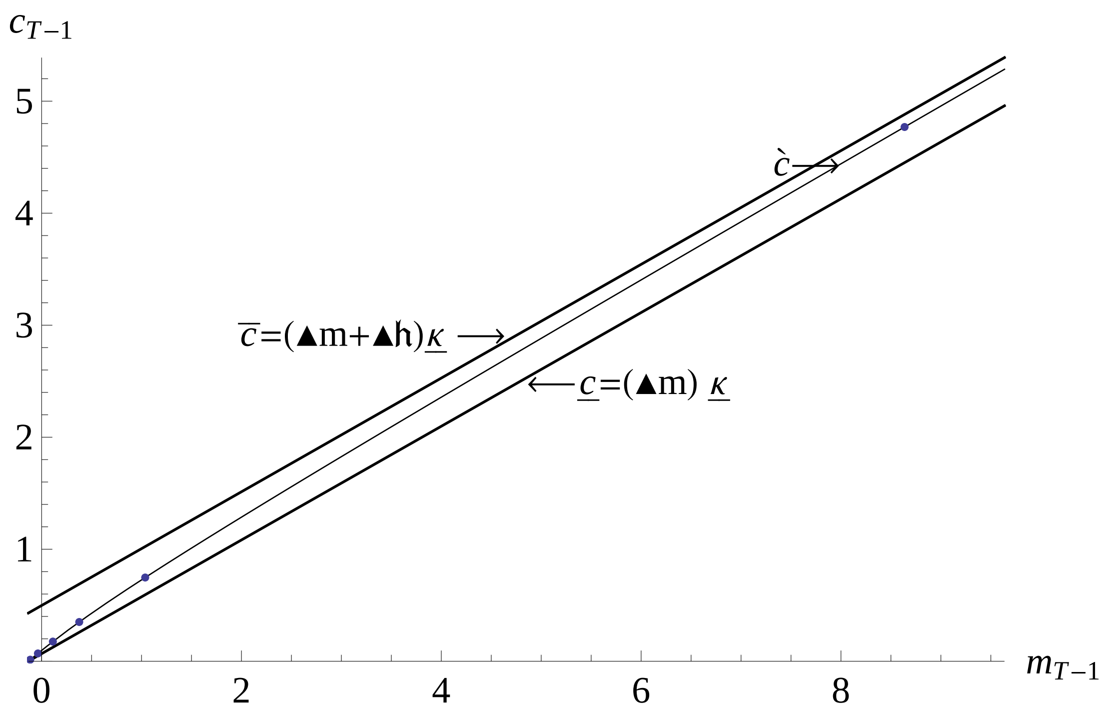

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Figure 1 Moderation Illustrated: $\underline{c}_{t-1} < \grave{c}_{t-1} < \overline{c}_{t-1}$