0.1 Model with Time Preference Heterogeneity

In this model, the economy consists of a continuum of households of mass one distributed on the unit interval. Households die with a constant probability $D = 1 - \cancel{D}$ between periods. This is different from the baseline model in which households only face probability of dying after they become unemployed. The income process of a household has been described in section 3.2. Each household maximizes expected discount utility from consumption:

$$\max \mathbb{E}_t \sum_{n=0}^{\infty} (\cancel{D}\beta)^n u(c_{t+n}) \tag{1}$$

The household consumption functions satisfies:

$$
\begin{aligned}
v(m_t) &= \max_{c_t} u(c_t(m_t)) + \beta\cancel{D}\mathbb{E}_t(\psi_{t+1})^{1-\rho} v(m_{t+1}), \tag{2} \\
&\text{s.t.,} \\
a_t &= m_t - c(m_t) \tag{3} \\
k_{t+1} &= \frac{a_t}{\cancel{D}\psi_{t+1}} \tag{4} \\
m_{t+1} &= (\daleth + r_t)k_{t+1} + \xi_{t+1} \tag{5} \\
a_t &\geq 0 \tag{6}
\end{aligned}
$$

where the variables are divided by the level of permanent income $\boldsymbol{p} = p_t \boldsymbol{W}$, so that when aggregate shocks are shut down, the only state variable is (normalized) cash-on hand m_t. The production function is Cobb-Douglass:

$$ZK^\alpha(\ell L)^{1-\alpha} \tag{7}$$

The aggregate wage rate $\boldsymbol{W}_t$ is determined by the aggregate productivity Z_t, capital stock K_t, and the aggregate supply of labor L_t:

$$\boldsymbol{W}_t = (1-\alpha)Z_t \left(\frac{K_t}{\ell L}\right)^\alpha \tag{8}$$

L_t is driven by two aggregate shocks:

$$L_t = P_t \Theta_t \tag{9}$$
$$P_t = P_{t-1}\Psi_t \tag{10}$$

where P_t is aggregate permanent productivity, Ψ_t is the aggregate permanent shock and Θ_t is the aggregate transitory shock.[1]

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[1]Note that Ψ is the capitalized version of the Greek letter ψ used for the idiosyncratic permanent shock; similarly Θ is the capitalized θ

References