A Tractable Model of Precautionary Reserves, Net Foreign Assets, or Sovereign Wealth Funds

October 2015

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Abstract

We model the motives for residents of a country to hold foreign assets, including the precautionary motive that has been omitted from much previous literature as intractable. Our model captures the principal insights from the existing literature on the precautionary motive with a novel and convenient formula for the economy's target asset ratio. The target is the value of assets that balances growth, impatience, prudence, risk, intertemporal substitution, and the rate of return. We use the model to shed light on two topical questions: "Upstream" flows of capital from developing to advanced countries, and the long-run impact of resorbtion of global financial imbalances.

Keywords	Buffer Stock Saving, Net Foreign Assets, Sovereign Wealth Funds, Foreign Exchange Reserves, Small Open Economy
JEL codes	C61

PDF:	http://econ.jhu.edu/people/ccarroll/papers/cjSOE.pdf
Slides:	http://econ.jhu.edu/people/ccarroll/papers/cjSOE-Slides.pdf
Web:	http://econ.jhu.edu/people/ccarroll/papers/cjSOE/
Archive:	http://econ.jhu.edu/people/ccarroll/papers/cjSOE.zip
Code:	http://econ.jhu.edu/people/ccarroll/code/Tractable.zip
	(Contains code solving this paper's model (and related models))

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0.1 Saving and Uncertainty in a More Realistic Model

Our tractable model's advantage is that the mechanisms at play are reflected in analytical expressions with straightforward interpretations. The cost of this tractability is that the model's treatment of uncertainty is so stylized as to be difficult to calibrate to real-world measures of uncertainty.

Without a clear mapping from our theory's stylized treatment of uncertainty to something readily measurable, a future empirical literature would find it difficult to test whether the model is even roughly 'true.'

The natural solution to this problem comes from a literature that has matured in the last couple of decades, in which numerical computational methods are used to solve models whose characterization of uncertainty is much more closely related to empirically measurable quantities. A conclusion of that literature is that an income process that has transitory and permanent components *a la* **?** does a remarkably good job of capturing the most important features of idiosyncratic income dynamics.

These measurement techniques have been most richly applied using U.S. data, but they are spreading. A recent example is the work of **?**, who are among the first to use national registry data (in his case, from Norway) that allows estimation of processes for income dynamics with an unprecedented degree of precision (and even the ability to construct fine-grained calculations of the differences in uncertainty across subgroups with the population). Combining such measures of uncertainty with data on saving rates (either across groups or across countries) should permit measurements of the empirical relationship between saving and uncertainty.

Those measurements can then be compared to predictions of a 'quantitative theorety' model that translates measured differences in uncertainty into implications about differences in saving rates.

We perform such an exercise using the parsimonious model exposited by **?** (henceforth, 'CSTW'), which is able to match the dynamics and distribution of microeconomic income, the distribution of wealth, and the personal saving rate in the U.S. with an income process in which annual income shocks are either transitory or permanent (with magnitudes calibrated to typical microeconomic estimates from U.S. data).

Our exercise is a simple one: Leaving all other aspects of the model the same as in the baseline model that matches U.S. data, we change a single parameter – a measure of uncertainty – over a range of values that are likely to encompass the plausible range of values across countries or subpopulations within countries. We then calculate the saving rate predicted by the model across that range of potential configurations of uncertainty. This produces a quantitative prediction of the magnitude of the relationship between uncertainty and saving that empirical researchers can compare to empirical estimates of the same quantity.

The remainder of this section describes such an exercise in detail, and presents the results.

0.1.1 Income Dynamics

CSTW assume that household income y_t is determined by aggregate wage rate W_t and two idiosyncratic components, the permanent component p_t and the transitory shock ξ_t:

$$y_t = p_t \xi_t \boldsymbol{W}_t \tag{1}$$

The permanent component follows:

$$p_t = p_{t-1} \psi_t \tag{2}$$

The transitory component is:

$$\begin{aligned} \xi_t &= \mu \text{ with probability } \mho_t, \\ &= (1 - \tau_t) \ell \theta_t \text{ with probability } 1 - \mho_t, \end{aligned} \tag{3}$$

Here μ is the unemployment insurance payment when unemployed. τ_t is the rate of tax collected to pay unemployment benefits, ℓ is time worked per employee. ψ and θ are lognormally distributed white noise shocks and $\mathbb{E}_t[\psi_{t+n}] = \mathbb{E}_t[\theta_{t+n}] = 1 \ \forall \ n > 0$. By changing the value of σ_ψ and σ_θ, we change the degree of uncertainty faced by households. The details of the model are described in appendix A.7.

The parameters are calibrated exactly as in CSTW except for the aggregate productivity growth rate and individual productivity growth rate. In CSTW, there is no aggregate and individual productivity growth; here we allow aggregate productivity growth $G = 1.015$, and individual productivity growth $X = 1.01$.

Following CSTW, we allow consumers to have heterogeneous time preference rates; the uniform distribution is calibrated to match the mean and the dispersion of wealth across U.S. households.

For this exercise to succeed, the model needs to be calibrated so that even the most patient consumer satisfies a 'growth impatience condition' that prevents the wealth-to-income ratio from asymptoting to infinity:

$$\boldsymbol{\Thorn}_{\acute{\Gamma}} \equiv \frac{(\beta R)^{1/\rho} \cancel{D}}{\underline{\Gamma}} \quad < \quad 1 \tag{4}$$

where $\underline{\Gamma} = \Gamma \acute{\psi}$ and $\acute{\psi} \equiv (\mathbb{E}[\psi^{-1}])^{-1}$. By computing the aggregate saving rate while increasing σ_ψ^2 and σ_θ^2, we derive the relationship between the aggregate saving rate and the degree of uncertainty. Increasing σ_ψ^2 will also cause $\underline{\Gamma}$ to increase. The growth impatience condition of the most patient agent restricts the maximum value of σ_ψ^2 we can choose. Here we allow σ_ψ^2 and σ_θ^2 to move from half of their benchmark values to twice the benchmark value.

Figure 1 shows how the aggregate saving rate changes with σ_ψ. The first row in Table 2 reports the slope and R^2 calculated from OLS regression between the aggregate saving rate and the standard deviation of the permanent income shock. Under the model calibration drawn from CSTW, the aggregate saving rate is almost perfectly correlated with the standard deviation of σ_ψ: adding 0.01 to σ_ψ will cause the aggregate saving rate to rise about 0.83 percent.

Figure 2 shows how the aggregate saving rate varies with the standard deviation of the

Table 1 Relation Between Measures of Uncertainty and Saving Rate in CSTW model

	Slope	R^2	max $\mathbf{Þ}_\Gamma$
σ_Ψ	0.826	0.998	0.998
σ_θ	0.0111	0.989	0.996

Table 2 presents the correlation between the aggregate saving rate and the degree of income uncertainty. The first two columns show the slope and R^2 from an OLS regression. The third column shows the growth impatience factor of the most patient agent in the economy. The first row shows the result of an OLS regression of the aggregate saving rate on the standard deviation of permanent income shocks σ_ψ. The second row shows the result of OLS regression of the aggregate saving rate on the standard deviation of transitory income shocks σ_θ. (No standard errors are reported because the precision of the estimates is a function of the size of the population that is simulated. The coefficients reported are stable when the simulated population is increased arbitrarily, so they are the 'true' implications of the model.

transitory income shock. The regression result is reported in the second row of Table 2. The impact of transitory uncertainty is negligible: Increasing σ_θ by 0.01 will cause the aggregate saving rate to increase by 0.011 percent. The last column reports the growth impatience factor of the most patient agent when σ_ψ^2 reaches its upper limit in each of the two exercises. We can see that the growth impatience condition is comfortably satisfied in both exercises.
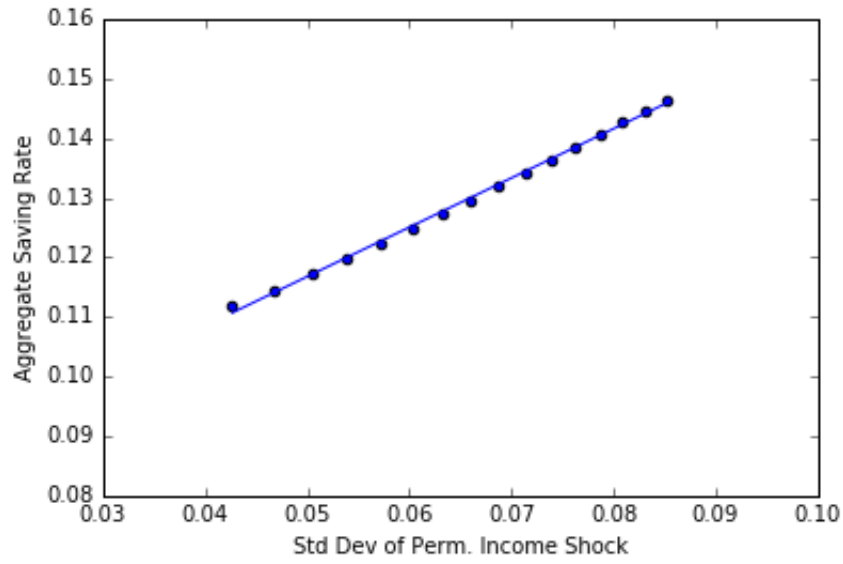
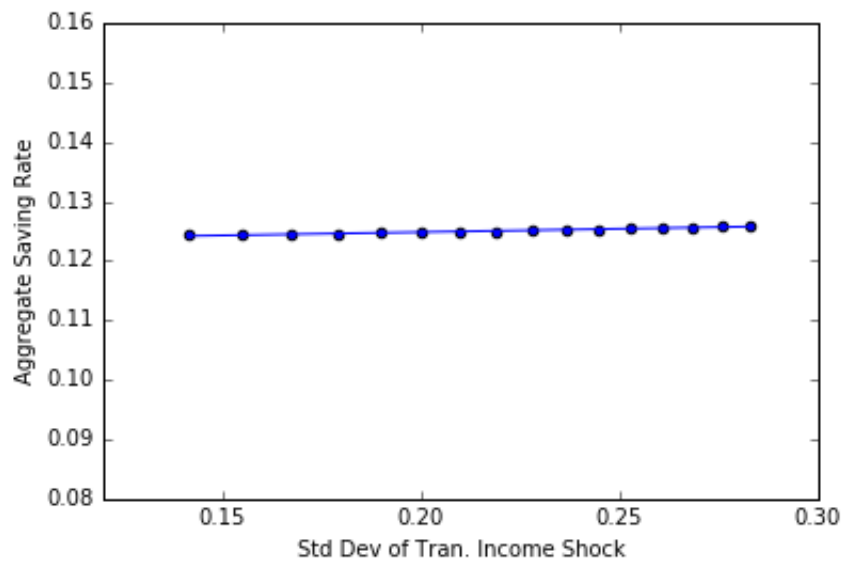**Figure 1** Change in Savings Following increasing in Permanent Income Uncertainty

Figure 2 Change in Savings Following increasing in Transitory Income Uncertainty

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