A Tractable Model of Precautionary Reserves, Net Foreign Assets, or Sovereign Wealth Funds

Christopher Carroll and Olivier Jeanne

Johns Hopkins University

February 25, 2019

Motivation

Three Hot Topics In International Macro:

- Huge Reserve Accumulation By Fast-Growing Developing Economies
  - China
- Surprising "Upstream" Capital Flows: Developing $\rightarrow$ Rich Countries
  - China – Following Japan, Korea, Taiwan, Singapore, Hong Kong
- Sovereign Wealth Funds
  - Mainly Oil-Rich Countries

Motivation

Three Hot Topics In International Macro:

- Huge Reserve Accumulation By Fast-Growing Developing Economies
  - China
- Surprising "Upstream" Capital Flows: Developing $\rightarrow$ Rich Countries
  - China – Following Japan, Korea, Taiwan, Singapore, Hong Kong, ...
- Sovereign Wealth Funds
  - Mainly Oil-Rich Countries

Motivation

Three Hot Topics In International Macro:

- Huge Reserve Accumulation By Fast-Growing Developing Economies
  - China
- Surprising "Upstream" Capital Flows: Developing $\rightarrow$ Rich Countries
  - China – Following Japan, Korea, Taiwan, Singapore, Hong Kong, ...
- Sovereign Wealth Funds
  - Mainly Oil-Rich Countries

Motivation

Three Hot Topics In International Macro:

- Huge Reserve Accumulation By Fast-Growing Developing Economies
  - China
- Surprising "Upstream" Capital Flows: Developing $\rightarrow$ Rich Countries
  - China – Following Japan, Korea, Taiwan, Singapore, Hong Kong, ...
- Sovereign Wealth Funds
  - Mainly Oil-Rich Countries

Motivation

Three Hot Topics In International Macro:

- Huge Reserve Accumulation By Fast-Growing Developing Economies
  - China
- Surprising "Upstream" Capital Flows: Developing $\rightarrow$ Rich Countries
  - China – Following Japan, Korea, Taiwan, Singapore, Hong Kong, ...
- Sovereign Wealth Funds
  - Mainly Oil-Rich Countries

Motivation

Three Hot Topics In International Macro:

- Huge Reserve Accumulation By Fast-Growing Developing Economies
  - China
- Surprising "Upstream" Capital Flows: Developing $\rightarrow$ Rich Countries
  - China – Following Japan, Korea, Taiwan, Singapore, Hong Kong, ...
- Sovereign Wealth Funds
  - Mainly Oil-Rich Countries

Motivation

Three Hot Topics In International Macro:

- Huge Reserve Accumulation By Fast-Growing Developing Economies
  - China
- Surprising “Upstream” Capital Flows: Developing $\rightarrow$ Rich Countries
  - China – Following Japan, Korea, Taiwan, Singapore, Hong Kong, ...
- Sovereign Wealth Funds
  - Mainly Oil-Rich Countries

Connection?

Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
  - Tractable. Tractable! TRACTABLE!!
  - The Natural Extension of the Ramsey Model
  - Shows Eqbm Relation Between Precautionary, Other Motives
- Two applications
  - Economic Growth and Capital Flows
  - Impact of Reducing Global Financial Imbalances

Connection?

Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
- Two applications

Connection?

Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
  - Tractable. Tractable! TRACTABLE!!!
  - The Natural Extension of the Ramsey Model
  - Shows Eqbm Relation Between Precautionary, Other Motives
- Two applications
  - Economic Growth and Capital Flows
  - Impact of Reducing Global Financial Imbalances

Connection?

Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
  - Tractable. Tractable! TRACTABLE!!!
  - The Natural Extension of the Ramsey Model
  - Shows Eqbm Relation Between Precautionary, Other Motives
- Two applications
  - Economic Growth and Capital Flows
  - Impact of Reducing Global Financial Imbalances

Connection?

Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
  - Tractable. Tractable! TRACTABLE!!!
  - The Natural Extension of the Ramsey Model
  - Shows Eqbm Relation Between Precautionary, Other Motives
- Two applications
  - Economic Growth and Capital Flows
  - Impact of Reducing Global Financial Imbalances

Connection?

Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
  - Tractable. Tractable! TRACTABLE!!!
  - The Natural Extension of the Ramsey Model
  - Shows Eqbm Relation Between Precautionary, Other Motives
- Two applications
  - Economic Growth and Capital Flows
  - Impact of Reducing Global Financial Imbalances

Connection?

Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
  - Tractable. Tractable! TRACTABLE!!!
  - The Natural Extension of the Ramsey Model
  - Shows Eqbm Relation Between Precautionary, Other Motives
- Two applications
  - Economic Growth and Capital Flows
  - Impact of Reducing Global Financial Imbalances

Connection?

Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
  - Tractable. Tractable! TRACTABLE!!!
  - The Natural Extension of the Ramsey Model
  - Shows Eqbm Relation Between Precautionary, Other Motives
- Two applications
  - Economic Growth and Capital Flows
  - Impact of Reducing Global Financial Imbalances

Connection?

Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
  - Tractable. Tractable! TRACTABLE!!!
  - The Natural Extension of the Ramsey Model
  - Shows Eqbm Relation Between Precautionary, Other Motives
- Two applications
  - Economic Growth and Capital Flows
  - Impact of Reducing Global Financial Imbalances

Connection?

Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
  - Tractable. Tractable! TRACTABLE!!!
  - The Natural Extension of the Ramsey Model
  - Shows Eqbm Relation Between Precautionary, Other Motives
- Two applications
  - Economic Growth and Capital Flows
  - Impact of Reducing Global Financial Imbalances

Connection?

Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
  - Tractable. Tractable! TRACTABLE!!!
  - The Natural Extension of the Ramsey Model
  - Shows Eqbm Relation Between Precautionary, Other Motives
- Two applications
  - Economic Growth and Capital Flows
  - Impact of Reducing Global Financial Imbalances

Connection?

Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
  - Tractable. Tractable! TRACTABLE!!!
  - The Natural Extension of the Ramsey Model
  - Shows Eqbm Relation Between Precautionary, Other Motives
- Two applications
  - Economic Growth and Capital Flows
  - Impact of Reducing Global Financial Imbalances

Connection?

Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
  - Tractable. Tractable! TRACTABLE!!!
  - The Natural Extension of the Ramsey Model
  - Shows Eqbm Relation Between Precautionary, Other Motives
- Two applications
  - Economic Growth and Capital Flows
  - Impact of Reducing Global Financial Imbalances

Connection?

Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
  - Tractable. Tractable! TRACTABLE!!!
  - The Natural Extension of the Ramsey Model
  - Shows Eqbm Relation Between Precautionary, Other Motives
- Two applications
  - Economic Growth and Capital Flows
  - Impact of Reducing Global Financial Imbalances

Literature

- Aggregated Micro Model
  - "Real" microfoundations!
- Builds on Toche (2005)
- Related: Fogli and Perri (2006), Mendoza, Quadrini, and Ríos-Rull (2009), Sandri (2014)
- Other Approaches: Caballero, Farhi, and Gourinchas (2008)

Literature

- Aggregated Micro Model
  - “Real” microfoundations!
- Builds on Toche (2005)
- Related: Fogli and Perri (2006), Mendoza, Quadrini, and Ríos-Rull (2009), Sandri (2014)
- Other Approaches: Caballero, Farhi, and Gourinchas (2008)

Literature

- Aggregated Micro Model
  - "Real" microfoundations!
- Builds on Toche (2005)
- Related: Fogli and Perri (2006), Mendoza, Quadrini, and Ríos-Rull (2009), Sandri (2014)
- Other Approaches: Caballero, Farhi, and Gourinchas (2008)

Literature

- Aggregated Micro Model
  - "Real" microfoundations!
- Builds on Toche (2005)
- Related: Fogli and Perri (2006), Mendoza, Quadrini, and Ríos-Rull (2009), Sandri (2014)
- Other Approaches: Caballero, Farhi, and Gourinchas (2008)

Literature

- Aggregated Micro Model
  - "Real" microfoundations!
- Builds on Toche (2005)
- Related: Fogli and Perri (2006), Mendoza, Quadrini, and Ríos-Rull (2009), Sandri (2014)
- Other Approaches: Caballero, Farhi, and Gourinchas (2008)

Structure

- Model
- Calibration and Simulation
- Applications
    - Growth and Capital Flows
    - Complete World Knowledge (General Equilibrium)

Structure

- Model
- Calibration and Simulation
- Applications
  - Growth and Capital Flows
  - Complete World Knowledge (General Equilibrium)

Structure

- Model
- Calibration and Simulation
- Applications
  - Growth and Capital Flows
  - Complete World Knowledge (General Equilibrium)

Structure

- Model
- Calibration and Simulation
- Applications
  - Growth and Capital Flows
  - Complete World Knowledge (General Equilibrium)

Structure

- Model
- Calibration and Simulation
- Applications
  - Growth and Capital Flows
  - Complete World Knowledge (General Equilibrium)

Overview

- Small Open Economy
- Balanced Growth Path With Population And Productivity Growth
- Accumulate Buffer Stock to Self-Insure Against Unemployment
- NFA: Aggregate Stock of Wealth Minus Domestic Capital Stock
- Closed-Form Solutions For Equilibrium

Overview

- Small Open Economy
- Balanced Growth Path With Population And Productivity Growth
- Accumulate Buffer Stock to Self-Insure Against Unemployment
- NFA: Aggregate Stock of Wealth Minus Domestic Capital Stock
- Closed-Form Solutions For Equilibrium

Overview

- Small Open Economy
- Balanced Growth Path With Population And Productivity Growth
- Accumulate Buffer Stock to Self-Insure Against Unemployment
- NFA: Aggregate Stock of Wealth Minus Domestic Capital Stock
- Closed-Form Solutions For Equilibrium

Overview

- Small Open Economy
- Balanced Growth Path With Population And Productivity Growth
- Accumulate Buffer Stock to Self-Insure Against Unemployment
- NFA: Aggregate Stock of Wealth Minus Domestic Capital Stock
- Closed-Form Solutions For Equilibrium

Overview

- Small Open Economy
- Balanced Growth Path With Population And Productivity Growth
- Accumulate Buffer Stock to Self-Insure Against Unemployment
- NFA: Aggregate Stock of Wealth Minus Domestic Capital Stock
- Closed-Form Solutions For Equilibrium

Macroeconomic Assumptions

- Domestic output is produced with the Cobb-Douglas function:

$$\boldsymbol{Y}_t = K_t^{\alpha}(z_t L_t)^{1-\alpha}, \tag{1}$$

- Labor productivity increases by G in every period,

$$z_{t+1} = \mathrm{G} z_t. \tag{2}$$

- Capital perfectly mobile internationally,

$$\overbrace{\daleth}^{\equiv 1-\delta} + \alpha \frac{\boldsymbol{Y}_t}{K_t} = \mathrm{R}, \tag{3}$$

- Capital-to-output ratio is constant and equal to,

$$\frac{K}{\boldsymbol{Y}} = \frac{\alpha}{\mathrm{R} - \daleth}. \tag{4}$$

Macroeconomic Assumptions

- Domestic output is produced with the Cobb-Douglas function:

$$\mathbf{Y}_t = K_t^{\alpha}(z_t L_t)^{1-\alpha}, \tag{1}$$

- Labor productivity increases by G in every period,

$$z_{t+1} = \mathsf{G} z_t. \tag{2}$$

- Capital perfectly mobile internationally,

$$\overbrace{\daleth}^{\equiv 1-\delta} + \alpha \frac{\mathbf{Y}_t}{K_t} = \mathsf{R}, \tag{3}$$

- Capital-to-output ratio is constant and equal to,

$$\frac{K}{\mathbf{Y}} = \frac{\alpha}{\mathsf{R} - \daleth}. \tag{4}$$

Macroeconomic Assumptions

- Domestic output is produced with the Cobb-Douglas function:

$$\mathbf{Y}_t = K_t^{\alpha}(z_t L_t)^{1-\alpha}, \tag{1}$$

- Labor productivity increases by G in every period,

$$z_{t+1} = \mathrm{G} z_t. \tag{2}$$

- Capital perfectly mobile internationally,

$$\overbrace{\daleth}^{\equiv 1-\delta} + \alpha \frac{\mathbf{Y}_t}{K_t} = \mathrm{R}, \tag{3}$$

- Capital-to-output ratio is constant and equal to,

$$\frac{K}{\mathbf{Y}} = \frac{\alpha}{\mathrm{R} - \daleth}. \tag{4}$$

Macroeconomic Assumptions

- Domestic output is produced with the Cobb-Douglas function:

$$\mathbf{Y}_t = K_t^{\alpha}(z_t L_t)^{1-\alpha}, \tag{1}$$

- Labor productivity increases by G in every period,

$$z_{t+1} = \mathrm{G} z_t. \tag{2}$$

- Capital perfectly mobile internationally,

$$\overbrace{\daleth}^{\equiv 1-\delta} + \alpha \frac{\mathbf{Y}_t}{K_t} = \mathrm{R}, \tag{3}$$

- Capital-to-output ratio is constant and equal to,

$$\frac{K}{\mathbf{Y}} = \frac{\alpha}{\mathrm{R} - \daleth}. \tag{4}$$

People and Populations

- Each worker is part of a single ‘generation’ born at the same time
- Size of generation born at t : Ξ^t.
- Life Stages:
  - Employment
  - Unemployment/Retirement
  - Death
- Transitions to unemployment and death are Poisson processes
  - Flow probabilities $\mho$ and D.
- Employed and Unemployed Populations:

$$
\begin{aligned}
\mathcal{E}_t &= \frac{\Xi^{t+1}}{\Xi - \cancel{\mho}} \\
\mathcal{U}_t &= \frac{\mho \Xi^{t+1}}{(\Xi - \cancel{\text{D}})(\Xi - \cancel{\mho})}.
\end{aligned}
$$

People and Populations

- Each worker is part of a single ‘generation’ born at the same time
- Size of generation born at t : Ξ^t.
- Life Stages:
  - Employment
  - Unemployment/Retirement
  - Death
- Transitions to unemployment and death are Poisson processes
  - Flow probabilities $\mho$ and D.
- Employed and Unemployed Populations:

$$\begin{aligned}
\mathcal{E}_t &= \frac{\Xi^{t+1}}{\Xi - \cancel{\mho}} \\
\mathcal{U}_t &= \frac{\mho \Xi^{t+1}}{(\Xi - \cancel{\mathsf{D}})(\Xi - \cancel{\mho})}.
\end{aligned}$$

People and Populations

- Each worker is part of a single ‘generation’ born at the same time
- Size of generation born at t : Ξ^t.
- Life Stages:
  - Employment
  - Unemployment/Retirement
  - Death
- Transitions to unemployment and death are Poisson processes
  - Flow probabilities $\mho$ and D.
- Employed and Unemployed Populations:

$$
\begin{aligned}
\mathcal{E}_t &= \frac{\Xi^{t+1}}{\Xi - \cancel{\mho}} \\
\mathcal{U}_t &= \frac{\mho \Xi^{t+1}}{(\Xi - \cancel{D})(\Xi - \cancel{\mho})}.
\end{aligned}
$$

People and Populations

- Each worker is part of a single 'generation' born at the same time
- Size of generation born at t : Ξ^t.
- Life Stages:
  - Employment
  - Unemployment/Retirement
  - Death
- Transitions to unemployment and death are Poisson processes
  - Flow probabilities $\mho$ and D.
- Employed and Unemployed Populations:

$$
\begin{aligned}
\mathcal{E}_t &= \frac{\Xi^{t+1}}{\Xi - \cancel{\mho}} \\
\mathcal{U}_t &= \frac{\mho \Xi^{t+1}}{(\Xi - \cancel{D})(\Xi - \cancel{\mho})}.
\end{aligned}
$$

People and Populations

- Each worker is part of a single ‘generation’ born at the same time
- Size of generation born at t : Ξ^t.
- Life Stages:
    - Employment
    - Unemployment/Retirement
    - Death
- Transitions to unemployment and death are Poisson processes
    - Flow probabilities $\mho$ and D.
- Employed and Unemployed Populations:

$$\mathcal{E}_t = \frac{\Xi^{t+1}}{\Xi - \cancel{\mho}}$$

$$\mathcal{U}_t = \frac{\mho \Xi^{t+1}}{(\Xi - \cancel{D})(\Xi - \cancel{\mho})}.$$

People and Populations

- Each worker is part of a single ‘generation’ born at the same time
- Size of generation born at t : Ξ^t.
- Life Stages:
  - Employment
  - Unemployment/Retirement
  - Death
- Transitions to unemployment and death are Poisson processes
  - Flow probabilities $\mho$ and D.
- Employed and Unemployed Populations:

$$\begin{aligned} \mathcal{E}_t &= \frac{\Xi^{t+1}}{\Xi - \cancel{\mho}} \\ \mathcal{U}_t &= \frac{\mho \Xi^{t+1}}{(\Xi - \cancel{\mathsf{D}})(\Xi - \cancel{\mho})}. \end{aligned}$$

People and Populations

- Each worker is part of a single 'generation' born at the same time
- Size of generation born at t : Ξ^t.
- Life Stages:
  - Employment
  - Unemployment/Retirement
  - Death
- Transitions to unemployment and death are Poisson processes
  - Flow probabilities $\mho$ and D.
- Employed and Unemployed Populations:

$$\begin{aligned} \mathcal{E}_t &= \frac{\Xi^{t+1}}{\Xi - \cancel{\mho}} \\ \mathcal{U}_t &= \frac{\mho \Xi^{t+1}}{(\Xi - \cancel{D})(\Xi - \cancel{\mho})}. \end{aligned}$$

People and Populations

- Each worker is part of a single 'generation' born at the same time
- Size of generation born at t : Ξ^t.
- Life Stages:
  - Employment
  - Unemployment/Retirement
  - Death
- Transitions to unemployment and death are Poisson processes
  - Flow probabilities $\mho$ and D.
- Employed and Unemployed Populations:

$$\mathcal{E}_t = \frac{\Xi^{t+1}}{\Xi - \cancel{\mho}}$$

$$\mathcal{U}_t = \frac{\mho \Xi^{t+1}}{(\Xi - \cancel{\mathsf{D}})(\Xi - \cancel{\mho})}.$$

People and Populations

- Each worker is part of a single 'generation' born at the same time
- Size of generation born at t : Ξ^t.
- Life Stages:
  - Employment
  - Unemployment/Retirement
  - Death
- Transitions to unemployment and death are Poisson processes
  - Flow probabilities $\mho$ and D.
- Employed and Unemployed Populations:

$$\begin{aligned} \mathcal{E}_t &= \frac{\Xi^{t+1}}{\Xi - \cancel{\mho}} \\ \mathcal{U}_t &= \frac{\mho \Xi^{t+1}}{(\Xi - \cancel{\mathsf{D}})(\Xi - \cancel{\mho})}. \end{aligned}$$

Balanced Growth

- Capital and output grow at constant rates
- Real wage grows by factor G in every period.
- Main variable of interest= N_t, the aggregate net foreign assets of the economy at the beginning of period t.

$$N_t = B_t - K_t. \tag{5}$$

Balanced Growth

- Capital and output grow at constant rates
- Real wage grows by factor G in every period.
- Main variable of interest= N_t, the aggregate net foreign assets of the economy at the beginning of period t.

$$N_t = B_t - K_t. \tag{5}$$

Balanced Growth

- Capital and output grow at constant rates
- Real wage grows by factor G in every period.
- Main variable of interest= N_t, the aggregate net foreign assets of the economy at the beginning of period t.

$$N_t = B_t - K_t. \tag{5}$$

The microeconomic consumer's problem

- Budget constraint of individual:

$$\frac{b_{t+1}}{R} + c_t = b_t + \overbrace{\xi_t \ell_t W_t}^{\text{labor income}}, \tag{6}$$

- Worker's labor supply ℓ grows by a factor X per period over his lifetime,

$$\ell_t = X^t \ell_0, \tag{7}$$

- For consumer who remains employed, labor income grows by

$$\Gamma \equiv GX.$$

The microeconomic consumer's problem

- Budget constraint of individual:

$$\frac{b_{t+1}}{\mathsf{R}} + c_t = b_t + \overbrace{\xi_t \ell_t \mathsf{W}_t}^{\text{labor income}}, \tag{6}$$

- Worker's labor supply ℓ grows by a factor X per period over his lifetime,

$$\ell_t = \mathsf{X}^t \ell_0, \tag{7}$$

- For consumer who remains employed, labor income grows by

$$\Gamma \equiv \mathsf{G}\mathsf{X}.$$

The microeconomic consumer's problem

- Budget constraint of individual:

$$\frac{b_{t+1}}{\mathsf{R}} + c_t = b_t + \overbrace{\xi_t \ell_t \mathsf{W}_t}^{\text{labor income}}, \tag{6}$$

- Worker's labor supply ℓ grows by a factor X per period over his lifetime,

$$\ell_t = \mathsf{X}^t \ell_0, \tag{7}$$

- For consumer who remains employed, labor income grows by

$$\Gamma \equiv \mathsf{G}\mathsf{X}.$$

The microeconomic consumer's problem

- Unemployment: Complete and permanent destruction of h
- CRRA felicity $\mathrm{u}(\bullet) = \bullet^{1-\rho}/(1-\rho)$; geometric discounting at β
- Unemployed convert their wealth into annuities.
- Solution to the unemployed consumer's optimization problem,

$$c_t^u = \kappa^u b_t,$$

where κ is the marginal propensity to consume,

$$\kappa^u \equiv 1 - \cancel{D}\frac{(\beta \mathrm{R})^{1/\rho}}{\mathrm{R}}.$$

The microeconomic consumer's problem

- Unemployment: Complete and permanent destruction of h
- CRRA felicity $\mathrm{u}(\bullet) = \bullet^{1-\rho}/(1-\rho)$; geometric discounting at β
- Unemployed convert their wealth into annuities.
- Solution to the unemployed consumer's optimization problem,

$$c_t^u = \kappa^u b_t,$$

where κ is the marginal propensity to consume,

$$\kappa^u \equiv 1 - \cancel{D}\frac{(\beta R)^{1/\rho}}{R}.$$

The microeconomic consumer's problem

- Unemployment: Complete and permanent destruction of h
- CRRA felicity $\mathrm{u}(\bullet) = \bullet^{1-\rho}/(1-\rho)$; geometric discounting at β
- Unemployed convert their wealth into annuities.
- Solution to the unemployed consumer's optimization problem,

$$c_t^u = \kappa^u b_t,$$

where κ is the marginal propensity to consume,

$$\kappa^u \equiv 1 - \cancel{D}\frac{(\beta \mathrm{R})^{1/\rho}}{\mathrm{R}}.$$

The microeconomic consumer's problem

- Unemployment: Complete and permanent destruction of h
- CRRA felicity $\mathrm{u}(\bullet) = \bullet^{1-\rho}/(1-\rho)$; geometric discounting at β
- Unemployed convert their wealth into annuities.
- Solution to the unemployed consumer's optimization problem,

$$c_t^u = \kappa^u b_t,$$

where κ is the marginal propensity to consume,

$$\kappa^u \equiv 1 - \not{D}\frac{(\beta R)^{1/\rho}}{R}.$$

The microeconomic consumer's problem

- 'Growth impatience condition':

$$Þ_{\Gamma} \equiv \frac{(\beta \mathsf{R})^{1/\rho}}{\Gamma} < 1$$

necessary for finite target ratio of wealth to income (Carroll (2016))

- Defining nonbold variables as, e.g., $c_t^e = \mathbf{c}_t^e/(\mathsf{W}_t \ell_t)$, we get

$$b_{t+1}^e = (\mathsf{R}/\Gamma)\left(b_t^e - c_t^e + 1\right). \tag{8}$$

$$c_{t+1}^e = Þ_{\Gamma} \mho^{1/\rho} c_t^e \left[1 - \mho \left(\frac{Þ_{\Gamma}}{\kappa^u} \frac{c_t^e}{\mathsf{R}/\Gamma(b_t^e - c_t^e + 1)} \right)^{\rho} \right]^{-1/\rho}. \tag{9}$$

- Saddle-point stable dynamics.

The microeconomic consumer's problem

- 'Growth impatience condition':

$$\text{Þ}_{\Gamma} \equiv \frac{(\beta \mathsf{R})^{1/\rho}}{\Gamma} < 1$$

  necessary for finite target ratio of wealth to income (Carroll (2016))
- Defining nonbold variables as, e.g., $c_t^e = \mathbf{c}_t^e/(\mathsf{W}_t \ell_t)$, we get

$$b_{t+1}^e = (\mathsf{R}/\Gamma)\left(b_t^e - c_t^e + 1\right). \tag{8}$$

$$c_{t+1}^e = \text{Þ}_{\Gamma} \not\mho^{1/\rho} c_t^e \left[1 - \mho \left(\frac{\text{Þ}_{\Gamma}}{\kappa^u} \frac{c_t^e}{\mathsf{R}/\Gamma(b_t^e - c_t^e + 1)}\right)^{\rho}\right]^{-1/\rho}. \tag{9}$$

- Saddle-point stable dynamics.

The microeconomic consumer's problem

- 'Growth impatience condition':

$$\text{Þ}_{\Gamma} \equiv \frac{(\beta \mathsf{R})^{1/\rho}}{\Gamma} < 1$$

necessary for finite target ratio of wealth to income (Carroll (2016))

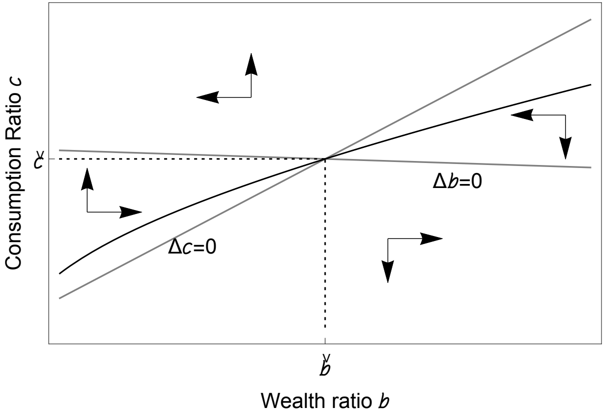
- Defining nonbold variables as, e.g., $c_t^e = \mathbf{c}_t^e/(\mathsf{W}_t \ell_t)$, we get

$$b_{t+1}^e = (\mathsf{R}/\Gamma)\left(b_t^e - c_t^e + 1\right). \tag{8}$$

$$c_{t+1}^e = \text{Þ}_{\Gamma} \cancel{\mho}^{1/\rho} c_t^e \left[1 - \mho \left(\frac{\text{Þ}_{\Gamma}}{\kappa^u} \frac{c_t^e}{\mathsf{R}/\Gamma(b_t^e - c_t^e + 1)}\right)^{\rho}\right]^{-1/\rho}. \tag{9}$$

- Saddle-point stable dynamics.

Phase Diagram



The Growth Impatience Condition

- Target wealth-to-income ratio: impatience vs prudence.
- Closed-form solution for the target wealth-to-income ratio

$$\check{b} = \left[\frac{\Gamma}{\mathsf{R}} - 1 + \kappa^u \left(1 + \frac{\text{Þ}_\Gamma^{-\rho} - 1}{\mho} \right)^{1/\rho} \right]^{-1} . \tag{10}$$

-

$$\frac{\partial \check{b}}{\partial \mho} > 0, \frac{\partial \check{b}}{\partial \beta} > 0, \frac{\partial \check{b}}{\partial \Gamma} < 0. \tag{11}$$

$$\frac{\partial \check{b}}{\partial \rho} > 0. \tag{12}$$

- The response of $\check{b}$ to R is ambiguous.

The Growth Impatience Condition

- Target wealth-to-income ratio: impatience vs prudence.
- Closed-form solution for the target wealth-to-income ratio

$$\check{b} = \left[\frac{\Gamma}{\mathsf{R}} - 1 + \kappa^{u} \left(1 + \frac{\text{Þ}_{\Gamma}^{-\rho} - 1}{\mho} \right)^{1/\rho} \right]^{-1}. \tag{10}$$

- $$\frac{\partial \check{b}}{\partial \mho} > 0, \frac{\partial \check{b}}{\partial \beta} > 0, \frac{\partial \check{b}}{\partial \Gamma} < 0. \tag{11}$$

$$\frac{\partial \check{b}}{\partial \rho} > 0. \tag{12}$$

- The response of $\check{b}$ to R is ambiguous.

The Growth Impatience Condition

- Target wealth-to-income ratio: impatience vs prudence.
- Closed-form solution for the target wealth-to-income ratio

$$\check{b} = \left[\frac{\Gamma}{\mathsf{R}} - 1 + \kappa^{u} \left(1 + \frac{\text{Þ}_{\Gamma}^{-\rho} - 1}{\mho} \right)^{1/\rho} \right]^{-1} . \tag{10}$$

-

$$\frac{\partial \check{b}}{\partial \mho} > 0, \frac{\partial \check{b}}{\partial \beta} > 0, \frac{\partial \check{b}}{\partial \Gamma} < 0. \tag{11}$$

$$\frac{\partial \check{b}}{\partial \rho} > 0. \tag{12}$$

- The response of $\check{b}$ to R is ambiguous.

The Growth Impatience Condition

- Target wealth-to-income ratio: impatience vs prudence.
- Closed-form solution for the target wealth-to-income ratio

$$\check{b} = \left[\frac{\Gamma}{\mathsf{R}} - 1 + \kappa^u \left(1 + \frac{\text{Þ}_\Gamma^{-\rho} - 1}{\mho} \right)^{1/\rho} \right]^{-1} . \tag{10}$$

-

$$\frac{\partial \check{b}}{\partial \mho} > 0, \frac{\partial \check{b}}{\partial \beta} > 0, \frac{\partial \check{b}}{\partial \Gamma} < 0. \tag{11}$$

$$\frac{\partial \check{b}}{\partial \rho} > 0. \tag{12}$$

- The response of $\check{b}$ to R is ambiguous.

Foreign Assets

- Ratio of employed workers' wealth to output,

$$B^e_t = \frac{B^e_t}{\boldsymbol{Y}_t} \quad = \quad (1-\alpha)\left(1 - \underbrace{\frac{\cancel{\mho}\mathrm{X}}{\Xi}}_{\equiv\Lambda}\right)\sum_{n=0}^{+\infty}\Lambda^n b^e_{t,t-n}, \tag{13}$$

where Λ is the factor by which the share of a generation in total labor supply shrinks every period.

- The Level of Unemployed Workers' Wealth is

$$B^u_{t+1} = \mathsf{R}(1-\kappa^u)B^u_t + \mho B^e_{t+1}. \tag{14}$$

Foreign Assets

- Ratio of employed workers' wealth to output,

$$B_t^e = \frac{B_t^e}{\mathbf{Y}_t} \quad = \quad (1-\alpha)\left(1 - \underbrace{\frac{\cancel{\mho}\mathcal{X}}{\Xi}}_{\equiv\Lambda}\right)\sum_{n=0}^{+\infty}\Lambda^n b_{t,t-n}^e, \tag{13}$$

where Λ is the factor by which the share of a generation in total labor supply shrinks every period.

- The Level of Unemployed Workers' Wealth is

$$B_{t+1}^u = \mathsf{R}(1-\kappa^u)B_t^u + \mho B_{t+1}^e. \tag{14}$$

Foreign Assets (cont)

- Steady state ratio of net foreign assets to GDP

$$\frac{N}{\boldsymbol{Y}} = \frac{\Xi \mathrm{G}}{\mathrm{R}} \left(1 + \frac{\mho \Xi \mathrm{G}}{\Xi \mathrm{G} - \text{Þ}(\beta \mathrm{R})^{1/\rho}} \right) \frac{B^e}{\boldsymbol{Y}} - \Xi \mathrm{G} \left(\frac{\alpha}{\mathrm{R} - \daleth} \right). \tag{15}$$

- Depends on Employed Workers' Target Savings

Foreign Assets (cont)

- Steady state ratio of net foreign assets to GDP

$$\frac{N}{\boldsymbol{Y}} = \frac{\Xi \mathrm{G}}{\mathrm{R}} \left(1 + \frac{\mho \Xi \mathrm{G}}{\Xi \mathrm{G} - \cancel{D}(\beta \mathrm{R})^{1/\rho}}\right) \frac{B^e}{\boldsymbol{Y}} - \Xi \mathrm{G} \left(\frac{\alpha}{\mathrm{R} - \daleth}\right). \tag{15}$$

- Depends on Employed Workers' Target Savings

'Stakes'

- Model with no stakes

$$B^e = \frac{B^e}{Y} = (1-\alpha)(1-\Lambda)\sum_{n=0}^{+\infty}\Lambda^n b^e(n). \tag{16}$$

- Model with stakes yielding a representative agent

$$\check{\check{B}} = \frac{B^e}{Y} = (1-\alpha)\check{\check{b}}. \tag{17}$$

where

$$\check{\check{b}} = \left[\frac{\Gamma}{\mathsf{R}} - \frac{1}{2-\Lambda} + \kappa^u\left(1+\frac{\text{Þ}_{\Gamma}^{-\rho}-1}{\mho}\right)^{1/\rho}\right]^{-1} \tag{18}$$

'Stakes'

- Model with no stakes

$$B^e = \frac{B^e}{\mathbf{Y}} = (1-\alpha)(1-\Lambda)\sum_{n=0}^{+\infty}\Lambda^n b^e(n). \tag{16}$$

- Model with stakes yielding a representative agent

$$\check{\check{B}} = \frac{B^e}{\mathbf{Y}} = (1-\alpha)\check{\check{b}}. \tag{17}$$

where

$$\check{\check{b}} = \left[\frac{\Gamma}{\mathsf{R}} - \frac{1}{2-\Lambda} + \kappa^u\left(1 + \frac{\text{Þ}_\Gamma^{-\rho} - 1}{\mho}\right)^{1/\rho}\right]^{-1} \tag{18}$$

Advantages Of Model With Stakes

- Closed-form solution for steady state
- Simple to characterize transition dynamics

Advantages Of Model With Stakes

- Closed-form solution for steady state
- Simple to characterize transition dynamics

Calibration and Simulation

Table 1

α	δ	Ξ	G	R	β^{-1}	Φ	$\mho$	ρ	d
0.3	0.06	1.01	1.04	1.04	1.04	1.01	0.025	2	0.05

- $N/Y = 0.17$ in the model with no stakes
- $N/Y = 0.79$ in the model with stakes

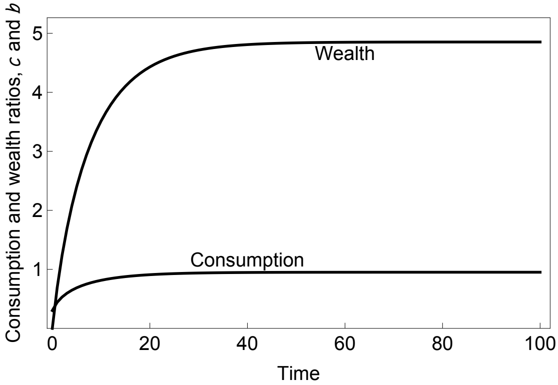
Calibration and Simulation

Table 1

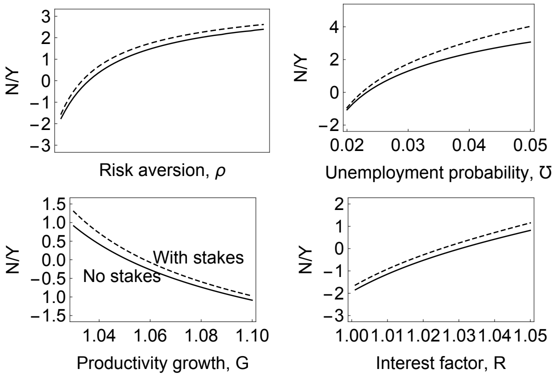
α	δ	Ξ	G	R	β^{-1}	Φ	$\mho$	ρ	d
0.3	0.06	1.01	1.04	1.04	1.04	1.01	0.025	2	0.05

- $N/\boldsymbol{Y} = 0.17$ in the model with no stakes
- $N/\boldsymbol{Y} = 0.79$ in the model with stakes

Paths


Wealth
Consumption
Consumption and wealth ratios, c and b
Time
0
20
40
60
80
100
1
2
3
4
5

Sensitivity analysis



Social Insurance

- Many countries have social transfers to unemployed/retired
- New assumption: labor income tax on the employed in order to finance transfers to the unemployed.
- Unemployed receive transfer whose value is a multiple ς of the labor income that they would have received if they had remained employed.
- New formula for target wealth-to-income ratio. Going through the same steps as before, we get

$$\check{b}(\varsigma) = \left\{ 1 - \varsigma \left[\frac{\mho}{\Xi} + \kappa^u \left(1 + \frac{\text{Þ}_\Gamma^{-\rho} - 1}{\mho} \right)^{1/\rho} \right] \right\} \check{b}, \qquad (19)$$

Social Insurance

- Many countries have social transfers to unemployed/retired
- New assumption: labor income tax on the employed in order to finance transfers to the unemployed.
- Unemployed receive transfer whose value is a multiple ς of the labor income that they would have received if they had remained employed.
- New formula for target wealth-to-income ratio. Going through the same steps as before, we get

$$\check{b}(\varsigma) = \left\{1 - \varsigma\left[\frac{\mho}{\Xi} + \kappa^u\left(1 + \frac{\text{Þ}_\Gamma^{-\rho} - 1}{\mho}\right)^{1/\rho}\right]\right\}\check{b}, \qquad (19)$$

Social Insurance

- Many countries have social transfers to unemployed/retired
- New assumption: labor income tax on the employed in order to finance transfers to the unemployed.
- Unemployed receive transfer whose value is a multiple ς of the labor income that they would have received if they had remained employed.
- New formula for target wealth-to-income ratio. Going through the same steps as before, we get

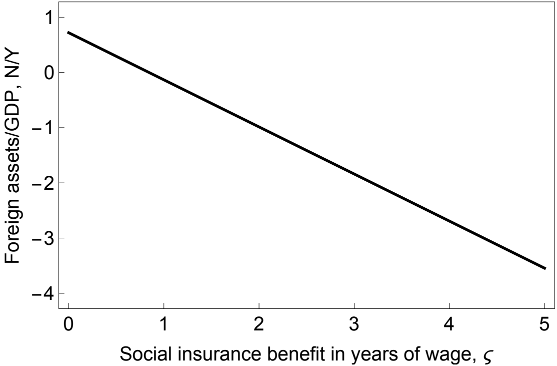
$$\check{b}(\varsigma) = \left\{1 - \varsigma\left[\frac{\mho}{\Xi} + \kappa^u\left(1 + \frac{\text{Þ}_{\Gamma}^{-\rho} - 1}{\mho}\right)^{1/\rho}\right]\right\}\check{b}, \qquad (19)$$

Social Insurance

- Many countries have social transfers to unemployed/retired
- New assumption: labor income tax on the employed in order to finance transfers to the unemployed.
- Unemployed receive transfer whose value is a multiple ς of the labor income that they would have received if they had remained employed.
- New formula for target wealth-to-income ratio. Going through the same steps as before, we get

$$\check{\grave{b}}(\varsigma) \quad = \quad \left\{1 - \varsigma\left[\frac{\mho}{\Xi} + \kappa^{u}\left(1 + \frac{\text{Þ}_{\Gamma}^{-\rho} - 1}{\mho}\right)^{1/\rho}\right]\right\}\check{b}, \qquad (19)$$

Social insurance



Growth And Saving

- Theory: Good Growth Prospects → Should Borrow to Invest
- Data: Fast-Growing Countries *Export* Capital
  - Carroll and Weil (1994); Loayza, Schmidt-Hebbel, and Servén (2000); Attanasio, Picci, and Scorcu (2000); Gourinchas and Jeanne, 2007, Prasad, Rajan and Subramanian (2007); Sandri (2008)
- Can this model shed light on this puzzle?
- Yes, if growth take-off entails idiosyncratic risk (both G and ℧ go up).

Growth And Saving

- Theory: Good Growth Prospects $\rightarrow$ Should Borrow to Invest
- Data: Fast-Growing Countries *Export* Capital
    - Carroll and Weil (1994); Loayza, Schmidt-Hebbel, and Servén (2000); Attanasio, Picci, and Scorcu (2000); Gourinchas and Jeanne, 2007, Prasad, Rajan and Subramanian (2007); Sandri (2008)
- Can this model shed light on this puzzle?
- Yes, if growth take-off entails idiosyncratic risk (both G and $\mho$ go up).

Growth And Saving

- Theory: Good Growth Prospects $\rightarrow$ Should Borrow to Invest
- Data: Fast-Growing Countries *Export* Capital
    - Carroll and Weil (1994); Loayza, Schmidt-Hebbel, and Servén (2000); Attanasio, Picci, and Scorcu (2000); Gourinchas and Jeanne, 2007, Prasad, Rajan and Subramanian (2007); Sandri (2008)
- Can this model shed light on this puzzle?
- Yes, if growth take-off entails idiosyncratic risk (both G and $\mho$ go up).

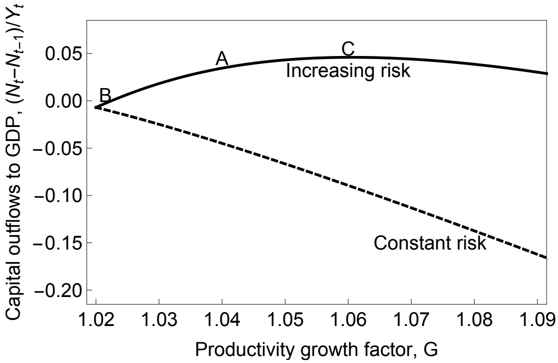
Growth And Saving

- Theory: Good Growth Prospects $\rightarrow$ Should Borrow to Invest
- Data: Fast-Growing Countries *Export* Capital
  - Carroll and Weil (1994); Loayza, Schmidt-Hebbel, and Servén (2000); Attanasio, Picci, and Scorcu (2000); Gourinchas and Jeanne, 2007, Prasad, Rajan and Subramanian (2007); Sandri (2008)
- Can this model shed light on this puzzle?
- Yes, if growth take-off entails idiosyncratic risk (both G and $\mho$ go up).

Growth And Saving

- Theory: Good Growth Prospects $\rightarrow$ Should Borrow to Invest
- Data: Fast-Growing Countries *Export* Capital
    - Carroll and Weil (1994); Loayza, Schmidt-Hebbel, and Servén (2000); Attanasio, Picci, and Scorcu (2000); Gourinchas and Jeanne, 2007, Prasad, Rajan and Subramanian (2007); Sandri (2008)
- Can this model shed light on this puzzle?
- Yes, if growth take-off entails idiosyncratic risk (both G and $\mho$ go up).

Growth and capital flows



World General Equilibrium

- Small economy assumption not appropriate to study global savings glut or adjustment of global financial imbalances.
- Study steady state equilibria in two-country extension of the model.
- Global interest rate R endogenous

$$N_h + N_f = 0, \qquad (20)$$

World General Equilibrium

- Small economy assumption not appropriate to study global savings glut or adjustment of global financial imbalances.
- Study steady state equilibria in two-country extension of the model.
- Global interest rate R endogenous

$$N_h + N_f = 0, \tag{20}$$

World General Equilibrium

- Small economy assumption not appropriate to study global savings glut or adjustment of global financial imbalances.
- Study steady state equilibria in two-country extension of the model.
- Global interest rate R endogenous

$$N_h + N_f = 0, \quad (20)$$

General Equilibrium

- Two countries identical except for size (h=20%, f=80%) and level of social insurance ($\varsigma_h = 1.5$, $\varsigma_f = 0.75$).
- This implies

$$\frac{N_h}{Y_h} = -0.5 \quad (21)$$

$$\frac{N_f}{Y_f} = 0.125 \quad (22)$$

- What is impact of increasing foreign social insurance to the home level?

General Equilibrium

- Two countries identical except for size (h=20%, f=80%) and level of social insurance ($\varsigma_h = 1.5$, $\varsigma_f = 0.75$).
- This implies

$$\frac{N_h}{\boldsymbol{Y}_h} = -0.5 \tag{21}$$

$$\frac{N_f}{\boldsymbol{Y}_f} = 0.125 \tag{22}$$

- What is impact of increasing foreign social insurance to the home level?

General Equilibrium

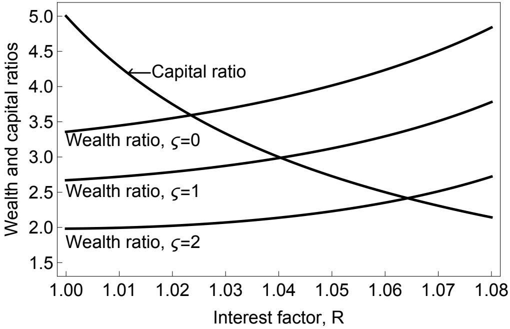
- Two countries identical except for size (h=20%, f=80%) and level of social insurance ($\varsigma_h = 1.5$, $\varsigma_f = 0.75$).
- This implies

$$\frac{N_h}{\boldsymbol{Y}_h} = -0.5 \tag{21}$$

$$\frac{N_f}{\boldsymbol{Y}_f} = 0.125 \tag{22}$$

- What is impact of increasing foreign social insurance to the home level?

General equilibrium



Conclusions

- Tractable model of net foreign assets of small open economy
- Two applications
  - Relationship between growth and capital flows
  - Long-run implications of reducing global imbalances.
- Extensions for future research: portfolio choice, real exchange rates, asset prices, etc.

Conclusions

- Tractable model of net foreign assets of small open economy
- Two applications
  - Relationship between growth and capital flows
  - Long-run implications of reducing global imbalances.
- Extensions for future research: portfolio choice, real exchange rates, asset prices, etc.

Conclusions

- Tractable model of net foreign assets of small open economy
- Two applications
    - Relationship between growth and capital flows
    - Long-run implications of reducing global imbalances.
- Extensions for future research: portfolio choice, real exchange rates, asset prices, etc.

Conclusions

- Tractable model of net foreign assets of small open economy
- Two applications
  - Relationship between growth and capital flows
  - Long-run implications of reducing global imbalances.
- Extensions for future research: portfolio choice, real exchange rates, asset prices, etc.

Conclusions

- Tractable model of net foreign assets of small open economy
- Two applications
  - Relationship between growth and capital flows
  - Long-run implications of reducing global imbalances.
- Extensions for future research: portfolio choice, real exchange rates, asset prices, etc.

References I

ATTANASIO, ORAZIO, LUCIO PICCI, AND ANTONELLO SCORCU (2000): "Saving, Growth, and Investment: A Macroeconomic Analysis Using a Panel of Countries," *Review of Economics and Statistics*, 82(1).

CABALLERO, RICARDO J., EMMANUEL FARHI, AND PIERRE-OLIVIER GOURINCHAS (2008): "An Equilibrium Model of "Global Imbalances" and Low Interest Rates," *American Economic Review*, 98(1), 358–388.

CARROLL, CHRISTOPHER D. (2016): "Theoretical Foundations of Buffer Stock Saving," manuscript, Department of Economics, Johns Hopkins University, Available at `http://econ.jhu.edu/people/ccarroll/papers/BufferStockTheory`.

CARROLL, CHRISTOPHER D., AND DAVID N. WEIL (1994): "Saving and Growth: A Reinterpretation," *Carnegie-Rochester Conference Series on Public Policy*, 40, 133–192, `http://econ.jhu.edu/people/ccarroll/CarrollWeilSavingAndGrowth.pdf`.

FOGLI, ALESSANDRA, AND FABRIZIO PERRI (2006): "The 'Great Moderation' and the U.S. External Imbalance," *NBER Working Paper Number w12708*.

LOAYZA, NORMAN, KLAUS SCHMIDT-HEBBEL, AND LUIS SERVÉN (2000): "What Drives Saving Across the World?," *Review of Economics and Statistics*, 82(1).

MENDOZA, ENRIQUE G., VINCENZO QUADRINI, AND JOSÉ-VÍCTOR RÍOS-RULL (2009): "Financial Integration, Financial Development, and Global Imbalances," *Journal of Political Economy*, 117(3).

SANDRI, DAMIANO (2014): "Growth and Capital Flows with Risky Entrepreneuship," *American Economic Journal: Macroeconomics*, 6(3), 102–23, http://llorracc.net/Papers/sandriGrowth.pdf.

TOCHE, PATRICK (2005): "A Tractable Model of Precautionary Saving in Continuous Time," *Economics Letters*, 87(2), 267–272, `http://ideas.repec.org/a/eee/ecolet/v87y2005i2p267-272.html`.