0.1 Model with Time Preference Heterogeneity

In this model, the economy consists of a continuum of households of mass one distributed on the unit interval. Households die with a constant probability $\mathrm{D} = 1 - \not{\mathrm{D}}$ between periods. (This is different from our baseline model in which households only face probability of dying after they become unemployed.) The income process was described in section **??**. Each household maximizes expected discounted utility from consumption:

$$\max \quad \mathbb{E}_t \sum_{n=0}^{\infty} (\not{\mathrm{D}}\beta)^n u(\mathrm{c}_{t+n}) \tag{1}$$

The household consumption function c satisfies:

$$\begin{aligned}
v(m_t) &= \max_{c_t} u(\mathrm{c}(m_t)) + \beta \not{\mathrm{D}} \mathbb{E}_t \psi_{t+1}^{1-\rho} v(m_{t+1}), \\
&\text{s.t.} \\
a_t &= m_t - \mathrm{c}(m_t) \\
k_{t+1} &= \frac{a_t}{\not{\mathrm{D}} \psi_{t+1}} \\
m_{t+1} &= (\daleth + r_t) k_{t+1} + \xi_{t+1} \\
a_t &\geq 0
\end{aligned}$$

where the variables are divided by the level of permanent income $\boldsymbol{p} = p_t \boldsymbol{W}$, so that when aggregate shocks are shut down, the only state variable is (normalized) cash-on hand m_t. The production function is Cobb-Douglas:

$$Z K^{\alpha} (\ell L)^{1-\alpha} \tag{2}$$

The aggregate wage rate $\boldsymbol{W}_t$ is determined by the aggregate productivity Z_t, capital stock K_t, and the aggregate supply of labor L_t:

$$\boldsymbol{W}_t = (1-\alpha) Z_t (\frac{K_t}{\ell L})^{\alpha} \tag{3}$$

L_t is driven by two aggregate shocks:

$$L_t = P_t \Theta_t \tag{4}$$

$$P_t = P_{t-1} \Psi_t \tag{5}$$

where P_t is aggregate permanent productivity, Ψ_t is the aggregate permanent shock and Θ_t is the aggregate transitory shock.[1]

[1] Note that Ψ is the capitalized version of the Greek letter ψ used for the idiosyncratic permanent shock; similarly Θ is the capitalized θ