## 0.1 Saving and Uncertainty in a More Realistic Model

Our tractable model's advantage is that the mechanisms at play are reflected in analytical expressions with straightforward interpretations. The cost of this tractability is that the model's treatment of uncertainty is so stylized as to be difficult to calibrate to real-world measures of uncertainty.

Without a clear mapping from our theory's stylized treatment of uncertainty to something readily measurable, a future empirical literature would find it difficult to test whether the model is even roughly 'true.'

The natural solution to this problem comes from a literature that has matured in the last couple of decades, in which numerical computational methods are used to solve models whose characterization of uncertainty is much more closely related to empirically measurable quantities. A conclusion of that literature is that an income process that has transitory and permanent components *a la* **?** does a remarkably good job of capturing the most important features of idiosyncratic income dynamics.

These measurement techniques have been most richly applied using U.S. data, but they are spreading. A recent example is the work of **?**, who are among the first to use national registry data (in his case, from Norway) that allows estimation of processes for income dynamics with an unprecedented degree of precision (and even the ability to construct fine-grained calculations of the differences in uncertainty across subgroups with the population). Combining such measures of uncertainty with data on saving rates (either across groups or across countries) should permit measurements of the empirical relationship between saving and uncertainty.

Those measurements can then be compared to predictions of a 'quantitative theorety' model that translates measured differences in uncertainty into implications about differences in saving rates.

We perform such an exercise using the parsimonious model exposited by **?** (henceforth, 'CSTW'), which is able to match the dynamics and distribution of microeconomic income, the distribution of wealth, and the personal saving rate in the U.S. with an income process in which annual income shocks are either transitory or permanent (with magnitudes calibrated to typical microeconomic estimates from U.S. data).

Our exercise is a simple one: Leaving all other aspects of the model the same as in the baseline model that matches U.S. data, we change a single parameter – a measure of uncertainty – over a range of values that are likely to encompass the plausible range of values across countries or subpopulations within countries. We then calculate the saving rate predicted by the model across that range of potential configurations of uncertainty. This produces a quantitative prediction of the magnitude of the relationship between uncertainty and saving that empirical researchers can compare to empirical estimates of the same quantity.

The remainder of this section describes such an exercise in detail, and presents the results.

### 0.1.1 Income Dynamics

CSTW assume that household income $y_t$ is determined by aggregate wage rate $W_t$ and two idiosyncratic components, the permanent component $p_t$ and the transitory shock $\xi_t$:

$$y_t = p_t \xi_t \boldsymbol{W}_t \tag{1}$$

The permanent component follows:

$$p_t = p_{t-1}\psi_t \tag{2}$$

The transitory component is:

$$\begin{aligned} \xi_t &= \mu \text{ with probability } \mho_t, \\ &= (1-\tau_t)\ell\theta_t \text{ with probability } 1-\mho_t, \end{aligned} \tag{3}$$

Here $\mu$ is the unemployment insurance payment when unemployed. $\tau_t$ is the rate of tax collected to pay unemployment benefits, $\ell$ is time worked per employee. $\psi$ and $\theta$ are lognormally distributed white noise shocks and $\mathbb{E}_t[\psi_{t+n}] = \mathbb{E}_t[\theta_{t+n}] = 1 \; \forall \; n > 0$. By changing the value of $\sigma_\psi$ and $\sigma_\theta$, we change the degree of uncertainty faced by households. The details of the model are described in appendix A.7.

The parameters are calibrated exactly as in CSTW except for the aggregate productivity growth rate and individual productivity growth rate. In CSTW, there is no aggregate and individual productivity growth; here we allow aggregate productivity growth $G = 1.015$, and individual productivity growth $X = 1.01$.

Following CSTW, we allow consumers to have heterogeneous time preference rates; the uniform distribution is calibrated to match the mean and the dispersion of wealth across U.S. households.

For this exercise to succeed, the model needs to be calibrated so that even the most patient consumer satisfies a 'growth impatience condition' that prevents the wealth-to-income ratio from asymptoting to infinity:

$$\text{Þ}_{\acute{\Gamma}} \equiv \frac{(\beta \mathsf{R})^{1/\rho}\cancel{\mathsf{D}}}{\underline{\Gamma}} \;\; < \;\; 1 \tag{4}$$

where $\underline{\Gamma} = \Gamma \acute{\psi}$ and $\acute{\psi} \equiv (\mathbb{E}[\psi^{-1}])^{-1}$. By computing the aggregate saving rate while increasing $\sigma^2_\psi$ and $\sigma^2_\theta$, we derive the relationship between the aggregate saving rate and the degree of uncertainty. Increasing $\sigma^2_\psi$ will also cause $\underline{\Gamma}$ to increase. The growth impatience condition of the most patient agent restricts the maximum value of $\sigma^2_\psi$ we can choose. Here we allow $\sigma^2_\psi$ and $\sigma^2_\theta$ to move from half of their benchmark values to twice the benchmark value.

Figure 1 shows how the aggregate saving rate changes with $\sigma_\psi$. The first row in Table 2 reports the slope and $R^2$ calculated from OLS regression between the aggregate saving rate and the standard deviation of the permanent income shock. Under the model calibration drawn from CSTW, the aggregate saving rate is almost perfectly correlated with the standard deviation of $\sigma_\psi$: adding 0.01 to $\sigma_\psi$ will cause the aggregate saving rate to rise about 0.83 percent.

Figure 2 shows how the aggregate saving rate varies with the standard deviation of the

**Table 1** Relation Between Measures of Uncertainty and Saving Rate in CSTW model

| | Slope | $R^2$ | max $\text{Þ}_\Gamma$ |
|---|---|---|---|
| $\sigma_\Psi$ | 0.826 | 0.998 | 0.998 |
| $\sigma_\theta$ | 0.0111 | 0.989 | 0.996 |

Table 2 presents the correlation between the aggregate saving rate and the degree of income uncertainty. The first two columns show the slope and $R^2$ from an OLS regression. The third column shows the growth impatience factor of the most patient agent in the economy. The first row shows the result of an OLS regression of the aggregate saving rate on the standard deviation of permanent income shocks $\sigma_\psi$. The second row shows the result of OLS regression of the aggregate saving rate on the standard deviation of transitory income shocks $\sigma_\theta$. (No standard errors are reported because the precision of the estimates is a function of the size of the population that is simulated. The coefficients reported are stable when the simulated population is increased arbitrarily, so they are the 'true' implications of the model.

transitory income shock. The regression result is reported in the second row of Table 2. The impact of transitory uncertainty is negligible: Increasing $\sigma_\theta$ by 0.01 will cause the aggregate saving rate to increase by 0.011 percent. The last column reports the growth impatience factor of the most patient agent when $\sigma_\psi^2$ reaches its upper limit in each of the two exercises. We can see that the growth impatience condition is comfortably satisfied in both exercises.

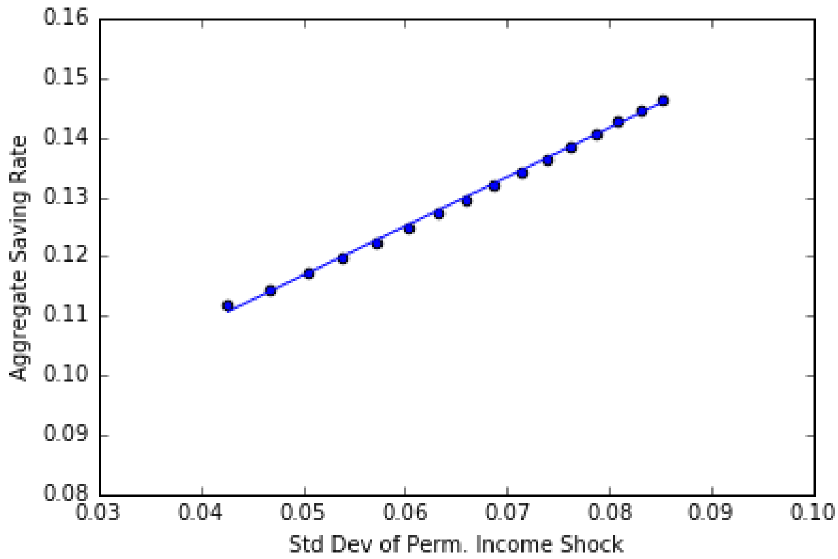


**Figure 1** Saving Rate as a Function of Variance of Permanent Shock

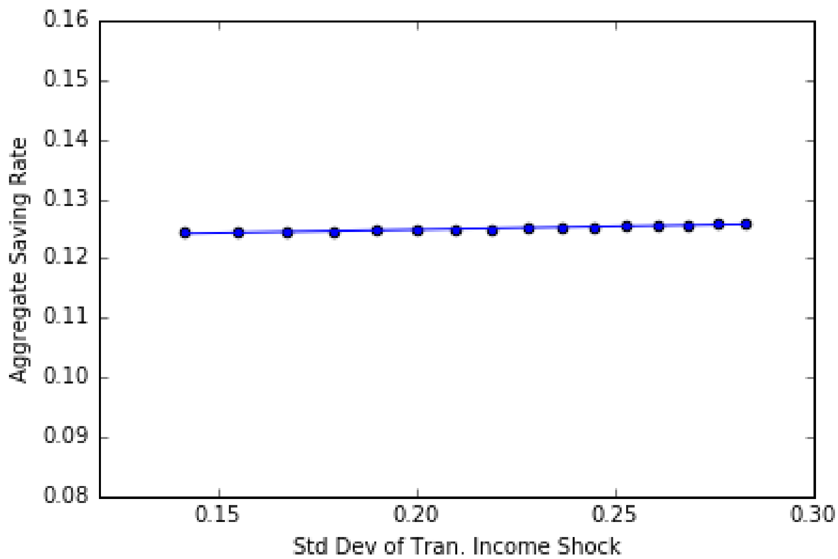


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**Figure 2** Saving Rate as a Function of Variance of Transitory Shock