A Tractable Model of Precautionary Reserves, Net Foreign Assets, or Sovereign Wealth Funds

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Motivation

Three Hot Topics In International Macro:

- Huge Reserve Accumulation By Fast-Growing Developing Economies
  - China
- Surprising “Upstream” Capital Flows: Developing → Rich Countries
  - China – Following Japan, Korea, Taiwan, Singapore, Hong Kong
- Sovereign Wealth Funds
  - Mainly Oil-Rich Countries

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Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
  - Tractable. Tractable! TRACTABLE!!
  - The Natural Extension of the Ramsey Model
  - Shows Eqbm Relation Between Precautionary, Other Motives
- Two applications
  - Economic Growth and Capital Flows
  - Impact of Reducing Global Financial Imbalances

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  - "Real" microfoundations!
- Builds on Toche (2005)
- Related: Fogli and Perri (2006), Mendoza, Quadrini, and Ríos-Rull (2009), Sandri (2014)
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- Small Open Economy
- Balanced Growth Path With Population And Productivity Growth
- Accumulate Buffer Stock to Self-Insure Against Unemployment
- NFA: Aggregate Stock of Wealth Minus Domestic Capital Stock
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Macroeconomic Assumptions

- Domestic output is produced with the Cobb-Douglas function:

$$\boldsymbol{Y}_t = \boldsymbol{K}_t^{\alpha}(z_t \boldsymbol{L}_t)^{1-\alpha}, \tag{1}$$

- Labor productivity increases by G in every period,

$$z_{t+1} = \mathrm{G} z_t. \tag{2}$$

- Capital perfectly mobile internationally,

$$\overbrace{\daleth}^{\equiv 1-\delta} + \alpha \frac{\boldsymbol{Y}_t}{\boldsymbol{K}_t} = \mathrm{R}, \tag{3}$$

- Capital-to-output ratio is constant and equal to,

$$\frac{\boldsymbol{K}}{\boldsymbol{Y}} = \frac{\alpha}{\mathrm{R} - \daleth}. \tag{4}$$

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People and Populations

- Each worker is part of a single ‘generation’ born at the same time
- Size of generation born at t : Ξ^t.
- Life Stages:
  - Employment
  - Unemployment/Retirement
  - Death
- Transitions to unemployment and death are Poisson processes
  - Flow probabilities $\mho$ and D.
- Employed and Unemployed Populations:

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\begin{aligned}
\mathcal{E}_t &= \frac{\Xi^{t+1}}{\Xi - \cancel{\mho}} \\
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Balanced Growth

- Capital and output grow at constant rates
- Real wage grows by factor G in every period.
- Main variable of interest= $\boldsymbol{N}_t$, the aggregate net foreign assets of the economy at the beginning of period t.

$$\boldsymbol{N}_t = \boldsymbol{B}_t - \boldsymbol{K}_t. \tag{5}$$

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The microeconomic consumer's problem

- Budget constraint of individual:

$$\frac{\boldsymbol{b}_{t+1}}{\mathsf{R}} + \boldsymbol{c}_t = \boldsymbol{b}_t + \overbrace{\xi_t \ell_t \mathsf{W}_t}^{\text{labor income}}, \tag{6}$$

- Worker's labor supply ℓ grows by a factor X per period over his lifetime,

$$\ell_t = \mathsf{X}^t \ell_0, \tag{7}$$

- For consumer who remains employed, labor income grows by

$$\Gamma \equiv \mathsf{G}\mathsf{X}.$$

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The microeconomic consumer's problem

- Unemployment: Complete and permanent destruction of h
- CRRA felicity $\mathrm{u}(\bullet) = \bullet^{1-\rho}/(1-\rho)$; geometric discounting at β
- Unemployed convert their wealth into annuities.
- Solution to the unemployed consumer's optimization problem,

$$\boldsymbol{c}_t^u = \kappa^u \boldsymbol{b}_t,$$

where κ is the marginal propensity to consume,

$$\kappa^u \equiv 1 - \cancel{D}\frac{(\beta \mathrm{R})^{1/\rho}}{\mathrm{R}}.$$

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- 'Growth impatience condition':

$$Þ_{\Gamma} \equiv \frac{(\beta \mathsf{R})^{1/\rho}}{\Gamma} < 1$$

necessary for finite target ratio of wealth to income (Carroll (2011))

- Defining nonbold variables as, e.g., $c_t^e = \boldsymbol{c}_t^e / (\mathsf{W}_t \ell_t)$, we get

$$b_{t+1}^e = (\mathsf{R}/\Gamma)\left(b_t^e - c_t^e + 1\right). \tag{8}$$

$$c_{t+1}^e = Þ_{\Gamma} \mho^{1/\rho} c_t^e \left[1 - \mho \left(\frac{Þ_{\Gamma}}{\kappa^u} \frac{c_t^e}{\mathsf{R}/\Gamma(b_t^e - c_t^e + 1)} \right)^{\rho} \right]^{-1/\rho}. \tag{9}$$

- Saddle-point stable dynamics.

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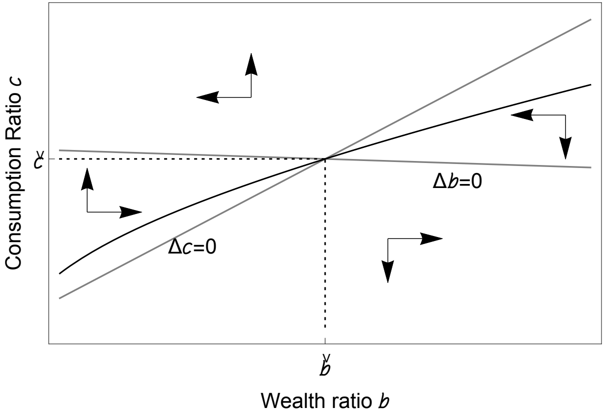
Phase Diagram



The Growth Impatience Condition

- Target wealth-to-income ratio: impatience vs prudence.
- Closed-form solution for the target wealth-to-income ratio

$$\check{b} = \left[\frac{\Gamma}{\mathsf{R}} - 1 + \kappa^{u} \left(1 + \frac{\text{Þ}_{\Gamma}^{-\rho} - 1}{\mho} \right)^{1/\rho} \right]^{-1}. \quad (10)$$

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- The response of $\check{b}$ to R is ambiguous.

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Foreign Assets

- Ratio of employed workers' wealth to output,

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where Λ is the factor by which the share of a generation in total labor supply shrinks every period.

- The Level of Unemployed Workers' Wealth is

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Table 1

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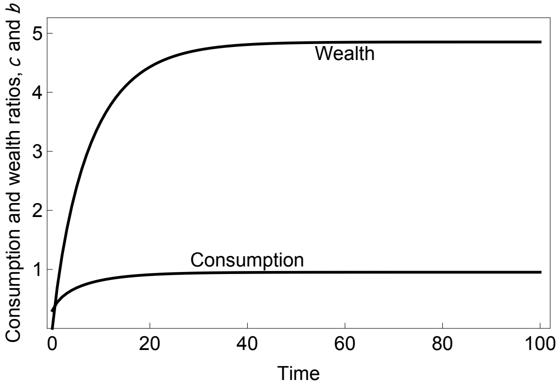
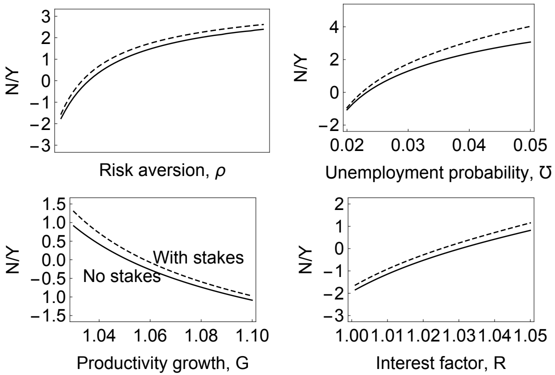

Wealth
Consumption
Consumption and wealth ratios, c and b
Time
0
20
40
60
80
100
1
2
3
4
5

Sensitivity analysis



Social Insurance

- Many countries have social transfers to unemployed/retired
- New assumption: labor income tax on the employed in order to finance transfers to the unemployed.
- Unemployed receive transfer whose value is a multiple ς of the labor income that they would have received if they had remained employed.
- New formula for target wealth-to-income ratio. Going through the same steps as before, we get

$$\check{b}(\varsigma) = \left\{1 - \varsigma\left[\frac{\mho}{\Xi} + \kappa^u\left(1 + \frac{\text{Þ}_\Gamma^{-\rho} - 1}{\mho}\right)^{1/\rho}\right]\right\}\check{b}, \qquad (19)$$

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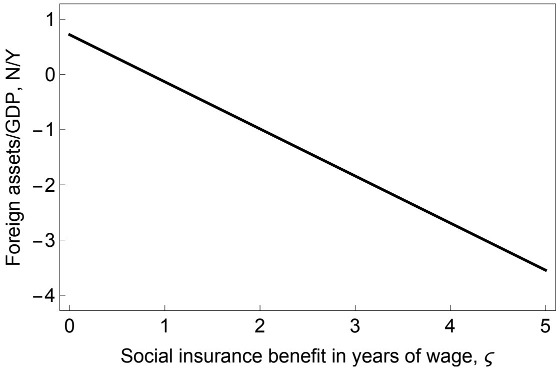

Growth And Saving

- Theory: Good Growth Prospects $\rightarrow$ Should Borrow to Invest
- Data: Fast-Growing Countries *Export* Capital
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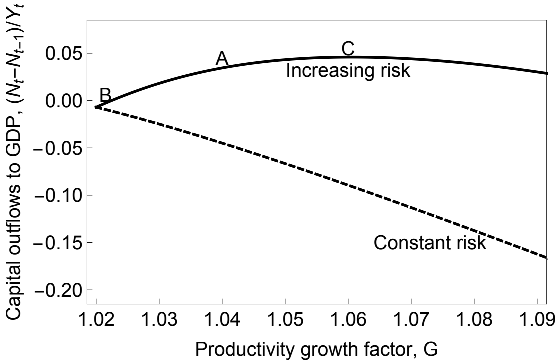
Growth and capital flows



World General Equilibrium

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- Two countries identical except for size (h=20%, f=80%) and level of social insurance ($\varsigma_h = 1.5$, $\varsigma_f = 0.75$).
- This implies

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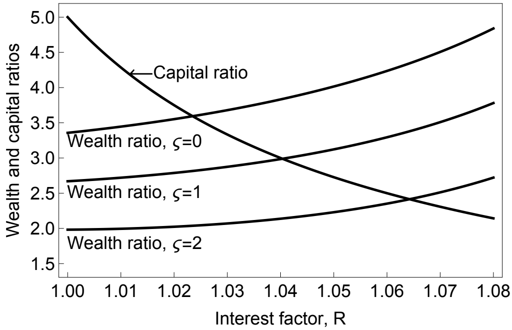

Conclusions

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