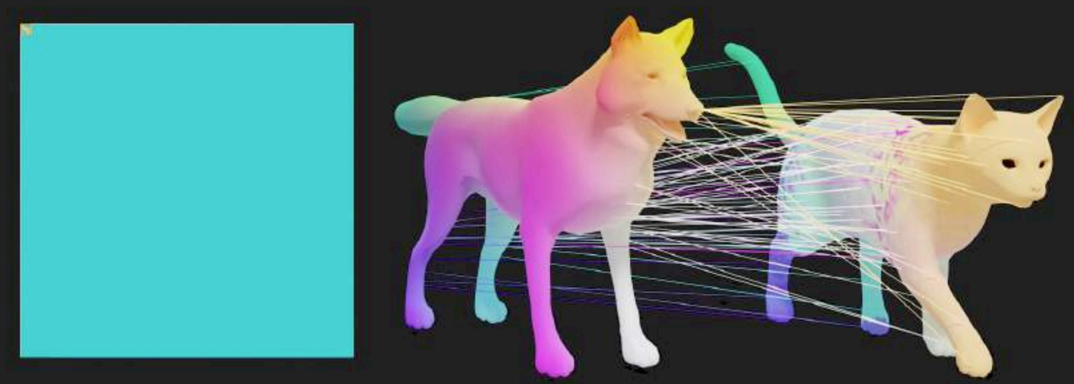
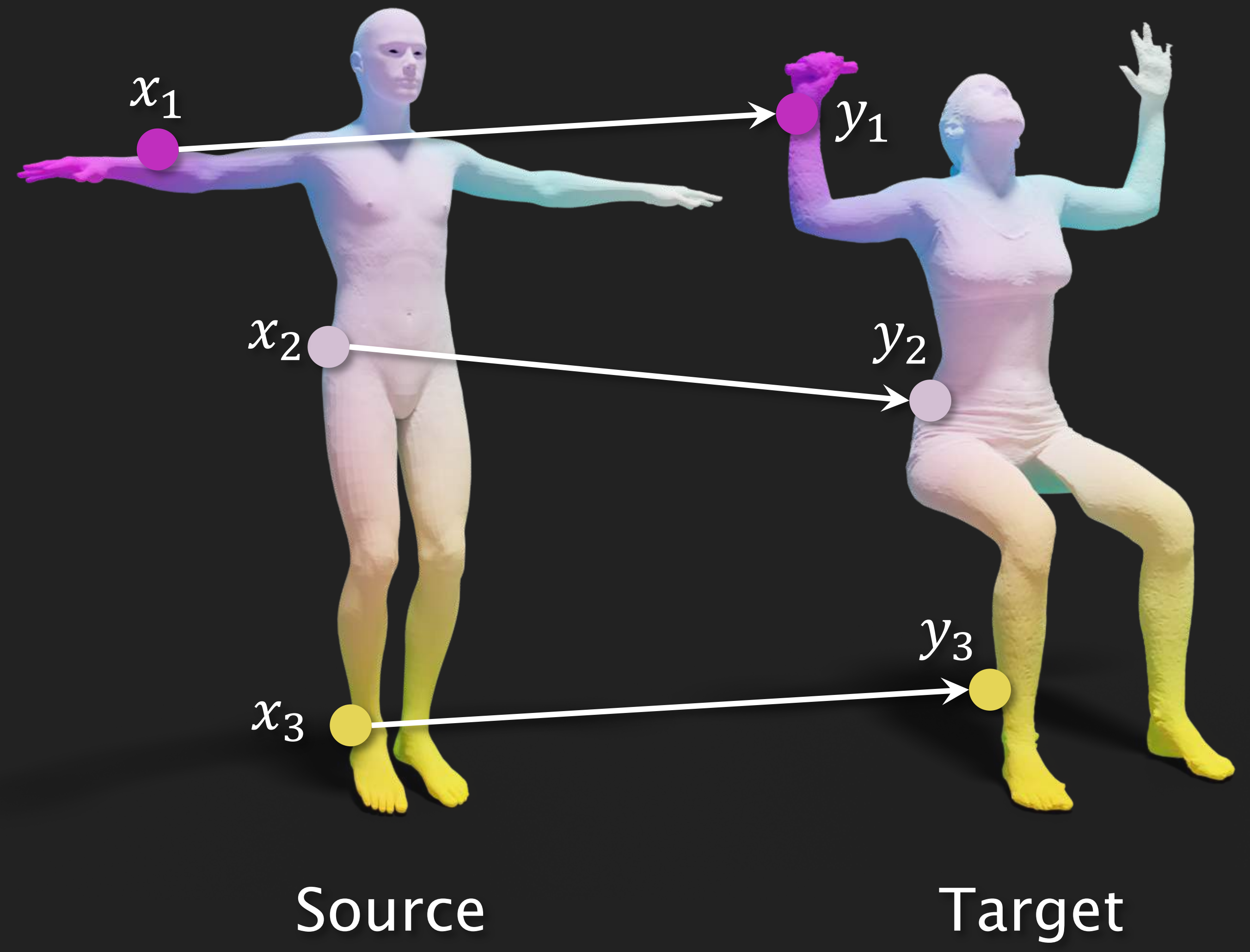
ZoomOut: Spectral Upsampling for Efficient Shape Correspondence

Simone Melzi*, University of Verona
Jing Ren*, KAUST
Emanuele Rodolà, Sapienza University of Rome
Abhishek Sharma, École Polytechnique
Peter Wonka, KAUST
Maks Ovsjanikov, École Polytechnique
(*equal contribution; Presenter: Jing Ren)

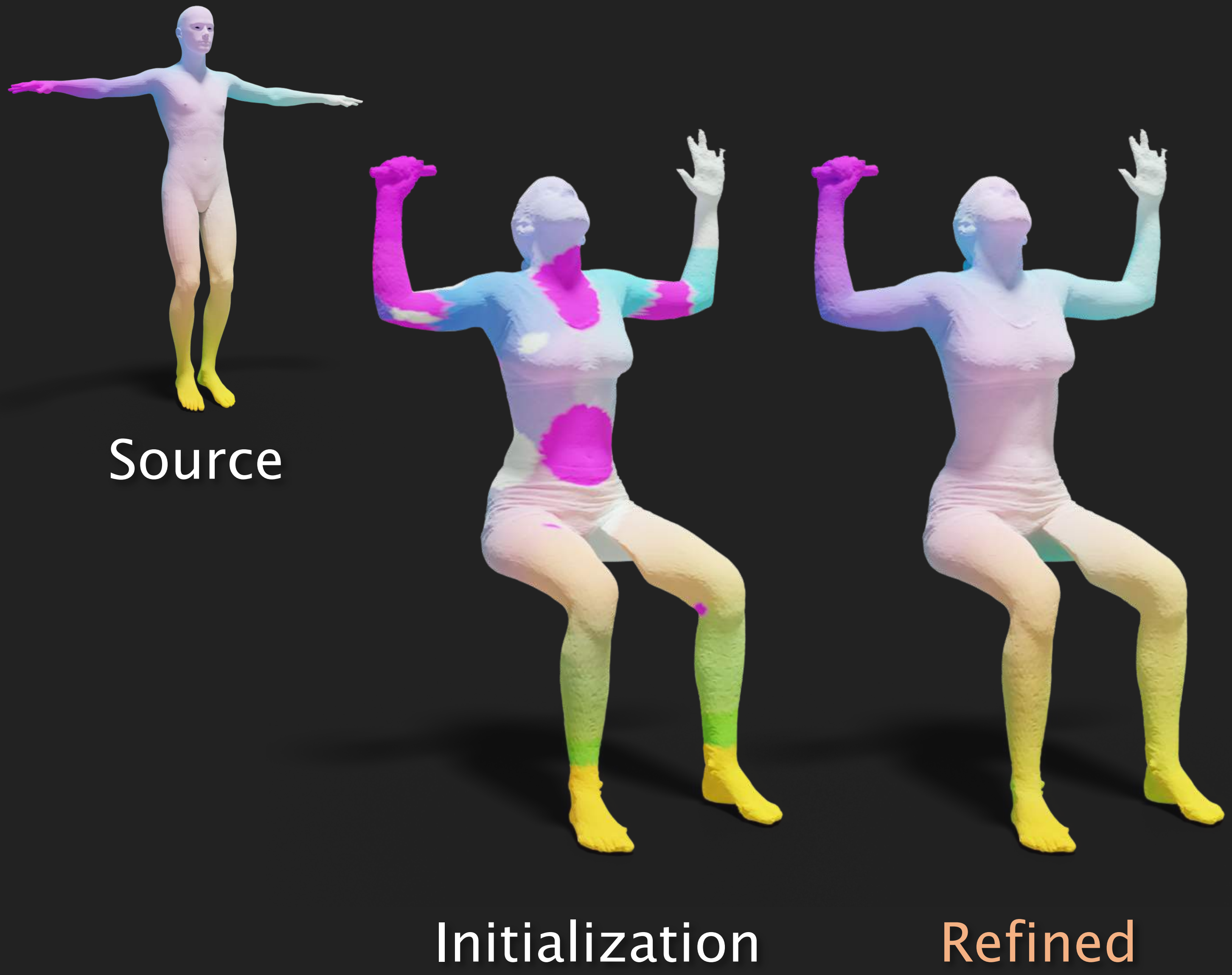


Shape Matching



- Point-based methods
  - [Bronstein et al. 2006],
  - [Huang et al. 2008]…
- Parametrization-based methods
  - [Lipman and Funkhouser 2009]
  - [Aigerman et al. 2017]…
- Optimal transport
  - [Solomon et al. 2016]
  - [Mandad et al. 2017]…
- Functional maps
  - [Ovsjanikov et al. 2012]
  - [Ezuz and Ben-Chen 2017]…
- ……

Map Refinement

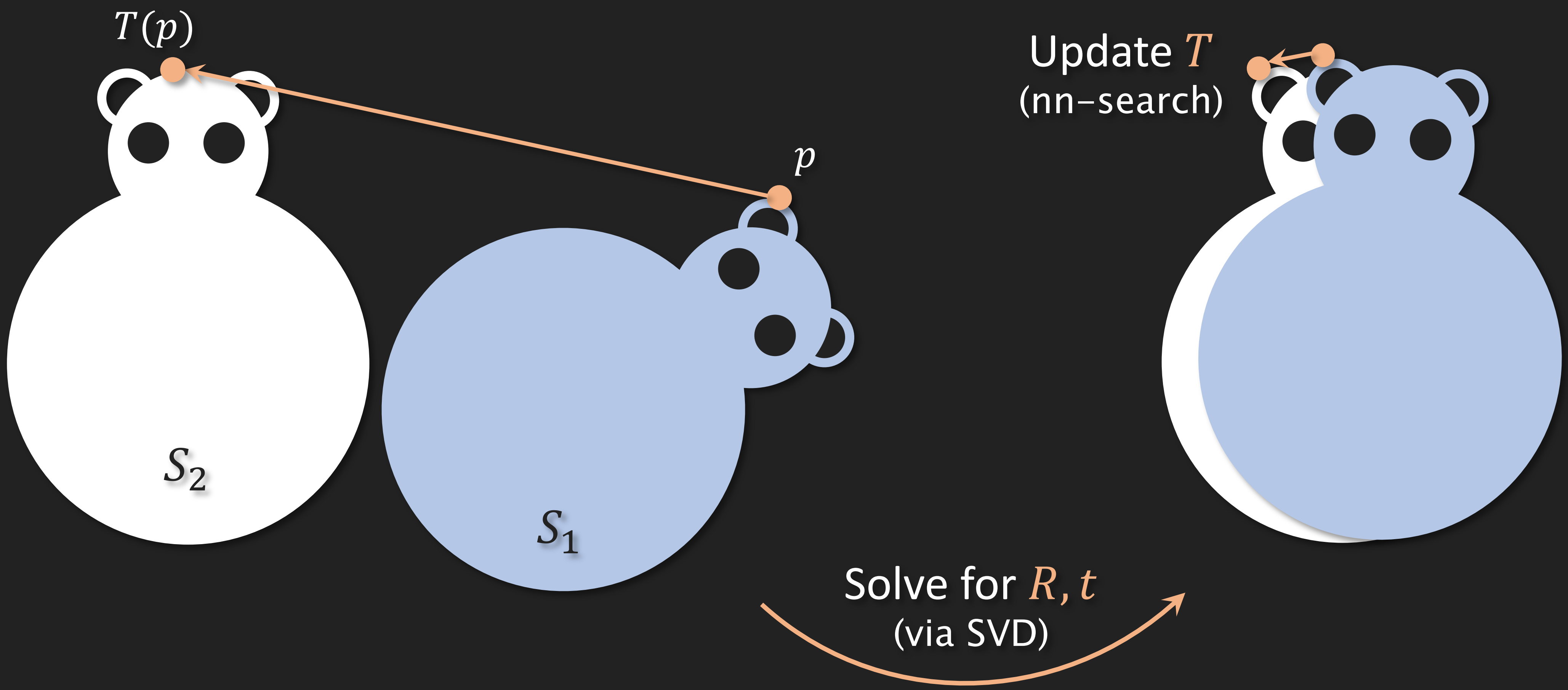


- ICP-based
  - [Besl and McKay 1992] (Spatial)
  - [Ovsjanikov et al. 2012] (Spectral)
  - [Ren et al. 2018] (Spatial & Spectral)
- Reversible Harmonic Maps
  - [Ezuz et al. 2019]
- Product Manifold Filter
  - [Vestner et al. 2017ab]
- Deblurring and Denoising
  - [Ezuz and Ben-Chen 2017]
- ……

Iterative Closest Point (ICP)

[Besl and McKay 1992]

$$\min_{\substack{R,t,T \\ R^T R = I}} \sum_{p \in S_1} \left\| R X_1(p) + t - X_2\big(T(p)\big) \right\|_F^2$$



Iterative Closest Point (ICP)

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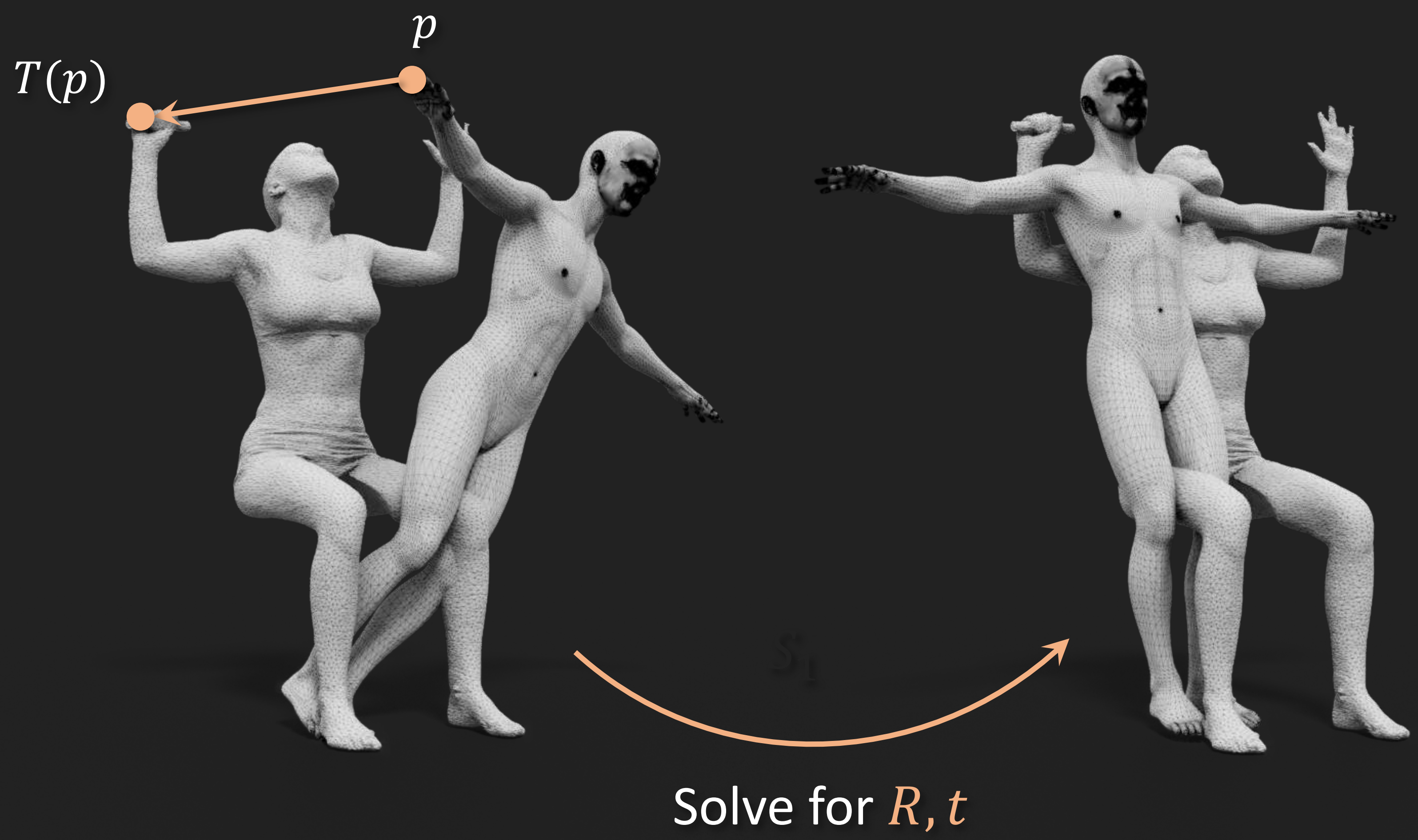


Problem:

- nn-search in spatial domain does not work
- Vertex positions (X_1, X_2) are extrinsic features

Solution:

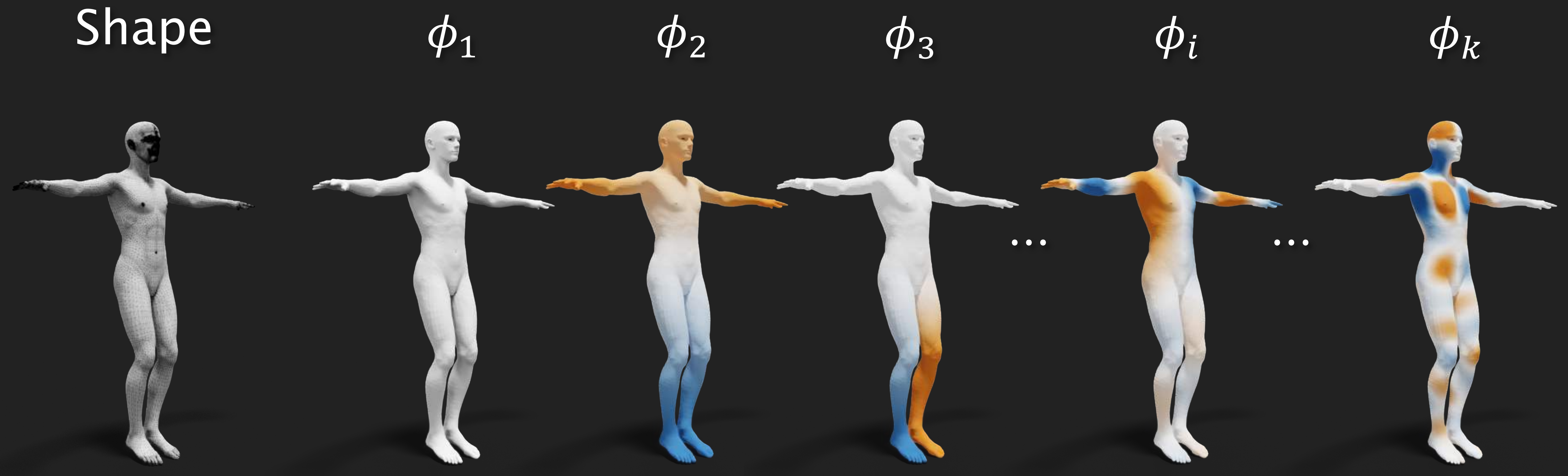
- Align intrinsic features

Laplace–Beltrami Operator Δ

Eigenfunctions & Eigenvalues

Helmholtz equation

$$\Delta\phi_i = \lambda_i\phi_i$$

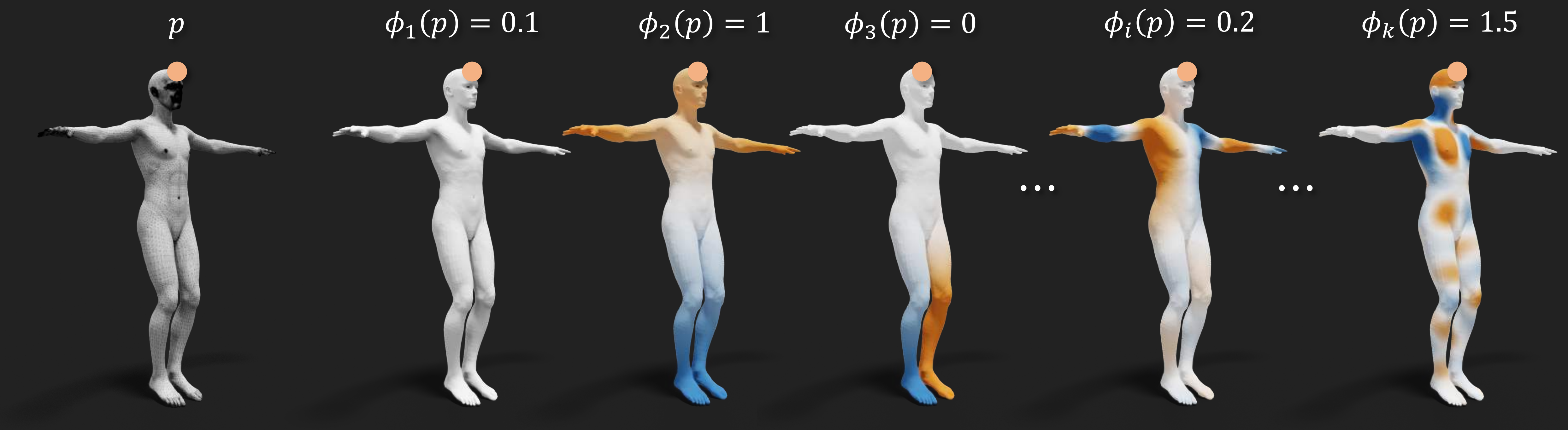


$$0 = \lambda_1 \quad \leq \quad \lambda_2 \quad \leq \quad \lambda_3 \quad \leq \cdots \quad \lambda_i \quad \cdots \leq \quad \lambda_k$$

Intrinsic features

Eigenfunctions of Δ

$\Phi(p) = (\ 0.1,\ \ 1,\ \ 0, \cdots,\ 0.2, \cdots,\ 1.5\ )$ intrinsic feature at p



ICP in the spectral domain

Spatial ICP [Besl and McKay 1992]

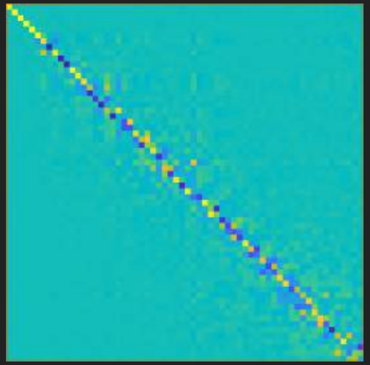
$$\min_{\substack{R,t,T \\ R^T R = I}} \sum_{p \in S_1} \left\| R X_1(p) + t - X_2(T(p)) \right\|_F^2$$

$X_1(\cdot), X_2(\cdot)$: vertex positions

Spectral ICP [Ovsjanikov et al.2012]

$$\min_{\substack{C,T \\ C^T C = I}} \sum_{p \in S_1} \left\| C \Phi_1(p) - \Phi_2(T(p)) \right\|_F^2$$

$\Phi_1(\cdot), \Phi_2(\cdot)$: LB eigenfunctions



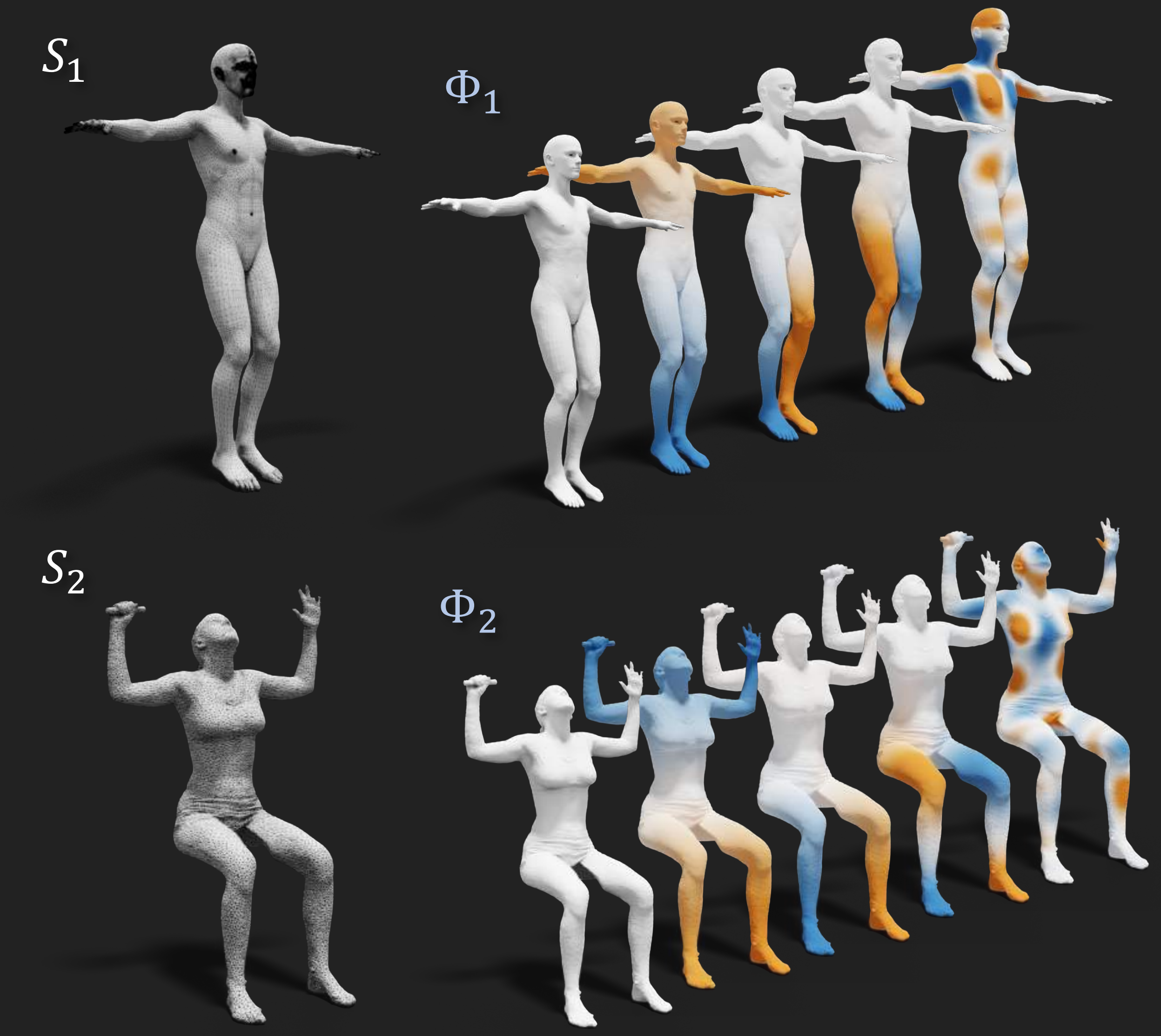
C: functional map

Optimize for a functional map

- [Ovsjanikov et al.2012]
- [Nogneng and Ovsjanikov 2017]
- [Ren et al. 2018]
- ……

ICP in the spectral domain

Align LB eigenfunctions



$$\min_{\substack{C,T \\ C^T C = I}} \sum_{p \in S_1} \left\| C\Phi_1(p) - \Phi_2\big(T(p)\big) \right\|_F^2$$

Observations

- Dimensionality of Φ_1, Φ_2: 50~250
- Solve for $C \in R^{250\times 250}$:
  - might be under–determined
  - easy to get trapped in a local minimum

Our solution: ZoomOut

- progressively registering the feature space
- without assumption on C

ZoomOut Algorithm

Some intuitions

Align the body

Align the head

Align the ear

Global structure → Local structure

ZoomOut Algorithm

Some intuitions

ϕ_2 ϕ_4 ϕ_8 ϕ_{11} ϕ_{26} ϕ_{70}

Low frequency → High frequency

ZoomOut Algorithm

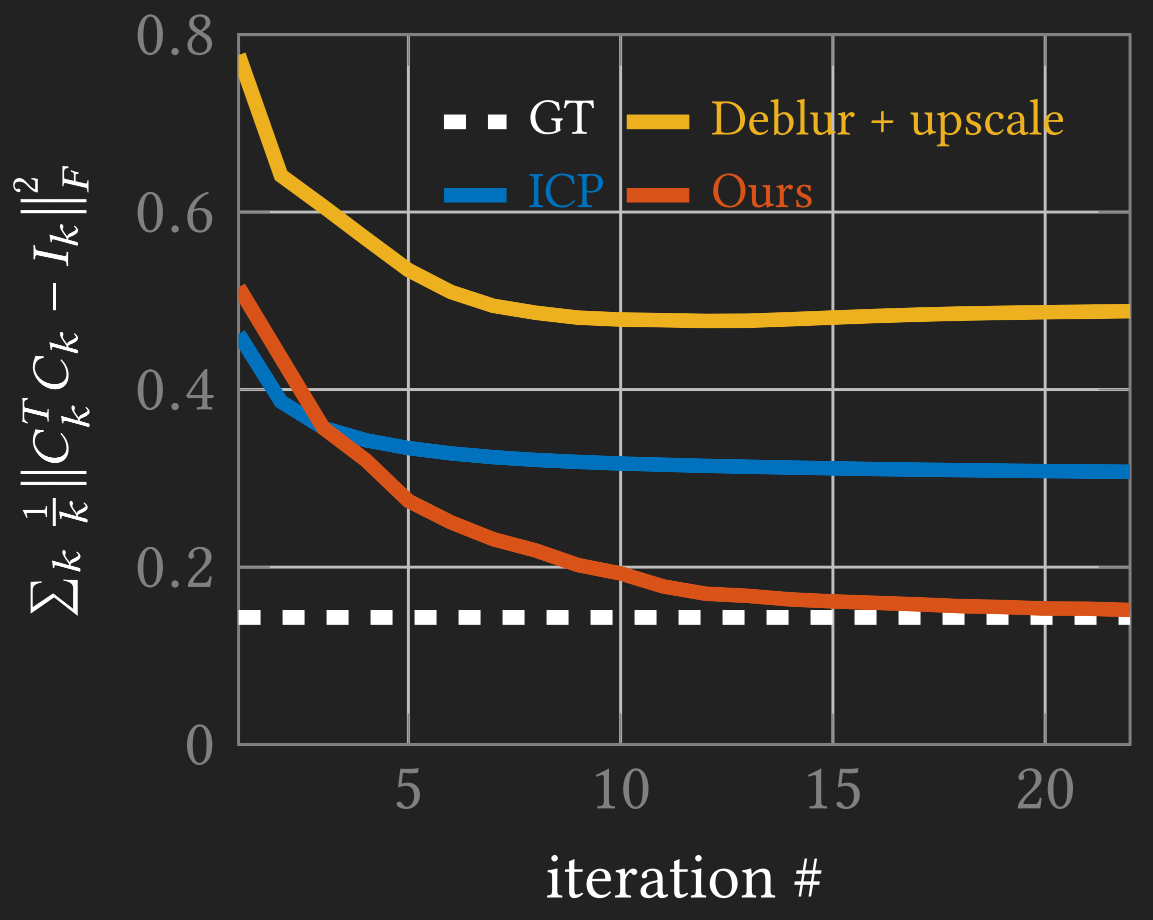
Notation

- Φ_i^k: stores the first k eigenfunctions of S_i

Algorithm

1. With initial T & k (use existing methods)
2. Solve $C^k = \underset{C}{\operatorname{argmin}} \sum_p \left\| C\Phi_1^k(p) - \Phi_2^k\big(T(p)\big)\right\|_F^2$ (align the first k features)
3. Update $T = \underset{T}{\operatorname{argmin}} \sum_p \left\| C^k\Phi_1^k(p) - \Phi_2^k\big(T(p)\big)\right\|_F^2$ (nn-search after alignment)
4. $k \leftarrow k + 1$ (increase the feature space to register)
5. Go to step 2 (with the first k dim well-aligned, align the $k + 1$ features)

ZoomOut: Theoretical Analysis



ZoomOut Energy

$$\boxed{\min_{C\in\mathbb{P}} \sum_k \frac{1}{k} \left\|C_k^T C_k - I_k\right\|_F^2}$$

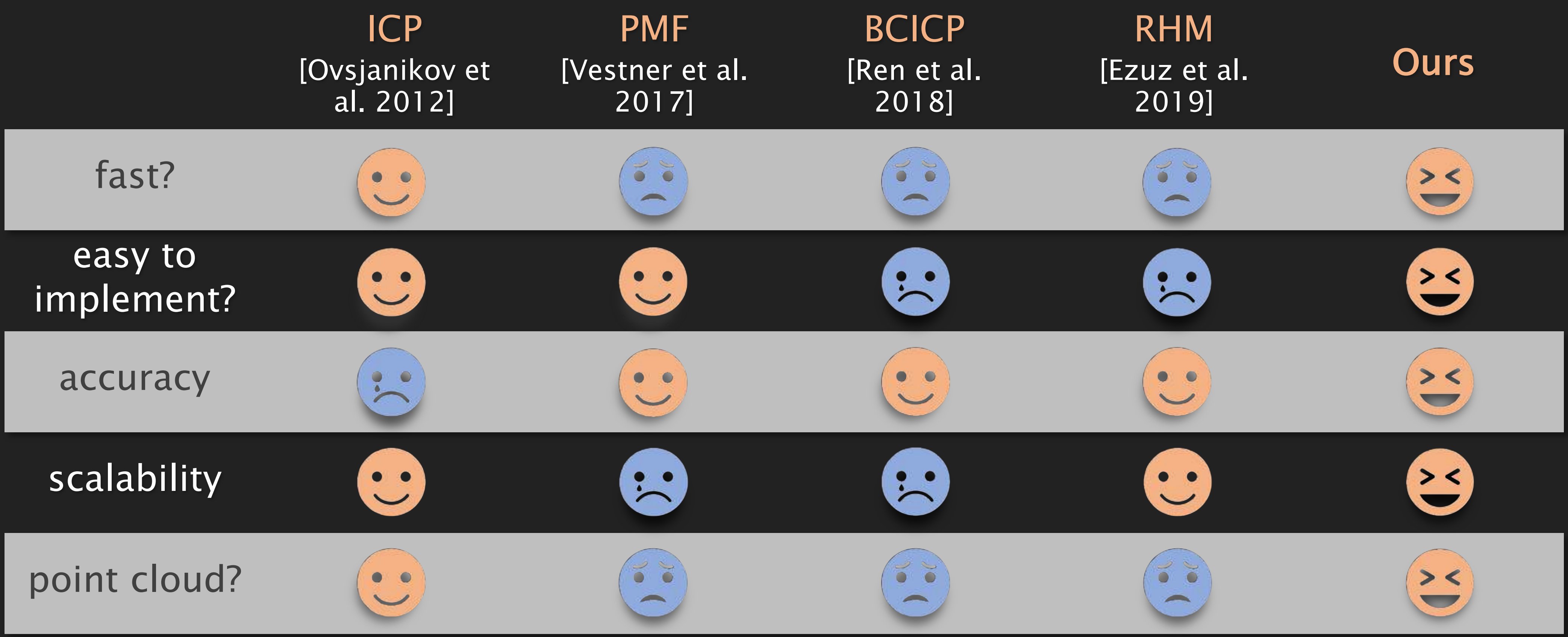
Notation

- C_k : principal sub-matrix of C
- I_k : $k\times k$ identity matrix
- $\mathbb{P}$: set of fMaps arising from pointwise maps

Theorem:

- Global minimum: isometry

Comparison to state-of-the-art



Property: easy to implement

Our method:

- 5 lines of code
- Similar complexity to ICP

```
function [C,P]=ZoomOut(M,N,C,k_final)

for k=size(C,1):k_final-1
 x = knnsearch(N.Phi(:,1:k)*C',M.Phi(:,1:k));
 P = sparse(1:M.n,x,1,M.n,N.n);
 C = M.Phi(:,1:k+1)'*M.A*P*N.Phi(:,1:k+1);
end
```

code

Other baselines

- PMF: kernel construction & linear assignment solver
- RHM: sophisticated energy function & optimization
- BCICP: ICP in both spatial & spectral domain with a lot of heuristics

Property: scalability

Speed-up by sampling

Recall:

1...

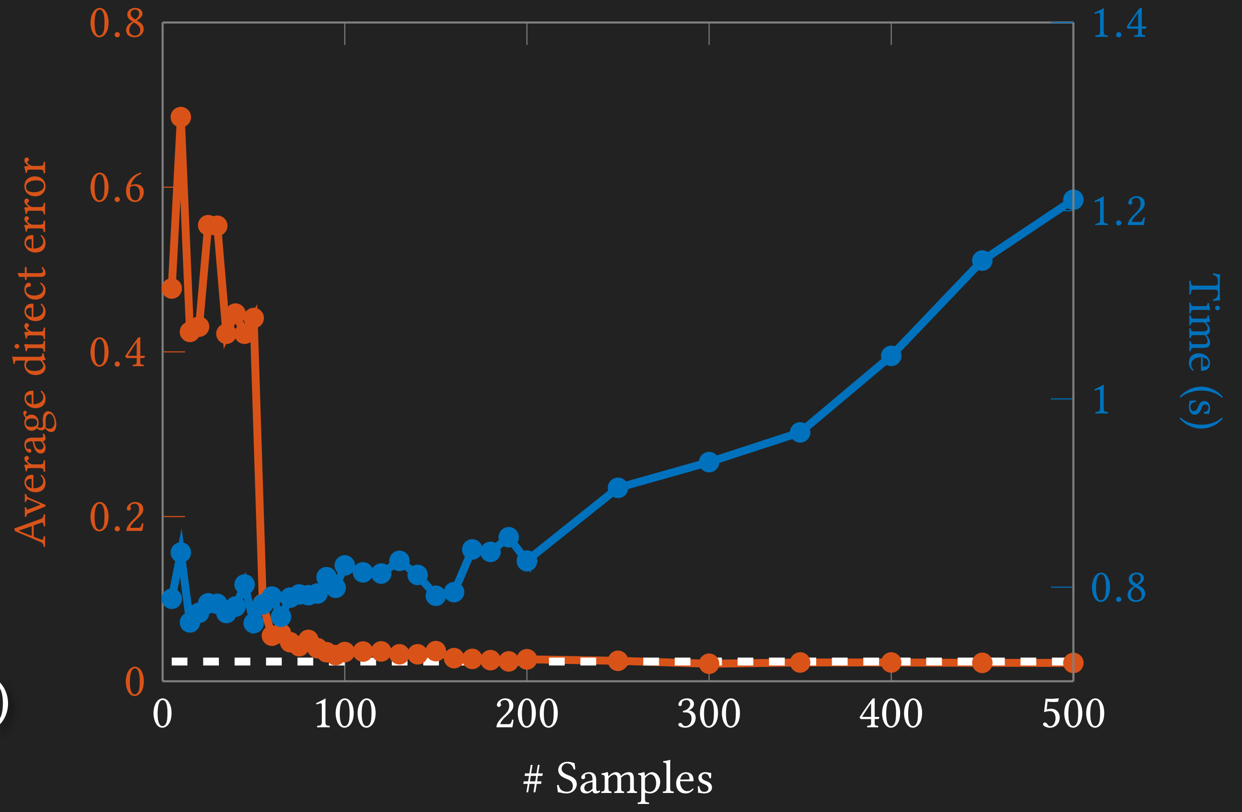
2. Solve $C^k = \text{argmin}_C \sum_p \left\| C\Phi_1^k(p) - \Phi_2^k(T(p)) \right\|_F^2$

3...

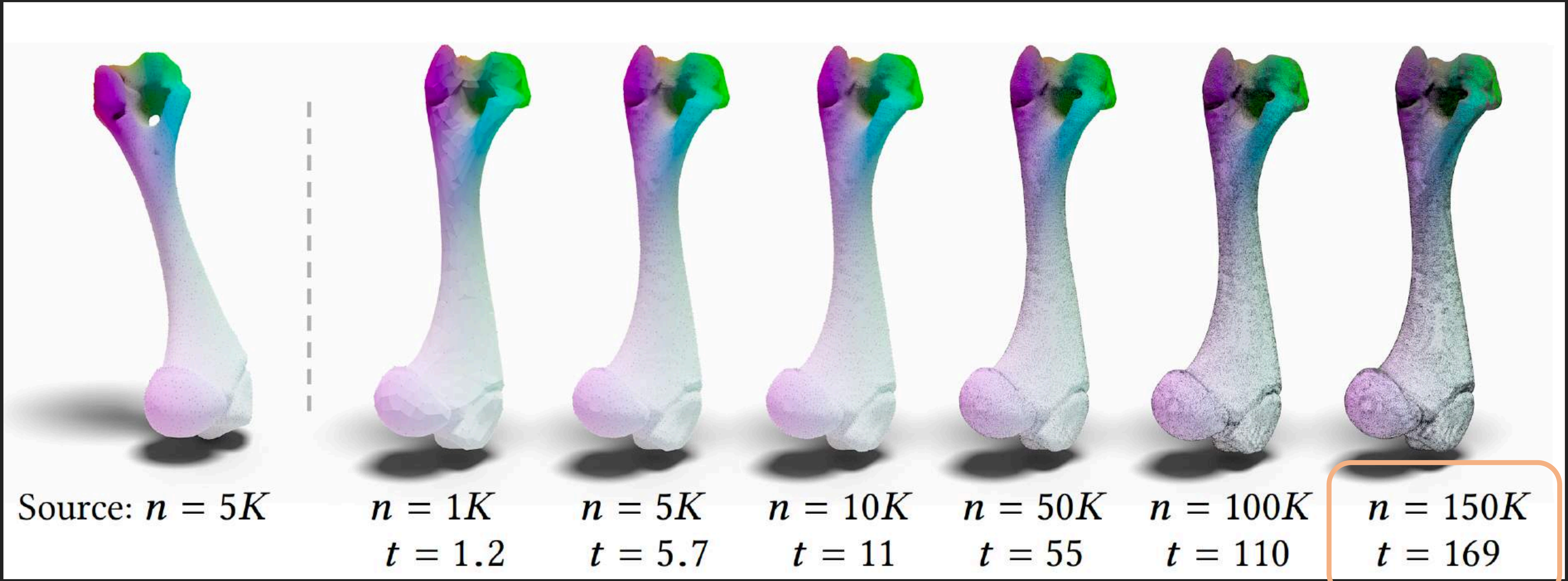
Observation

- do NOT need all p to compute $C^k \in R^{k\times k}$
- A small subset is enough! (e.g., 200 samples)

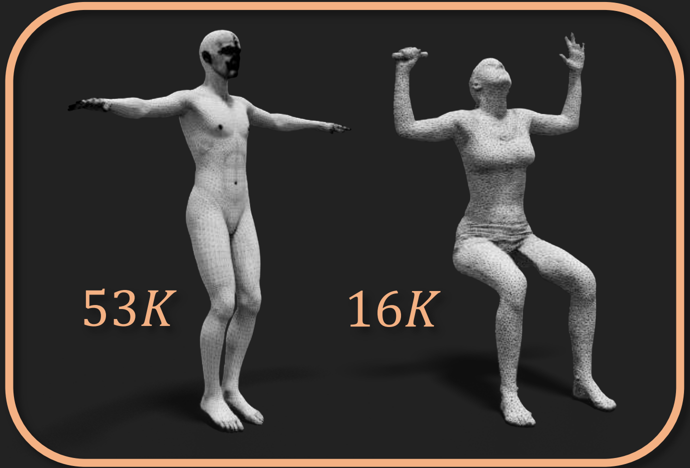
Source 20K, Target 5K



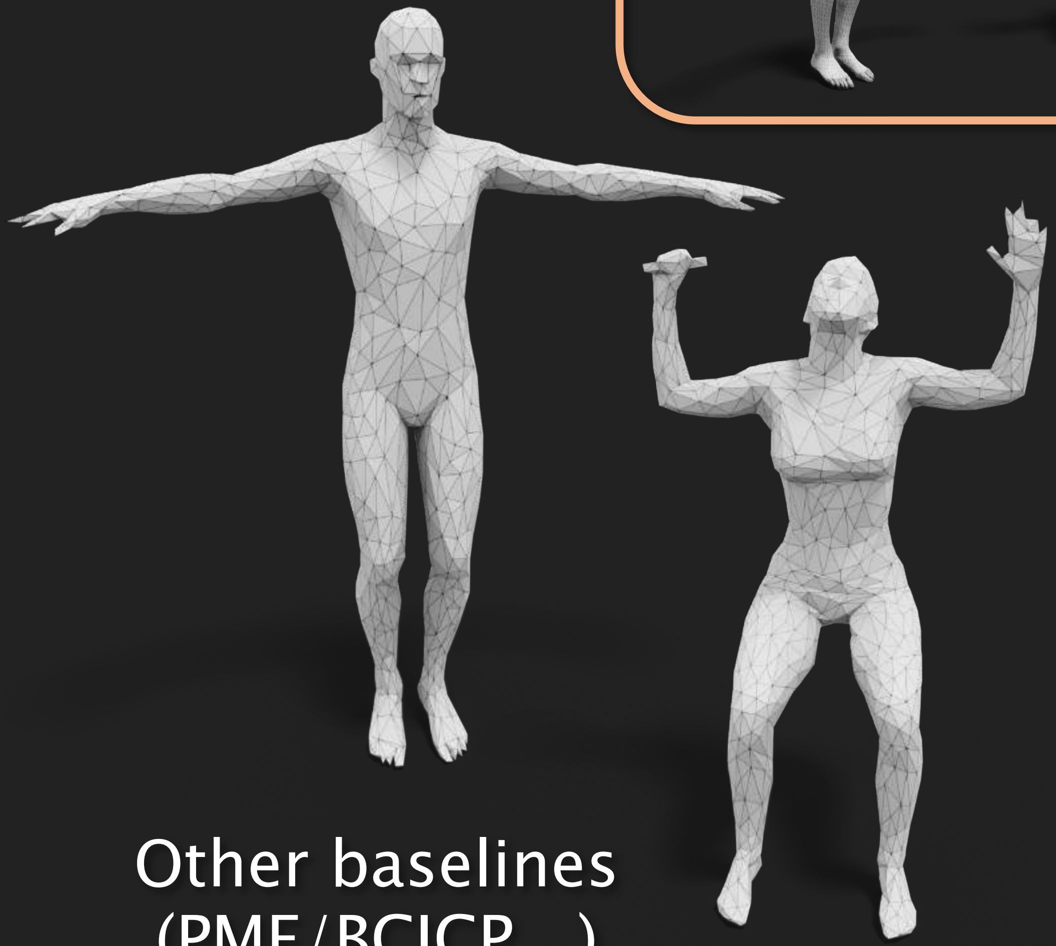
Property: scalability



Property: sample without remeshing



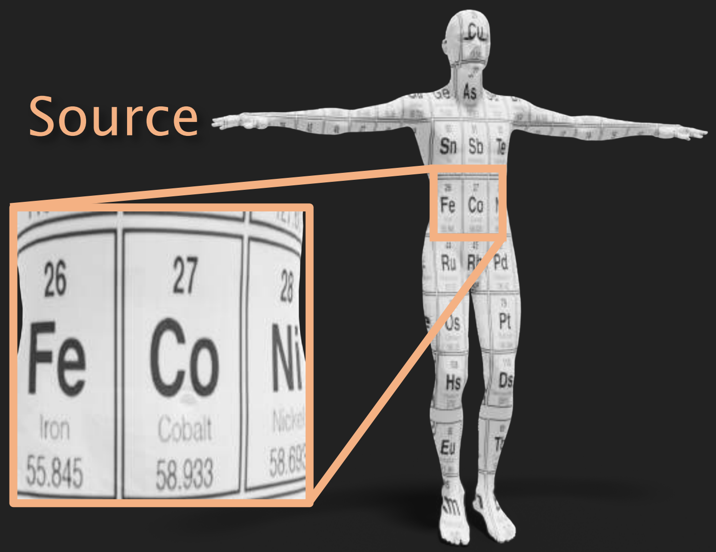
Ours

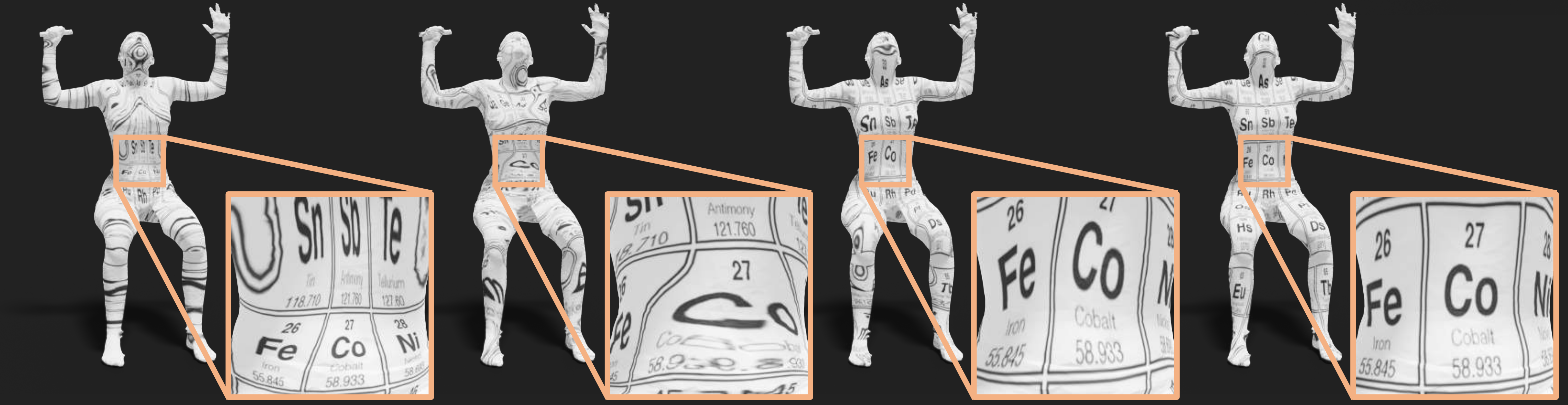


Other baselines
(PMF/BCICP…)

Property: sample without remeshing

#samples = 5000



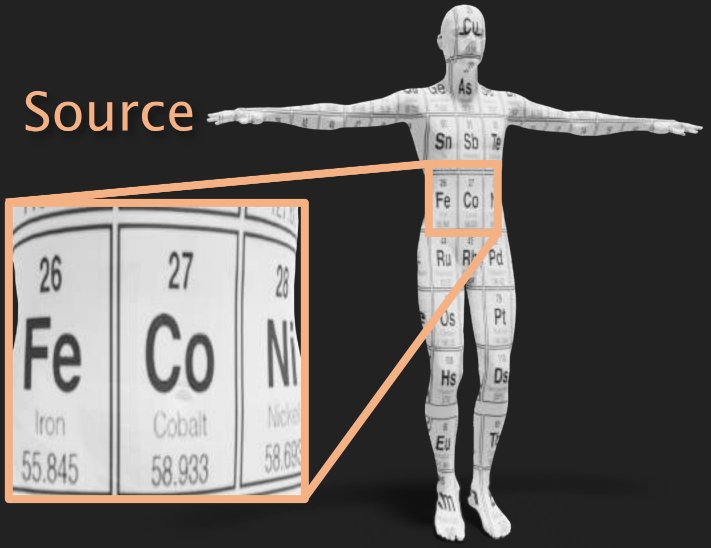


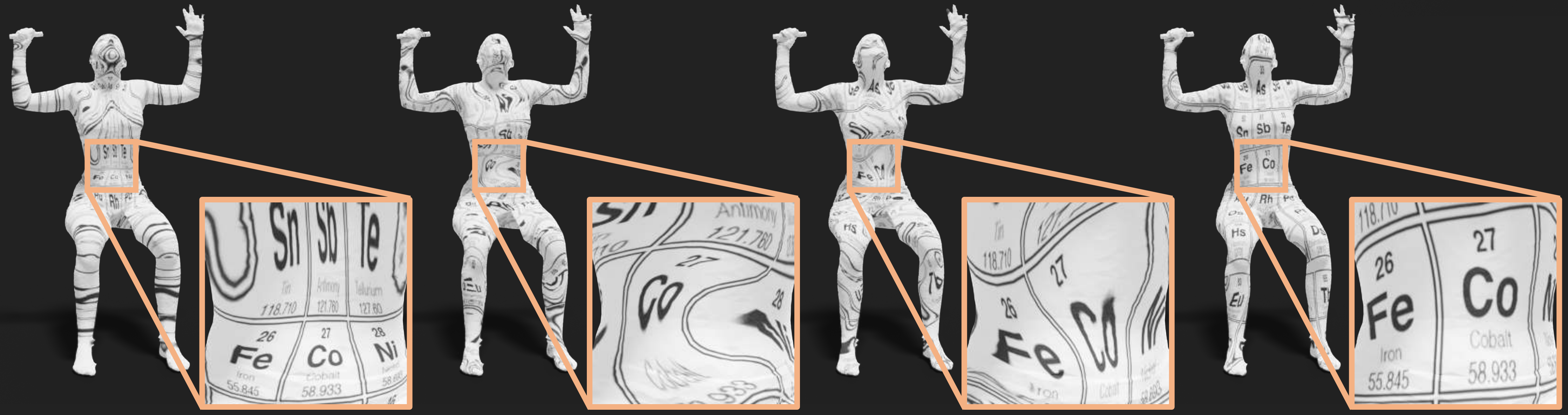
Initialization PMF BCICP Ours

Property: sample without remeshing

#samples = 500

Source





Initialization

PMF
72.9s

BCICP
90.0s

Ours
1.5s

Property: state-of-the-art accuracy

Refinement for shape matching

Accuracy: 15% ~ 30% improvement, Runtime: 10 ~ 500× improvement

		Average Error ($\times 10^{-3}$)			Average Runtime (s)		
Method \ Dataset		Original	Remeshed	Resampled	Original	Remeshed	Resampled
Ini		67.3	44.0	46.5	-	-	-
ICP		54.0	36.3	29.3	10.2	10.1	5.32
Deblur		61.9	38.6	44.4	10.9	11.7	10.4
RHM		41.9	33.3	32	41.4	42.5	47.4
PMF		26.4	25.9	86.4	736.5	780.2	311.5
BCICP		21.6	19.5	26	183.7	117.8	364.2
Ours		**15.8**	**13.3**	**21.7**	9.60	9.64	6.49
Ours*		17.5	14.5	24.6	**1.14**	**1.15**	**0.68**
Improv.	Ours	**26.9%**	**31.8%**	**16.5%**	19×	12×	56×
	Ours*	19.0%	25.6%	5.4%	**160×**	**100×**	**535×**

Property: state-of-the-art accuracy

Refinement for shape matching

Source	Initial	ICP	PMF	RHM	BCICP	Ours	GT

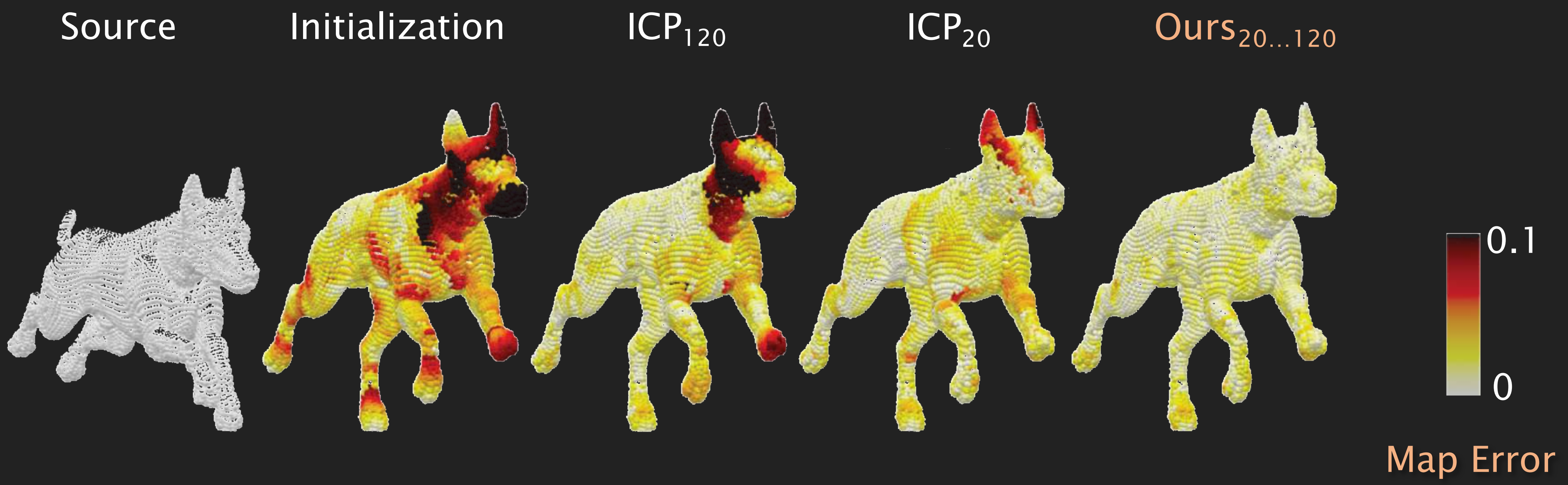
	ICP	PMF	RHM	BCICP	Ours
Runtime	3s	312s	56s	301s	0.47s
Error	0.069	0.108	0.039	0.028	0.024

Property: state-of-the-art accuracy

Challenging cases: topological noise

Property: applicable to point clouds

[Belkin et al. 2009]: estimation of Laplace Operator on point clouds



Summary

- Goal: refine given maps
- ZoomOut: progressively refine the correspondences in the spectral domain (no extra heuristics)
- Properties:
  - Simplicity: 5 lines of code
  - State-of-the-art accuracy
  - Efficiency: (compared to the best baseline)
    - Without sampling: $(4{\sim}50)\times$ faster (with better accuracy)
    - With sampling: $(100{\sim}500)\times$ faster (with comparable results!)
  - Scalability: works with a mesh of $150K+$ vertices
  - Applicable to point clouds directly
- Limitations:
  - Need initial correspondences

جامعة الملك عبدالله
للعلوم والتقنية
King Abdullah University of
Science and Technology

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Thank you for your attention

ZoomOut:
Spectral Upsampling for Efficient Shape Correspondence

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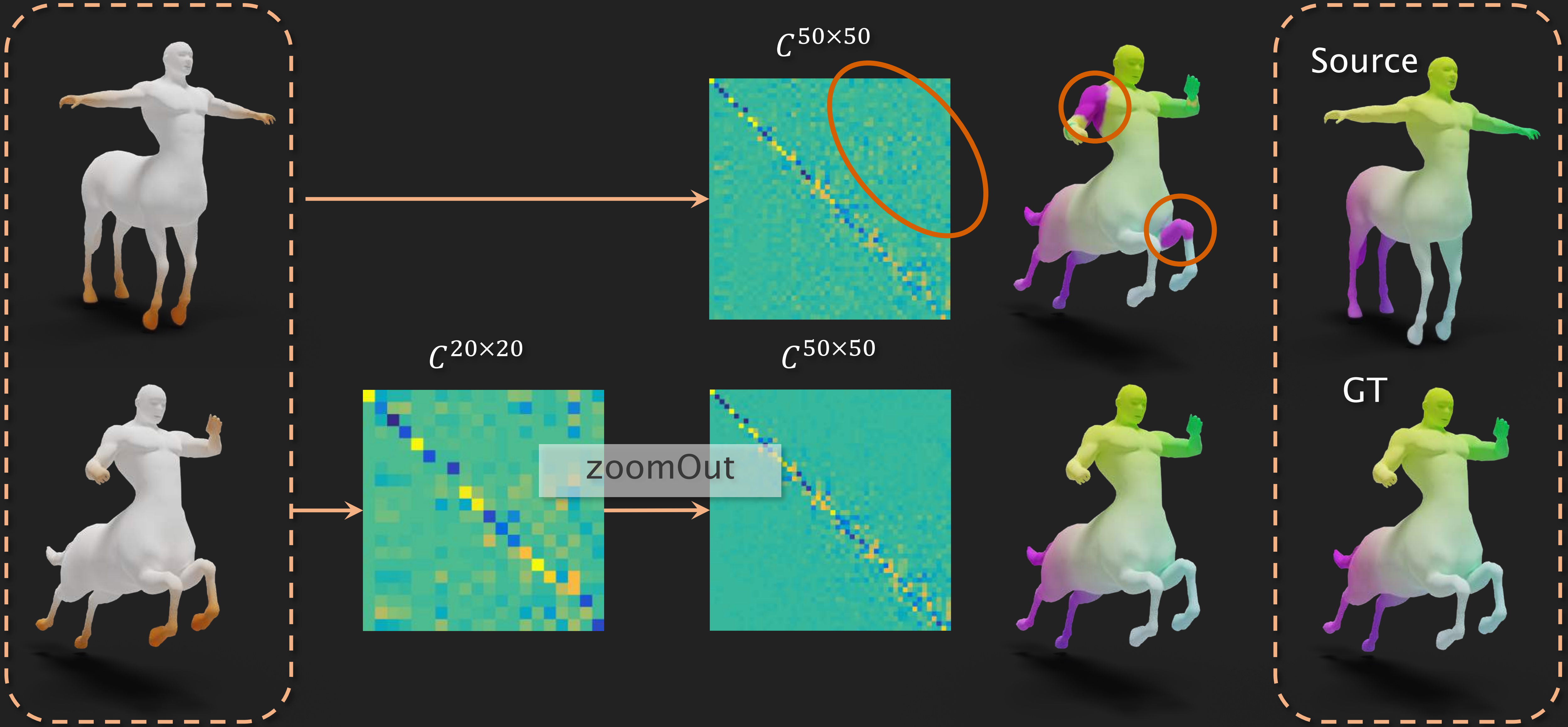
code

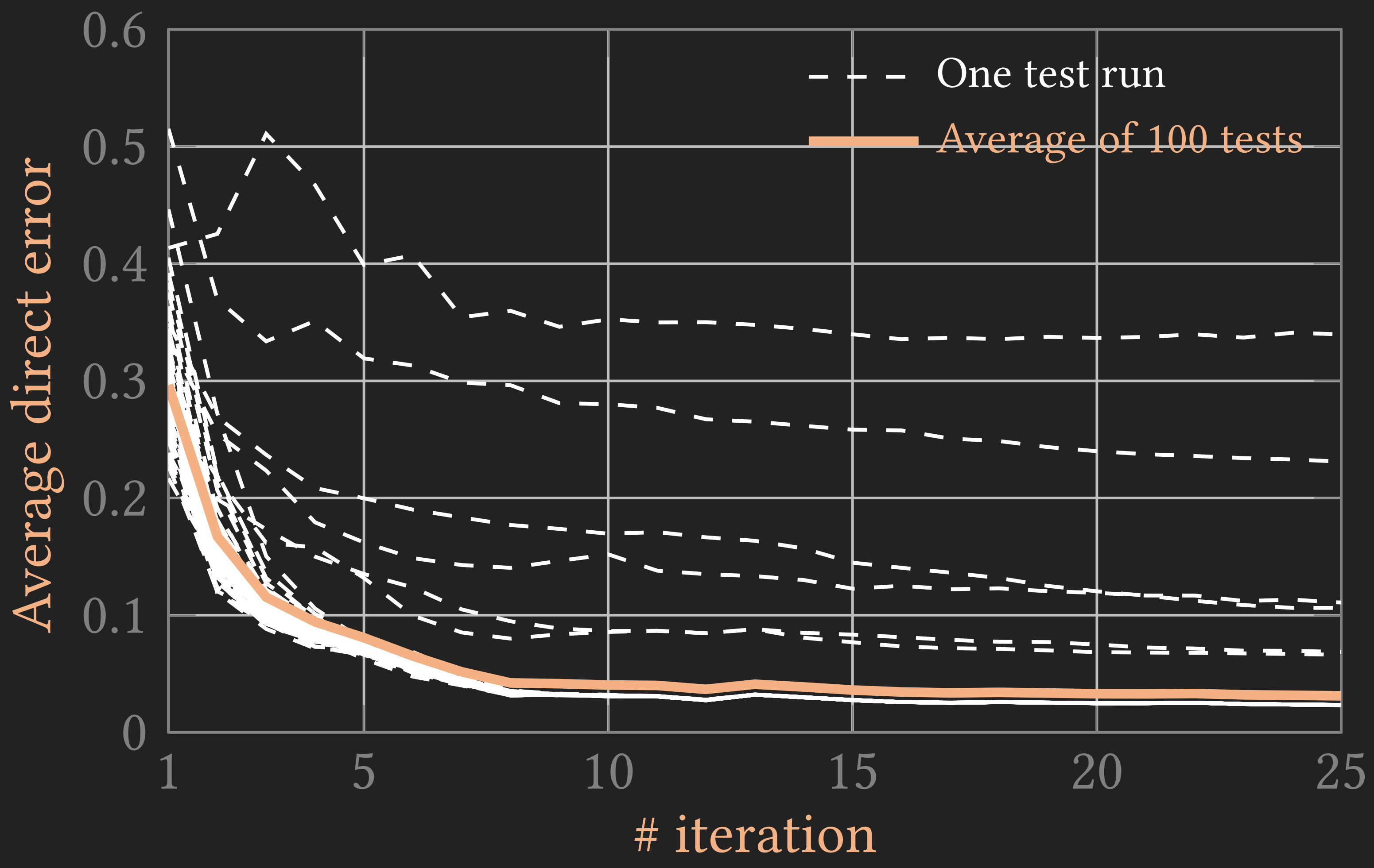
https://github.com/llorz/SGA19_zoomOut/

Additional materials

Property: robust to noise



Property: robust to noise



Adding white noise to initialization (4×4 functional map):
96 out of 100 converged to exactly the same solution

More Applications

Symmetry detection

Source

Ground-truth

BIM [Kim et al. 2011]

GroupRep [Wang and Huang 2017]

IntSymm [Nagar and Raman 2018]

OrientRev [Ren et al. 2018]

BCICP [Ren et al. 2018]

Ours

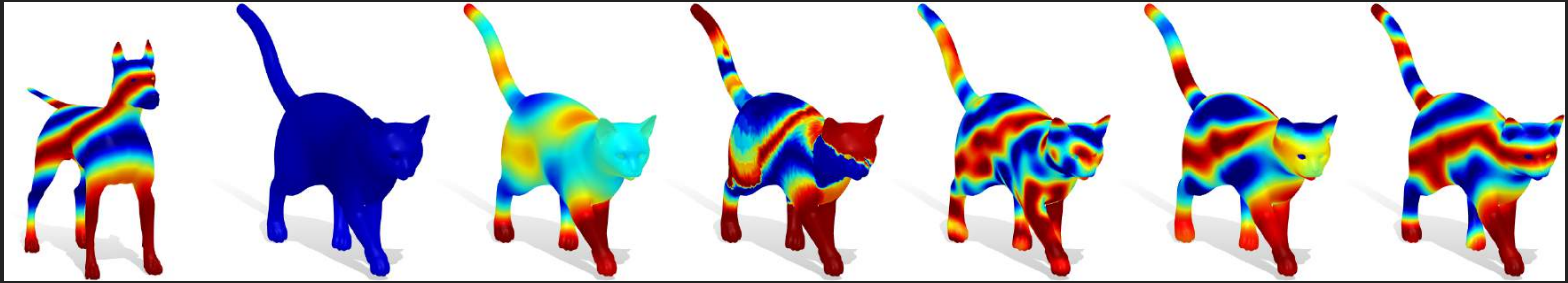
More Applications

Symmetry detection

Measurement		Average Error ($\times 10^{-3}$)		Average Runtime (s)	
Method \ Dataset		FAUST	SCAPE	FAUST	SCAPE
BIM [Kim et al. 2011]		65.4	133	34.6	41.7
GroupRep [Wang and Huang 2017]		224	347	8.48	16.7
IntSymm [Nagar and Raman 2018]		33.9	60.3	1.35	1.81
OrientRev (Ini) [Ren et al. 2018]		68.0	110	0.59	1.07
Ini + BCICP [Ren et al. 2018]		29.2	49.7	195.1	525.6
Ini + Ours		**16.1**	**46.2**	22.6	62.7
Ini + Ours*		18.5	46.6	**1.78**	**3.66**
Improv. w.r.t state-of-the-art	Ini + Ours	**44.9%**	**7.0%**	8×	8×
	Ini + Ours*	36.6%	6.2%	**110×**	**140×**

More Applications

Function Transfer



Original f | Initialization | ICP | p2p† | ICP_{300} | Prod† | Ours

Challenging case

Partial Shape Matching

Failure case