[illegible]

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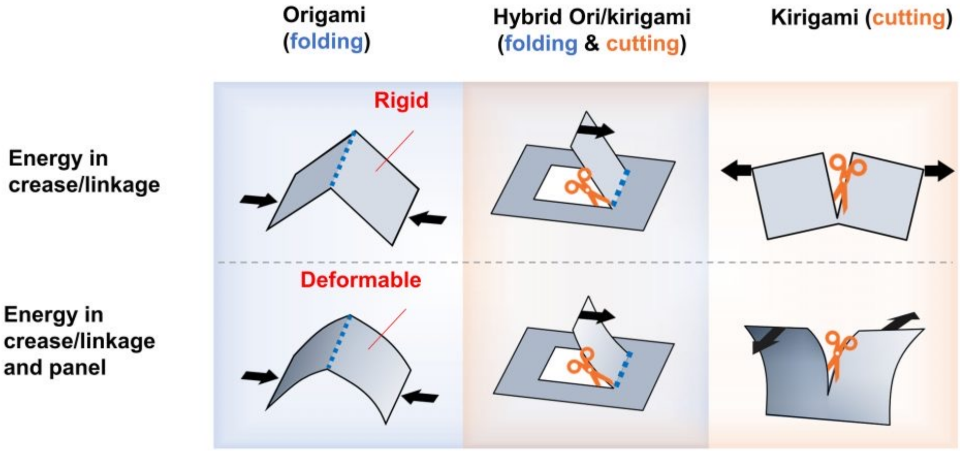
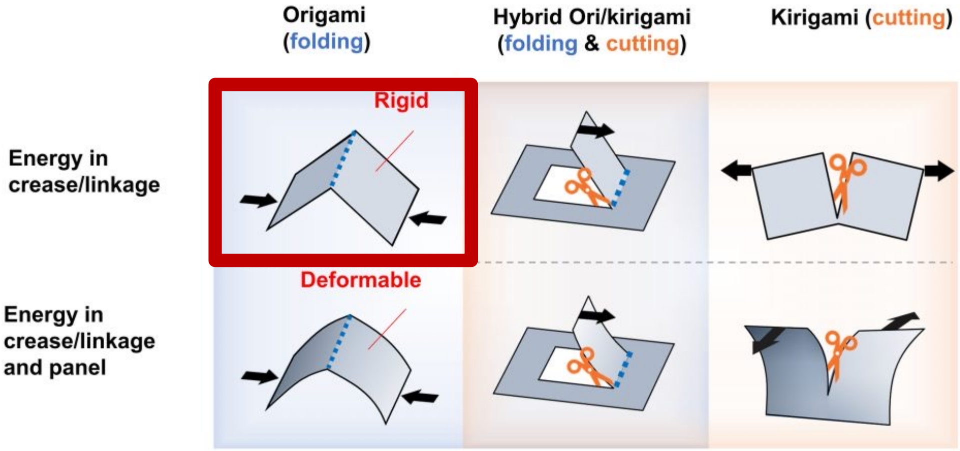
Kirigami



“Mechanical metamaterials based on origami and kirigami”, Zhai et al., 2021

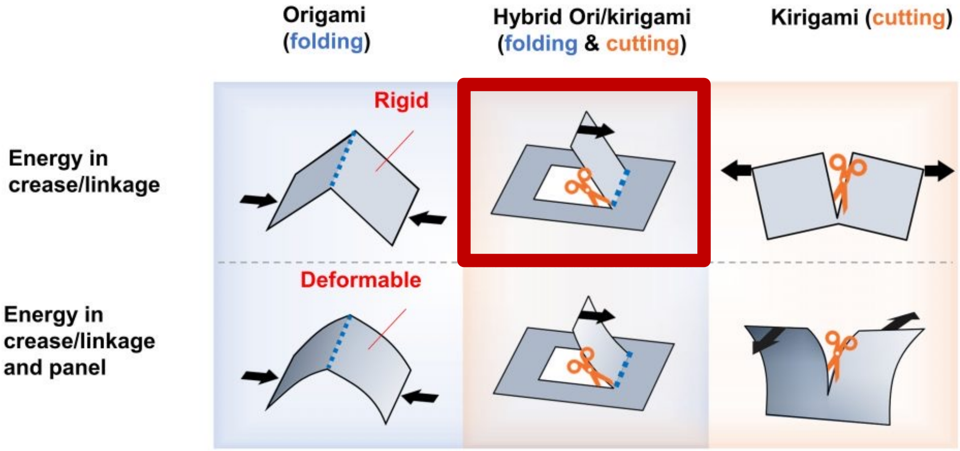
Kirigami



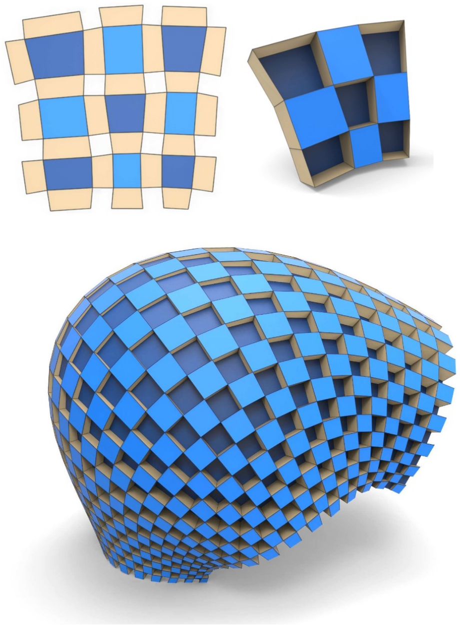
“Mechanical metamaterials based on origami and kirigami”, Zhai et al., 2021

“Inkjet 4D print: self-folding tessellated origami objects by inkjet uv printing”,
Narumi et al. *SIGGRAPH 2023*

Kirigami

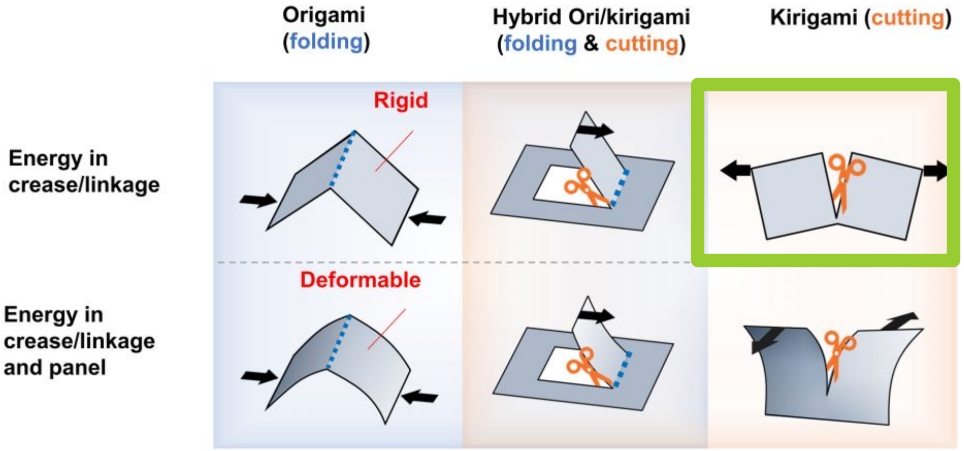


“Mechanical metamaterials based on origami and kirigami”, Zhai et al., 2021

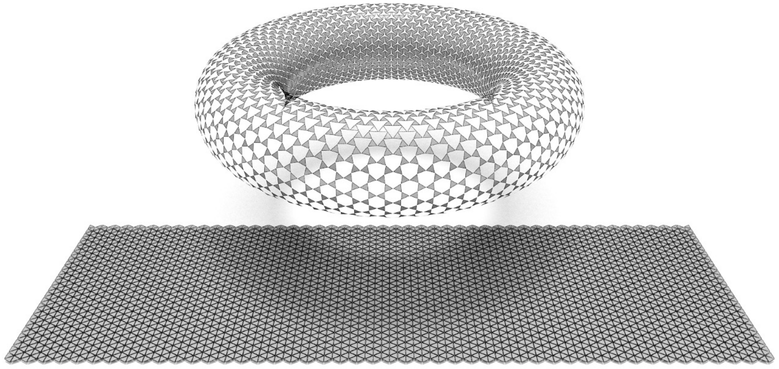


“Freeform quad-based kirigami”,
Jiang et al. *SIGGRAPH 2020*

Kirigami

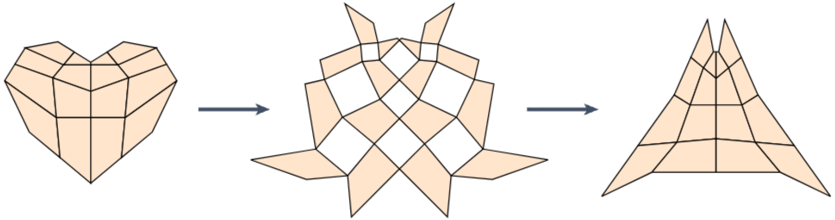


“Mechanical metamaterials based on origami and kirigami”, Zhai et al., 2021

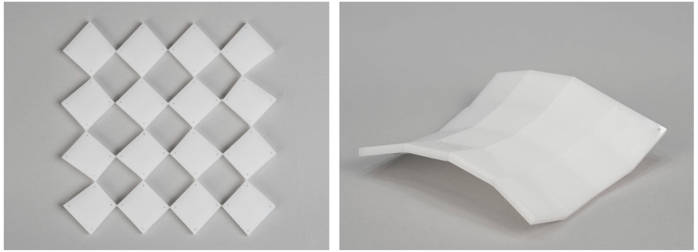


“Beyond developable: computational design and fabrication with auxetic materials”, Konaković-Luković et al. *SIGGRAPH 2016*

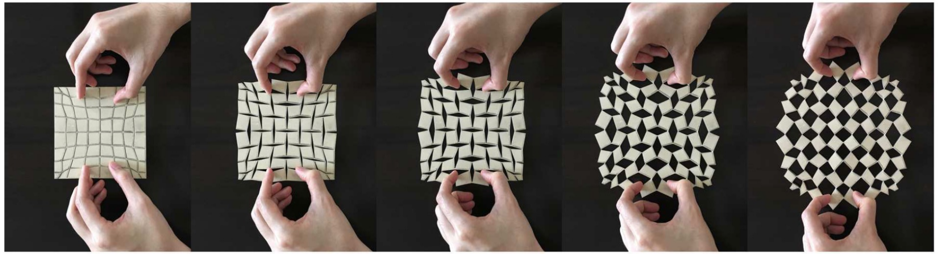
Hinged kirigami



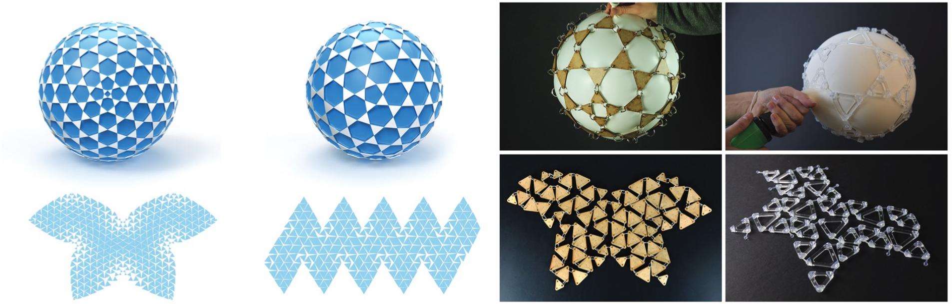
“**An additive framework for kirigami design**”, Dudte et al., 2023



“**Shape-morphing mechanical metamaterials**”, Jiang et al., 2022

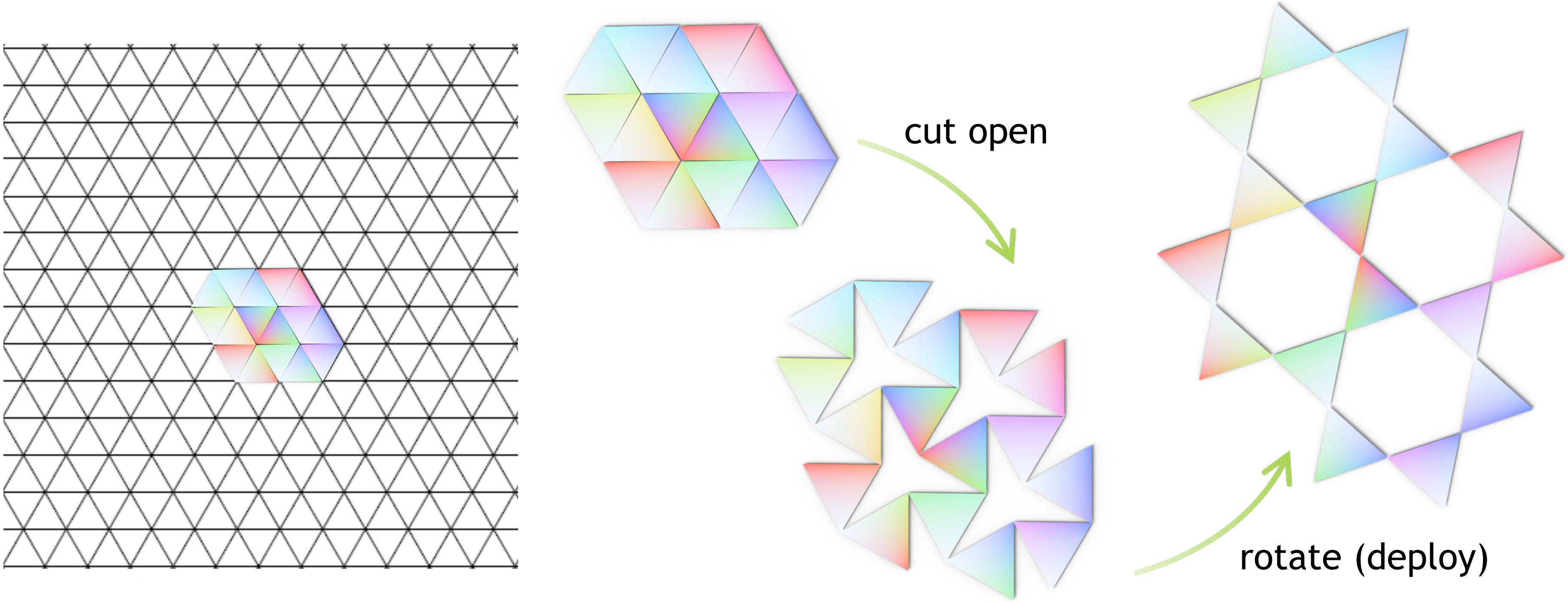


“**Programming shape using kirigami tessellations**”, Choi et al., 2019

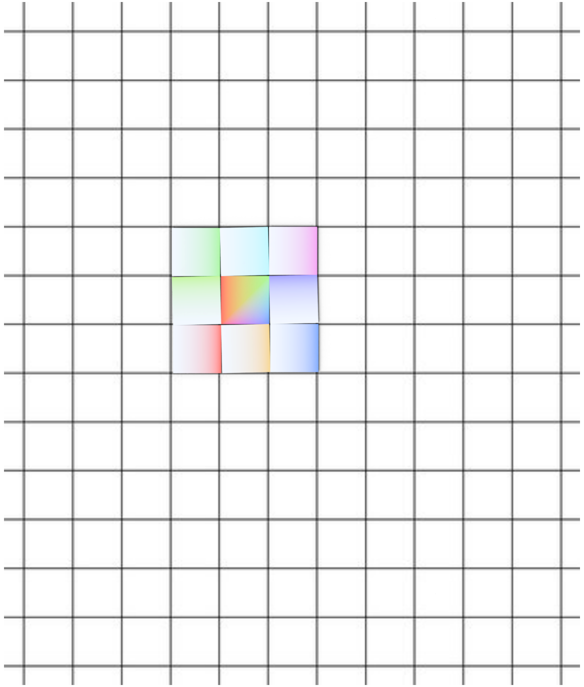


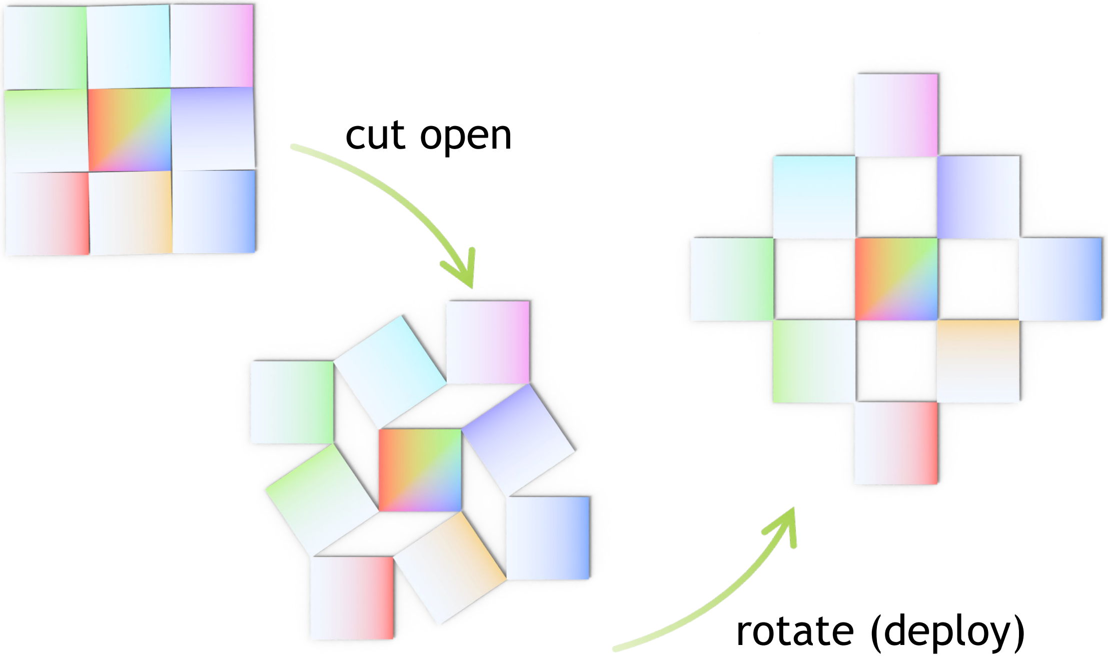
“**Rapid deployment of curved surfaces via programmable auxetics**”,
Konaković-Luković et al., 2018

Patterns from triangular tilings

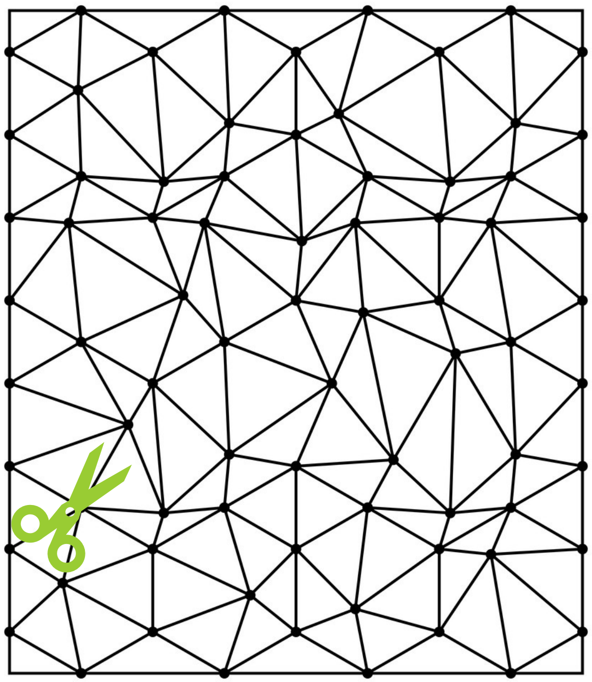


Patterns from quad tilings





Generalized kirigami patterns?

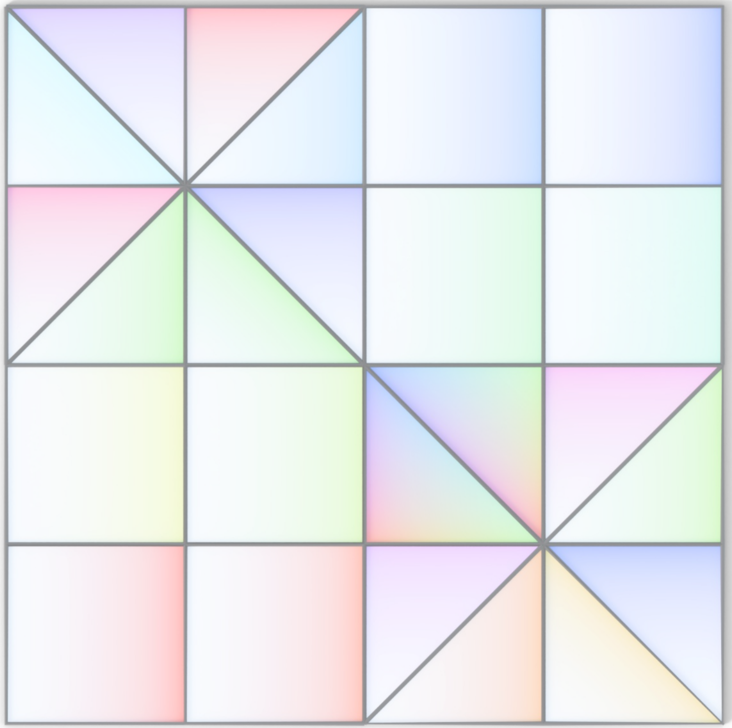


For an arbitrary planar tiling (2D manifold polygonal mesh with disk topology contains no holes)

- Can we cut it into a valid hinged kirigami structure?
- Is it deployable?
- How can we mathematically characterize its rotational deployment process?

Generalized kirigami patterns?

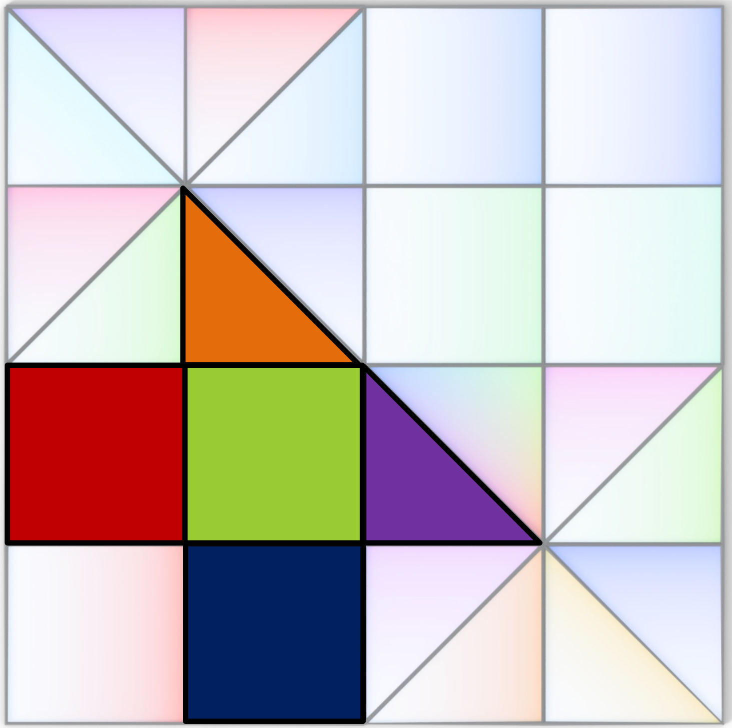
Cutting rules:



Generalized kirigami patterns?

Cutting rules:

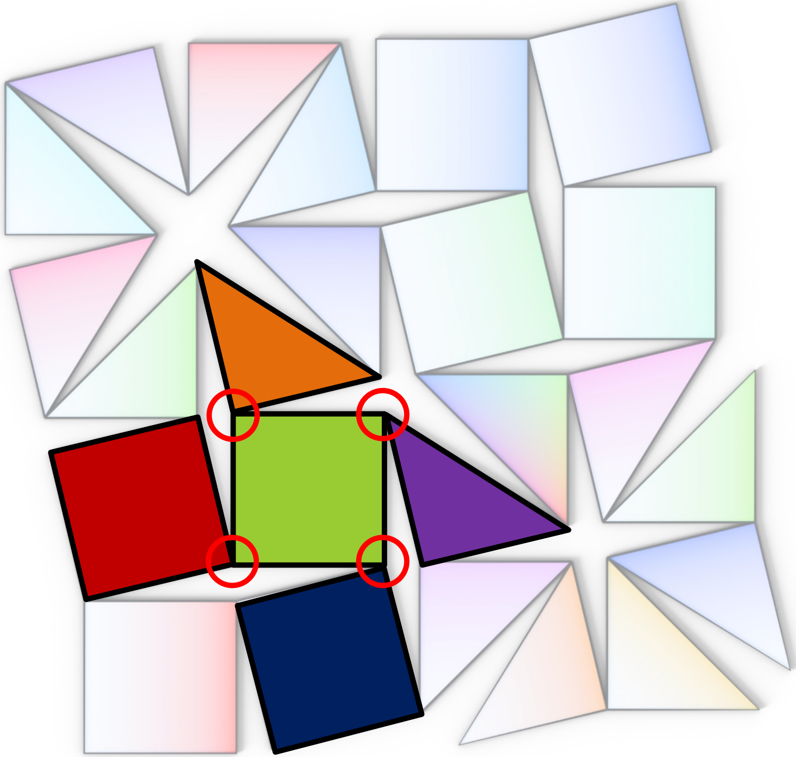
- Adjacent faces share a hinge



Generalized kirigami patterns?

Cutting rules:

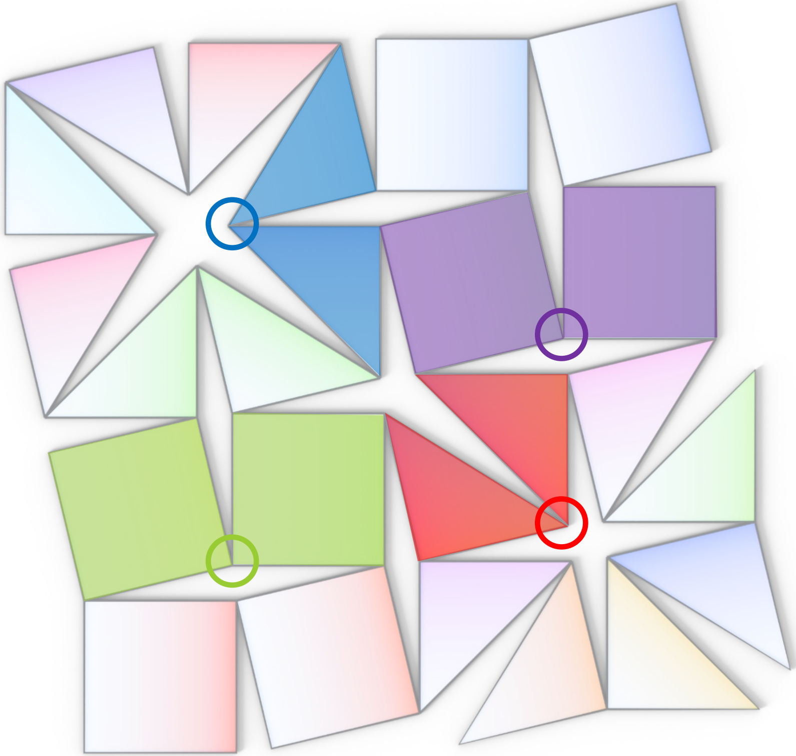
- Adjacent faces share a hinge



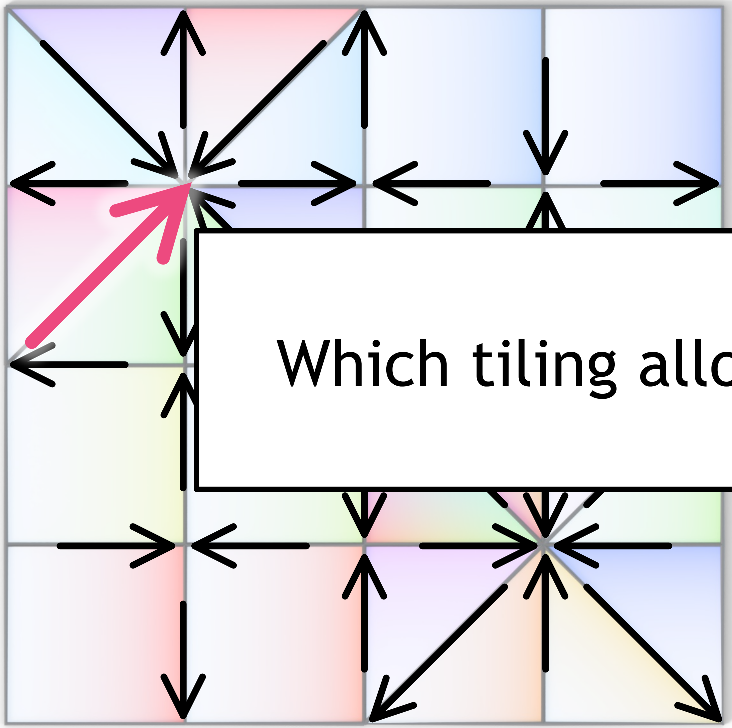
Generalized kirigami patterns?

Cutting rules:

- Adjacent faces share a hinge
- Each hinge vertex is shared by exactly two faces

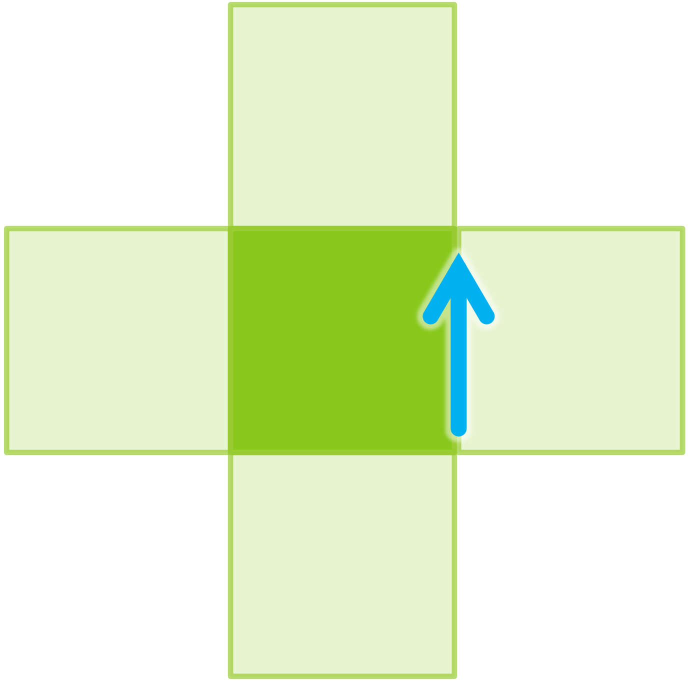


Definition : complete cut of a tiling

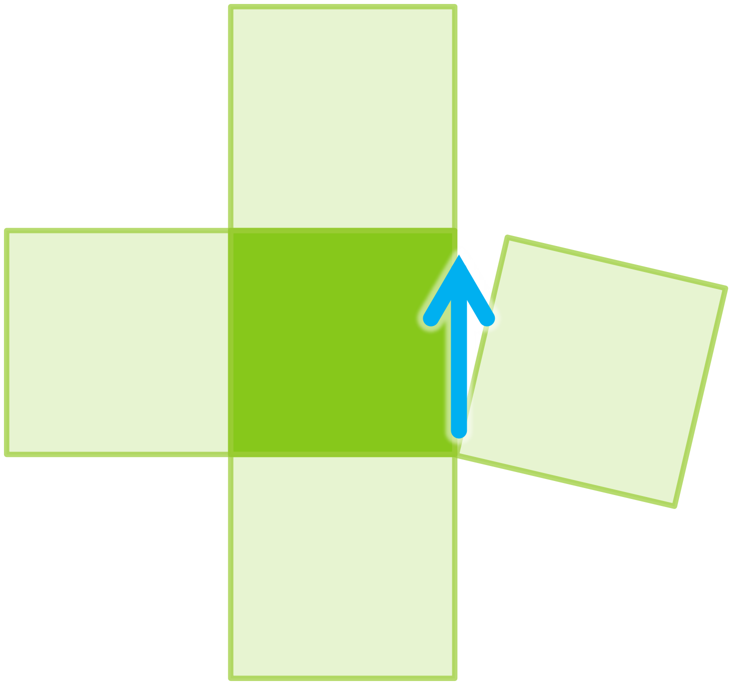


Which tiling allows for a valid complete cut?

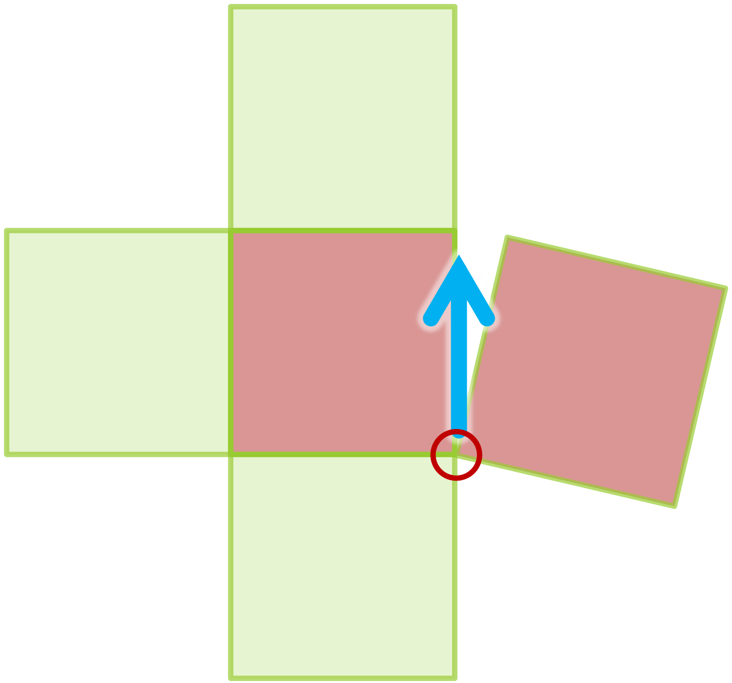
Combinatorial condition



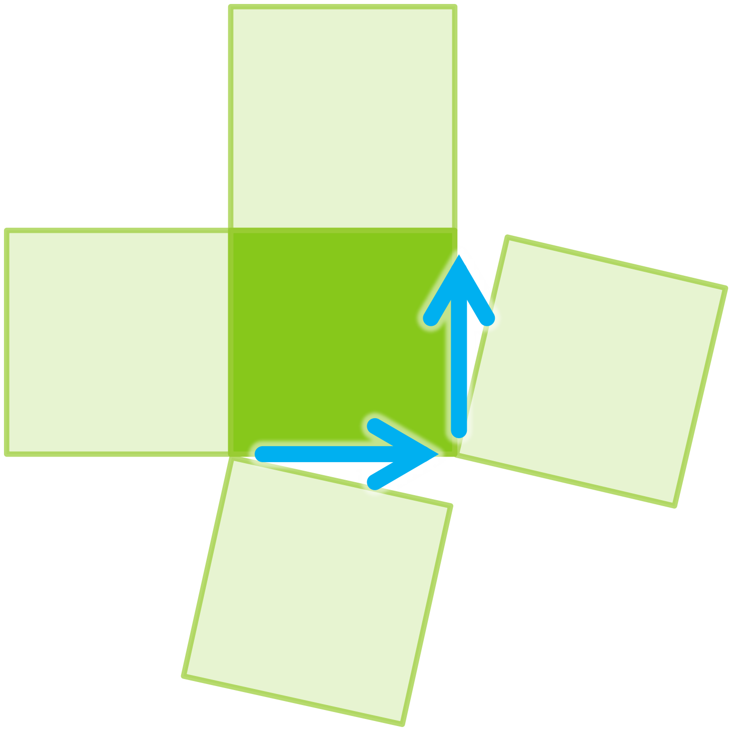
Combinatorial condition



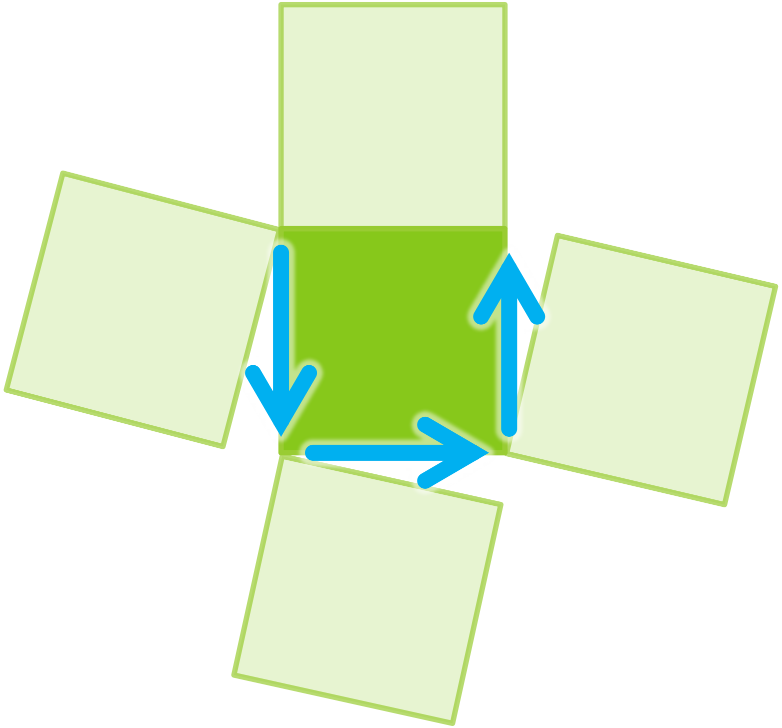
Combinatorial condition



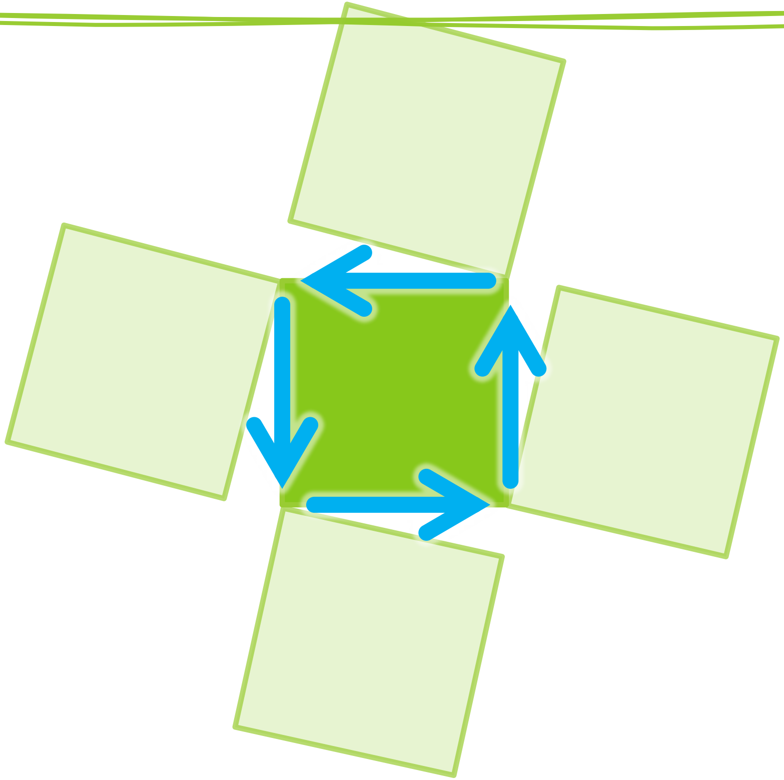
Combinatorial condition



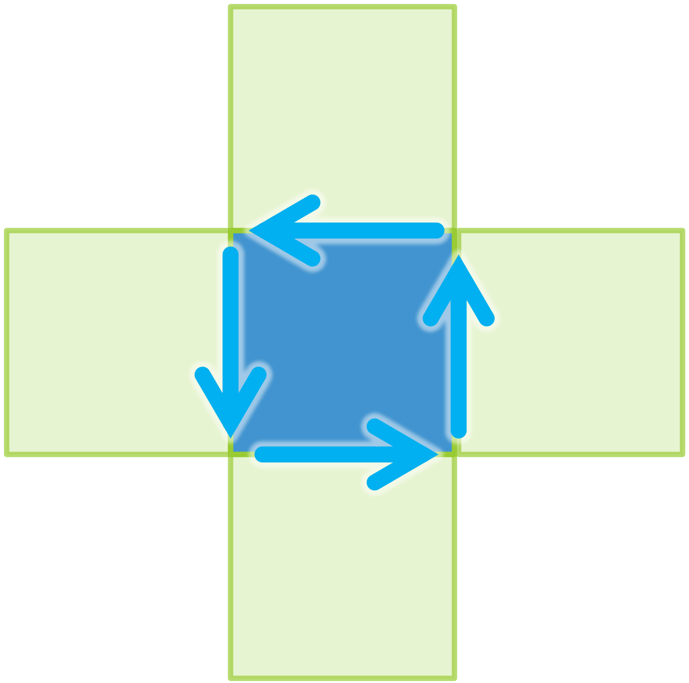
Combinatorial condition



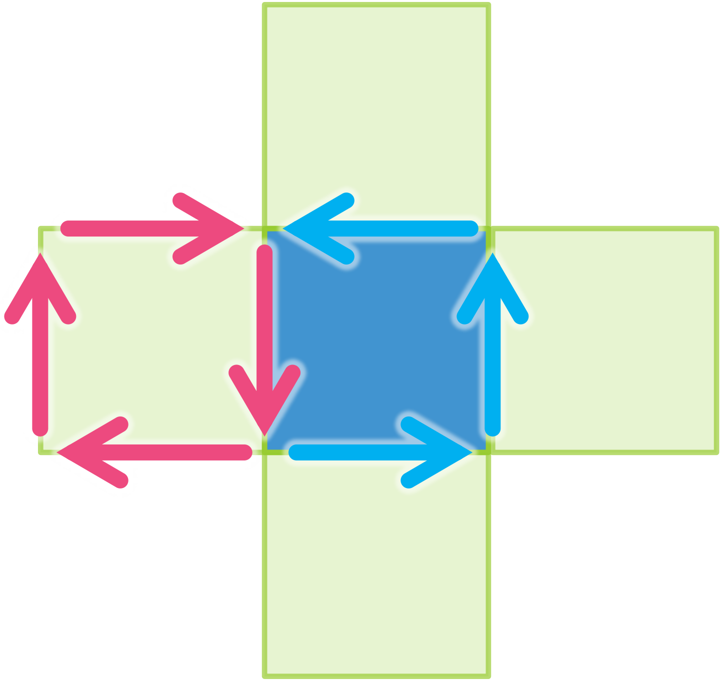
Combinatorial condition



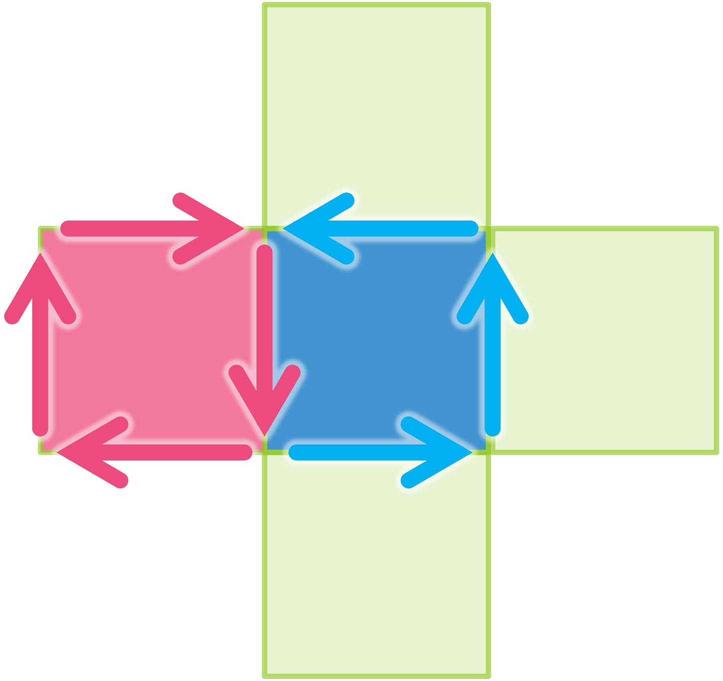
Combinatorial condition



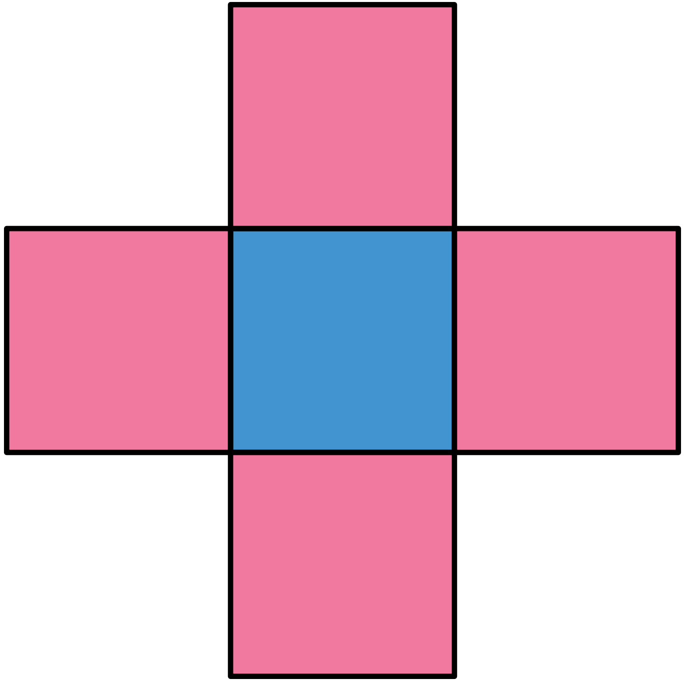
Combinatorial condition



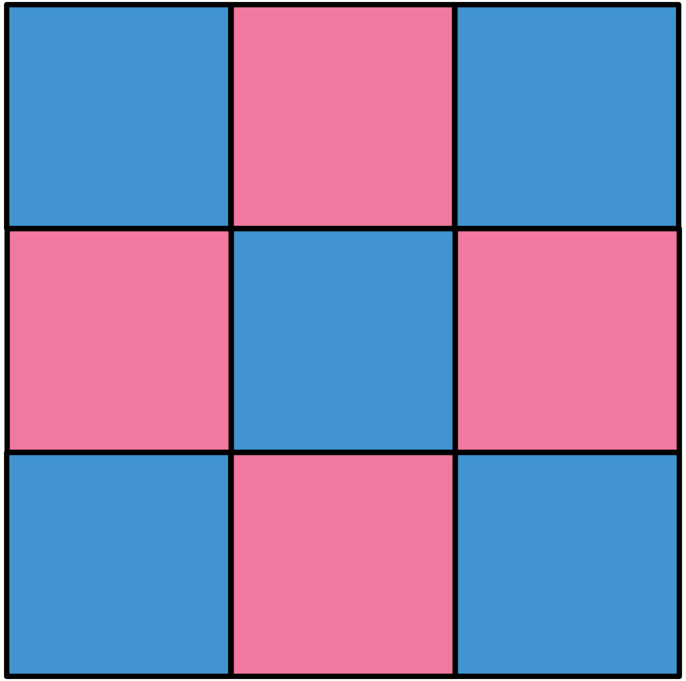
Combinatorial condition



Combinatorial condition



Combinatorial condition

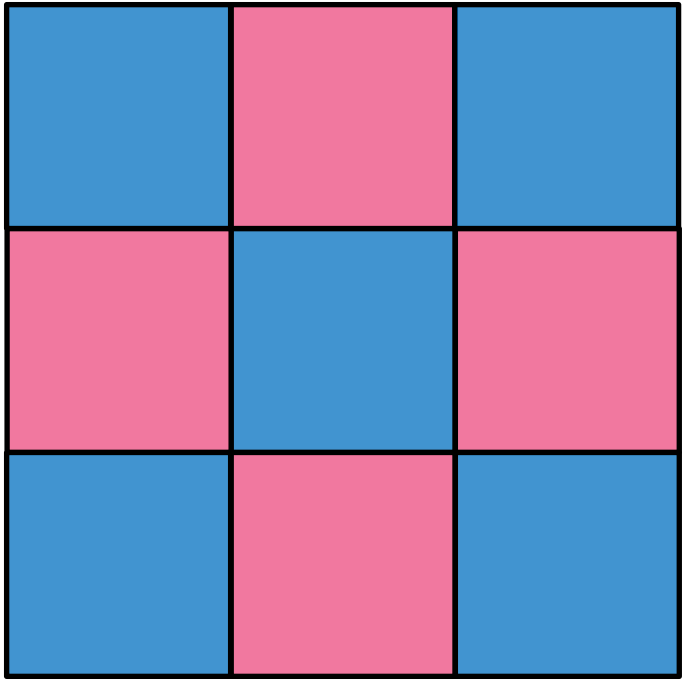


Combinatorial condition

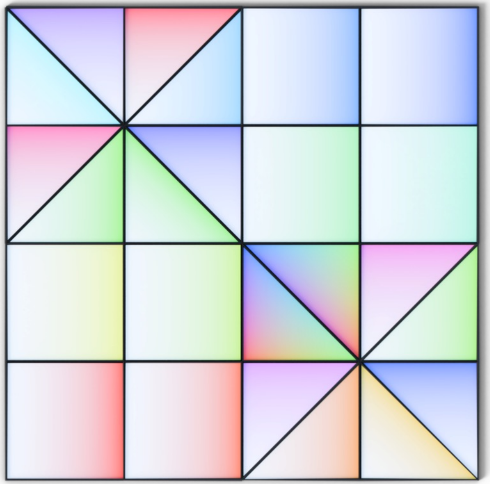
The cutting pattern can be applied if the tiling is
2-face colorable

⬍

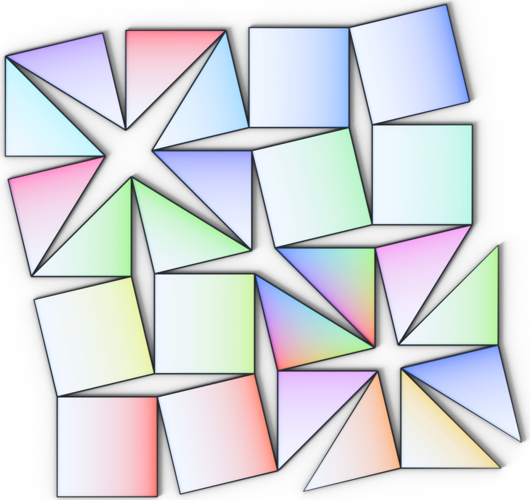
Each interior vertex has
even valency



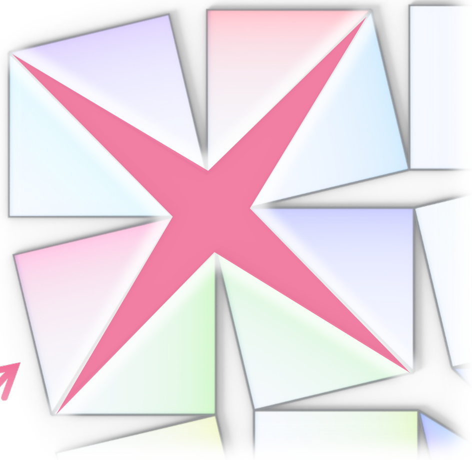
Geometric condition



apply the valid complete cut



Is the resulting hinged kirigami pattern deployable in 2D?

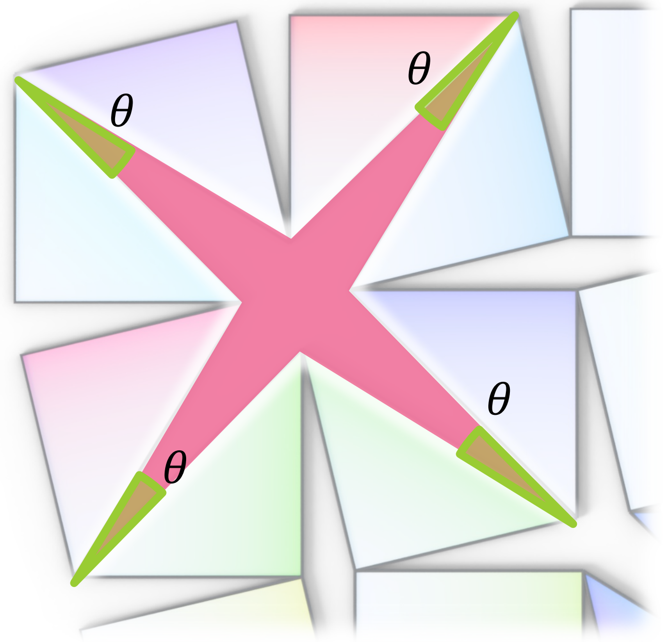


can this hole shape be embedded in 2D?

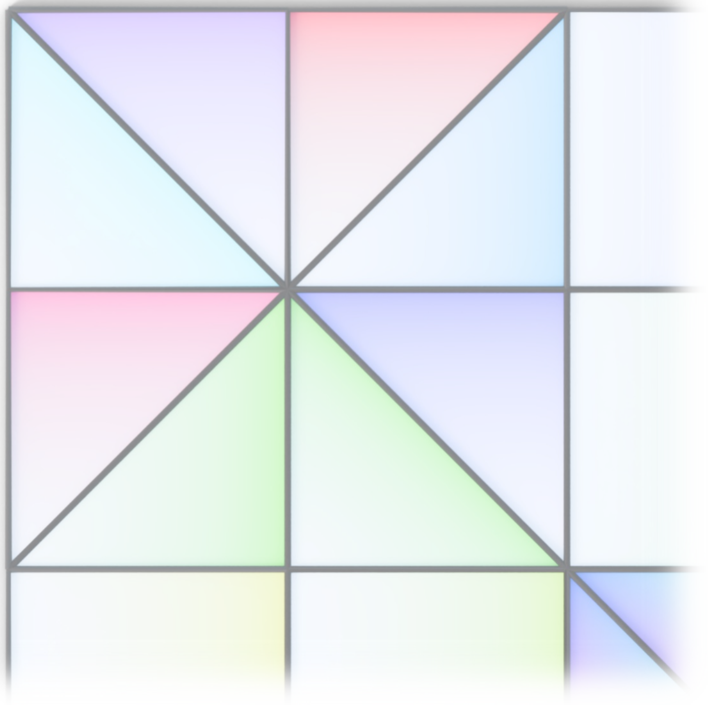
Geometric condition

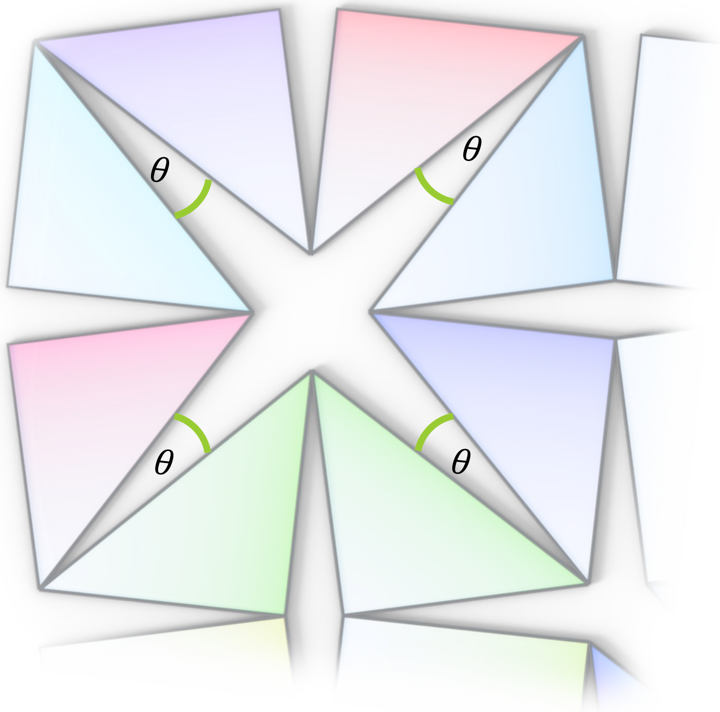
Is the resulting hinged kirigami pattern **uniformly deployable** in 2D?

Each hinge rotates at the same speed

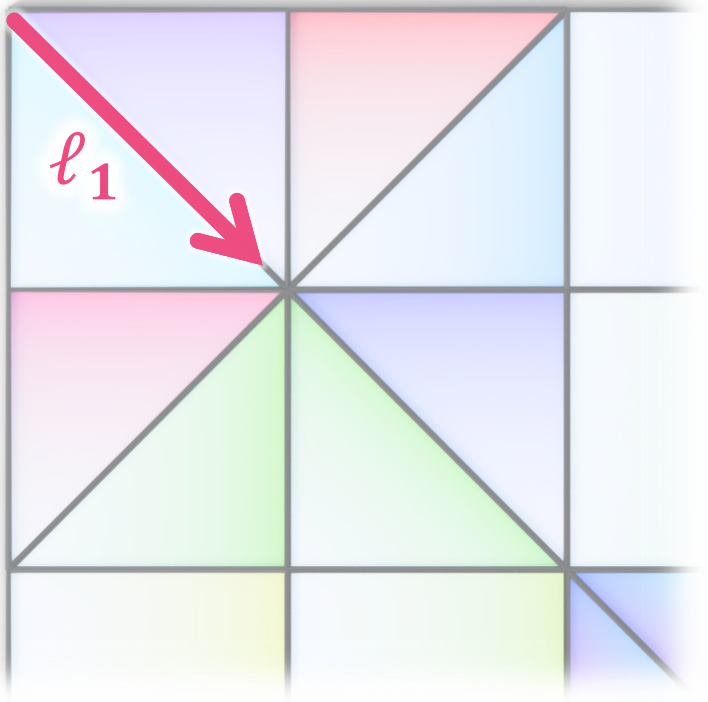


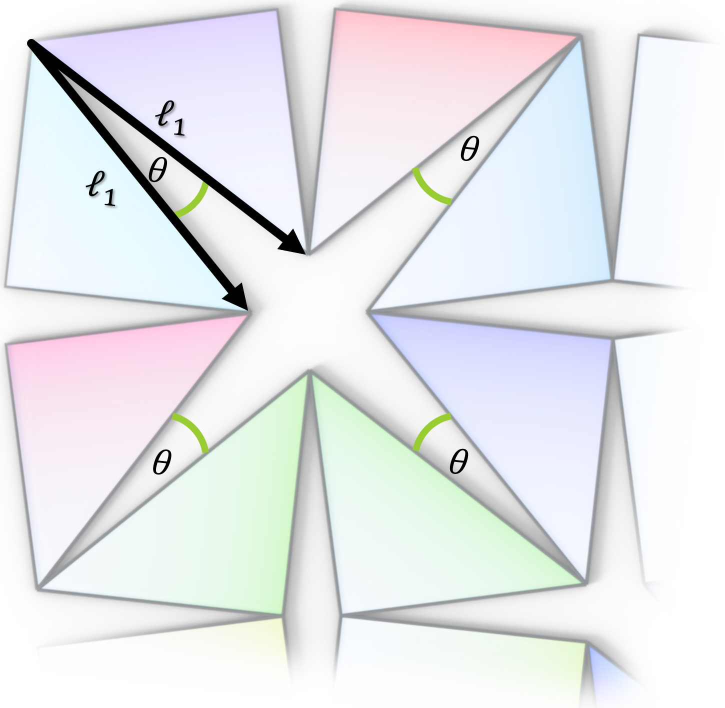
Geometric condition



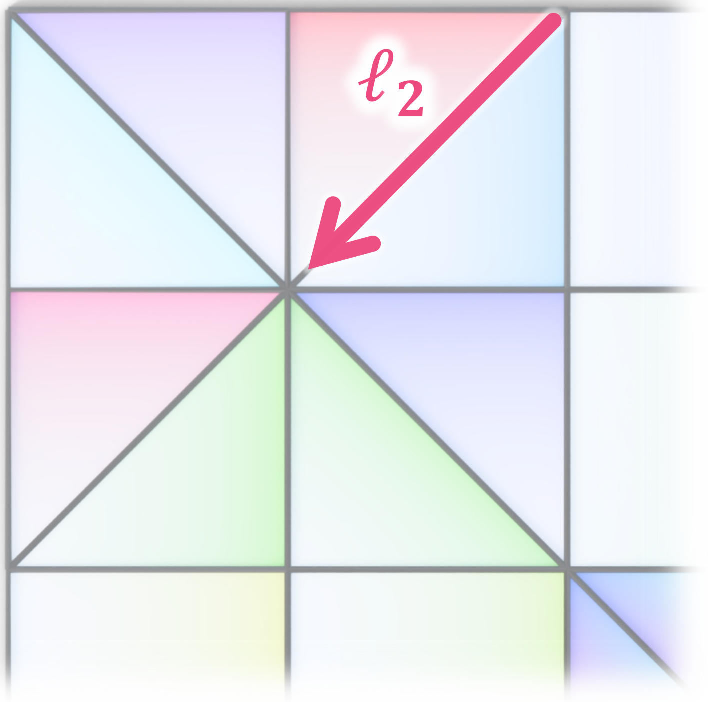


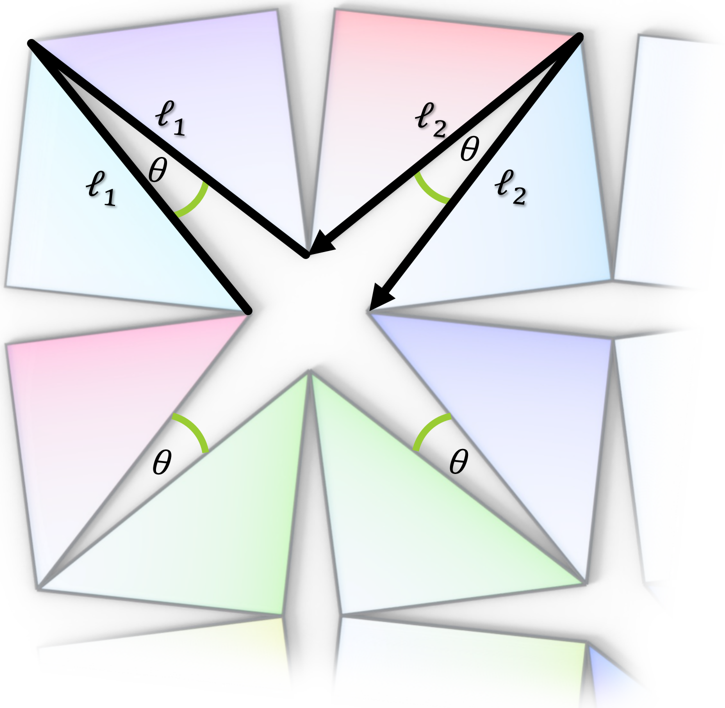
Geometric condition



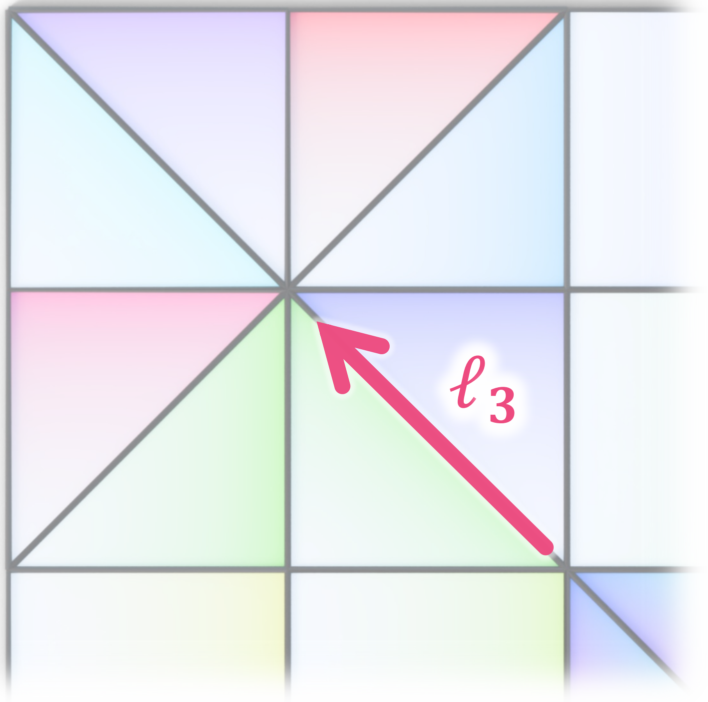


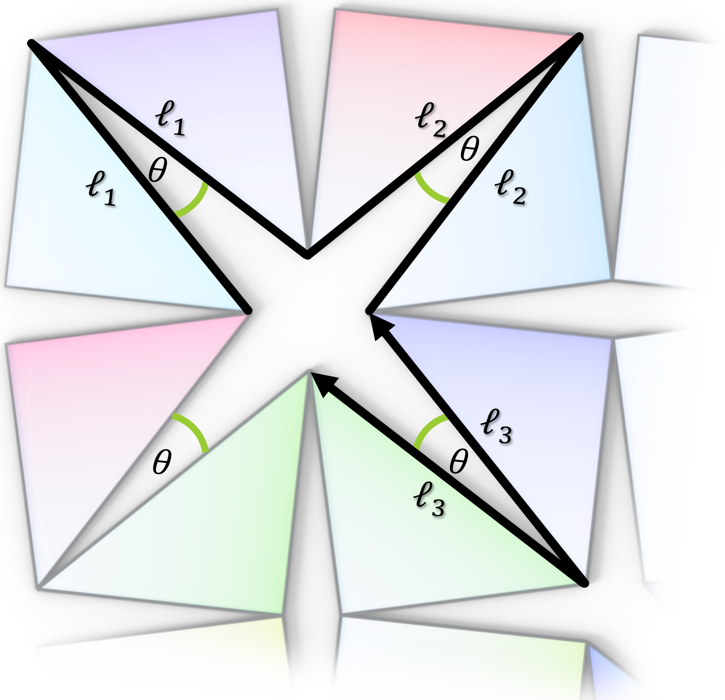
Geometric condition



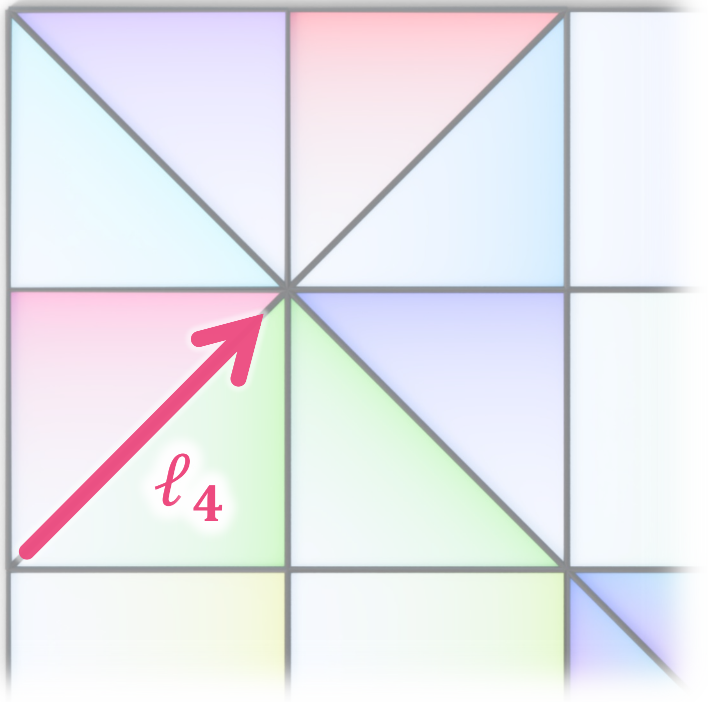


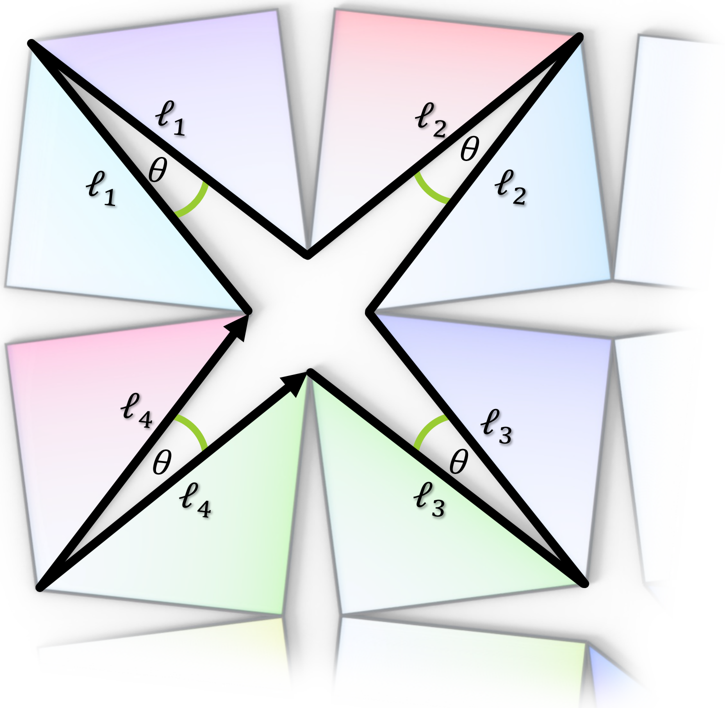
Geometric condition



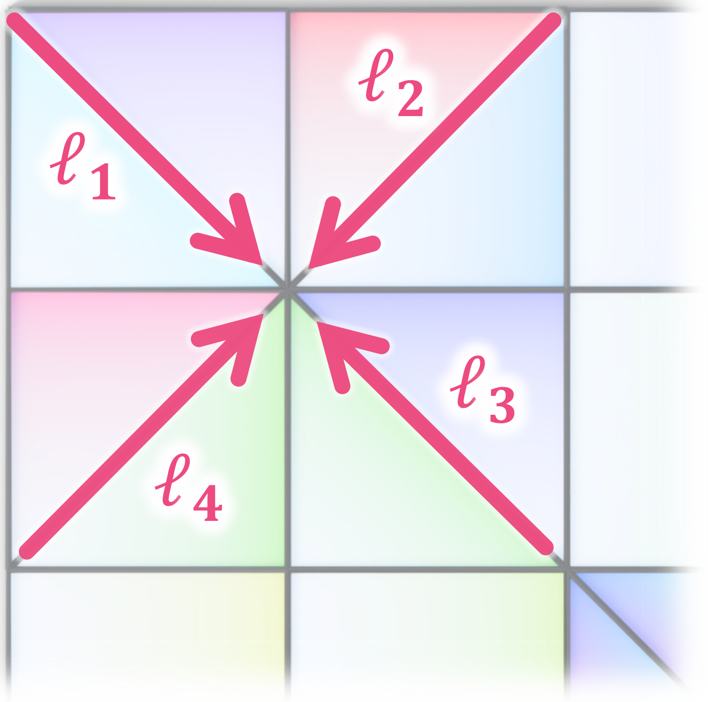


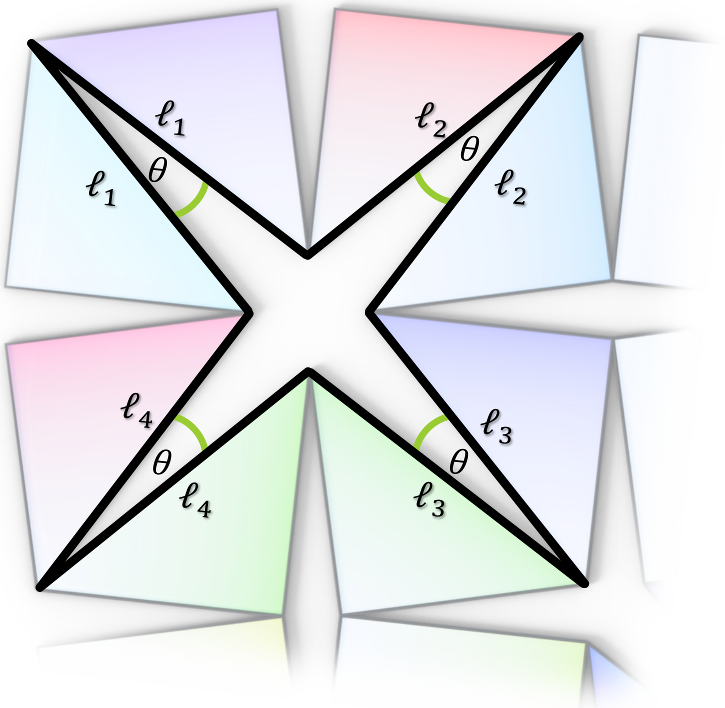
Geometric condition



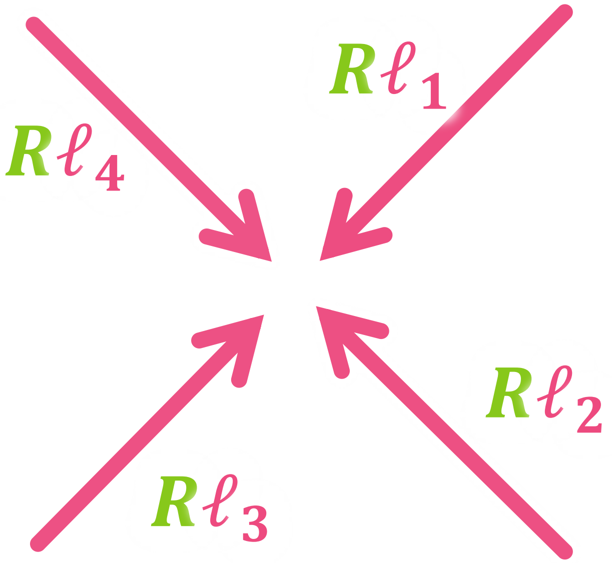


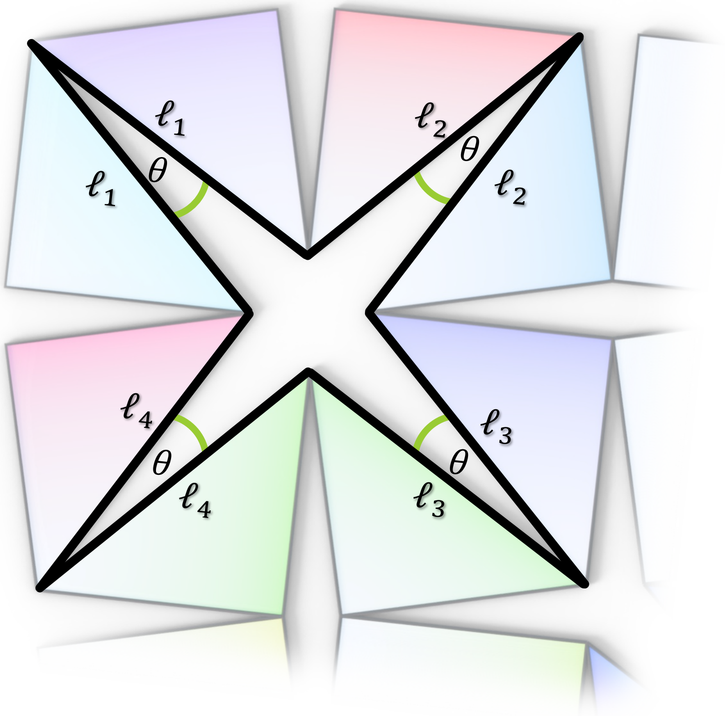
Geometric condition





Geometric condition





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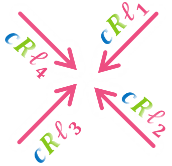
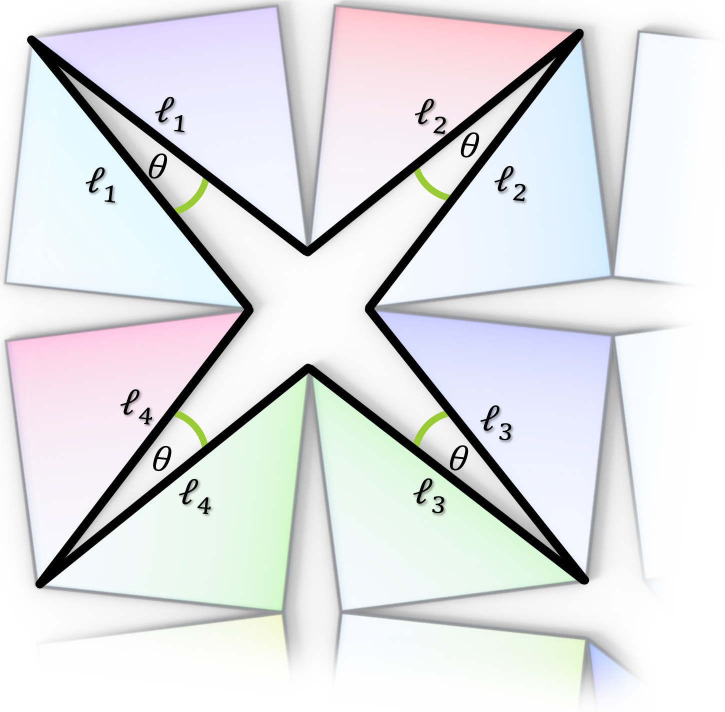




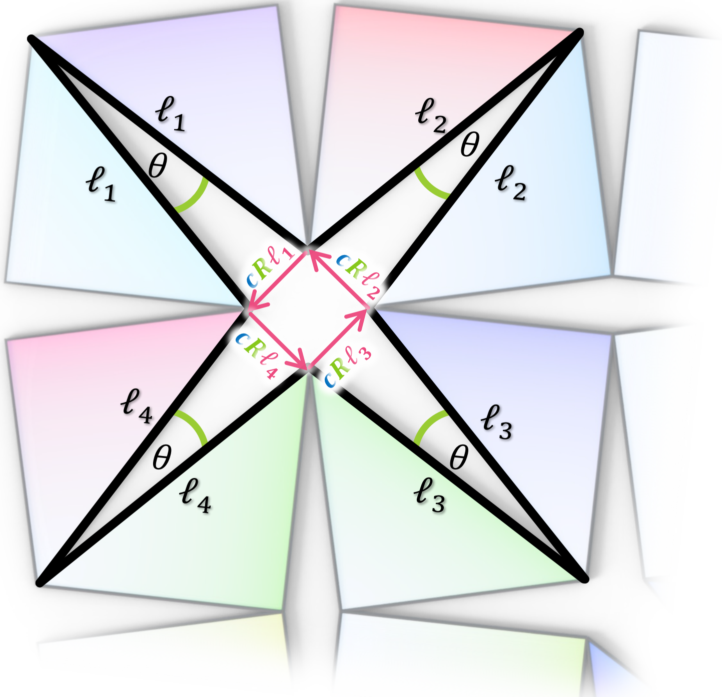
Geometric condition

Can be rigidly embedded if

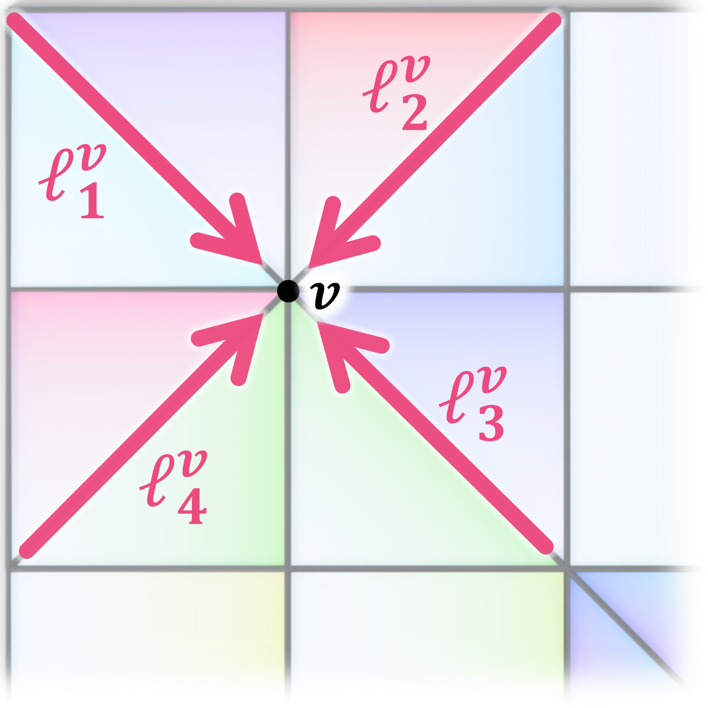
$$\sum_i cR\overrightarrow{\ell_i} = \vec{0}$$

$$\Updownarrow$$

$$\sum_i \overrightarrow{\ell_i} = \vec{0}$$



Geometric condition

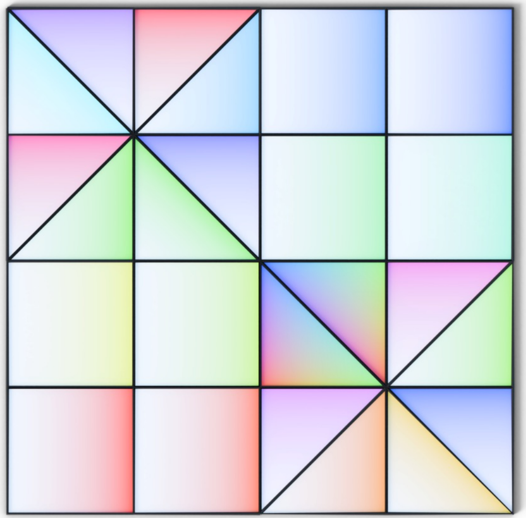


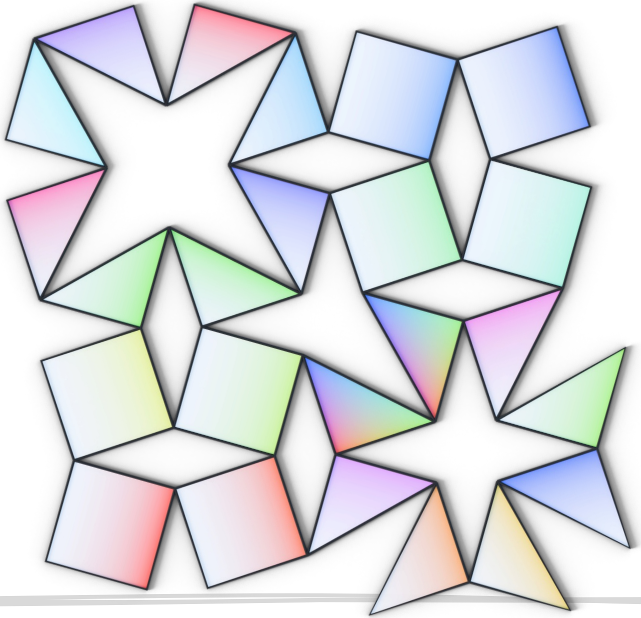
A vertex v is deployment-friendly if

$$\sum_i \overrightarrow{\ell_i^v} = \vec{0}$$

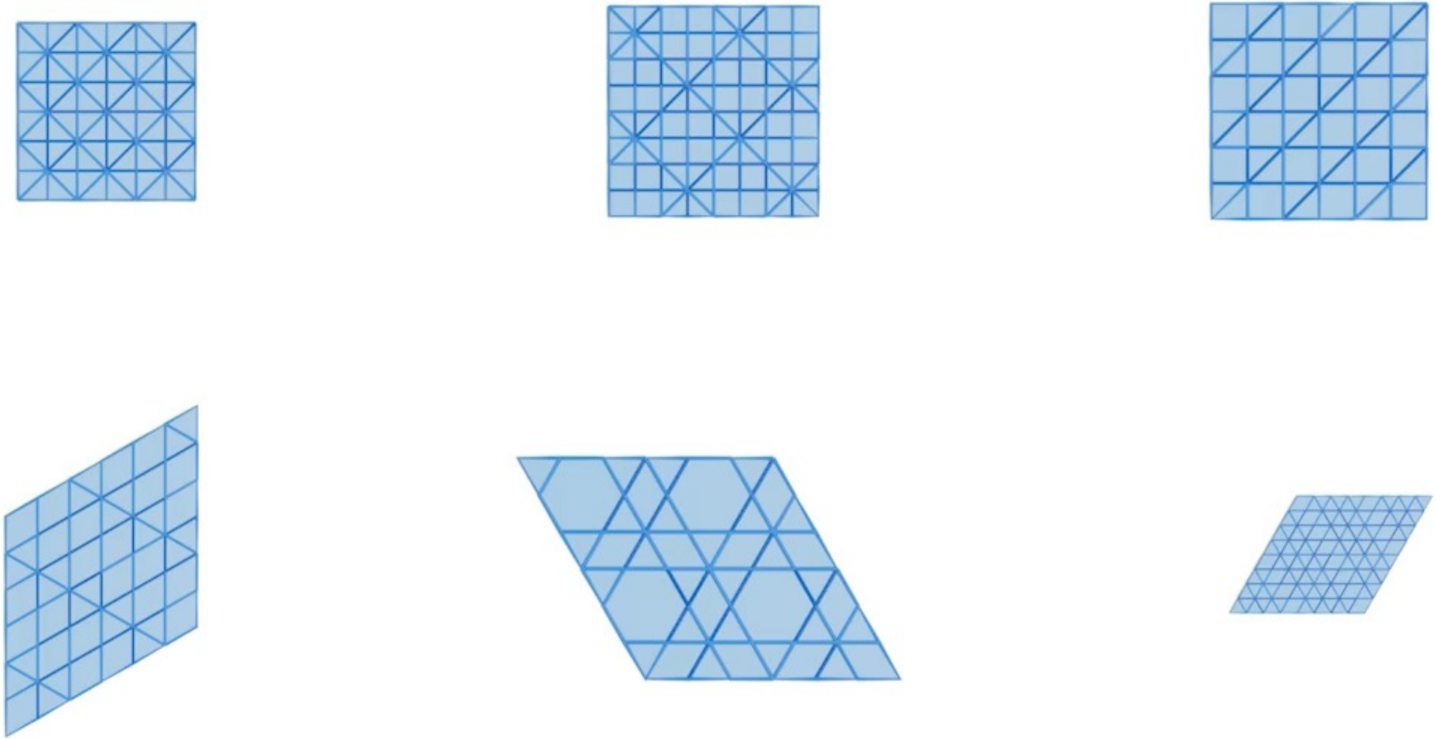
necessary & sufficient conditions

“every interior vertex is deployment-friendly” ⇔ “the resulting kirigami is uniformly deployable”



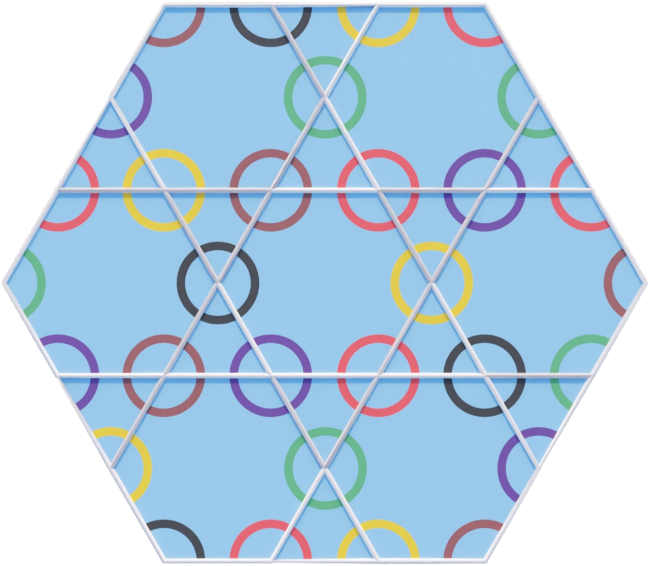


Deployable patterns

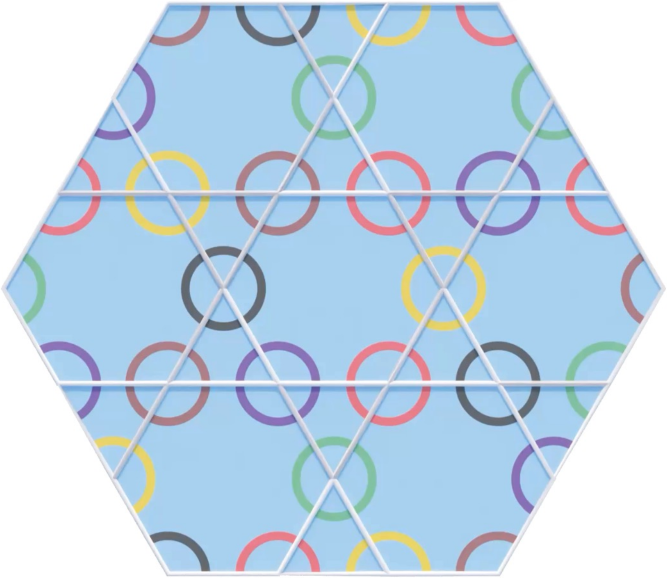


Inverse design

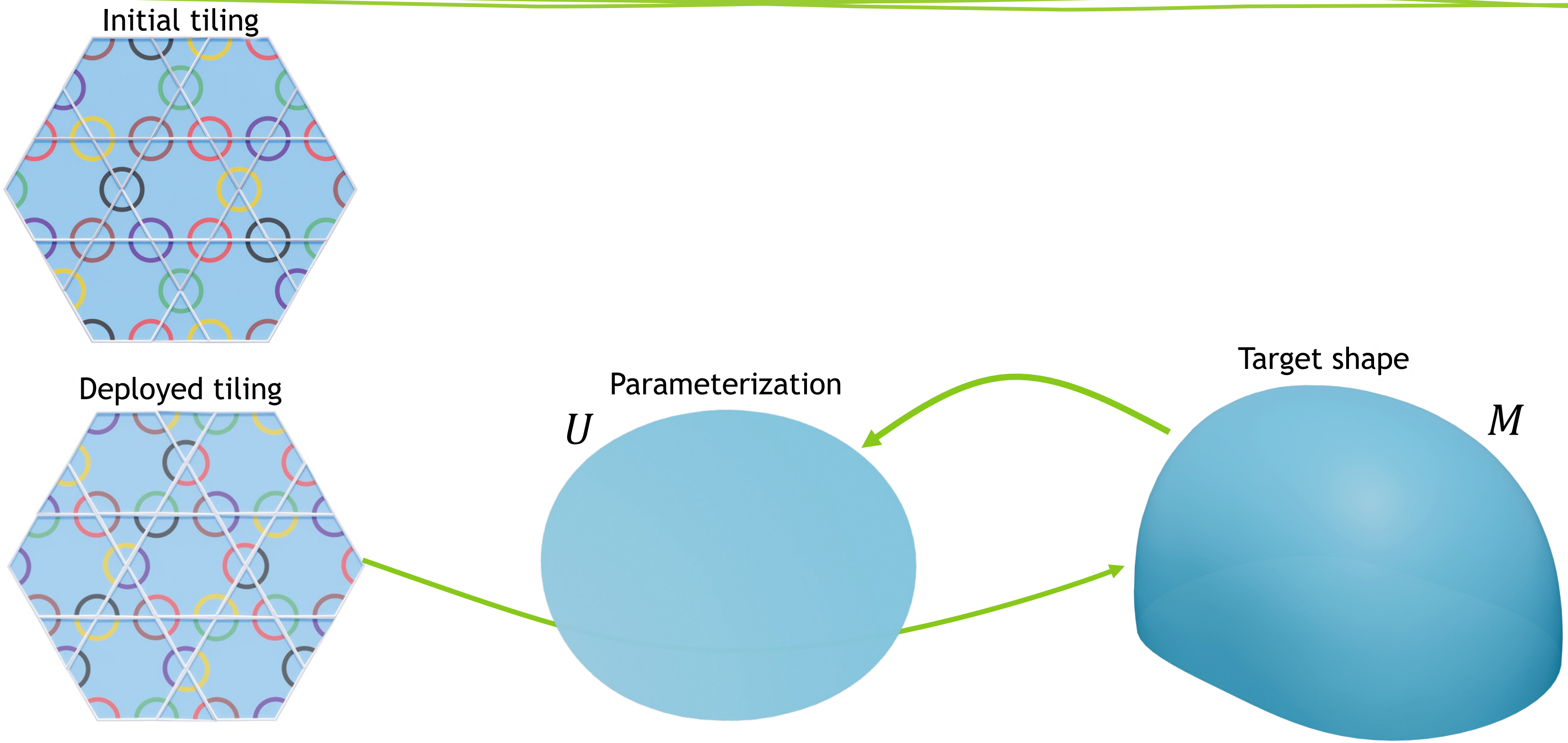
Initial tiling



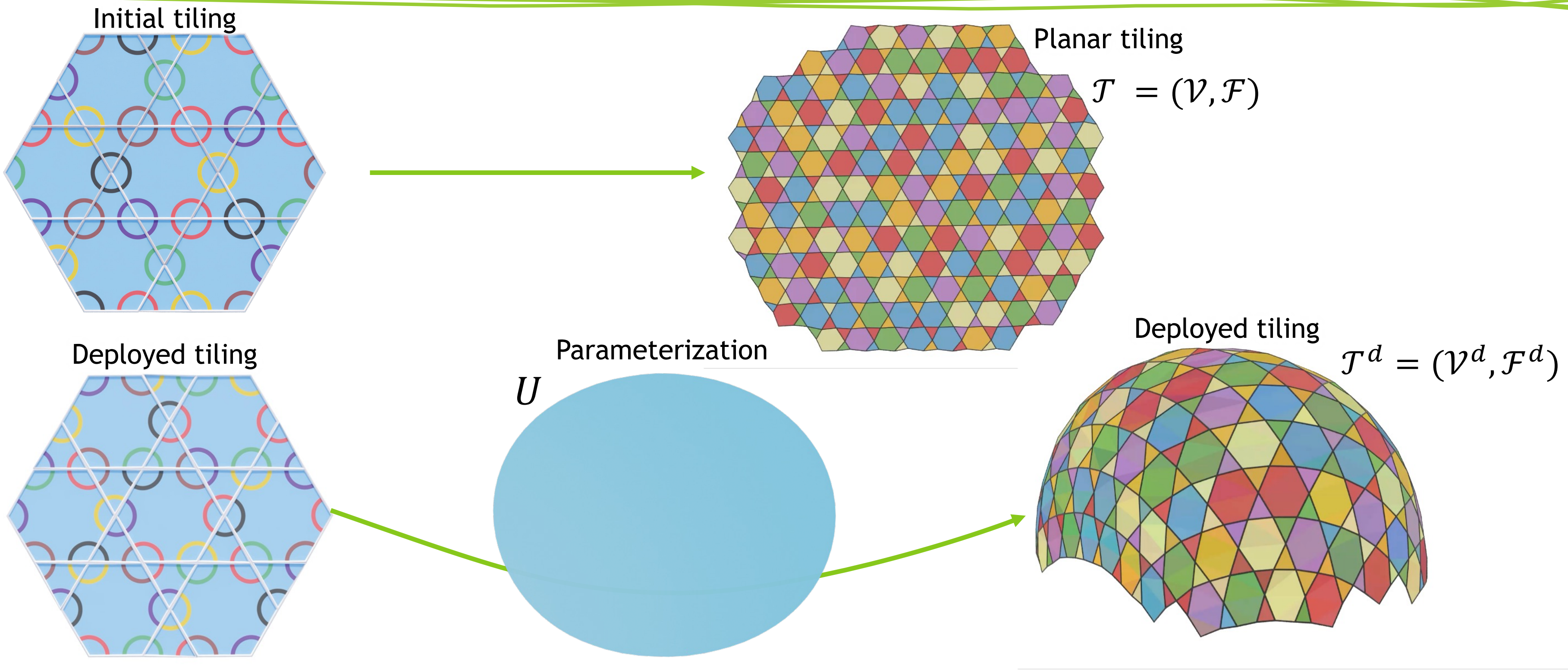
Deployed tiling



Inverse design

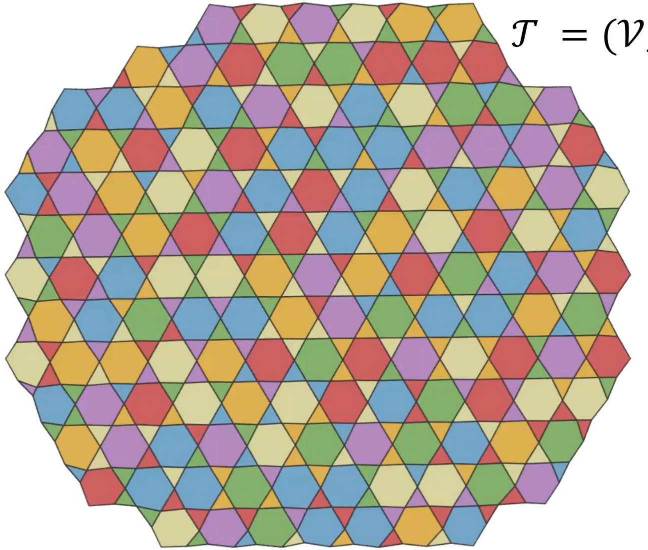


Inverse design



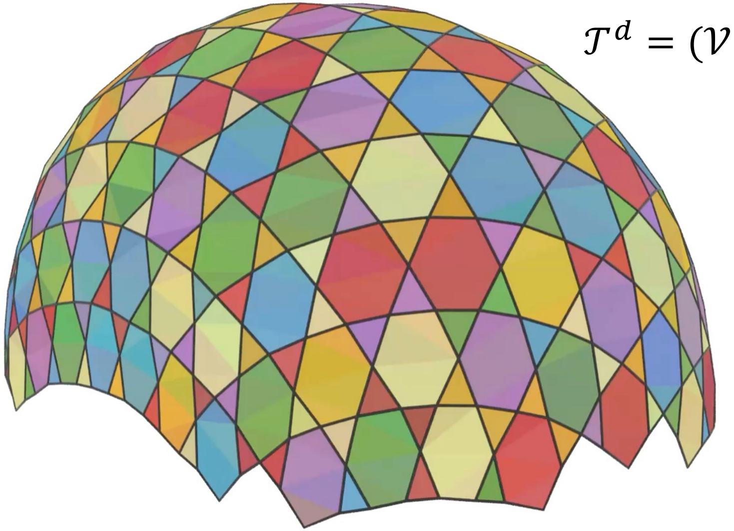
Inverse design

Planar tiling



$\mathcal{T} = (\mathcal{V}, \mathcal{F})$

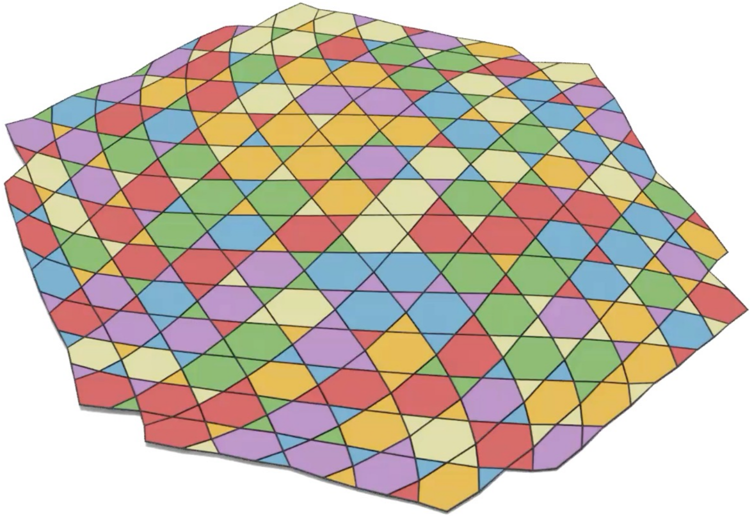
Deployed tiling



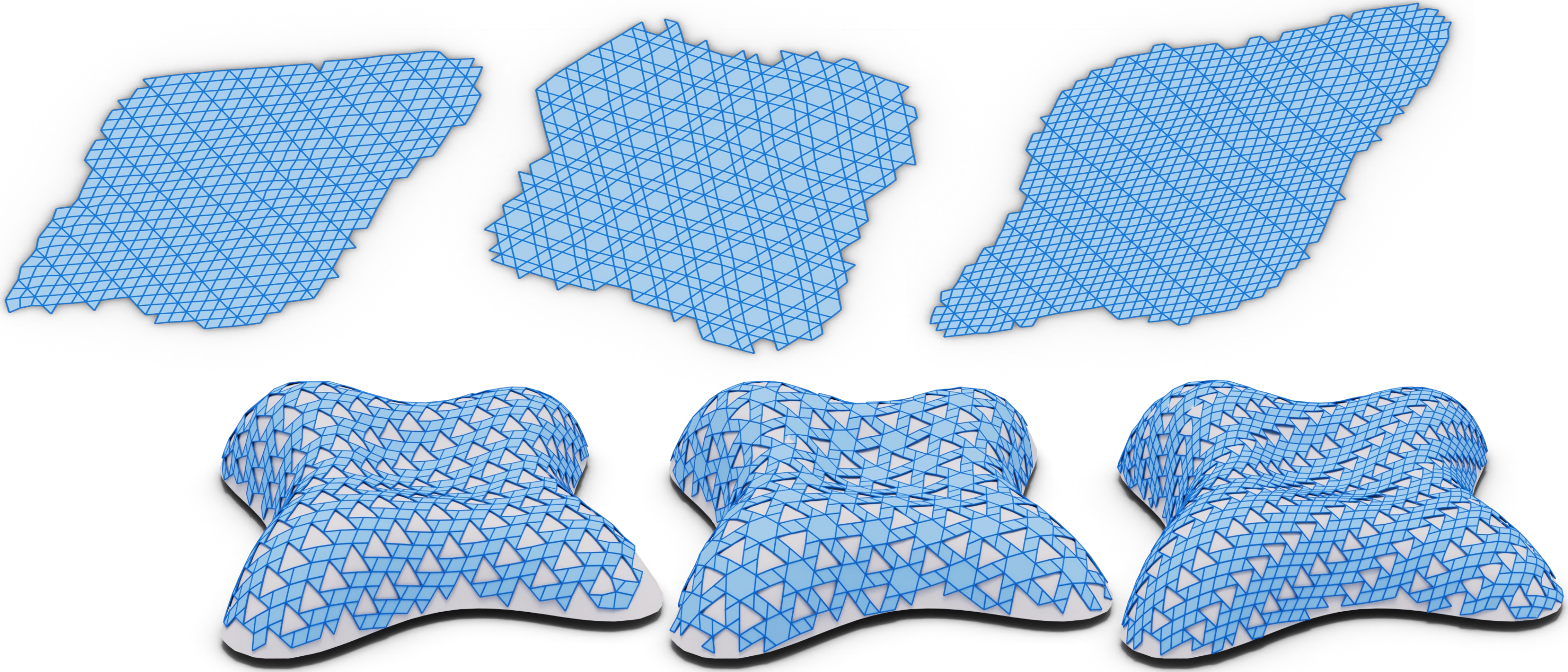
$\mathcal{T}^d = (\mathcal{V}^d, \mathcal{F}^d)$

$$\min_{V,V^d} \omega_1 \boxed{\mathbb{E}_{rigid}\left(V, V^d\right)} + \omega_2 \boxed{\mathbb{E}_{planar}\left(V^d\right)} + \omega_3 \boxed{\mathbb{E}_{shape}\left(V^d \middle| \mathcal{M}\right)} + \omega_4 \boxed{\mathbb{E}_{fairness}\left(V^d\right)}$$

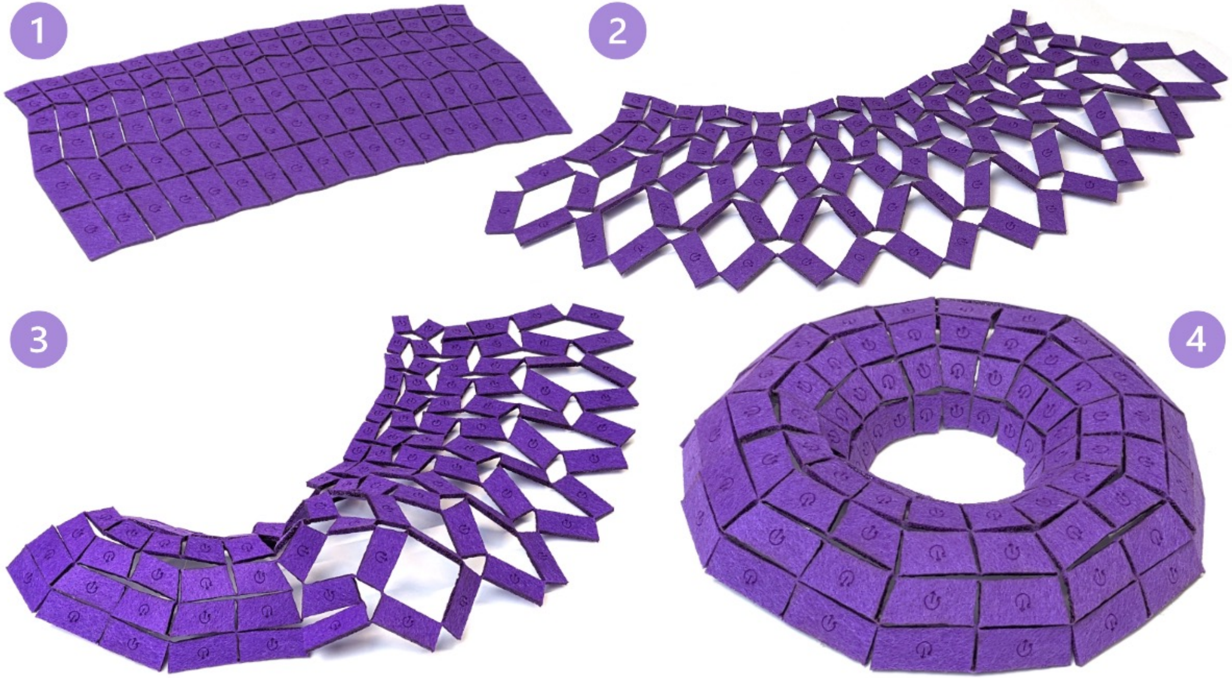
Deployment



Results: different patterns

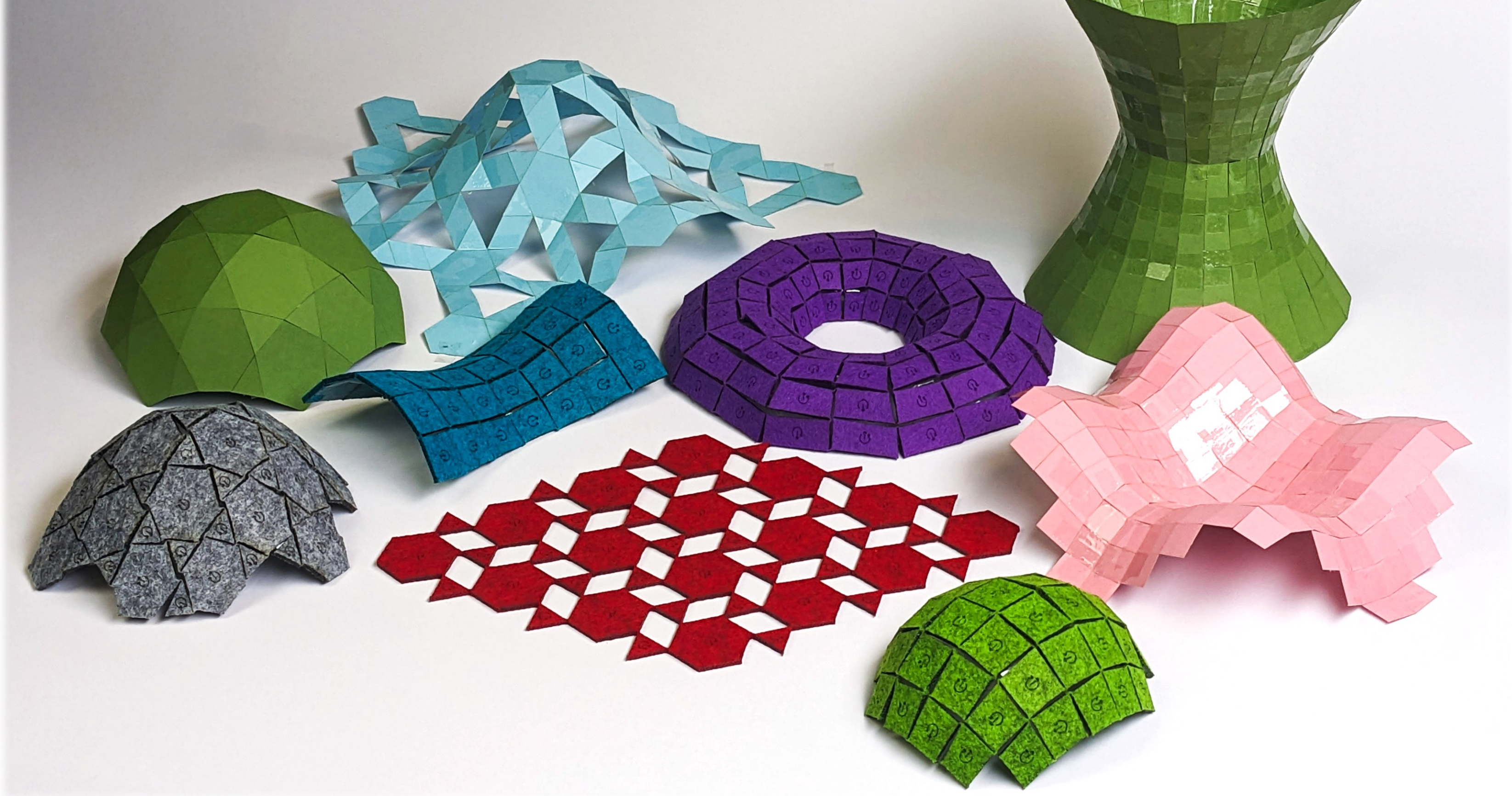


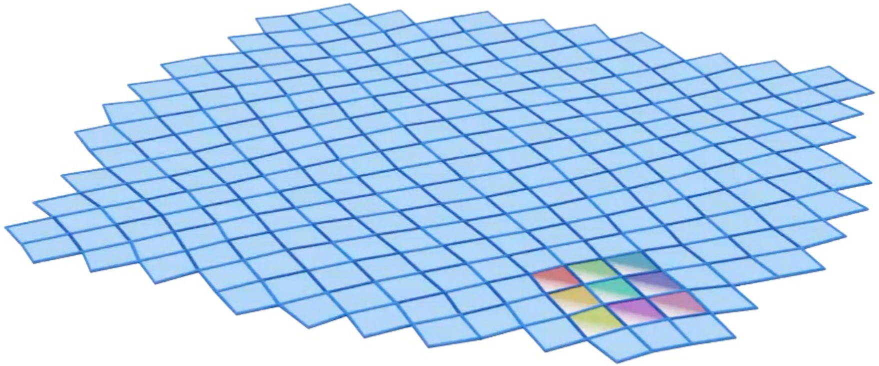
Results: fabrications

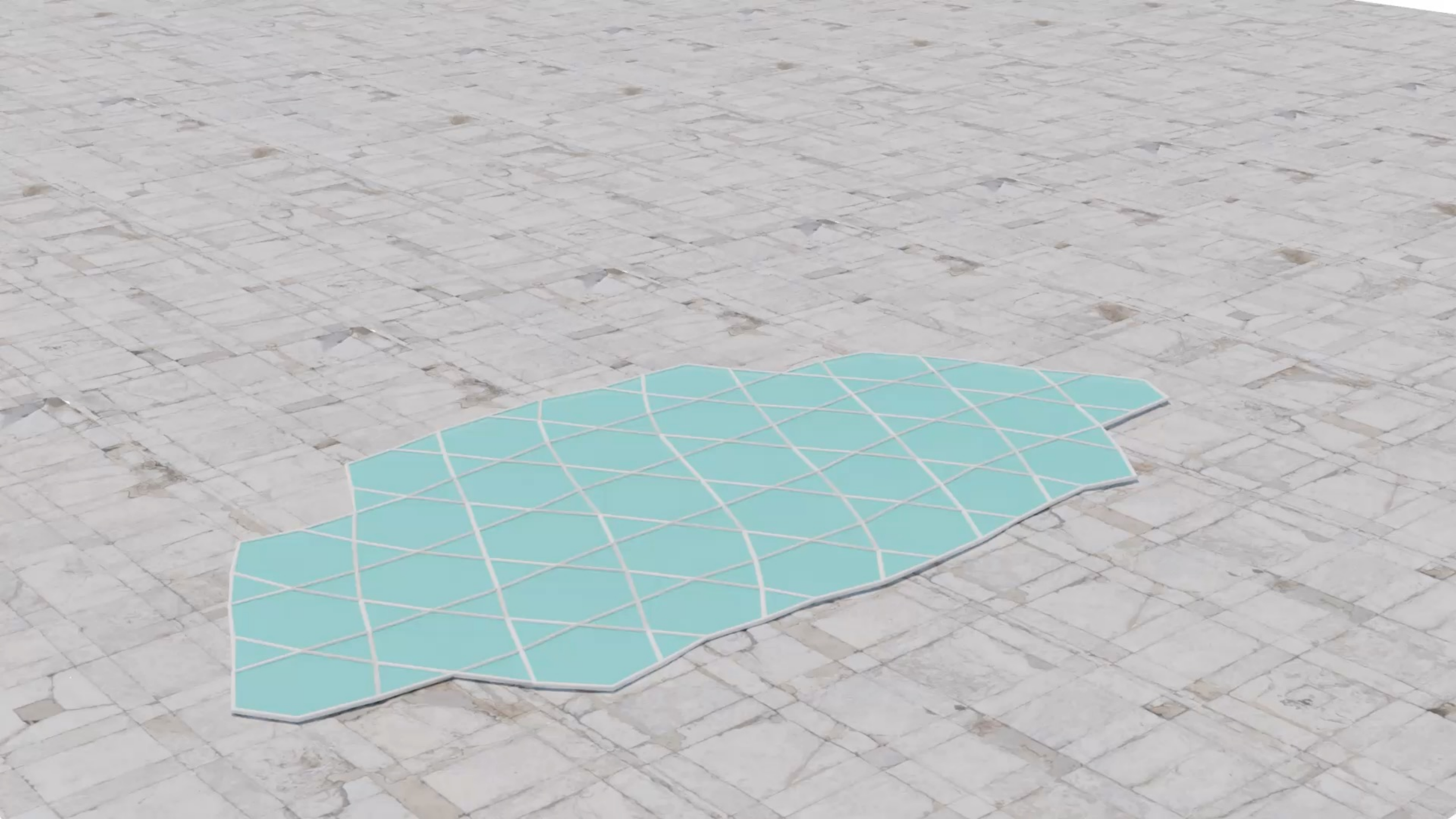


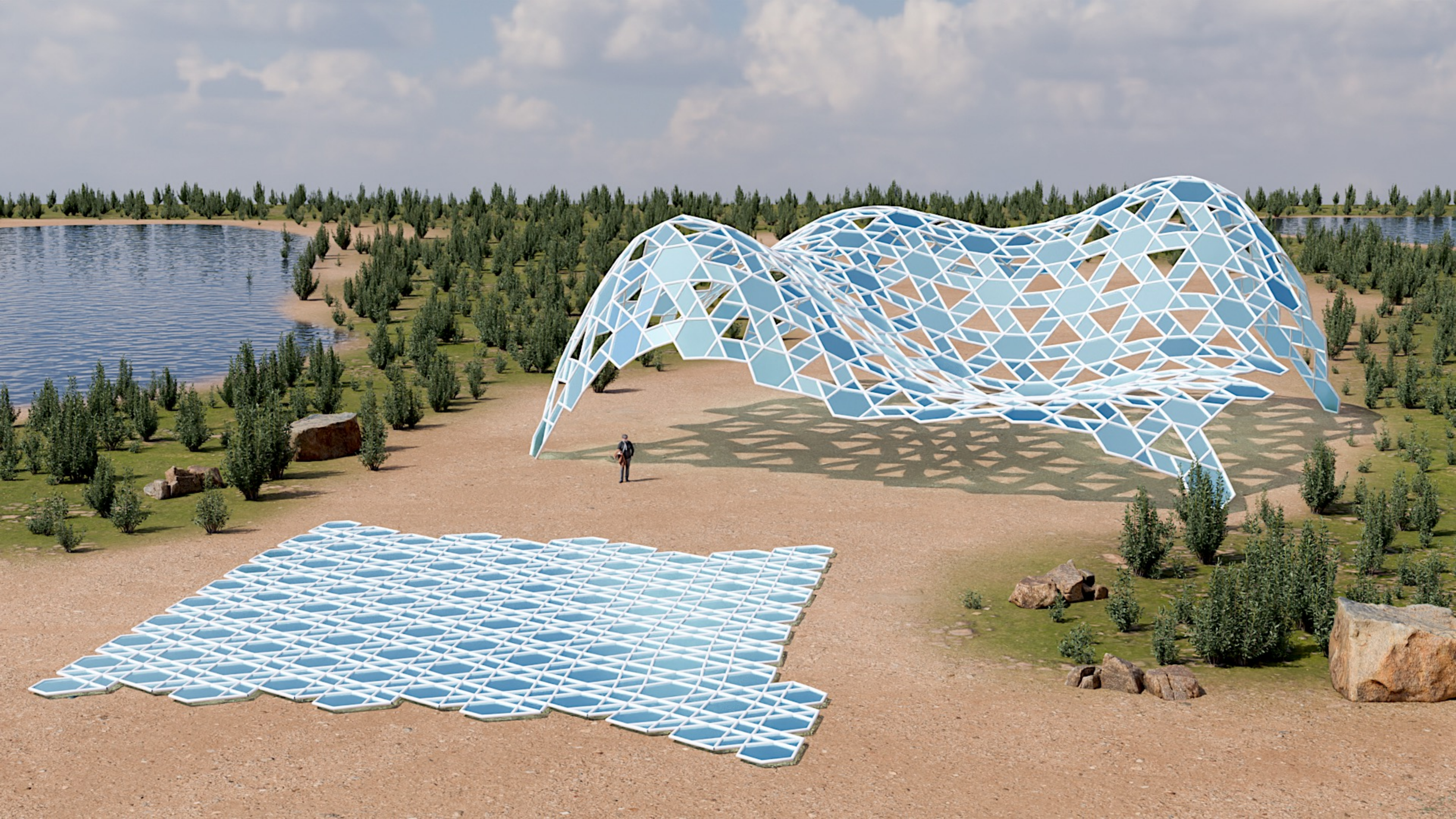
Fabrications

(paper & felt, fixed with tapes)









Limitations

Fabrication is not easy

- Cylinder hinges -> no rigid motion
  - Elastic energy stored in hinges/panels
  - Only the final configuration is rigid
- Spherical joints -> too much freedom
  - Requires gluing

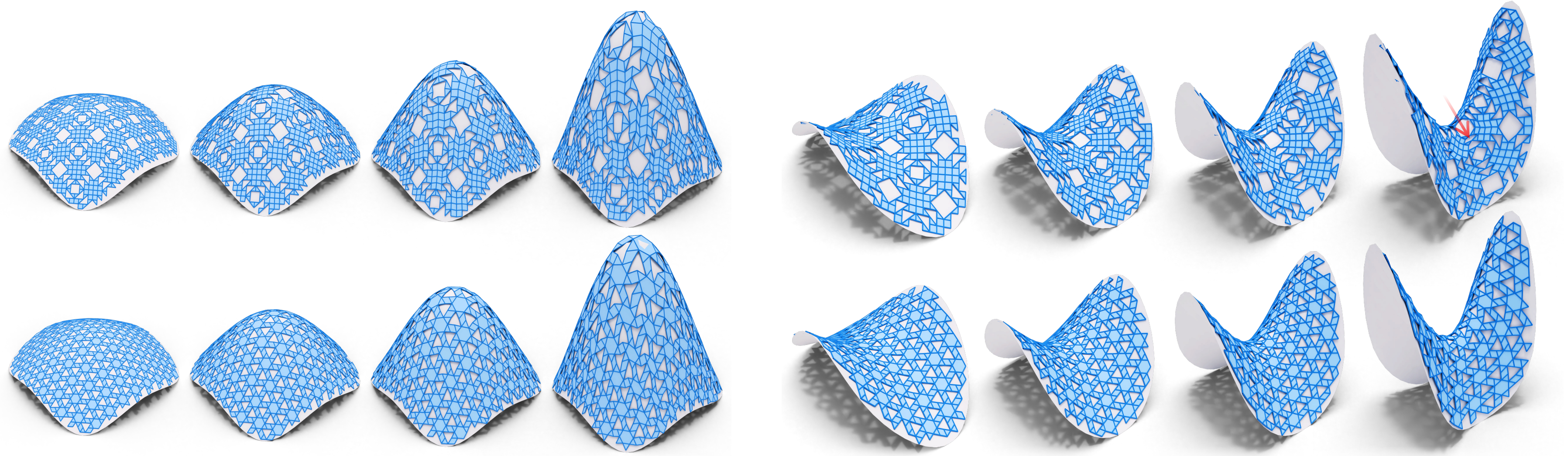
Limitations

Fabrication is not easy

- Spherical joints with constrained axis trajectory
  - Hard to fabricate
  - Allows for a full rigid motion

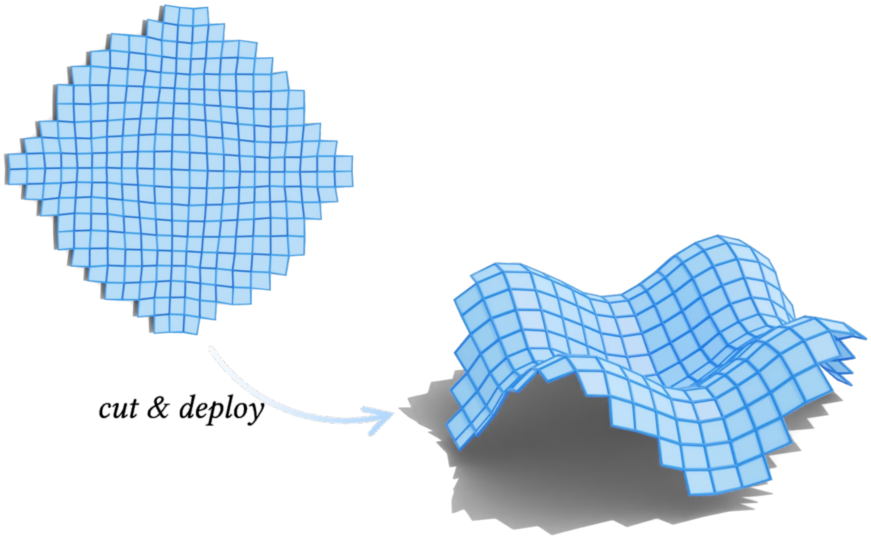
Limitations

Shape space?

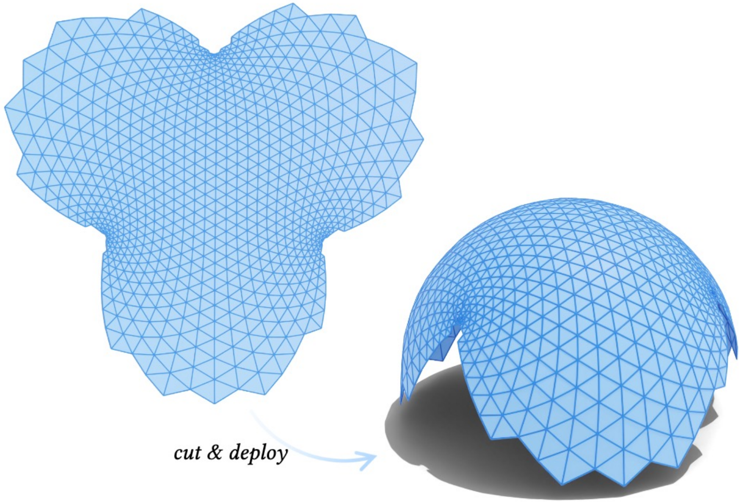


Limitations

Shape space?



Conjugate net?

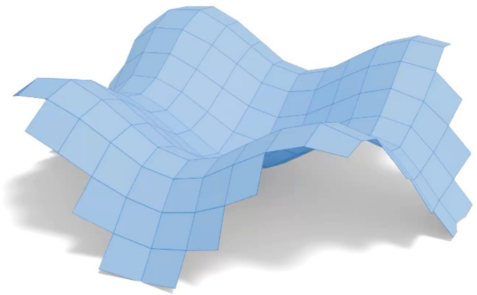


Semi-conformal map?

Limitations

Stability analysis of deployed shapes

- Is it possible to achieve bi-stability?
- Analyze equilibrium points assuming non-negligible masses



Thank You

INTERACTIVE GEOMETRY LAB

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich