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Reconfigurable Hinged Kirigami Tessellations

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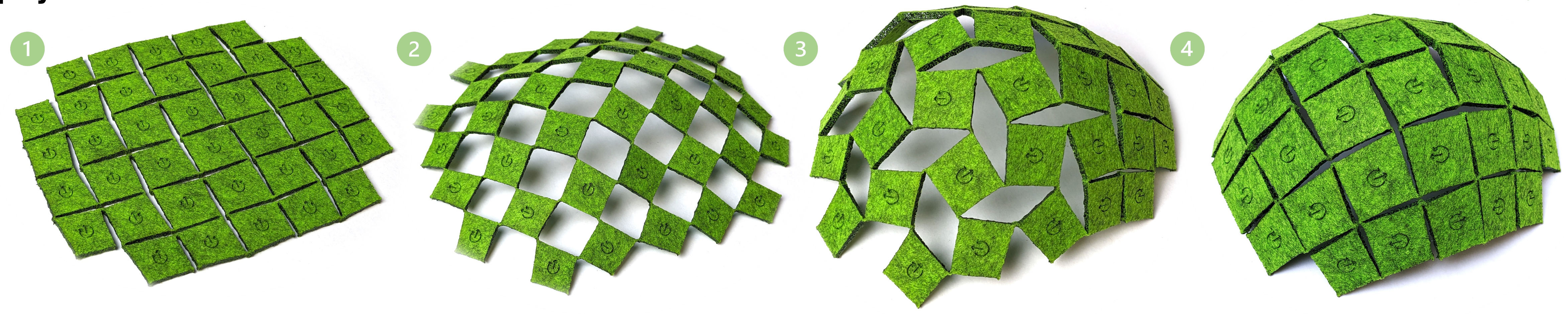
Goal can we transform a flat, fully-closed 2D pattern into a target 3D shape via rotations only?

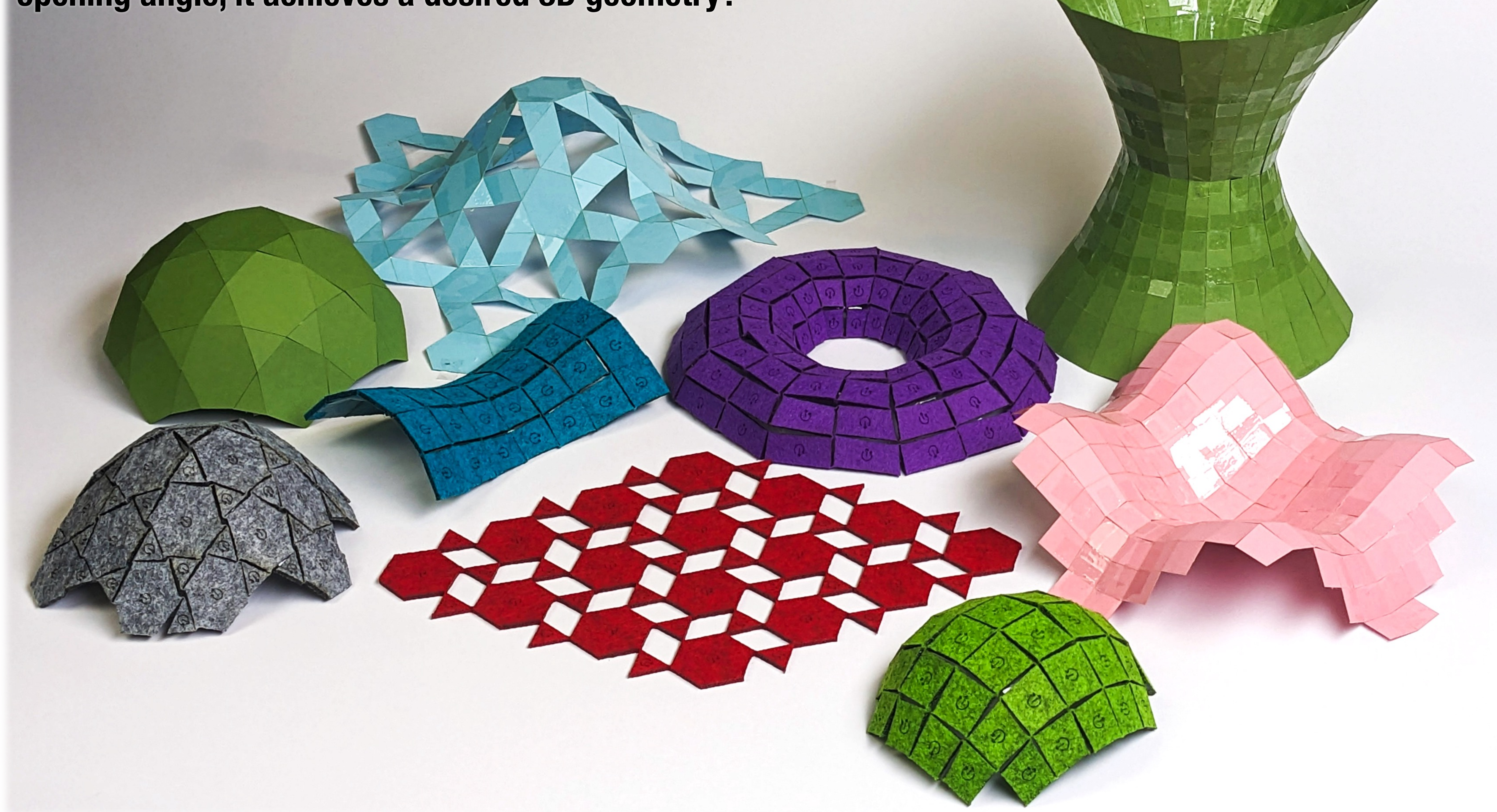


Problem 1 (Deployability of tilings). How can we determine whether a tiling is deployable, i.e., whether there exists a complete cut that transforms the tiling into a hinged kirigami structure whose face can rotate rigidly about hinge vertices without collision?

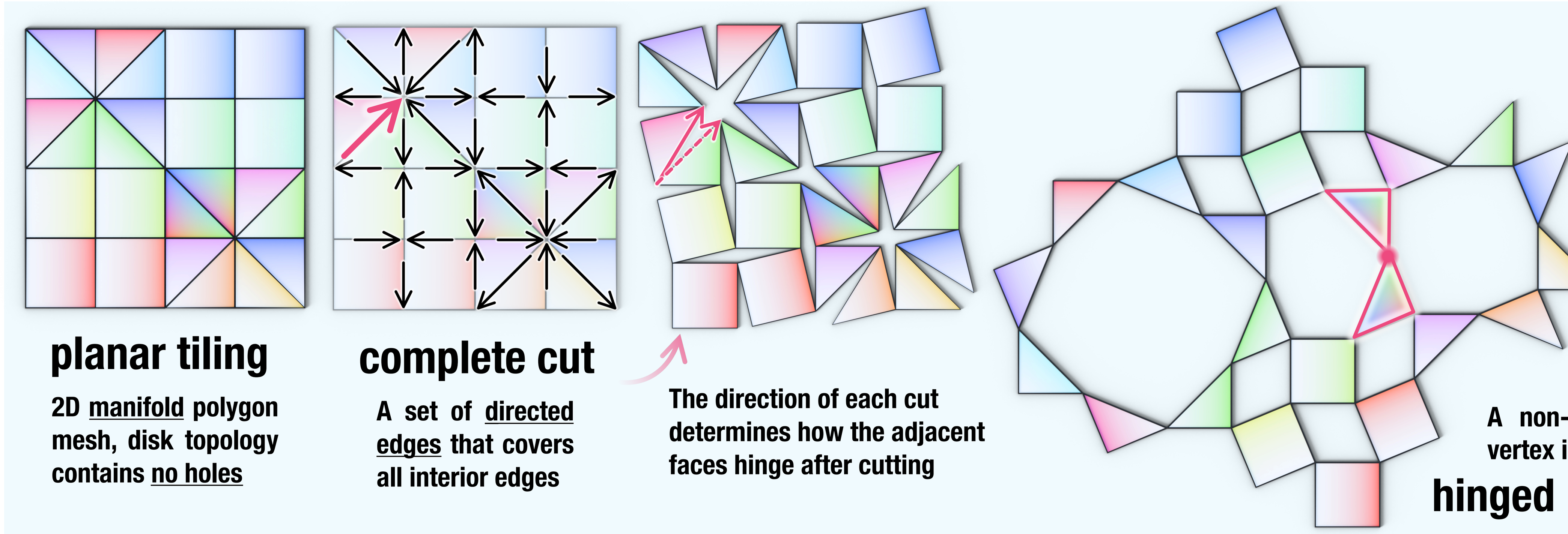
Problem 2 (Rotational deployment analysis). Given a deployable hinged kirigami pattern, how can we mathematically characterize the bounds of face rotations about hinges, i.e., its opening angles that define its maximally open configuration during deployment?

Proposition (Necessary and sufficient condition for uniform deployability). For a planar tiling with all interior vertices of even valency, the following two statements are equivalent: (1) Every interior vertex in the tiling is deployment-friendly. (2) The resulting hinged kirigami structure is uniformly deployable in 2D

Problem 3 (Inverse design). How can we computationally modify a 2D manifold tiling such that, when cut and rotated to the maximal opening angle, it achieves a desired 3D geometry?



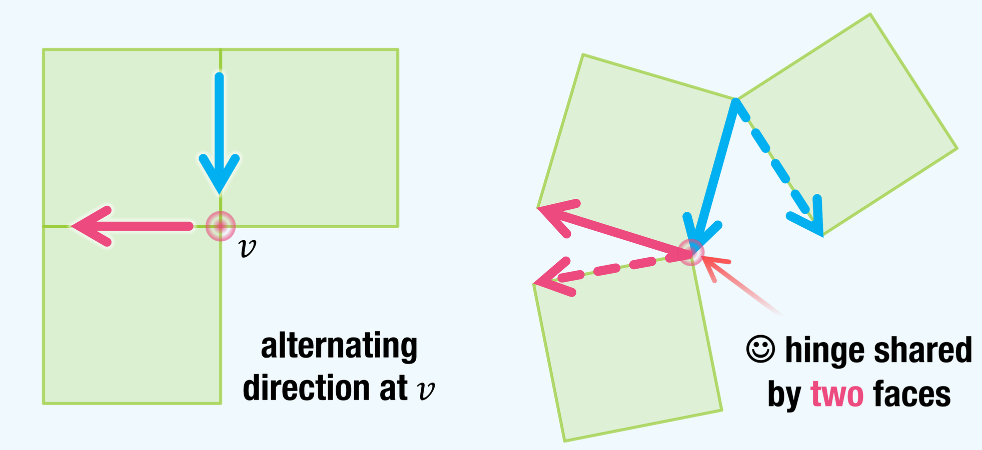
Definitions



deployable, if its faces can rotate rigidly around hinge vertices without collision with neighboring faces

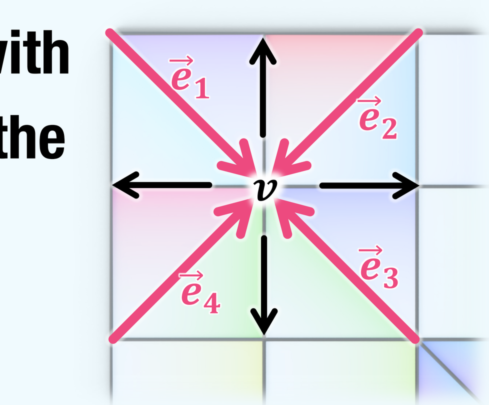
Remark a planar tiling can be cut into a valid hinged kirigami pattern if and only if all its interior vertices have even valency.

Note: to understand why, consider two adjacent cuts (see right). They must have alternating directions to ensure the hinge vertex is shared by exactly two faces after cutting. If both cuts point into or away from the vertex v, the resulting structure violates the hinged kirigami condition. This would either create a dangling vertex or connect more than two faces at v. See Fig.7 of our paper. Since this "having alternating directions" condition must be satisfied for every pair of adjacent cuts, it is equivalent to the condition of being two-colorable.



Definition (deployment-friendly vertex) let $\mathcal{T} = (\mathcal{V}, \mathcal{F})$ be a planar tiling with a complete cut $\mathcal{E}_c$. Let $v \in \mathcal{V}$ be a vertex of even valency $2k$. Denote by $\vec{e}_1 \cdots \vec{e}_k$ the directed edges (cuts) in $\mathcal{E}_c$ ending at v, ordered cyclically. Vertex v is said to be deployment-friendly if

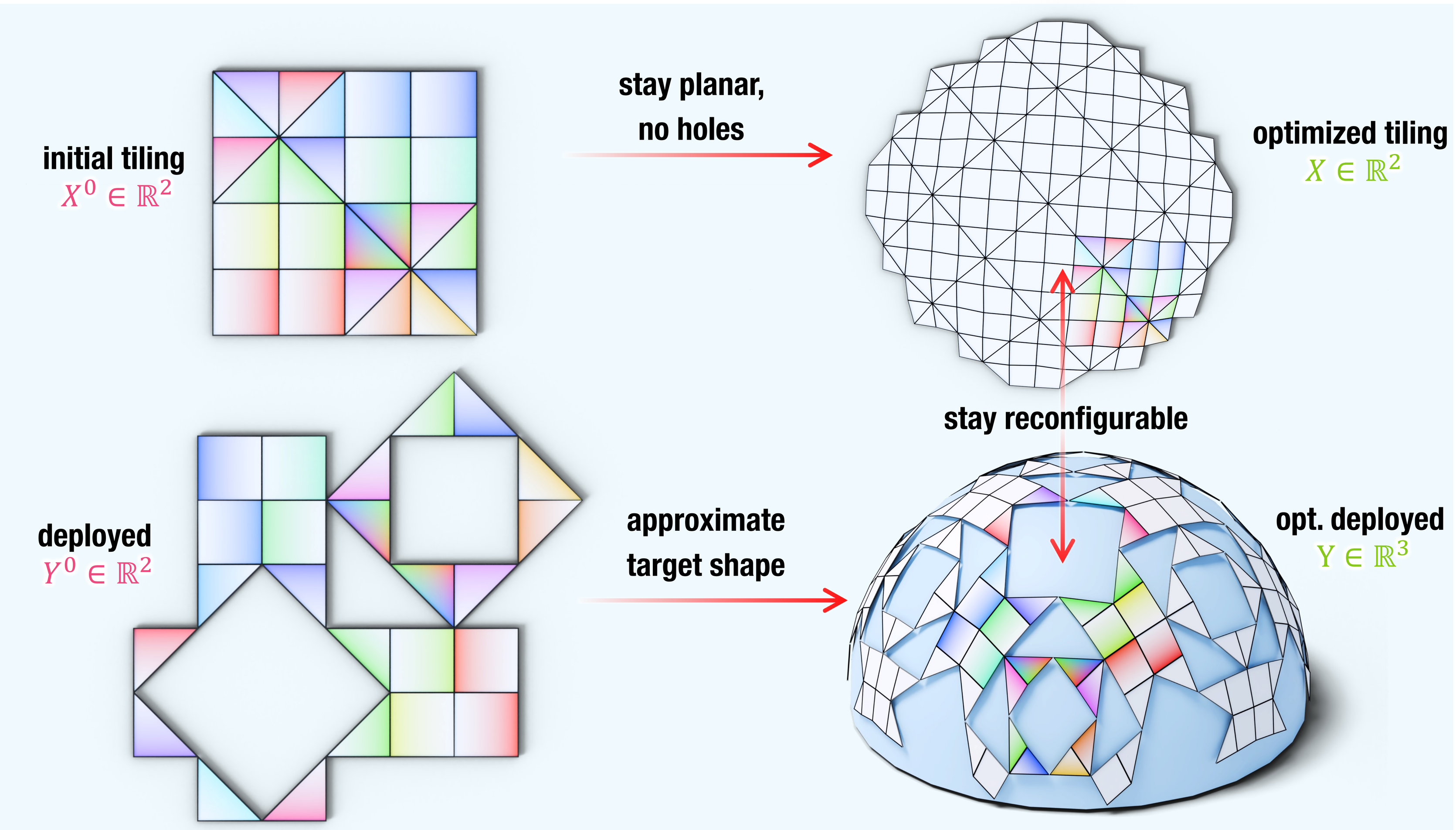
$$\sum_{i=1}^{k} \vec{e}_i = \vec{0}$$



Inverse design

input a specified tiling and 3D target shape
output optimized tiling with its
- 2D embedding before deployment
- 3D embedding after deployment

objectives
- **planarity**: deployed faces are planar
- **reconfigurability**: corresponding faces before and after deployment stay rigid
- **shape approximation**: 3D embedding after deployment well approximates the target geometry
- **fairness**: the optimized tiling does not deviate too much from its initial shape



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