MUS420/EE367A Lecture 5A
Elementary Digital Waveguide Models for Vibrating Strings

Julius O. Smith III (jos@ccrma.stanford.edu)
Center for Computer Research in Music and Acoustics (CCRMA)
Department of Music, Stanford University
Stanford, California 94305

February 15, 2011

Outline

- Ideal vibrating string
- Sampled traveling waves
- Terminated string
- Plucked and struck string
- Damping and dispersion
- String Loop Identification
- Nonlinear “overdrive” distortion

Ideal Vibrating String Model

We know already how to model a string as a *bidirectional delay line* with

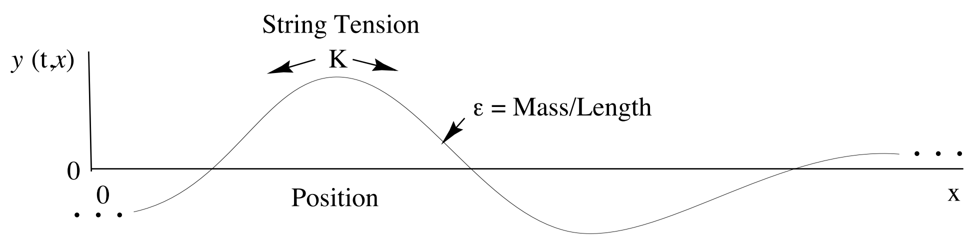
- inverting reflecting terminations (for displacement)
- filters for loss and dispersion
- outputs as sums of traveling-wave components

This model is based on *traveling waves* and the *superposition* of traveling waves as *experimental fact*. In such a model, sound-speed must be measured experimentally.

We now take our string model to the next level based on the *physics* of ideal strings:

- Sound speed becomes a *predicted* quantity
- The very useful concept of *wave impedance* is derived

Ideal String Physics



Wave Equation

$$\boxed{Ky'' = \epsilon \ddot{y}}$$

$$
\begin{aligned}
K &\triangleq \text{string tension} & y &\triangleq y(t,x) \\
\epsilon &\triangleq \text{linear mass density} & \dot{y} &\triangleq \frac{\partial}{\partial t} y(t,x) \\
y &\triangleq \text{string displacement} & y' &\triangleq \frac{\partial}{\partial x} y(t,x)
\end{aligned}
$$

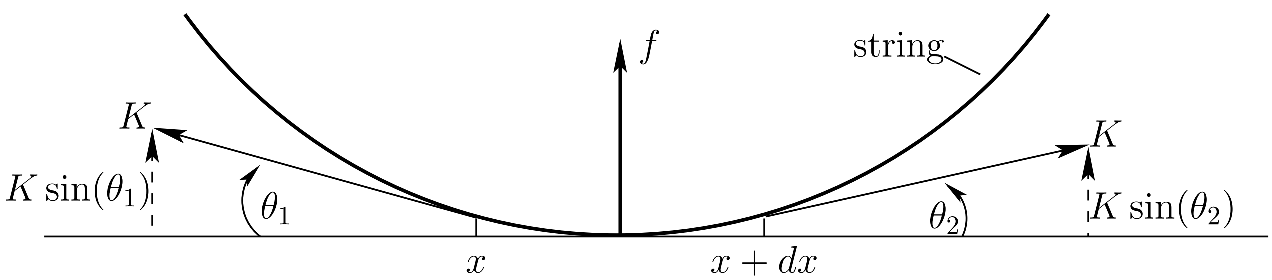
Newton's second law

$$\boxed{\text{Force} = \text{Mass} \times \text{Acceleration}}$$

Assumptions

- Lossless
- Linear
- Flexible (no "Stiffness")
- Slope $y'(t,x) \ll 1$

String Wave Equation Derivation



Force diagram for length dx string element

Total upward force on length dx string element:

$$\begin{aligned}
f(x+dx/2) &= K\sin(\theta_1) + K\sin(\theta_2) \\
&\approx K\left[\tan(\theta_1) + \tan(\theta_2)\right] \\
&= K\left[-y'(x) + y'(x+dx)\right] \\
&\approx K\left[-y'(x) + y'(x) + y''(x)dx)\right] \\
&= Ky''(x)dx
\end{aligned}$$

Mass of length dx string segment: $m = \epsilon\, dx$.

By Newton's law, $f = ma = m\ddot{y}$, we have

$$Ky''(t,x)dx = (\epsilon\, dx)\ddot{y}(t,x)$$

or

$$\boxed{Ky''(t,x) = \epsilon\ddot{y}(t,x)}$$

Traveling-Wave Solution

One-dimensional lossless wave equation:

$$\boxed{Ky'' = \epsilon \ddot{y}}$$

Plug in *traveling wave to the right*:

$$\begin{aligned} y(t,x) &= y_r(t - x/c) \\ \Rightarrow \quad y'(t,x) &= -\frac{1}{c}\dot{y}(t,x) \\ y''(t,x) &= \quad \frac{1}{c^2}\ddot{y}(t,x) \end{aligned}$$

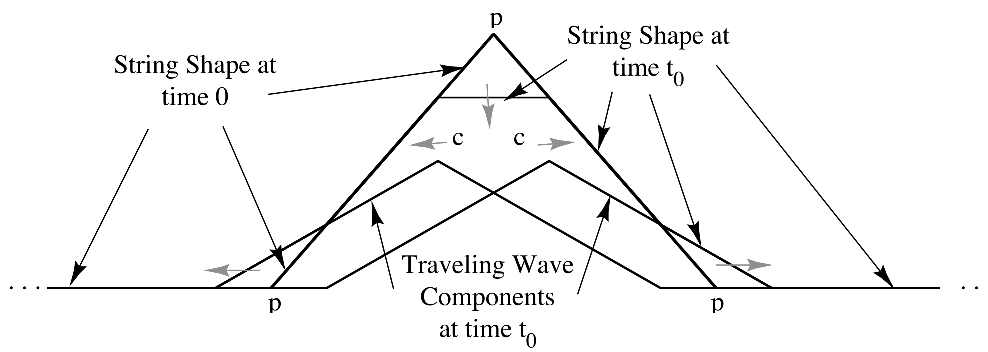
- Given $c \stackrel{\Delta}{=} \sqrt{K/\epsilon}$, the wave equation is satisfied for *any shape traveling to the right at speed* c (but remember slope $\ll 1$)
- Similarly, any *left-going* traveling wave at speed c, $y_l(t + x/c)$, statisfies the wave equation (show)

- General solution to lossless, 1D, second-order wave equation:

$$y(t, x) = y_r(t - x/c) + y_l(t + x/c)$$

- $y_l(\cdot)$ and $y_r(\cdot)$ are arbitrary twice-differentiable functions (slope $\ll 1$)
- **Important point:** Function of two variables $y(t, x)$ is replaced by two functions of a single (time) variable $\Rightarrow$ *reduced computational complexity*.
- Published by d'Alembert in 1747
(wave equation itself introduced in same paper)

Infinitely long string plucked simultaneously at three points marked 'p'



- Initial displacement = sum of two identical triangular pulses
- At time t_0, traveling waves centers are separated by $2ct_0$ meters
- String is not moving where the traveling waves overlap at same slope.
- Animation[1]

[1] http://ccrma.stanford.edu/~jos/rsadmin/TravellingWaveApp.swf

Sampled Traveling Waves in a String

For discrete-time simulation, we must *sample* the traveling waves

- Sampling interval $\stackrel{\Delta}{=} T$ seconds
- Sampling rate $\stackrel{\Delta}{=} f_s$ Hz $= 1/T$
- Spatial sampling interval $\stackrel{\Delta}{=} X$ m/s $\stackrel{\Delta}{=} cT$
  $\Rightarrow$ *systolic grid*

For a vibrating string with length L and fundamental frequency f_0,

$$c = f_0 \cdot 2L \qquad \left(\frac{\text{meters}}{\text{sec}} = \frac{\text{periods}}{\text{sec}} \cdot \frac{\text{meters}}{\text{period}}\right)$$

so that

$$X = cT = (f_0 2L)/f_s = L[f_0/(f_s/2)]$$

Thus, the number of *spatial samples* along the string is

$$\boxed{L/X = (f_s/2)/f_0}$$

or

$$\boxed{\text{Number of spatial samples} = \text{Number of string harmonics}}$$

Examples:

- Spatial sampling interval for CD-quality digital model of Les Paul electric guitar (strings ≈ 26 inches)
  - $X = Lf_0/(f_s/2) = L82.4/22050 \approx 2.5$ mm for low E string
  - $X \approx 10$ mm for high E string (two octaves higher and the same length)
  - Low E string: $(f_s/2)/f_0 = 22050/82.4 = 268$ harmonics (spatial samples)
  - High E string: 67 harmonics (spatial samples)
- Number of harmonics = number of oscillators required in *additive synthesis*
- Number of harmonics = number of two-pole filters required in *subtractive, modal*, or *source-filter decomposition synthesis*
- Digital waveguide model needs only *one delay line* (length $2L$)

Examples (continued):

- Sound propagation in *air*:
  - Speed of sound $c \approx 331$ meters per second
  - $X = 331/44100 = 7.5$ mm
  - Spatial sampling rate $= \nu_s = 1/X = 133$ samples/m
  - Sound speed in air is *comparable* to that of transverse waves on a guitar string (faster than some strings, slower than others)
  - Sound travels much faster in most solids than in air
  - Longitudinal waves in strings travel faster than transverse waves
    - typically an order of magnitude faster

Sampled Traveling Waves in any Digital Waveguide

$$\begin{aligned} x &\to x_m = mX \\ t &\to t_n = nT \end{aligned}$$

$\Rightarrow$

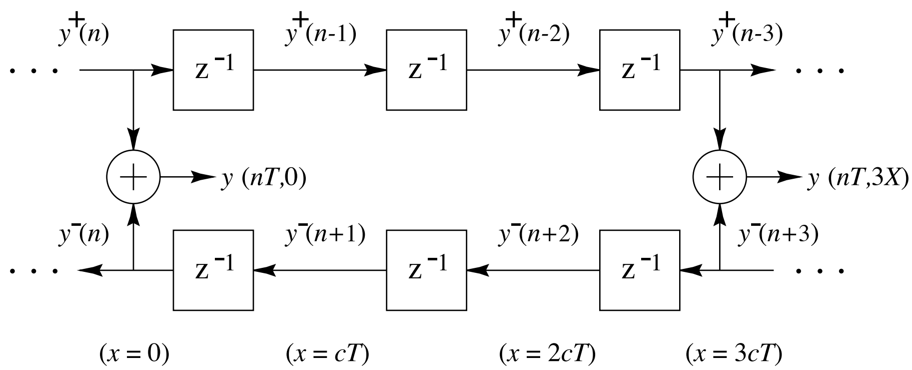
$$\begin{aligned} y(t_n, x_m) &= y_r(t_n - x_m/c) + y_l(t_n + x_m/c) \\ &= y_r(nT - mX/c) + y_l(nT + mX/c) \\ &= y_r\left[(n-m)T\right] + y_l\left[(n+m)T\right] \\ &= y^+(n-m) + y^-(n+m) \end{aligned}$$

when $X = cT$, where we defined

$$y^+(n) \stackrel{\Delta}{=} y_r(nT) \qquad\qquad y^-(n) \stackrel{\Delta}{=} y_l(nT)$$

- "$+$" superscript $\Longrightarrow$ *right-going*
- "$-$" superscript $\Longrightarrow$ *left-going*
- $y_r\left[(n-m)T\right] = y^+(n-m) =$ output of m-sample delay line with input $y^+(n)$
- $y_l\left[(n+m)T\right] \stackrel{\Delta}{=} y^-(n+m) =$ *input* to an m-sample delay line whose *output* is $y^-(n)$

Lossless digital waveguide with observation points at $x = 0$ and $x = 3X = 3cT$



- Recall:

$$
\begin{aligned}
y(t,x) &= y^+\left(\frac{t - x/c}{T}\right) + y^-\left(\frac{t + x/c}{T}\right) \\
&\downarrow \\
y(nT, mX) &= y^+(n-m) + y^-(n+m)
\end{aligned}
$$

- *Position* $x_m = mX = mcT$ is *eliminated* from the simulation
- Position x_m remains laid out from left to right
- Left- and right-going traveling waves must be *summed* to produce a *physical* output

$$y(t_n, x_m) = y^+(n-m) + y^-(n+m)$$

- Similar to *ladder* and *lattice digital filters*

Important point: Discrete time simulation is *exact* at the sampling instants, to within the numerical precision of the samples themselves.

To avoid *aliasing* associated with sampling:

- Require all initial waveshapes be *bandlimited* to $(-f_s/2, f_s/2)$
- Require all external driving signals be similarly bandlimited
- Avoid nonlinearities or keep them "weak"
- Avoid time variation or keep it slow
- Use plenty of lowpass filtering with rapid high-frequency roll-off in severely nonlinear and/or time-varying cases
- Prefer "feed-forward" over "feed-back" around nonlinearities and/or modulations when possible

Interactive simulation of a vibrating string:
`http://www.colorado.edu/physics/phet/simulations/-stringwave/stringWave.swf`

Other Wave Variables

Transverse Velocity Waves:

$$\begin{aligned} v^+(n) &\stackrel{\Delta}{=} \dot{y}^+(n) \\ v^-(n) &\stackrel{\Delta}{=} \dot{y}^-(n) \end{aligned}$$

Wave Impedance (we'll derive later):

$$\boxed{R = \sqrt{K\epsilon} = \frac{K}{c} = \epsilon c}$$

Force Waves:

$$\begin{aligned} f^+(n) &\stackrel{\Delta}{=} R\,v^+(n) \\ f^-(n) &\stackrel{\Delta}{=} -R\,v^-(n) \end{aligned}$$

Ohm's Law for Traveling Waves:

$$\boxed{\begin{array}{rcrl} f^+(n) & = & & R\,v^+(n) \\ f^-(n) & = & - & R\,v^-(n) \end{array}}$$

Acoustic Plane Waves

Pressure Plane Waves:

$$
\begin{aligned}
p^+(n) &\stackrel{\Delta}{=} R_a\, u^+(n) \\
p^-(n) &\stackrel{\Delta}{=} -R_a\, u^-(n)
\end{aligned}
$$

where u^+, u^- are
Longitudinal Particle-Velocity Waves

Ohm's Law for Traveling Acoustic Plane Waves:

$$
\boxed{
\begin{aligned}
p^+(n) &= \; R_a u^+(n) \\
p^-(n) &= -\; R_a u^-(n)
\end{aligned}
}
$$

where

$$\boxed{R_a = \rho c}$$

is the wave impedance of air in terms of mass density ρ (kg/m^3) and sound speed c.

Acoustic Tubes

In acoustic tubes, we again work with
Pressure Plane Waves:

$$p^+(n) \stackrel{\Delta}{=} R_{\scriptscriptstyle T} U^+(n)$$
$$p^-(n) \stackrel{\Delta}{=} -R_{\scriptscriptstyle T} U^-(n)$$

However, now U^+, U^- are
Longitudinal Volume-Velocity Waves:

$$U^+(n) \stackrel{\Delta}{=} A\, u^+(n)$$
$$U^-(n) \stackrel{\Delta}{=} A\, u^-(n)$$

where A is the cross-sectional area of the tube. In an acoustic tube, it is volume velocity that is conserved from one tube section to the next.

Ohm's Law for Traveling Plane Waves in an Acoustic Tube:

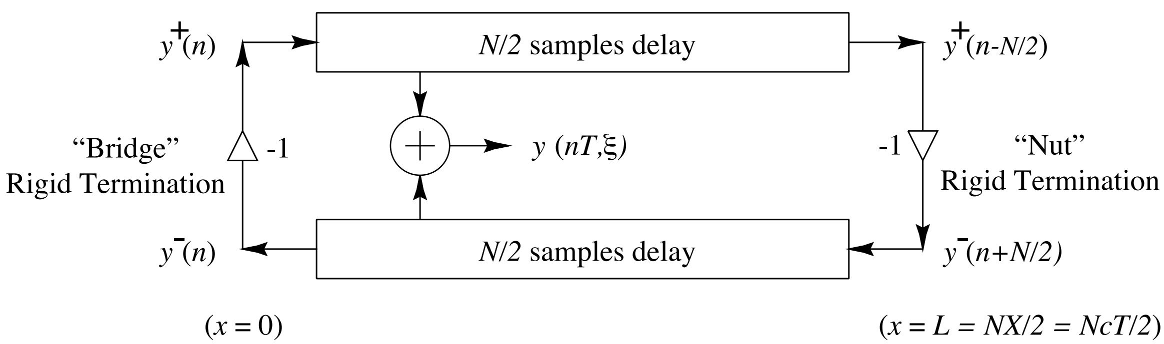
$$\boxed{\begin{array}{rcr} p^+(n) & = & R_{\scriptscriptstyle T} U^+(n) \\ p^-(n) & = & -\; R_{\scriptscriptstyle T} U^-(n) \end{array}}$$

where

$$\boxed{R_{\scriptscriptstyle T} = \frac{\rho c}{A}}$$

is the wave impedance of air in terms of mass density ρ, sound speed c, and tube cross-section area A.

Rigidly Terminated Ideal String



- Reflection *inverts* for displacement, velocity, or acceleration waves (proof below)
- Reflection *non-inverting* for slope or force waves

Boundary conditions:

$$y(t,0) \equiv 0 \qquad y(t,L) \equiv 0 \qquad (L = \text{string length})$$

Expand into Traveling-Wave Components:

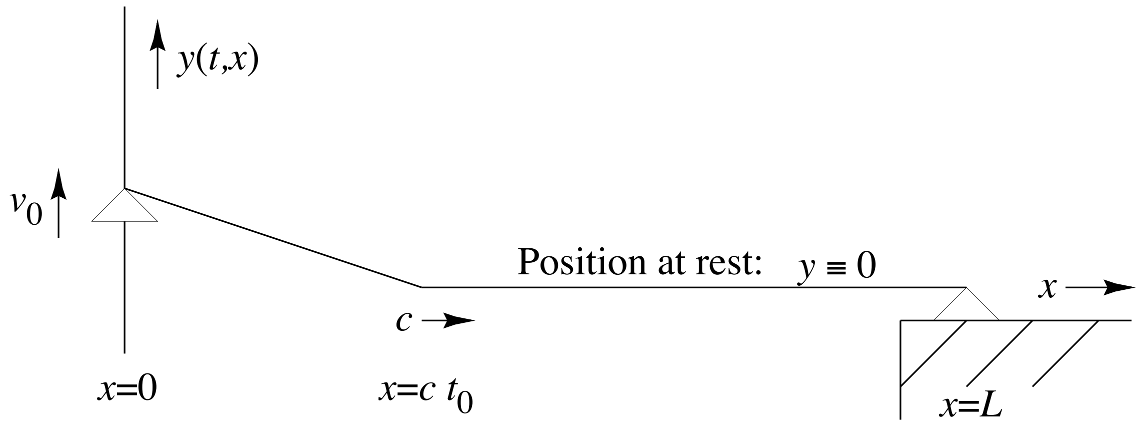
$$\begin{aligned} y(t,0) &= y_r(t) + y_l(t) = y^+(t/T) + y^-(t/T) \\ y(t,L) &= y_r(t-L/c) + y_l(t+L/c) \end{aligned}$$

Solving for outgoing waves gives

$$\begin{aligned} y^+(n) &= -y^-(n) \\ y^-(n+N/2) &= -y^+(n-N/2) \end{aligned}$$

$N \stackrel{\Delta}{=} 2L/X =$ *round-trip* propagation time in samples

Moving Termination: Ideal String



Uniformly moving rigid termination for an ideal string (tension K, mass density ϵ) at time $0 < t_0 < L/c$.

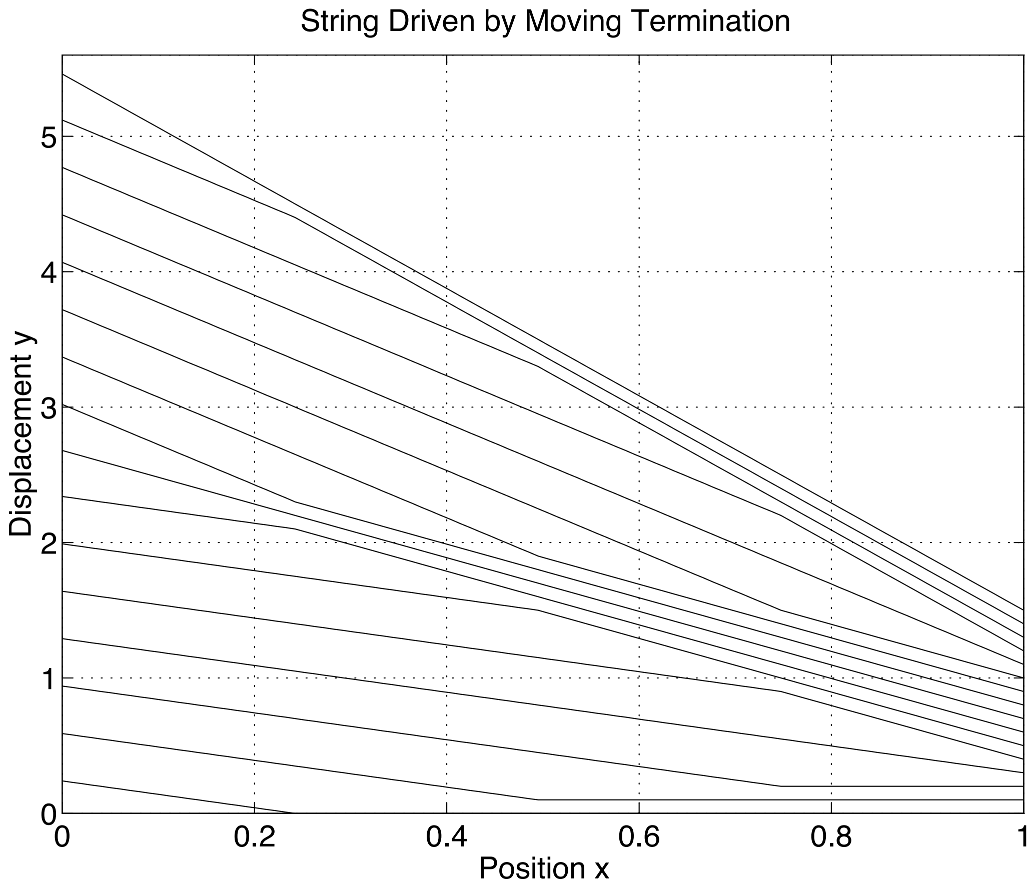
Driving-Point Impedance:

$$y'(t,0) \;=\; -\frac{v_0 t_0}{c t_0} = -\frac{v_0}{c} = -\frac{v_0}{\sqrt{K/\epsilon}}$$

$$\Rightarrow \quad f_0 \;=\; -K\sin(\theta) \approx -K y'(t,0) = \sqrt{K\epsilon}\, v_0 \stackrel{\Delta}{=} R v_0$$

- If the left endpoint moves with constant velocity v_0 then the external applied force is $f_0 = R v_0$
- $R \stackrel{\Delta}{=} \sqrt{K\epsilon} \stackrel{\Delta}{=}$ *wave impedance* (for transverse waves)
- Equivalent circuit is a *resistor* (dashpot) $R > 0$
- We have the simple relation $f_0 = R v_0$ only in the *absence of return waves*, i.e., until time $t_0 = 2L/c$.

- Interactive Animation[2]

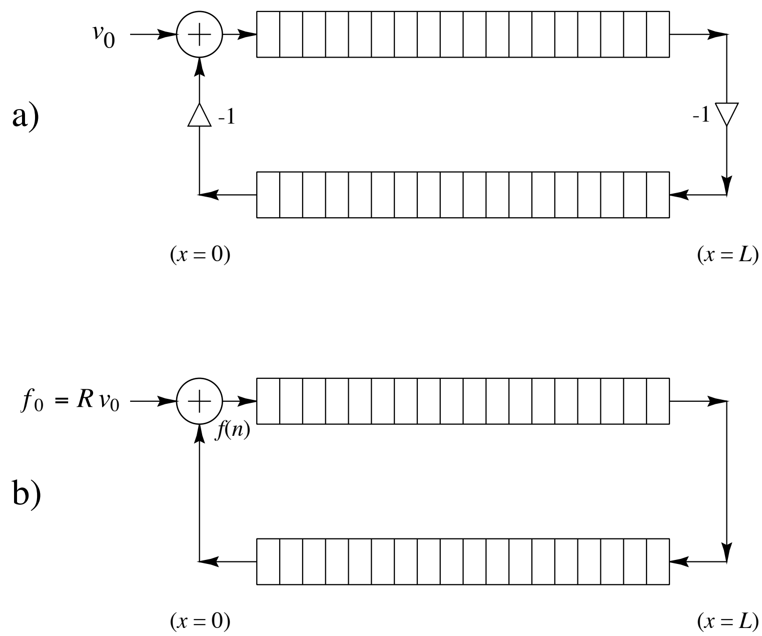


- Successive snapshots of the ideal string with a uniformly moving rigid termination
- Each plot is offset slightly higher for clarity
- GIF89A animation at

  `http://ccrma.stanford.edu/~jos/swgt/movet.html`

[2] `http://phet.colorado.edu/simulations/sims.php?sim=Wave_on_a_String`

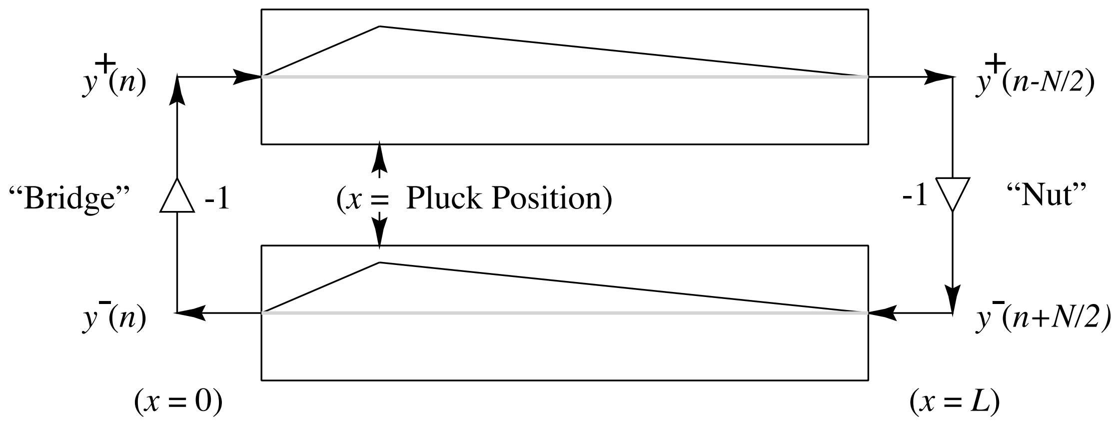
Waveguide "Equivalent Circuits" for the Uniformly Moving Rigid String Termination



a) Velocity waves b) Force waves

- String moves with speed v_0 or 0 only
- String is always one or two straight segments
- "Helmholtz corner" (slope discontinuity) shuttles back and forth at speed c
- String slope increases without bound
- Applied force at termination steps up to infinity
  - Physical string force is labeled $f(n)$
  - $f_0 = Rv_0 =$ *incremental* force per period

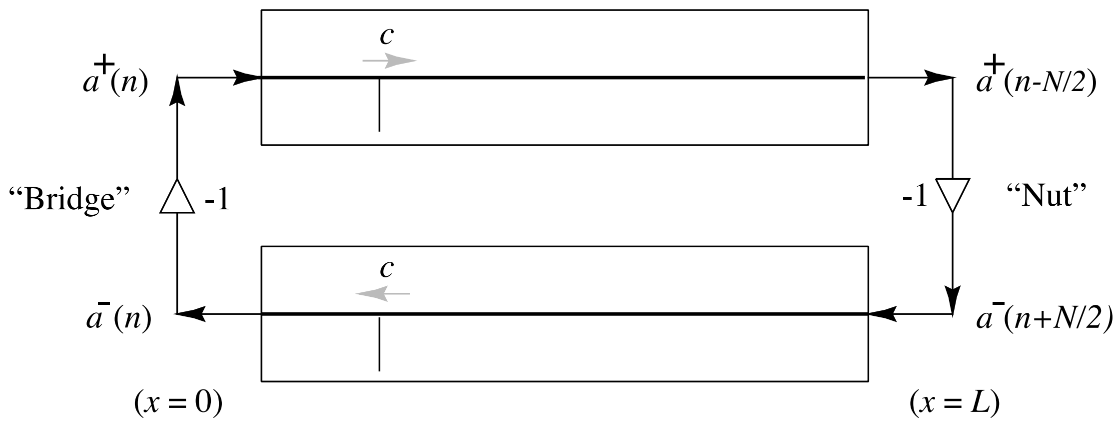
Digital Waveguide Plucked-String Model Using Initial Conditions



Initial conditions for the ideal plucked string.

- Amplitude of each traveling-wave $= 1/2$ initial string displacement.
- Sum of the upper and lower delay lines $=$ initial string displacement.

Acceleration-Wave Simulation



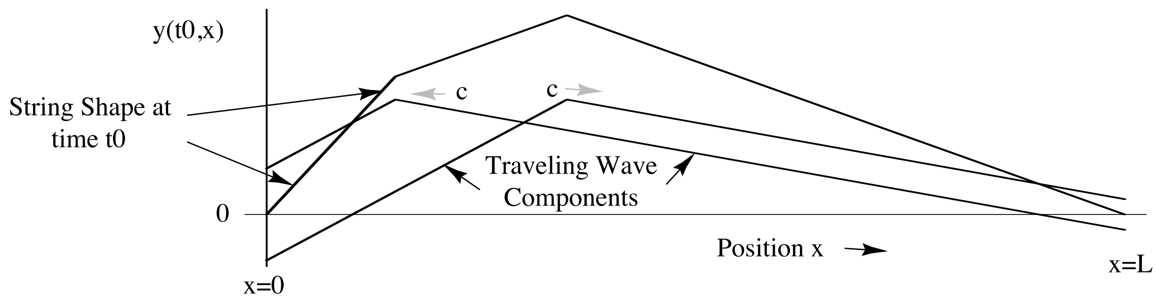
Initial conditions for the ideal plucked string: acceleration or curvature waves.

Recall:

$$\boxed{y'' = \frac{1}{c^2}\ddot{y}}$$

Acceleration waves are proportional to "curvature" waves.

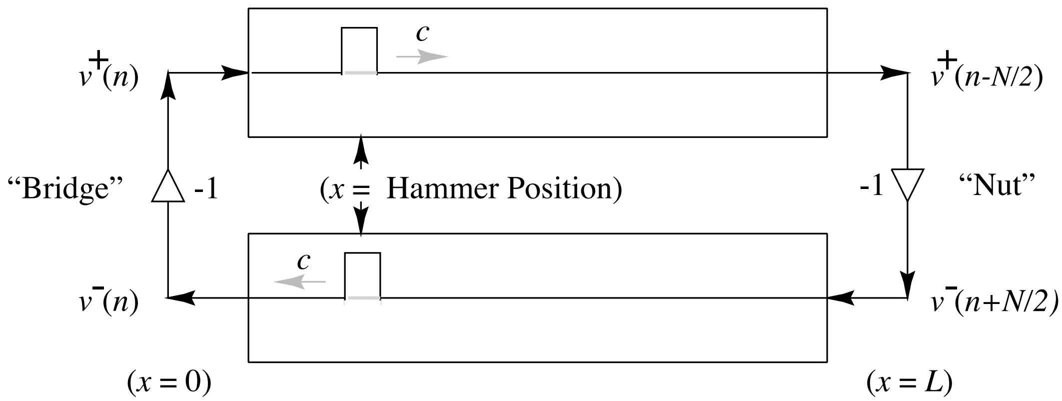
Doubly Terminated Ideal Plucked String



A doubly terminated string, “plucked” at 1/4 its length.

- Shown short time after pluck event.
- Traveling-wave components and physical string-shape shown.
- Note traveling-wave components sum to zero at terminations. (Use image method.)

Ideal Struck-String Velocity-Wave Simulation

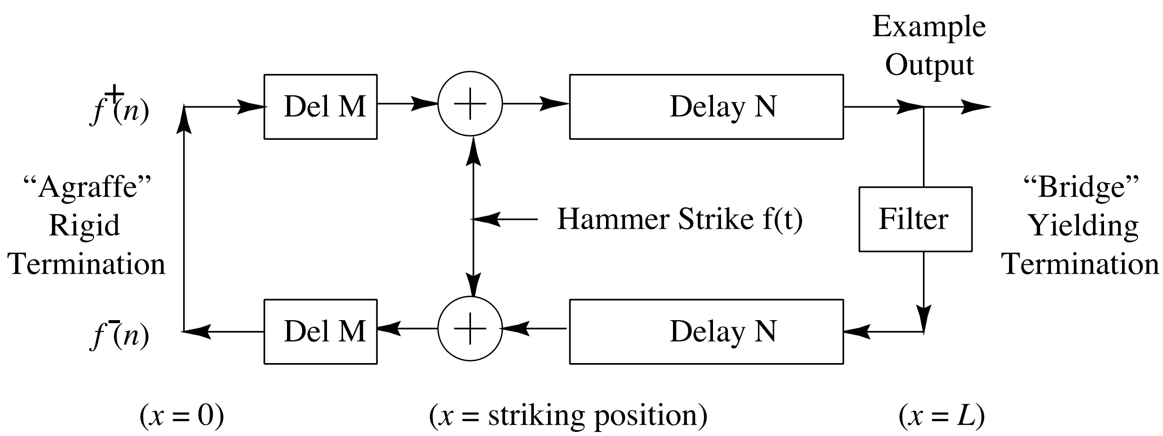


Initial conditions for the ideal struck string in a *velocity wave* simulation.

Hammer strike = *momentum transfer* = velocity step:

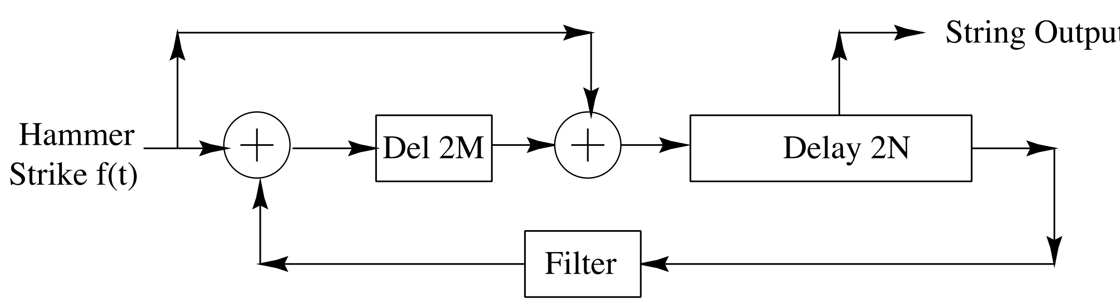
$$m_h v_h(0-) = (m_h + m_s) v_s(0+)$$

External String Excitation at a Point



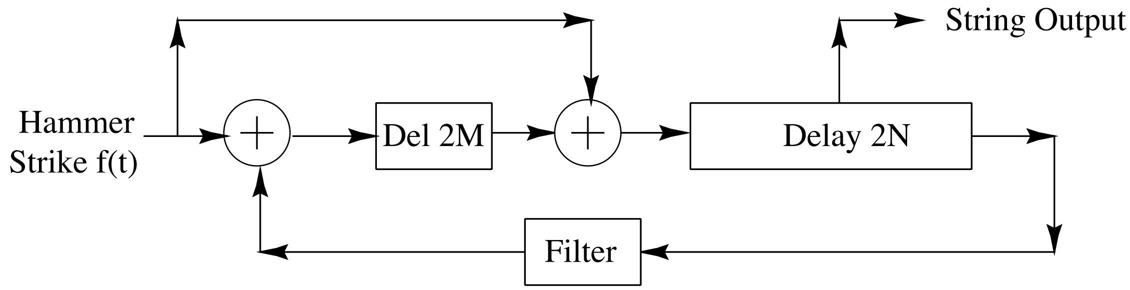
"Waveguide Canonical Form"

Equivalent System: Delay Consolidation

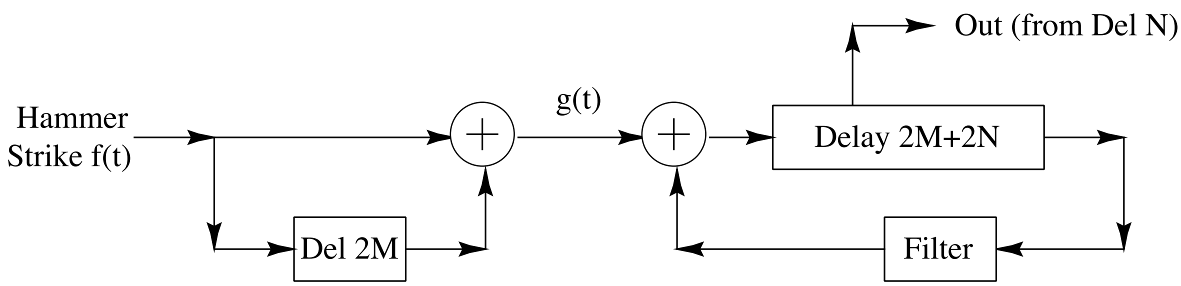


Finally, we "pull out" the comb-filter component:

Delay Consolidated System (Repeated):

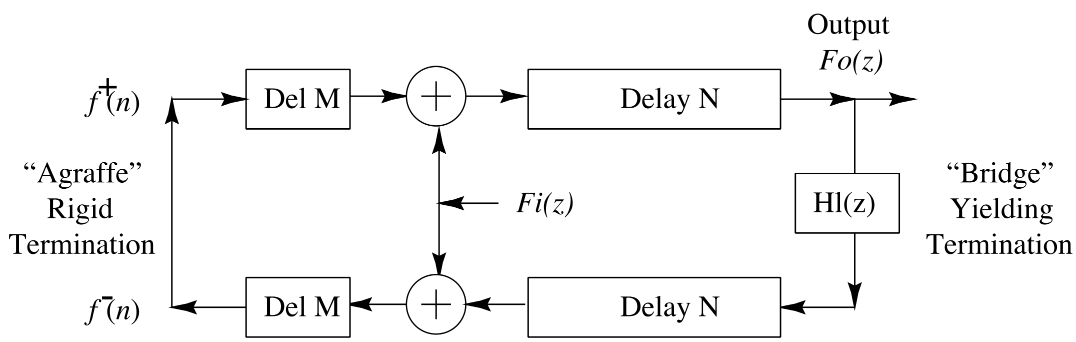


Equivalent System: FFCF Factored Out:



- Extra memory needed.
- Output “tap” can be moved to delay-line output.

Algebraic Derivation

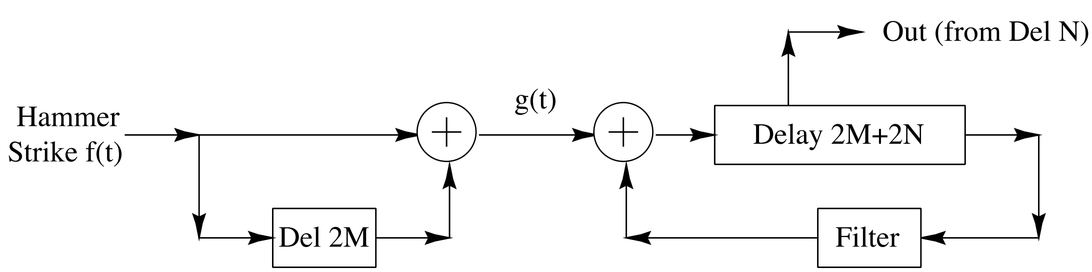


By inspection:

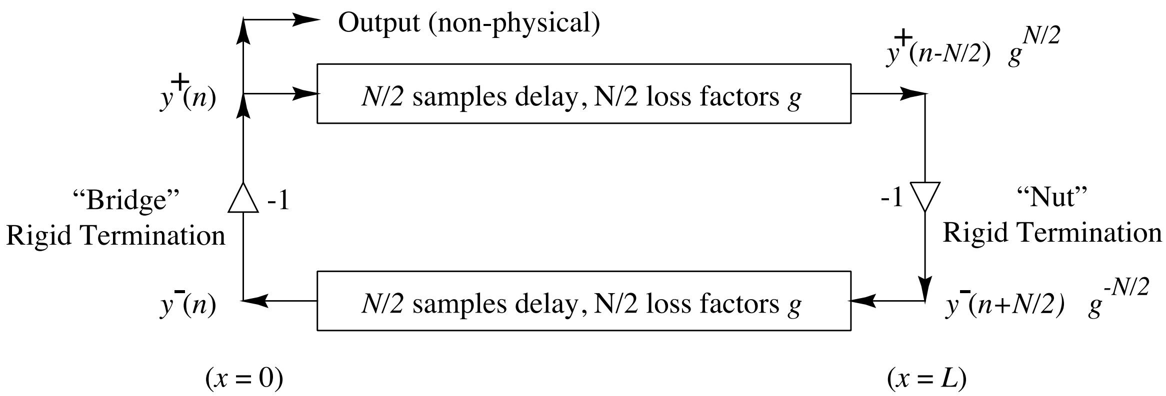
$$F_o(z) = z^{-N}\left\{F_i(z) + z^{-2M}\left[F_i(z) + z^{-N}H_l(z)F_o(z)\right]\right\}$$

$$\Rightarrow \quad H(z) \triangleq \frac{F_o(z)}{F_i(z)} = z^{-N}\frac{1+z^{-2M}}{1-z^{-(2M+2N)}}$$

$$= \left(1+z^{-2M}\right)\frac{z^{-N}}{1-z^{-(2M+2N)}}$$



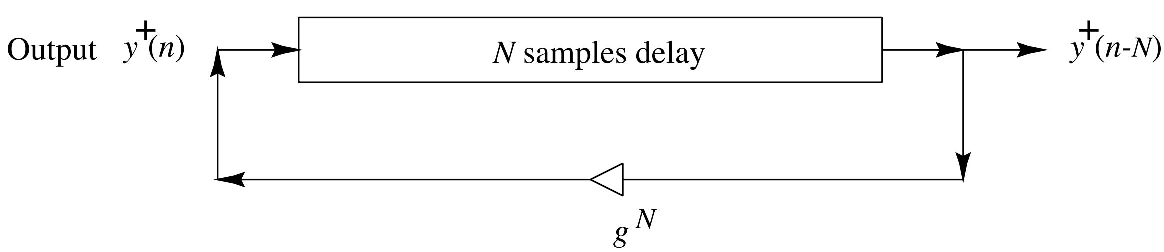
Damped Plucked String



Rigidly terminated string with distributed resistive losses.

- N loss factors g are embedded between the delay-line elements.

Equivalent System: Gain Elements Commuted



All N loss factors g have been "pushed" through delay elements and combined at a *single* point.

Computational Savings

- $f_s = 50$kHz, $f_1 = 100Hz \Rightarrow$ delay $= 500$
- Multiplies reduced by two orders of magnitude
- Input-output transfer function unchanged
- Round-off errors reduced

Frequency-Dependent Damping

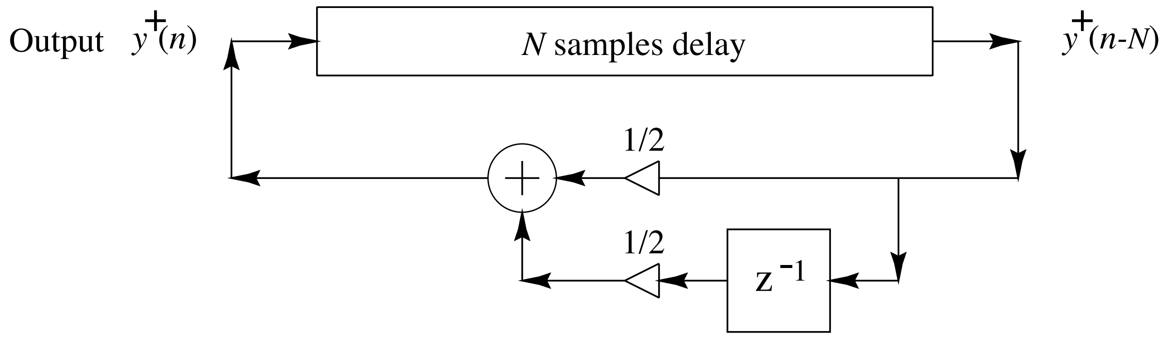
- Loss factors g should really be digital filters
- Gains in nature typically decrease with frequency
- Loop gain may not exceed 1 (for stability)
- Gain filters commute with delay elements (LTI)
- Typically only *one* gain filter used per loop

Simplest Frequency-Dependent Loop Filter

$$\boxed{H_l(z) = b_0 + b_1 z^{-1}}$$

- Linear phase $\Rightarrow b_0 = b_1$ ($\Rightarrow$ delay $= 1/2$ sample)
- Zero damping at dc $\Rightarrow b_0 + b_1 = 1$
  $\Rightarrow b_0 = b_1 = 1/2$
  $\Rightarrow$
  $$\boxed{H_l(e^{j\omega T}) = \cos(\omega T/2)\,, \quad |\omega| \le \pi f_s}$$

Karplus-Strong Algorithm



- To play a note, the delay line is initialized with random numbers ("white noise")

KS Physical Interpretation

- Rigidly terminated ideal string with the simplest damping filter
- Damping consolidated at one point and replaced by a one-zero filter approximation
- String *shape* = *sum* of upper and lower delay lines
- The *difference* of upper and lower delay lines corresponds to a nonzero initial string *velocity*. To show this, recall that $f \triangleq -Ky'$ so that

$$y' = -\frac{1}{K}(f^+ + f^-) = -\frac{R}{K}(v^+ - v^-) = \frac{1}{c}(v^- - v^+)$$

implying

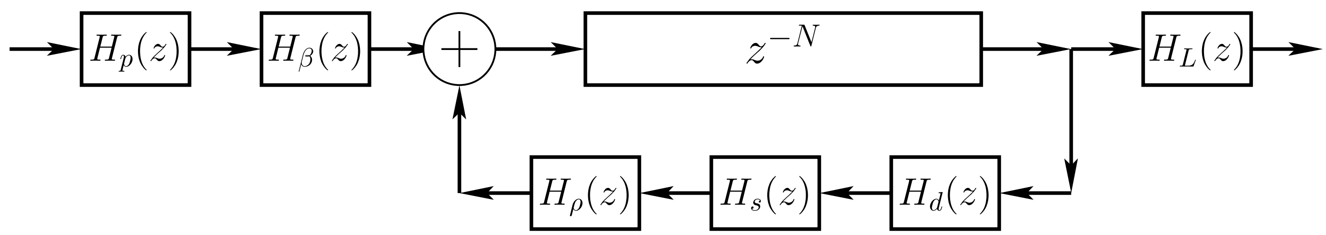
$$\boxed{v^+ = -c(y^+)' \qquad v^- = c(y^-)'}$$

- Karplus-Strong string is both "plucked" and "struck" by random amounts along entire length of string!
- A "splucked" string?

KS Sound Examples

- "Vintage" 8-bit sound examples:
  - Original Plucked String: (AIFF) (MP3)
  - Drum: (AIFF) (MP3)
  - Stretched Drum: (AIFF) (MP3)
- STK Plucked String: (WAV) (MP3)
- Extended KS (EKS) Scale: (WAV) (MP3)

Extended Karplus-Strong (EKS) Algorithm



$$N = \text{pitch period } (2\times \text{ string length) in samples}$$

$$H_p(z) = \frac{1-p}{1-p\,z^{-1}} = \text{pick-direction lowpass filter}$$

$$H_\beta(z) = 1 - z^{-\beta N} = \text{pick-position comb filter, } \beta \in (0,1)$$

$$H_d(z) = \text{string-damping filter (one/two poles/zeros typical)}$$

$$H_s(z) = \text{string-stiffness allpass filter (several poles and zeros)}$$

$$H_\rho(z) = \frac{\rho(N) - z^{-1}}{1-\rho(N)\,z^{-1}} = \text{first-order string-tuning allpass filter}$$

$$H_L(z) = \frac{1-R_L}{1-R_L\,z^{-1}} = \text{dynamic-level lowpass filter}$$

EKS Sound Example

Bach A-Minor Concerto—Orchestra Part: (WAV) (MP3)

- Executes in real time on one Motorola DSP56001 (20 MHz clock, 128K SRAM)
- Developed for the NeXT Computer introduction at Davies Symphony Hall, San Francisco, 1989
- Solo violin part was played live by Dan Kobialka of the San Francisco Symphony

Simplest Frequency-Dependent Loss

Recall that the two-point average used in the Karplus-Strong algorithm can be interpreted as the simplest possible frequency-dependent loss filter for the otherwise ideal vibrating string:

$$H_l(z) = \frac{1 + z^{-1}}{2}$$

Next Simplest Case: Length 3 FIR Loop Filter

$$\boxed{H_l(z) = b_0 + b_1 z^{-1} + b_2 z^{-2}}$$

- Linear phase $\Rightarrow b_0 = b_2$ ($\Rightarrow$ delay $=$ 1 sample)
- Unity dc gain $\Rightarrow b_0 + b_1 + b_2 = 2b_0 + b_1 = 1 \Rightarrow$

$$H_l(e^{j\omega T}) = e^{-j\omega T} \left[(1 - 2b_0) + 2b_0 \cos(\omega T)\right]$$

- Remaining degree of freedom $=$ *damping control*

Length 3 FIR Loop Filter with Variable DC Gain

Relaxing the unity-dc-gain restriction, but keeping linear phase, we have

$$H_l(z) = b_0 + b_1 z^{-1} + b_0 z^{-2} \qquad \text{(linear phase)}$$

We can use the remaining two degrees of freedom for *brightness* B & *sustain* S:

$$\begin{aligned} g_0 &\stackrel{\Delta}{=} e^{-6.91P/S} \\ b_0 &= g_0(1-B)/4 = b_2 \\ b_1 &= g_0(1+B)/2 \end{aligned}$$

where

$$\begin{aligned} P &= \text{period in seconds (total loop delay)} \\ S &= \text{desired sustain time in seconds} \\ B &= \text{brightness parameter in the interval } [0,1] \end{aligned}$$

Sustain time S is defined here as the time to decay 60 dB (or 6.91 time-constants) when brightness B is maximum ($B=1$). At minimum brightness ($B=0$), we have

$$|H_l(e^{j\omega T})| \;=\; g_0 \frac{1+\cos(\omega T)}{2} \;=\; g_0 \cos^2(\omega T)$$

Loop Filter Identification

For loop-filter design, we wish to minimize the error in

- partial decay time (set by amplitude response)
- partial overtone tuning (set by phase response)

Simple and effective method (MUS421 style):

- Estimate pitch (elaborated next page)
- Set Hamming FFT-window length to four periods
- Compute the short-time Fourier transform (STFT)
- Detect peaks in each spectral frame
- Connect peaks through time (amplitude envelopes)
- Amplitude envelopes must decay *exponentially*
- On a dB scale, exponential decay is a *straight line*
- Slope of straight line determines decay time-constant
- Can use 1st-order `polyfit` in Matlab or Octave
- For beating decay, connect amplitude envelope peaks
- Decay rates determine ideal amplitude response
- Partial tuning determines ideal phase response

Plucked/Struck String Pitch Estimation

- Take FFT of middle third of plucked string tone
- Detect spectral peaks
- Form histogram of peak spacing Δf_i
- Pitch estimate $\hat{f}_0 \stackrel{\Delta}{=}$ most common spacing Δf_i
- Refine $\hat{f}_0$ with gradient search using harmonic comb:

$$\begin{aligned}\hat{f}_0 &\stackrel{\Delta}{=} \arg\max_{\hat{f}_0} \sum_{k=1}^{K} \log\left|X(k\hat{f}_0)\right| \\ &= \arg\max_{\hat{f}_0} \prod_{k=1}^{K} \left|X(k\hat{f}_0)\right|\end{aligned}$$

where

$$\begin{aligned}K &= \text{number of peaks, and} \\ k &= \text{harmonic number of } k\text{th peak} \\ & \quad\ \text{(valid method for non-stiff strings)}\end{aligned}$$

Must skip over any missing harmonics,
i.e., omit k whenever $|X(k\hat{f}_0)| \approx 0$.

The text provides further details and pointers to recent papers on pitch estimation.

Nonlinear "Overdrive"

A popular type of distortion, used in *electric guitars*, is *clipping* of the guitar waveform.

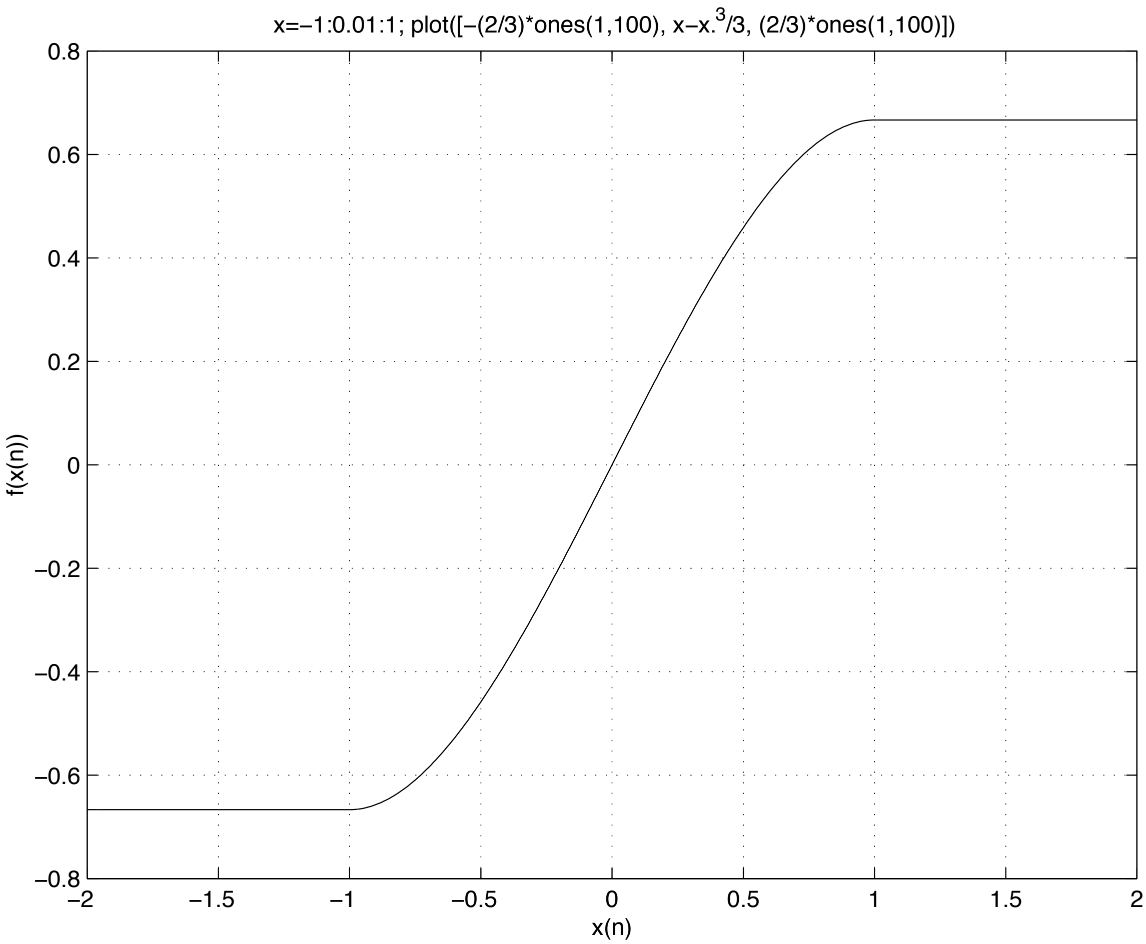
Hard Clipper

$$f(x) = \begin{cases} -1, & x \leq -1 \\ x, & -1 \leq x \leq 1 \\ 1, & x \geq 1 \end{cases}$$

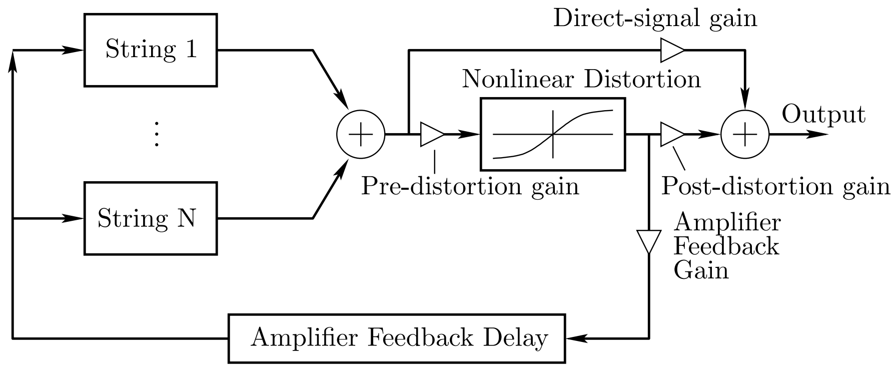
where x denotes the current input sample $x(n)$, and $f(x)$ denotes the output of the nonlinearity.

Soft Clipper

$$f(x) = \begin{cases} -\frac{2}{3}, & x \le -1 \\ x - \frac{x^3}{3}, & -1 \le x \le 1 \\ \frac{2}{3}, & x \ge 1 \end{cases}$$



Amplifier Distortion + Amplifier Feedback



Simulation of a basic distorted electric guitar with amplifier feedback.

- Distortion should be preceded and followed by *EQ*
  E.g., integrator "pre" and differentiator "post"

- Distortion output signal often further filtered by an *amplifier cabinet filter*, representing speaker cabinet, driver responses, etc.

- In Class A tube amplifiers, there should be *duty-cycle modulation* as a function of signal level[3]
  - 50% at low levels (no duty-cycle modulation)
  - 55-65% duty cycle observed at high levels
    $\Rightarrow$ even harmonics come in
  - Example: Distortion input can *offset by a constant* (e.g., input RMS level times some scaling)

[3] `http://www.trueaudio.com/at_eetjlm.htm`