Product Rule :

You are given a task to do where there are n number of ways to do the perticular task. We will call this A represented by n(A) for all possible ways to do this Single task.

Lets expand this example into something we can use using Independence

Independence - the number of ways to do a perticular task isn't dictated by any other external influence.

Now, lets redefine the product rule. We have 3 tasks A, B, C and each of them has a unique number of ways we can do a perticular task Where A, B, C don't determine the outcome of the other How many different ways can we do all three tasks when each of them is independent?

Ex: 1

$n(A) \cdot m(B) \cdot k(C) =$ Number of different Combinations

Ex: 2 how many ways can you order the numbers 1 - 50 for A, B, C including duplicates?

A = 50 possible Combinations

B = 50 possible Combinations

C = 50 possible Combinations

$50_A \cdot 50_B \cdot 50_C = 125{,}000$ possible Combinations of the numbers for A, B, C.

Sum Rule:

Very Similar to product rule .... but, instead of including the different Combinations its a "Straight Line" Solution. Think of it Like flights. You want to fly from Chicago to Miami. Each Flight must fly throug Atlanta. There are 4 flights from Chicago to Atlanta a day and 6 from Atlanta to Miami a day. Then there are 4 + 6 flights to Miami Meaning there are 10 possible ways to get from Chicago to Miami Via Atlanta.

Ex: 3 How many possible 4 to 8 digit passcodes are there that Can be made from the 26 Leters of the alphabet?

$$4^{26} + 5^{26} + 6^{26} + 7^{26} + 8^{26} = 311,791,011,589,203,638,326,570$$

Combinations. but.... this includes duplicates. (A, A, A, B), (A, A, B, A) (A, B, A, A), (B, A, A, A)

Side Note

There is a Concept Called Replacement.

You are given a deck of Cards 52 to be exact.

You want to know what the probability of you drawing a perticular Card is.

With Replacement you have a 1/52 Chance for every draw.

Without Replacement your odds change as you remove Cards

1st = 1/52 2nd = 1/51 3rd = 1/50 4th = 1/49 ....

Replacement is exactly that. Replaceing what you took.

Permutations:

Permutations is a quick and efficient way to Calculate the possible ways/Combinations Something Can be arranged.

*This still includes duplicates. ie 1201, 1021

size of the set → n; Permutation → P; # selected → k

$$ {}_{n}P_{k} = \frac{n!}{(n-k)!} $$

Your removing the numbers you used from the top of the Set. 70-4=66, ... 1

$(n-k)!$ — The set that is left behind.

* When order matters like a race between 5 cars and you want to know the possible outcome of 1st, 2nd, 3rd. This equation will include all orders possible.

Ex:4 We have Cars 5 possible cars or 1, 2, 3, 4, 5. and 3 positions. We want to fill where we have accounted for every possible Combination

$$ {}_{n}P_{k} = \frac{n!}{(n-k)!} $$

$$ {}_{5}P_{3} = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 60 $$

possible Combinations of 1st, 2nd, 3rd place.

Permutations is great for finding all possible Combos when the order they appear doesn't matter.

Combination:

In Contrast to Permutations, Combinations excludes duplicates. That is to Say it removes sets with the Same numbers (1, 2, 3) are the Same as (3, 1, 2) and (3, 2, 1)....

size of the set — Combo — number selected

$${}_{n}C_{k} = \frac{n!}{n!\,(n-k)!}$$

Set that is Left behind

Accounts for duplicate Combinations.

Ex: 5. Lotto

We need to Choose 5 numbers that range from 1–70 without replacement.

kinda Understood

With Permutations.

$$70P_5 = \frac{70!}{(70-5)!} = \frac{70!}{65!} = 1{,}452{,}361{,}680$$

This is a ton of possible Combos.

We Can narrow it down using Combination

$$70C_5 = \frac{70!}{5!(70-5)!} = \frac{70!}{5!\,65!} = 12{,}103{,}014$$ Combinations that are **Unique!**

Bonus!

Permutations With Replacement:

$$n^k$$

Combinations With Replacement:

$$_nC^R_k = \frac{(n+k-1)!}{k!\,(n-k)!}$$