Simulation of Exponential RVs to Illustrate the CLT

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Simulation of Exponential RV's:

```
set.seed(1)
sims <- 10000 #trials
n <- 40 #samples per trial
lambda <- 0.2
data <- matrix(rexp(sims * n, lambda), sims)
sample_means <- apply(data, 1, mean)
true_mean <- 1/lambda
estimated_mean <- mean(sample_means)
true_sd <- 1/(lambda * sqrt(n))
estimated_sd <- sd(sample_means)
```

True and estimated means, followed by true and estimated variances, of the sampling distribution:

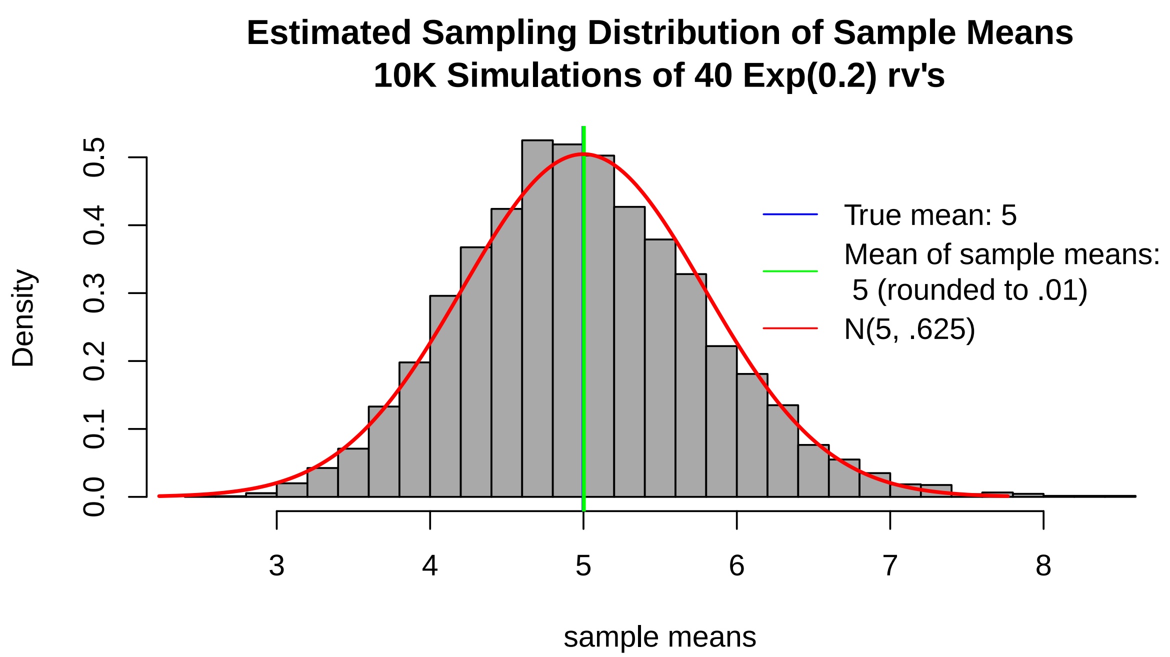
```
## [1] 5
```

```
## [1] 5.003
```

```
## [1] 0.625
```

```
## [1] 0.6262
```

A density plot showing the distribution of 10000 sample means, the true and estimated mean, and the normal distribution our sampling distribution approaches according to the CLT, namely N(5, 0.625):



Confidence interval coverage:

We have 10000 sample means, so we may calculate 10000 95% confidence intervals, $\bar{x} \pm 1.96\,\frac{s}{\sqrt{n}}$.

```
sample_sds <- apply(data, 1, sd)
CIs_sample_means <-
  matrix(c(sample_means - qnorm(.975) * sample_sds / sqrt(n),
           sample_means + qnorm(.975) * sample_sds / sqrt(n)), nrow = sims)
res <- apply(CIs_sample_means, 1, function(x) 5 >= x[1] & 5 <= x[2] )
p <- length(res[res == T])/sims * 100 #percent coverage for all trials
p
```

```
## [1] 92.43
```

Thus with this simulation of 10000 trials, about 92.43% of the confidence intervals actually contain the true mean $\frac{1}{\lambda} = 5$