Tessellation README

2023-01-23

Metropolis Hastings Derivation

First, following the steps in AdaptSPEC, we rewrite the Whittle Likelihood as follows using the notation change that $y_n(\omega_j) = \log(I_n(\omega_j))$:

$$
\begin{aligned}
p(x|f) &= (2\pi)^{n/2} \prod_{j=0}^{n-1} \exp\{-\frac{1}{2}[\log f(\omega_j) + I_n(\omega_j)/f(\omega_j)]\} \\
&= (2\pi)^{-n/2} \prod_{j=0}^{n-1} \exp\left\{-\frac{1}{2}\left[\log(f(\omega_j)) + \frac{\exp\{\log(I_n(\omega_j))\}}{\exp\{\log(f(\omega_j))\}}\right]\right\}
\end{aligned}
$$

Then we use the equation given in the paper: $\log(f_j) = g(\omega_j) = \alpha_0 + h(\omega_j) = \alpha_0 + X_j^T\beta$ in order to replace the unknown spectral density with α_0's and β's

$$
\begin{aligned}
&= (2\pi)^{-n/2} \prod_{j=0}^{n-1} \exp\left\{-\frac{1}{2}\left[\log(f(\omega_j)) + \frac{\exp\{y_n(\omega_j)\}}{\exp\{\log(f(\omega_j))\}}\right]\right\} \\
\mathcal{L}(\alpha_0, \beta, \tau^2|y) &= (2\pi)^{-n/2} \prod_{j=0}^{n-1} \exp\left\{-\frac{1}{2}\left[\alpha_0 + X_j^T\beta + \frac{\exp\{y_j\}}{\exp\{\alpha_0 + X_j^T\beta\}}\right]\right\} \\
&= (2\pi)^{-n/2} \prod_{j=0}^{n-1} \exp\left\{-\frac{1}{2}\left[\alpha_0 + X_j^T\beta + \exp\{y_j - \alpha_0 - X_j^T\beta\}\right]\right\} \\
&= (2\pi)^{-n/2} \exp\left\{-\frac{1}{2}[\alpha_0 n + 1_n X\beta + \sum_{j=0}^{n-1} \exp\{y_j - \alpha_0 - X_j^T\beta\}]\right\}
\end{aligned}
$$

Priors

Following closely* the priors outlined in AdaptSPEC, we give the parameters the following priors:

$$
\begin{aligned}
\pi(\alpha) &\sim N(0, \sigma_\alpha^2) \\
\pi(\beta) &\sim N(0, \tau^2, D_K) \\
\pi(\tau^2|\lambda) &\sim IG\left(\frac{\nu_0}{2}, \frac{\nu_0}{\lambda}\right) \\
\pi(\lambda) &\sim IG\left(\frac{1}{2}, \frac{1}{\eta_0^2}\right)
\end{aligned}
$$

Such that D_K is a positive definite diagonal matrix of dimension K.

*Instead of giving τ^2 a Uniform prior as in AdaptSPEC we have given it a half-t prior. Following the work laid out in Wand et.al. we are able to write the half-t prior as an Inverse-Gamma PDF with the parameters as written above.

Posterior Distribution of the Whittle Likelihood

Now, we go through the derivation of calculating the full posterior distribution of the Whittle Likelihood:

$$\pi(y|\beta, \tau^2, \alpha_0, \lambda) \propto \mathcal{L}(\beta, \tau^2, \alpha_0, \lambda|y) \cdot \pi(\beta) \cdot \pi(\tau^2|\lambda) \cdot \pi(\lambda) \cdot \pi(\alpha_0)$$