

Tessellation README

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Metropolis Hastings Derivation

First, following the steps in AdaptSPEC, we rewrite the Whittle Likelihood as follows using the notation change that $y_n(\omega_j) = \log(I_n(\omega_j))$:

$$\begin{aligned} p(x|f) &= (2\pi)^{n/2} \prod_{j=0}^{n-1} \exp\left\{-\frac{1}{2}[\log f(\omega_j) + I_n(\omega_j)/f(\omega_j)]\right\} \\ &= (2\pi)^{-n/2} \prod_{j=0}^{n-1} \exp\left\{-\frac{1}{2}\left[\log(f(\omega_j)) + \frac{\exp\{\log(I_n(\omega_j))\}}{\exp\{\log(f(\omega_j))\}}\right]\right\} \end{aligned}$$

Then we use the equation given in the paper: $\log(f_j) = g(\omega_j) = \alpha_0 + h(\omega_j) = \alpha_0 + X_j^T \beta$ in order to replace the unknown spectral density with α_0 's and β 's

$$\begin{aligned} &= (2\pi)^{-n/2} \prod_{j=0}^{n-1} \exp\left\{-\frac{1}{2}\left[\log(f(\omega_j)) + \frac{\exp\{y_n(\omega_j)\}}{\exp\{\log(f(\omega_j))\}}\right]\right\} \\ \mathcal{L}(\alpha_0, \beta, \tau^2|y) &= (2\pi)^{-n/2} \prod_{j=0}^{n-1} \exp\left\{-\frac{1}{2}\left[\alpha_0 + X_j^T \beta + \frac{\exp\{y_j\}}{\exp\{\alpha_0 + X_j^T \beta\}}\right]\right\} \\ &= (2\pi)^{-n/2} \prod_{j=0}^{n-1} \exp\left\{-\frac{1}{2}\left[\alpha_0 + X_j^T \beta + \exp\{y_j - \alpha_0 - X_j^T \beta\}\right]\right\} \\ &= (2\pi)^{-n/2} \exp\left\{-\frac{1}{2}[\alpha_0 n + 1_n X \beta + \sum_{j=0}^{n-1} \exp\{y_j - \alpha_0 - X_j^T \beta\}]\right\} \end{aligned}$$

Priors

Following closely* the priors outlined in AdaptSPEC, we give the parameters the following priors:

$$\begin{aligned} \pi(\alpha) &\sim N(0, \sigma_\alpha^2) \\ \pi(\beta) &\sim N(0, \tau^2, D_K) \\ \pi(\tau^2|\lambda) &\sim IG\left(\frac{\nu_0}{2}, \frac{\nu_0}{\lambda}\right) \\ \pi(\lambda) &\sim IG\left(\frac{1}{2}, \frac{1}{\eta_0^2}\right) \end{aligned}$$

Such that D_K is a positive definite diagonal matrix of dimension K .

*Instead of giving τ^2 a Uniform prior as in AdaptSPEC we have given it a half-t prior. Following the work laid out in Wand et.al. we are able to write the half-t prior as an Inverse-Gamma PDF with the parameters as written above.