Bayesian Analysis On Monetary Policy & Exchange Rate Using A Multivariate Linear Regression

Start Date: 04/06/2024

by

Lebo Sango
Lebo.Sango@slu.edu

Under the supervision of

Dr. Fei Tan

Saint Louis University

Contents

1 Introduction

The interpolation of monetary policy and exchange rates is quite peculiar. The exchange rate does not directly pertain to the United States central bank monetary policy dual mandates inflation and unemployment. However, the exchange rate is considered by some economists as a transmission mechanism in many of the policy evaluation models on microeconomic and macroeconomic levels. According to Taylor (2001), The exchange rate usually enters as part of an arbitrage equation relating the interest rate in one country to the interest rates in other countries through the expected rate of appreciation of the exchange rate. As the value of one currency is susceptible to appreciation or depreciation against another country's currency caused by macroeconomic factors, its stability allows capital mobility and the flow of exports and imports. Though the exchange rate is not a direct mandate of the central bank's monetary policy, it does influence other aspects of one's domestic economic state and international trade.

In essence, the United States' monetary policy differs from conventional measures. Even though its dual mandates are inflation and unemployment, the Federal Reserve Bank conducts unconventional policies including controlling the size of its balance sheet through large asset purchases and interest rate announcements (Inoue et. al., 2019). These tools at their disposal support the domestic economy by providing maximum employment, price stability, and moderate long-term interest rates. Moreover, large asset purchases or LSA consist of purchasing long-term government bonds, mortgage-backed securities, debt issued by government-sponsored entities like Fannie Mae and Freddie Mac, and corporate bonds to support the corporate sector. The overarching goal of these monetary tools is to inject liquidity into the financial system, lower interest rates and aims to stimulate economic growth, job creation, and inflation. For instance, the US Central Bank engaged in LSA in times of crisis during the 2008 financial crisis and the COVID-19 pandemic by increasing the money supply and lowering interest rates. According to the Federal Reserve Bank of Saint Louis, M1 grew by more than 200 %, and the security repurchase agreement operations reached as much as $ 2.3 trillion (2024) compared to the rounds of large-scale asset purchases between 2008 and 2014 of 1.6 $ trillion per the Federal Reserve Bank of New York (2018).

Monetary policy interventions relative to exchange rates ought to carefully seek the effects of their response, as they could create a domino effect in the economy, before implementing them. Mindford (1981)

discusses the division of exchange rates as nominal and real rates. Real rate policy is one for the labor market to aid in reducing government interventions through benefits, reducing union interventions, and improving the workings of closely related markets. On the flip side, a monetary policy would ideally deliver strong competition with a low minimal exchange rate to encourage manufacturing and a stable inflation price level. Moreover, he further argues that popular economic commentary would deliver high competitiveness for manufacturing and zero inflation, yet low monetary targets can lead to low competition. An alternative policy would favor tight monetary policy by raising the nominal exchange rate leading to a higher real change rate; therefore, a rift in trains of thought arises when one side favors inflation control with a high nominal exchange rate - the highest priority of the Central Bank and one which rates the health of manufacturing, so low exchange rates. It is of relevance to point out that the United States ran a trade deficit of more than $ 700 billion in 2023 Q3 per the Federal Reserve Bank of Saint Louis (2024). A low exchange rate for the health of manufacturing in the US does not pertain amongst the Central Bank mandates.

This study explores the response of exchange rates to monetary policy through a basic Bayesian multivariate linear regression while reproducing Taylor's theoretical policy rules equation. Nevertheless, the structure of the equation has been altered by also incorporating previous short-term interest rates and observing the response of the current nominal exchange rate. The results will further guide not only theoretically but empirically the apprehension of the relation between monetary policy and exchange rate.

2 Literature Reviews

Taylor (2001) theoretically explores the long-run relationship between short-term interest rates and exchange rates. He argues higher exchange rates cause lower inflation with imported goods becoming relatively cheaper as the currency appreciates and reduces real output with expenditures switching. He highlights the inertia effect because inflation and real output react with a lag relative to the exchange rate. Thus, the expectation of a lower inflation rate and lower output level provokes a decline in short-term interest rates even though the exchange rate is not directly in the monetary policy rule. The absence of a direct effect shows that a change in exchange rate should have a minimal influence on interest rate than a direct change in interest rate which will have adverse effects on real output and inflation (Taylor,2001). A drastic rise in

short-term interest rates in response to the nominal exchange rate undermines other economic indicators such as fiscal debt. The federal government will be subject to repay its debts with higher interest. Inoue et. al. observed the effects of conventional and unconventional monetary policies on exchange rates and concluded that US monetary policy easing results in a depreciation of the US dollar exchange, although the exchange rate response differs based on the effects of the monetary policy on people's expectation of the interest rate path and risk premium in the short, medium, and long term (2019). In quintessence, the study covers monetary policy in its full spectrum by looking at term structure interest rates as they englobe enough information on future interest rates and market rate expectations through risks and uncertainty in the economy. Moreover, Mindford mentioned that other market participants directly influence the exchange rate such as government or competitive agents and monetary policy intervenes through a mechanism such as a tough regime or pre-announcement of interest rates to offset the exchange rate (1981). Those interventions should be implemented cautiously as other relevant economic indicators rely on them such as a government fiscal regime, etc.

3 Bayesian Model, Hypotheses, and Methodology

3.1 Bayesian model

- Monetary Policy Equation

$$I_t = \beta_0 \ + \beta_1 I_{t-1} \ + \beta_2 EXCH_t + \beta_3 INFL_t + \beta_4 GDP_t + e_t \tag{1}$$

3.2 Hypotheses

The Federal Reserve monetary policy controls the stability of the overall price levels in the United States. Inflation in itself can be an indicator of growth when its level is moderate followed by an expansion in the output gap. Conversely, higher levels of inflation can create price stickiness and have a negative perennial effect on the domestic economy by hindering growth, so an increase in inflation will force the Central Bank to hike interest rates to control price levels. Moreover, during economic expansion, the interest rate is expected to decrease or remain unchanged when the real gross domestic product widens. In addition, the exchange rate has an indirect relationship with monetary policy as the Federal Reserve Bank cannot directly raise rates in response to a high level in the nominal exchange rate attributable to other economic indicators will be directly affected. As the nominal exchange rate advances, the Fed will decrease interest rates. Previous rates are susceptible to positively influencing future rates. An uncertain or unpredictable interest rate can disrupt the functioning of the whole economy which will conflict with the central bank's dual mandates.

3.3 Methodology

Exploring the effects of exchange rates on monetary policy is achieved through a Bayesian multivariate linear regression. A multivariate linear regression intends to isolate the responses of multiple variables on the dependent variable. The methods utilized to estimate the expected effects are the maximum likelihood and the Bayesian method. The Bayesian method compounds the information coming from the prior and the likelihood. Thus, it yields an expected response of interest which is the posterior. In the empirical analysis, the mean value of the hyper-parameters will be evaluated to confirm the aforementioned hypotheses.

The Bayesian set up through a matrix form:

Given that,

$$Y_i^* = X_i^{'}\beta_i + \mu_i,\ where\ u_i|X_i \sim iid\ tv(0, \sigma^2)$$
$$\mathrm{E}(\mathrm{Y}_i|X_i) = G(X_i^{'}\beta)\ ,\ i = 1, ..., n(2)$$

$E(Y_i|X_i)$ as the expected response capturing the posterior distribution updated by the conditional conjugate prior to the Gaussian errors presented. The Gaussian errors follow the assumption that the residual errors are normally distributed and the conditional prior is the conditional beliefs about the parameters before observing the data.

For the regression coefficients or hyper-parameters, β_i follows a normal distribution as the prior:

$$\beta_i \sim\ N(\beta_0, B_0) \tag{3}$$

As for the variance errors, σ^2 is an inverse-gamma distribution (conjugate prior) which serves to increase the precision of the posterior $Y_i|X_i$, given as :

$$\sigma^2 \sim\ IG(\frac{\alpha_0}{2}, \frac{\gamma_0}{2}) \tag{4}$$

The Markov Chain Monte Carlo or MCMC facilitates the samples from the posterior by iterating from the conditional distribution of each parameter holding others constant through the following :

For β's distribution:

$$\beta^{(g)} \sim \mathcal{N}(\beta_1^{(g+1)}, B_1^{(g+1)})$$

For σ^2:

$$\sigma^2(g+1) \sim IG - 2\left(\frac{\alpha_1}{2}, \frac{s_1^{(g+1)}}{2}\right)$$

These distributions are calculated using:

$$\begin{aligned} B_1^{(g+1)} &= (\sigma^{-2(g)}X'X + B_0^{-1})^{-1} \\ \beta_1^{(g+1)} &= B_1^{(g+1)}(\sigma^{-2(g)}X'y + B_0^{-1}\beta_0) \\ \alpha_1 &= \alpha_0 + n \\ s_1^{(g+1)} &= s_0 + (y - X\beta^{(g+1)})'(y - X\beta^{(g+1)}) \end{aligned}$$

$\beta_1^{(g+1)}$ corresponds to the computed posterior precision matrix for the coefficients incorporating both the data shown as $X^{'}X$ and the prior (β_0^{-1}). It is also the updated covariance for β at iteration g+1, which is the updated value at the next iteration. $B_1(g+1)$ corresponds to the posterior mean vector for the coefficients by weighting the information from the data $(X^{'}Y)$and the prior mean (β_0). In other words, the updated covariance matrix at iteration g+1 using the mean covariance matrix $B_1(g+1)$, the precision of the errors σ^2, the design matrix X, the response vector Y, and the prior mean vector β_0. Therefore each step of the Gibbs sampling using the MCMC process updates values of β and σ^2 based on the full conditional distribution derived from the posterior distribution.

4 Data Sources

The data ranges from 2004 to 2023 quarterly and originates from the following sources:

- Yahoo Finance, **YF.**
- Federal Reserve Bank of Saint Louis,**FRED.**
- US Bureau of Labor and Statistics,**BLS.**

Table 1: Data Sources Table

Attribute	**Attribute Definition**	**Source**	**Unit**
I_t	Federal Funds Interest Rate	FRED	Percentage
$EXCH_t$	Nominal EURUSD Exchange Rate	YF	Difference change
GDP_t	Output Gap	FRED	Difference change
$INFL_t$	US Inflation	BLS	Percentage change

As stated above in equation (1), the revised Taylor's monetary equation is composed of I_t is the Federal fund rates or short-term interest rates in percentage points, $INFL_t$ represents the percentage change of inflation in the US minus the Central Bank inflation policy target, $EXCH_t$ is described as the nominal EURUSD taken at difference to adjust for variations and only observe the change in EURUSD exchange rate level, GDP_t embodies the output gap between gross domestic product and potential gross domestic product observed at difference change to adjust for fluctuations. Table 2 GDP_t demonstrates a high Kurtosis value of 25. That is, it indicates that the distribution has a longer tail and is more sensitive to outliers.

Table 2: Descriptive Statistics

	EXCH	**INT**	**GDP**	**INF**
Mean	1.239674	1.495986875	0.51418125	2.37
Median	1.2207	0.47674	0.60363	2.1
Maximum	1.5758	5.33	7.75963	6.3
Minimum	0.98296	0.05879	-7.8914	0.73333
Std. Dev.	0.129985174	1.777865229	1.390731737	1.22718413
Skewness	0.420163008	1.091327654	-1.170911998	2.006146624
Kurtosis	-0.453323987	-0.210357249	25.47290734	3.528609349
Observations	80	80	80	80

Table 3 portrays low correlation coefficients among exogenous and endogenous variables. Low correlation coefficients do not show signs of multicollinearity which could hinder coefficient estimates.

Table 3: Correlation statistics

Correlation	EXCH	INT	GDP	INF
EXCH	1.000000			
INT	-0.039409216	1.000000		
GDP	-0.04808549	0.038861277	1.000000	
INF	-0.451806855	0.351698459	0.032969408	1.000000

5 Empirical Analysis and Results

5.1 Empirical analysis

The Bayesian posteriors from the hyper-parameters depict the mean estimates obtained via a multivariate linear regression. Table 4 captures the responses of the endogenous variable short-term interest rate, I_t relative to exogenous variables. Previous short-term rate interest increases current or future interest rates by 99 basis points. The empirical result exhibits the predictive pattern of the Federal Reserve Bank. An unpredictable pattern can lead to an uncertain interest environment which will impede financial market developments alongside a volatile term structure of interest rate. Moreover, the nominal exchange rate causes a decrease in the federal fed funds rate of 3 basis points. In other terms, the Federal Reserve will slightly decrease the short-term interest rate in response to a rise in the nominal exchange rate. The cautious move in rates takes into account that other economic indicators directly depend on short-term interest rates. Furthermore, an increase in inflation results in cutting interest rates down by less than one basis point. The negative coefficient is contrary to economic theory relative to the relationship between inflation and interest rate. Commonly, it is known that in navigating a high inflation environment the Central Bank will raise short-term interest rates as their inflation mandate is 2%. Last, an acceleration in the output gap results in a positive increase of short-term interest rates by less than 1 basis point. The slight increase in rates demonstrates that during the period of economic expansion, GDP will raise the demand for money as individuals will need more money to make necessary transactions. In other words, real money demand rises due to the transaction demand effect. Therefore, the Fed will raise the rate to control real money demand.

Table 4: Bayesian Multivariate Linear Regression Output Table

Posterior	Posterior estimates	Standard error	T-statistics
I_{t-1}	0.9965	0.0189	52.8041
$EXCH_t$	-0.0333	0.6590	-0.0505
INF_t	-0.0061	0.0272	-0.2253
GDP_t	0.0013	0.0223	0.0561

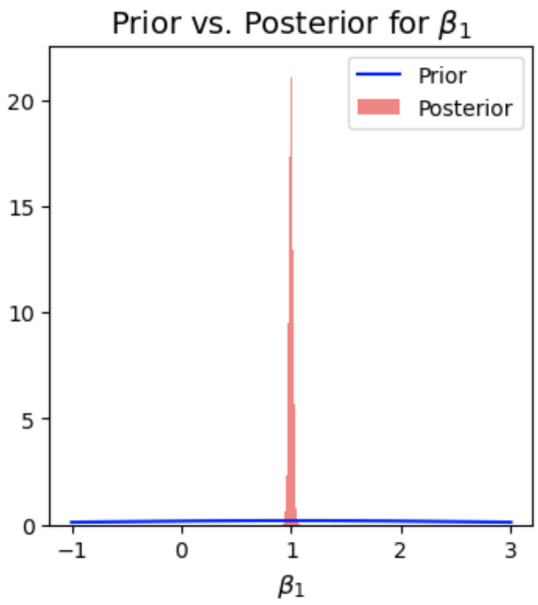

Figure 1: I_{t-1}

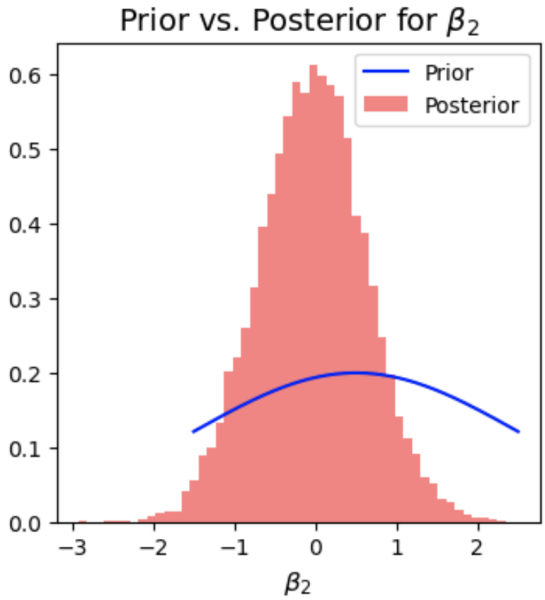

Figure 2: $EXCH_t$

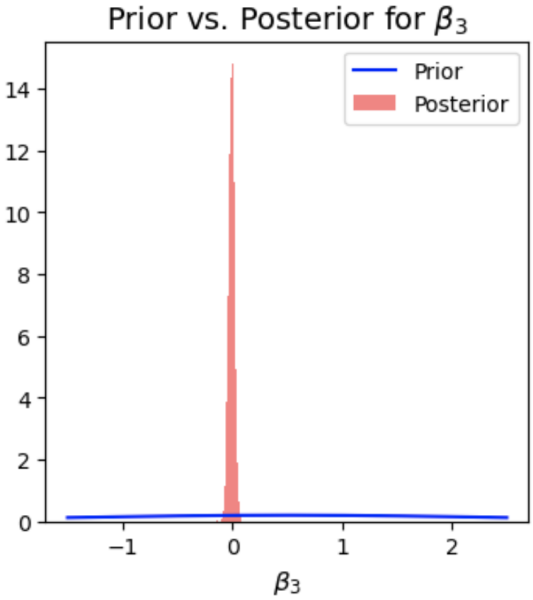

Figure 3: $INFL_t$

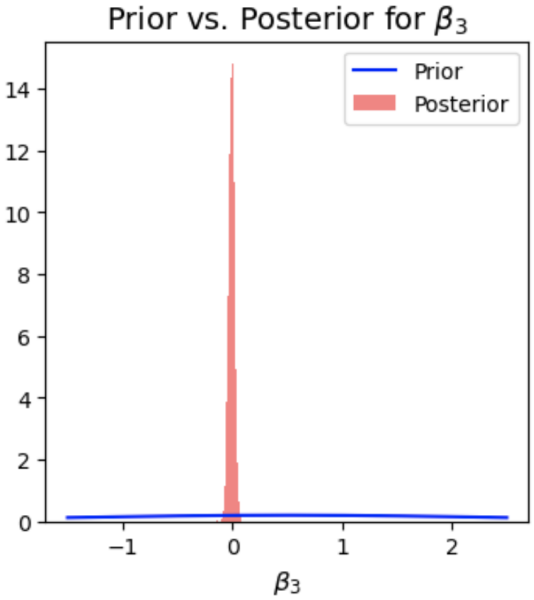

Figure 4: GDP_t

5.1.1 Serial correlation test

Table 5 does not exhibit any serial correlation in the lag term residuals and confirms that the hyper-parameters are stationary and do not suffer from endogeneity.

Table 5: Serial Correlation Test Results

Lag	ACF	PACF	Q-Stat	p-value
1	-0.014	-0.014	0.016	0.8991
2	0.029	0.030	0.087	0.9574
3	0.039	0.041	0.217	0.9748
4	-0.012	-0.012	0.229	0.9939

6 Conclusion

The study has effectively utilized a Bayesian multivariate linear regression to explore the nuanced interactions between monetary policy and exchange rates. Through this approach, it becomes evident that exchange rates, while not a primary focus of central banks, play a crucial role as a transmission mechanism within monetary policy frameworks. The Bayesian model provided a robust framework for assessing the impact of various economic indicators, including interest rates, inflation, and GDP on exchange rates.

Key findings from the empirical analysis suggest that previous short-term interest rates have a significant influence on current and future rates, indicating a strong inertia effect. The study also highlights that exchange rate adjustments, prompted by monetary policy measures, have subtle but significant impacts on the Federal Reserve's decisions regarding short-term interest rates. For example, an appreciating nominal exchange rate tends to lead to a minor reduction in federal funds rates, aiming to balance the economic impact.

Moreover, the study underscores the Federal Reserve's strategic use of unconventional monetary tools, such as large-scale asset purchases, especially during periods of economic downturn. These tools are employed to stabilize the economy by influencing liquidity and interest rates, thereby indirectly affecting exchange rates. Importantly, the Bayesian analysis emphasizes the need for careful consideration of monetary policy decisions. These decisions do not operate as a whole but are part of a complex system where exchange rates and other economic indicators interplay. The results caution against hasty monetary adjustments which could trigger

unintended consequences across the economy.

In conclusion, this analysis contributes to the broader understanding of monetary policy's indirect effects on exchange rates. It provides empirical support for policy formulations that consider the broader macroeconomic implications of exchange rate adjustments. Future research should continue to refine these models, perhaps integrating more dynamic economic factors such as the yield curve which captures interest expectations, to better predict and manage the ripple effects of monetary policy decisions on exchange rates.

References

1. Bureau of Labor Statistics (2024). Consumer Price Index for All Urban Consumers. *US Bureau of Labor Statistics* Retrieved April 19, 2024.https://beta.bls.gov/dataViewer/view/timeseries/CUUR0000SA0

2. Federal Reserve Bank of New York. (2018). Large-scale asset purchases. *Board of Governors of the Federal Reserve System (US)*. https://www.newyorkfed.org/markets/programs-archive/large-scale-asset-purchases

3. Federal Reserve Bank of St. Louis. (2024). Federal Funds Effective Rate. *Board of Governors of the Federal Reserve System (US)*. https://fred.stlouisfed.org/series/FEDFUNDS

4. Federal Reserve Bank of St. Louis. (2024). Money Market Funds; Security Repurchase Agreements (Federal Reserve Reverse Repurchase Agreement Operations); Asset, Level. *Board of Governors of the Federal Reserve System (US)*. https://fred.stlouisfed.org/series/BOGZ1FL632051103Q

5. Inoue, A., & Rossi B. (2019). The effects of conventional and unconventional monetary policy on exchange rates. *Journal of International Economics*, 118, 419–447. https://doi.org/10.1016/j.jinteco.2019.01.015

6. Mindford, P. (1981). The exchange rate and monetary policy. *Oxford Economic Papers, 33(Supplement: The Money Supply and the Exchange Rate*, 120-142. https://www.jstor.org/stable/2662786

7. Taylor, J. B. (2001). The Role of the Exchange Rate in Monetary-Policy Rules. *American Economic Review*, 91(2), 263-267.

8. Yahoo Finance. (2024). EUR/USD (EURUSD=X) Live Rate. *Yahoo Finance* Retrieved April 19, 2024.https://finance.yahoo.com/quote/EURUSD=X/