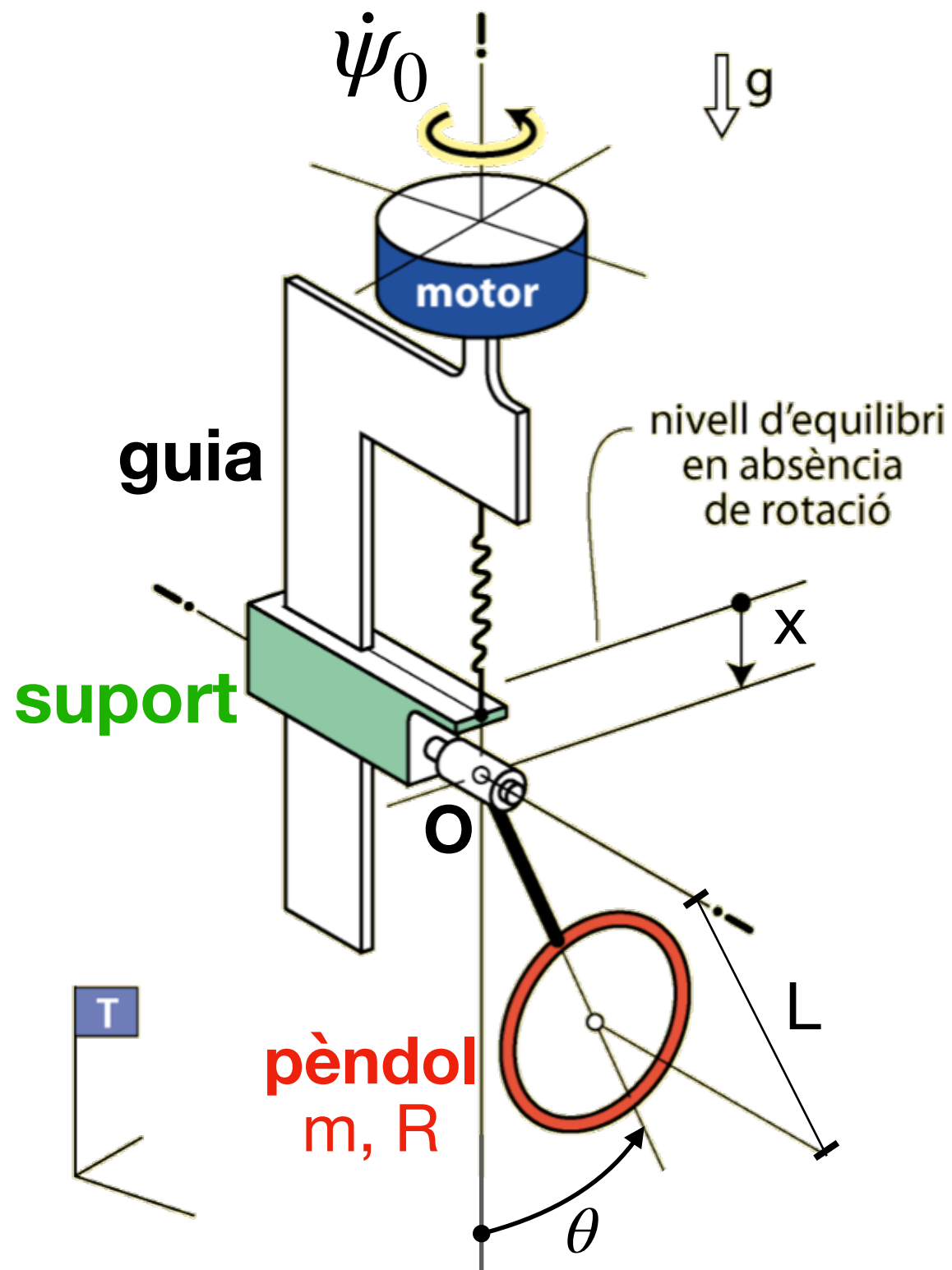


13P

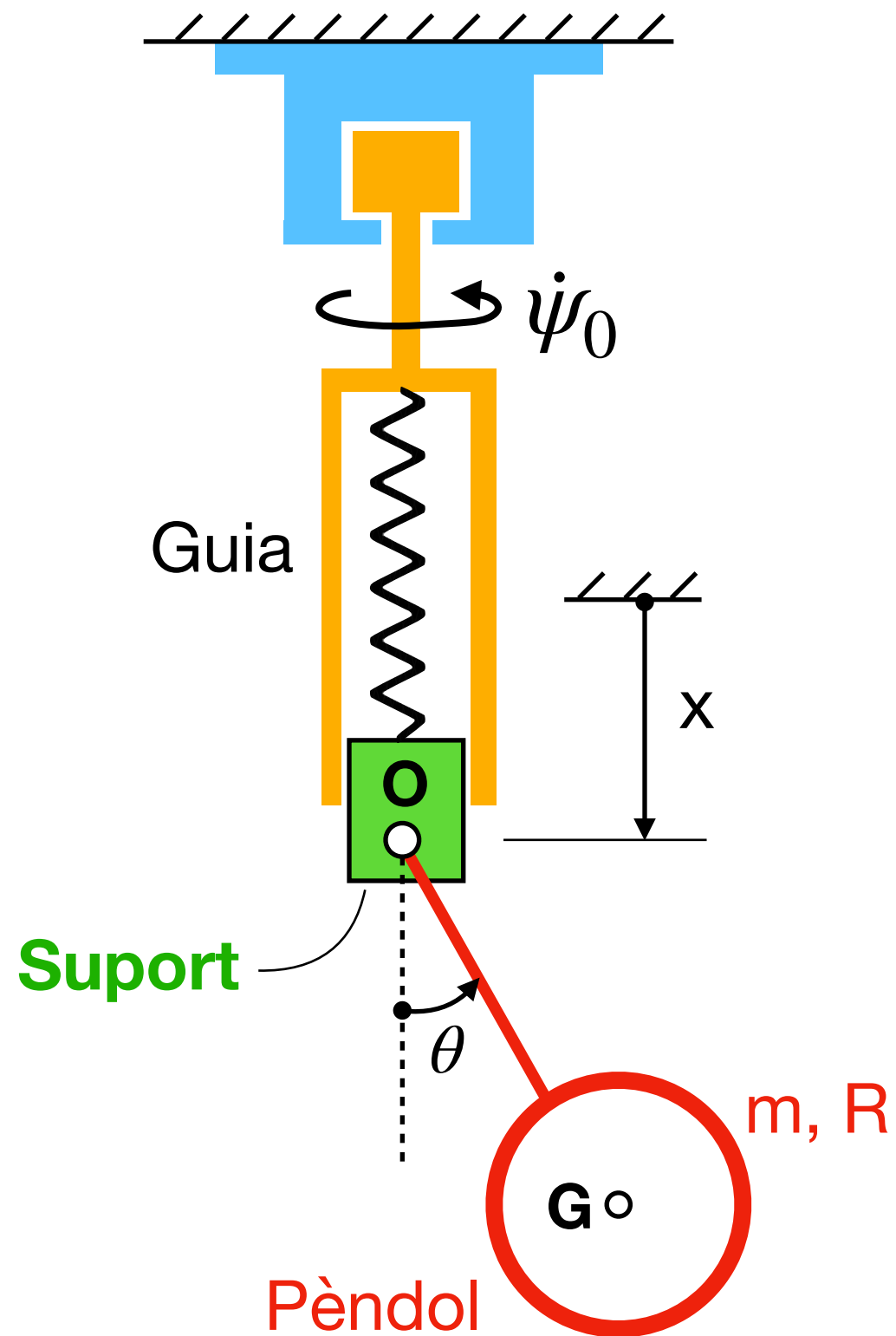
Teoremes vectorials III

Exemples 3D

- DGI
- Eqs. mov. per a x i θ
- Parell motor Γ per mantenir $\dot{\psi}_0 = \text{ct}$

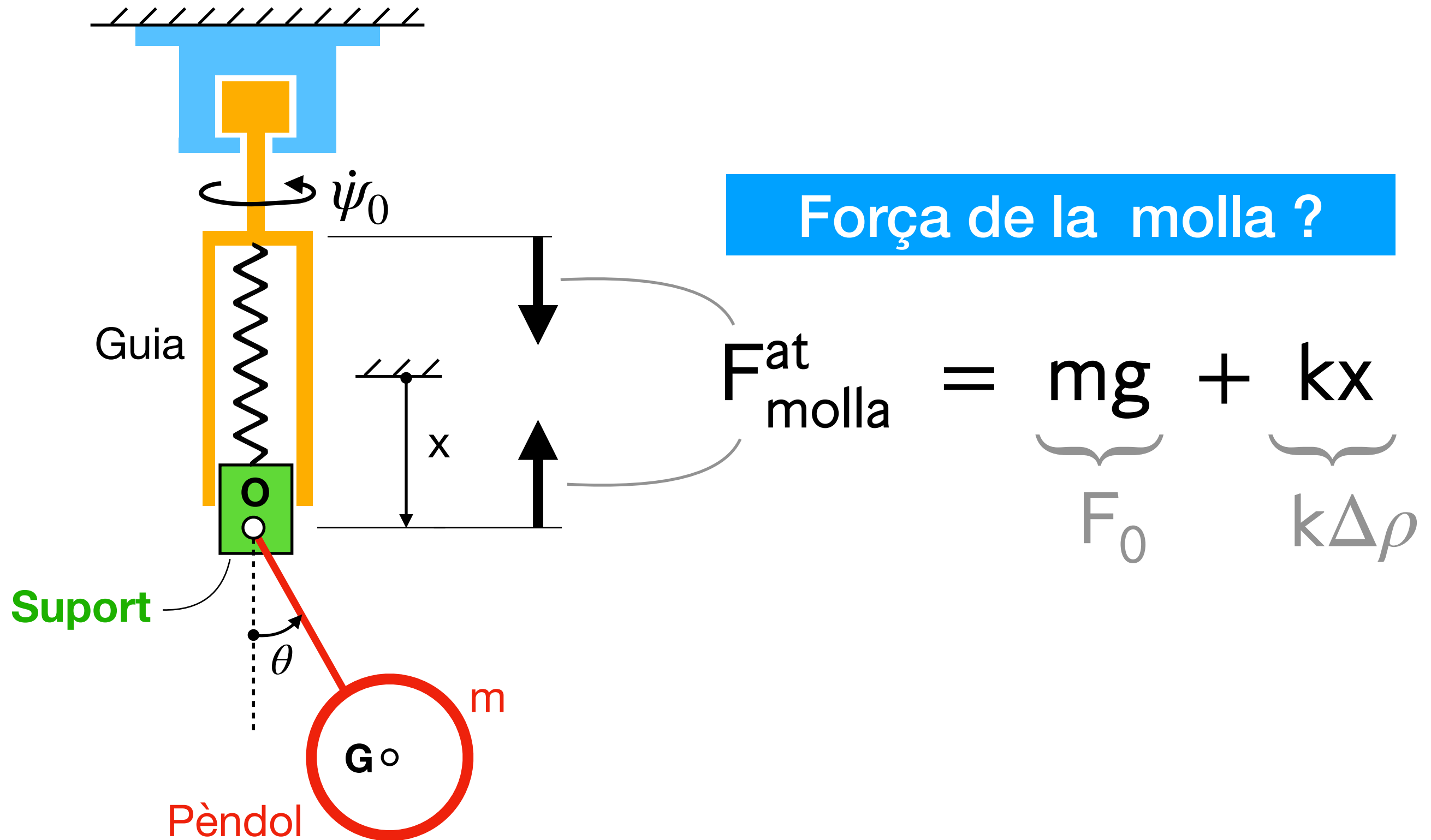


Amb motor aturat: $x = 0$, $\theta = 0$ és config. d'equilibri



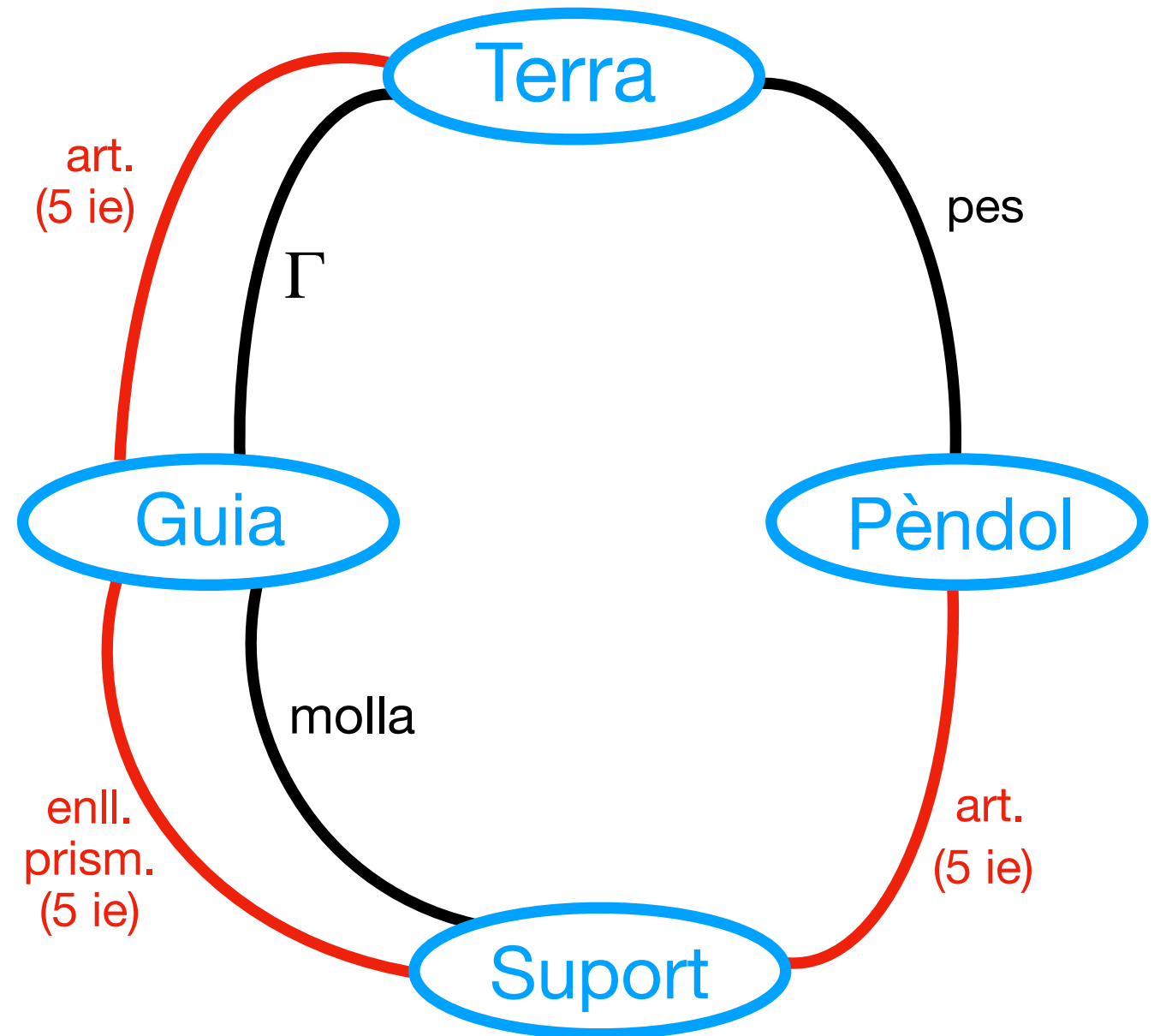
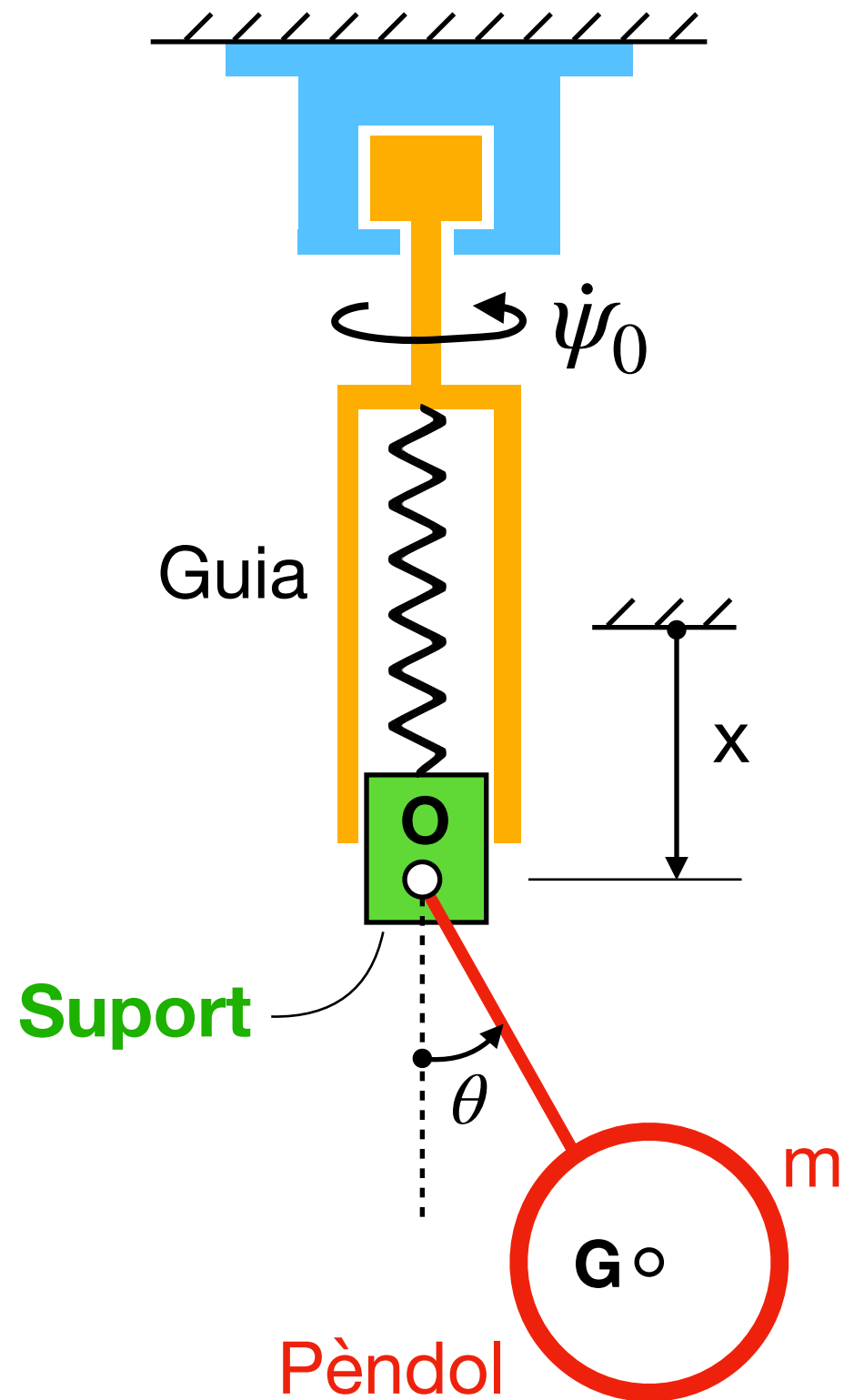
- DGI
- Eqs. mov. per a x i θ
- Parell motor Γ per mantenir $\dot{\psi}_0 = ct$

Amb motor aturat: $x = 0$, $\theta = 0$ és config. d'equilibri



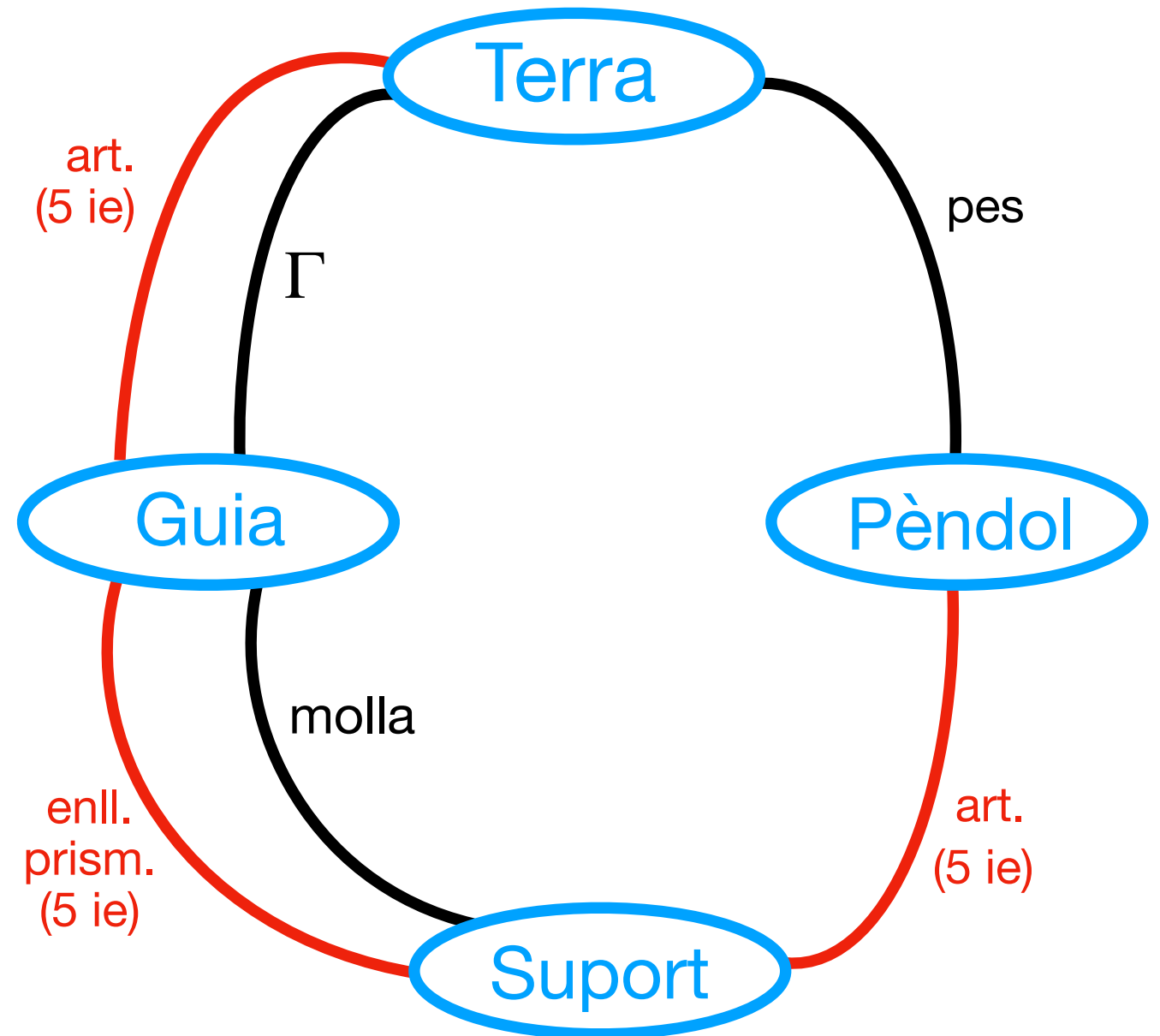
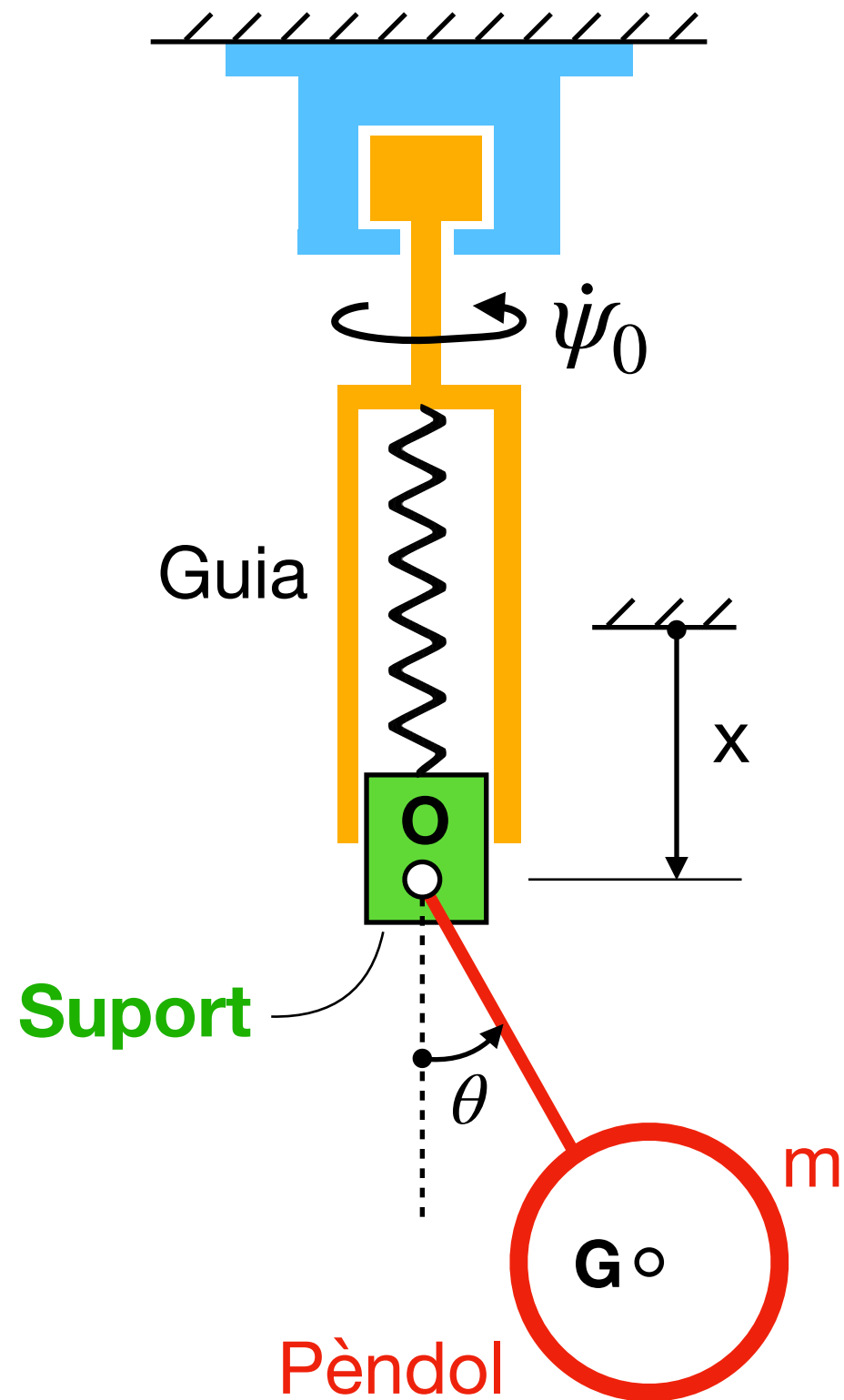
Amb motor aturat: $x = 0$, $\theta = 0$ és config. d'equilibri

DGI



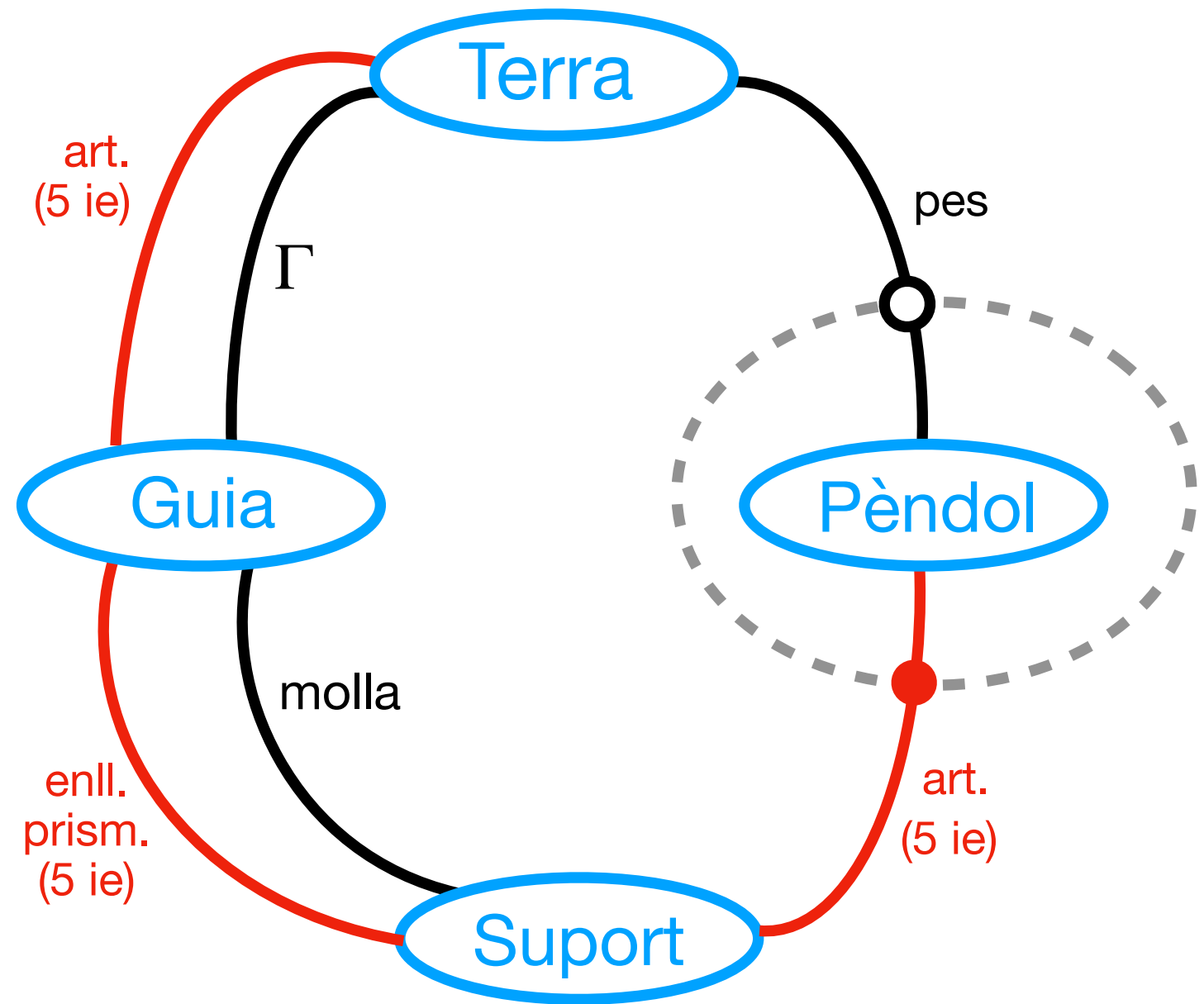
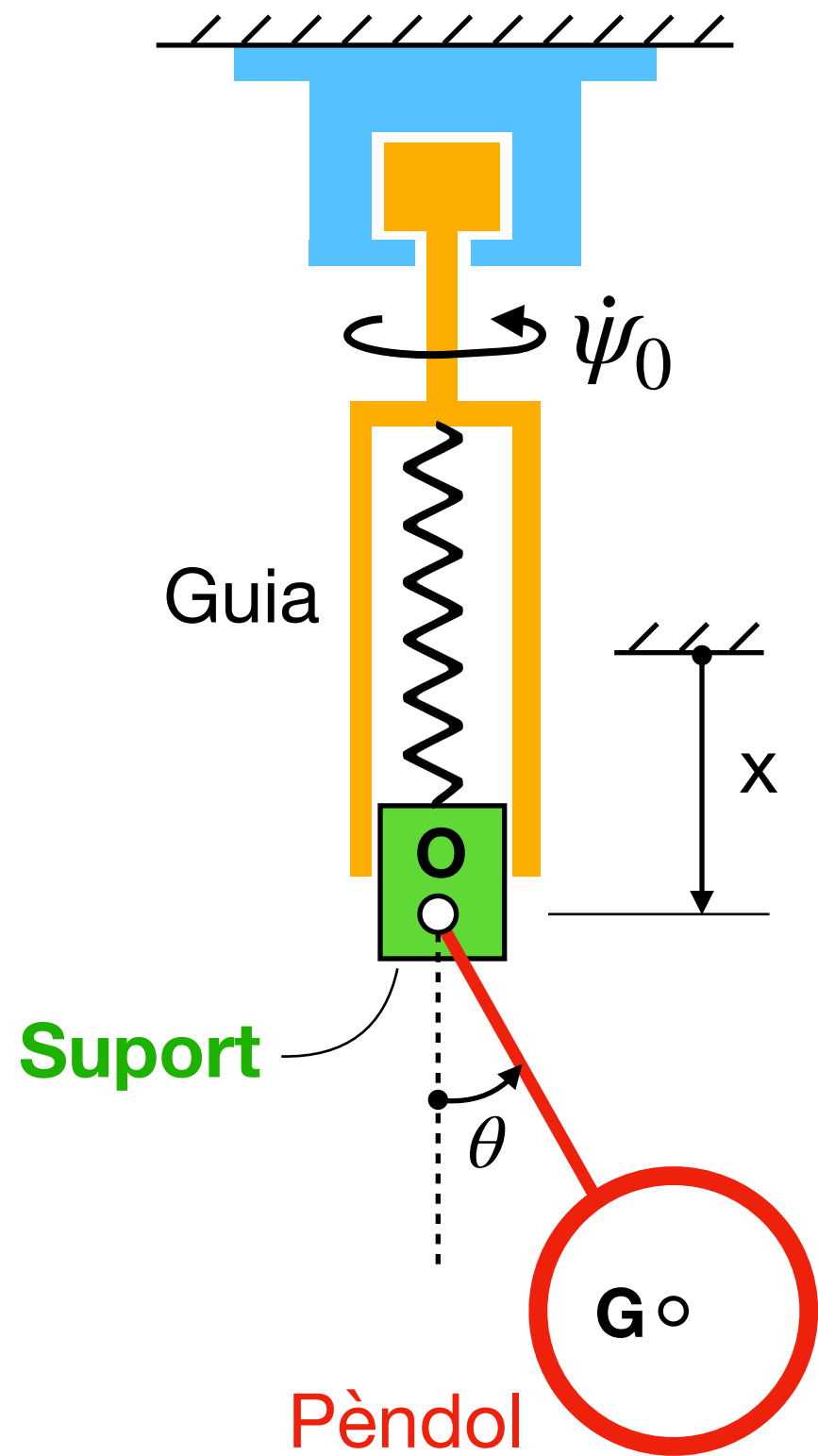
$$\left. \begin{array}{l} 15 \text{ ie}, \Gamma, \ddot{x}, \ddot{\theta} \Rightarrow 18 \text{ incòg} \\ 3 \text{ sòlids} \cdot 6 \text{ eqs/sòlid} \Rightarrow 18 \text{ eqs} \end{array} \right\} \text{DET}$$

DGI

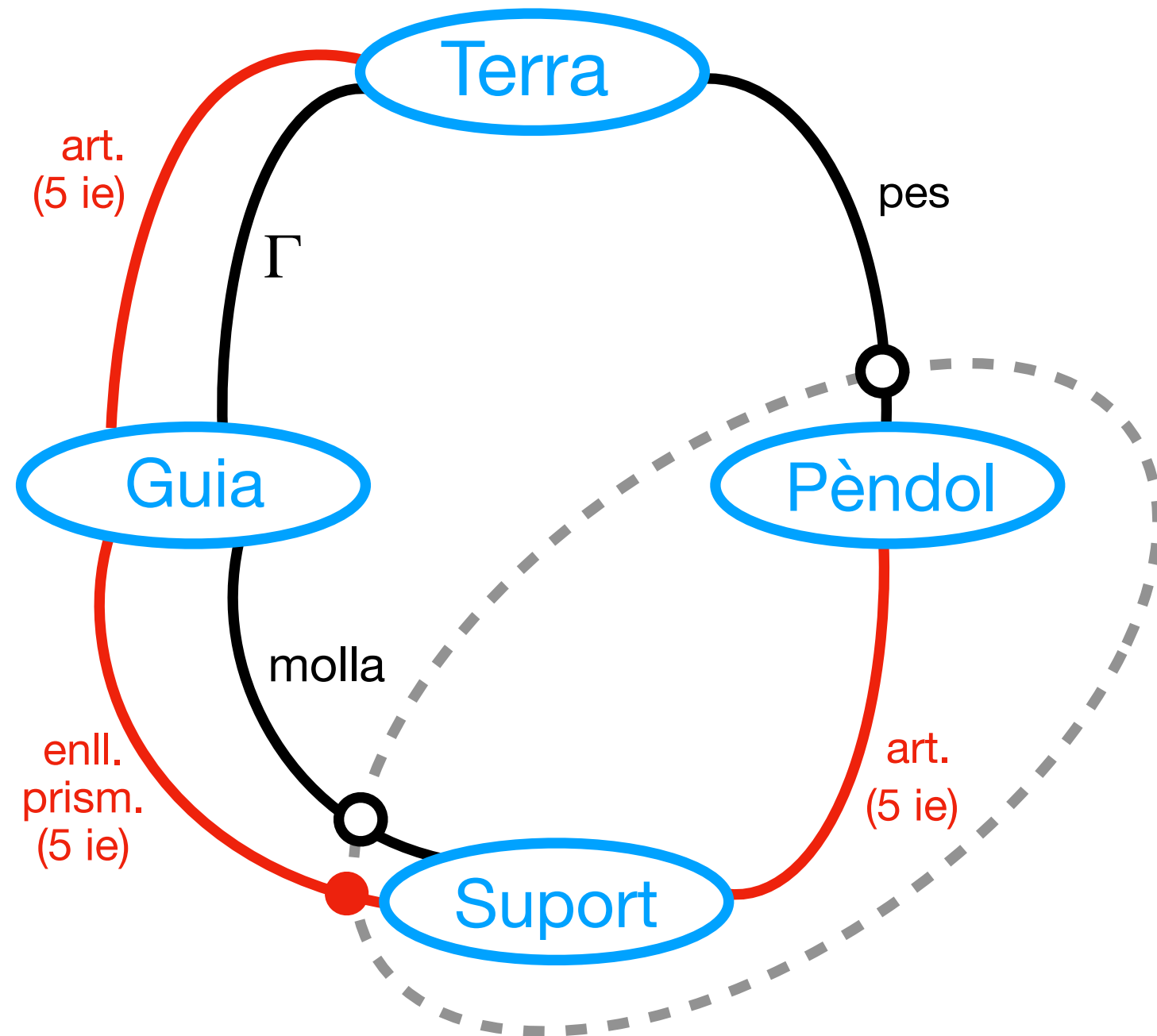
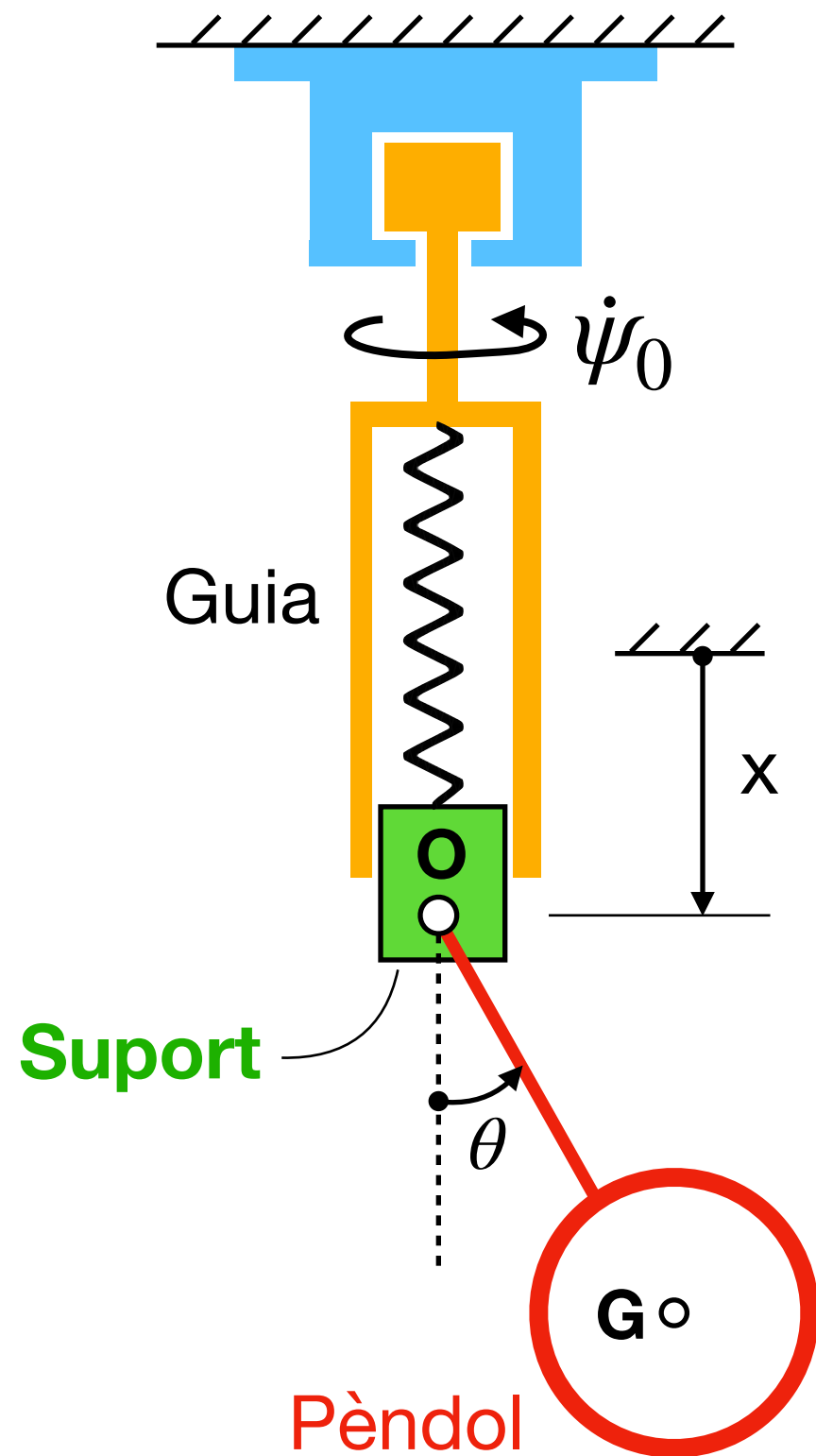


x i θ només afecten **pèndol** i **suport**

Explorem sistemes que els incloguin



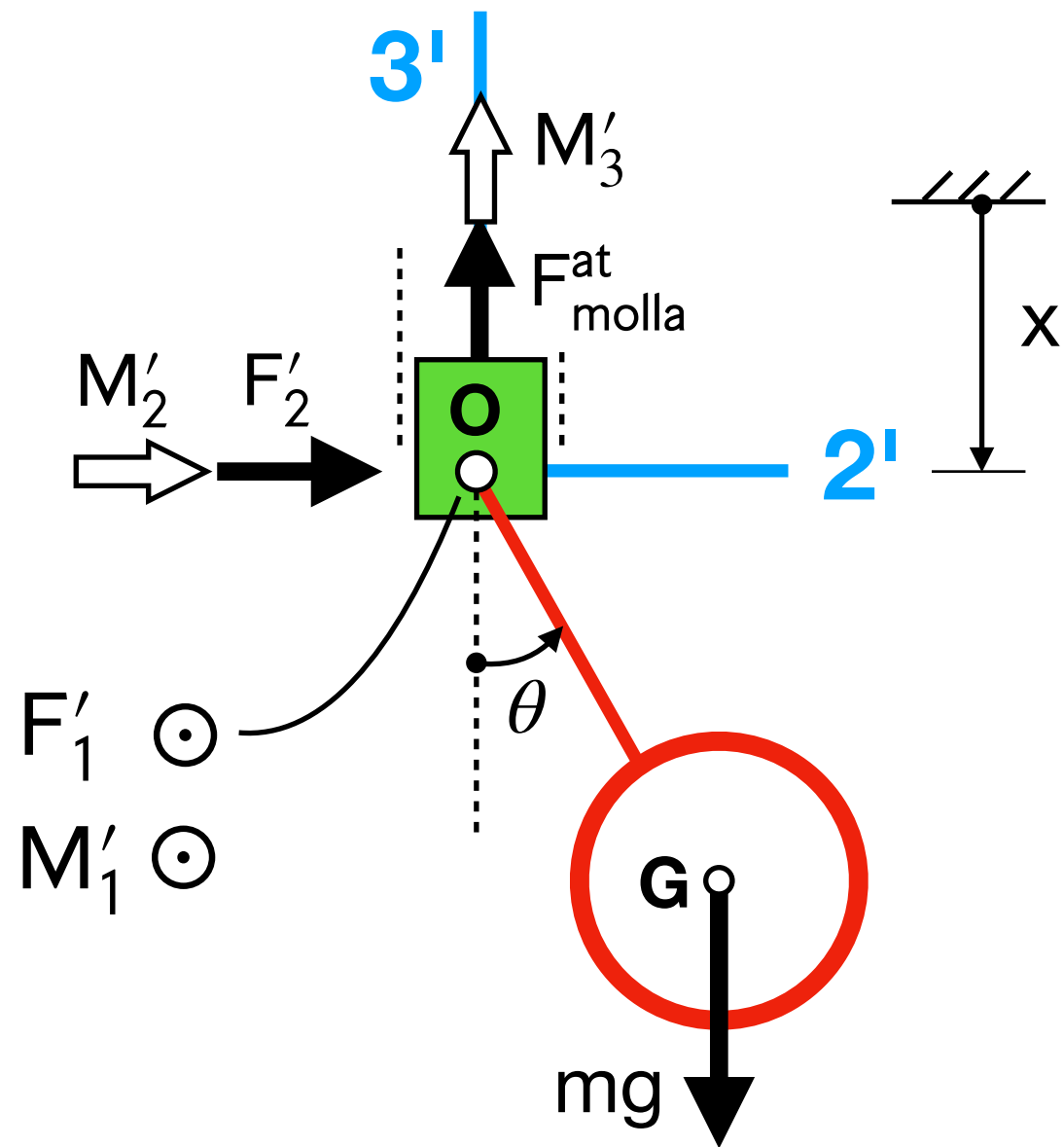
Sistema	Incògn.	Problema
Pèndol	5 ie, \ddot{x} , $\ddot{\theta}$	INDET



Sistema	Incògn.	Problema
Pèndol	5 ie, \ddot{x} , $\ddot{\theta}$	INDET
Pènd. + sup.	5 ie, \ddot{x} , $\ddot{\theta}$	INDET

Els altres sistemes tenen + incògnites !

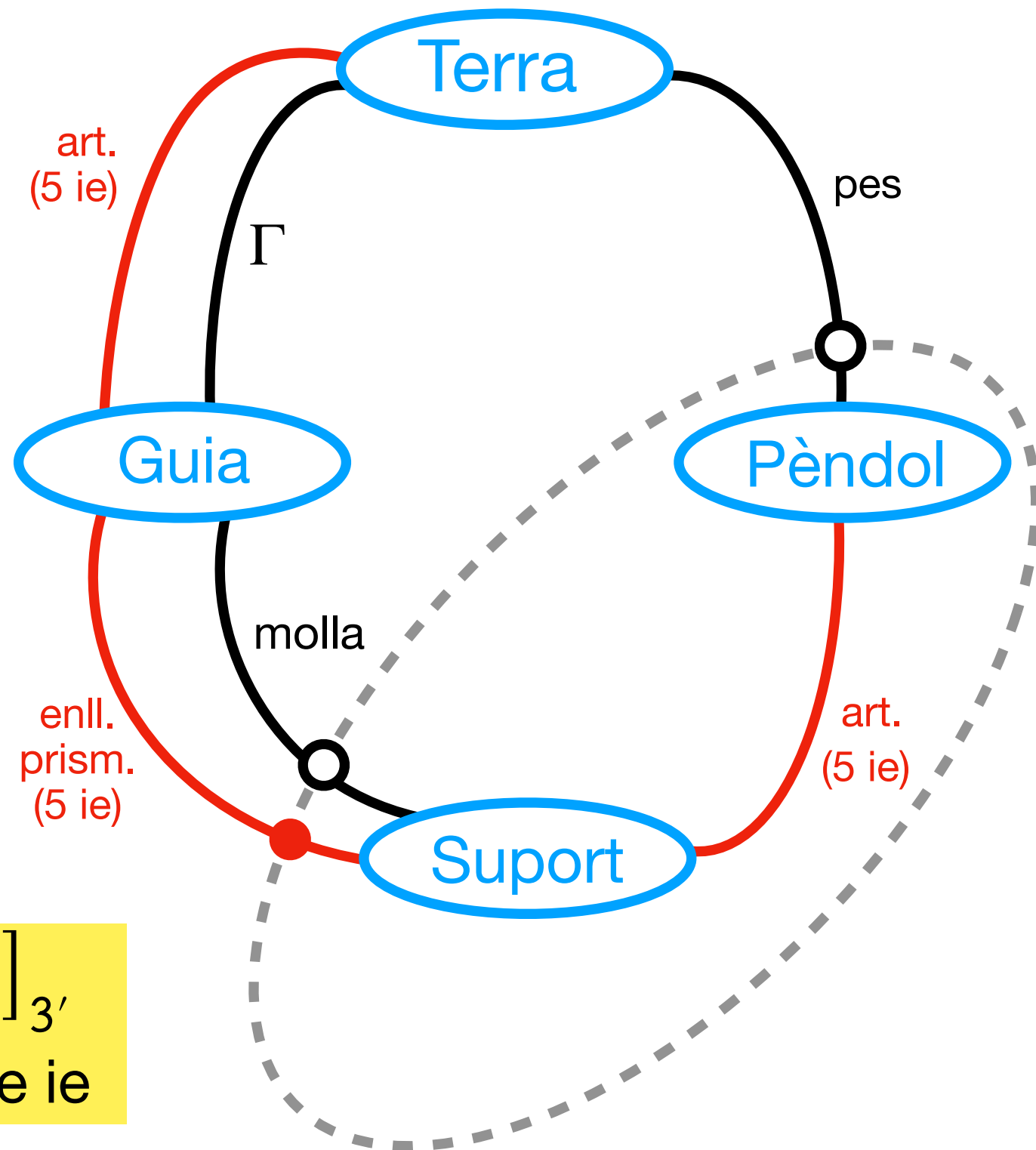
Sist = Pèndol + Suport



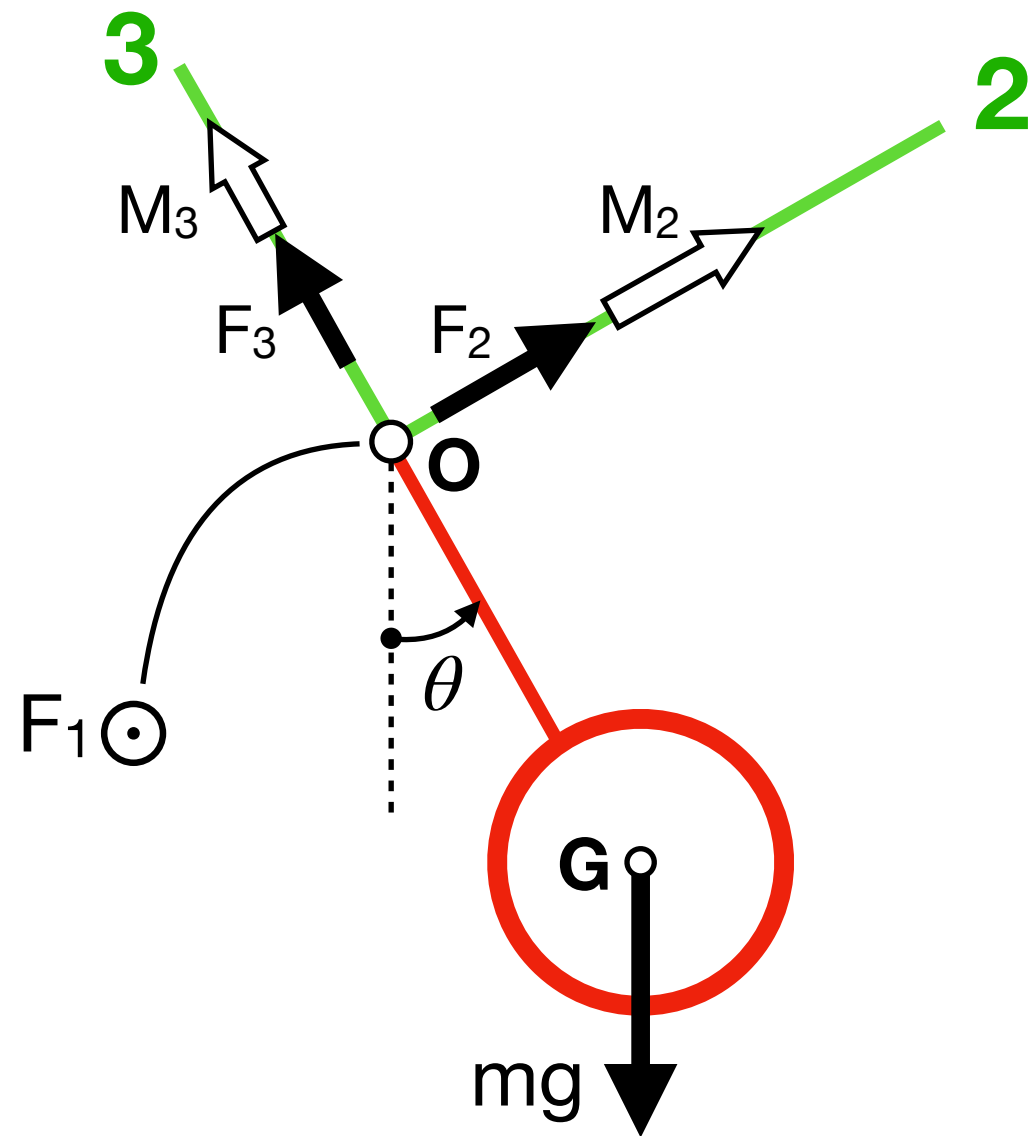
$$\left\{ \bar{F}_{Guia \rightarrow Sup} \right\}_{B'} = \begin{Bmatrix} F'_1 \\ F'_2 \\ 0 \end{Bmatrix}$$

$$\left\{ \bar{M}_{Guia \rightarrow Sup} (O) \right\}_{B'} = \begin{Bmatrix} M'_1 \\ M'_2 \\ M'_3 \end{Bmatrix}$$

$TQM]_{3'}$
lliure de ie

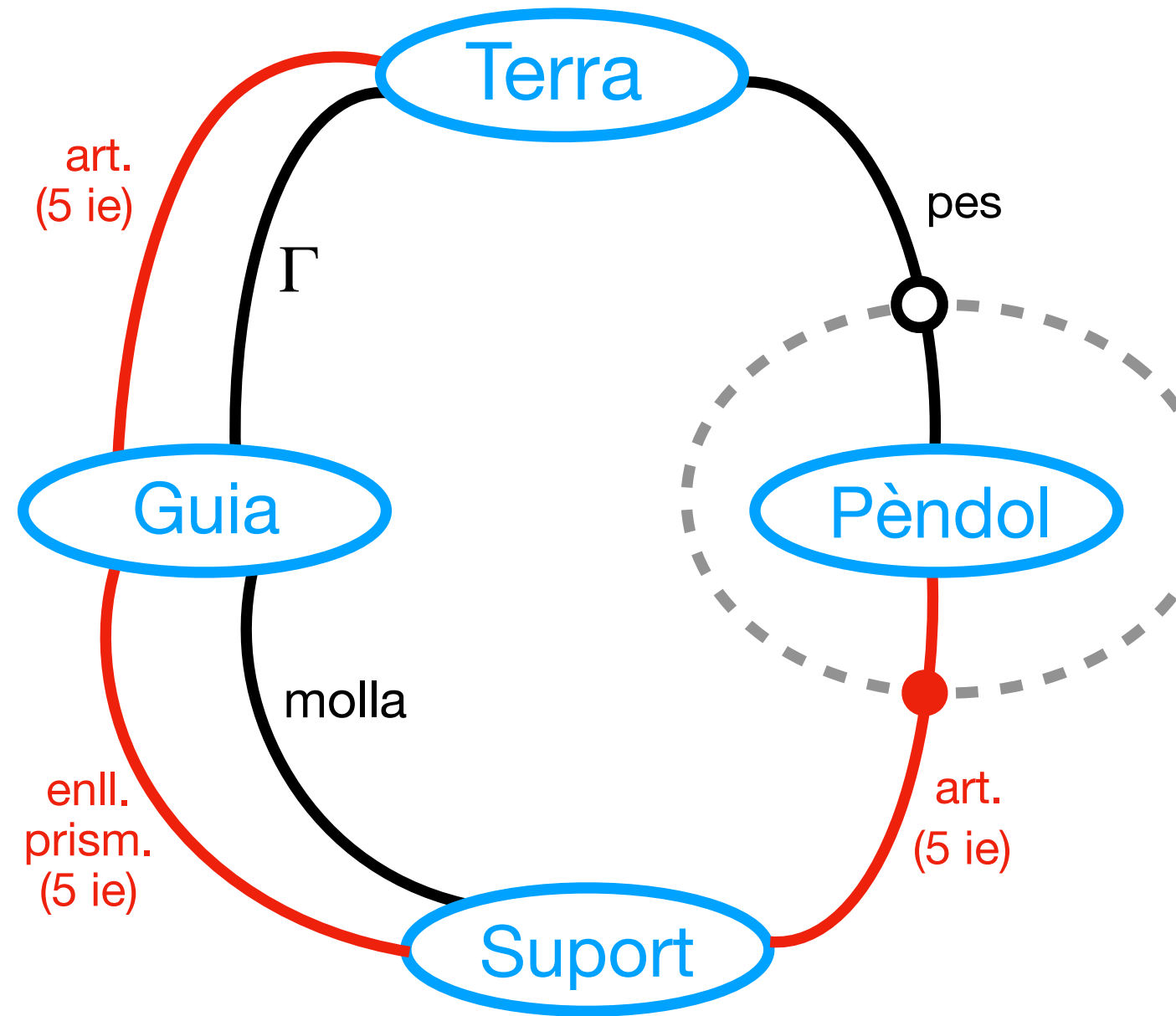


Sist = Pèndol

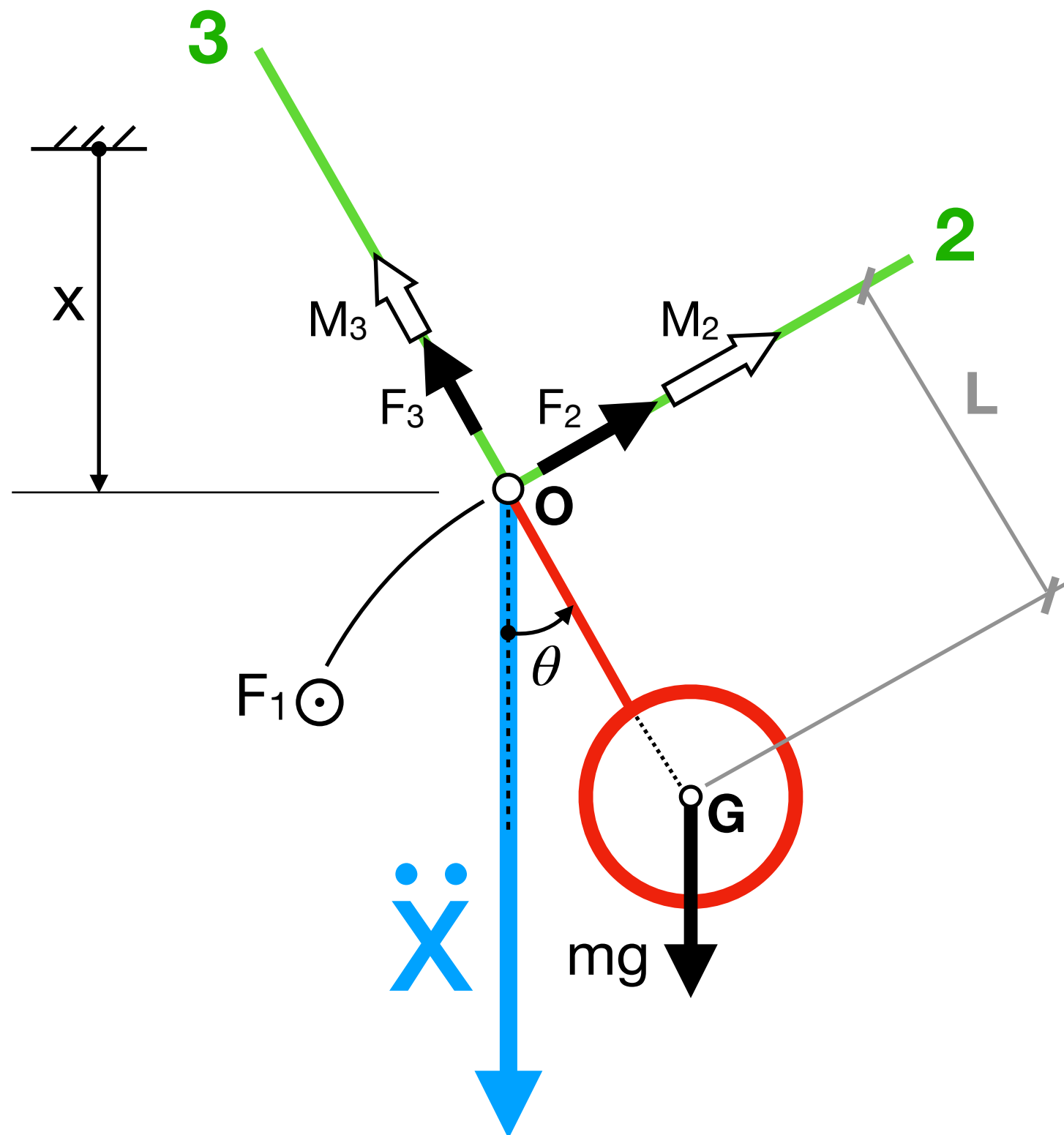


$$\left\{ \bar{\mathbf{F}}_{\text{Sup} \rightarrow \text{Pendol}} \right\}_{\mathbf{B}} = \begin{Bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \\ \mathbf{F}_3 \end{Bmatrix}$$

$$\left\{ \bar{\mathbf{M}}_{\text{Sup} \rightarrow \text{Pendol}} (\mathbf{O}) \right\}_{\mathbf{B}} = \begin{Bmatrix} 0 \\ M_2 \\ M_3 \end{Bmatrix} \leftarrow \begin{array}{l} \mathbf{TMC}(\mathbf{O})]_1 \\ \text{lliure de ie} \end{array}$$



TMC(O)]₁ sobre SIST = Pèndol



TMC(O)]₁ sobre SIST = Pèndol

$$\left\{ \dot{\ddot{H}}_{\text{RTO}}(\mathbf{O}) \right\}_{\text{B}} = \left\{ \begin{array}{l} I_{11} \ddot{\theta} + (I_{33} - I_{22}) \dot{\psi}_0^2 \sin \theta \cos \theta \\ (I_{11} + I_{22} - I_{33}) \dot{\psi}_0 \dot{\theta} \cos \theta \\ (I_{22} - I_{11} - I_{33}) \dot{\psi}_0 \dot{\theta} \sin \theta \end{array} \right\} \quad \text{(III)}$$

Pas final

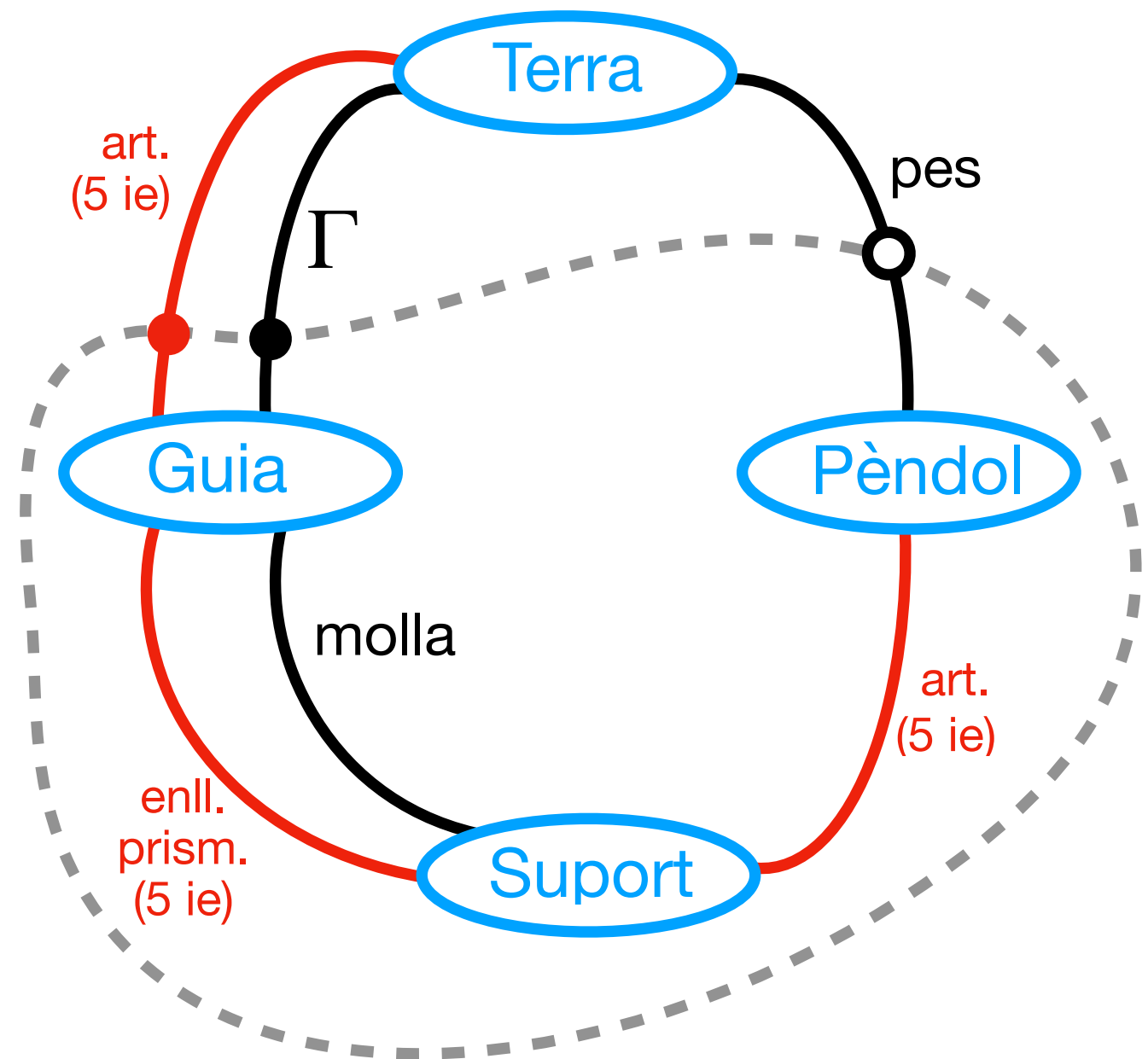
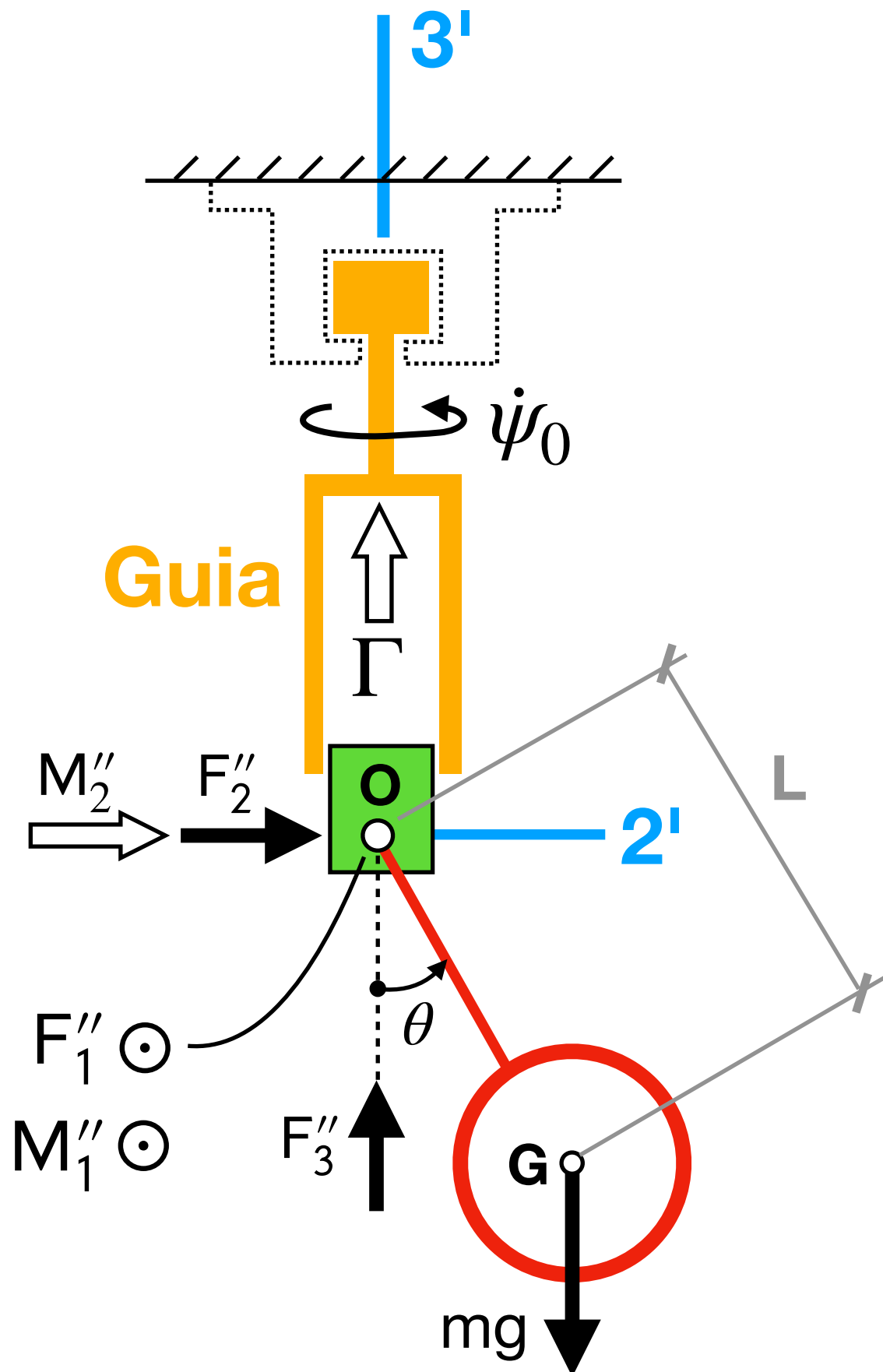
$$\begin{cases} (R^2 + L^2) \ddot{\theta} - (L \sin \theta) \ddot{x} = (L \dot{\psi}_0^2 \cos \theta - g) L \sin \theta \\ - (L \sin \theta) \ddot{\theta} + \ddot{x} = -\frac{k}{m} x + L \dot{\theta}^2 \cos \theta \end{cases}$$

Aillant $\ddot{\theta}$ i \ddot{x}

$$\begin{cases} \ddot{\theta} = F_1(\theta, \dot{\theta}, x, \dot{x}) & \text{Eq. mov. } \theta \\ \ddot{x} = F_2(\theta, \dot{\theta}, x, \dot{x}) & \text{Eq. mov. } x \end{cases}$$

No cal
que el feu

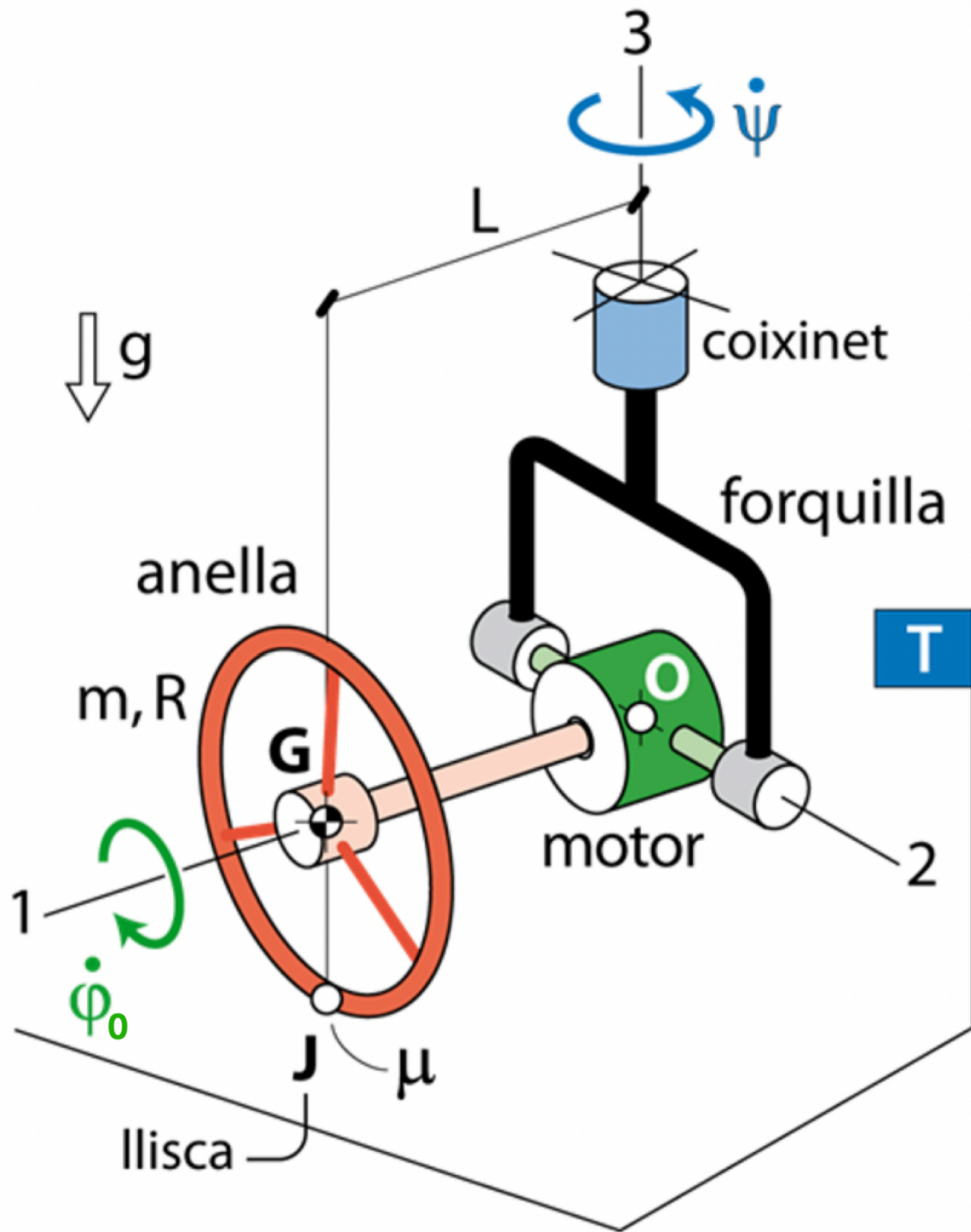
Parell motor per mantenir $\dot{\psi}_0 = ct$



5 ie, Γ , \ddot{x} , $\ddot{\theta}$ \Rightarrow 8 incòg } **DET**
 Ara ja sabem \ddot{x} , $\ddot{\theta}$

D'abans

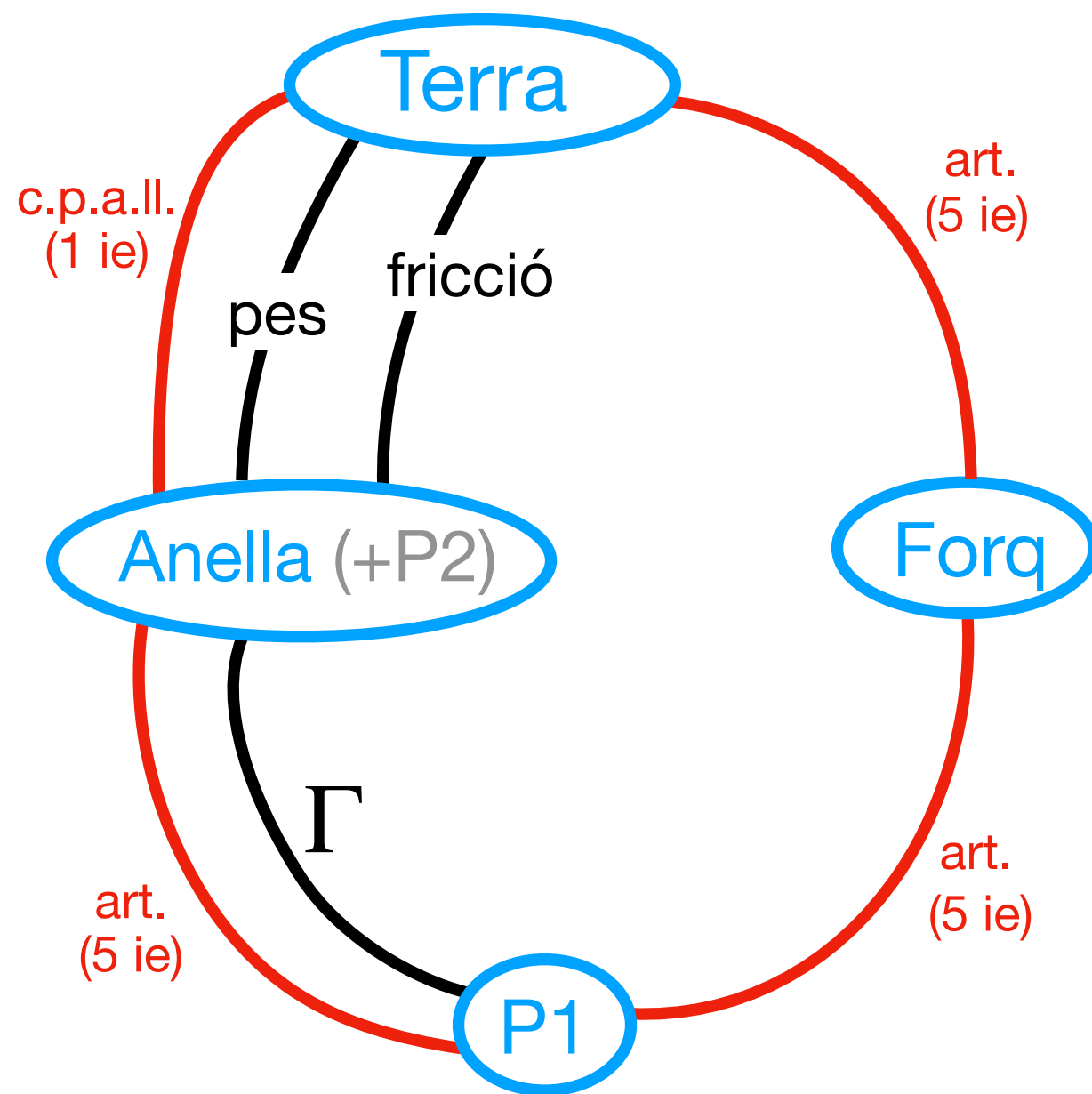
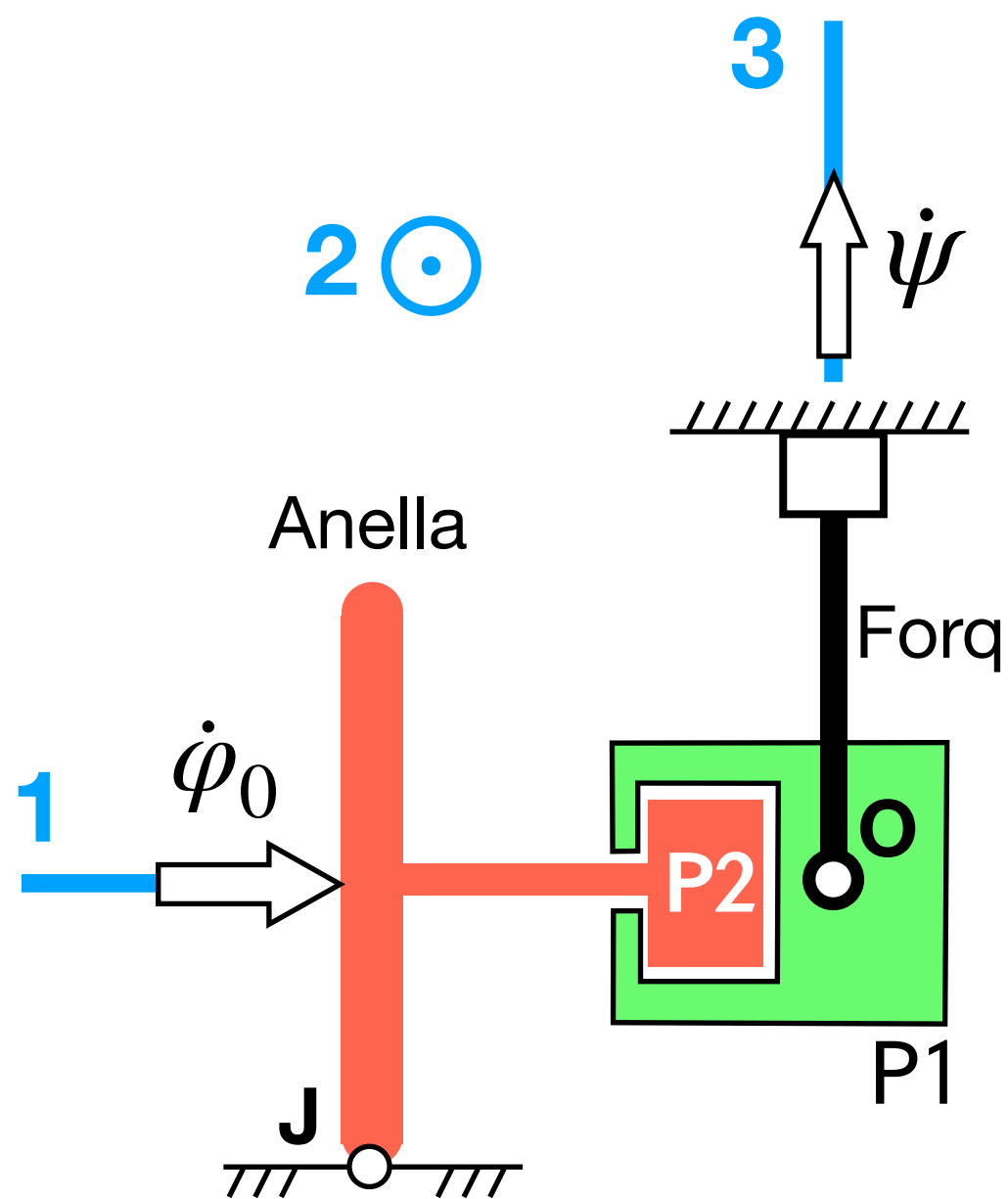
$$\left\{ \dot{\ddot{\mathbf{H}}}_{\text{RTO}}(\mathbf{O}) \right\}_{\text{B}} = \begin{Bmatrix} I_{11}\ddot{\theta} + (I_{33} - I_{22})\dot{\psi}_0^2 \sin\theta \cos\theta \\ (I_{11} + I_{22} - I_{33})\dot{\psi}_0 \dot{\theta} \cos\theta \\ (I_{22} - I_{11} - I_{33})\dot{\psi}_0 \dot{\theta} \sin\theta \end{Bmatrix} = \begin{Bmatrix} \dot{\mathbf{H}}_1 \\ \dot{\mathbf{H}}_2 \\ \dot{\mathbf{H}}_3 \end{Bmatrix}$$



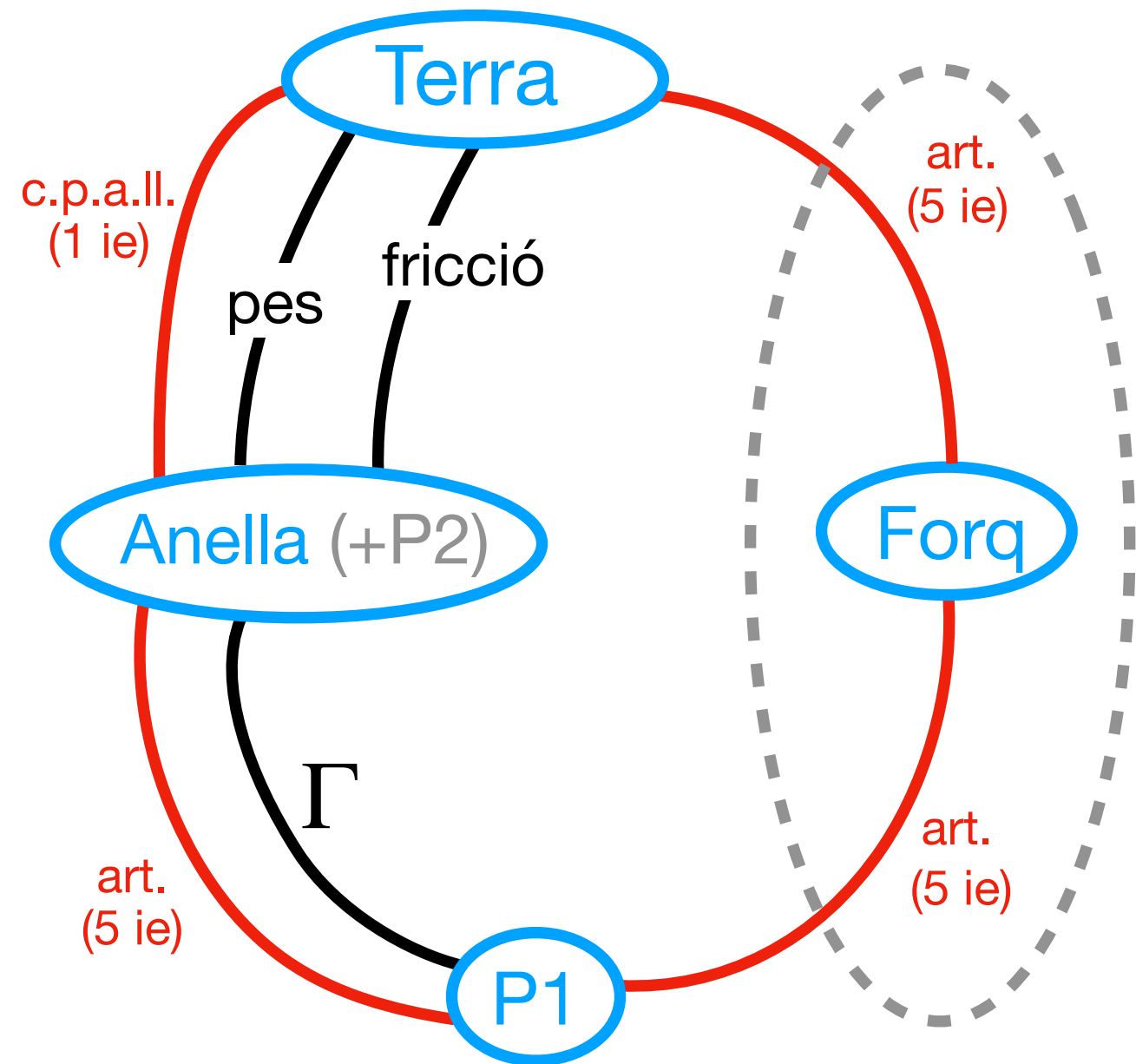
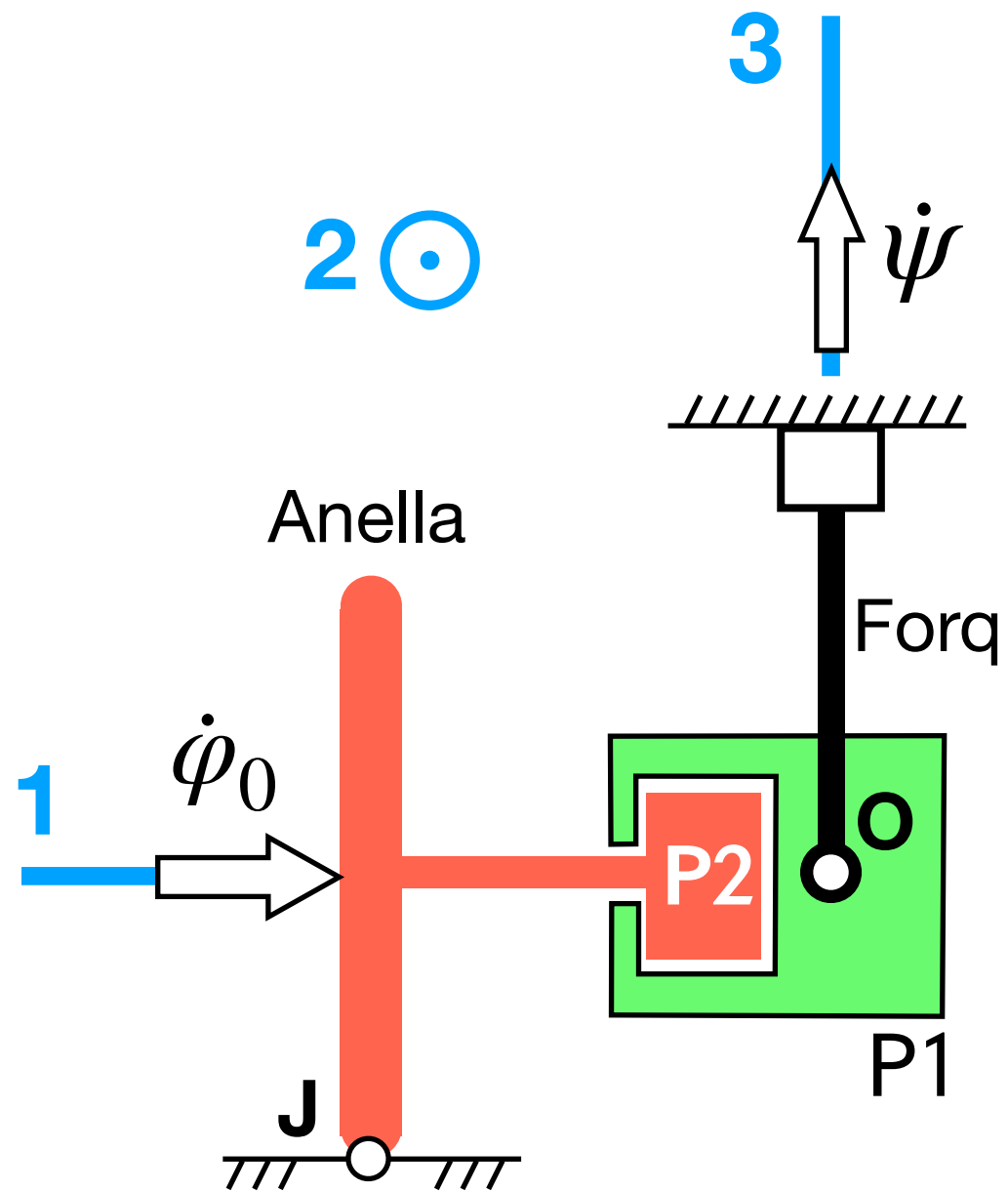
- GL sistema?
- DGI
- Caracterització de torsors?
- Eq. mov per a la coord. ψ
- Força normal a J
- Parell motor per mantenir $\dot{\phi}_0$

Manté contacte puntual a J

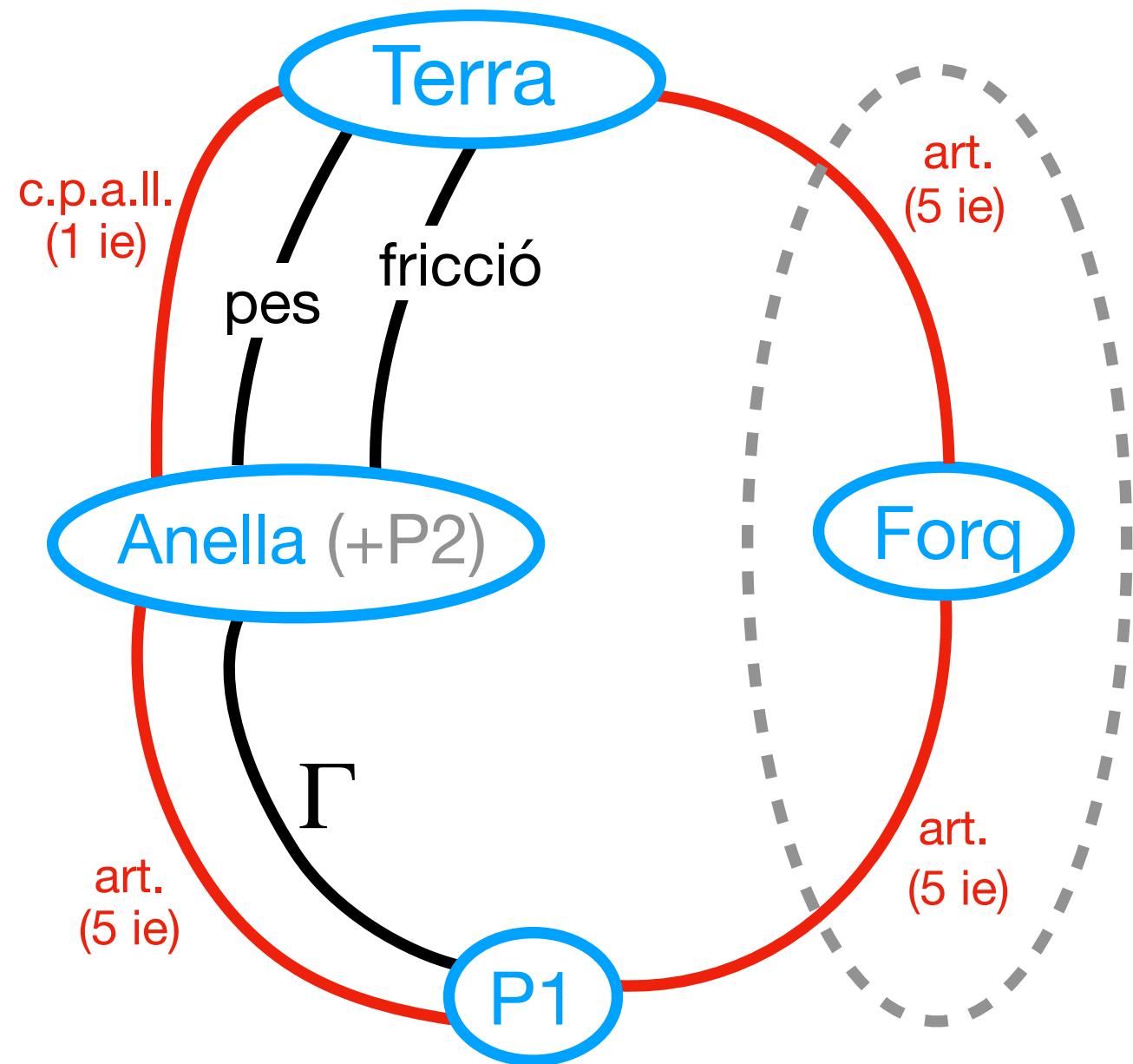
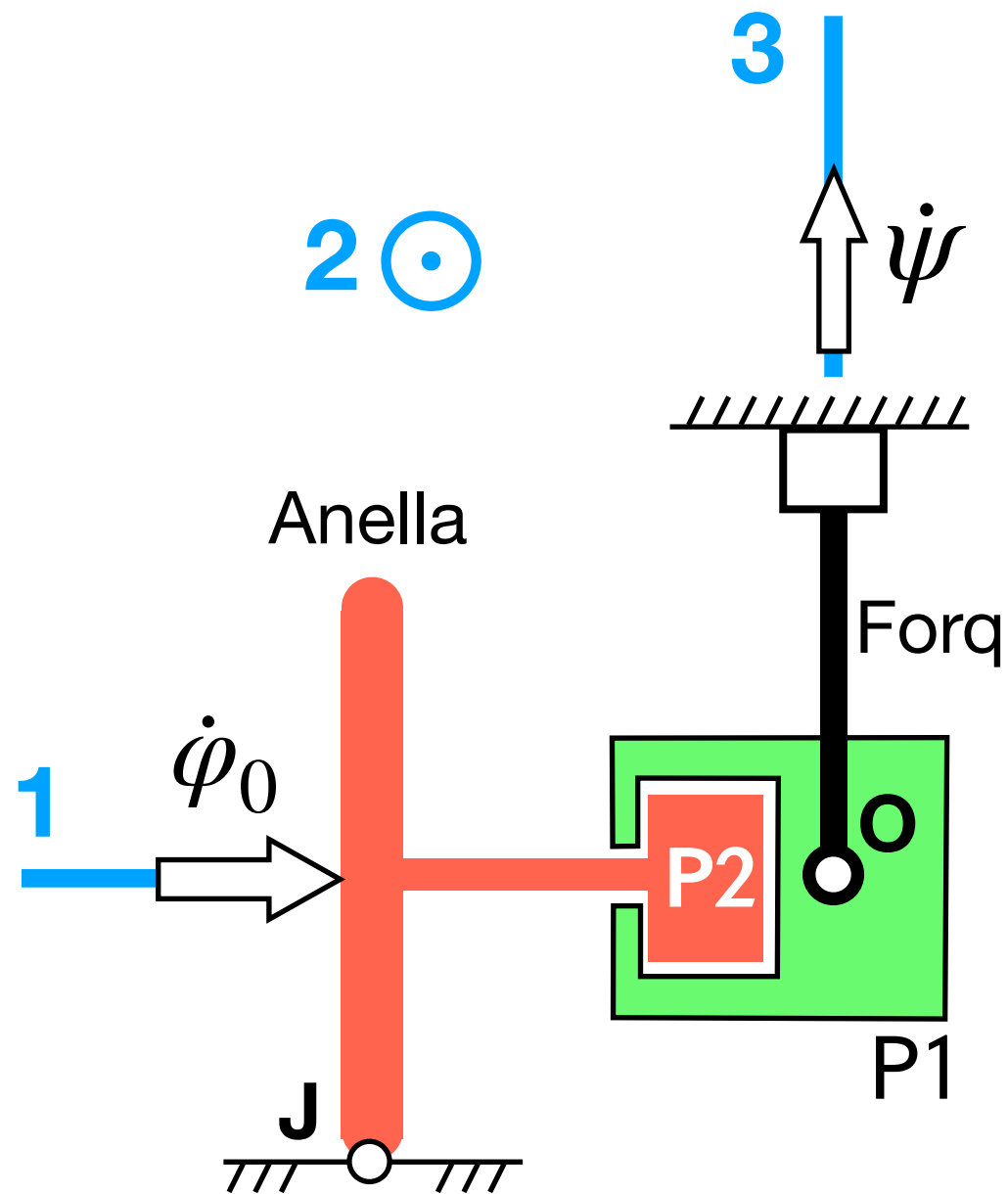
Motor manté $\dot{\phi}_0 = \text{ct}$



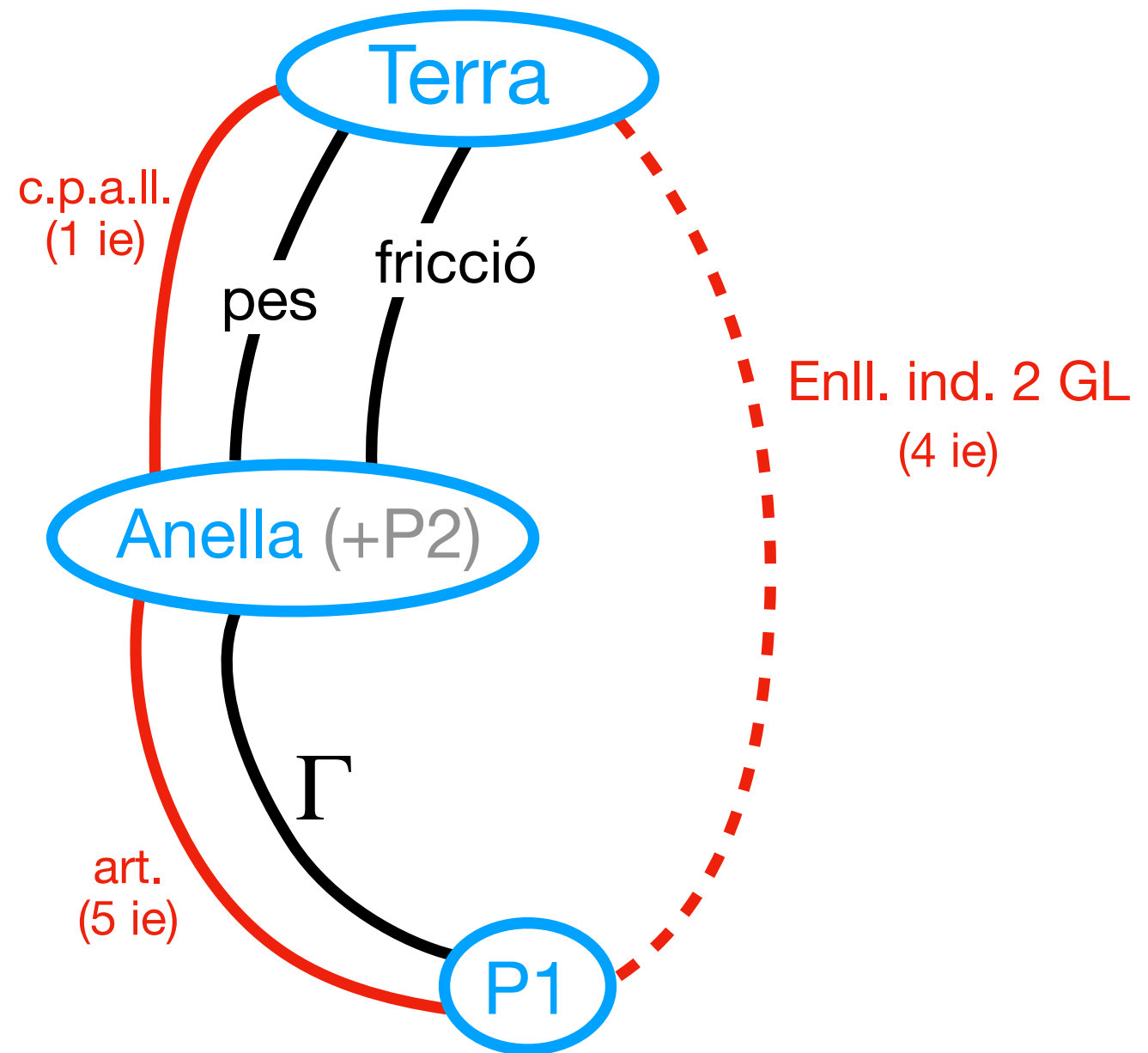
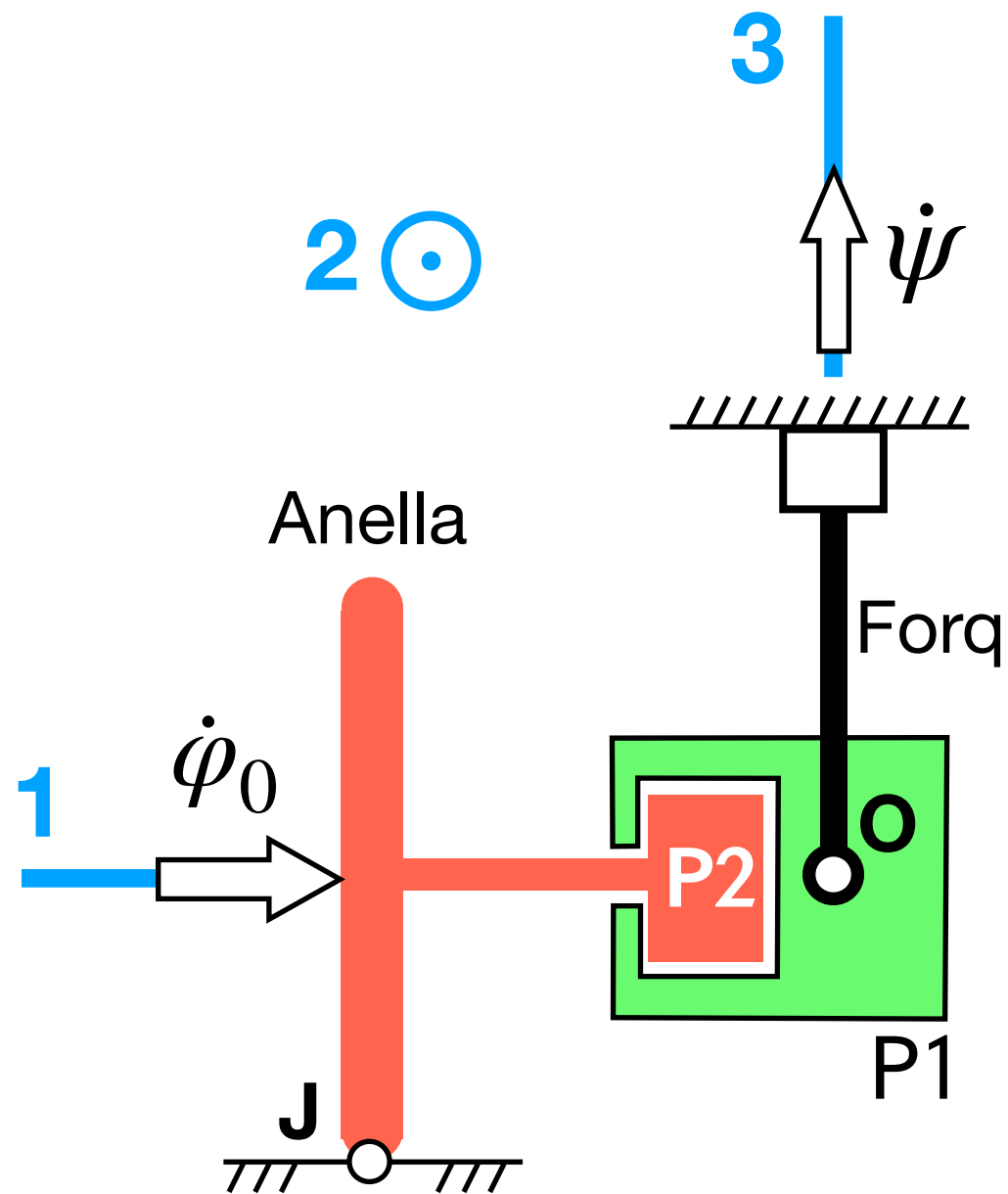
Forq és SAE ...



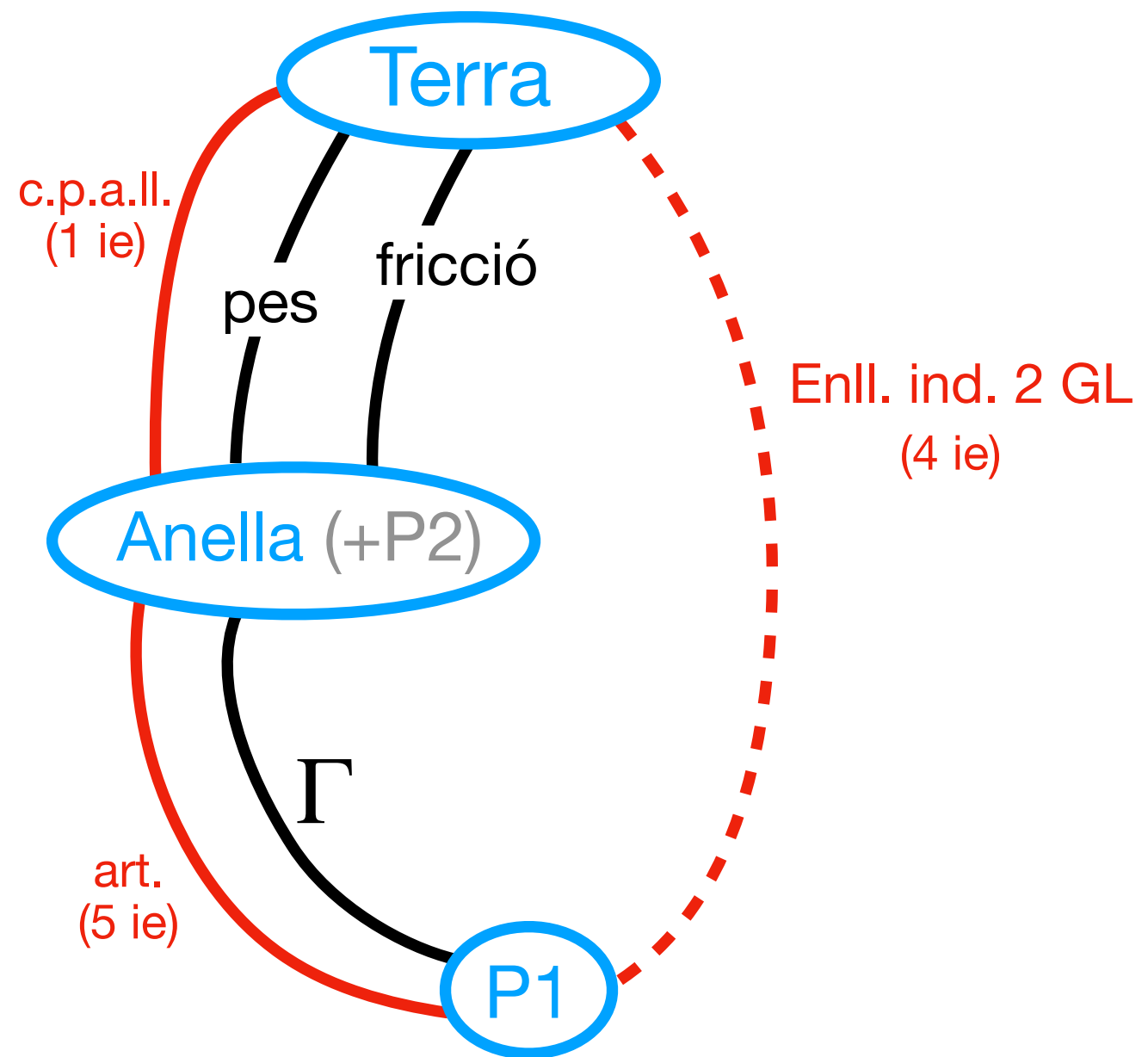
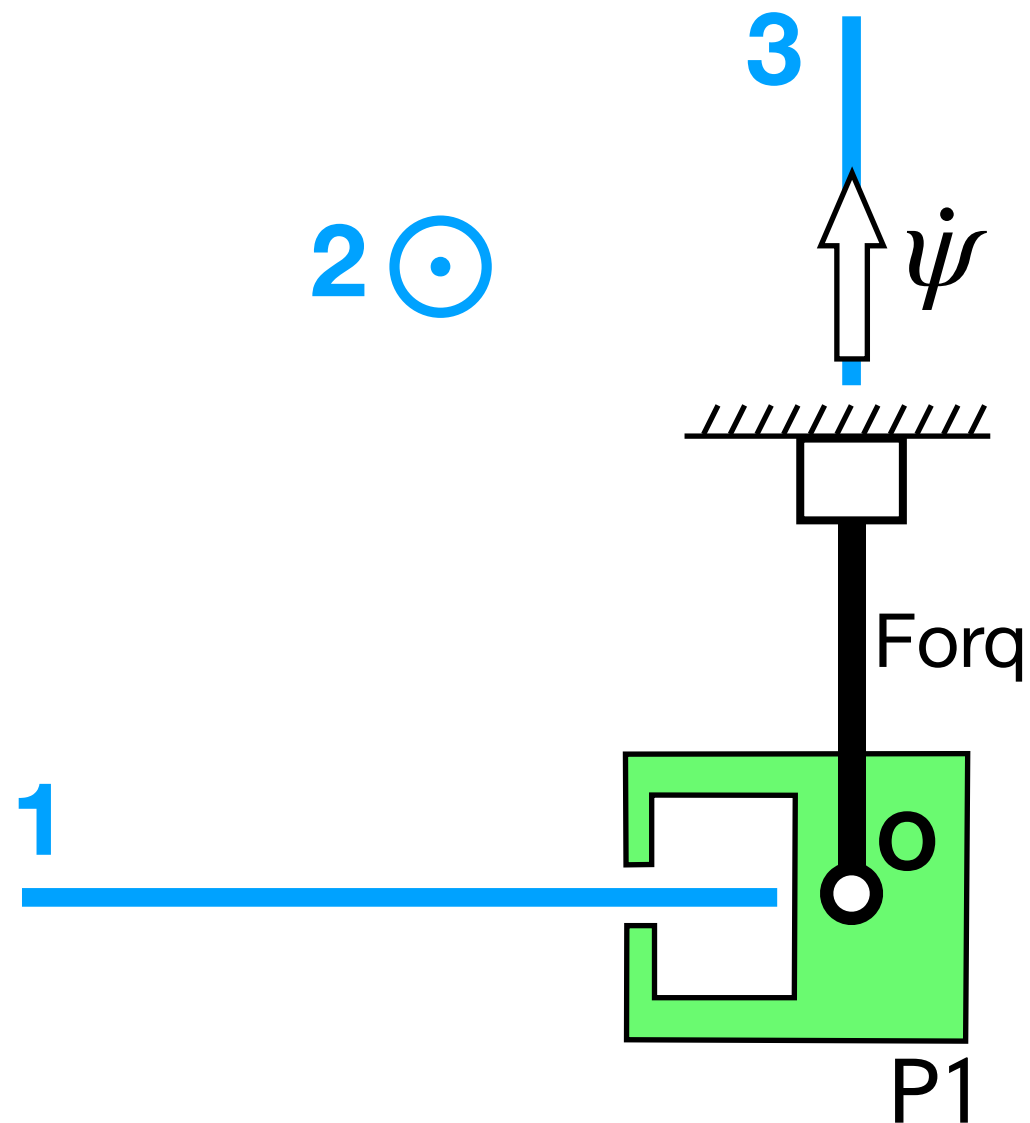
Substituim **Forq** per enllaç indirecte $T \rightarrow P_1$



Substituim **Forq** per enllaç indirecte $T \rightarrow P_1$



Substituïm **Forq** per enllaç indirecte $T \rightarrow P_1$

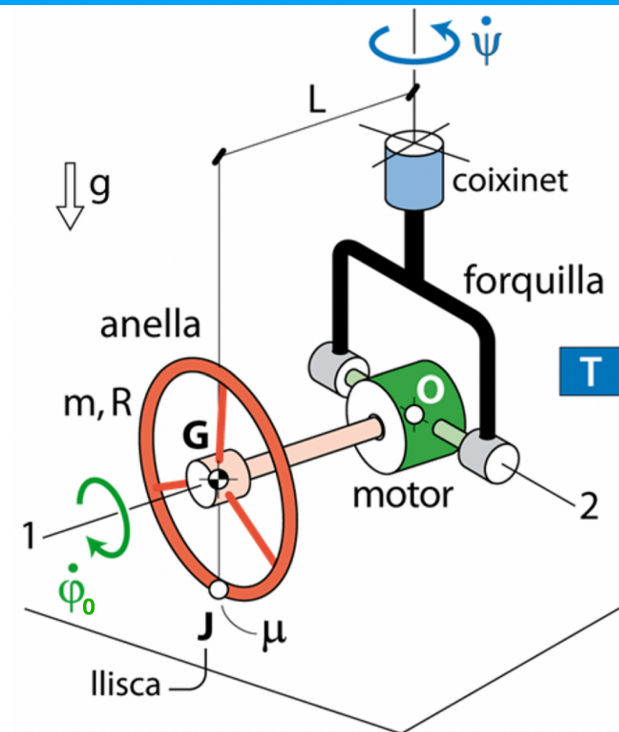


Torsor d'enllaç indirecte $T \rightarrow P1$ al punt **O**:

$$\left\{ \bar{\mathbf{F}}_{T \rightarrow (\text{forq}) \rightarrow \text{Sup}} \right\}_{\mathbf{B}} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \quad \mathbf{O} \text{ fix a } T$$

$$\left\{ \bar{\mathbf{M}}_{T \rightarrow (\text{forq}) \rightarrow \text{Sup}} (\mathbf{O}) \right\}_{\mathbf{B}'} = \begin{Bmatrix} M_1 \\ 0 \\ 0 \end{Bmatrix} \quad \begin{array}{l} 0 \text{ en les rotacions} \\ \text{permeses de } P1 \\ \text{resp } T \end{array}$$

Full ruta per eq. del mov. ψ

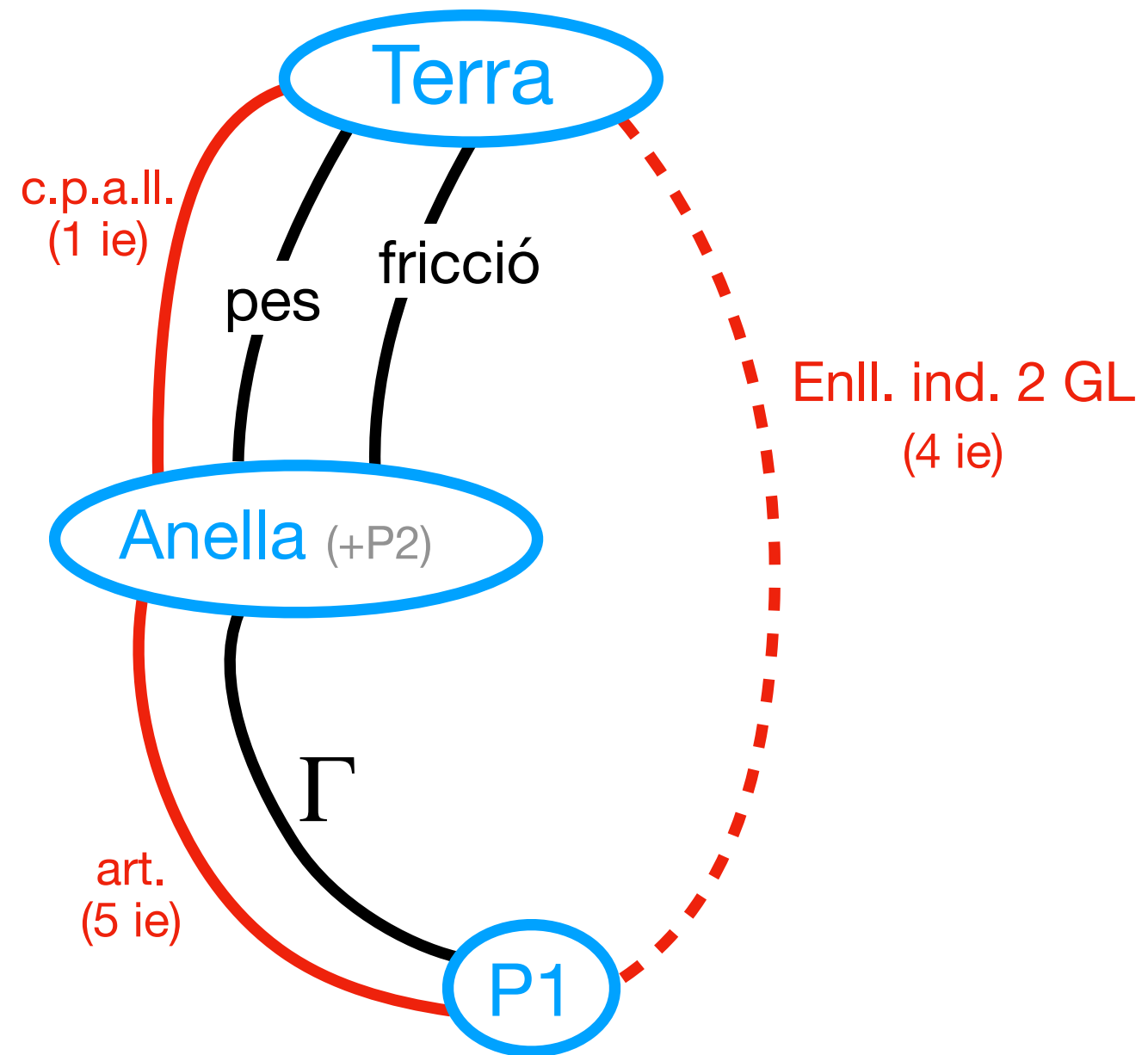


ψ afecta la cinemàtica de **P1** i **Anella**



Sist ha d'incloure **P1** o **Anella** (o ambdós)

Sistema	Incògn.	Problema
Anella	6 ie, Γ , $\ddot{\psi}$	INDET
P1	9 ie, Γ , $\ddot{\psi}$	INDET
Anella + P1	5 ie, $\ddot{\psi}$	DET



\Rightarrow Triem SIST = Anella + P1