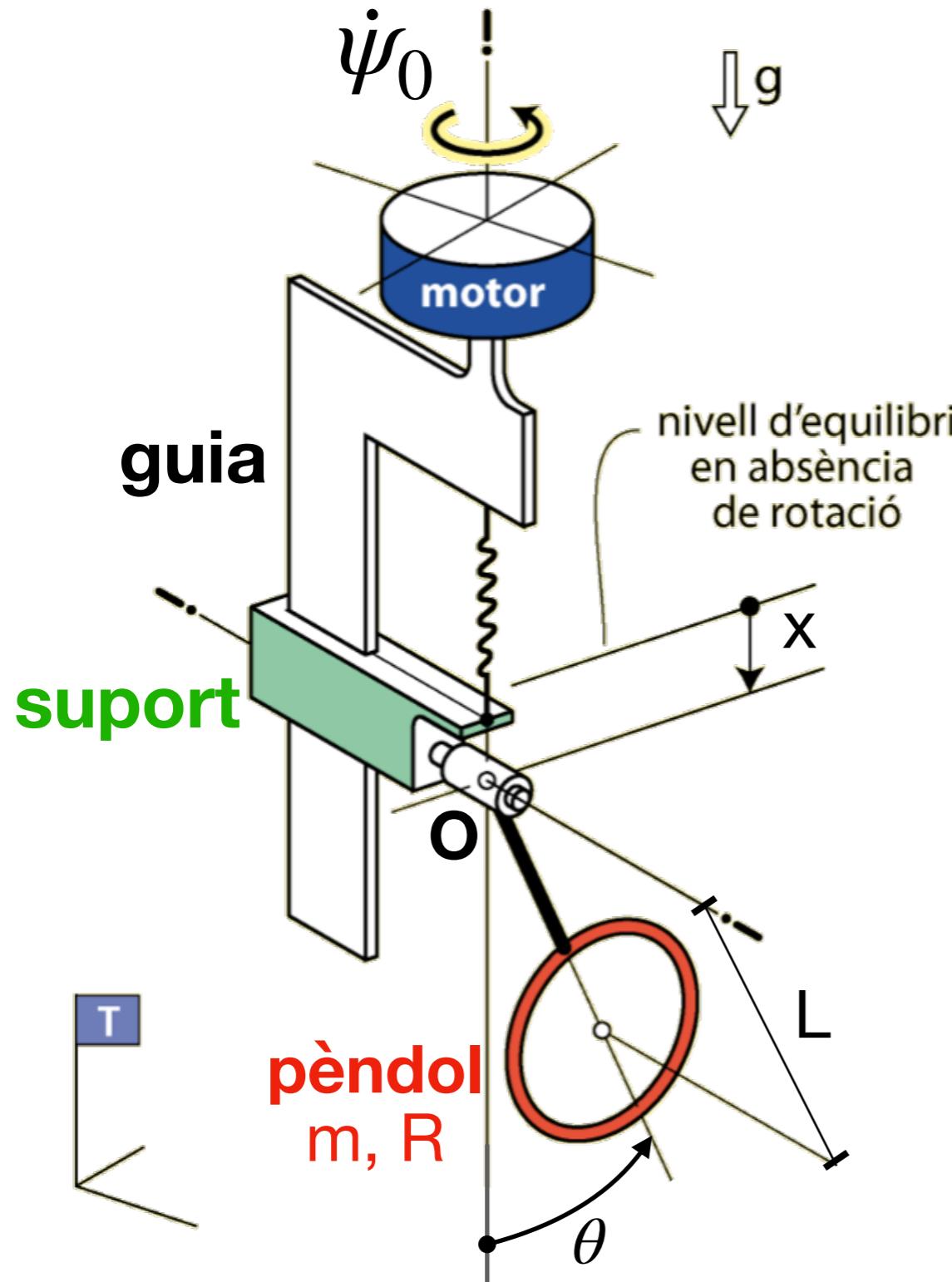


# 13P

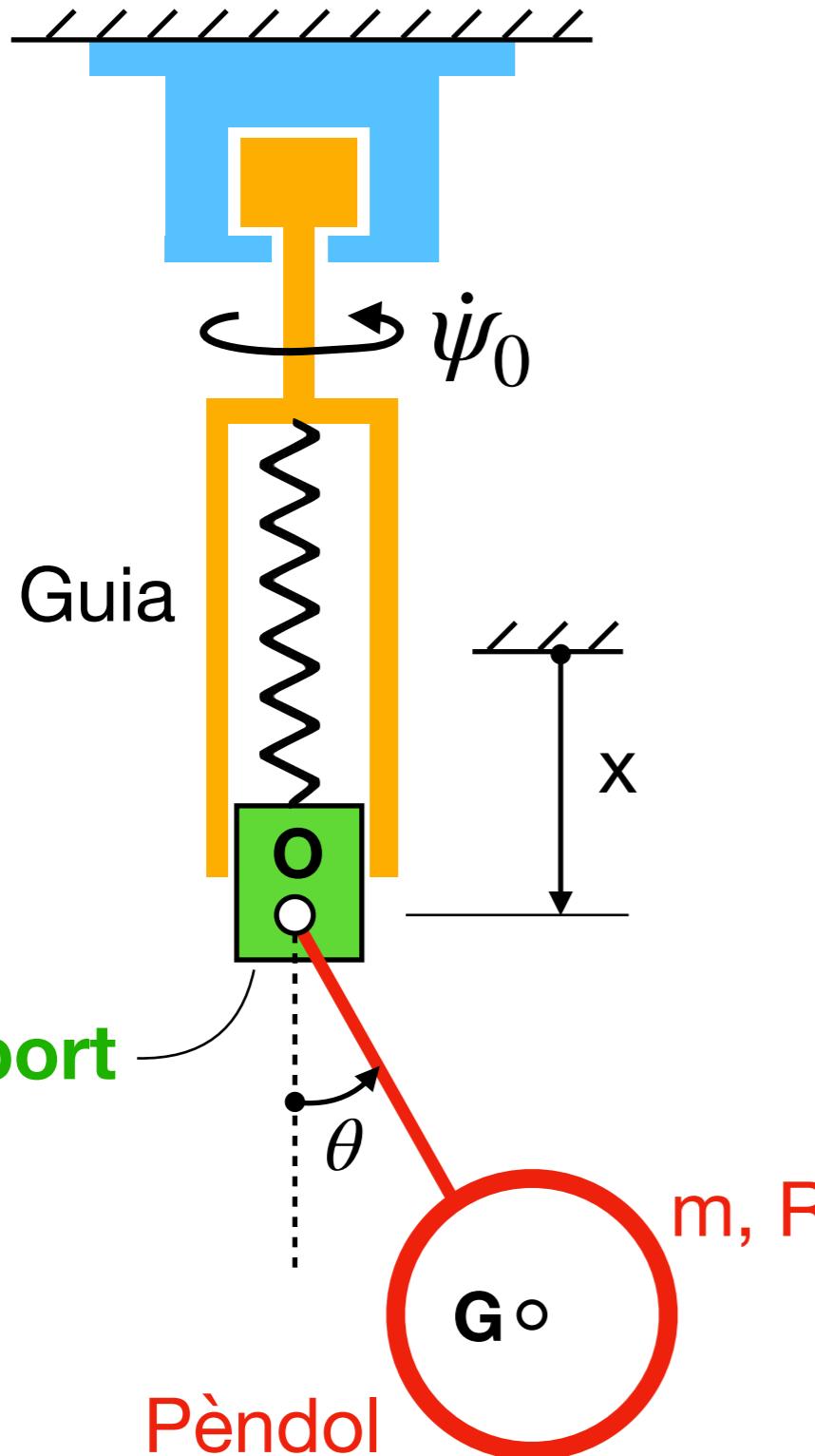
## Teoremes vectorials III

Exemples 3D



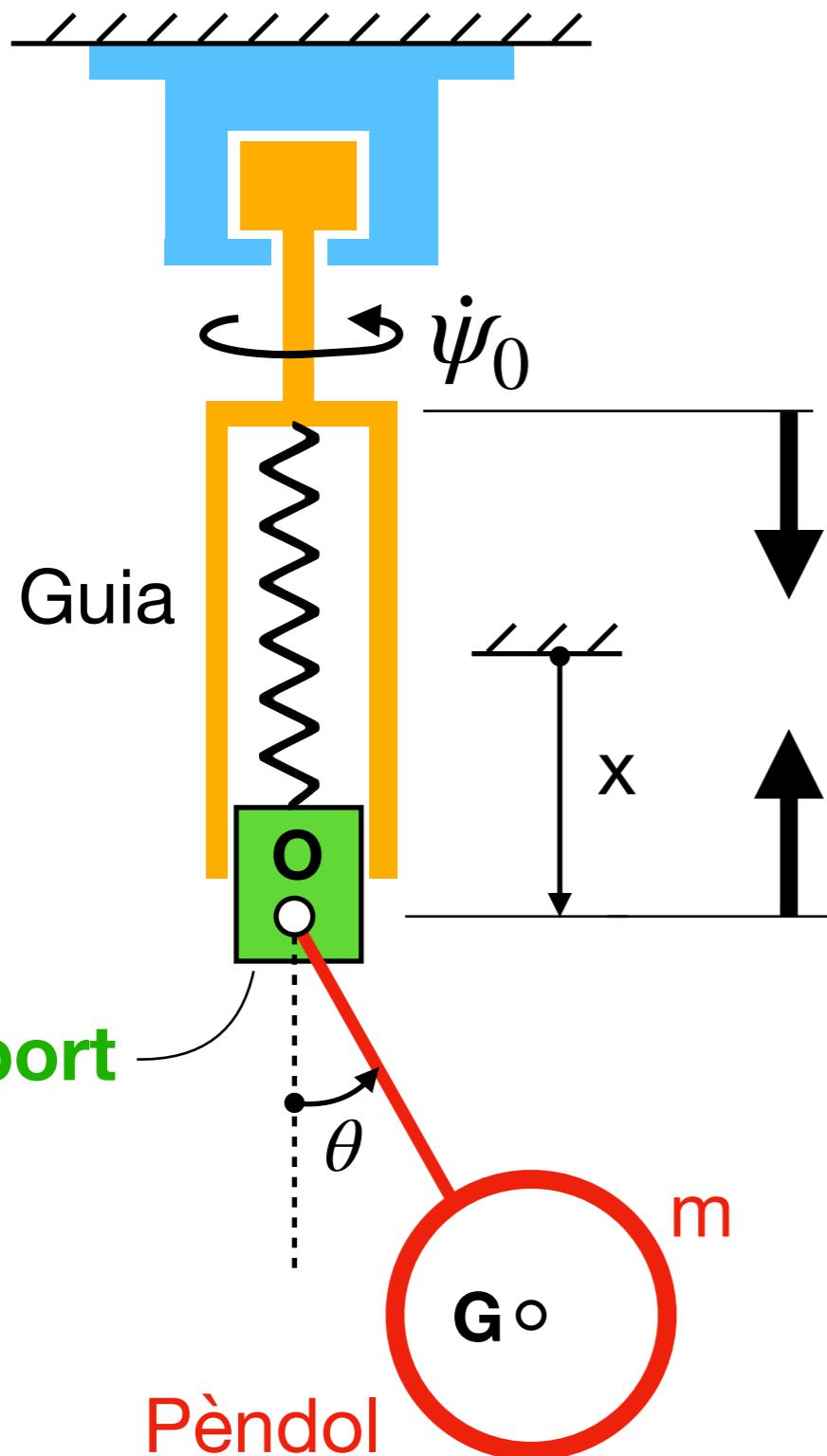
- DGI
- Eqs. mov. per a  $x$  i  $\theta$
- Parell motor  $\Gamma$  per mantenir  $\dot{\psi}_0 = \text{ct}$

Amb motor aturat:  $x = 0, \theta = 0$  és config. d'equilibri



- DGI
- Eqs. mov. per a x i  $\theta$
- Parell motor  $\Gamma$  per mantenir  $\dot{\psi}_0 = \text{ct}$

Amb motor aturat:  $x = 0, \theta = 0$  és config. d'equilibri

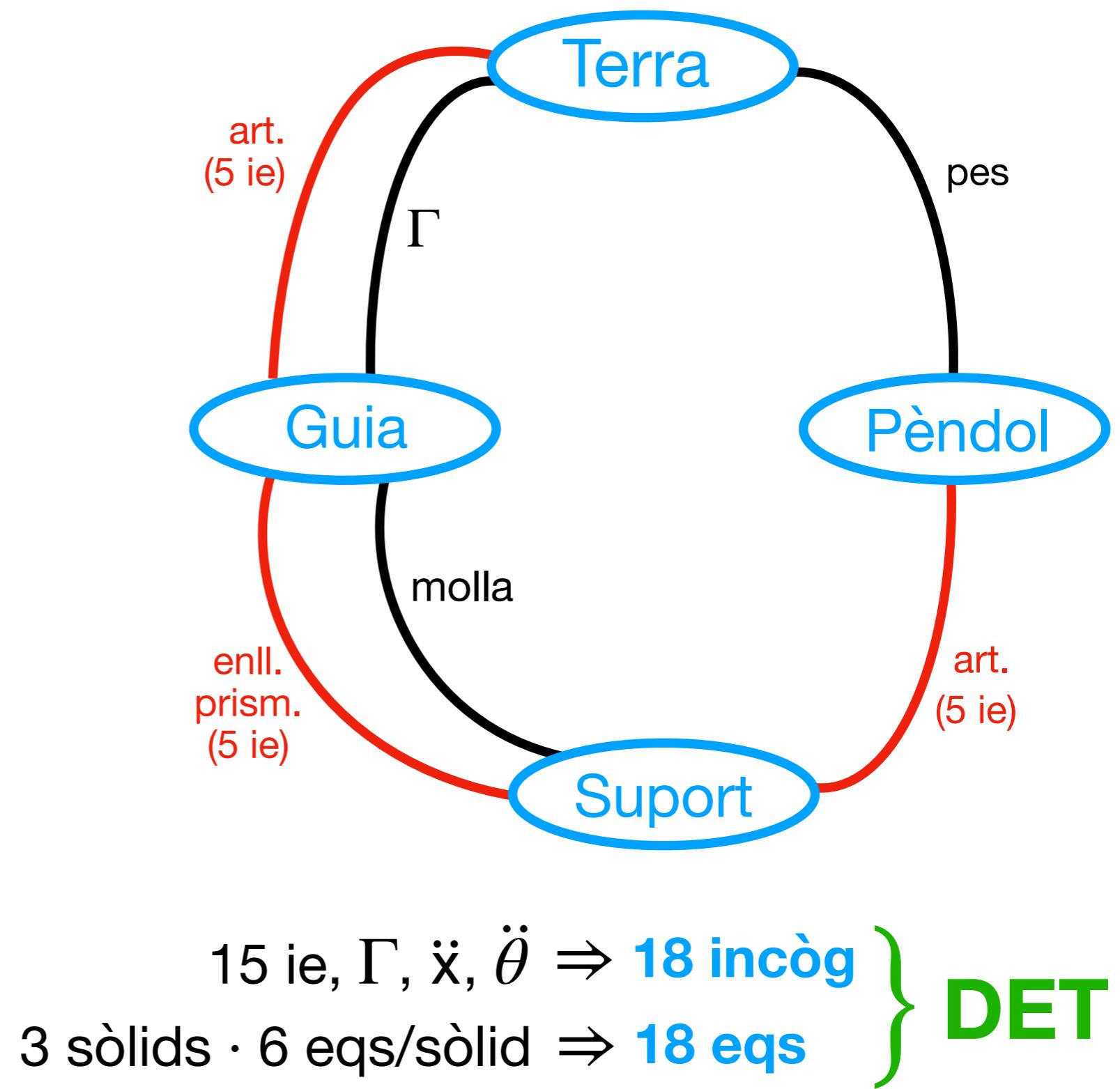
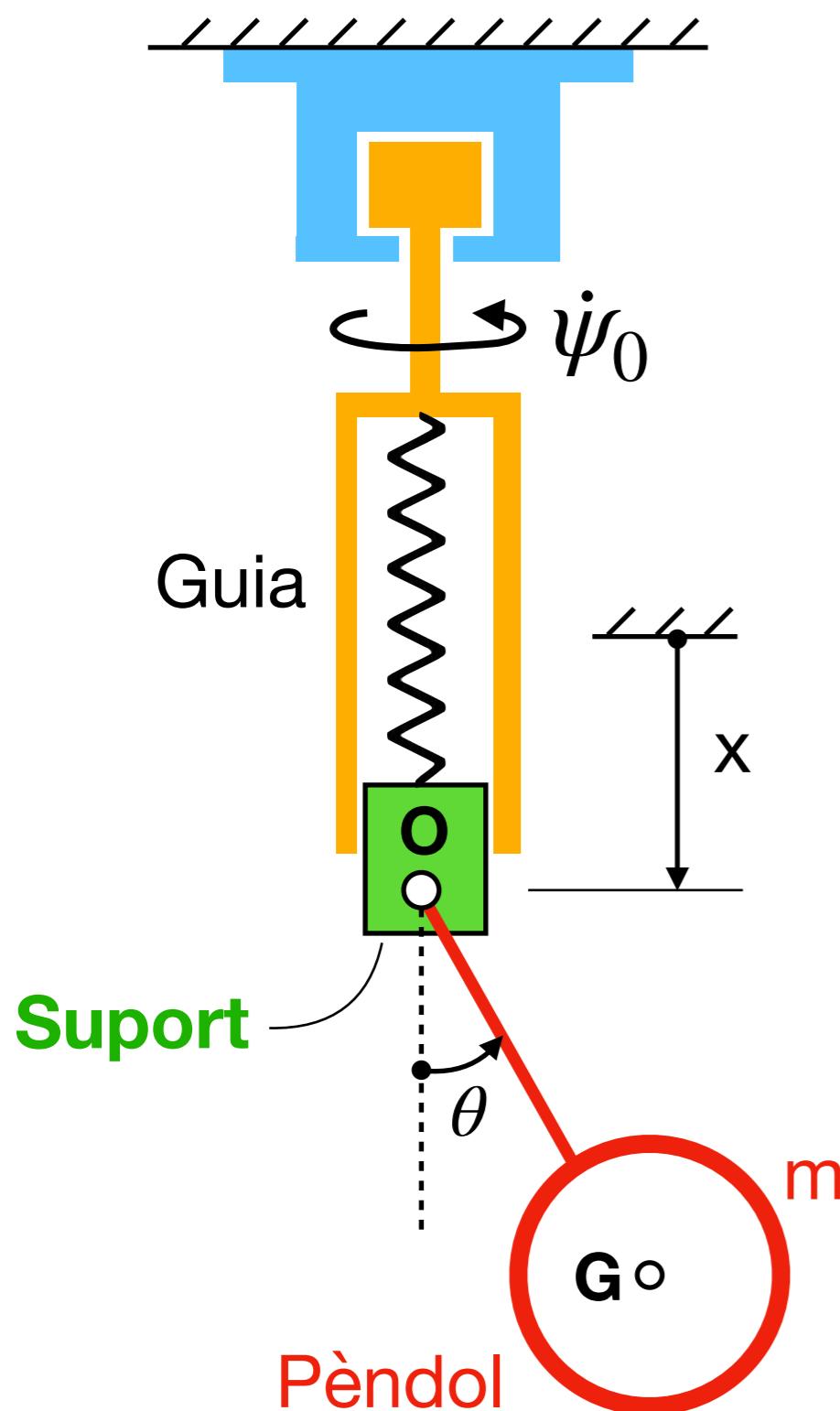


Força de la molla ?

$$F_{\text{at molla}} = \underbrace{mg}_{F_0} + \underbrace{kx}_{k\Delta\rho}$$

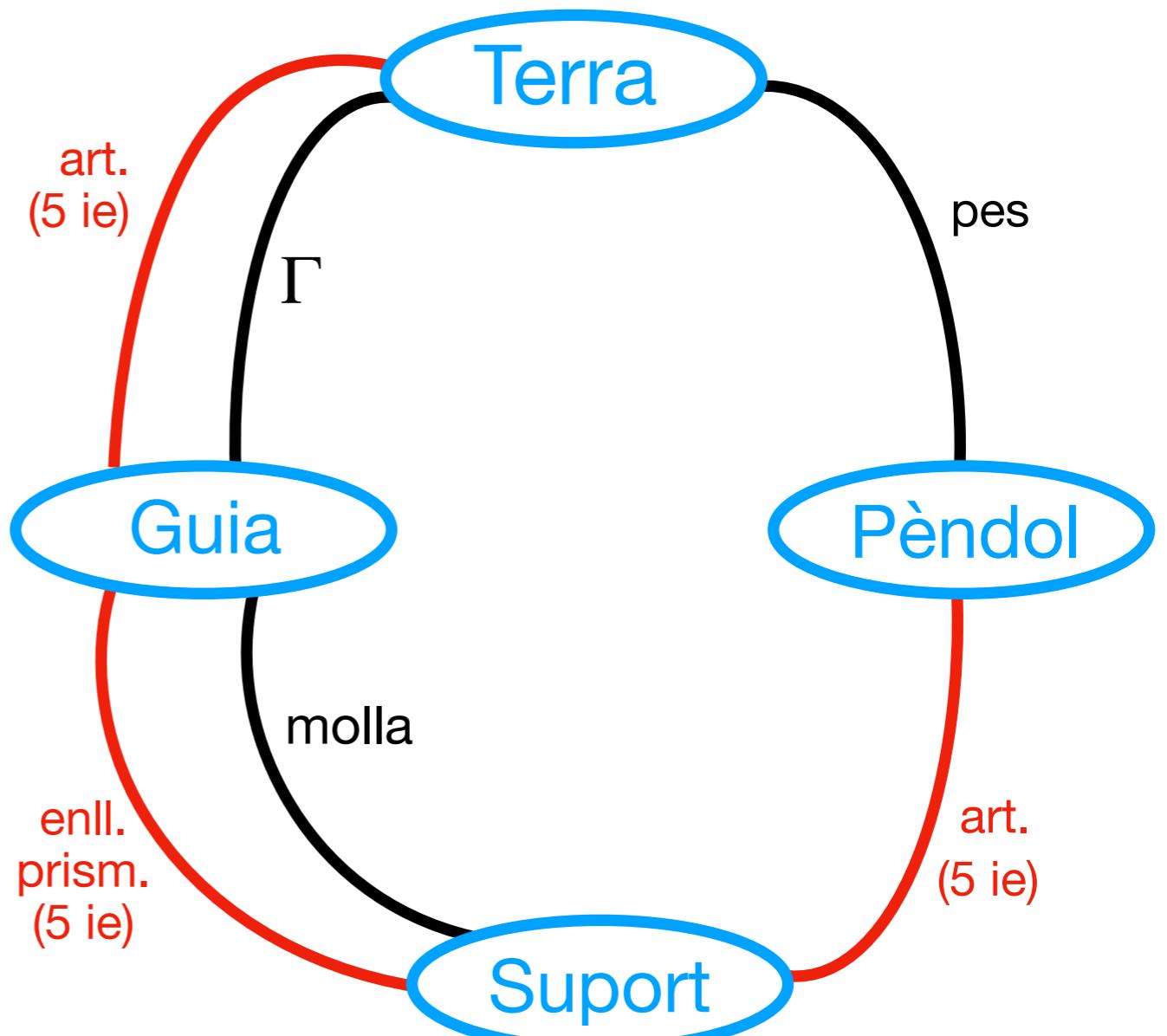
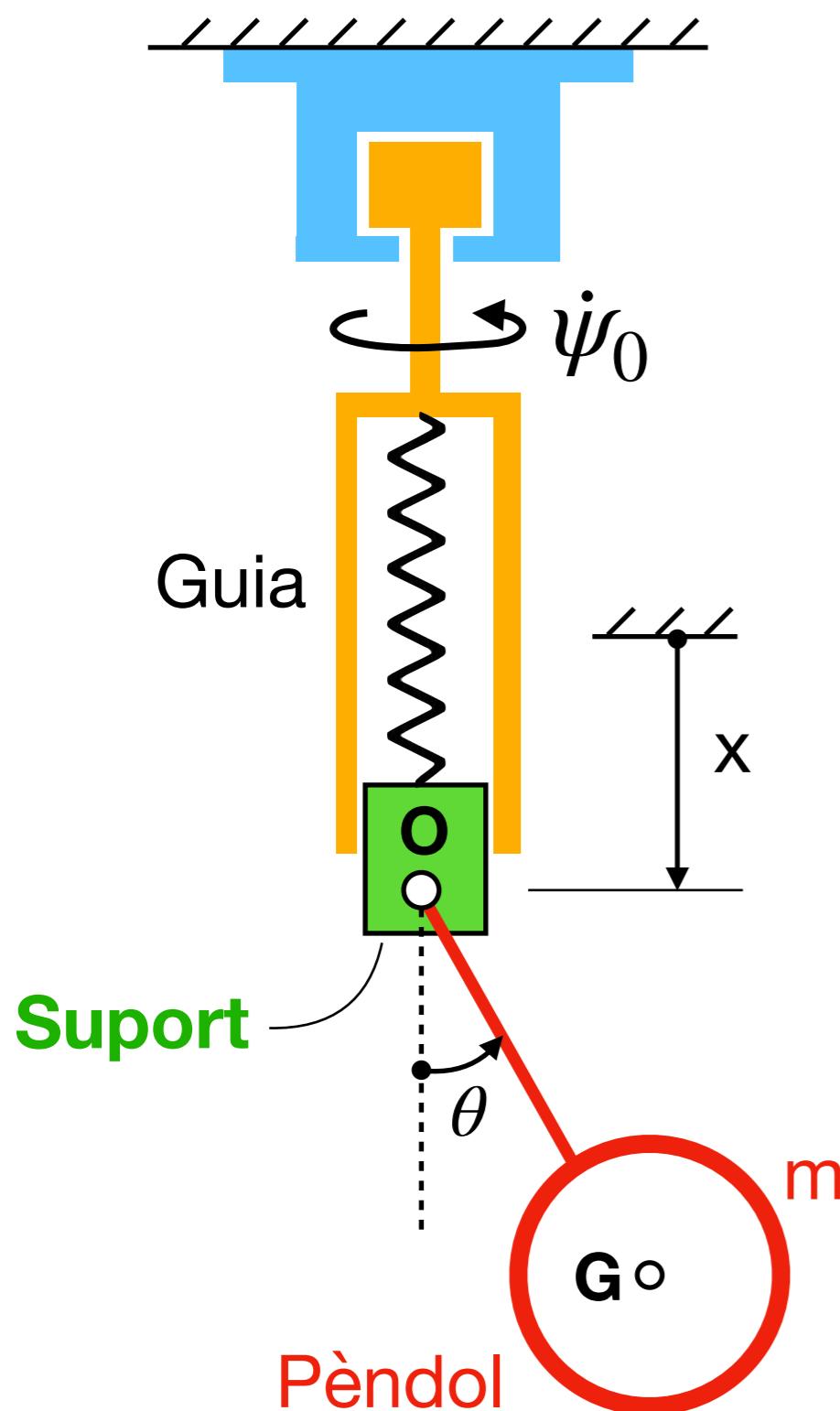
Amb motor aturat:  $x = 0, \theta = 0$  és config. d'equilibri

# DGI



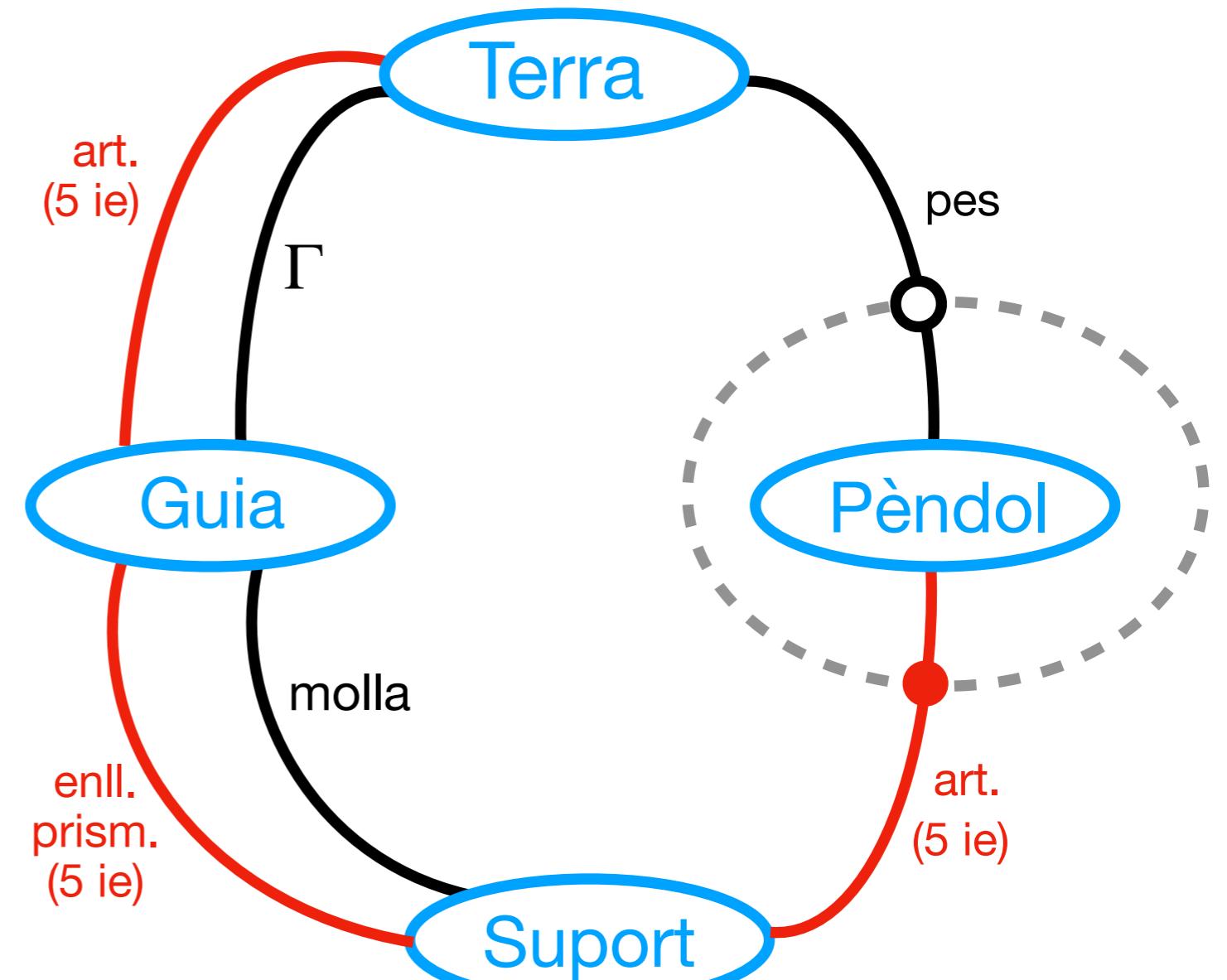
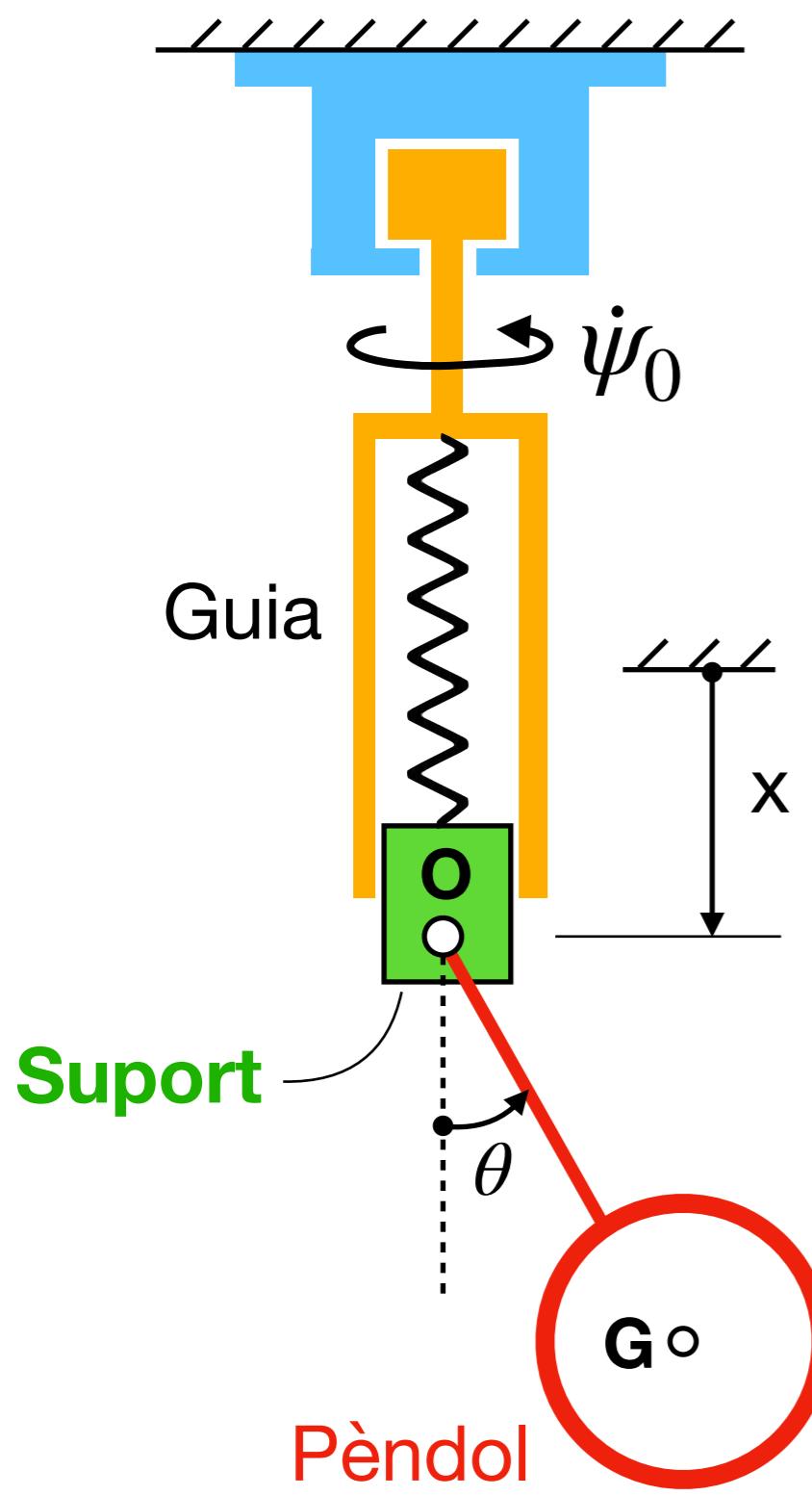
$15 \text{ ie}, \Gamma, \ddot{x}, \ddot{\theta} \Rightarrow 18 \text{ incòg}$  } **DET**  
 $3 \text{ sòlids} \cdot 6 \text{ eqs/sòlid} \Rightarrow 18 \text{ eqs}$

# DGI



$x$  i  $\theta$  només afecten **pèndol** i **suport**

Explorem sistemes que els incloguin



Sistema

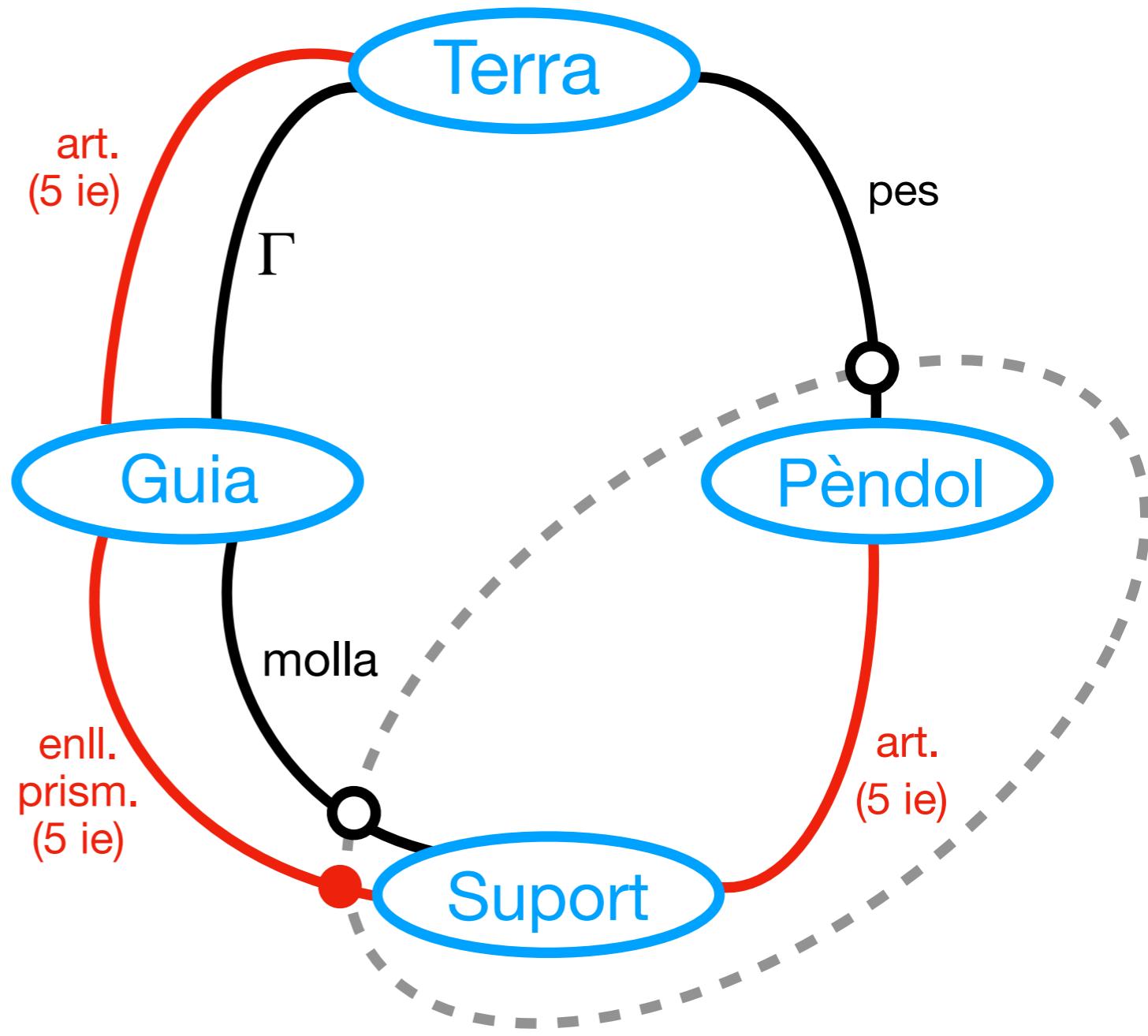
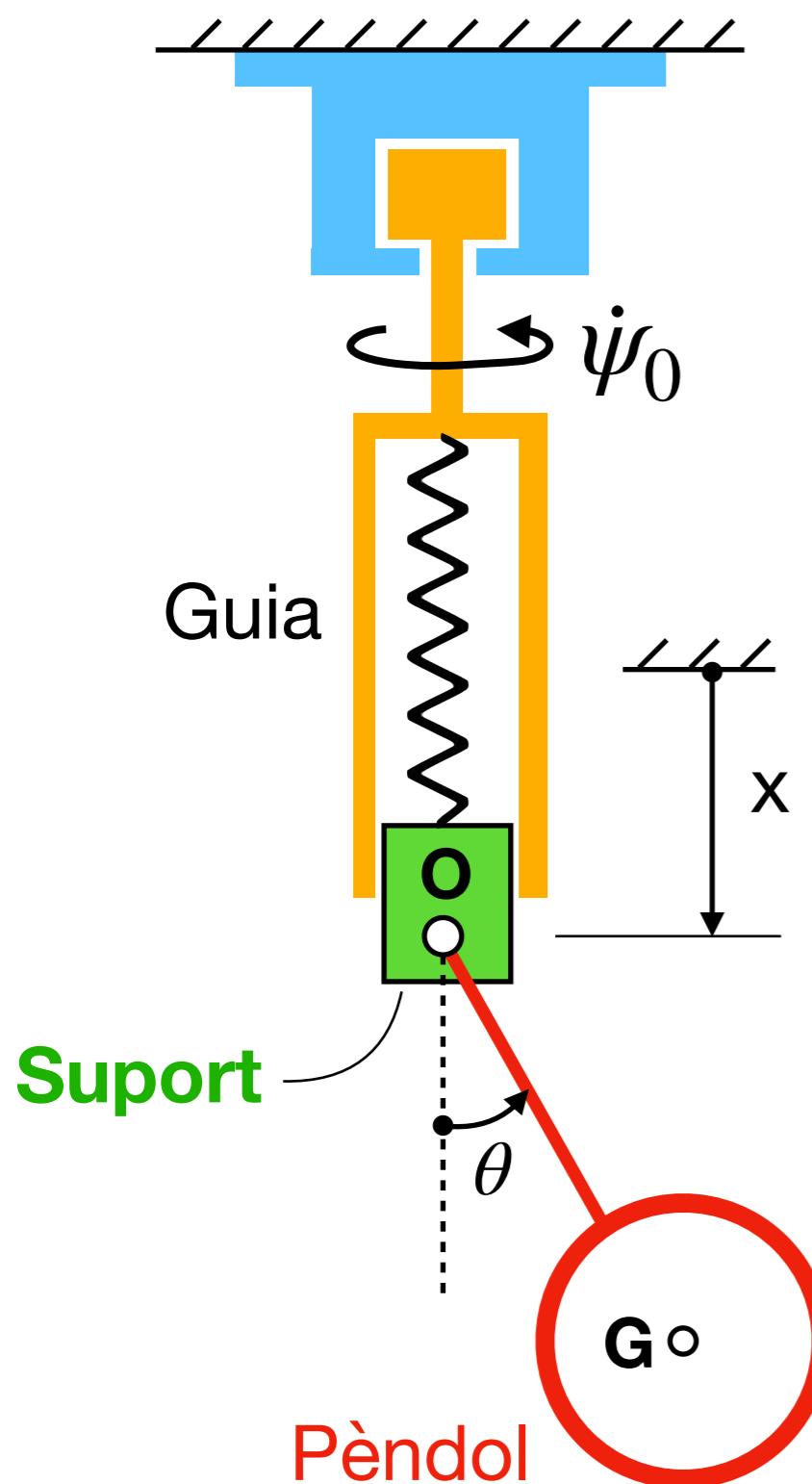
Pèndol

Incògn.

$5 \text{ ie}, \ddot{x}, \ddot{\theta}$

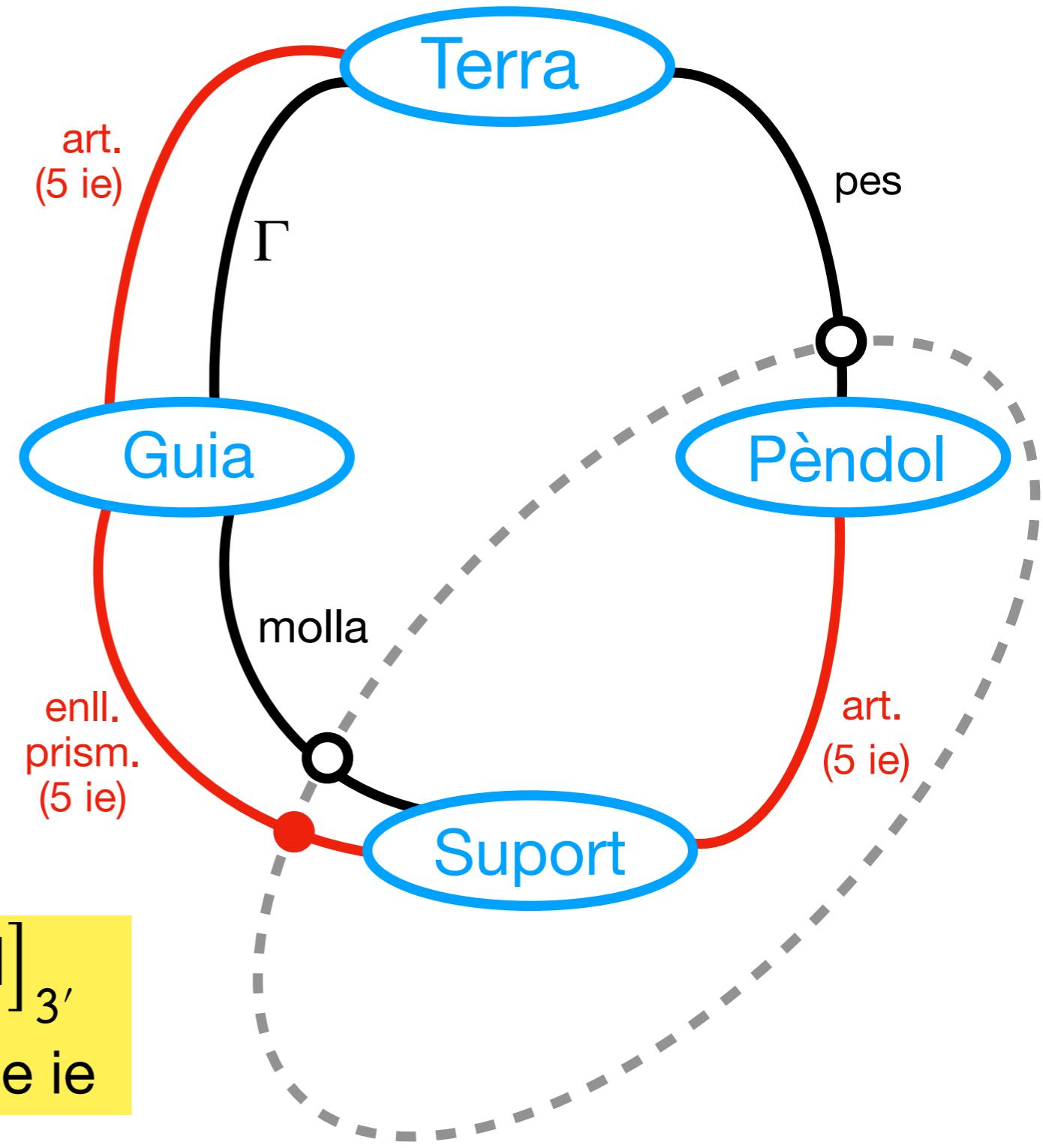
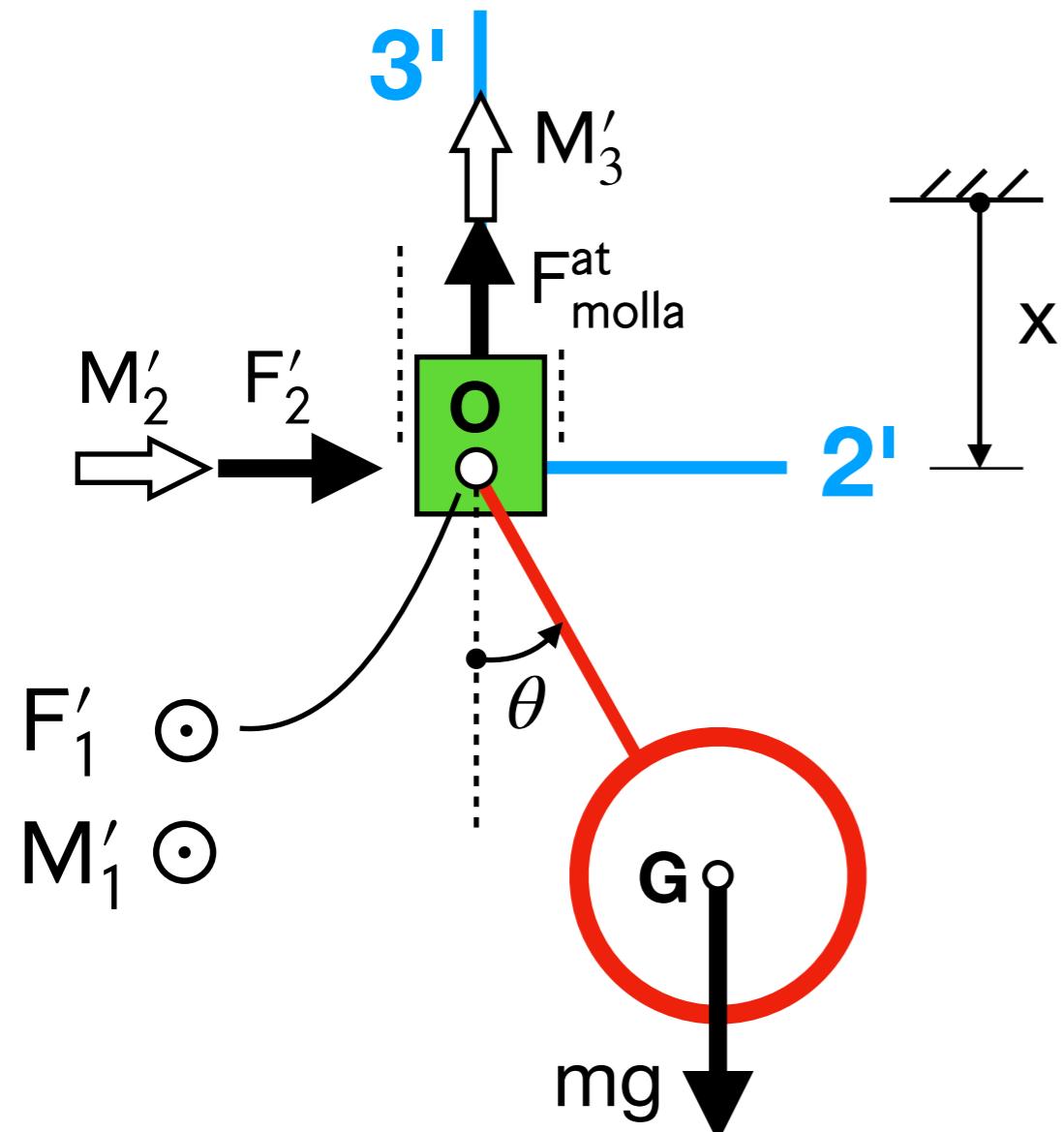
Problema

**INDET**



Sistema	Incògn.	Problema
Pèndol	$5\ ie, \ddot{x}, \ddot{\theta}$	<b>INDET</b>
Pènd. + sup.	$5\ ie, \ddot{x}, \ddot{\theta}$	<b>INDET</b>
Els altres sistemes tenen + incògnites !		

# Sist = Pèndol + Suport

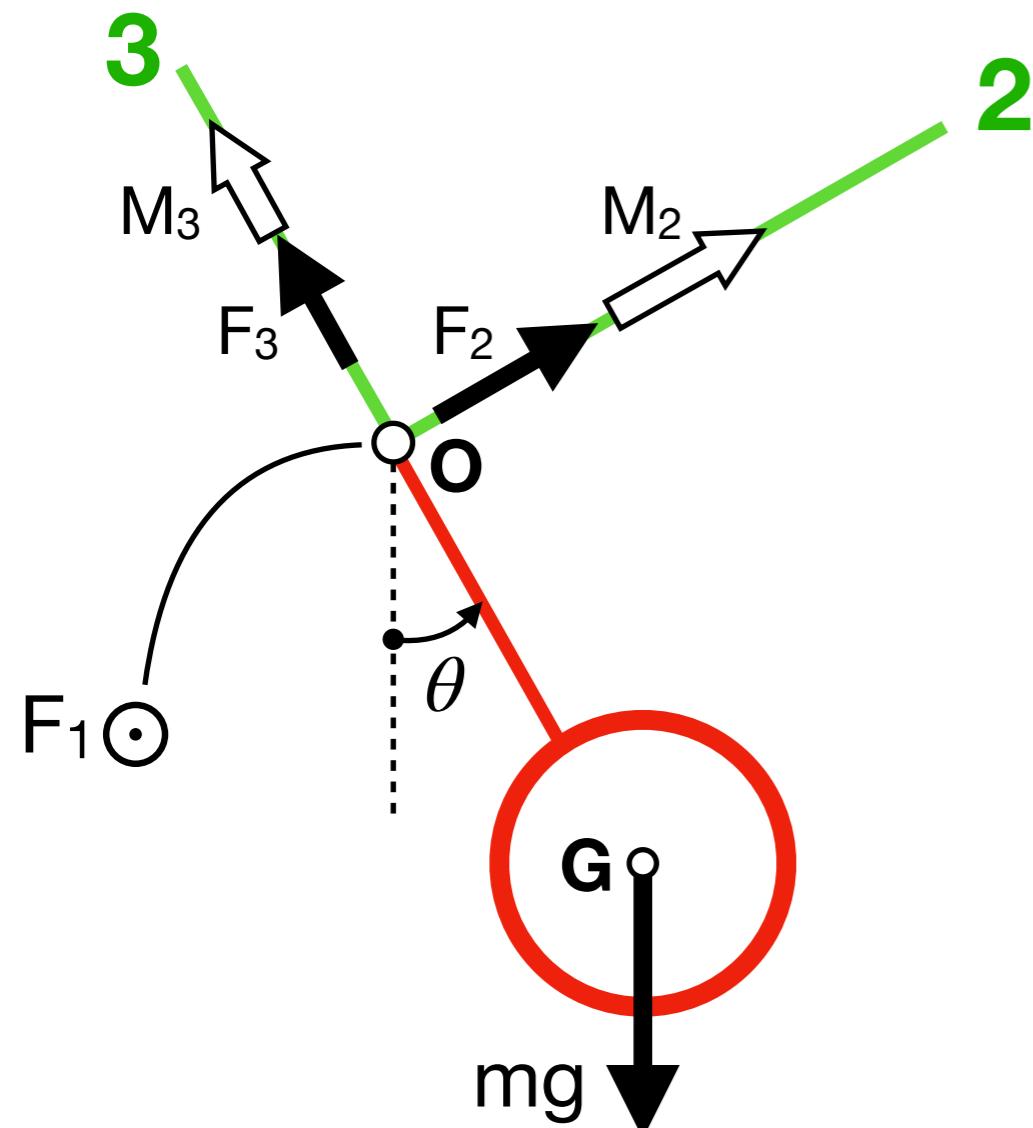


$$\left\{ \bar{F}_{\text{Guia} \rightarrow \text{Sup}} \right\}_{B'} = \begin{Bmatrix} F'_1 \\ F'_2 \\ 0 \end{Bmatrix}$$

TQM]  $_{3'}$   
lliure de ie

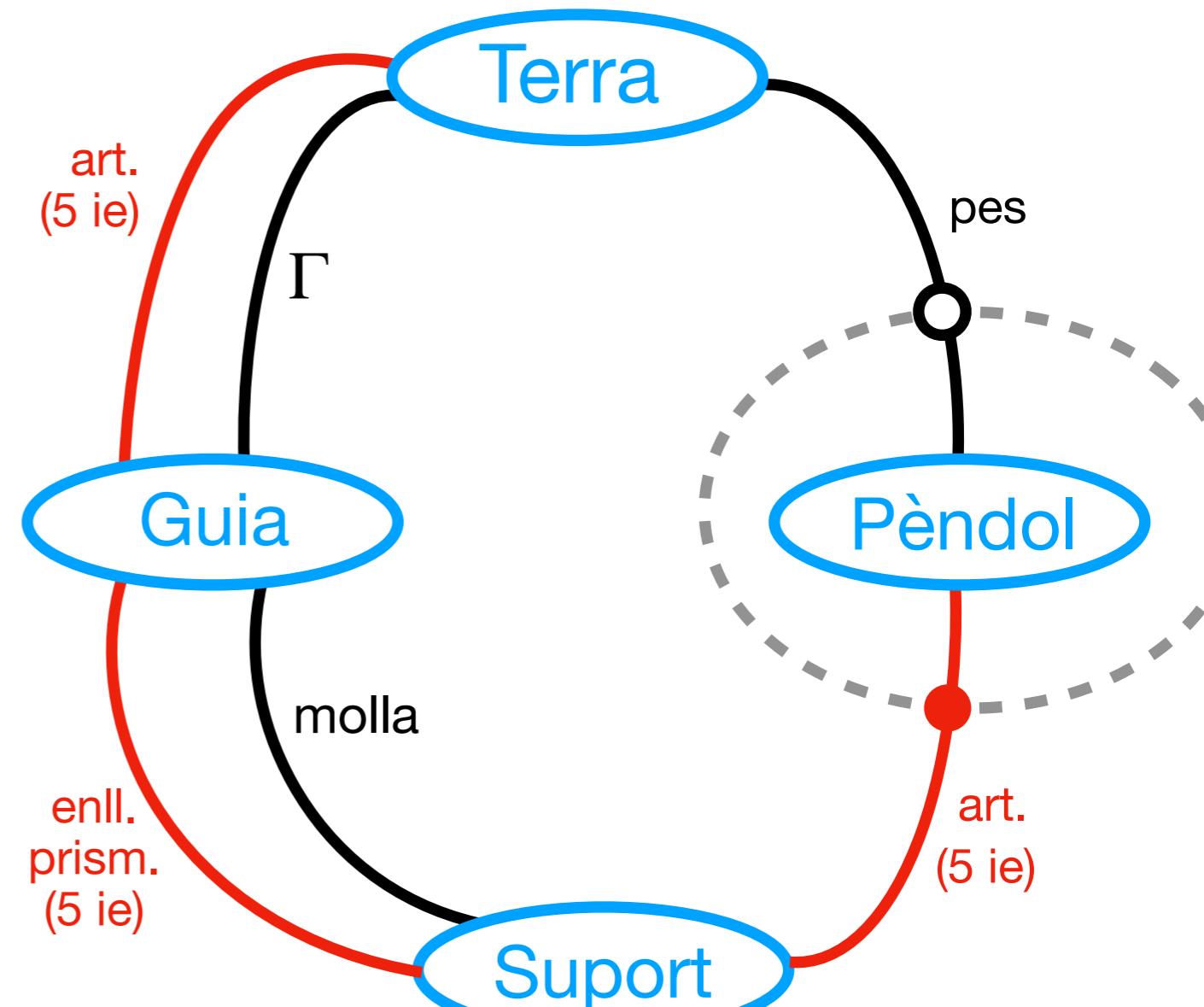
$$\left\{ \bar{M}_{\text{Guia} \rightarrow \text{Sup}} (\mathbf{O}) \right\}_{B'} = \begin{Bmatrix} M'_1 \\ M'_2 \\ M'_3 \end{Bmatrix}$$

# Sist = Pèndol

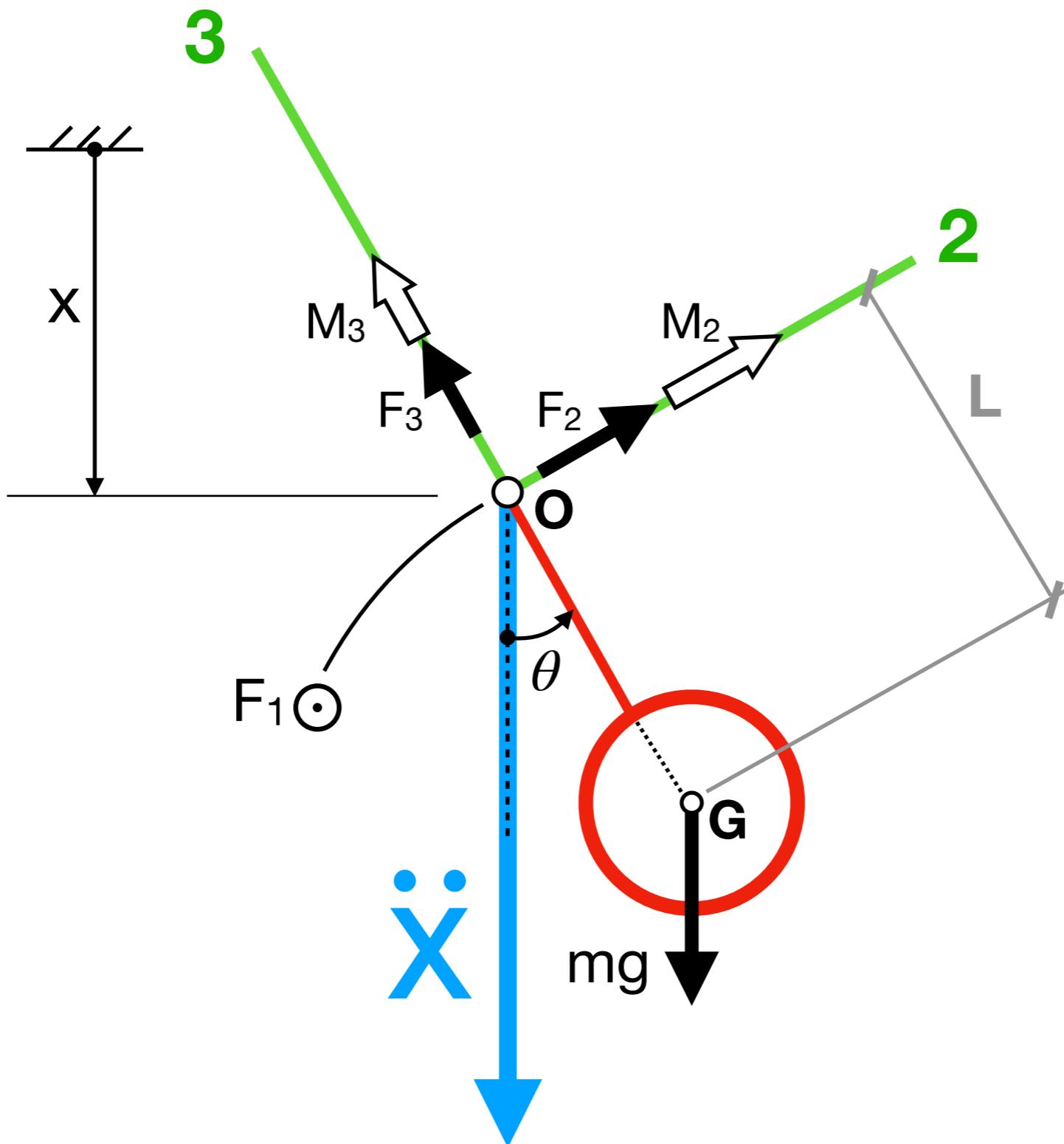


$$\left\{ \bar{F}_{\text{Sup} \rightarrow \text{Pendol}} \right\}_B = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\left\{ \bar{M}_{\text{Sup} \rightarrow \text{Pendol}}(O) \right\}_B = \begin{Bmatrix} 0 \\ M_2 \\ M_3 \end{Bmatrix} \leftarrow \boxed{\text{TMC}(O)]_1} \quad \text{Illiure de ie}$$



# TMC(O) ]<sub>1</sub> sobre SIST = Pèndol



# TMC(O) ]<sub>1</sub> sobre SIST = Pèndol

$$\left\{ \dot{\bar{H}}_{RTO}(O) \right\}_B = \begin{Bmatrix} I_{11}\ddot{\theta} + (I_{33} - I_{22})\dot{\psi}_0^2 \sin \theta \cos \theta \\ (I_{11} + I_{22} - I_{33})\dot{\psi}_0 \dot{\theta} \cos \theta \\ (I_{22} - I_{11} - I_{33})\dot{\psi}_0 \dot{\theta} \sin \theta \end{Bmatrix} \quad (\text{III})$$

# Pas final

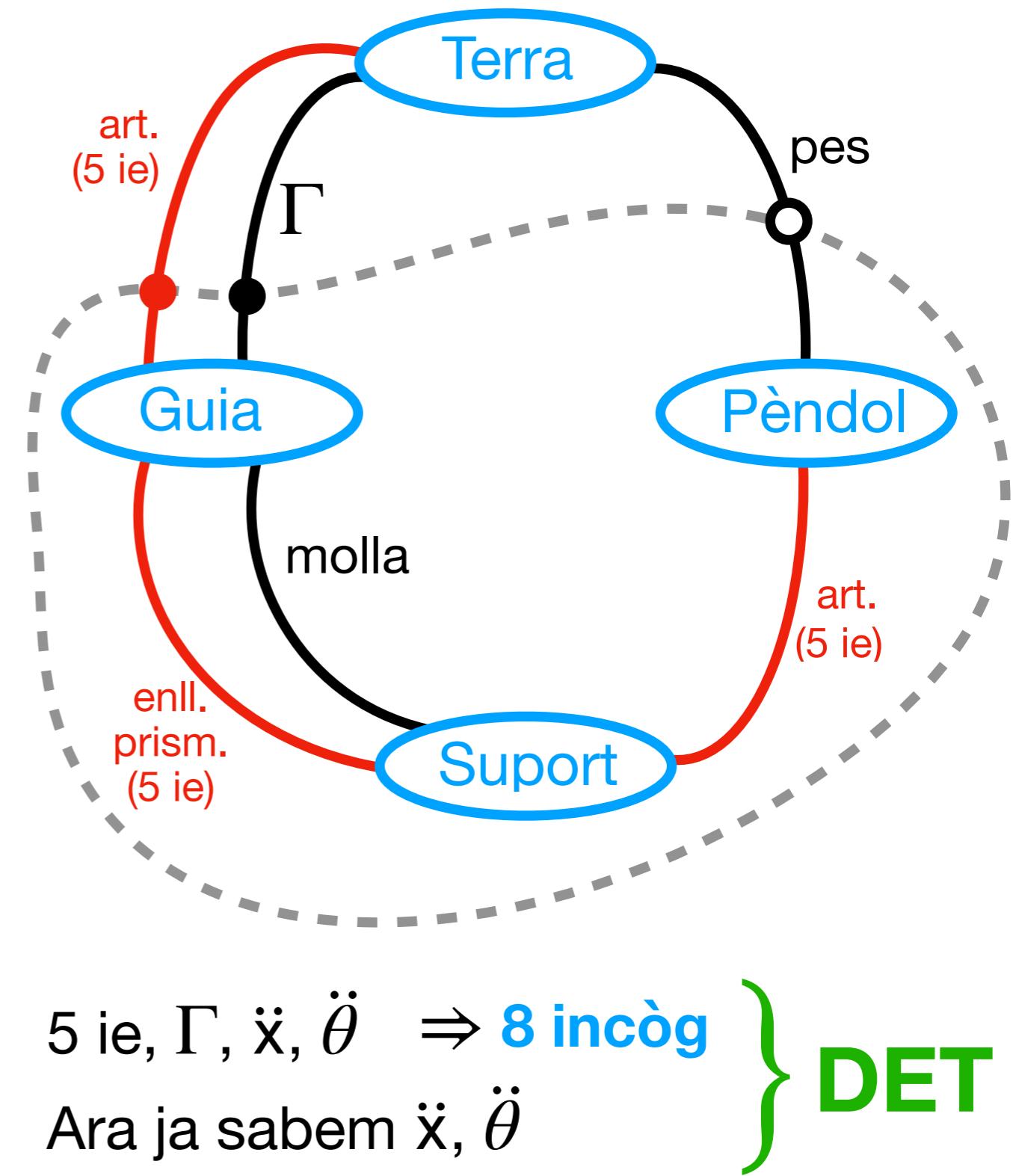
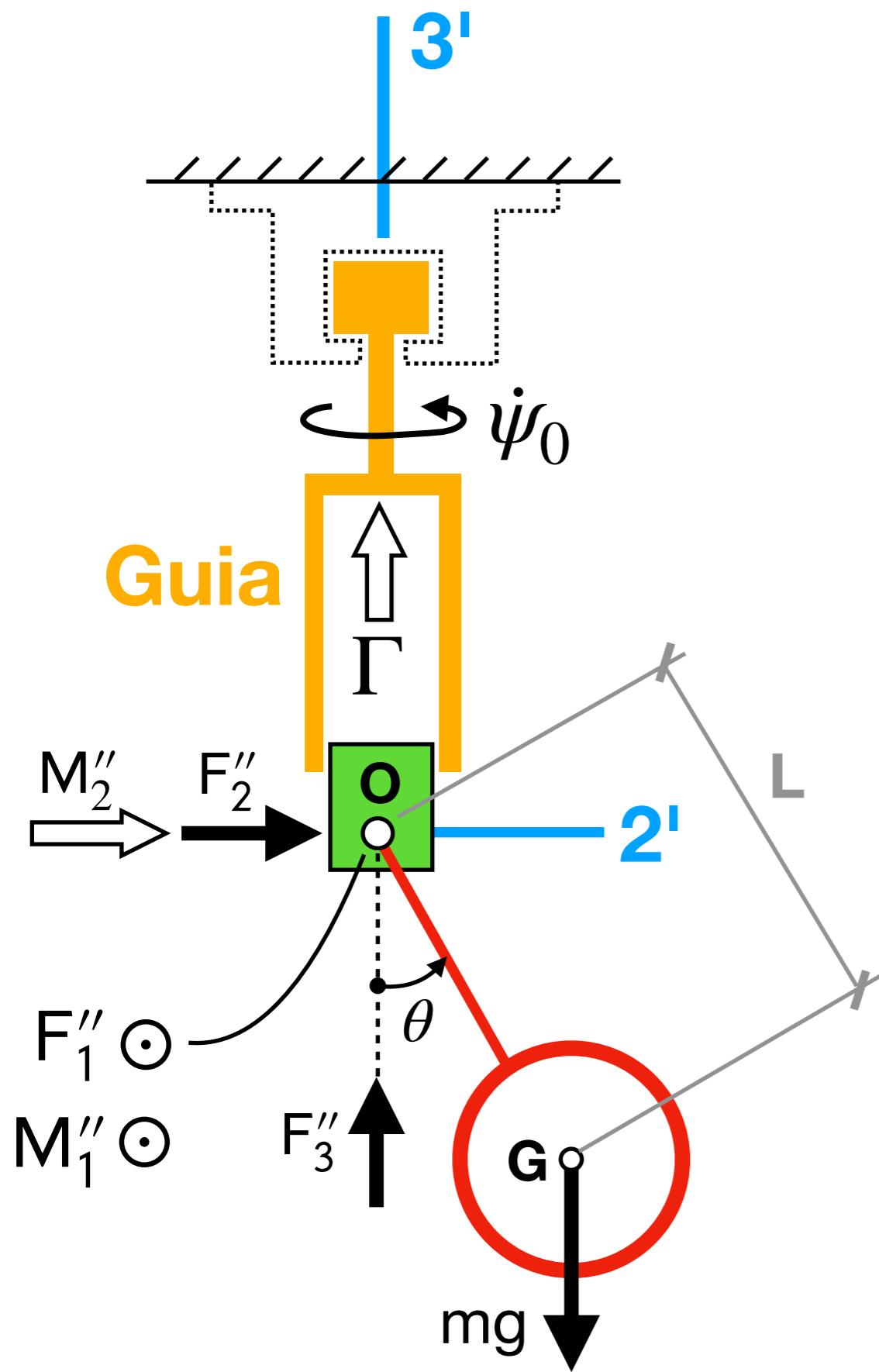
$$\left\{ \begin{array}{l} (R^2 + L^2) \ddot{\theta} - (L \sin \theta) \ddot{x} = (L \dot{\psi}_0^2 \cos \theta - g) L \sin \theta \\ - (L \sin \theta) \ddot{\theta} + \ddot{x} = - \frac{k}{m} x + L \dot{\theta}^2 \cos \theta \end{array} \right.$$

Aillant  $\ddot{\theta}$  i  $\ddot{x}$

$$\left\{ \begin{array}{l} \ddot{\theta} = F_1(\theta, \dot{\theta}, x, \dot{x}) \\ \ddot{x} = F_2(\theta, \dot{\theta}, x, \dot{x}) \end{array} \right. \quad \begin{array}{l} \text{Eq. mov. } \theta \\ \text{Eq. mov. } x \end{array}$$

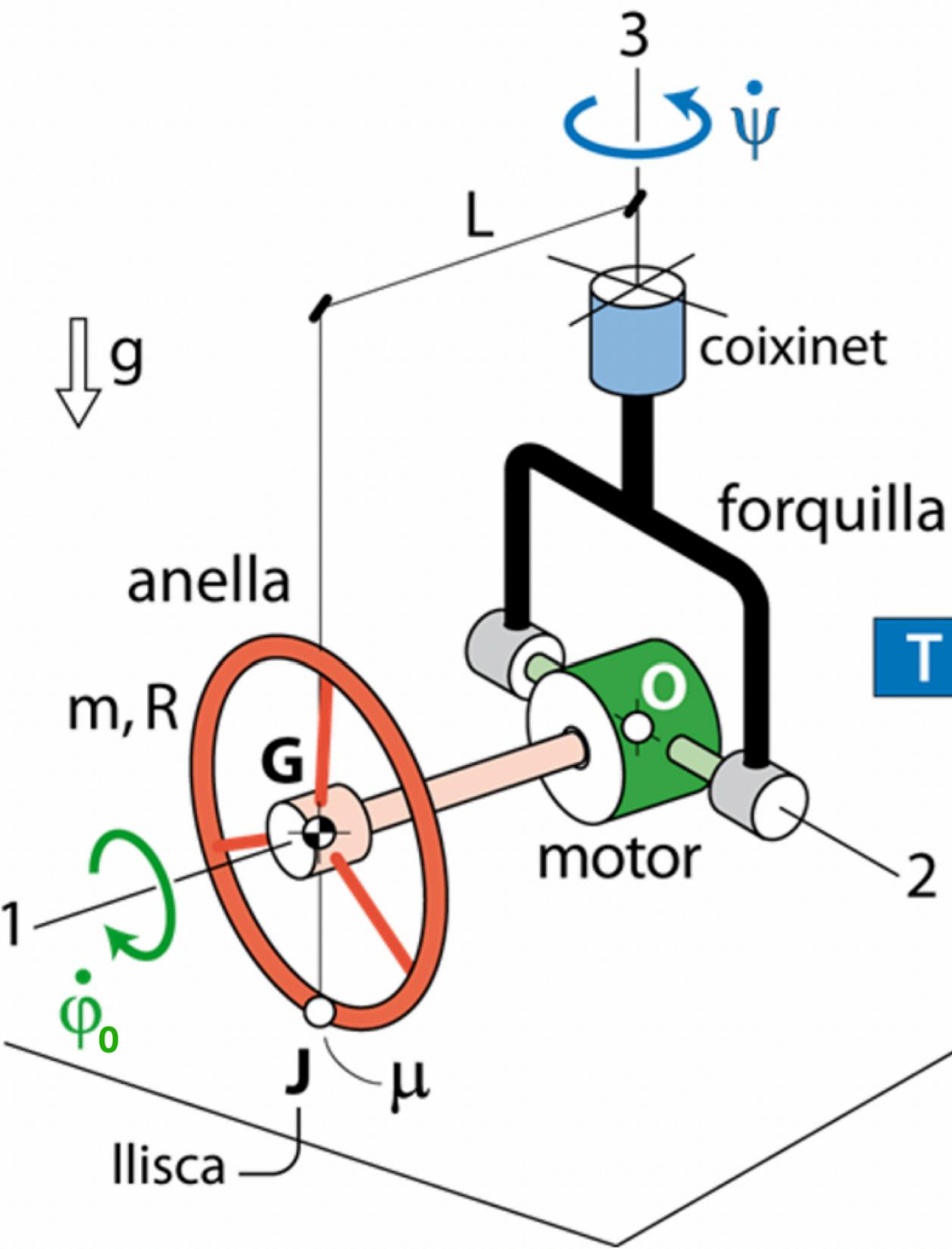
No cal  
que el feu

# Parell motor per mantenir $\dot{\psi}_0 = \text{ct}$



# D'abans

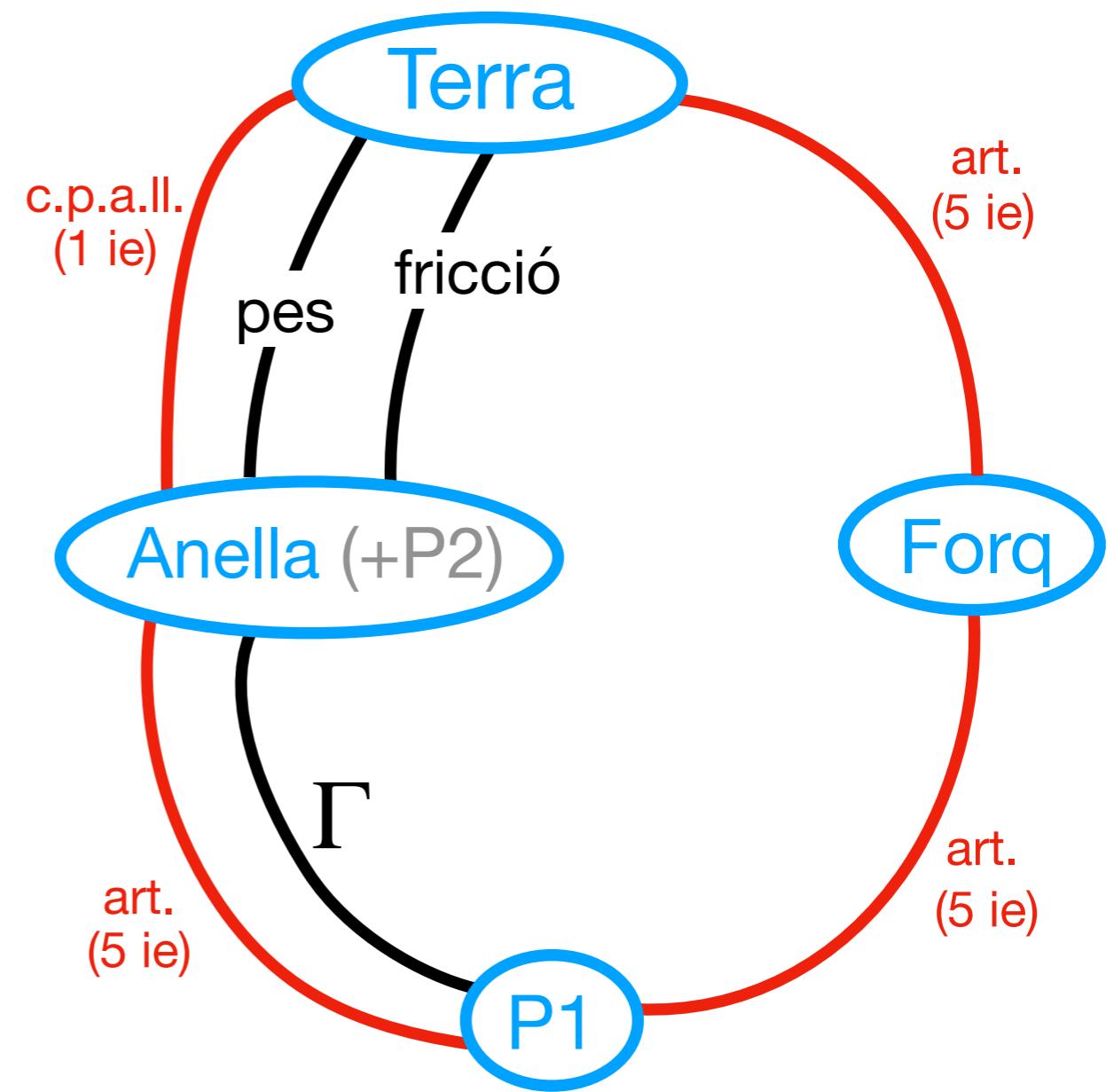
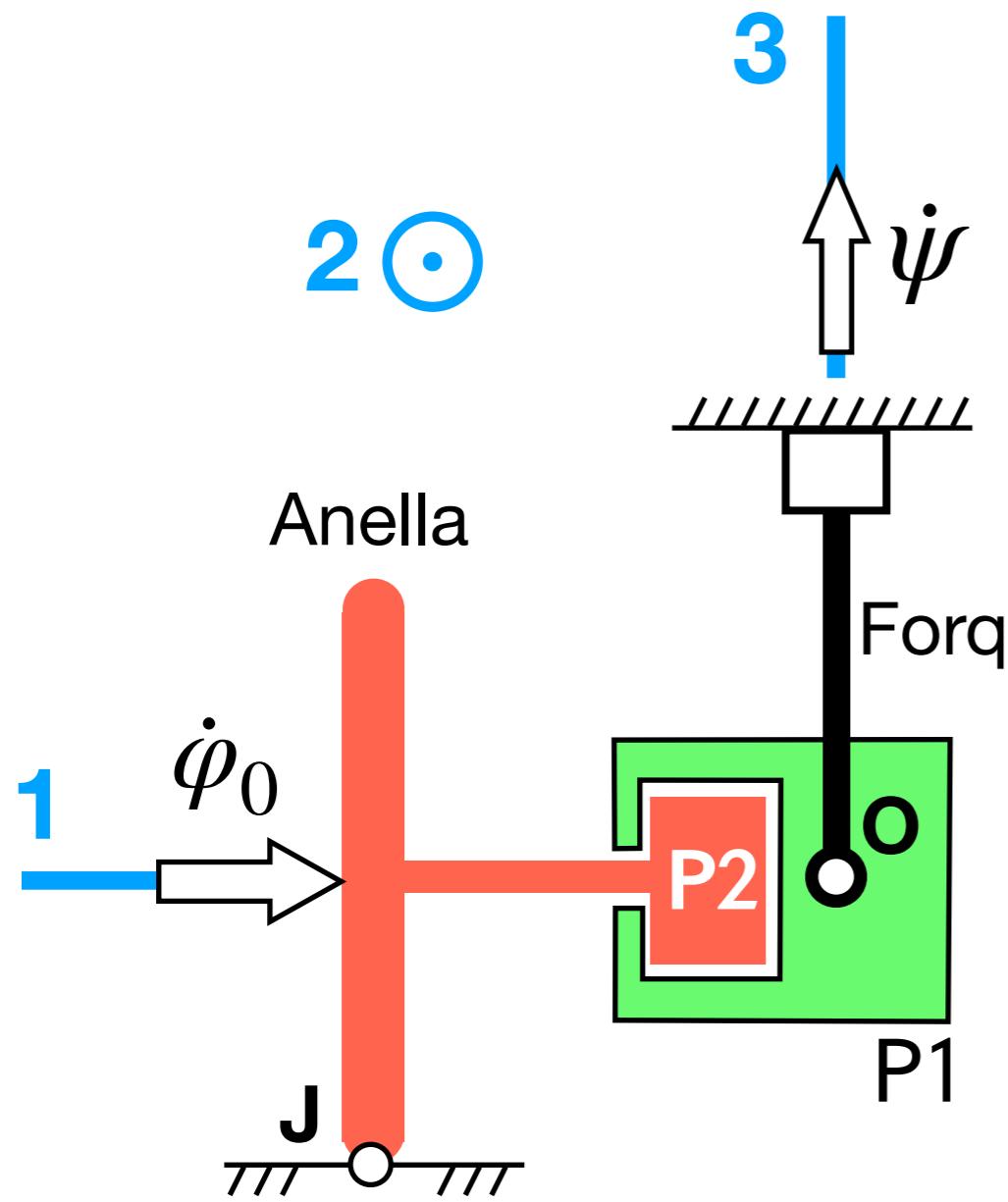
$$\left\{ \dot{\bar{H}}_{RTO}(\mathbf{O}) \right\}_B = \begin{Bmatrix} I_{11} \ddot{\theta} + (I_{33} - I_{22}) \dot{\psi}_0^2 \sin \theta \cos \theta \\ (I_{11} + I_{22} - I_{33}) \dot{\psi}_0 \dot{\theta} \cos \theta \\ (I_{22} - I_{11} - I_{33}) \dot{\psi}_0 \dot{\theta} \sin \theta \end{Bmatrix} = \begin{Bmatrix} \dot{H}_1 \\ \dot{H}_2 \\ \dot{H}_3 \end{Bmatrix}$$



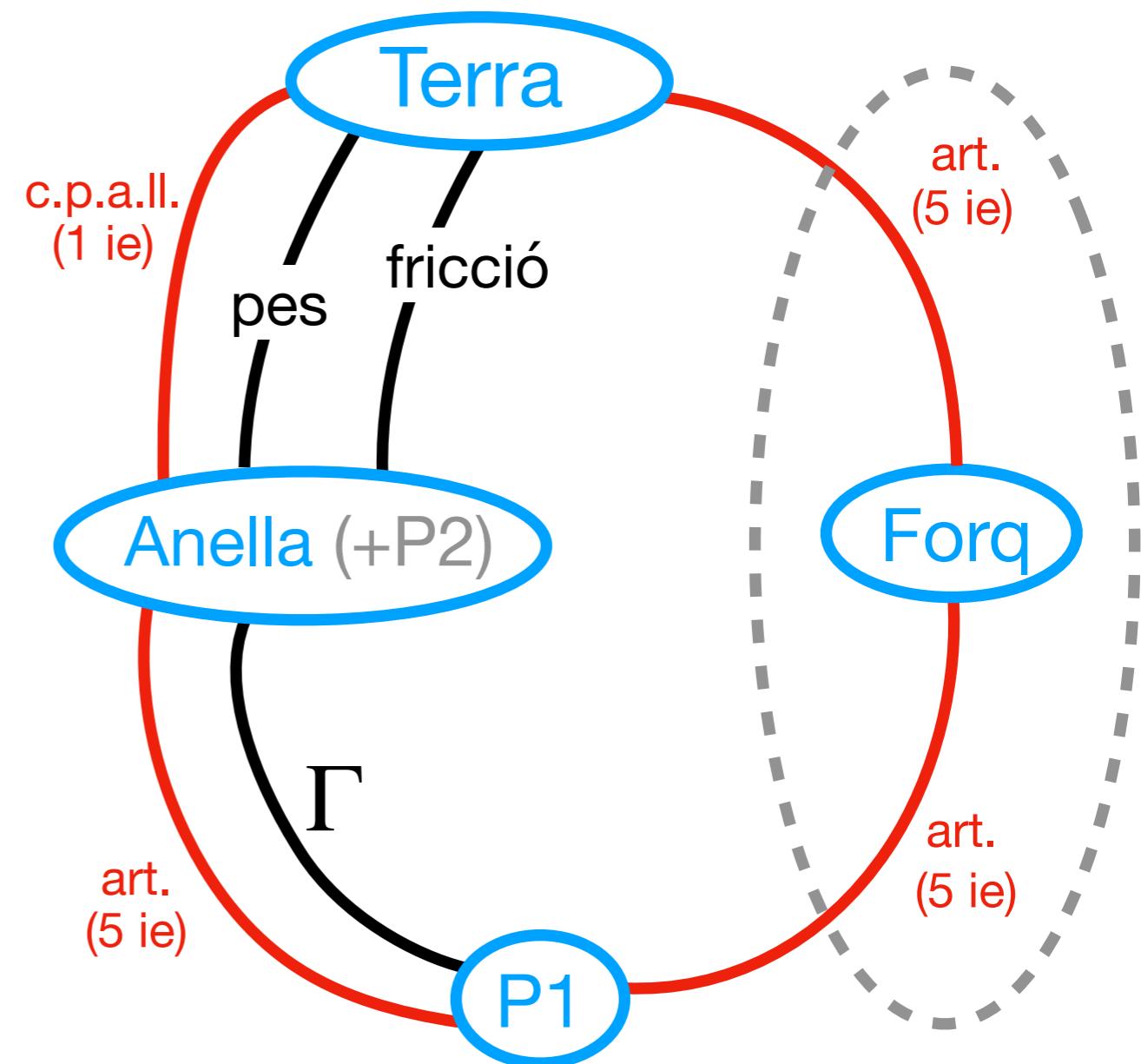
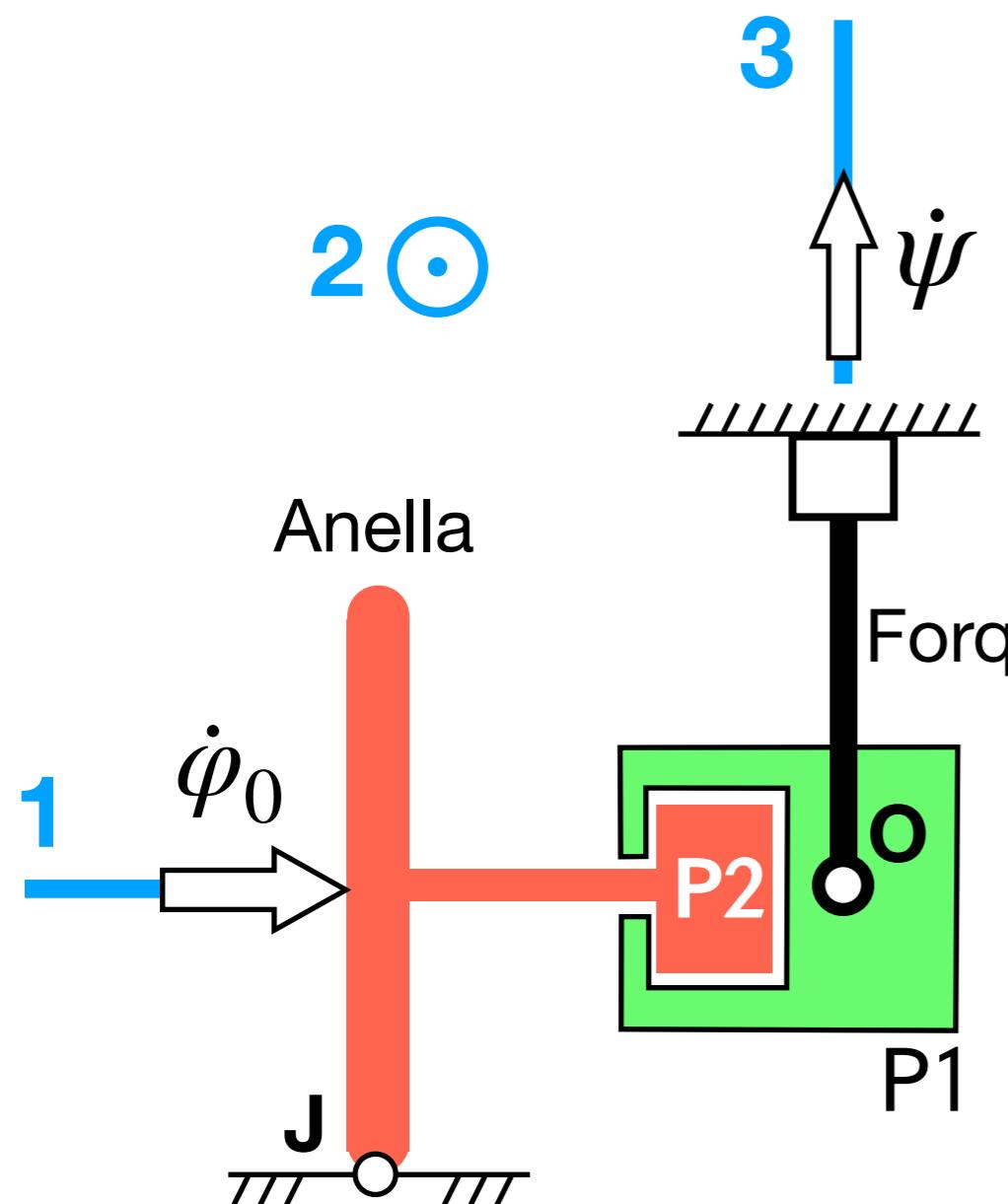
- GL sistema?
- DGI
- Caracterització de torsors?
- Eq. mov per a la coord.  $\psi$
- Força normal a J
- Parell motor per mantenir  $\dot{\phi}_0$

Manté contacte puntual a J

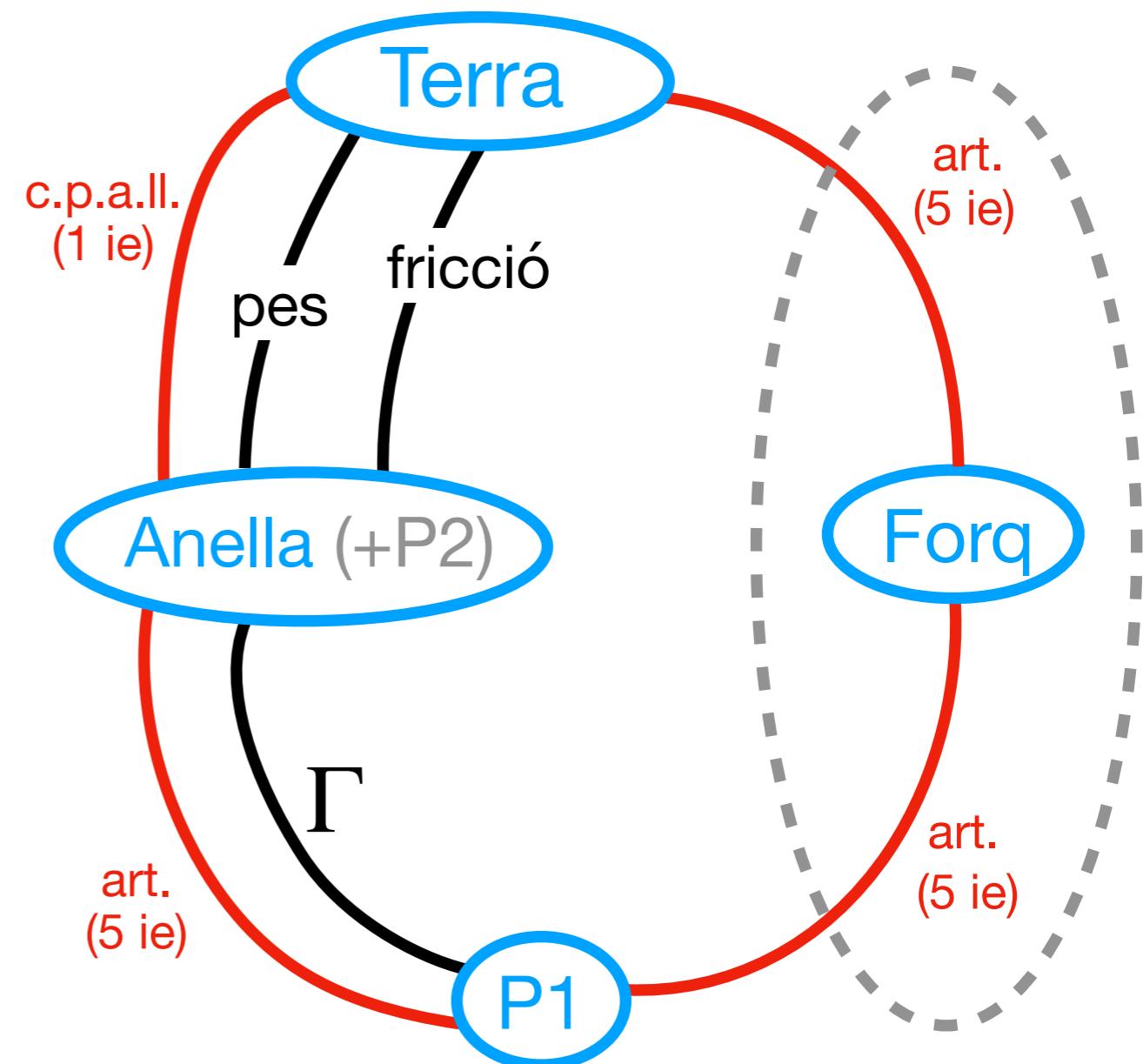
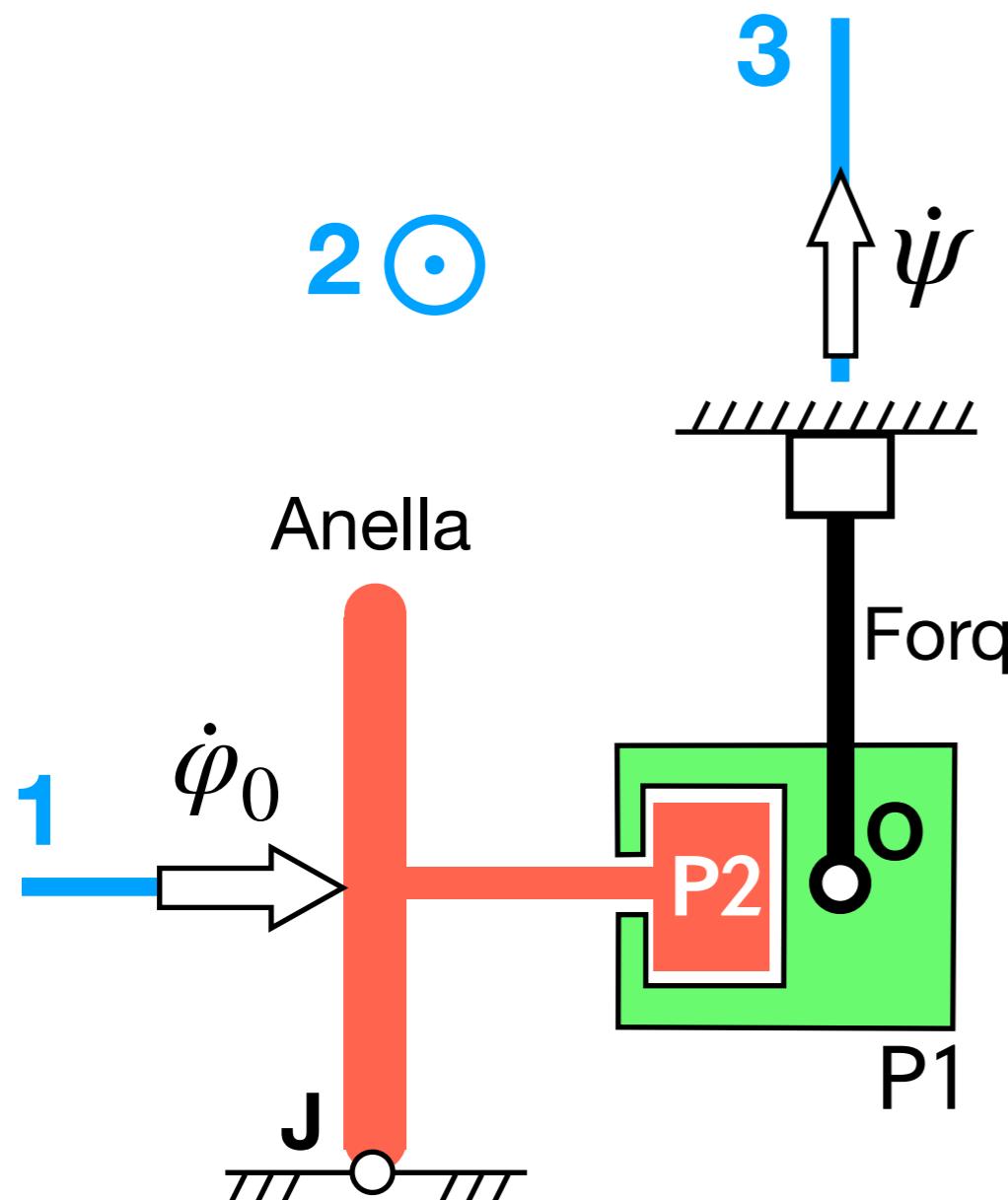
Motor manté  $\dot{\phi}_0 = \text{ct}$



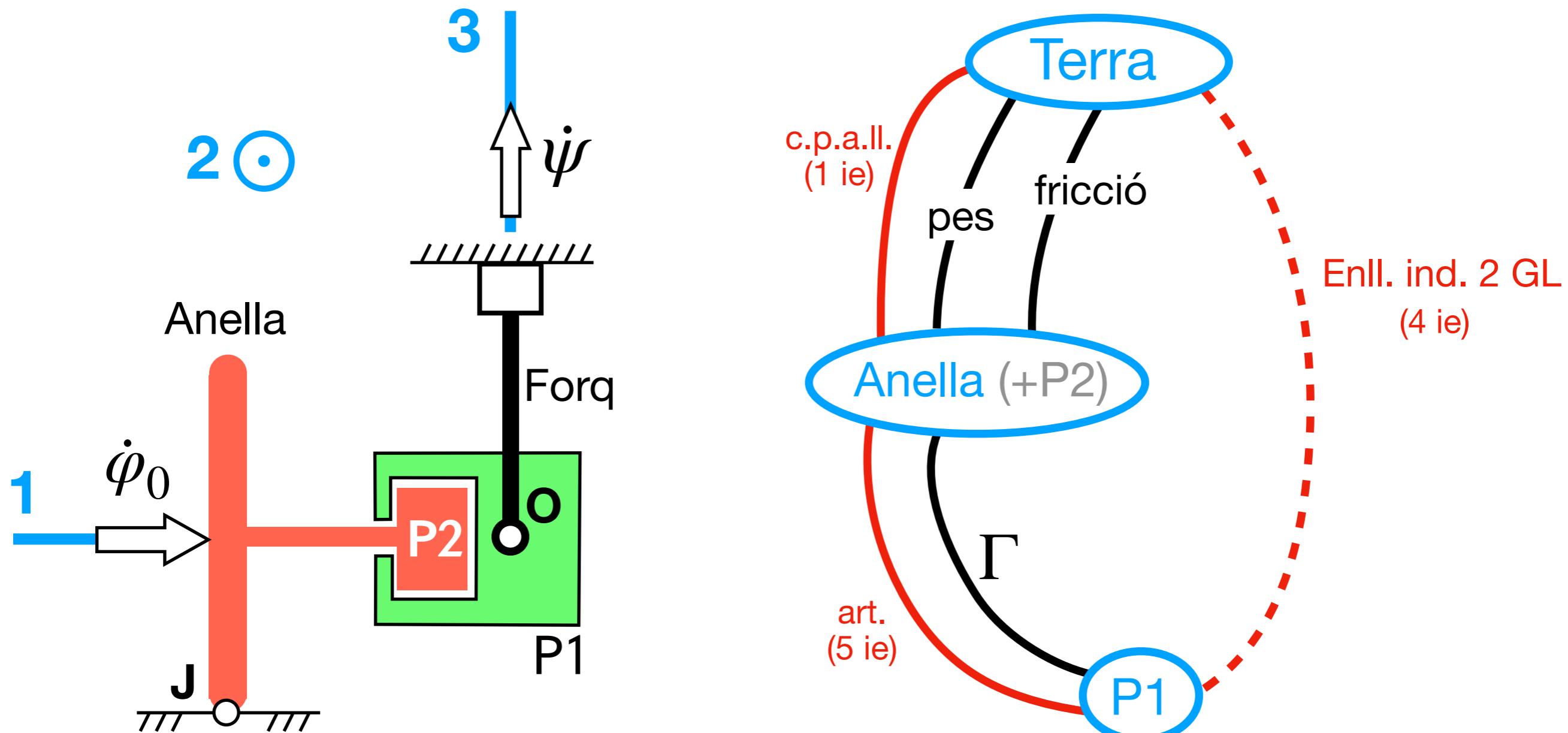
Forq és SAE ...



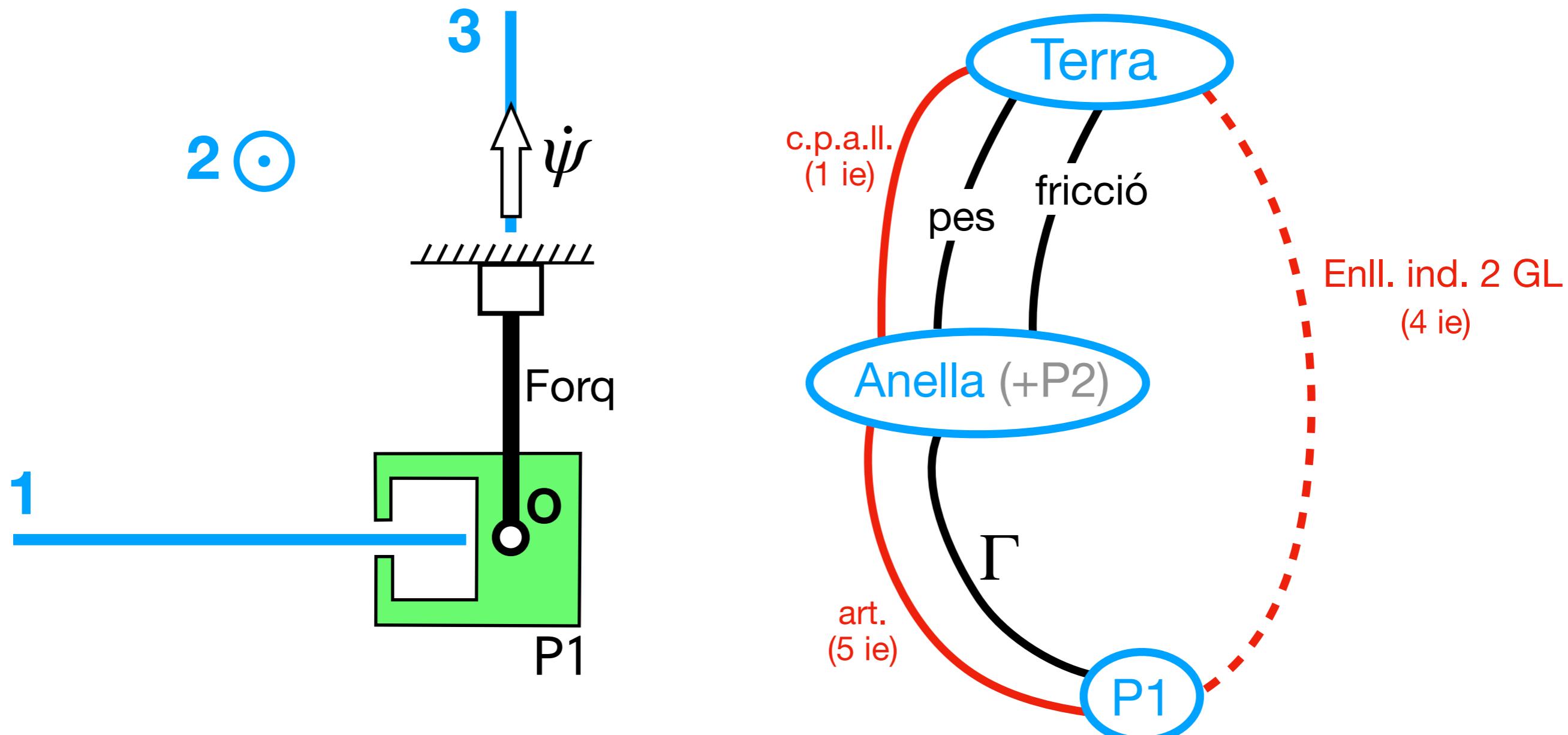
# Substituim Forq per enllaç indirecte $T \rightarrow P_1$



# Substituim Forq per enllaç indirecte $T \rightarrow P_1$



# Substituim Forq per enllaç indirecte $T \rightarrow P_1$

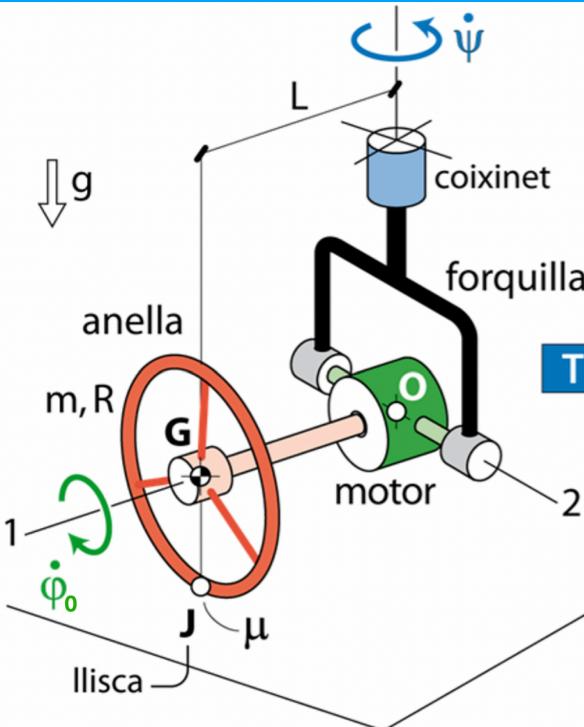


Torsor d'enllaç indirecte  $T \rightarrow P_1$  al punt O:

$$\left\{ \bar{F}_{T \rightarrow (\text{forq}) \rightarrow \text{Sup}} \right\}_B = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \quad O \text{ fix a } T$$

$$\left\{ \bar{M}_{T \rightarrow (\text{forq}) \rightarrow \text{Sup}} (O) \right\}_{B'} = \begin{Bmatrix} M_1 \\ 0 \\ 0 \end{Bmatrix} \quad 0 \text{ en les rotacions permeses de } P_1 \text{ resp } T$$

# Full ruta per eq. del mov. $\psi$

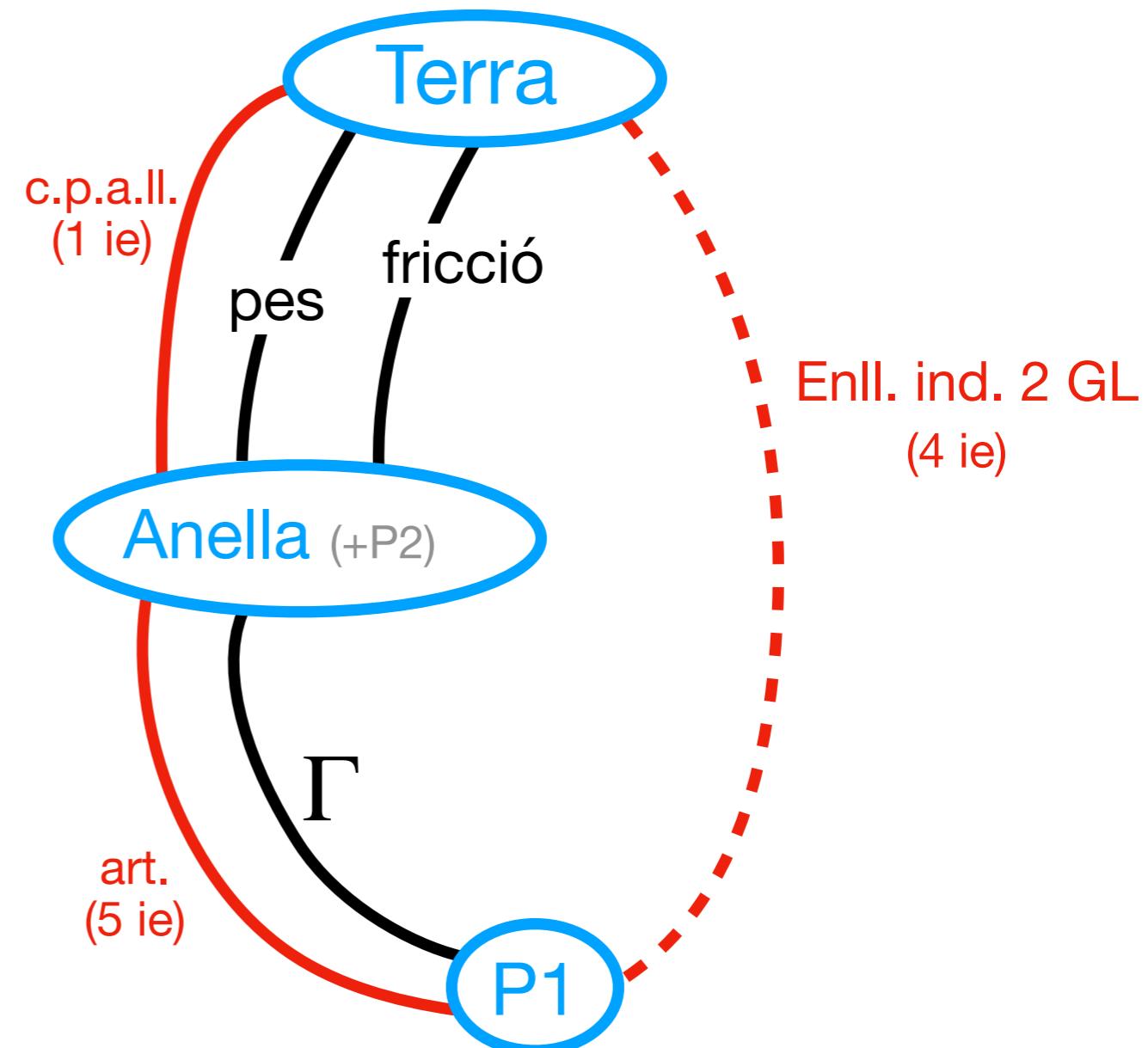


$\psi$  afecta la cinemàtica  
de **P1** i **Anella**



Sist ha d'incloure **P1** o **Anella**  
(o ambdós)

Sistema	Incògn.	Problema
Anella	6 ie, $\Gamma$ , $\ddot{\psi}$	<b>INDET</b>
P1	9 ie, $\Gamma$ , $\ddot{\psi}$	<b>INDET</b>
Anella + P1	5 ie, $\ddot{\psi}$	<b>DET</b>



⇒ Triem SIST = Anella + P1