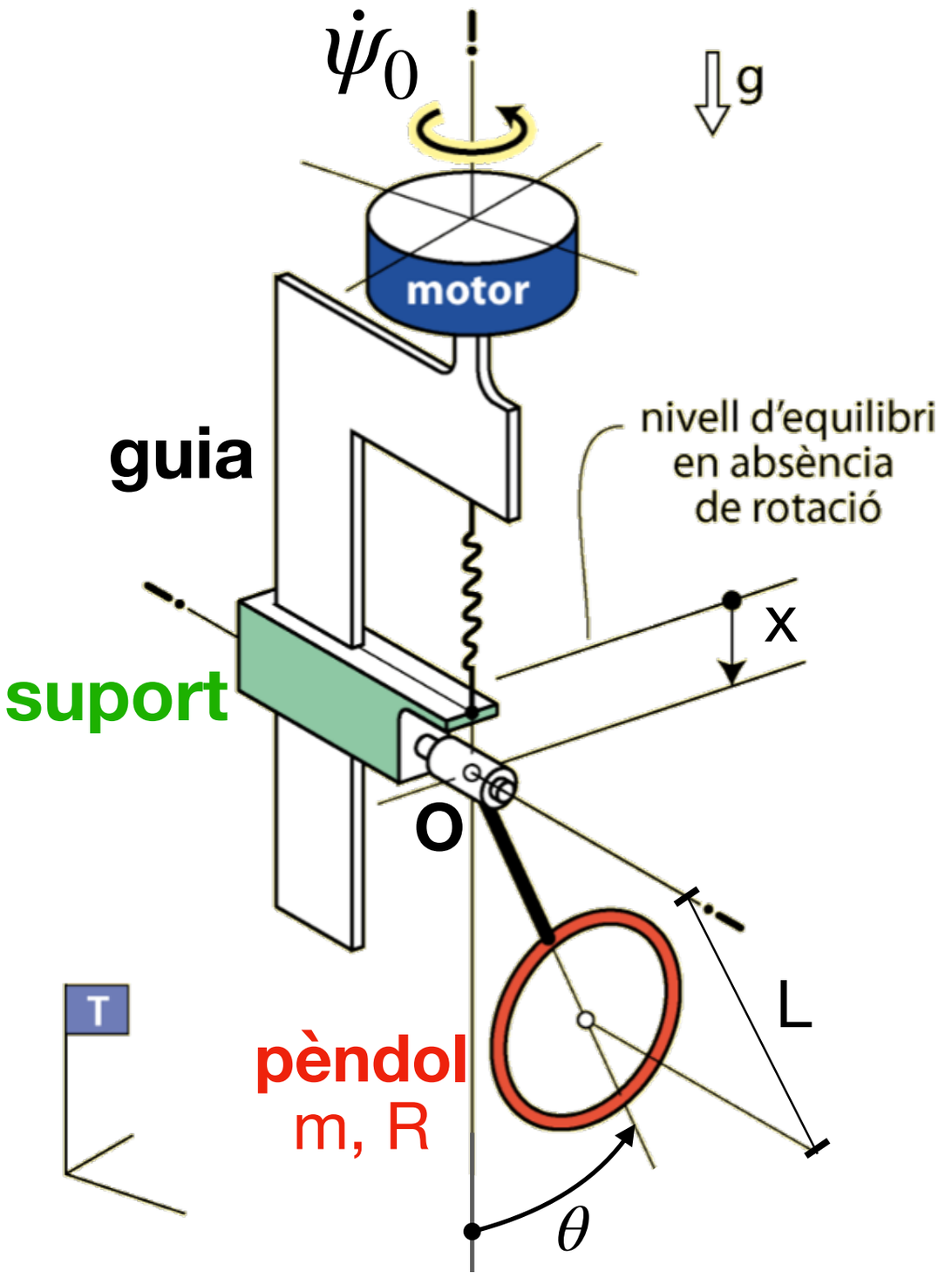
13P

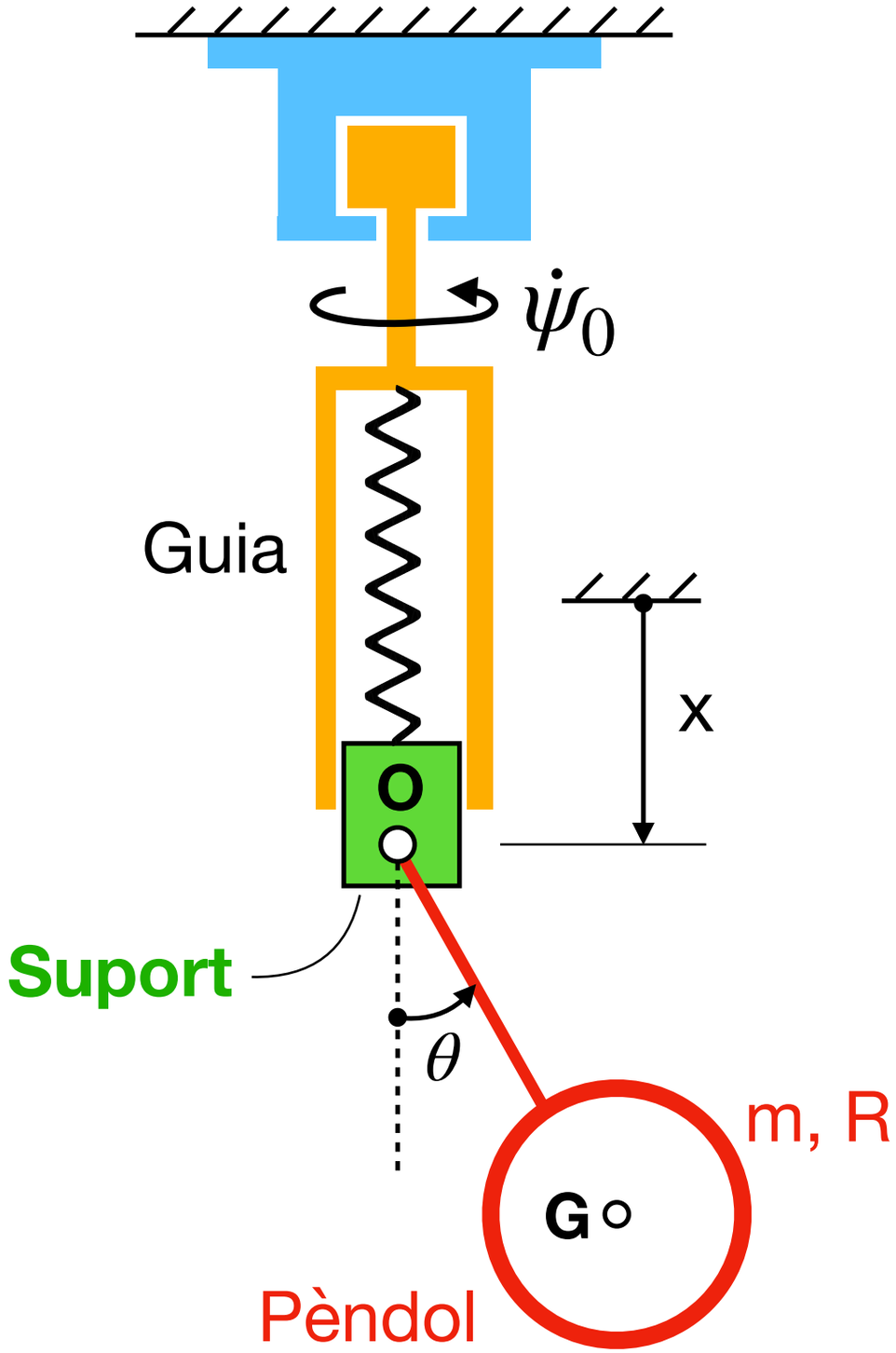
Teoremes vectorials III

Exemples 3D



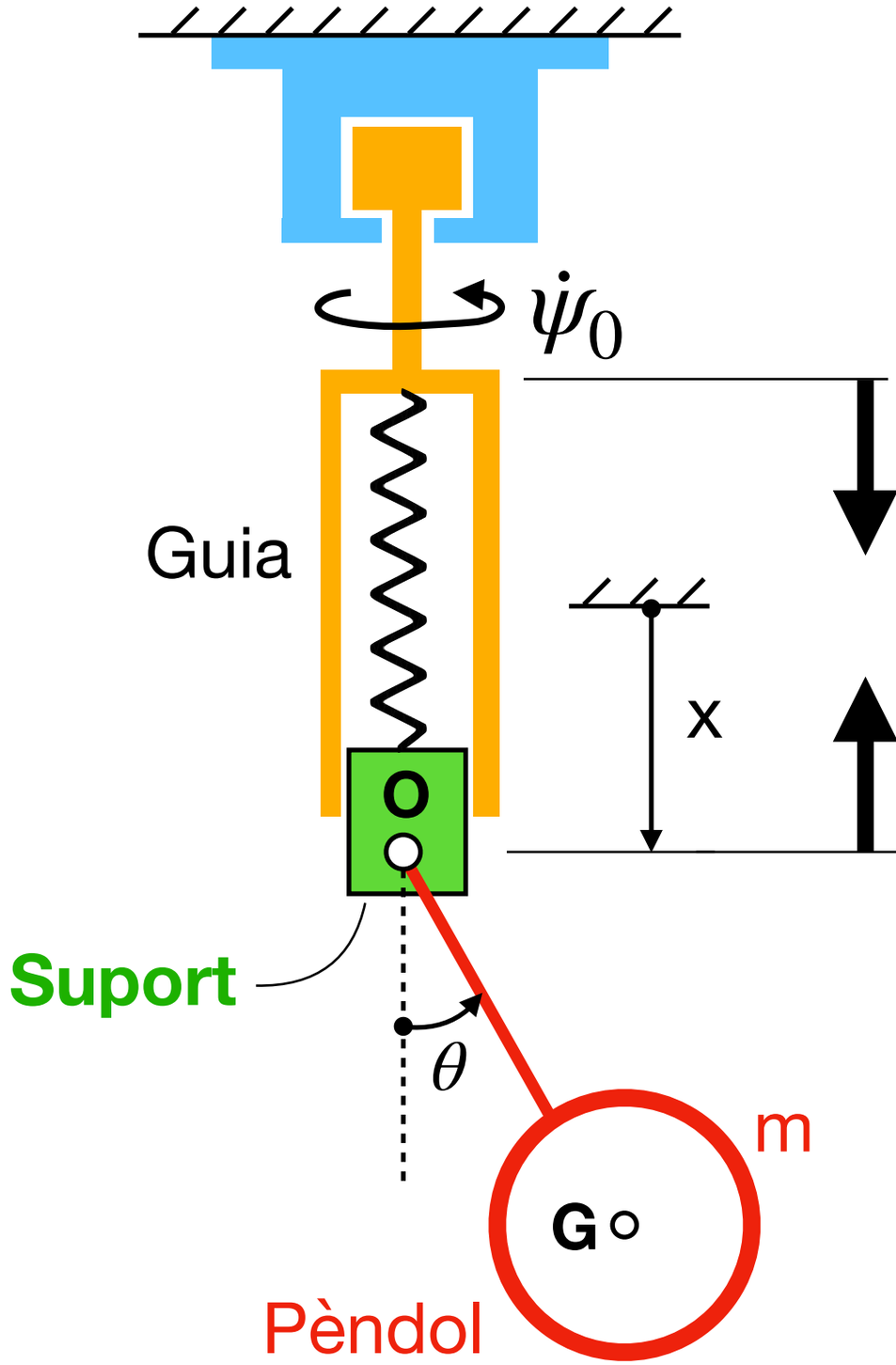
- DGI
- Eqs. mov. per a x i θ
- Parell motor Γ per mantenir $\dot{\psi}_0$ = ct

Amb motor aturat: x = 0, $\theta = 0$ és config. d'equilibri



- DGI
- Eqs. mov. per a x i θ
- Parell motor Γ per mantenir $\dot{\psi}_0$ = ct

Amb motor aturat: x = 0, $\theta = 0$ és config. d'equilibri

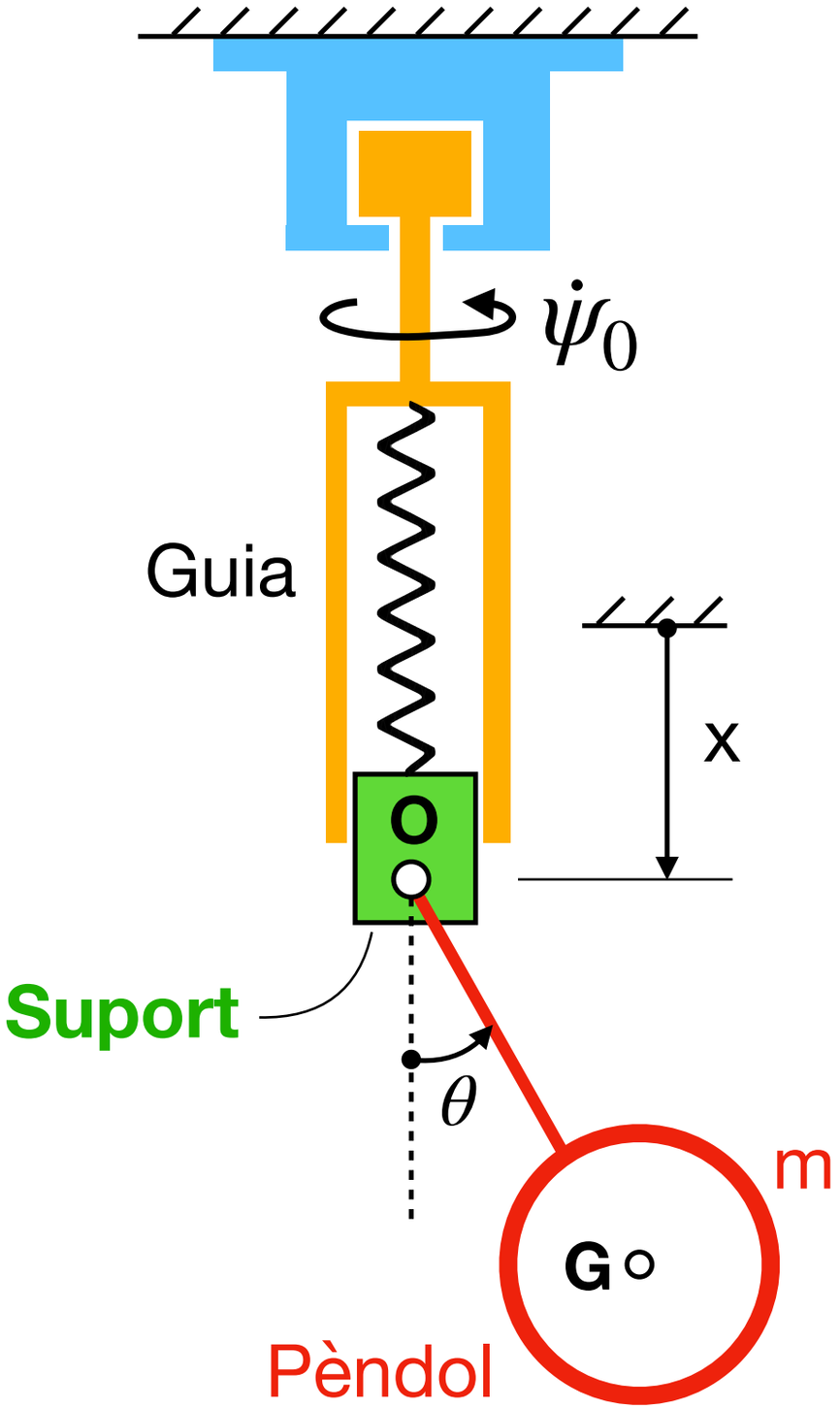


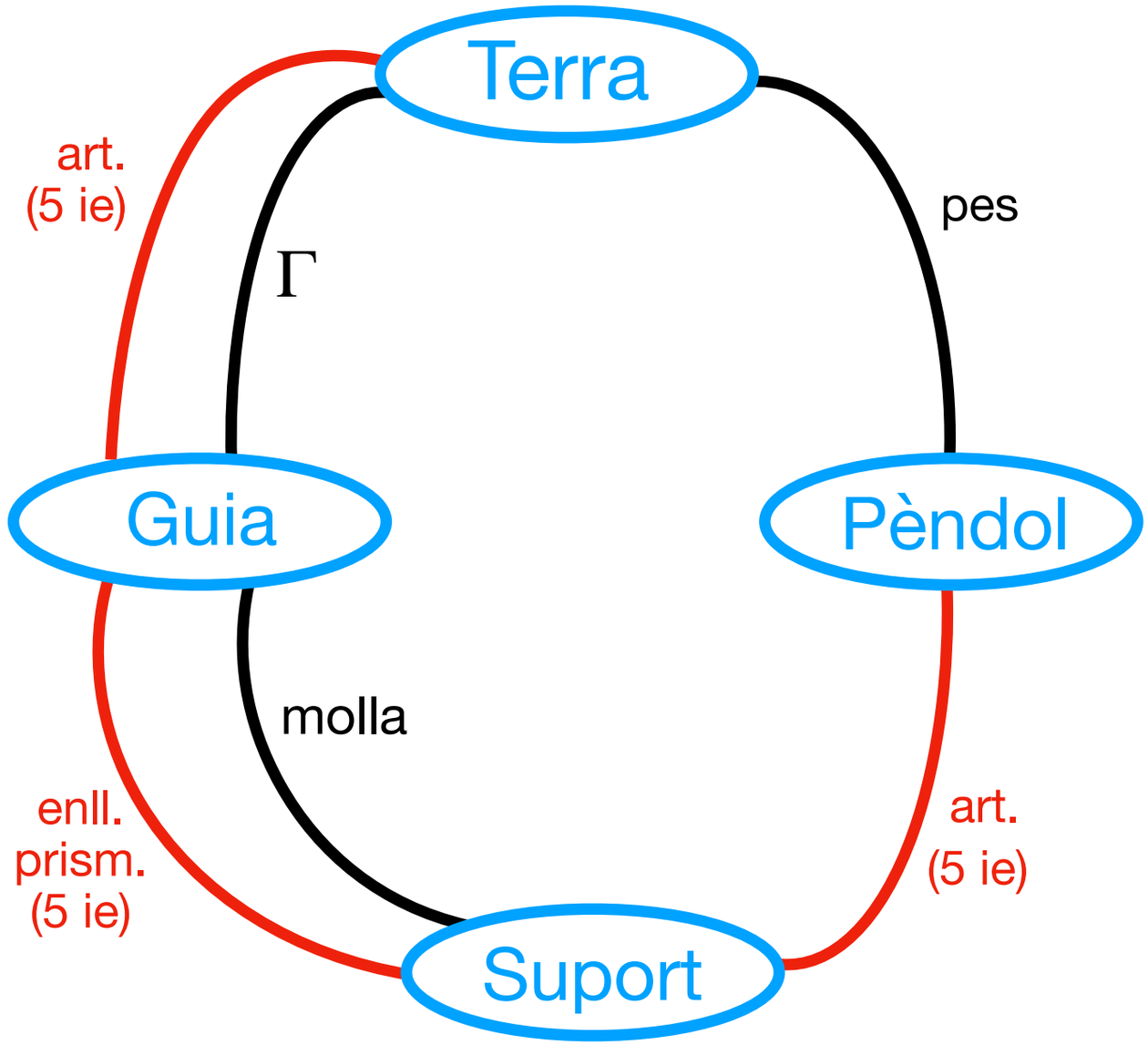
Força de la molla ?

$$F^{at}_{molla} = \underbrace{mg}_{F_0} + \underbrace{kx}_{k\Delta\rho}$$

Amb motor aturat: x = 0, $\theta = 0$ és config. d'equilibri

DGI



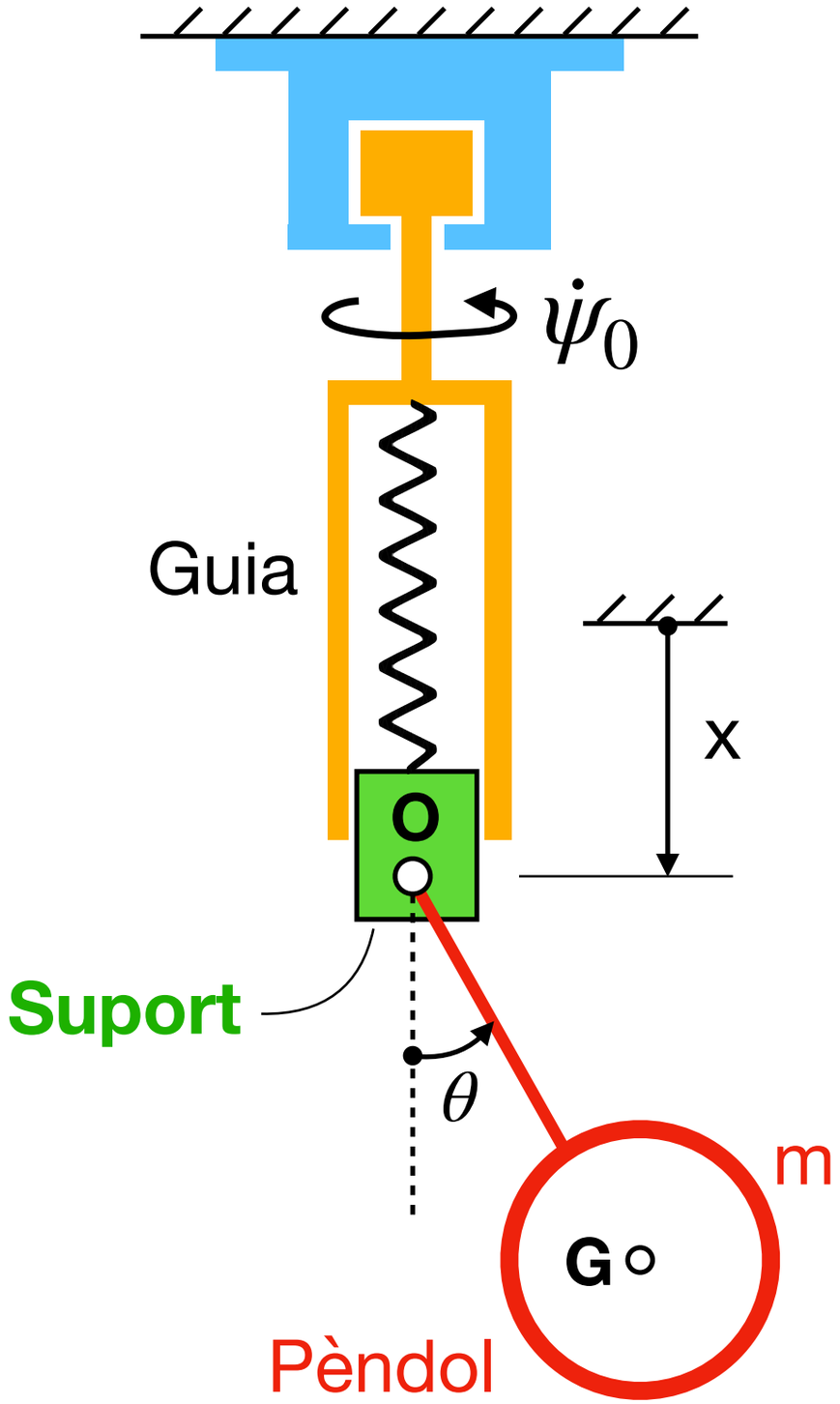


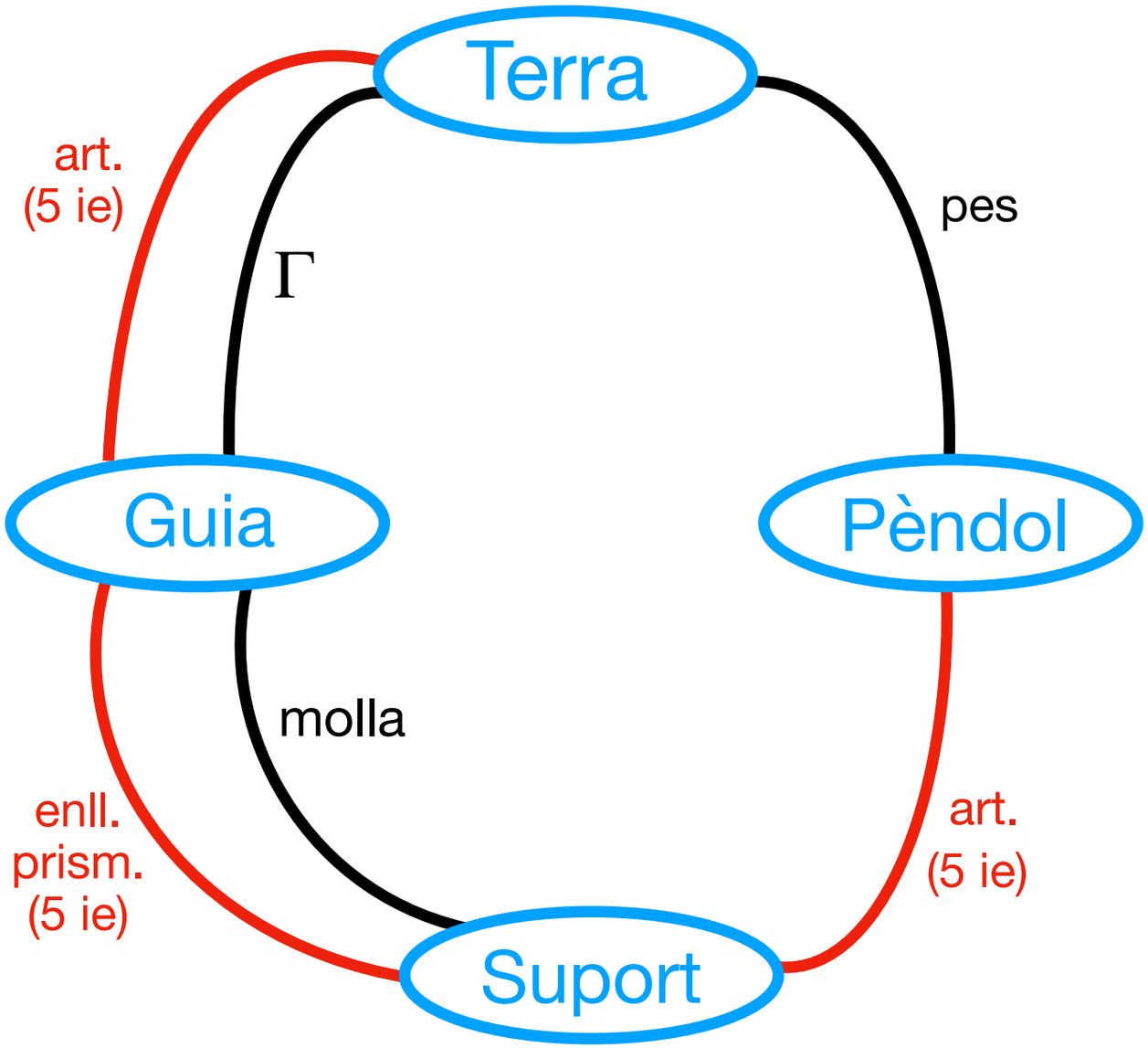
15 ie, Γ, $\ddot{x}$, $\ddot{\theta}$ $\Rightarrow$ **18 incòg**

3 sòlids · 6 eqs/sòlid $\Rightarrow$ **18 eqs**

} **DET**

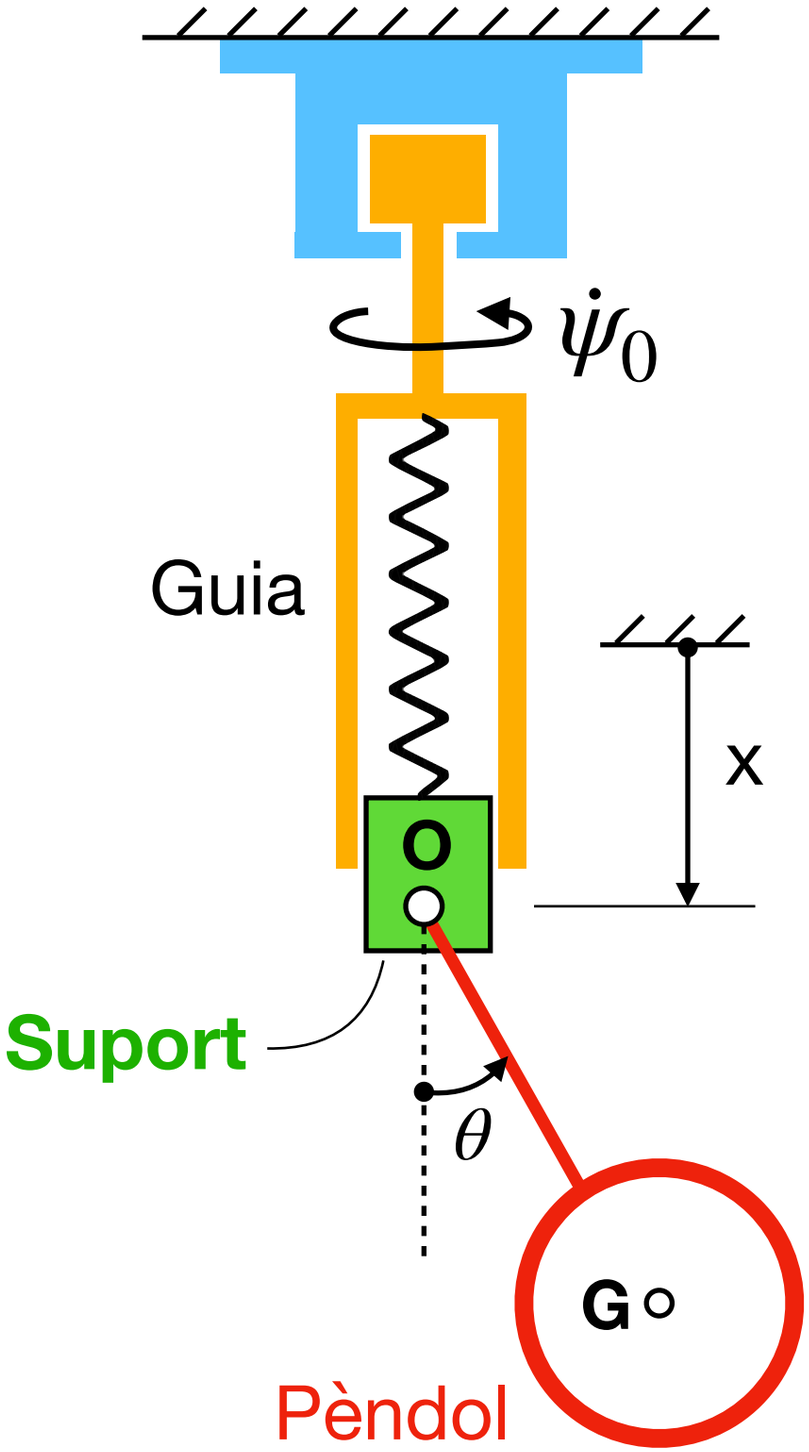
DGI

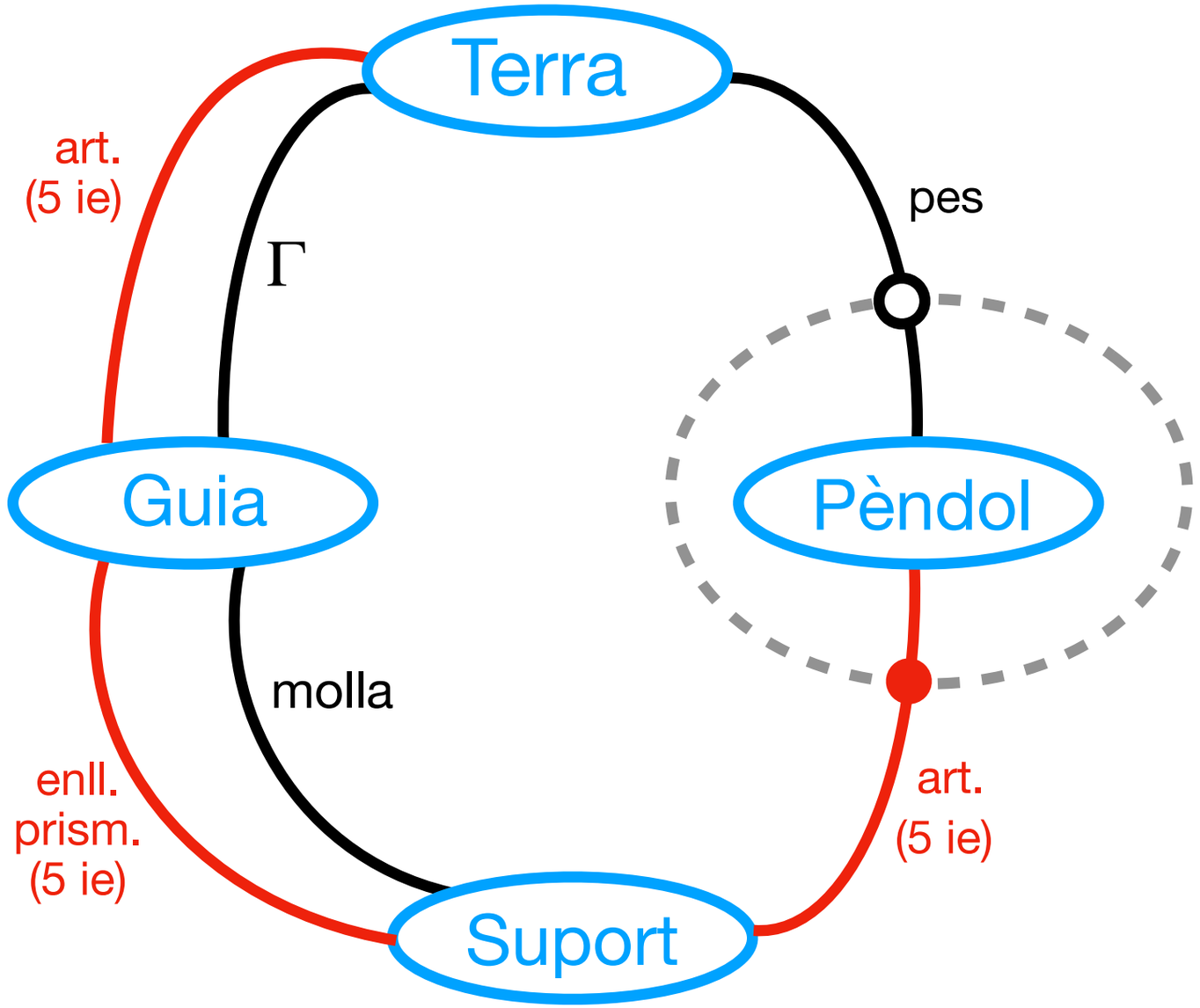




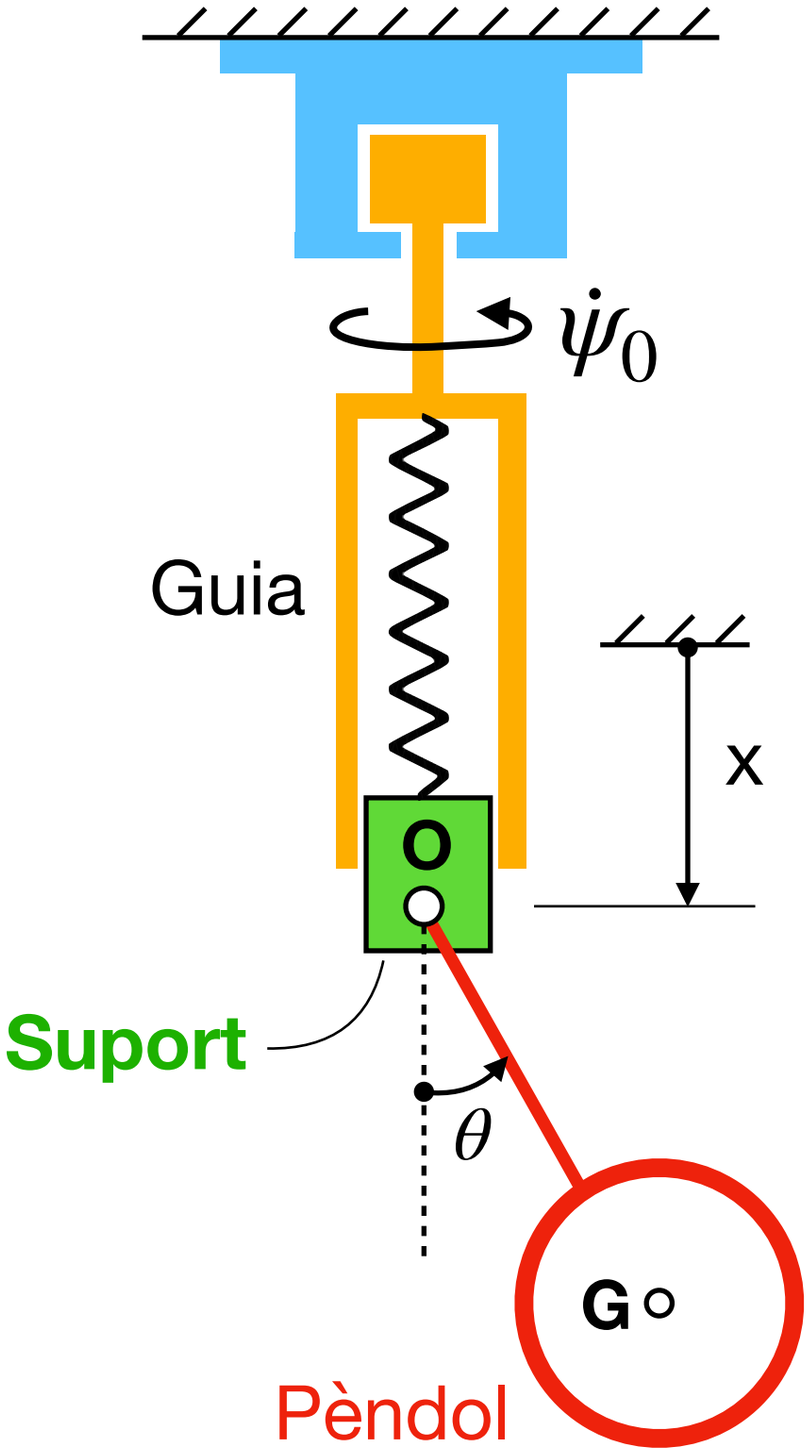
x i θ només afecten **pèndol** i **suport**

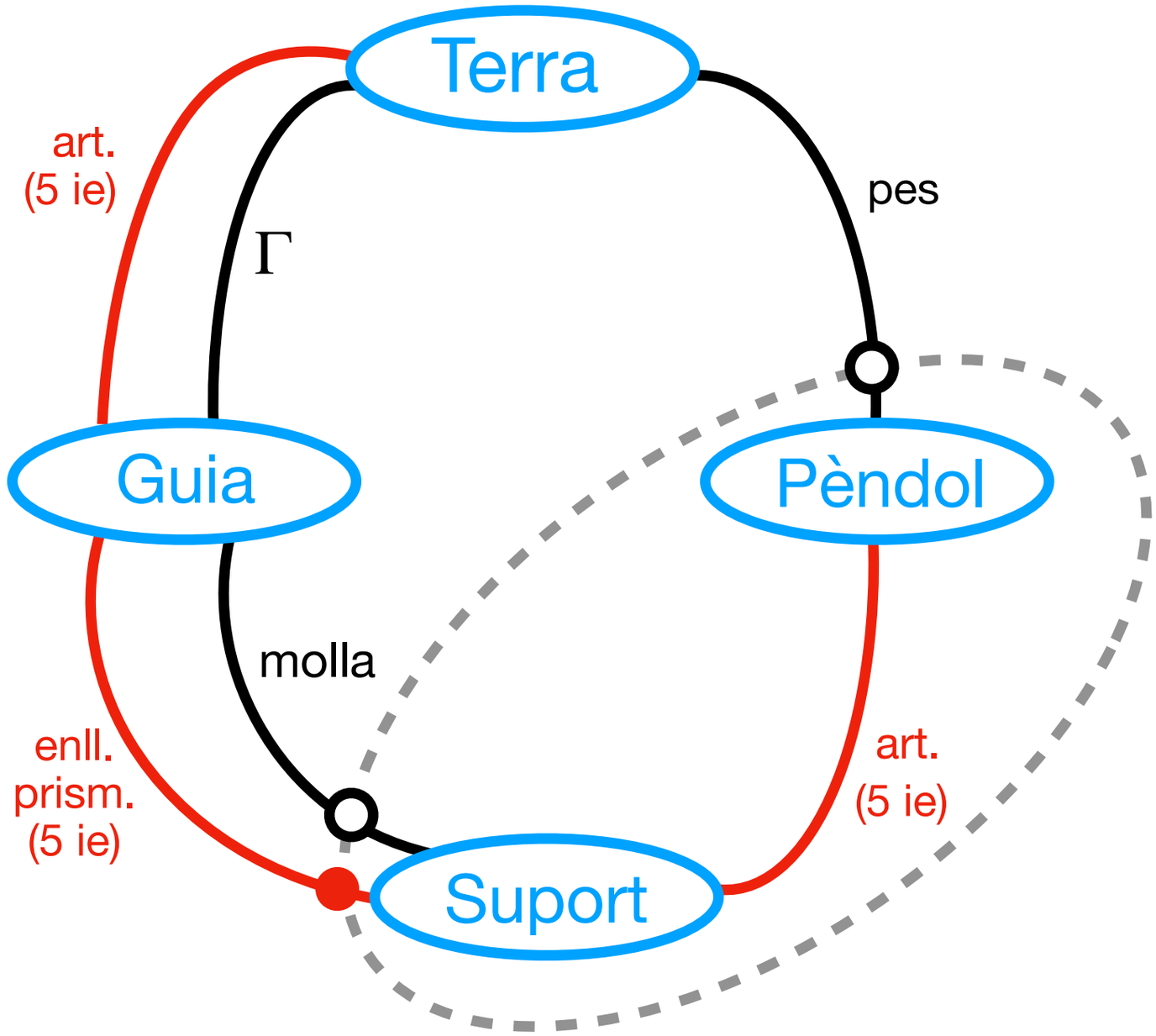
Explorem sistemes que els incloguin





Sistema	Incògn.	Problema
Pèndol	5 ie, $\ddot{x}$, $\ddot{\theta}$	**INDET**

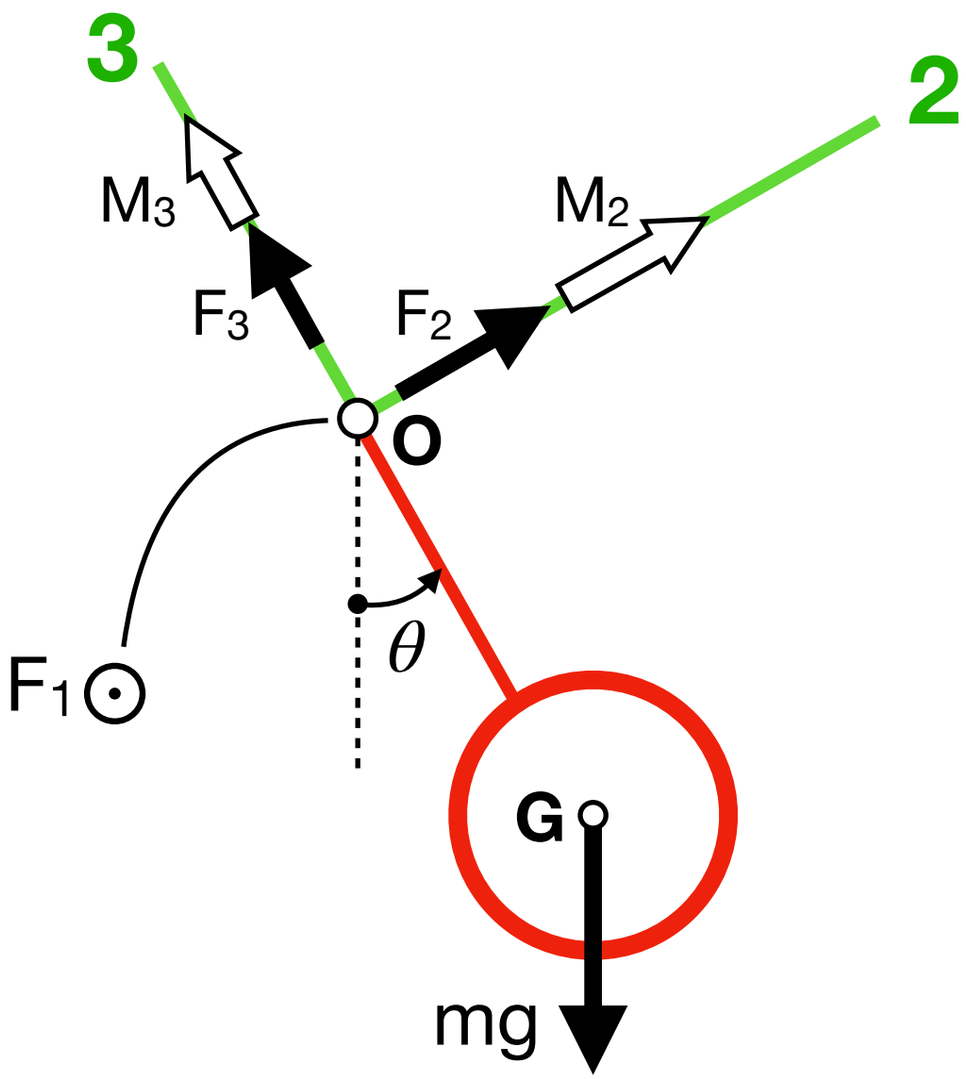


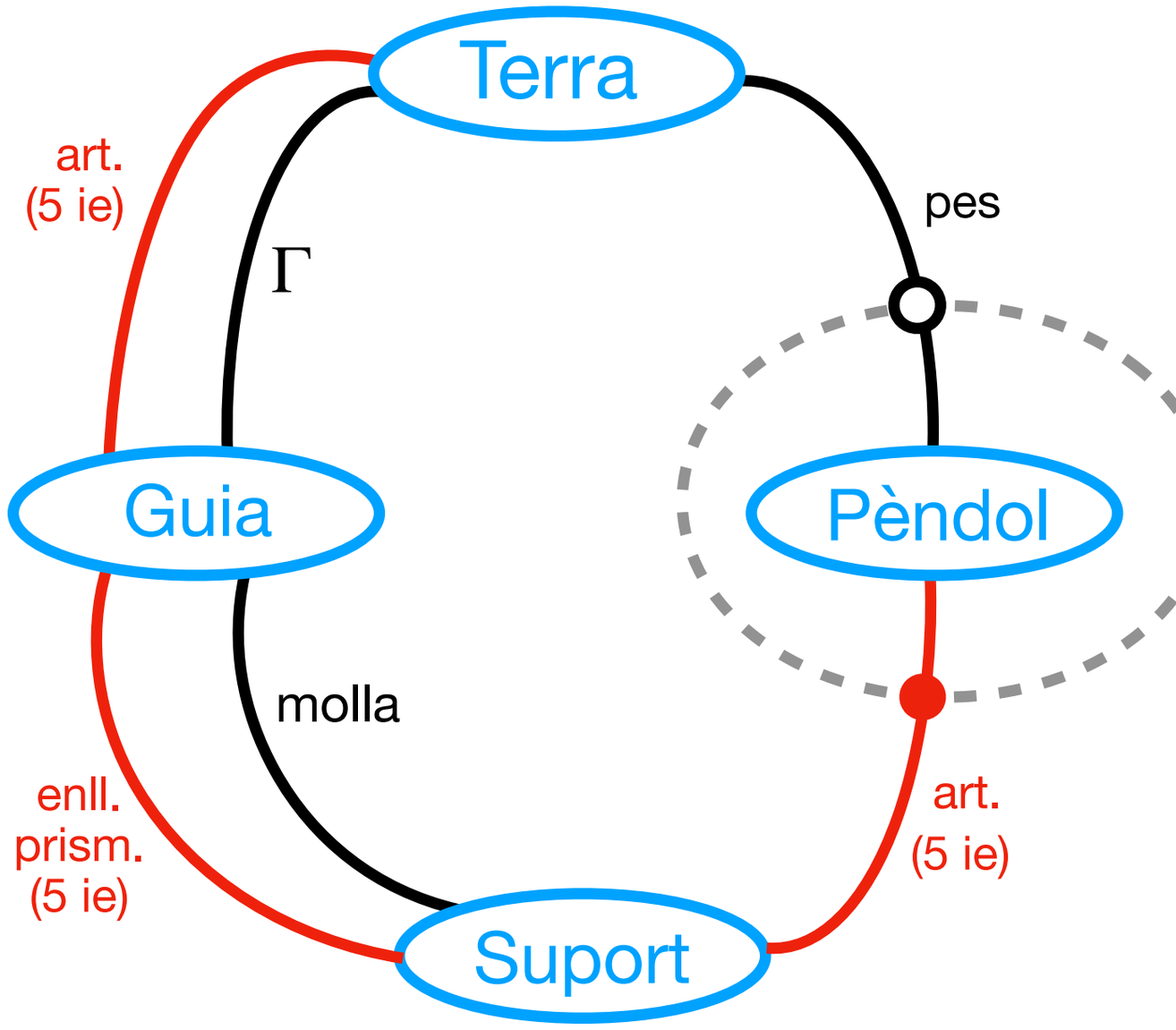


Sistema	Incògn.	Problema
Pèndol	5 ie, $\ddot{x}$, $\ddot{\theta}$	**INDET**
Pènd. + sup.	5 ie, $\ddot{x}$, $\ddot{\theta}$	**INDET**

Els altres sistemes tenen + incògnites !

Sist = Pèndol



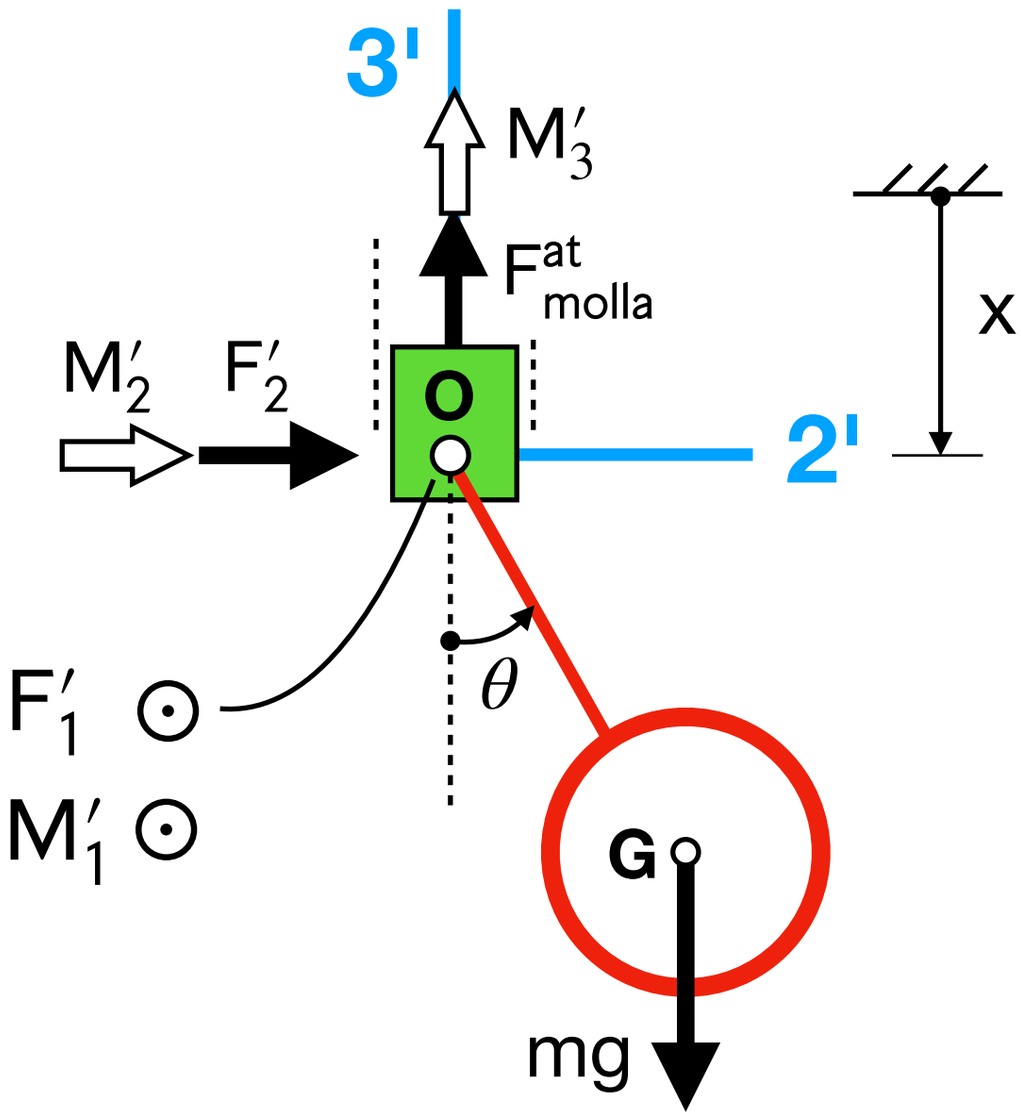


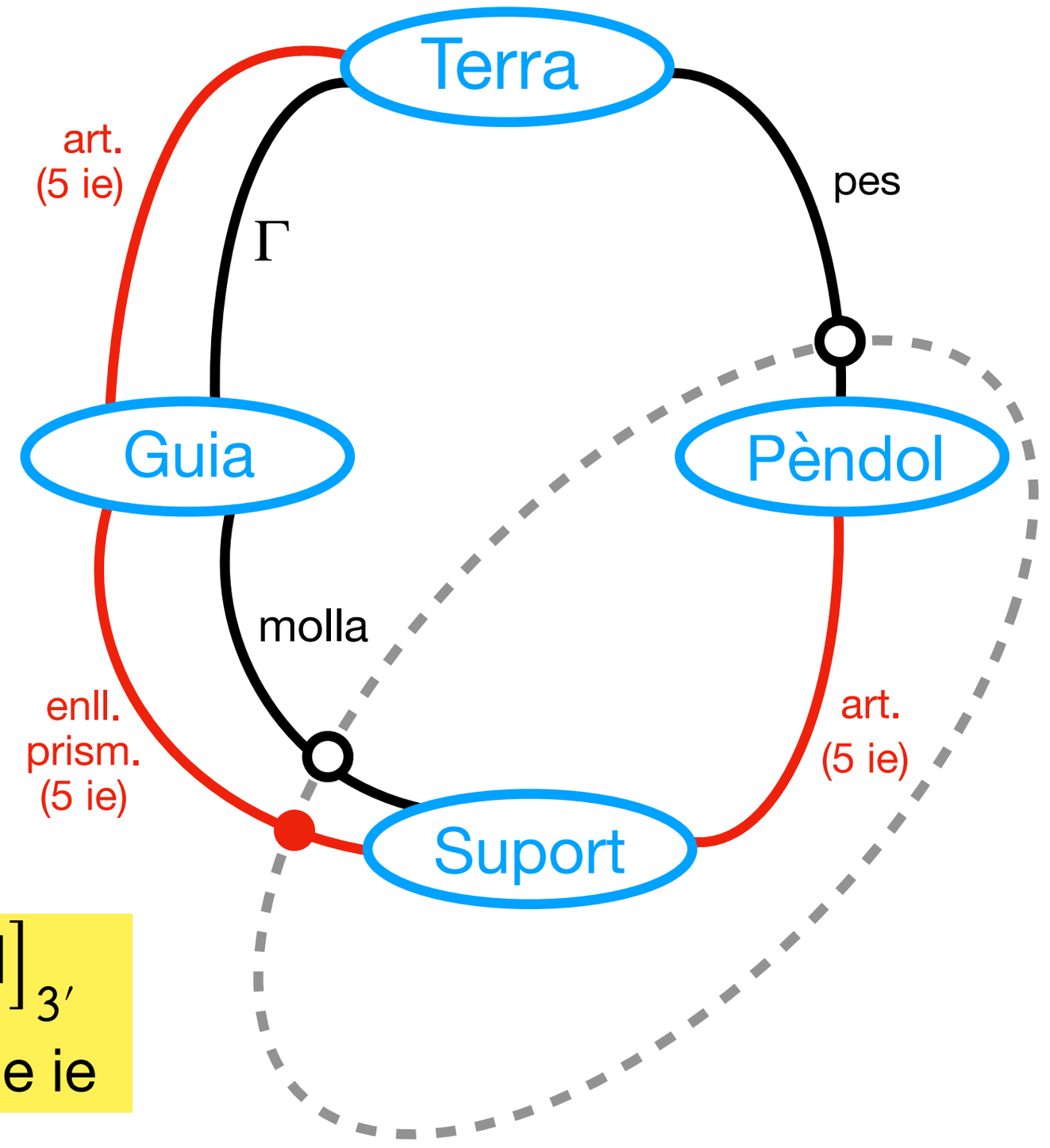
$$\left\{ \bar{F}_{Sup \to Pendol} \right\}_B = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\left\{ \bar{M}_{Sup \to Pendol}(\mathbf{O}) \right\}_B = \begin{Bmatrix} 0 \\ M_2 \\ M_3 \end{Bmatrix} \leftarrow \text{TMC}(\mathbf{O})\big]_1$$

lliure de ie

Sist = Pèndol + Suport



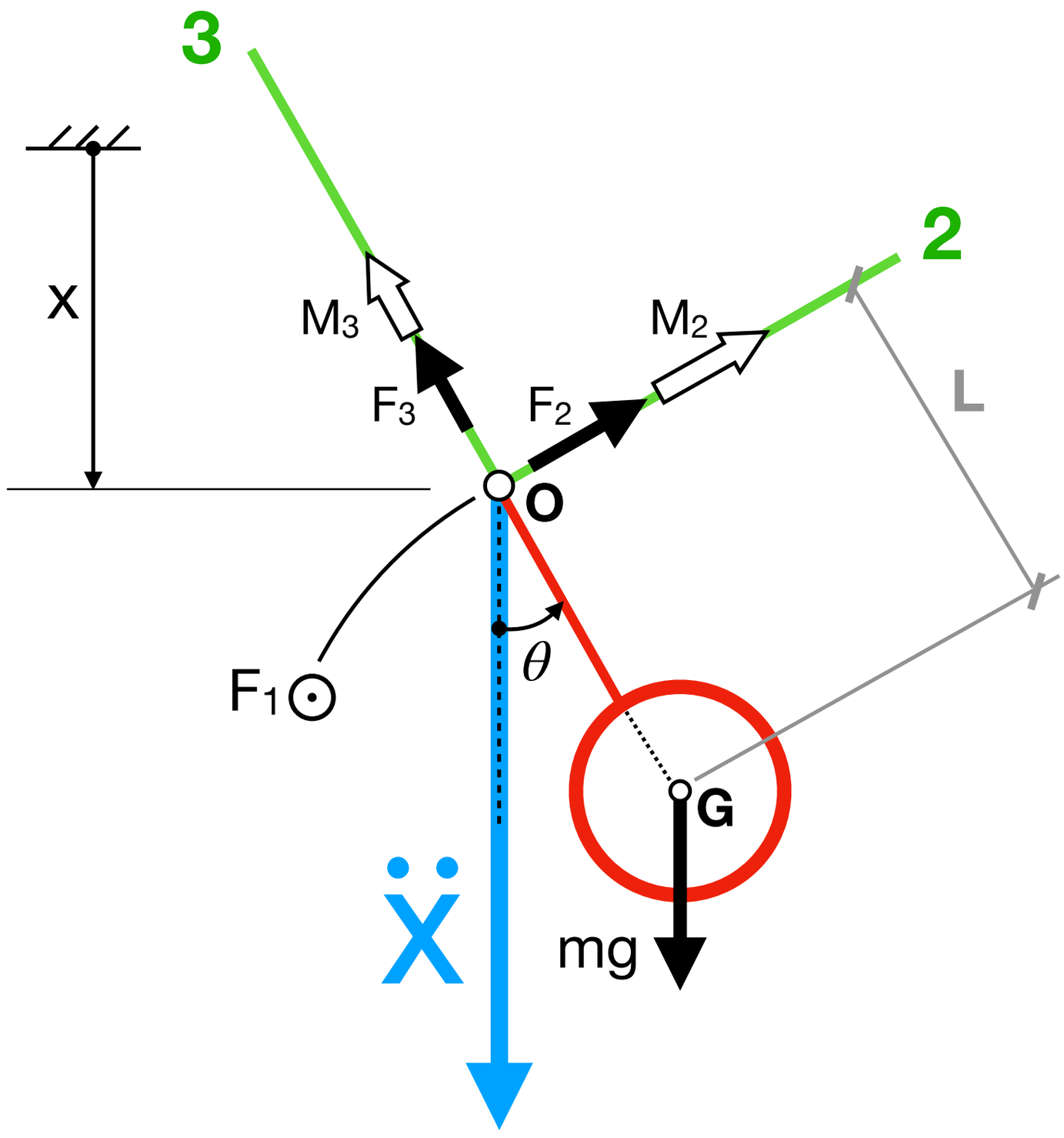


$$\left\{\bar{F}_{Guia\to Sup}\right\}_{B'} = \begin{Bmatrix} F'_1 \\ F'_2 \\ 0 \end{Bmatrix}$$

$$\left\{\bar{M}_{Guia\to Sup}\,(\mathbf{O})\right\}_{B'} = \begin{Bmatrix} M'_1 \\ M'_2 \\ M'_3 \end{Bmatrix}$$

$TQM]_{3'}$ lliure de ie

$TMC(O)\ ]_1$ sobre SIST = Pèndol



TMC(O) $]_1$ sobre SIST = Pèndol

$$\left\{\dot{\bar{H}}_{RTO(\mathbf{O})}\right\}_B = \left\{\begin{array}{c} I_{11}\ddot{\theta} + (I_{33} - I_{22})\,\dot{\psi}_0^2\,\sin\theta\,\cos\theta \\ (I_{11} + I_{22} - I_{33})\,\dot{\psi}_0\,\dot{\theta}\cos\theta \\ (I_{22} - I_{11} - I_{33})\,\dot{\psi}_0\,\dot{\theta}\sin\theta \end{array}\right\} \qquad \text{(III)}$$

Pas final

$$\begin{cases} \left(R^2 + L^2\right) \ddot{\theta} - \left(L \sin\theta\right) \ddot{x} = \left(L\dot{\psi_0}^2 \cos\theta - g\right) L \sin\theta \\ -\left(L \sin\theta\right) \ddot{\theta} + \ddot{x} = -\dfrac{k}{m} x + L\dot{\theta}^2 \cos\theta \end{cases}$$

Aillant $\ddot{\theta}$ i $\ddot{x}$

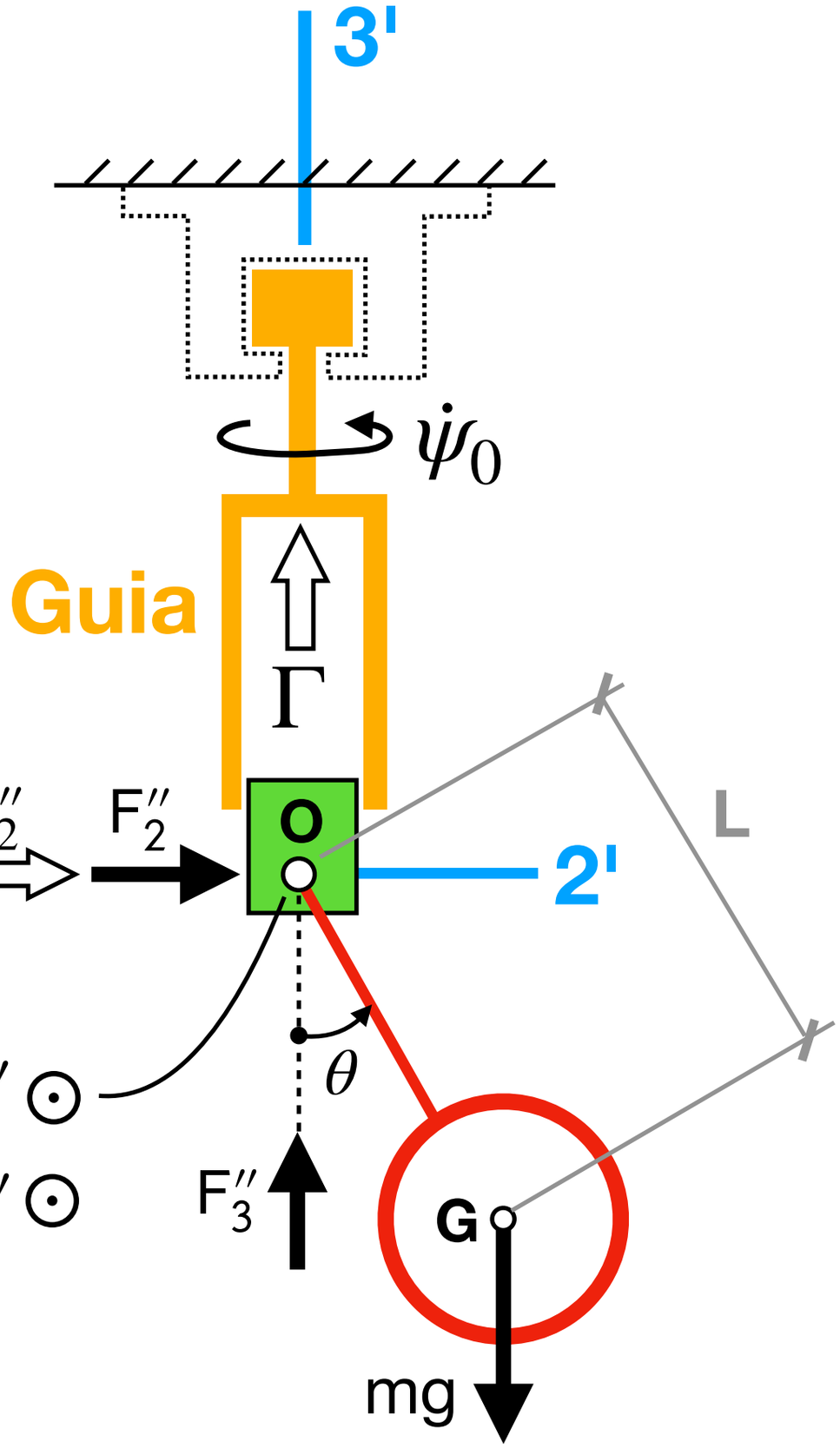
$$\begin{cases} \ddot{\theta} = F_1\left(\theta, \dot{\theta}, x, \dot{x}\right) \\ \ddot{x} = F_2\left(\theta, \dot{\theta}, x, \dot{x}\right) \end{cases}$$

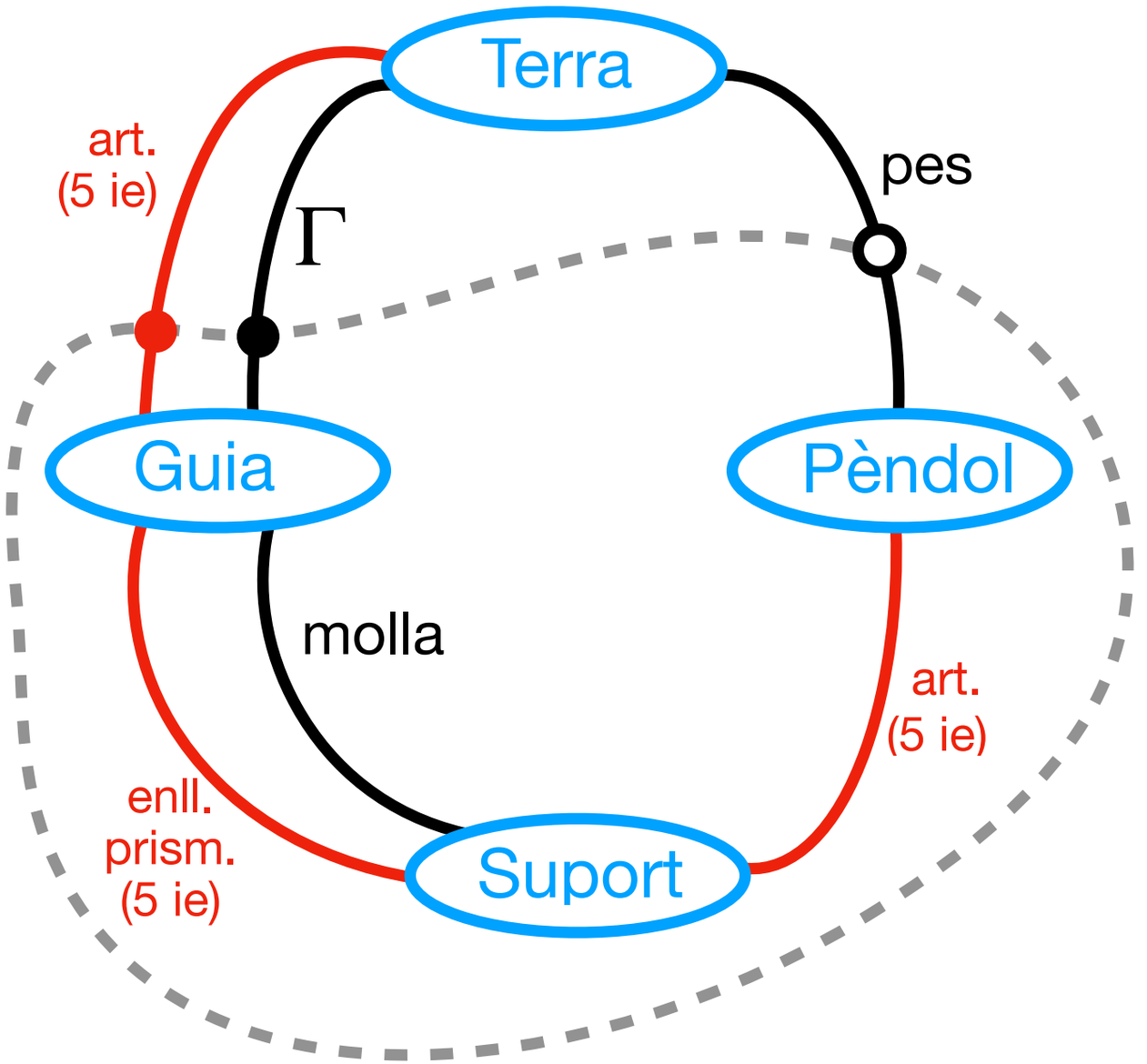
Eq. mov. θ

Eq. mov. x

No cal que el feu

Parell motor per mantenir $\dot{\psi}_0 = \text{ct}$





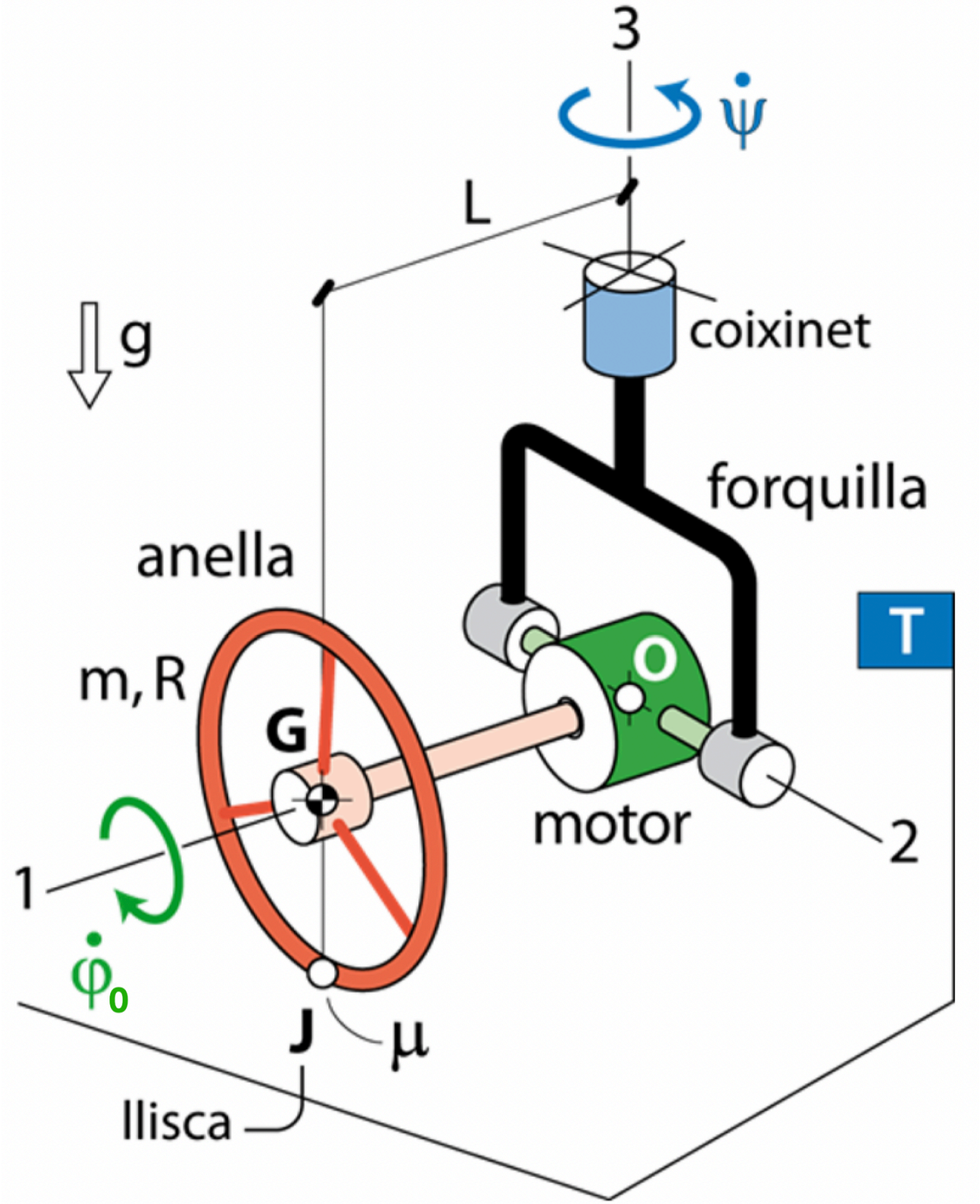
5 ie, Γ, $\ddot{x}$, $\ddot{\theta}$ $\Rightarrow$ **8 incòg**

Ara ja sabem $\ddot{x}$, $\ddot{\theta}$

} **DET**

D'abans

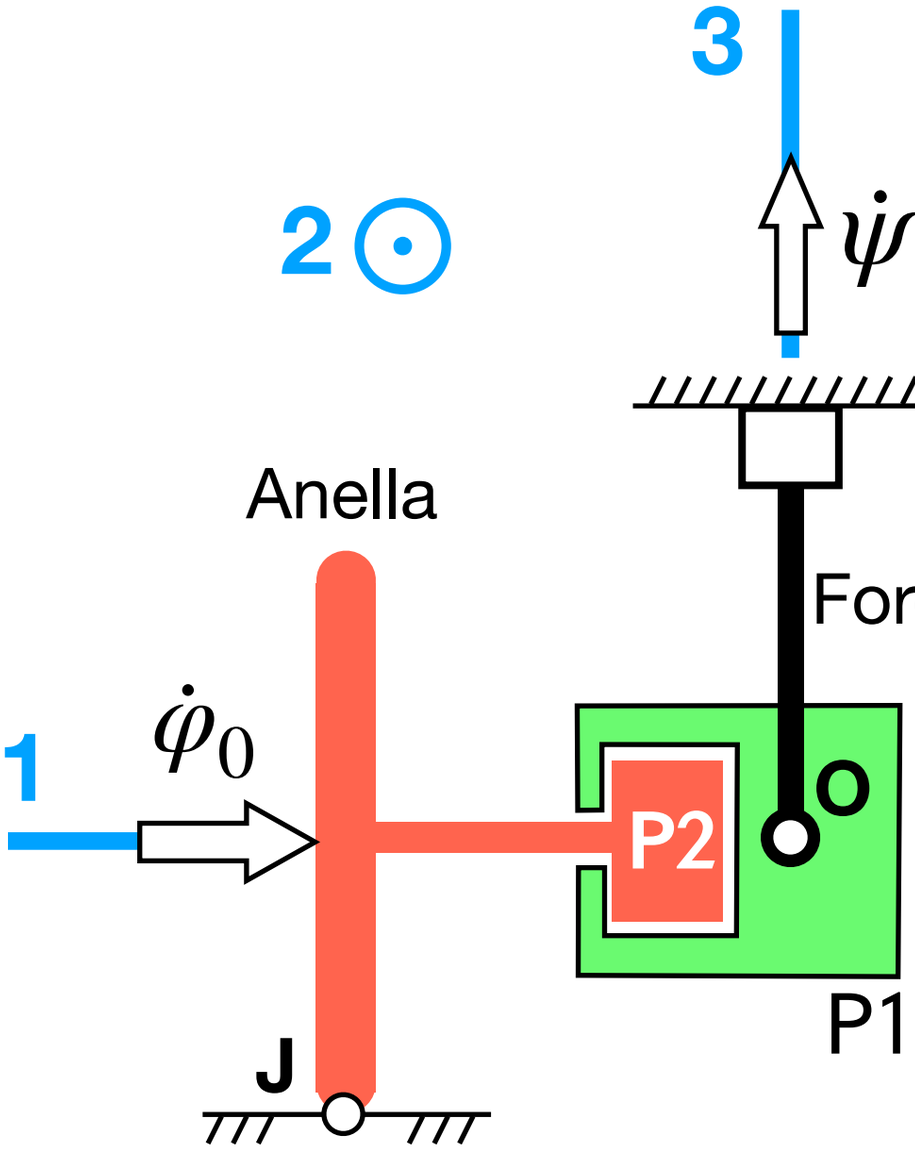
$$\left\{\dot{\bar{H}}_{RTO(\mathbf{O})}\right\}_B = \left\{\begin{array}{c} I_{11}\ddot{\theta} + (I_{33} - I_{22})\,\dot{\psi}_0^2\,\sin\theta\,\cos\theta \\ (I_{11}+I_{22}-I_{33})\,\dot{\psi}_0\,\dot{\theta}\cos\theta \\ (I_{22}-I_{11}-I_{33})\,\dot{\psi}_0\,\dot{\theta}\sin\theta \end{array}\right\} = \left\{\begin{array}{c} \dot{H}_1 \\ \dot{H}_2 \\ \dot{H}_3 \end{array}\right\}$$

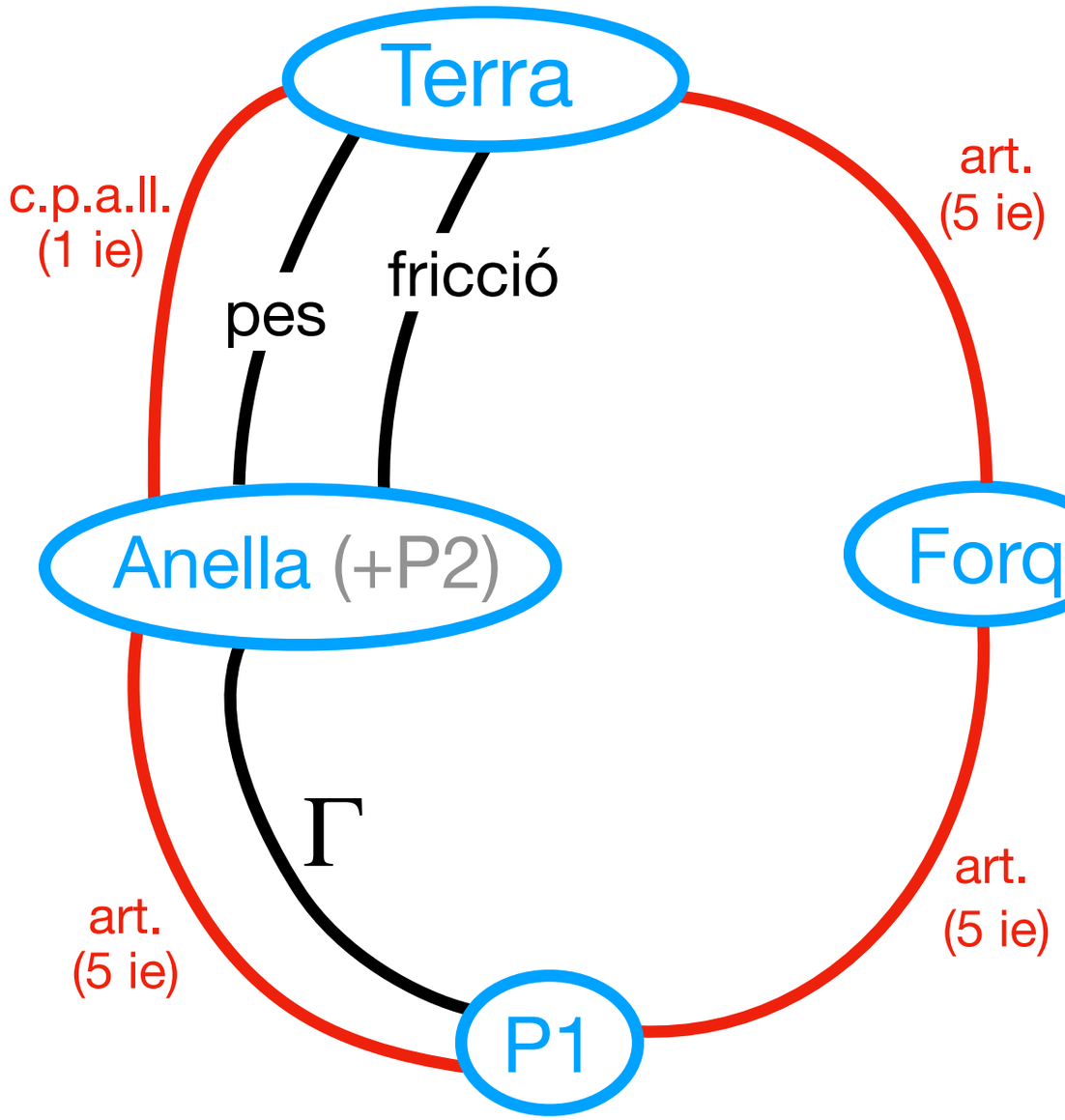


- GL sistema?
- DGI
- Caracterització de torsors?
- Eq. mov per a la coord. ψ
- Força normal a J
- Parell motor per mantenir $\dot{\varphi}_0$

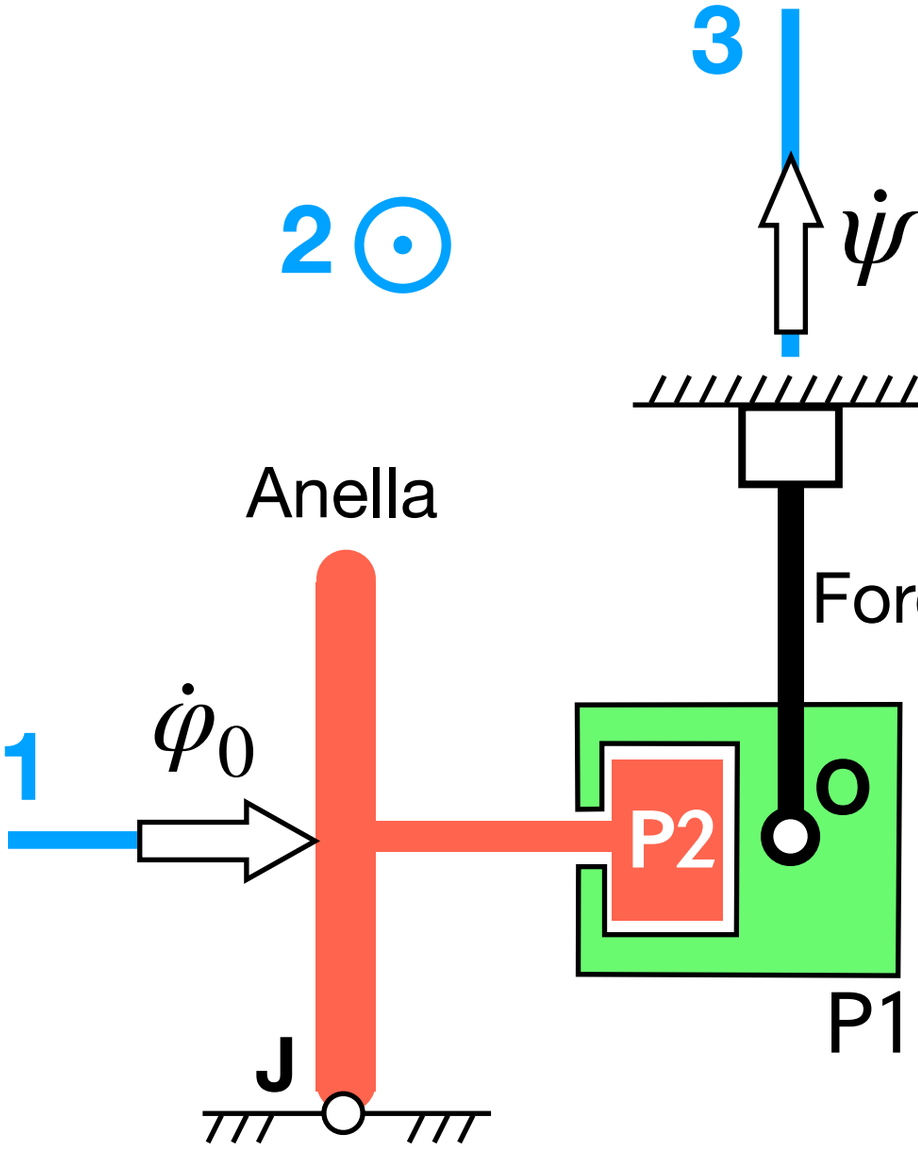
Manté contacte puntual a J

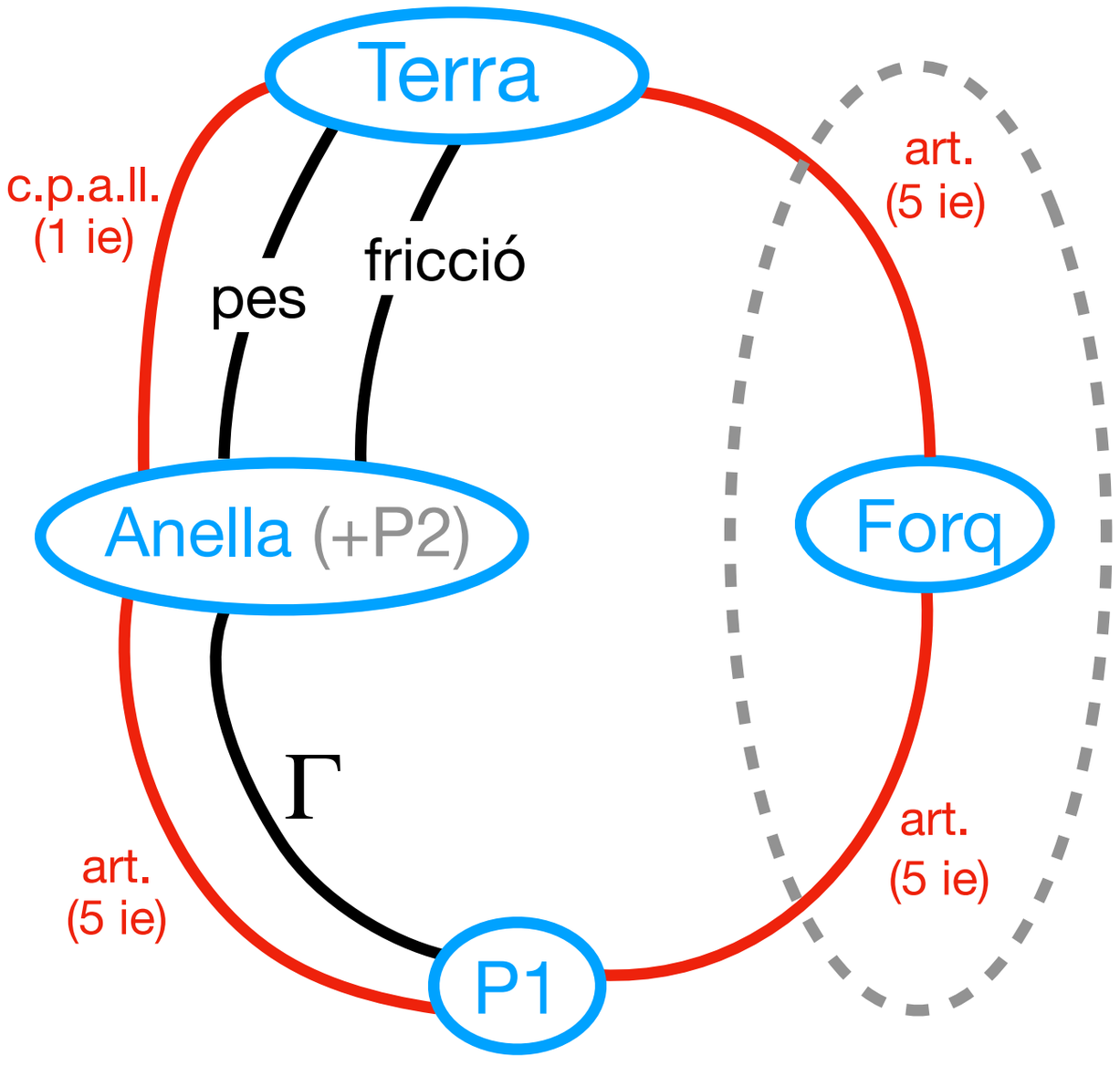
Motor manté $\dot{\varphi}_0$ = ct


3
2
$\dot{\psi}$
Anella
Forq
$\dot{\varphi}_0$
1
P2
P1
J

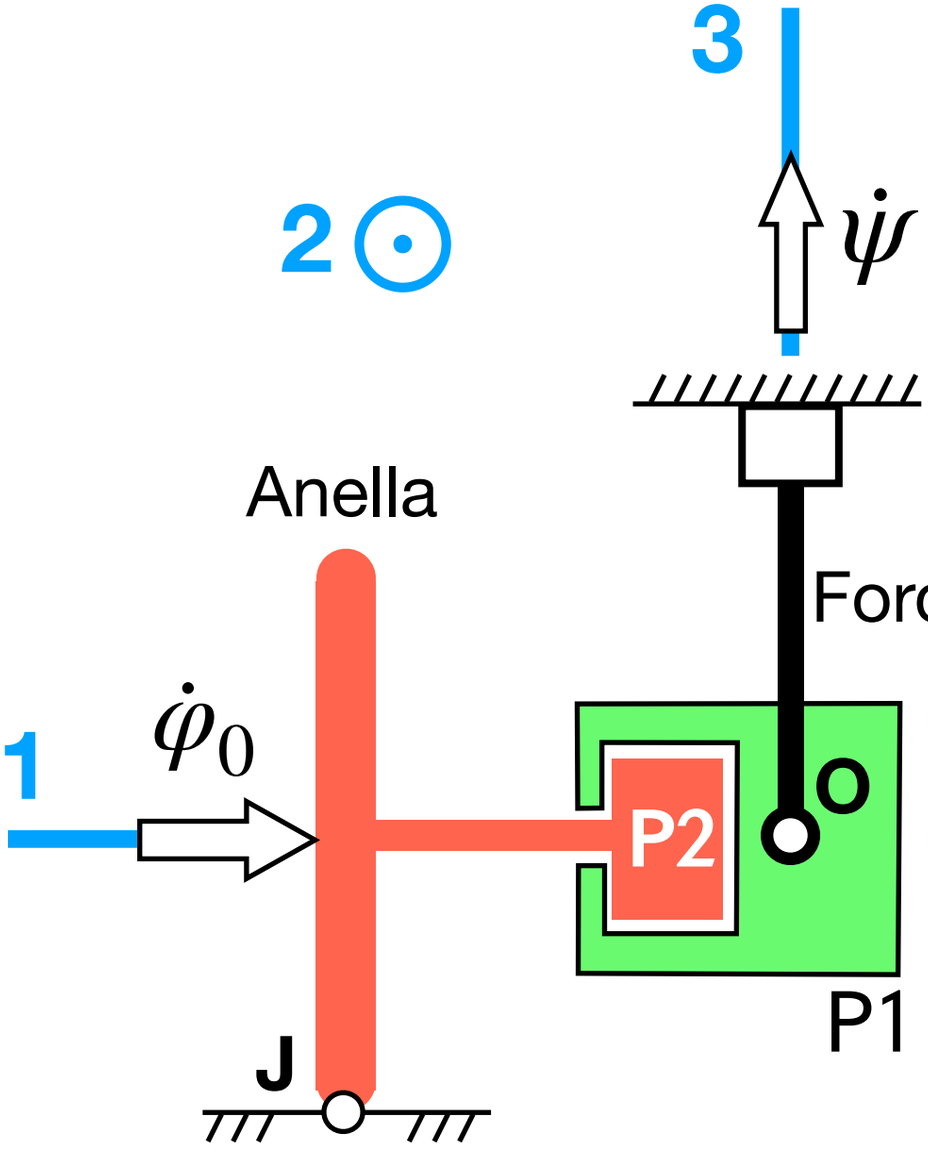

Terra
c.p.a.ll.
(1 ie)
pes
fricció
art.
(5 ie)
Anella (+P2)
Forq
Γ
art.
(5 ie)
art.
(5 ie)
P1

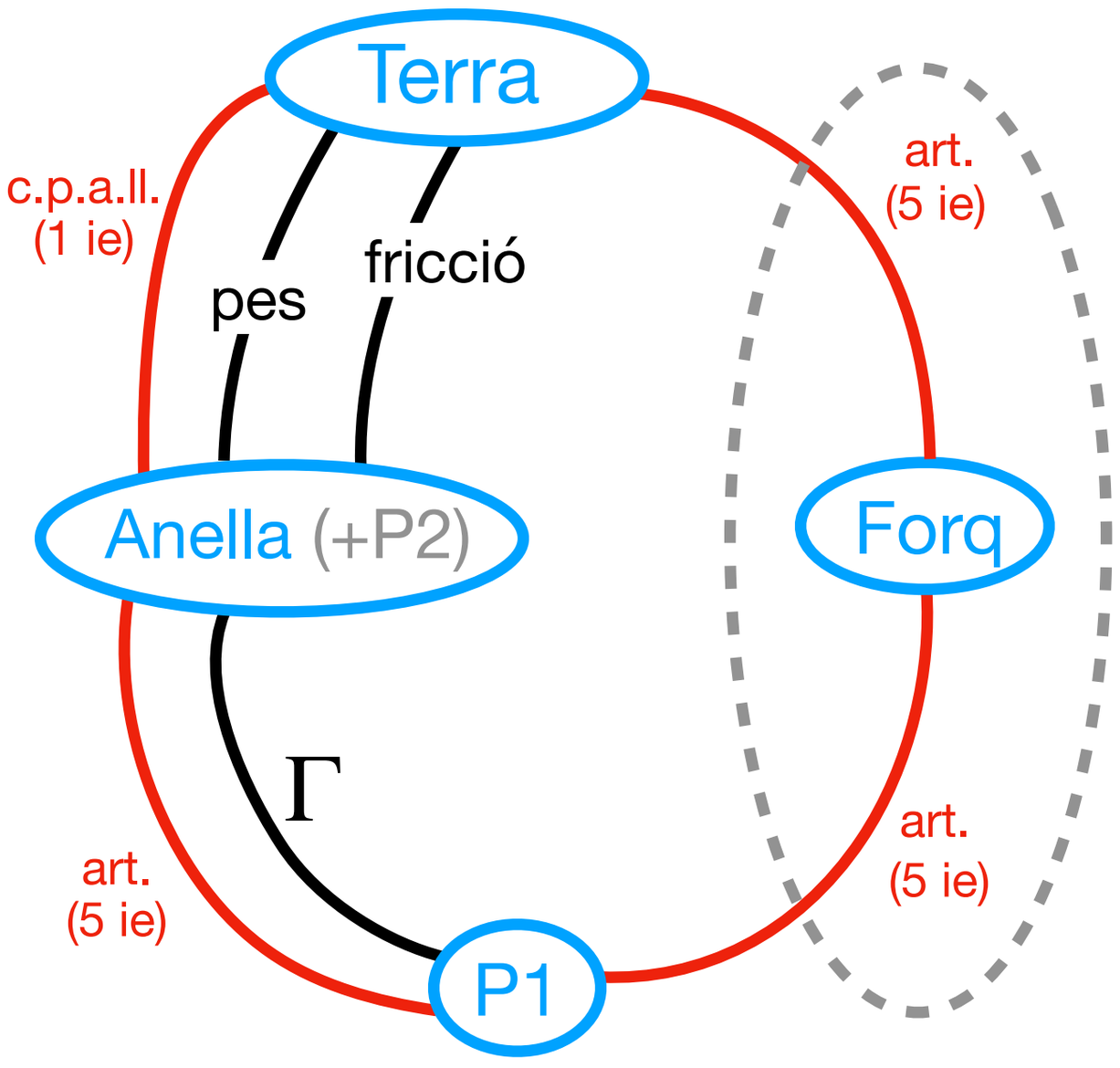
Forq és SAE ...


3
2
$\dot{\psi}$
Anella
Forq
$\dot{\varphi}_0$
1
P2
P1
J

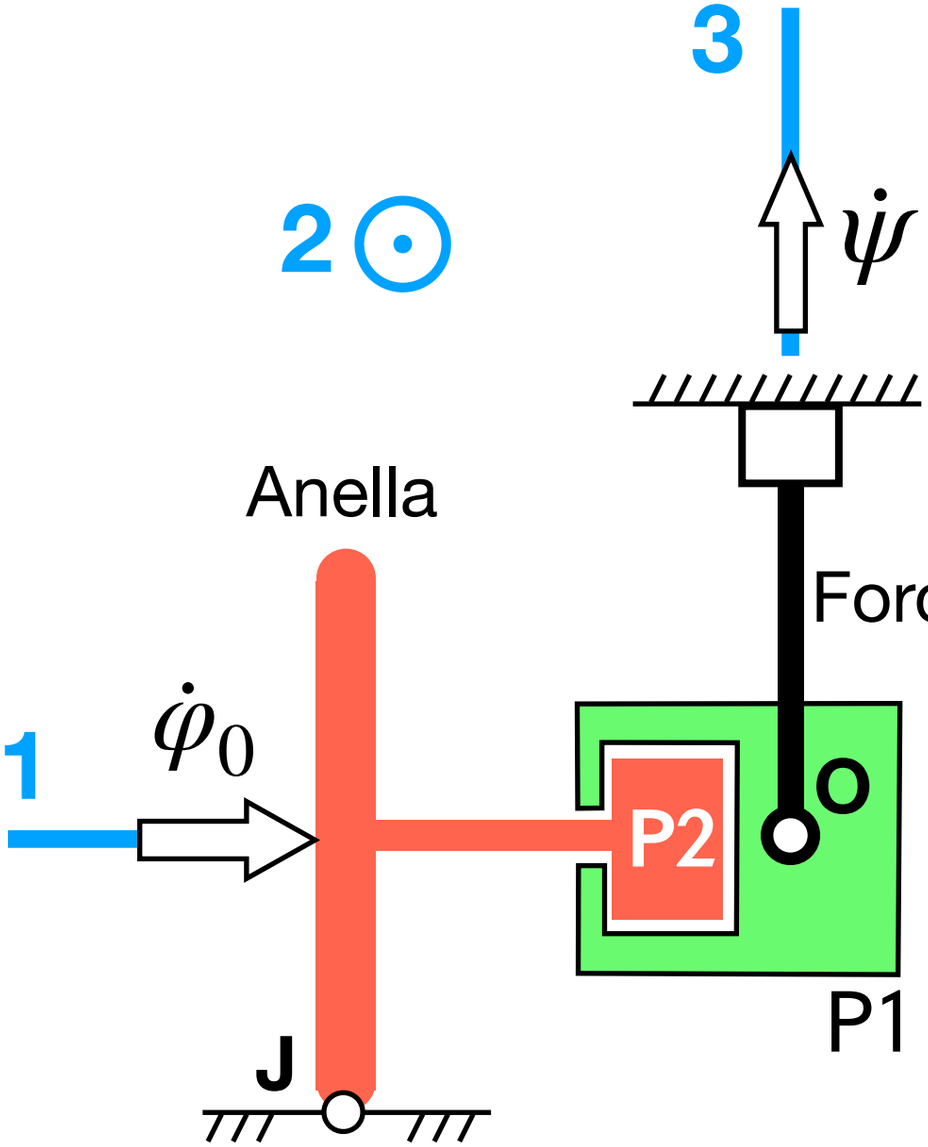

Terra
c.p.a.ll.
(1 ie)
pes
fricció
art.
(5 ie)
Anella (+P2)
Forq
Γ
art.
(5 ie)
art.
(5 ie)
P1

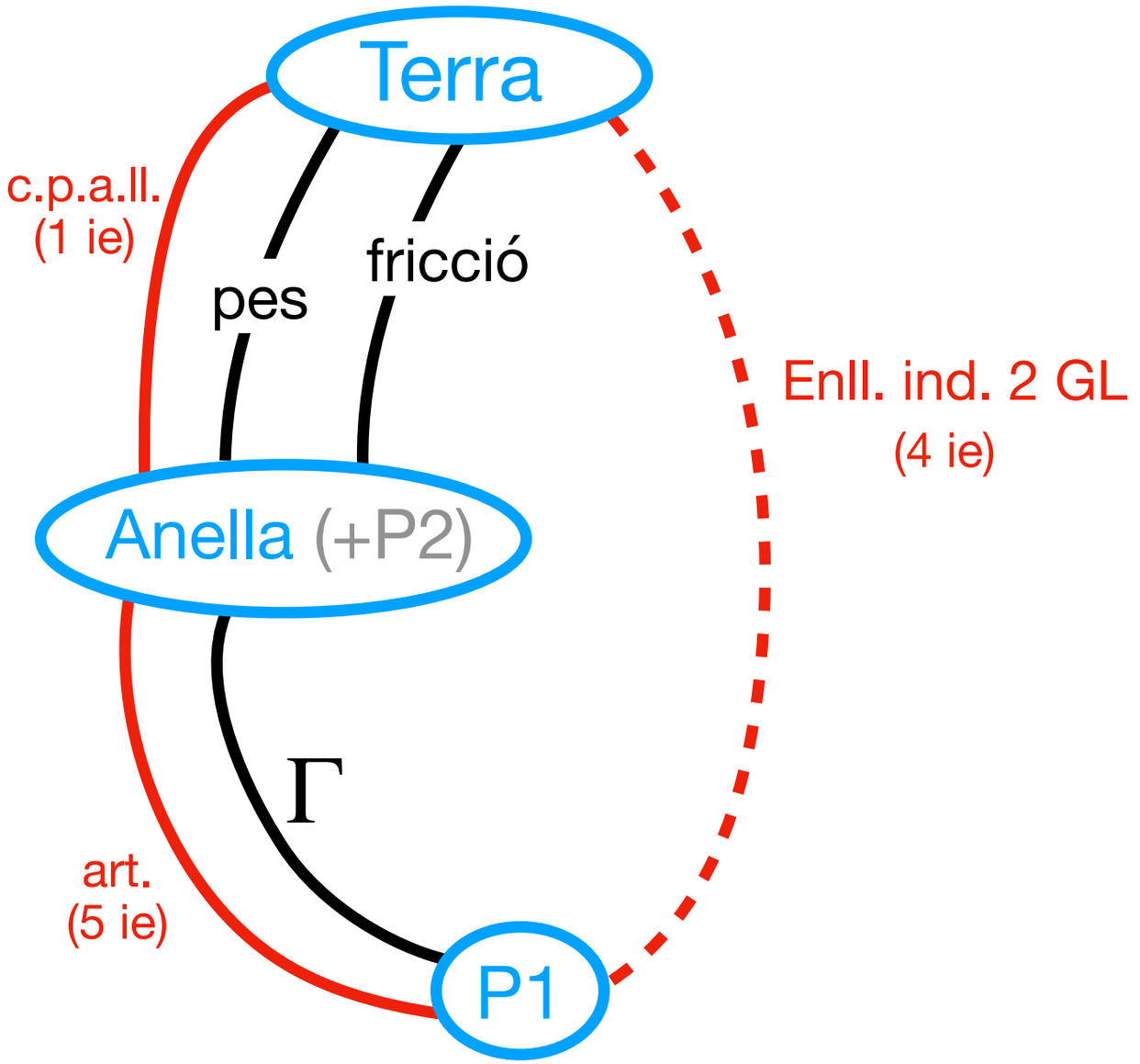
Substituim **Forq** per enllaç indirecte $T \rightarrow P_1$



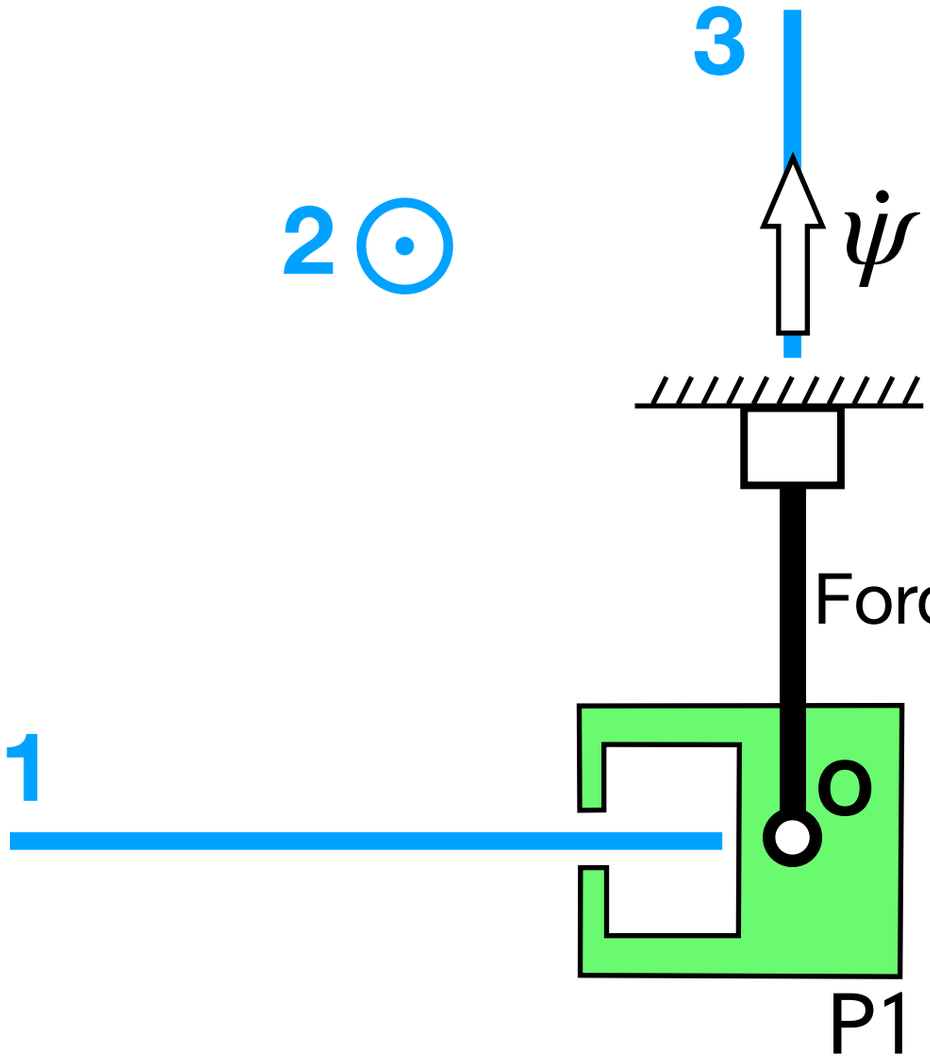


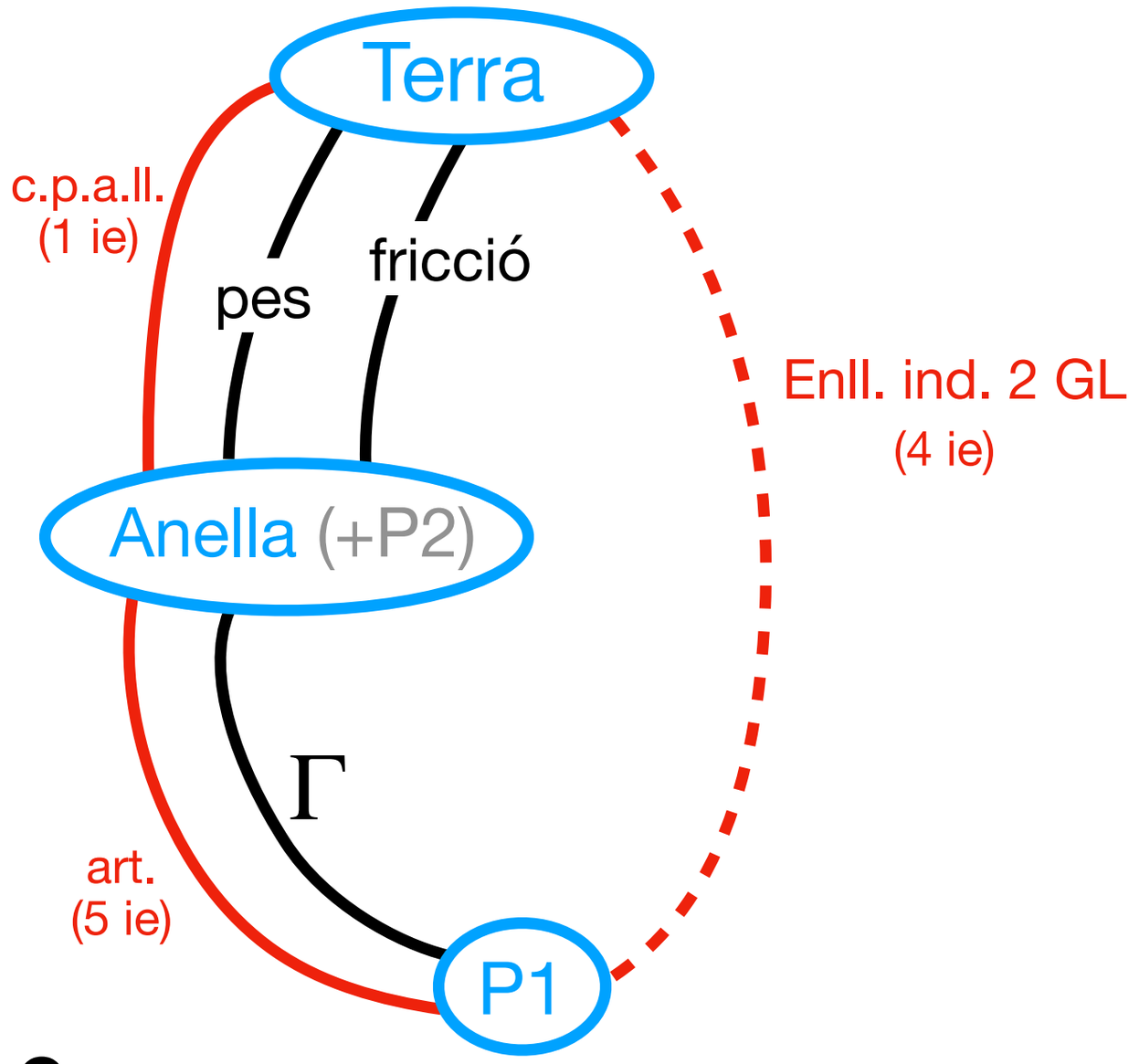
Substituim **Forq** per enllaç indirecte $T \rightarrow P_1$





Substituim **Forq** per enllaç indirecte $T \rightarrow P_1$



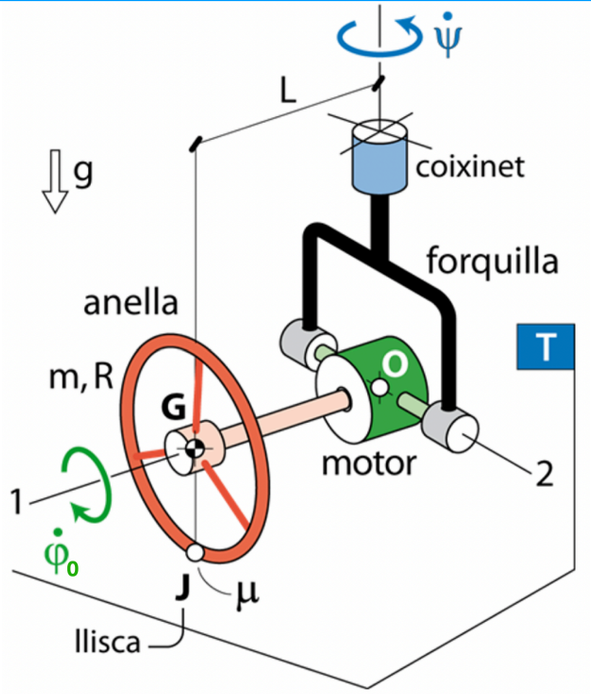


Torsor d'enllaç indirecte T → P1 al punt **O**:

$$\left\{ \bar{F}_{T\rightarrow(forq)\rightarrow Sup} \right\}_B = \left\{ \begin{matrix} F_1 \\ F_2 \\ F_3 \end{matrix} \right\}$$ **O** fix a T

$$\left\{ \bar{M}_{T\rightarrow(forq)\rightarrow Sup}\,(\mathbf{O}) \right\}_{B'} = \left\{ \begin{matrix} M_1 \\ 0 \\ 0 \end{matrix} \right\}$$ 0 en les rotacions permeses de P1 resp T

Full ruta per eq. del mov. ψ

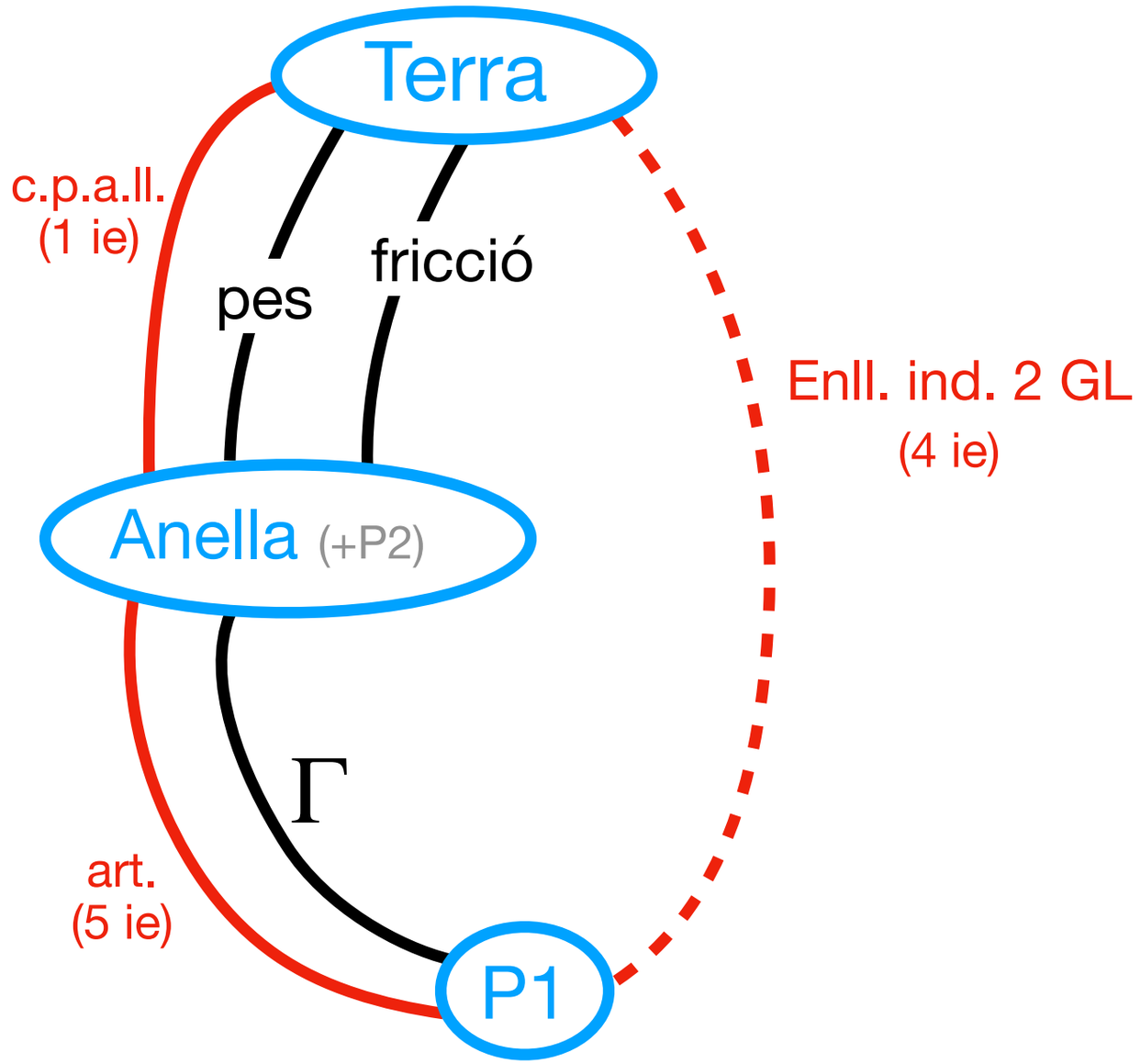


ψ afecta la cinemàtica de **P1** i **Anella**

⇓

Sist ha d'incloure **P1** o **Anella** (o ambdós)

Sistema	Incògn.	Problema
Anella	6 ie, Γ, $\ddot{\psi}$	**INDET**
P1	9 ie, Γ, $\ddot{\psi}$	**INDET**
Anella + P1	5 ie, $\ddot{\psi}$	**DET**



⇒ Triem SIST = Anella + P1