Finding All Valid Hand Configurations for a Given Precision Grasp

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Problem statement

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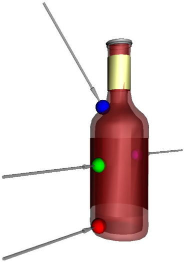

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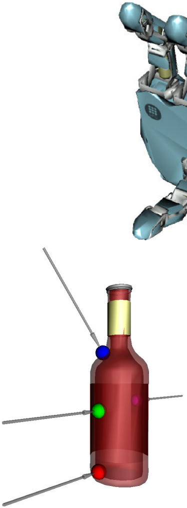

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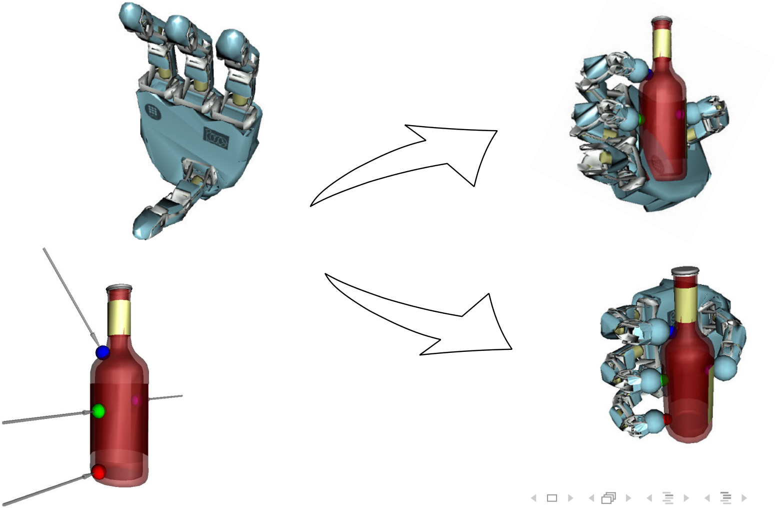

Grasping and manipulation tasks

Usually tackled in two steps:

1. Find the grasping points:
   Largely solved, e.g. force/form closure, etc.
2. Solving inverse kinematics:
   Previous work

[Borst *et al.*, 2002] Unconstrained optimization, penalty terms
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Shortcomings of previous works

- Need an initial estimation
- May diverge
- Converge to only one solution
- Incomplete

Contribution over previous works

The proposed approach is an inverse kinematic technique that:

- Does not require an initial estimation
- Is *complete* (converges to all solutions)
- Is applicable to other hand structures

Approach

Formulation :

formulate kinematic loop closure constraints algebraically

Numerical solution :

solve the resulting equations via a branch-and-prune technique based on linear relaxations

Formulation

The formulation is tailored to the numerical solution adopted:

- Algebraic equations directly
- Involving monomials of linear, bilinear and quadratic type

System of equations

$$\mathbf{x}_j - \sum_{i=1}^{4} \mathbf{q}_{j,i} = \mathbf{x}_k - \sum_{i=1}^{4} \mathbf{q}_{k,i} \quad (1)$$

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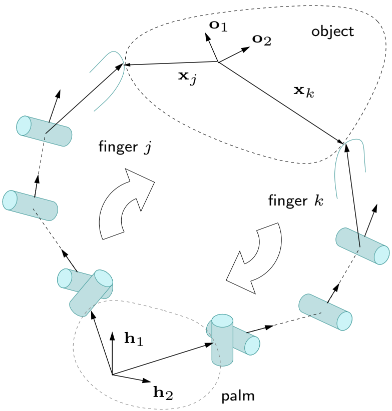
Loop closure constraints



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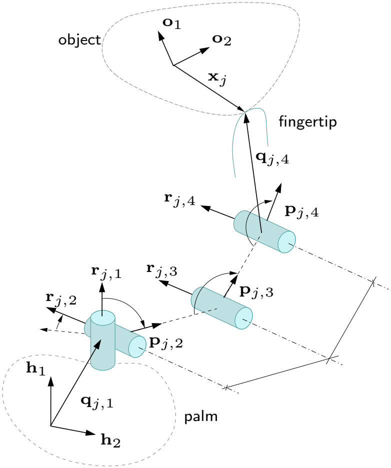
Reference frame constraints



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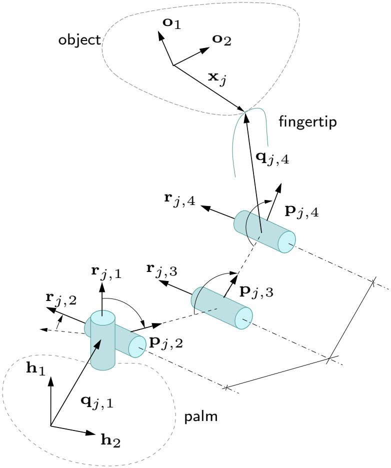
Joint position constraints



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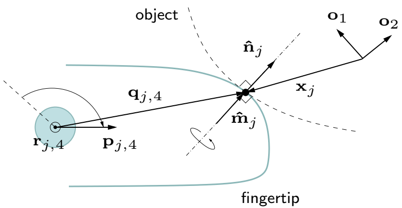
Contact constraints



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Introducing joint limits constraints

Joint angles are constrained by limiting their sine and cosine

To limit ϕ to $[-\alpha, \alpha]$ we define

$$c = \cos(\phi),$$
$$s = \sin(\phi),$$

then, introduce two new constraints

$$c = \mathbf{u} \cdot \mathbf{v},$$
$$s \cdot \mathbf{w} = \mathbf{u} \times \mathbf{v},$$

with $\mathbf{u}, \mathbf{v}, \mathbf{w}$ appropriate finger vectors, and finally set

$$c \in [c_{\min}, c_{\max}],$$
$$s \in [s_{\min}, s_{\max}].$$

Numerical solution

1. System of polynomials to be solved
2. Note all monomials are of the form x_i, x_i^2 or $x_i x_j$
3. Change of variables $q_i = x_i^2$ and $b_k = x_i x_j$
4. New system:

$$L(\mathbf{x}) = 0 \quad (9)$$
$$Q(\mathbf{x}) = 0 \quad (10)$$
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5. Search space:
Rectangular box defined by the ranges of the variables

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Solving with two basic operations

1. **Shrink box:** Reduce the size of the box along x_i

2. **Split box:** Trivial bisection of the box along x_i

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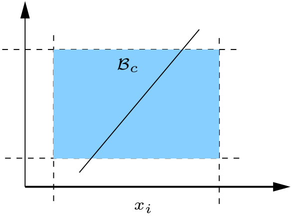

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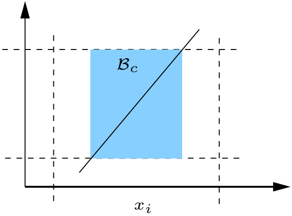

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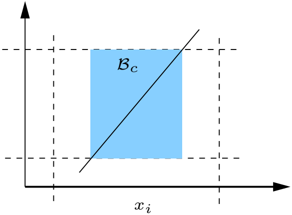

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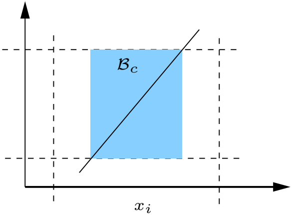

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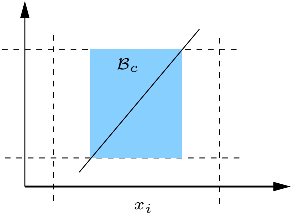

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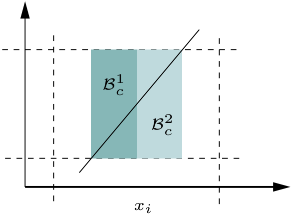
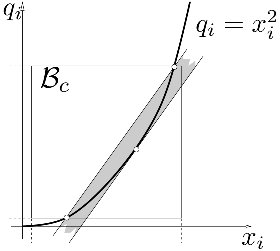


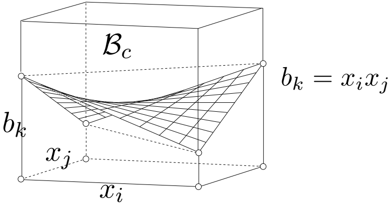
Shrink Box

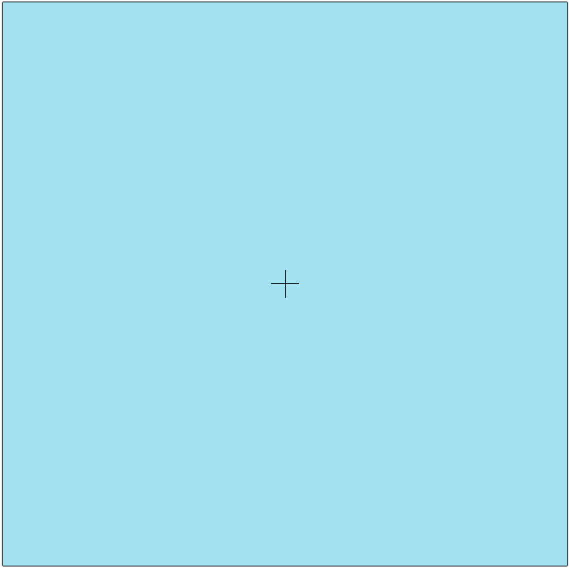
A linear programming problem:

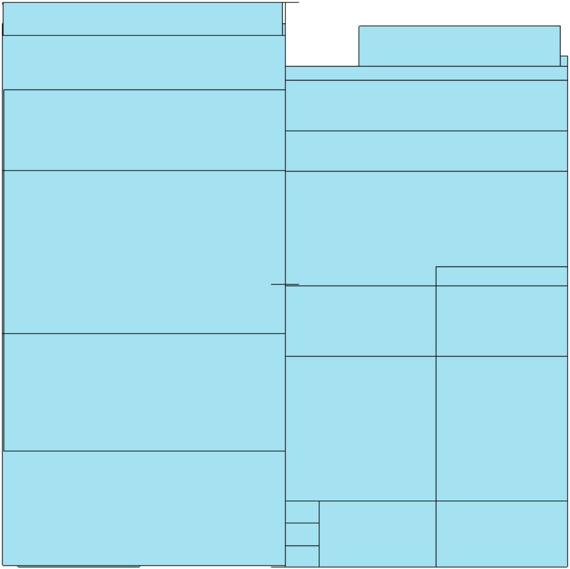
$$\textbf{LP1:} \text{ Minimize } x_i, \text{ subject to: } L(\mathbf{x}) = 0, \mathbf{x} \in \mathcal{B}_c$$
$$\textbf{LP2:} \text{ Maximize } x_i, \text{ subject to: } L(\mathbf{x}) = 0, \mathbf{x} \in \mathcal{B}_c$$

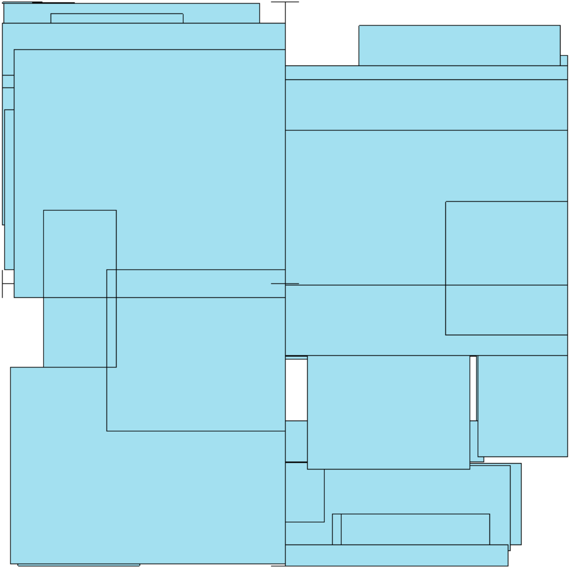
Quadratic and bilinear equations treated via linear relaxations:

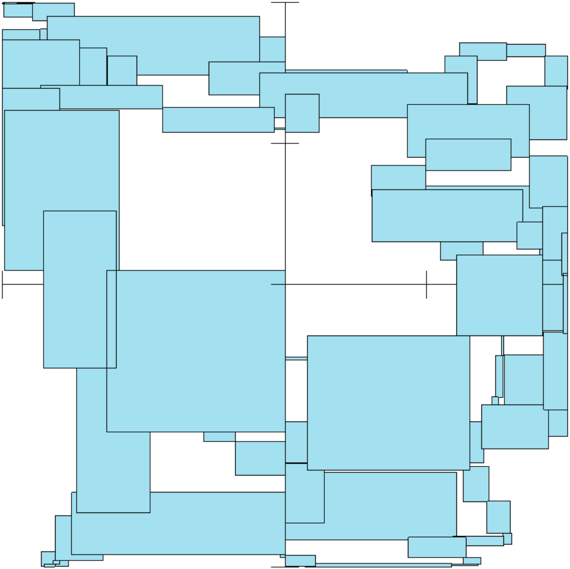


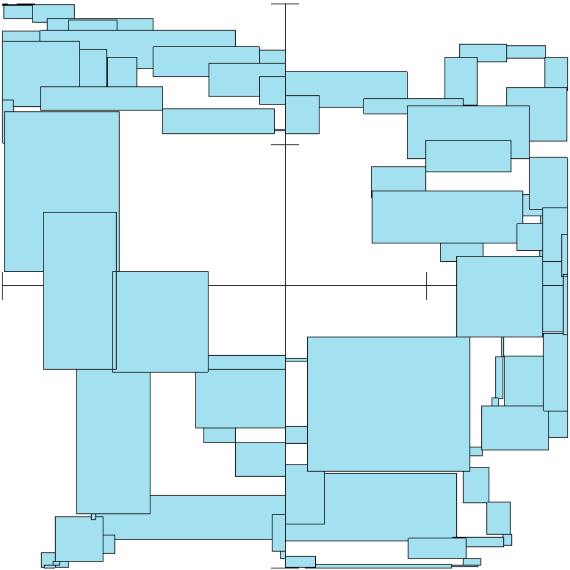


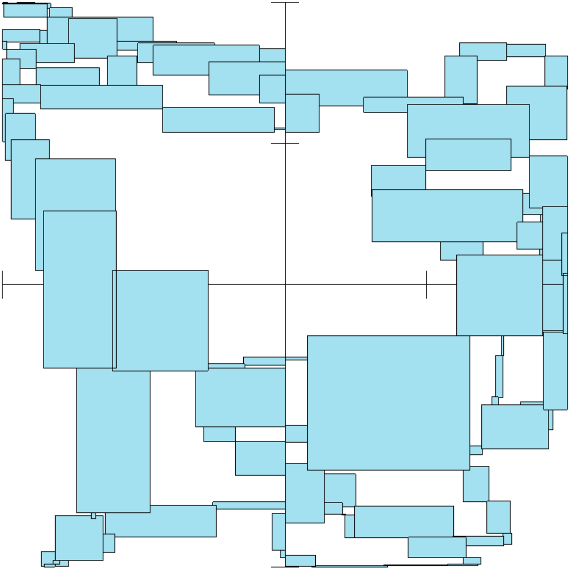


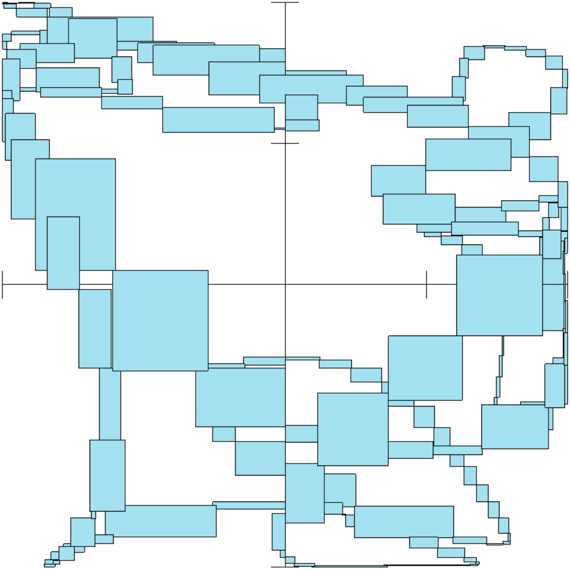


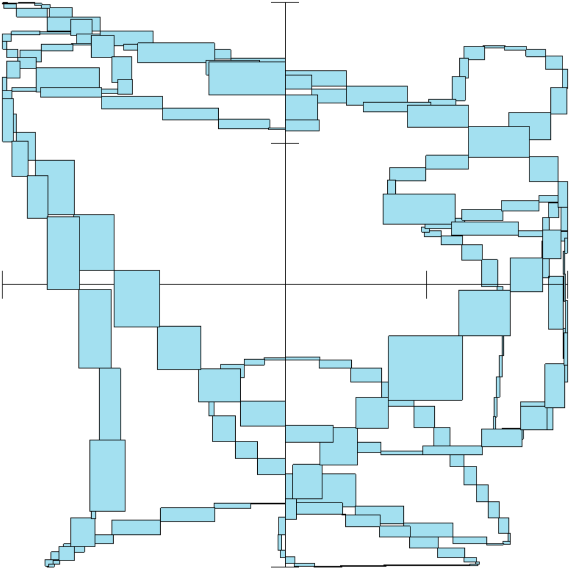


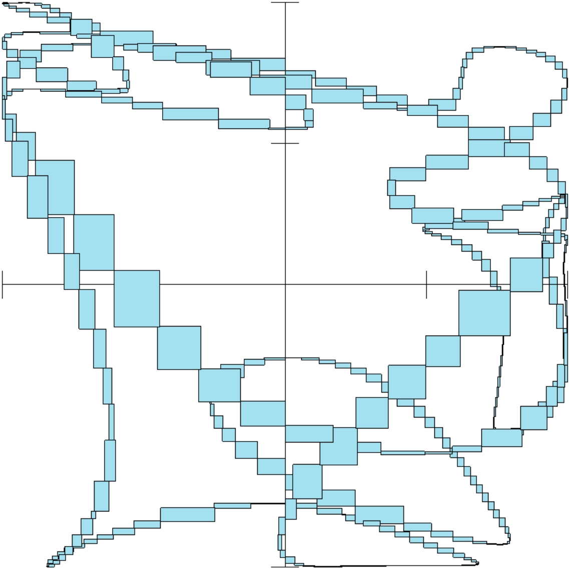


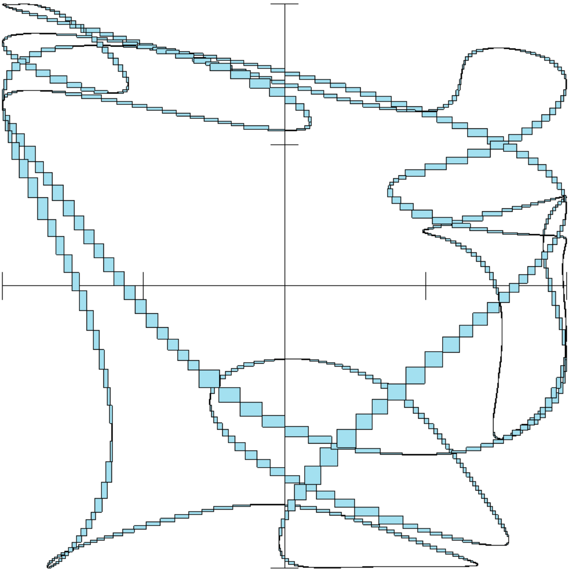


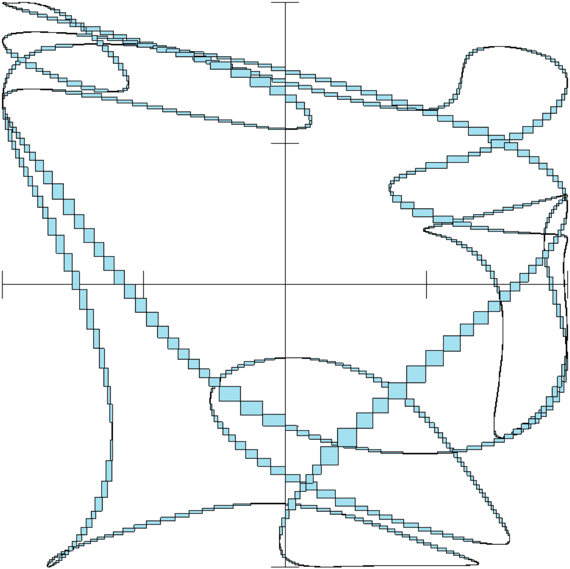


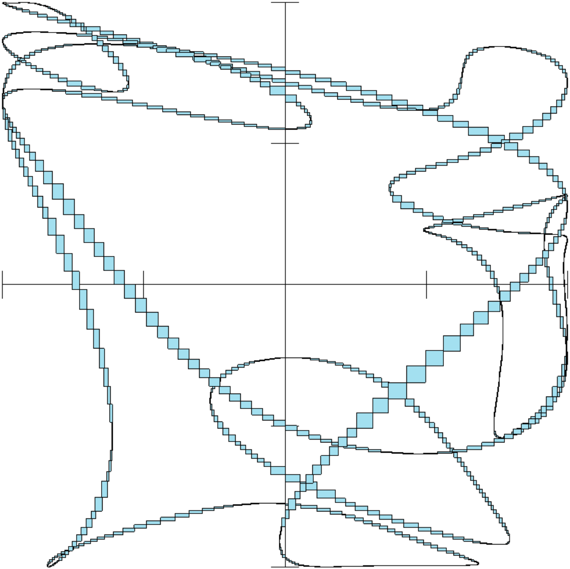


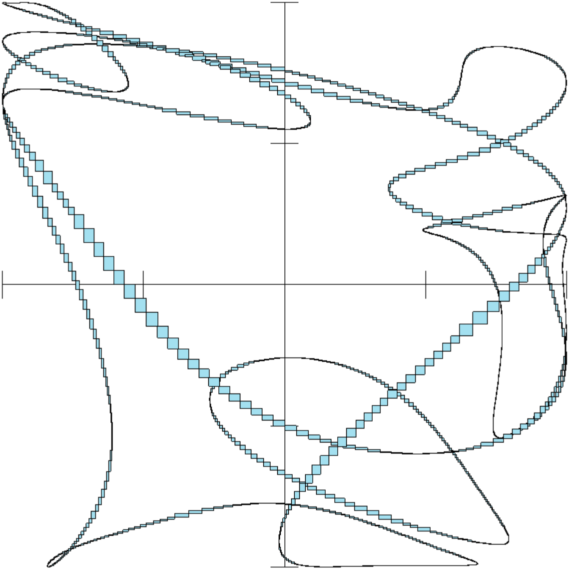


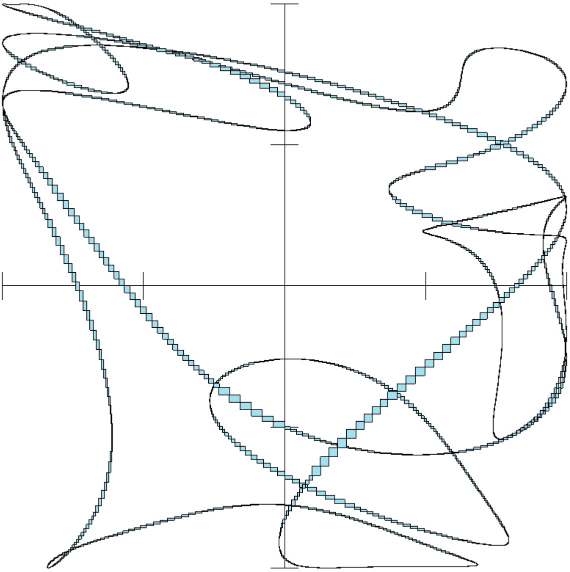


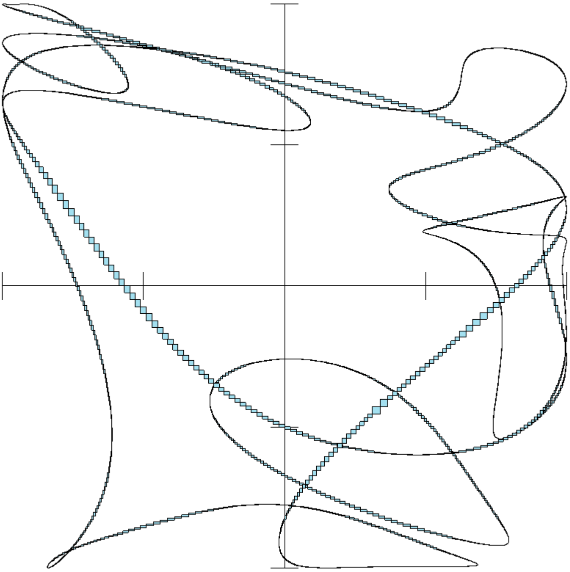


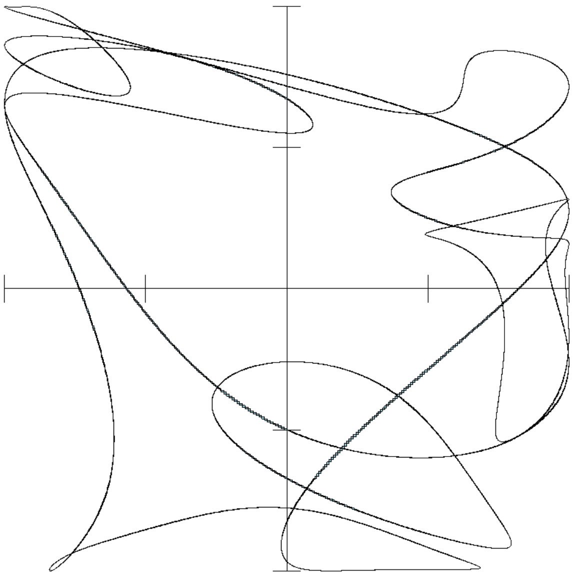












Dimension of the solution space

For a grasp performed by the hand MA-I using n fingers:

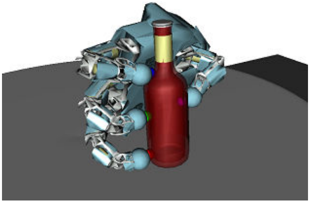
- $f = 5n$ degrees of freedom
- $r = 6(n-1)$ constraints
- By the Grübler-Kutzbach criterion, the dimension of the solution space will be $d = f - r = 6 - n$

Additional constraints can be included, if plausible

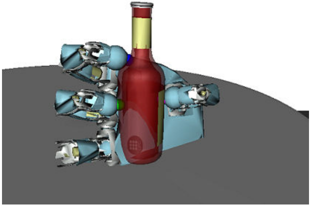
0-dimensional solutions

Added constraints: Coupling the proximal and distal joints of the *ring* and *middle* fingers

Resulting system: 54 variables, 54 equations



(a) A valid solution.

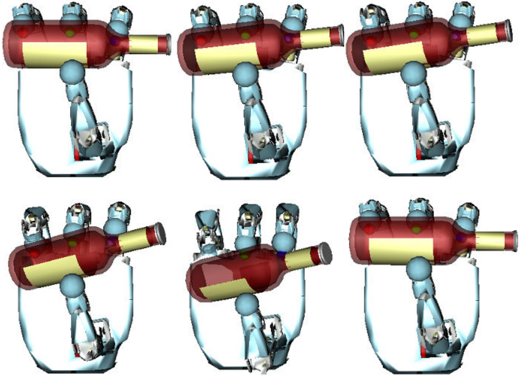


(b) A non-valid solution due to collision.

1-dimensional solutions

Added constraint: Coupling the proximal and distal joints of the *ring* finger only

Resulting system: 54 variables, 53 equations



Conclusions

Summary:

- An inverse kinematic technique for anthropomorphic hands
- Does not require an initial estimation
- Is *complete* (converges to all solutions)
- Is applicable to other hand structures

Future work:

- To integrate the given kinematic loop closure constraints with additional force closure and mobility constraints, so as to achieve a reachable, prehensile and manipulable grasp simultaneously.

Thanks for your attention

Feel free to ask questions, I will do my best to answer them!

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