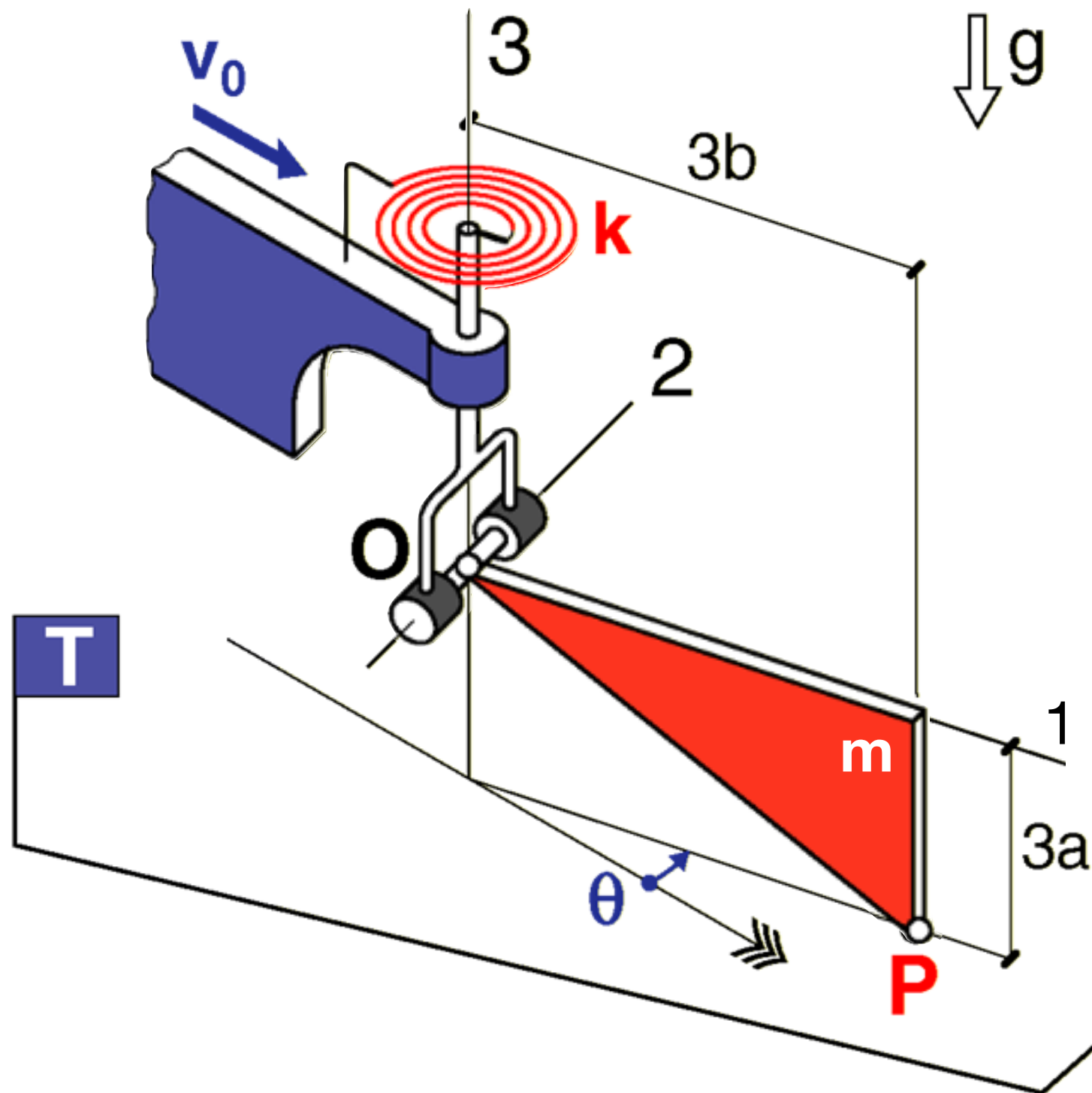


# 12P

## Teoremes vectorials II

Exemples 3D

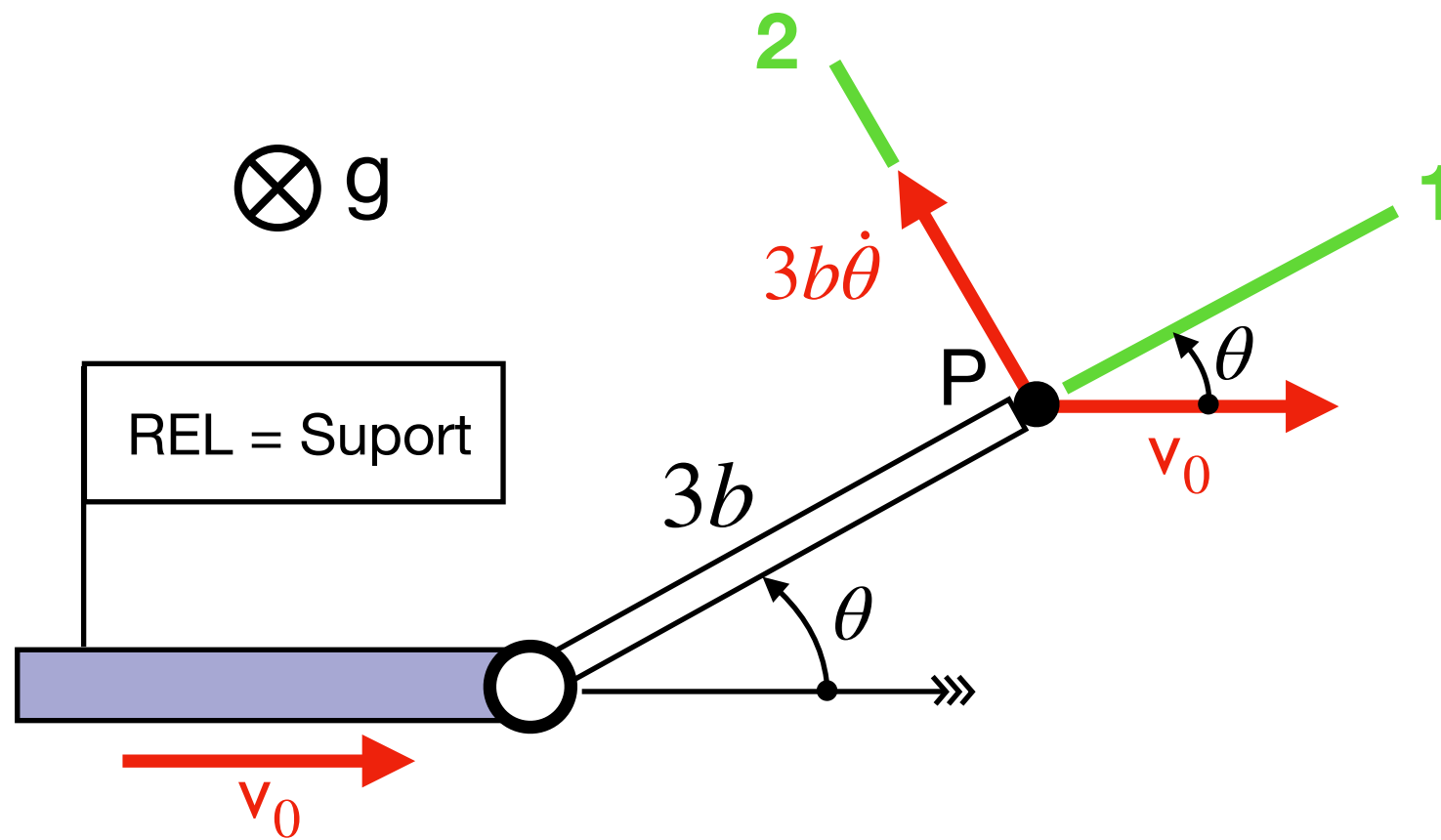


- Eq. mov. per a  $\theta$  ?
- $k_{\min}$  per a que  $\theta_{\text{eq}} = 0$  sigui **ESTABLE** ?

$\exists$  frec viscós  $T \rightarrow P$   
(de coef  $c$ )

Per  $\theta = 0$  la molla  
està distesa

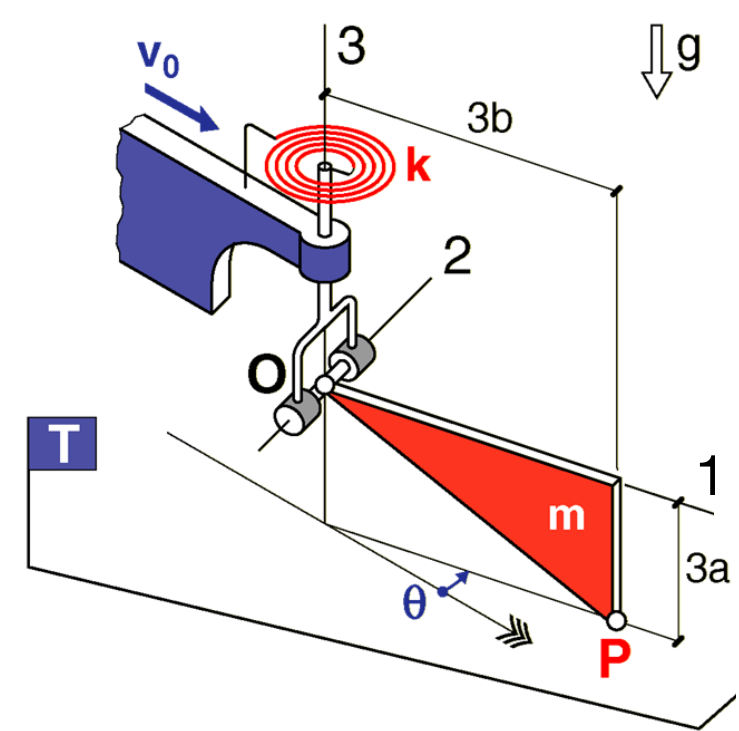
# Força de frec viscós $T \rightarrow P$



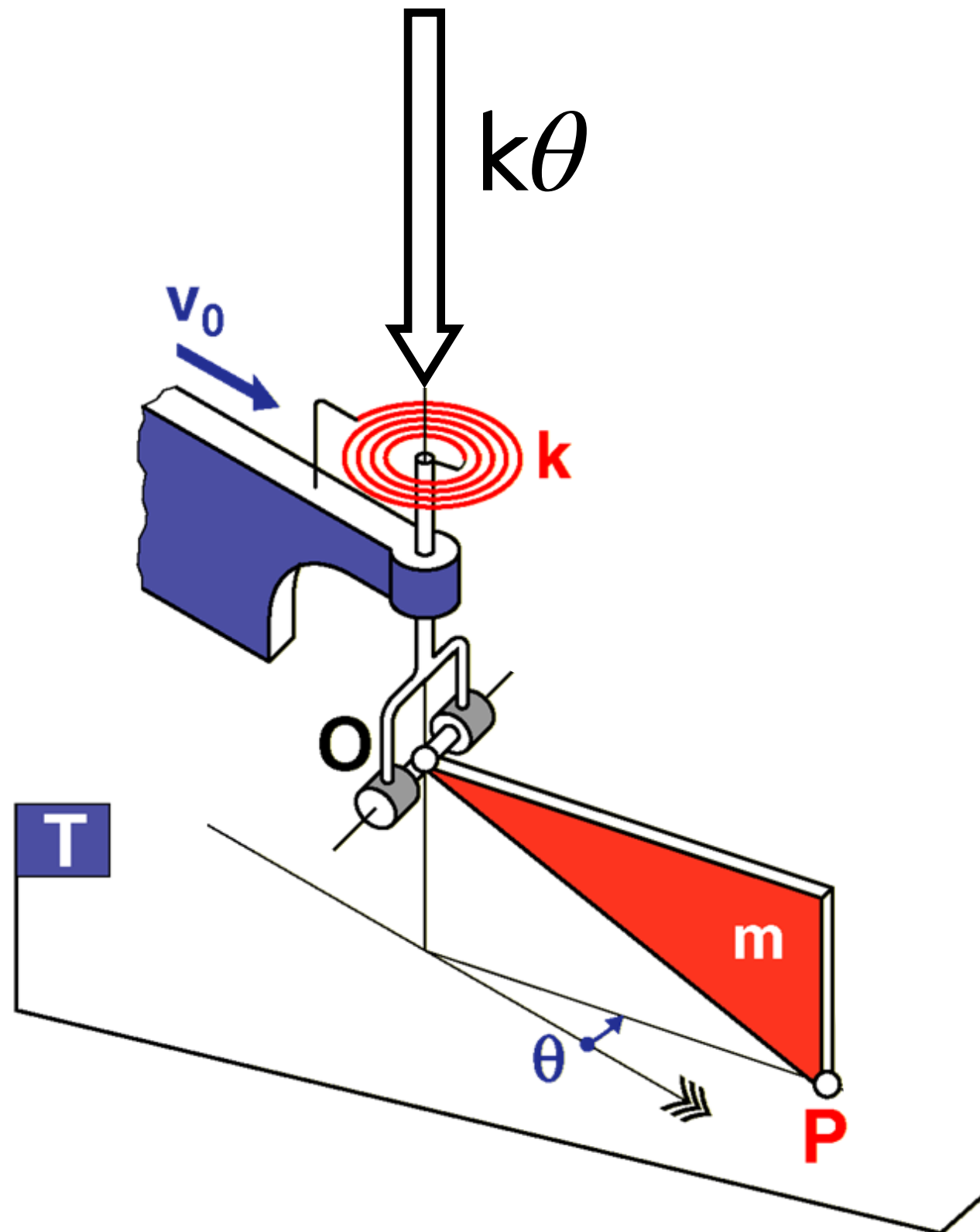
$$\bar{v}_T(\mathbf{P}) = \bar{v}_{REL}(\mathbf{P}) + \bar{v}_{ar}(\mathbf{P}) = \begin{Bmatrix} v_0 \cos \theta \\ -v_0 \sin \theta + 3b\dot{\theta} \\ 0 \end{Bmatrix}_{B=(1,2,3)}$$

$$\bar{F}_{fv} = -c \bar{v}_T(\mathbf{P}) = \begin{Bmatrix} -cv_0 \cos \theta \\ cv_0 \sin \theta - 3cb\dot{\theta} \\ 0 \end{Bmatrix}_B$$

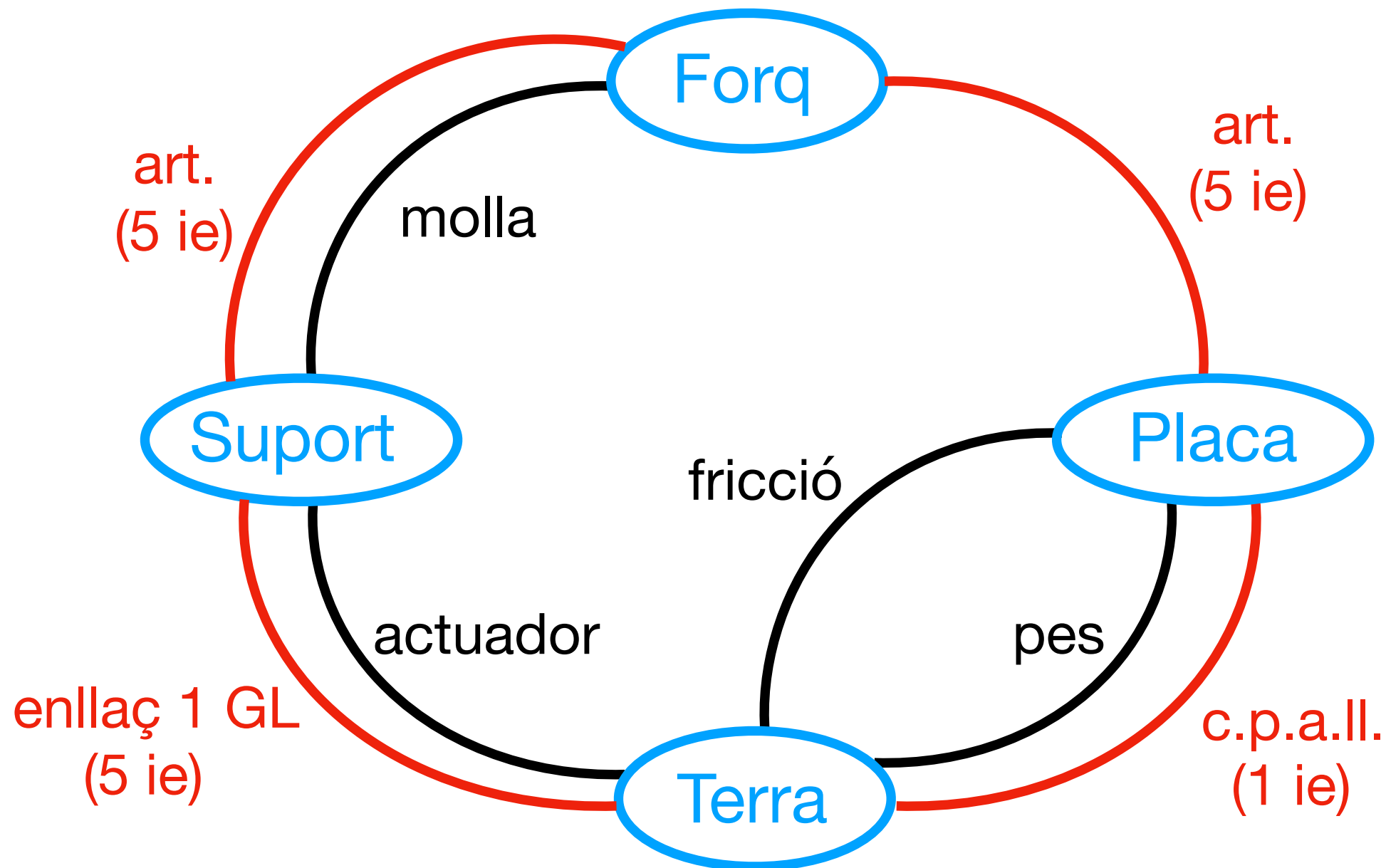
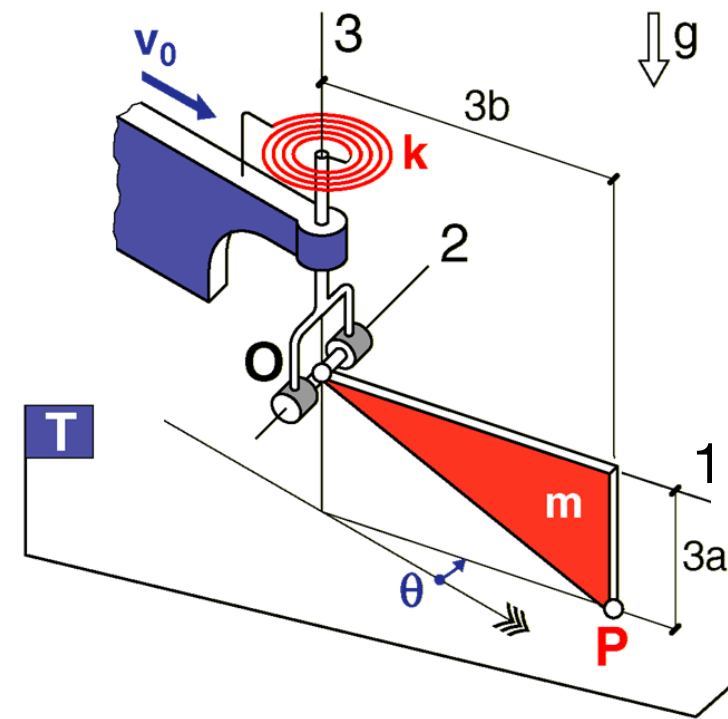
$F_{fv1}$   
 $F_{fv2}$



Parell molla torsional  $\rightarrow$  forq



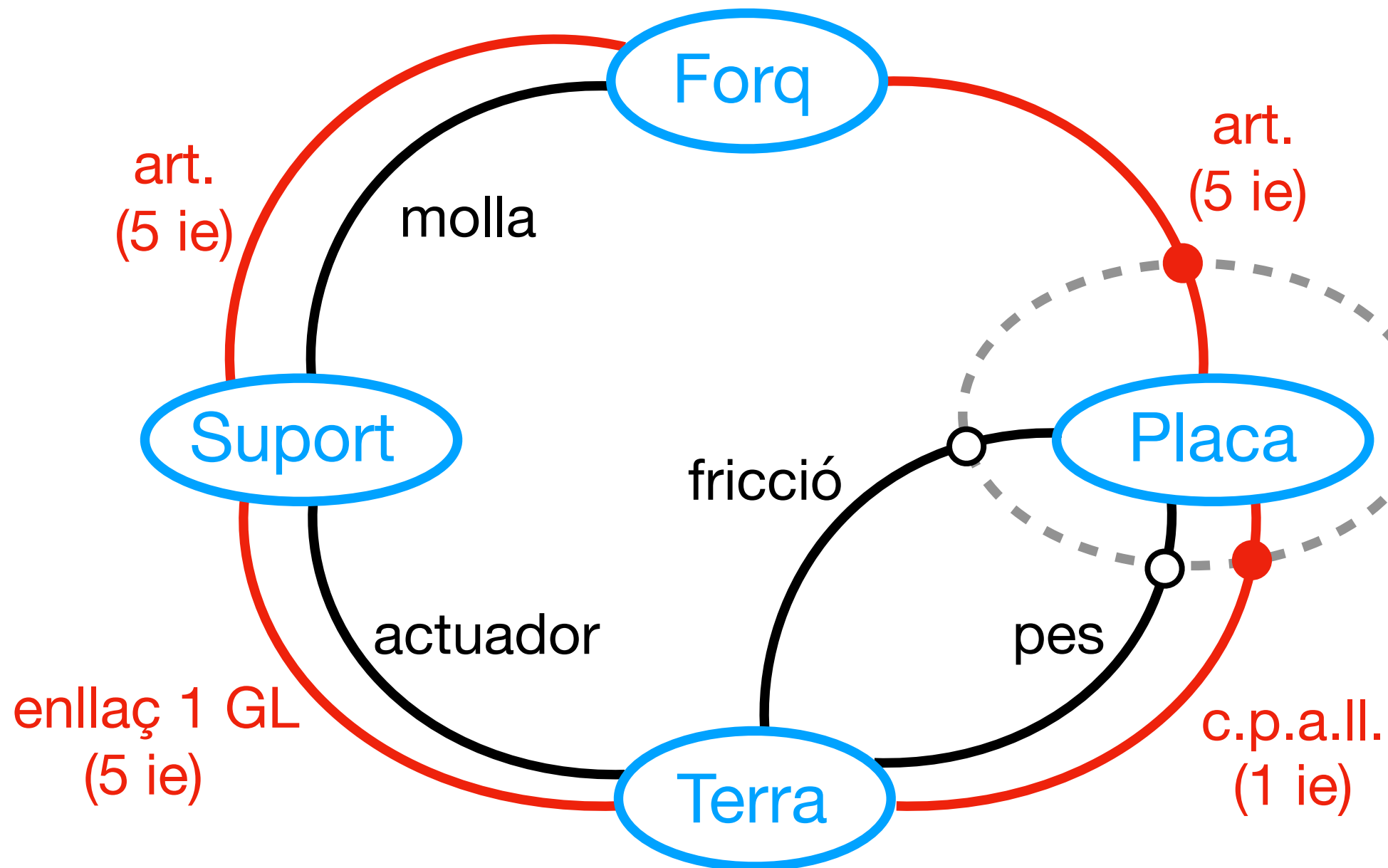
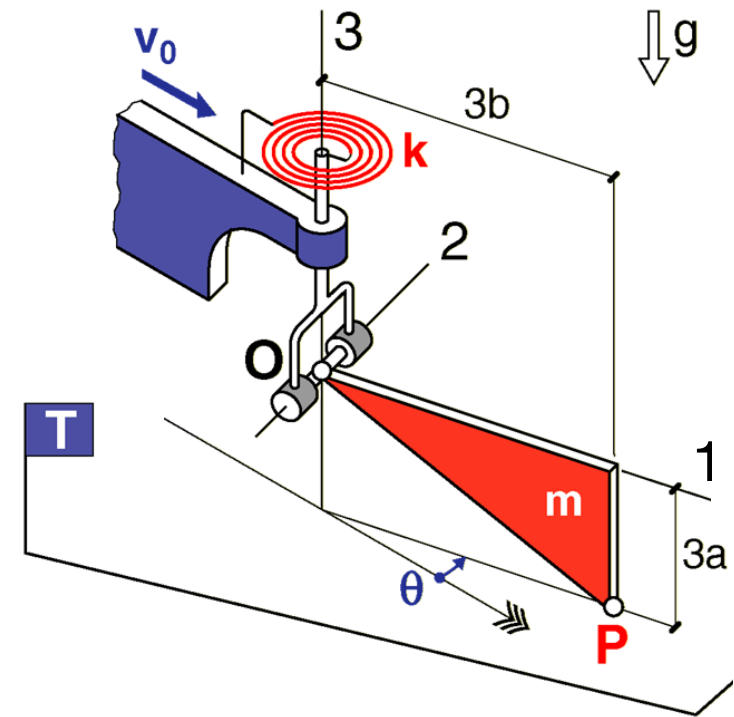
**DGI** = Diagrama general d'interaccions



**INDET**

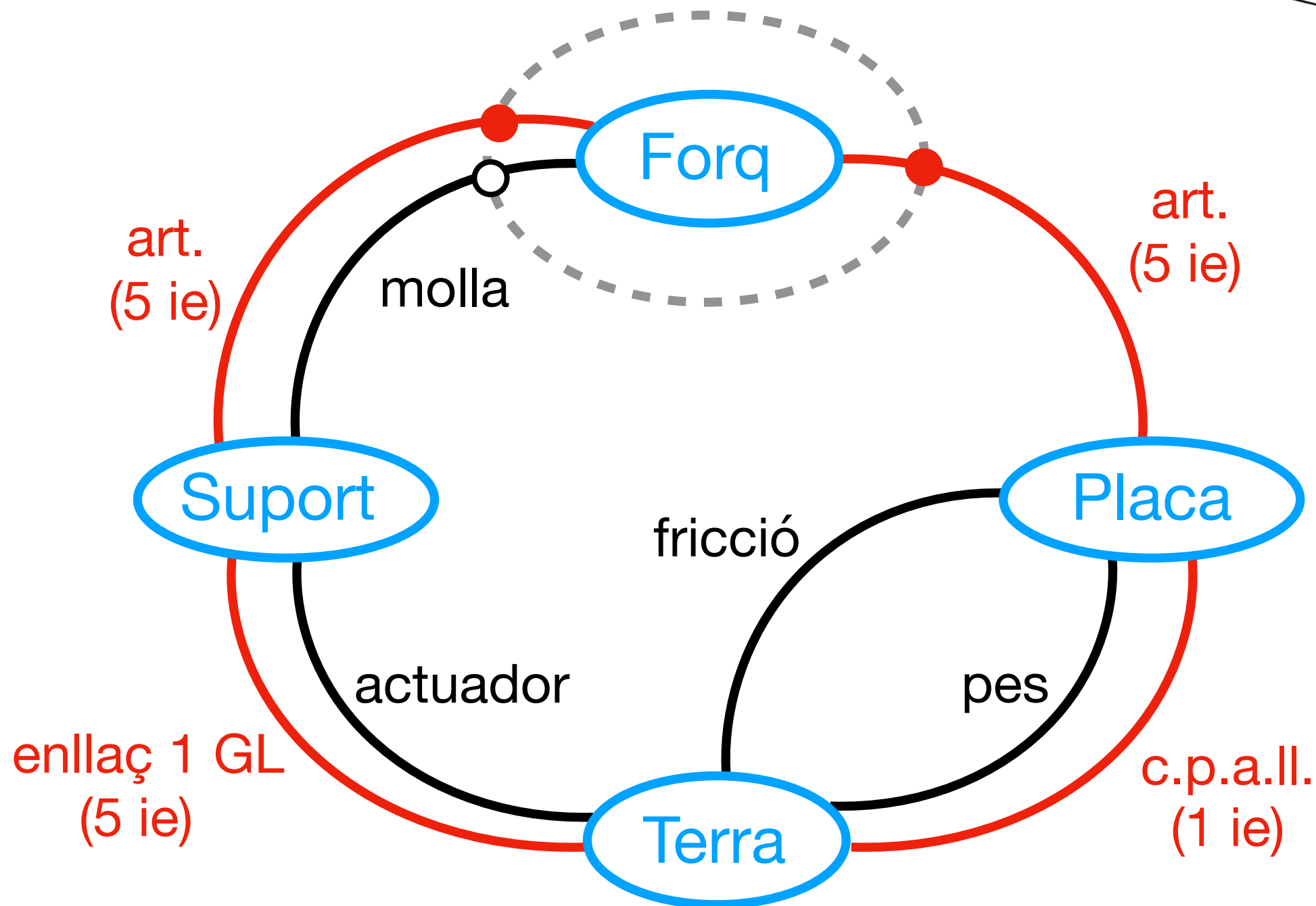
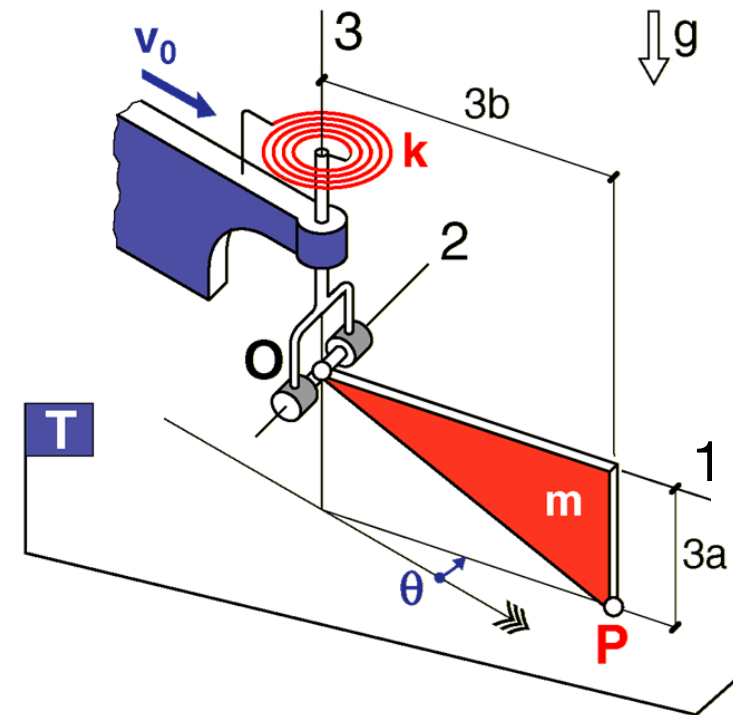
Sist = Placa

6 ie +  $\ddot{\theta}$  = 7 incòg



INDET

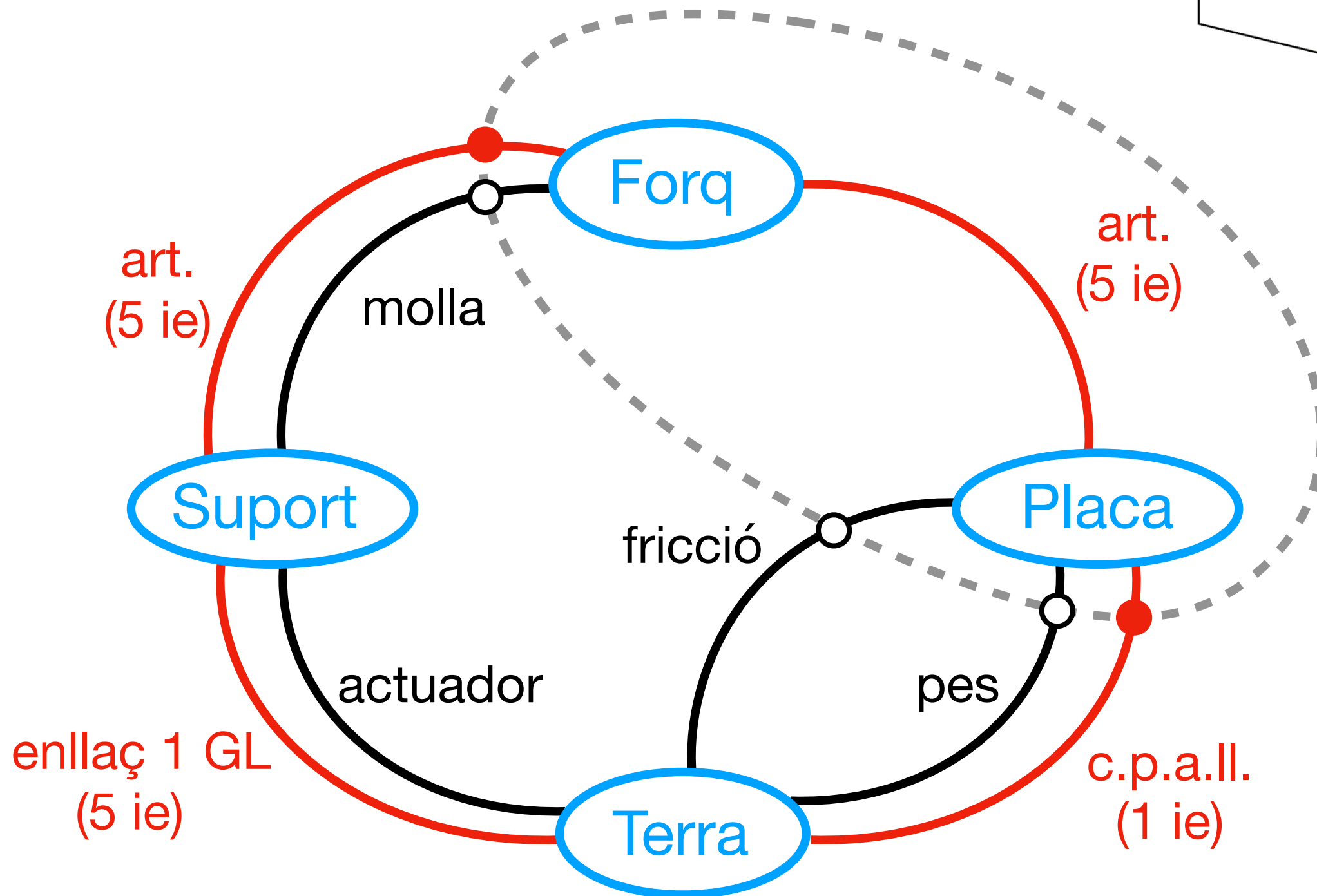
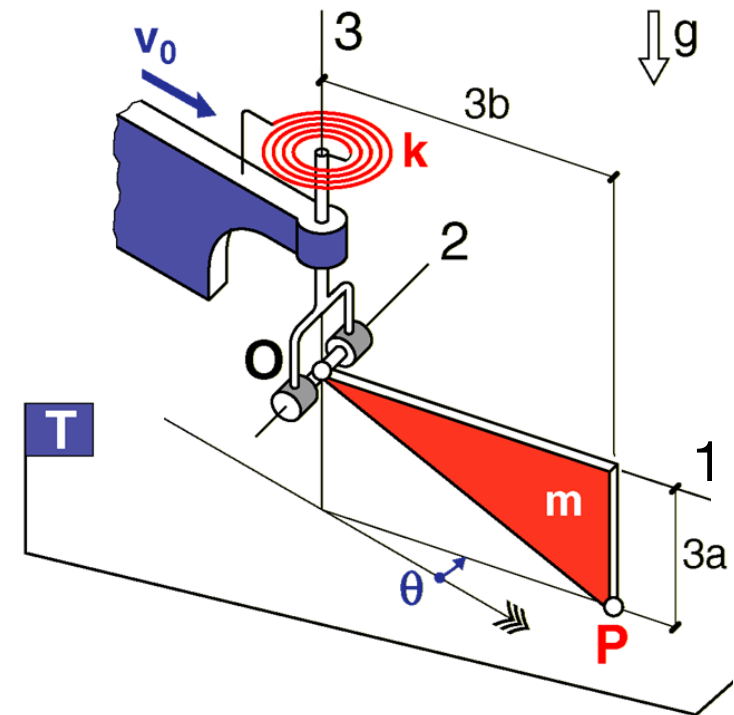
$$\left\{ \begin{array}{l} \text{Sist} = \text{Forq} \\ 10 \text{ ie} + \ddot{\theta} = 11 \text{ incòg} \end{array} \right.$$



**INDET**

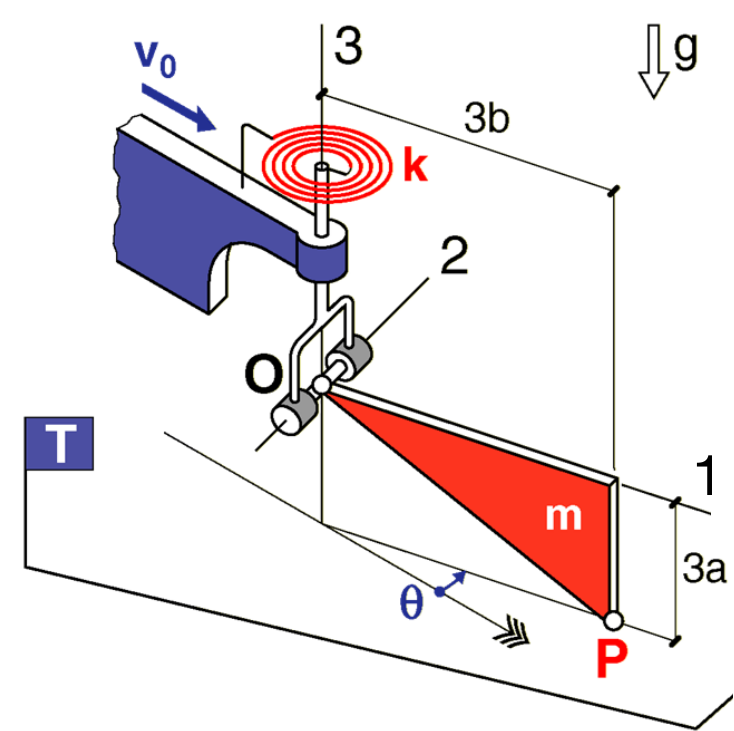
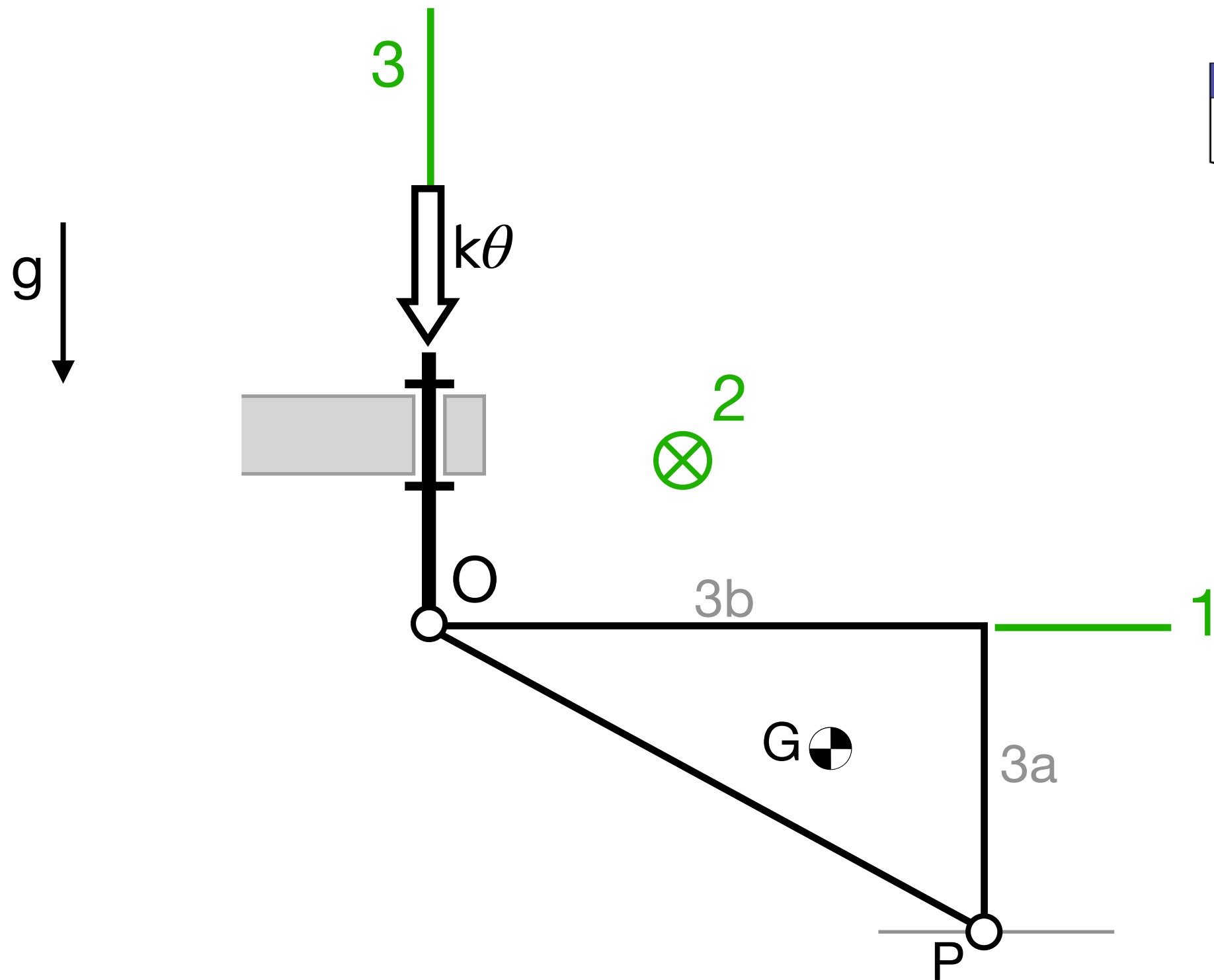
Sist = Placa + Forq

$$6 \text{ ie} + \ddot{\theta} = 7 \text{ incòg}$$

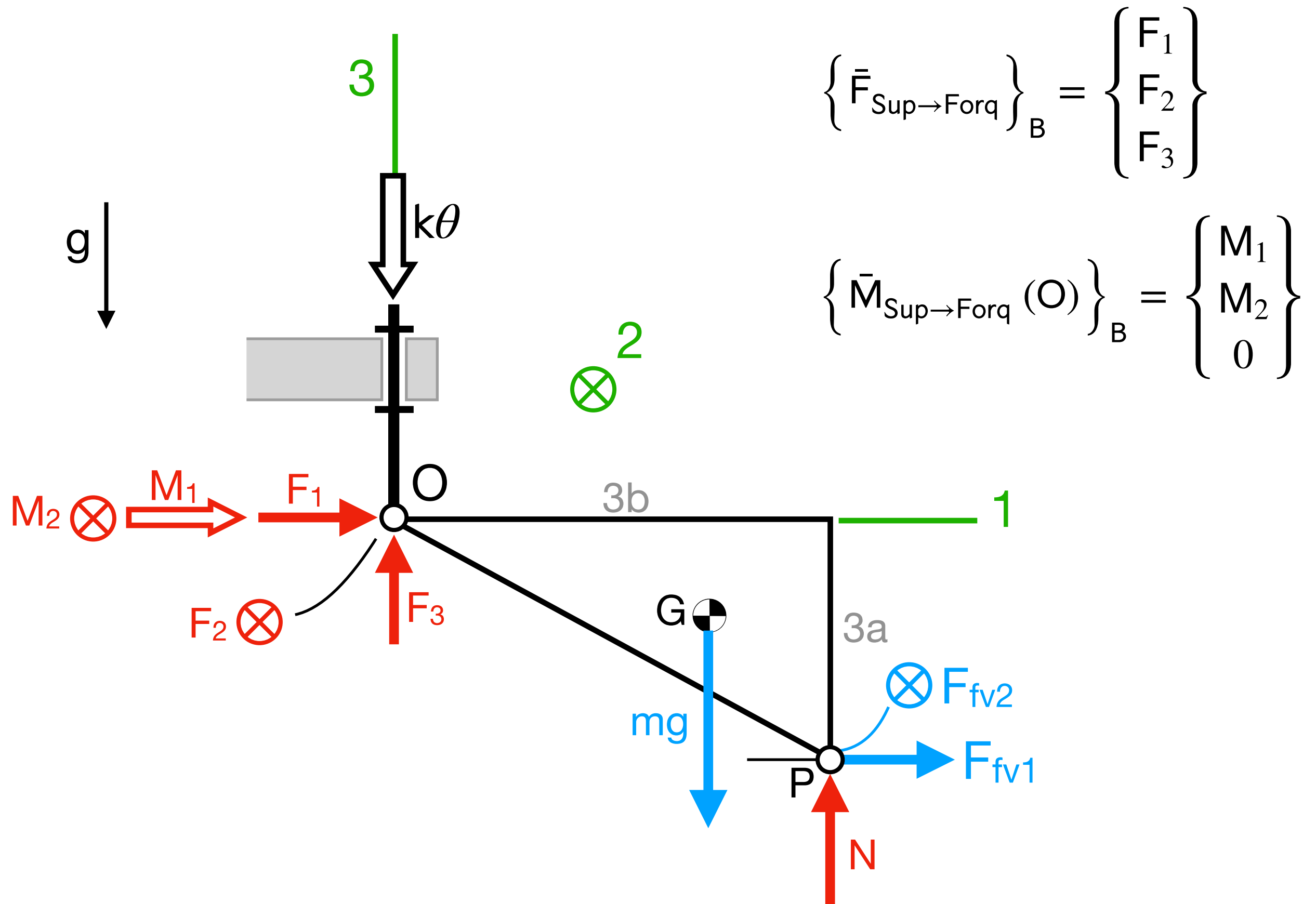




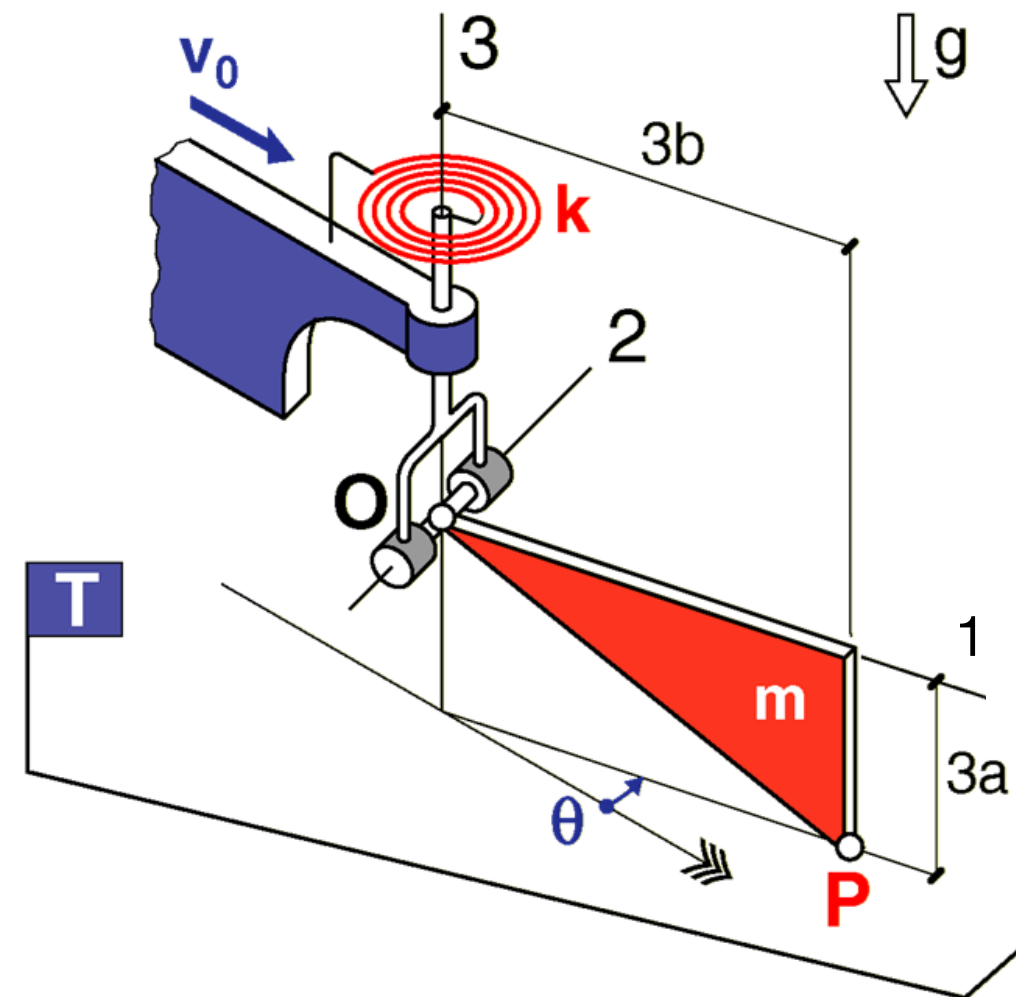
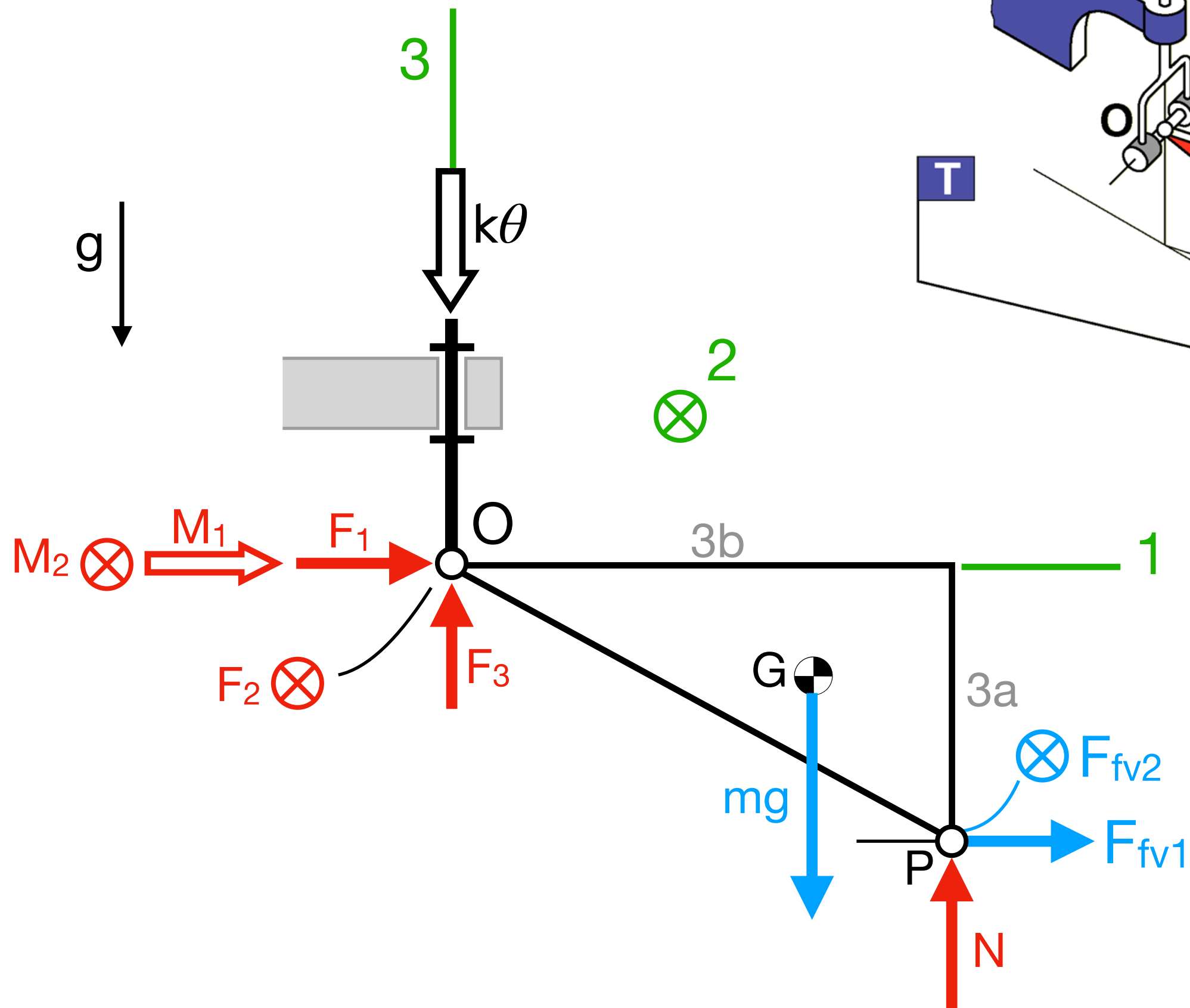
# Forces sobre "Placa + Forq"



# Forces sobre "Placa + Forq"



# Forces sobre "Placa + Forq"



# Anàlisi de l'estabilitat de $\theta_{eq} = 0$

**3 passos**  
com al pèndol simple

$$I_{33} \ddot{\theta} + 9cb^2 \dot{\theta} + k\theta - 3bcv_0 \sin \theta = 0$$

Obtenim EDO de l'error  $\varepsilon$

$$\theta = \theta_{eq} + \varepsilon = \varepsilon$$

$$\dot{\theta} = \dot{\varepsilon}$$

$$\ddot{\theta} = \ddot{\varepsilon}$$

en aquest exemple

$$I_{33} \ddot{\varepsilon} + 9cb^2 \dot{\varepsilon} + k\varepsilon - 3bcv_0 \sin \varepsilon = 0$$

La linealitzem

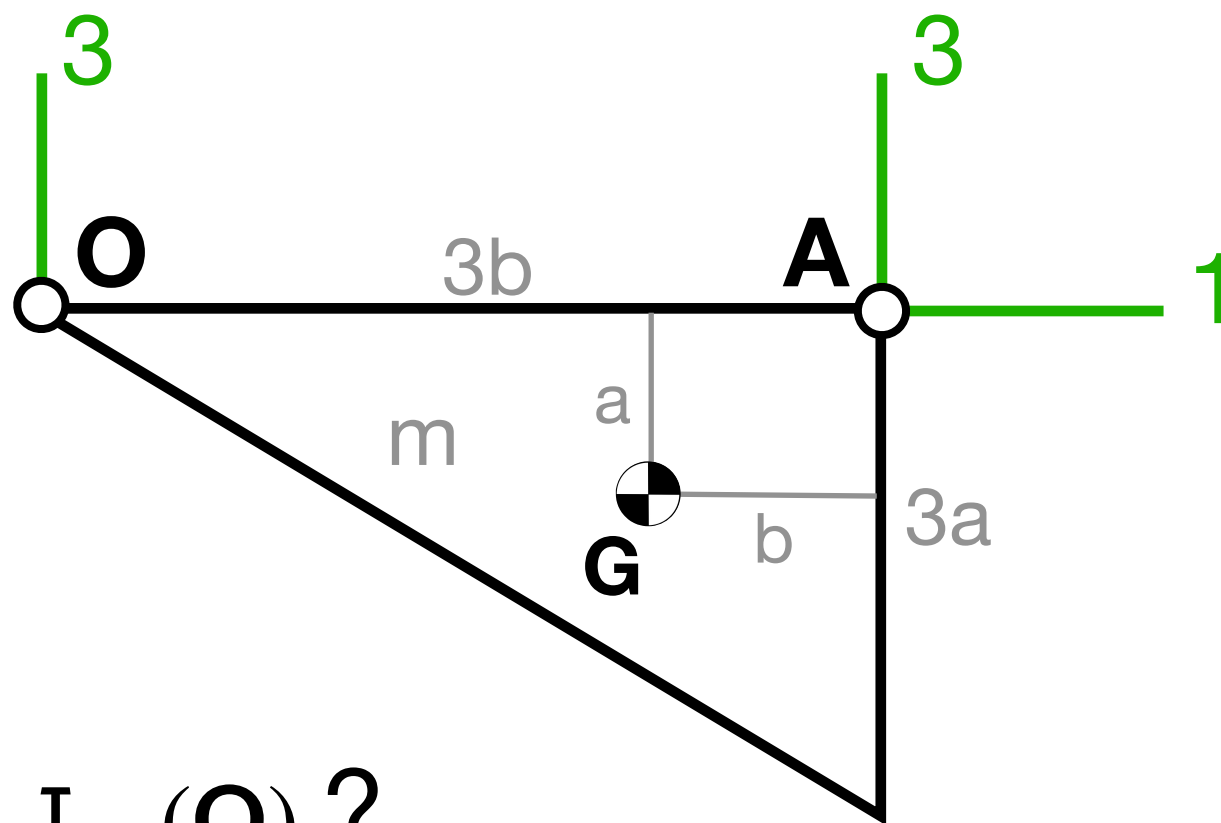
$$\sin \varepsilon \approx \varepsilon$$

$$I_{33} \ddot{\varepsilon} + \underbrace{9cb^2}_A \dot{\varepsilon} + \underbrace{(k - 3bcv_0)}_B \varepsilon = 0$$

$$\ddot{\varepsilon} = -\underbrace{\frac{B}{I_{33}}}_K \varepsilon - \underbrace{\frac{A}{I_{33}}}_{C > 0} \dot{\varepsilon}$$

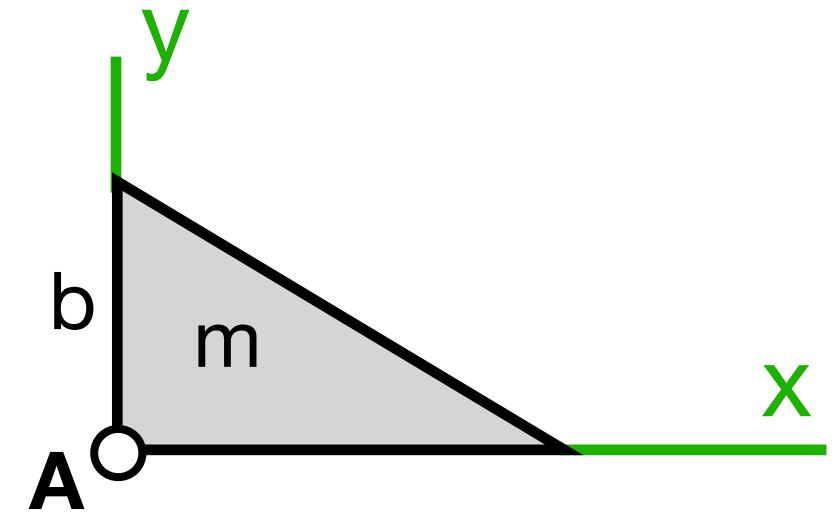
$K > 0?$

$$K > 0 \iff B > 0 \iff k > 3bcv_0$$



$I_{33}(\mathbf{O})$  ?

Taules



$$I_{xx}(\mathbf{A}) = \frac{1}{6}mb^2$$

$I_{33}(\mathbf{A})$  de taules + **doble Steiner** per passar a  $\mathbf{O}$ :

$$(a) \quad I_{33}(\mathbf{O}) = I_{33}(\mathbf{G}) + I_{33}^{\oplus}(\mathbf{O})$$

$$(b) \quad I_{33}(\mathbf{A}) = I_{33}(\mathbf{G}) + I_{33}^{\oplus}(\mathbf{A})$$

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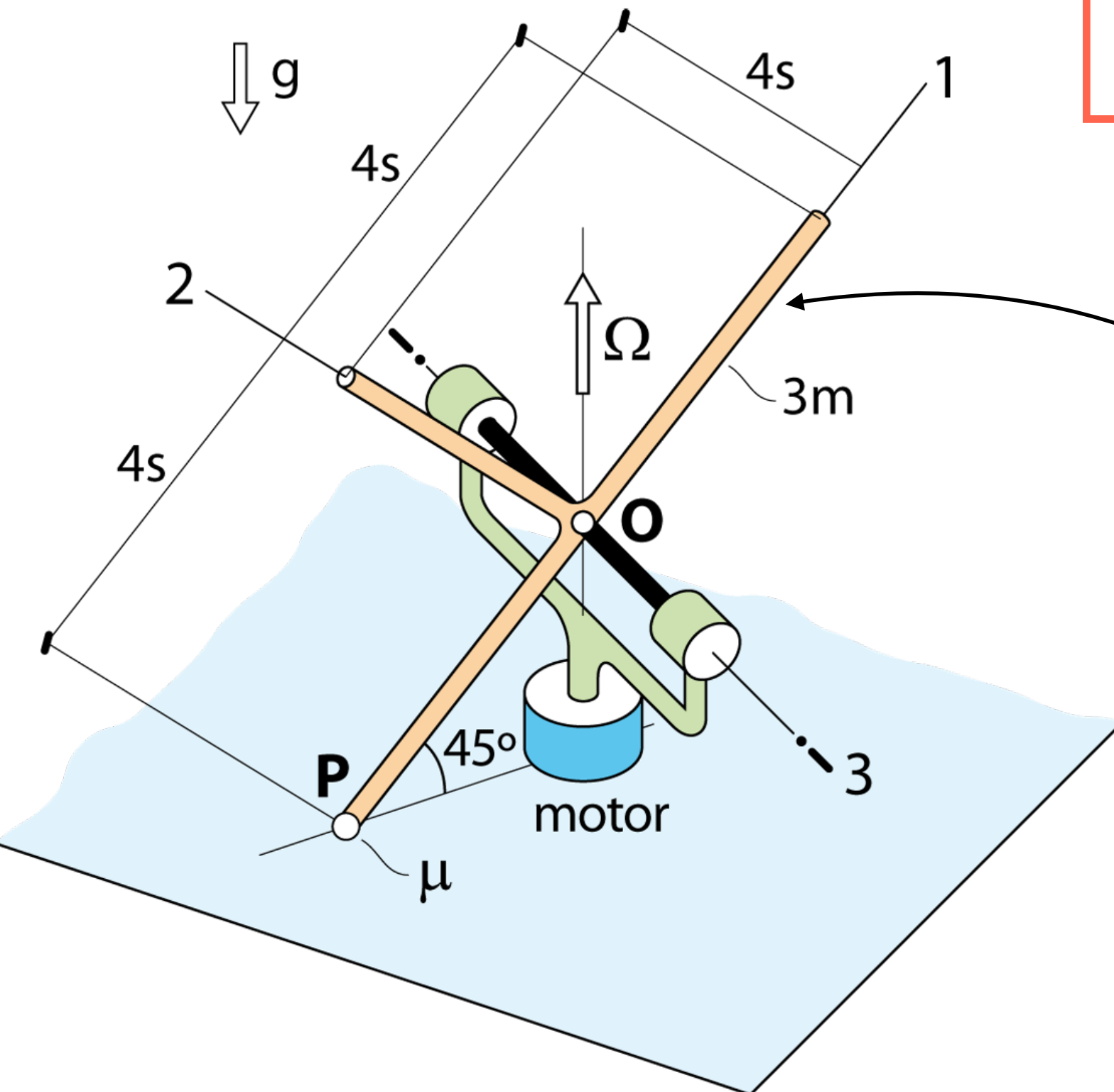

$$(a - b) \quad I_{33}(\mathbf{O}) = I_{33}(\mathbf{A}) + I_{33}^{\oplus}(\mathbf{O}) - I_{33}^{\oplus}(\mathbf{A})$$

$$I_{33}(\mathbf{O}) = \frac{1}{6}m(3b^2) + m(2b)^2 - mb^2 = \frac{9}{2}mb^2$$

$$\Omega = ct$$

$\Omega_{\text{critica}}$  pèrdua contacte a P?

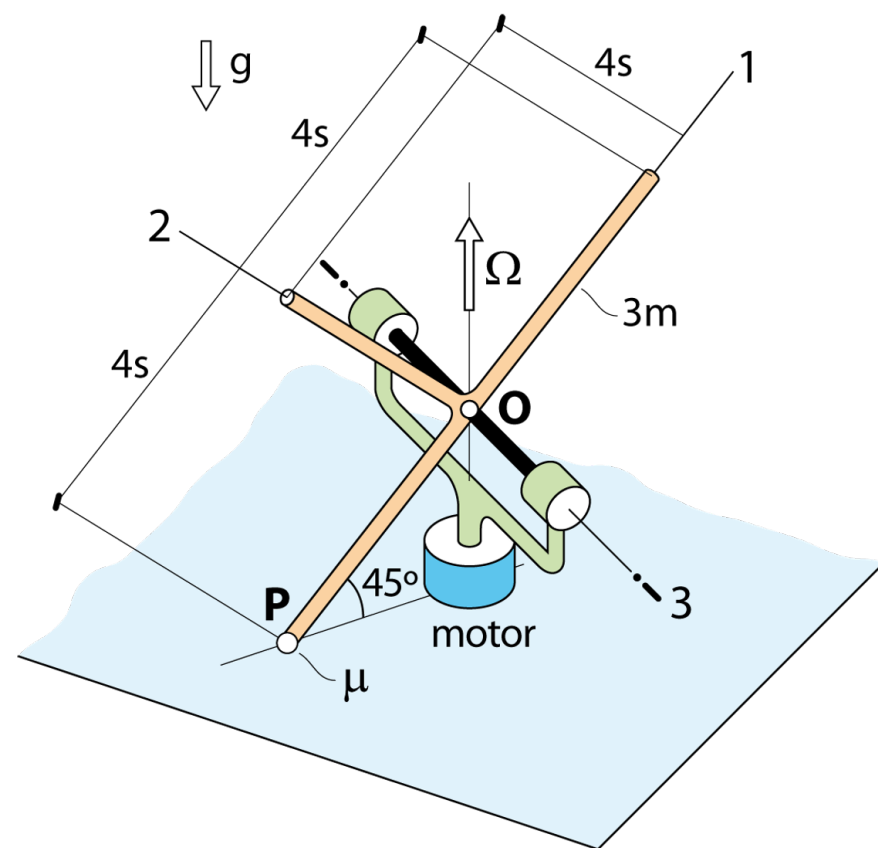
Equació del mov. quan  
 $\Omega > \Omega_{\text{critica}}$



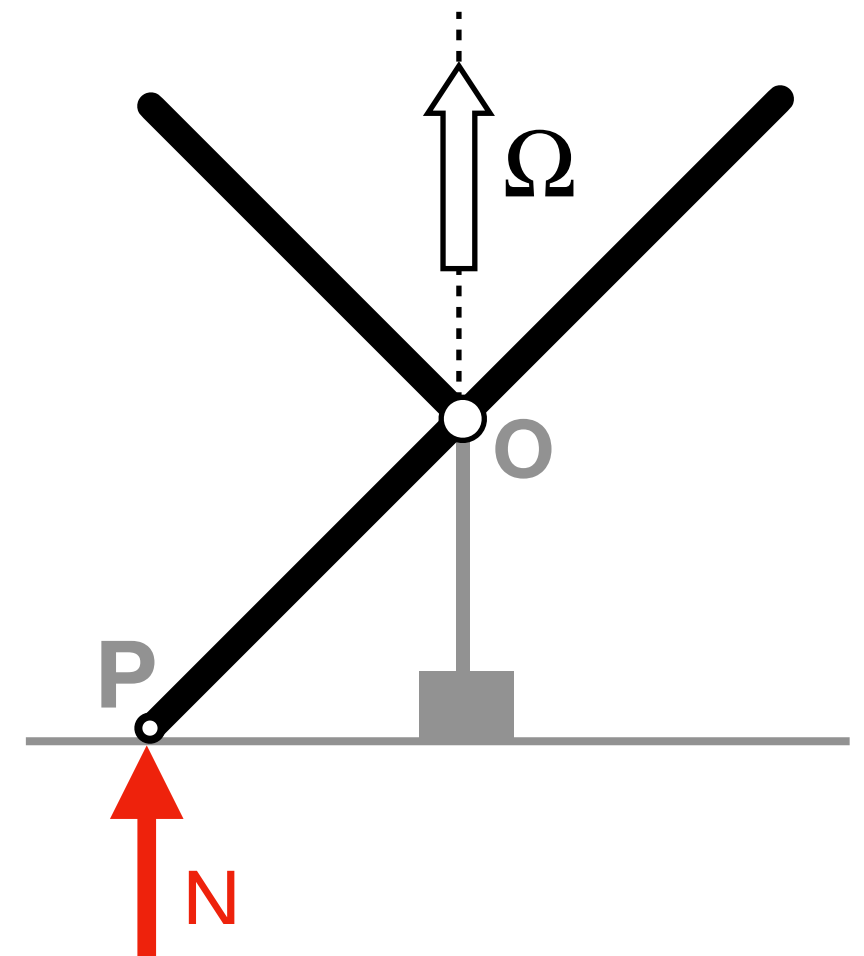
"Peça"

# GL?

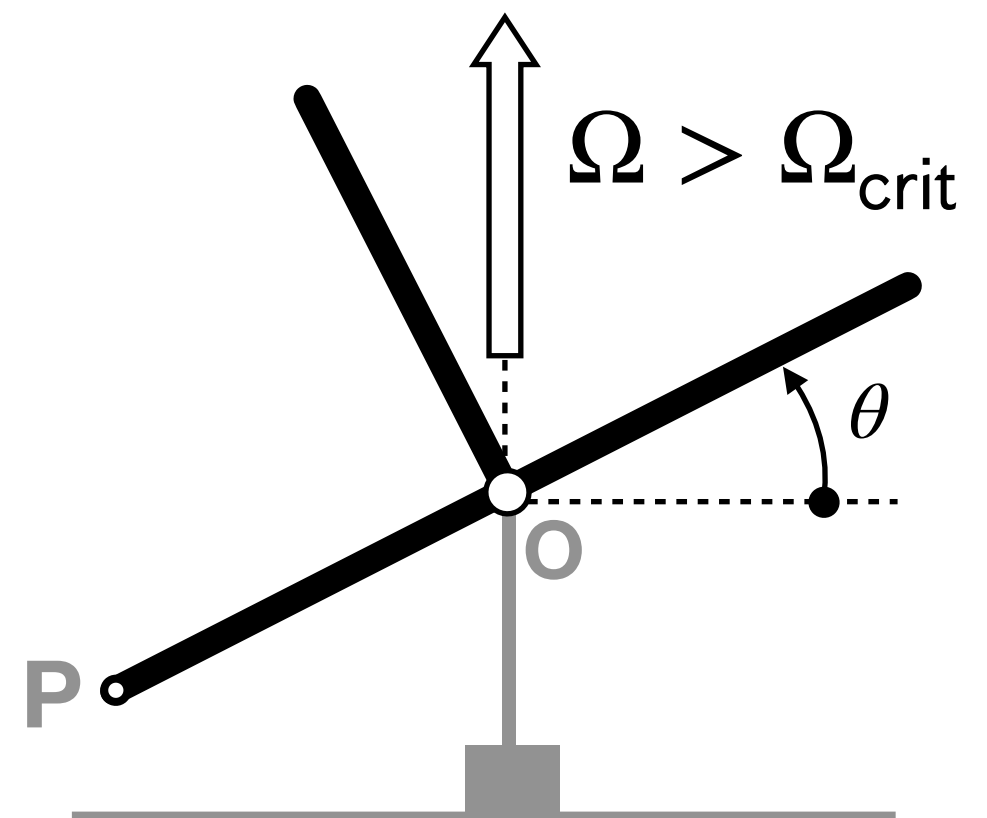
## 2 situations!

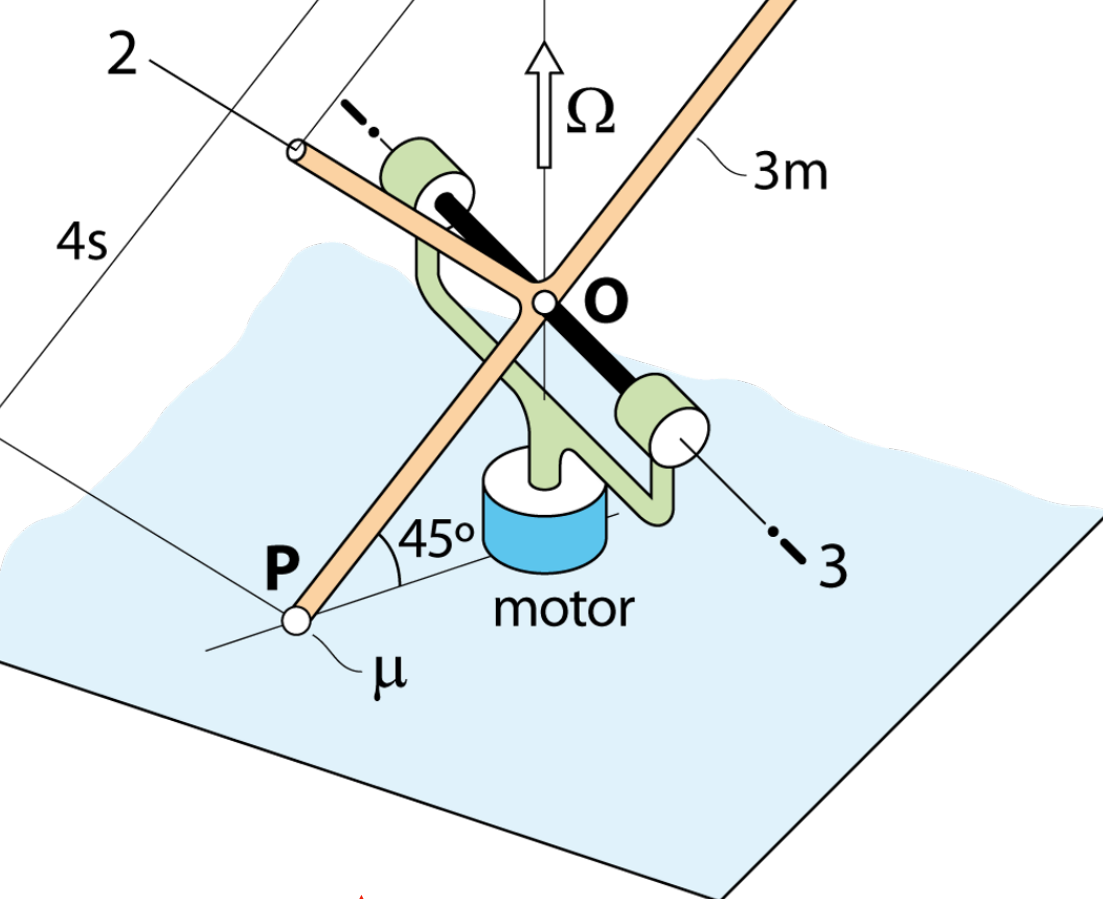


**P** manté  
contacte  
amb T



Contacte  
perdut

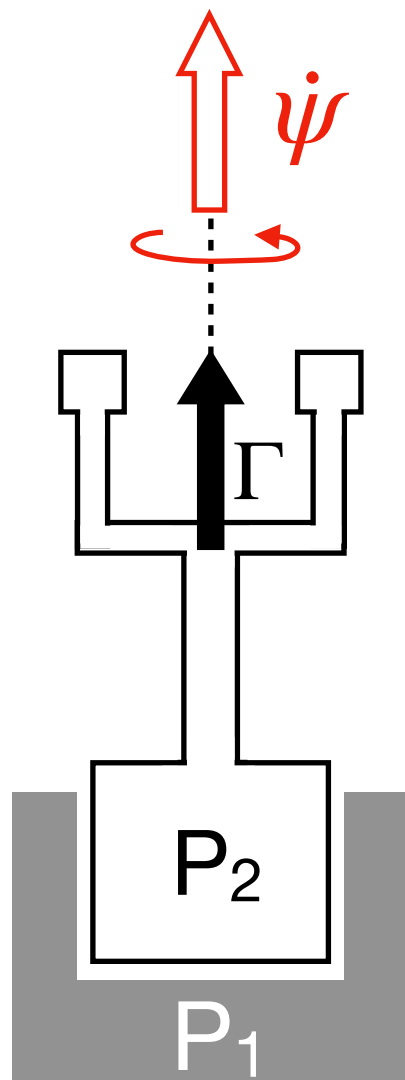




Recordeu

$\Gamma$  conegut  $\Rightarrow \ddot{\psi}$  és incògnita

$\ddot{\psi}$  coneguda  $\Rightarrow \Gamma$  és incògnita

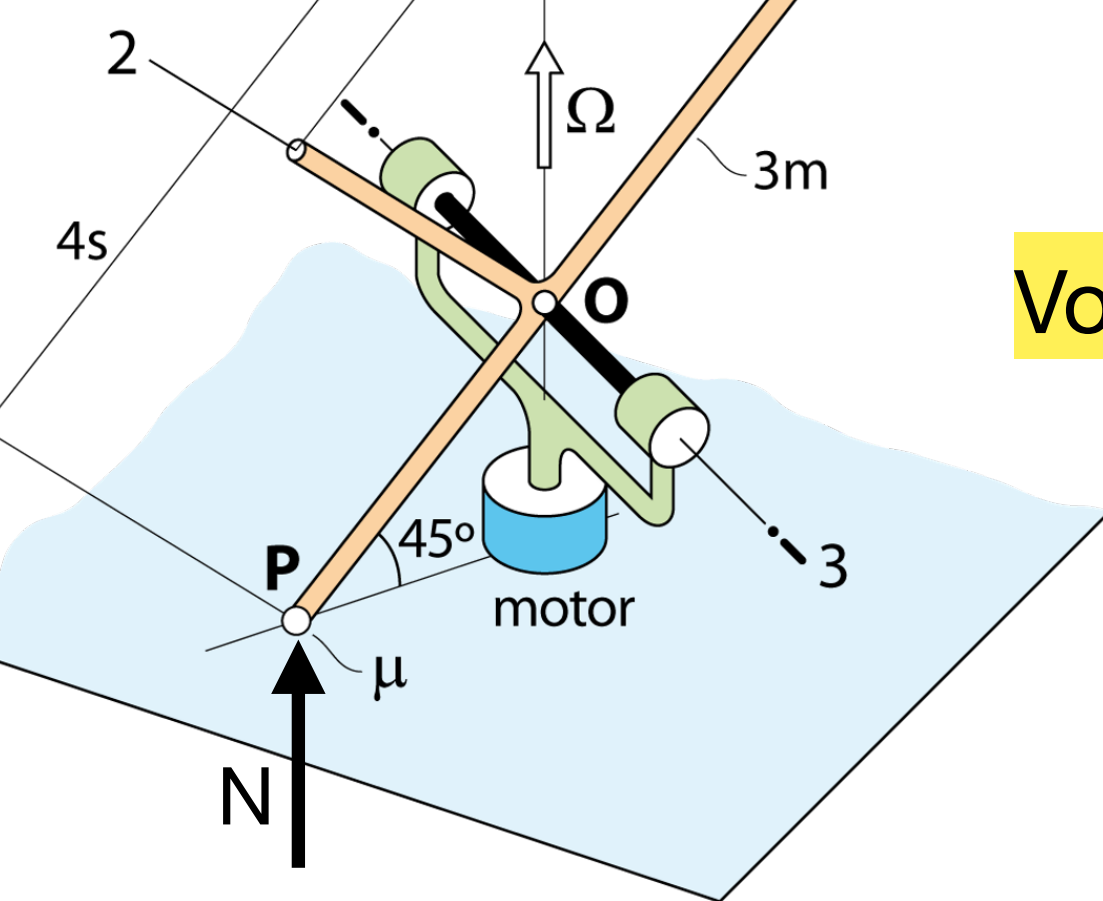


En aquest exercici

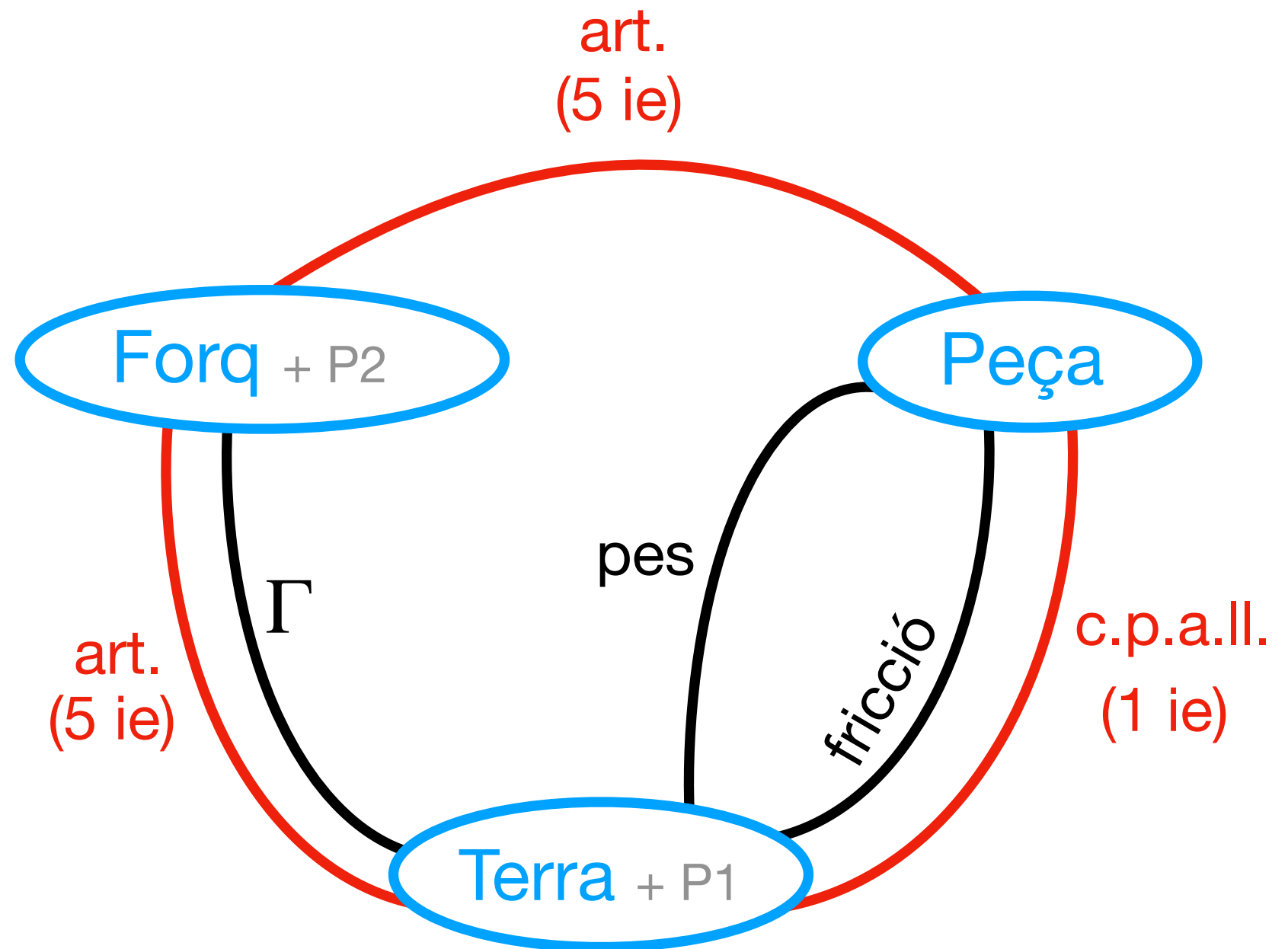
$\ddot{\psi} = \Omega = ct \Rightarrow \ddot{\psi} = 0$  (coneguda)

$\Gamma$  serà incògnita



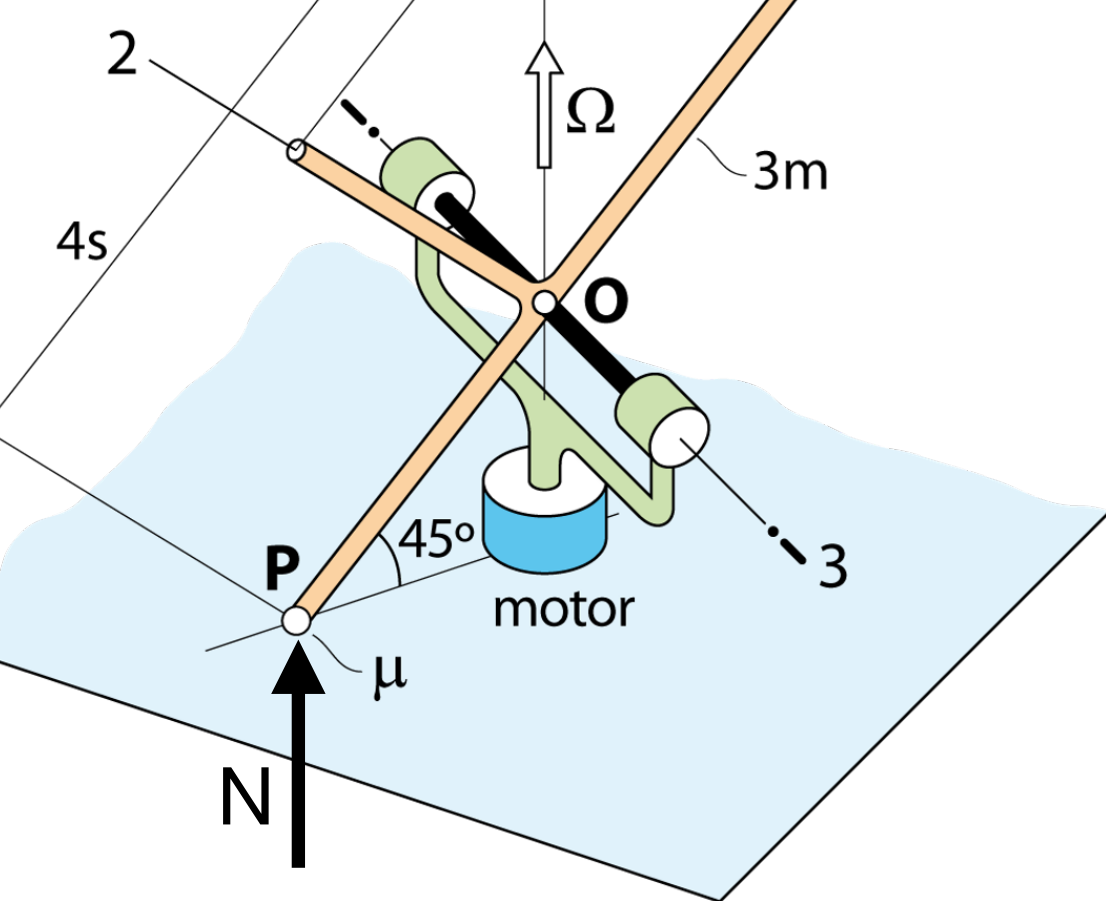


Volem  $N \implies$  SIST ha d'incloure la peça!



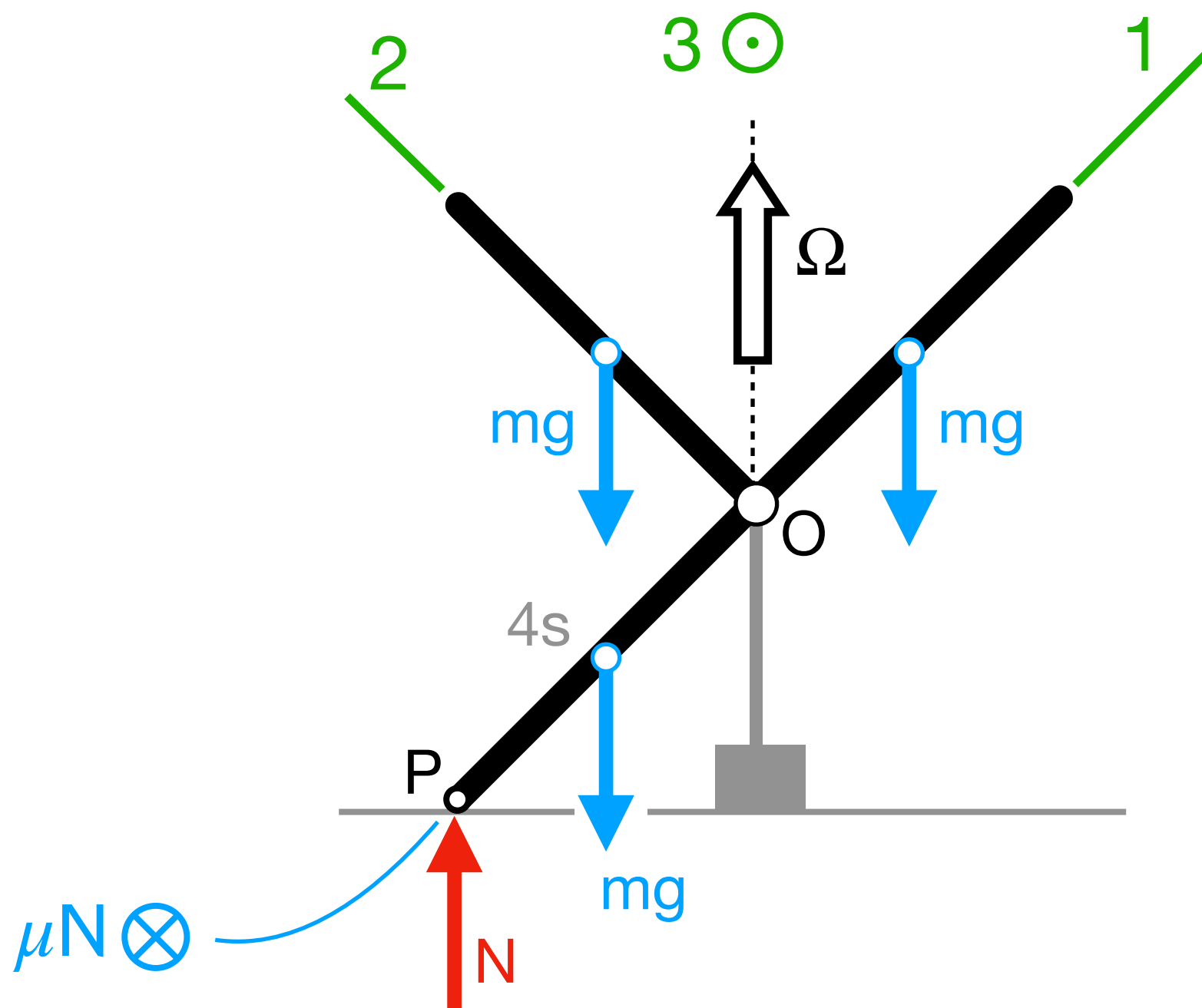






$$\left\{ \bar{\mathbf{F}}_{\text{Forq} \rightarrow \text{Peça}} \right\}_{\text{B}} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\left\{ \bar{\mathbf{M}}_{\text{Forq} \rightarrow \text{Peça}} (\text{O}) \right\}_{\text{B}} = \begin{Bmatrix} M_1 \\ M_2 \\ 0 \end{Bmatrix}$$



# DEURES

Determineu

- Parell motor  $\Gamma$  per mantenir  $\Omega = ct$
- Eq. mov. per al cas en que el contacte a P ja s'ha perdut ( $\Omega > \Omega_{\text{critica}}$ )