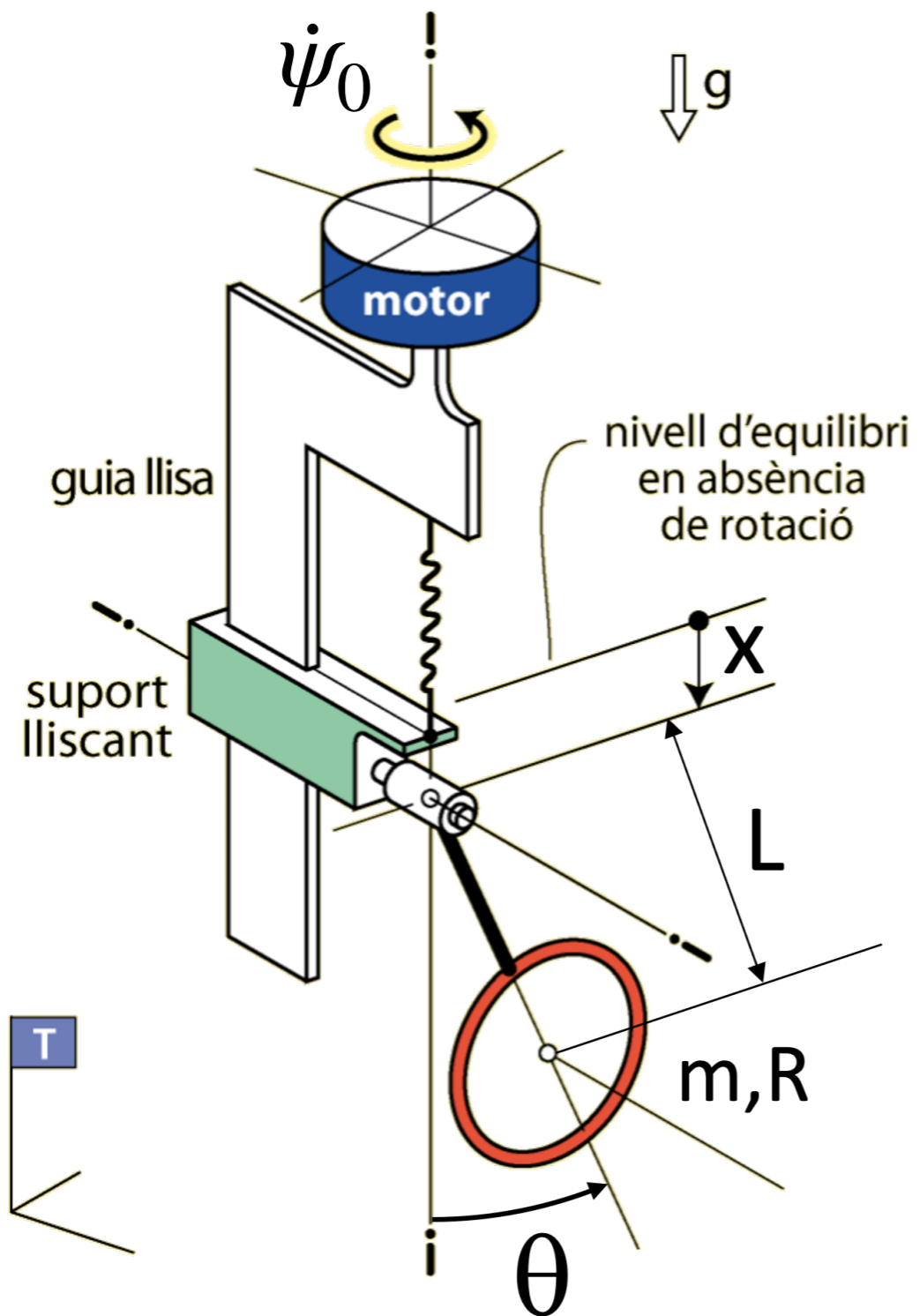


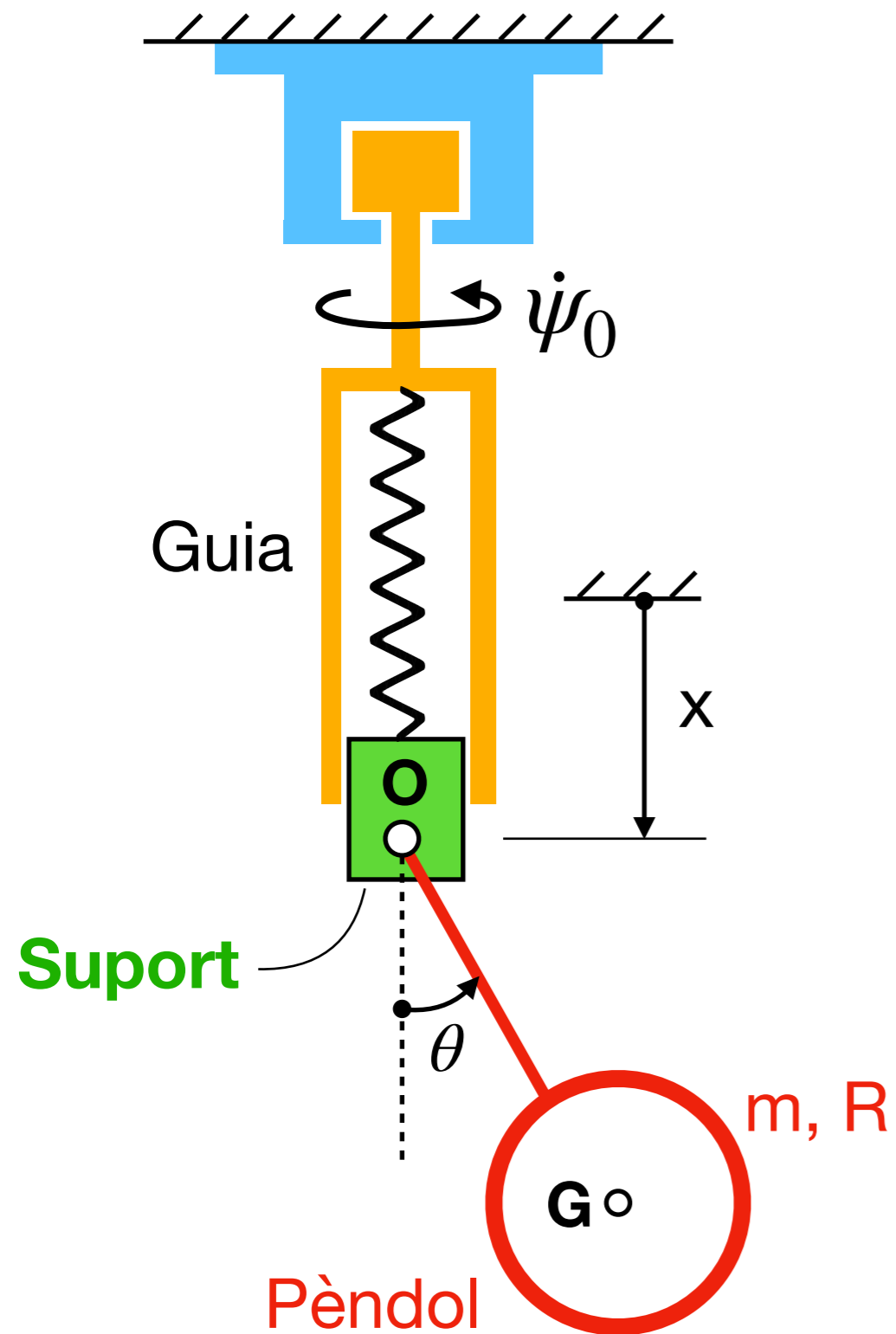
# 13P

## Teoremes vectorials III

Exemples 3D

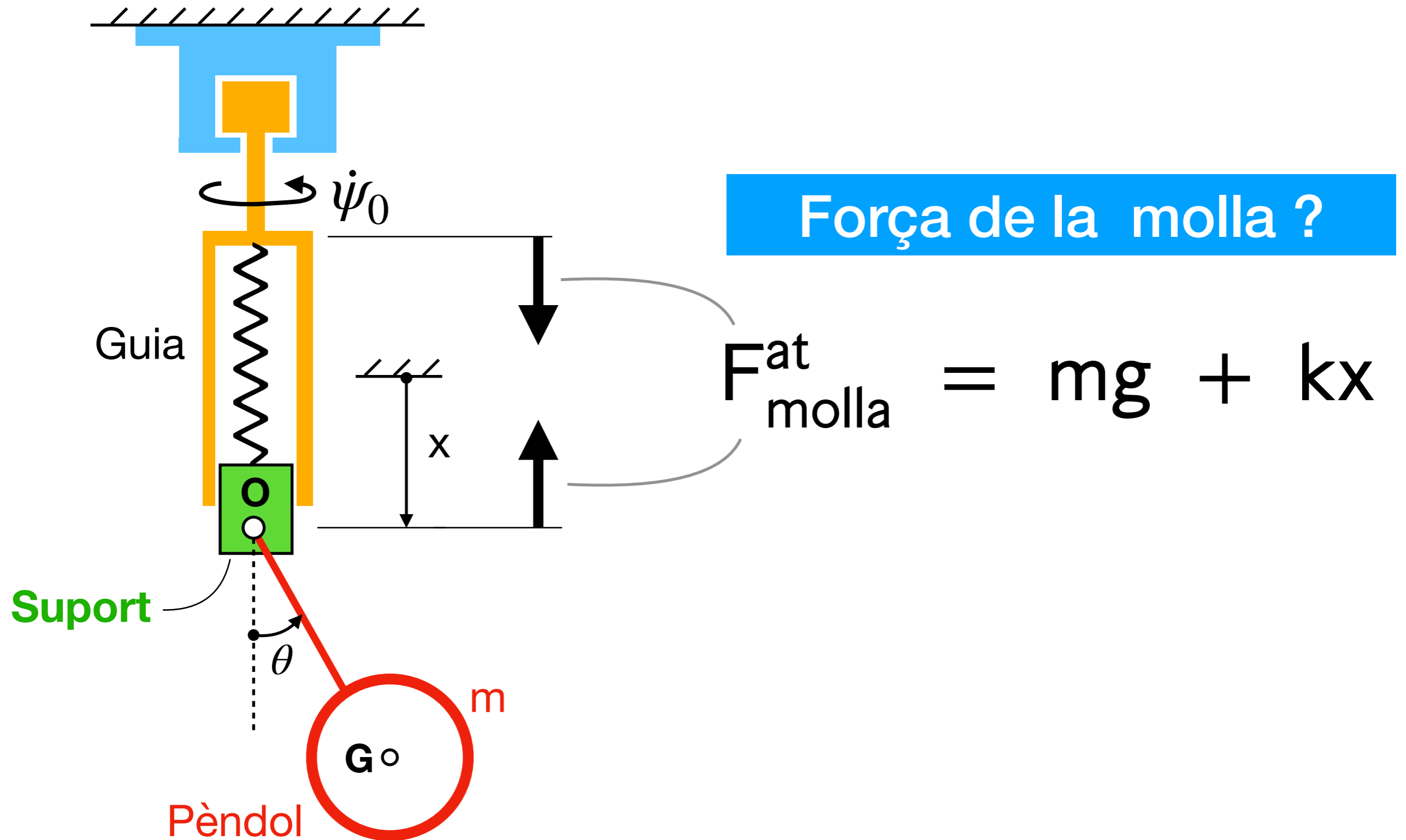
- DGI
- Eqs. mov. per a  $x$  i  $\theta$
- Parell motor  $\Gamma$  per mantenir  $\dot{\psi}_0$



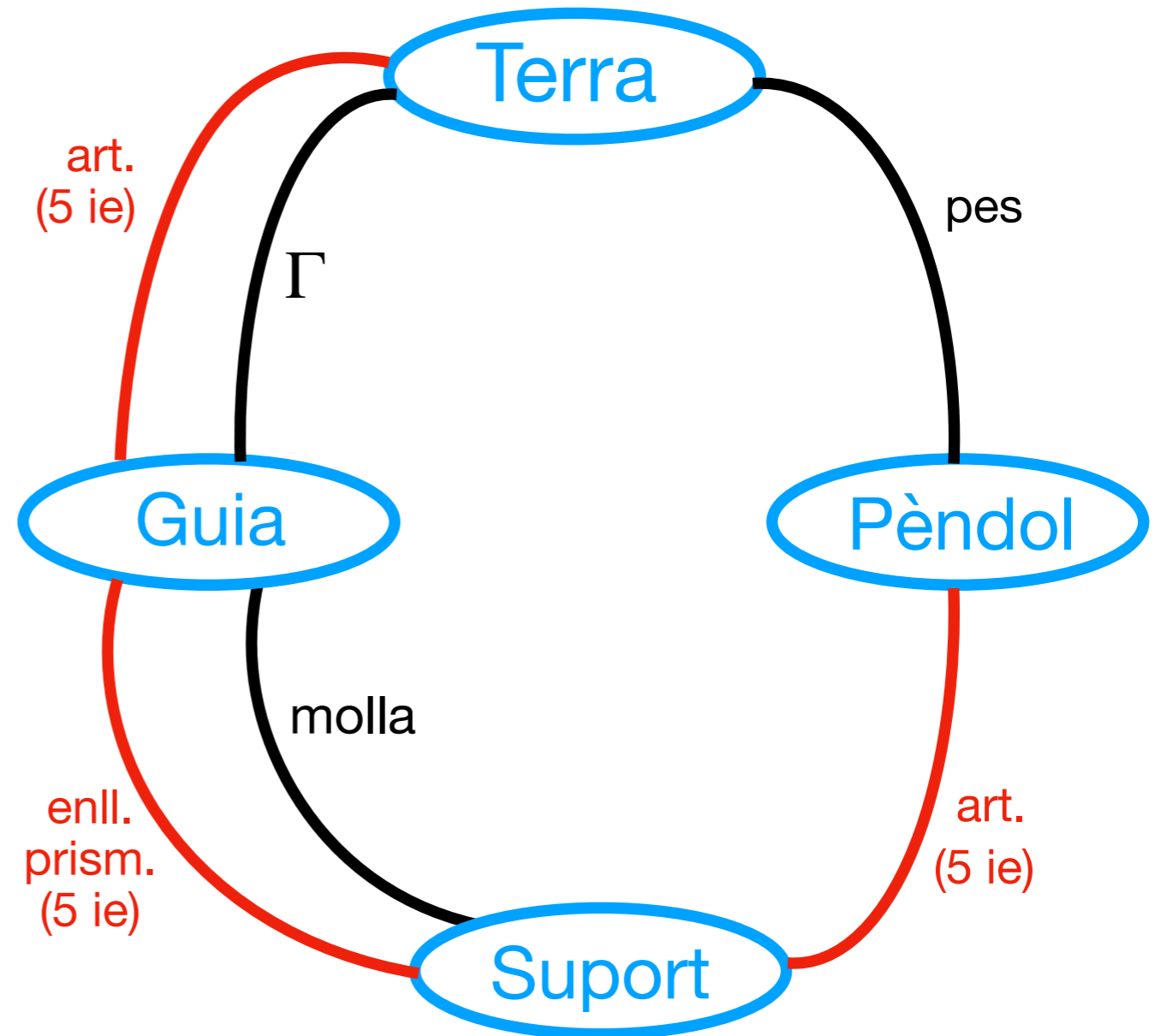
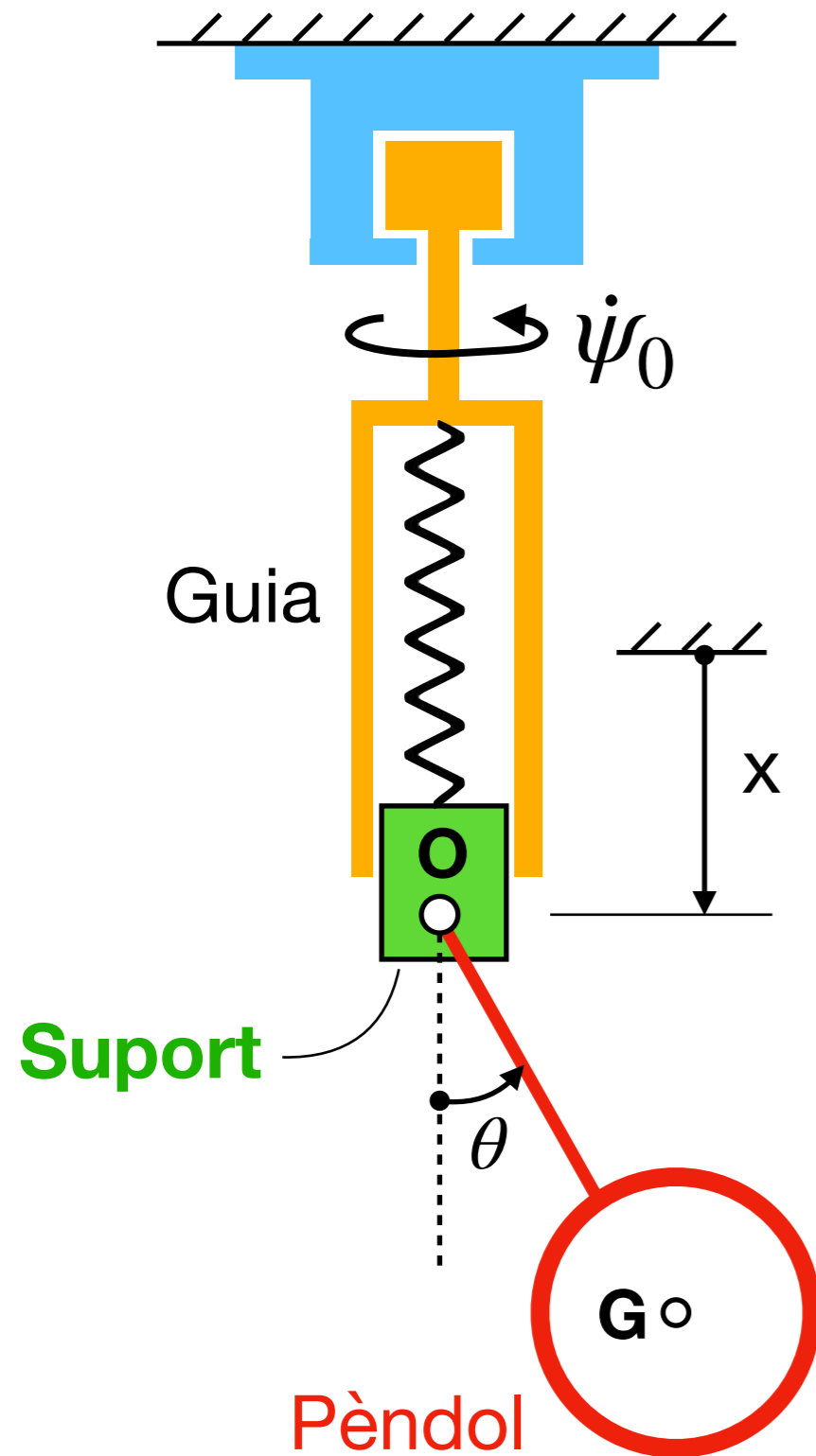


- DGI
- Eqs. mov. per a  $x$  i  $\theta$
- Parell motor  $\Gamma$  per mantenir  $\dot{\psi}_0$

Amb motor aturat:  $x = 0$ ,  $\theta = 0$  és configuració d'equilibri.

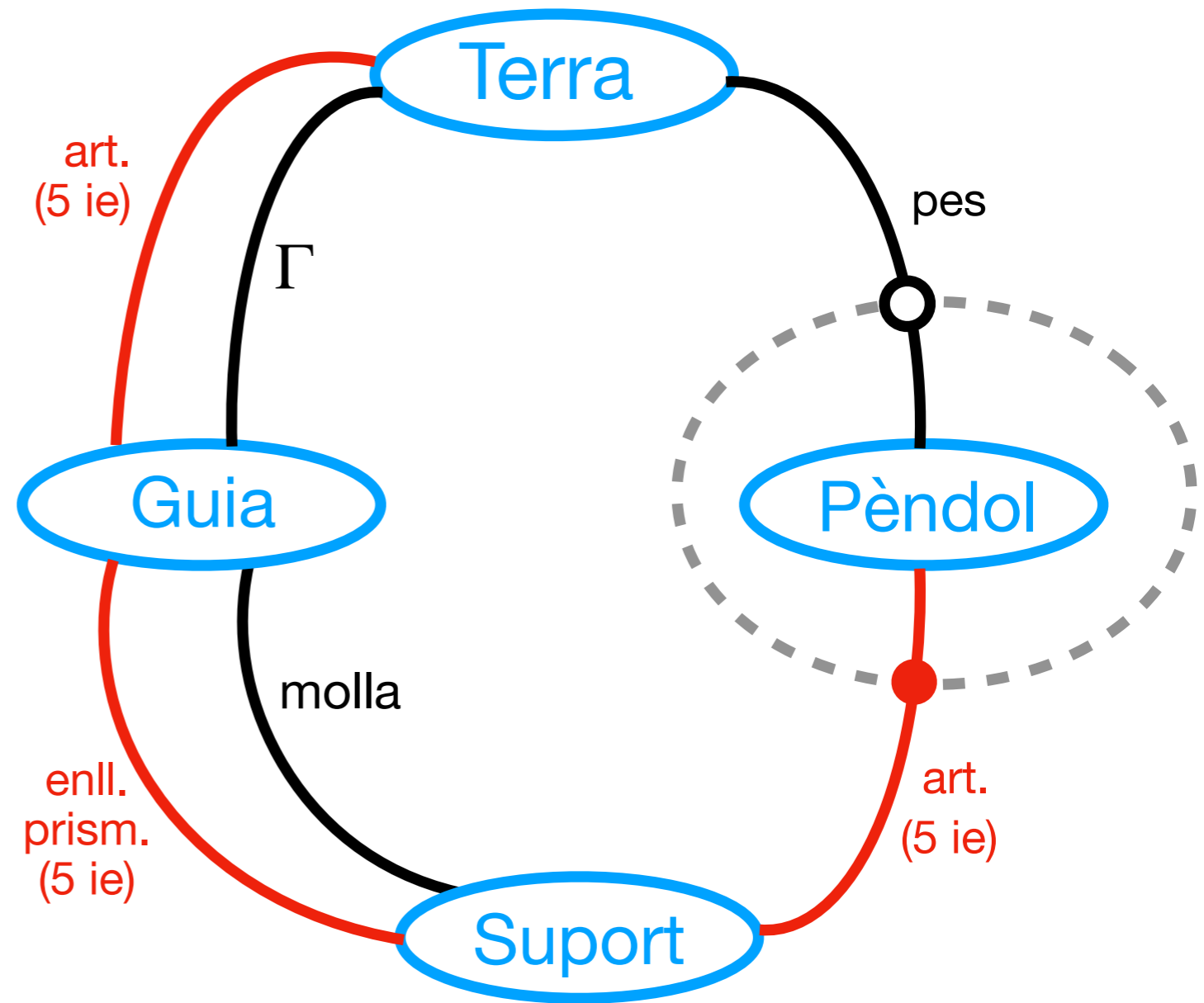
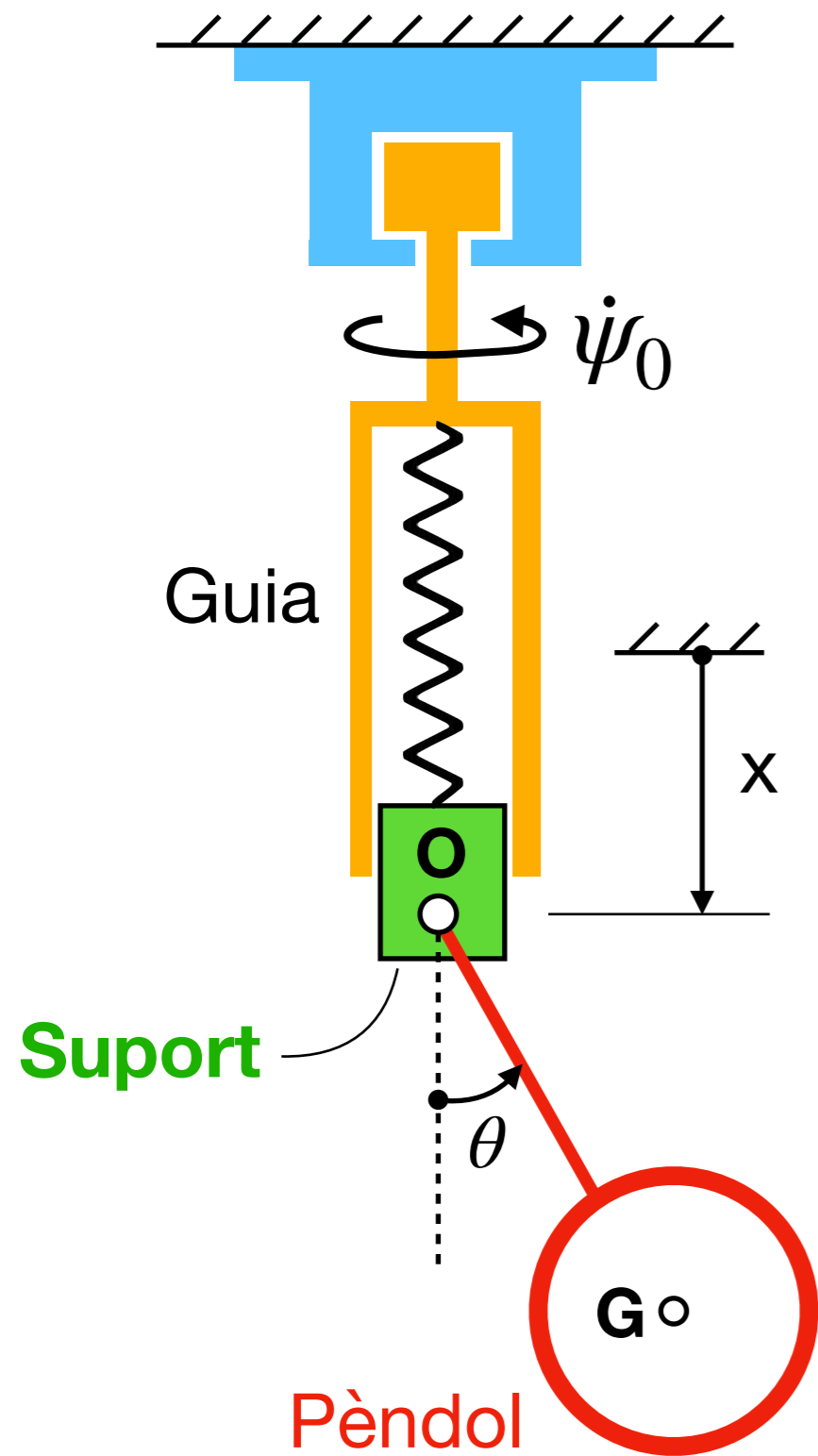


Amb motor aturat:  $x = 0$ ,  $\theta = 0$  és configuració d'equilibri.

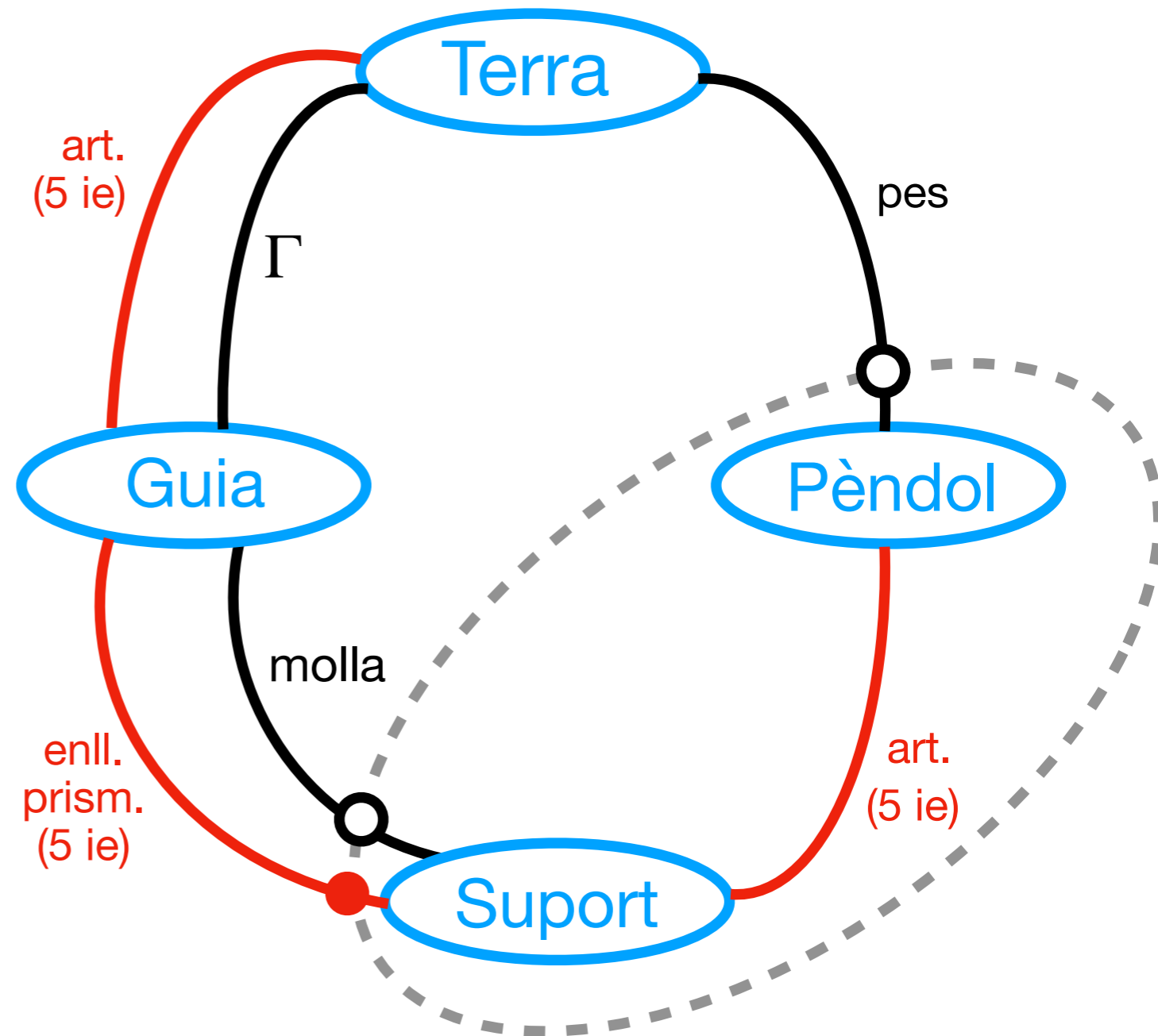
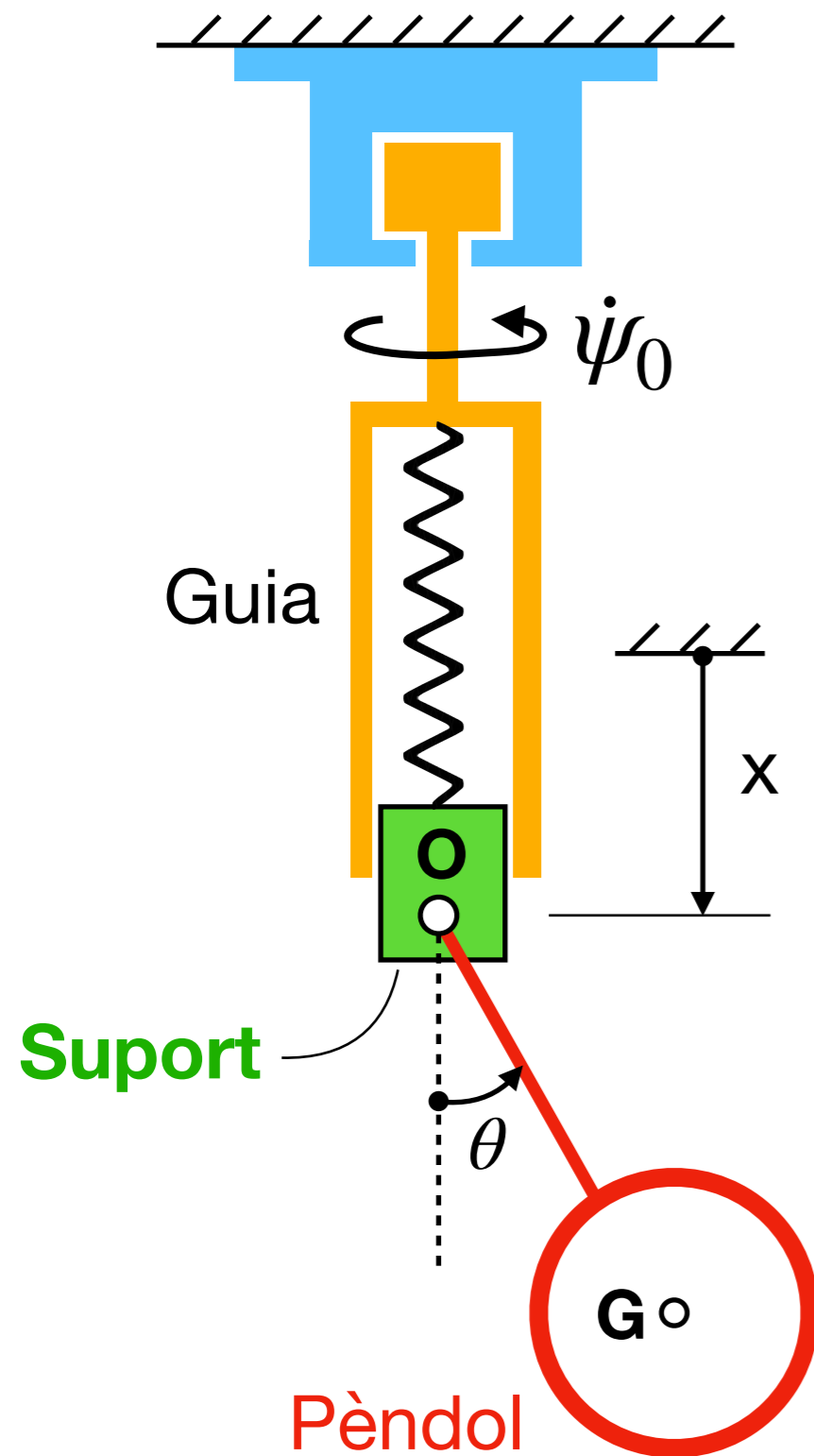


$x$  i  $\theta$  només afecten **pèndol** i **suport**

Explorem sistemes que els incloguin



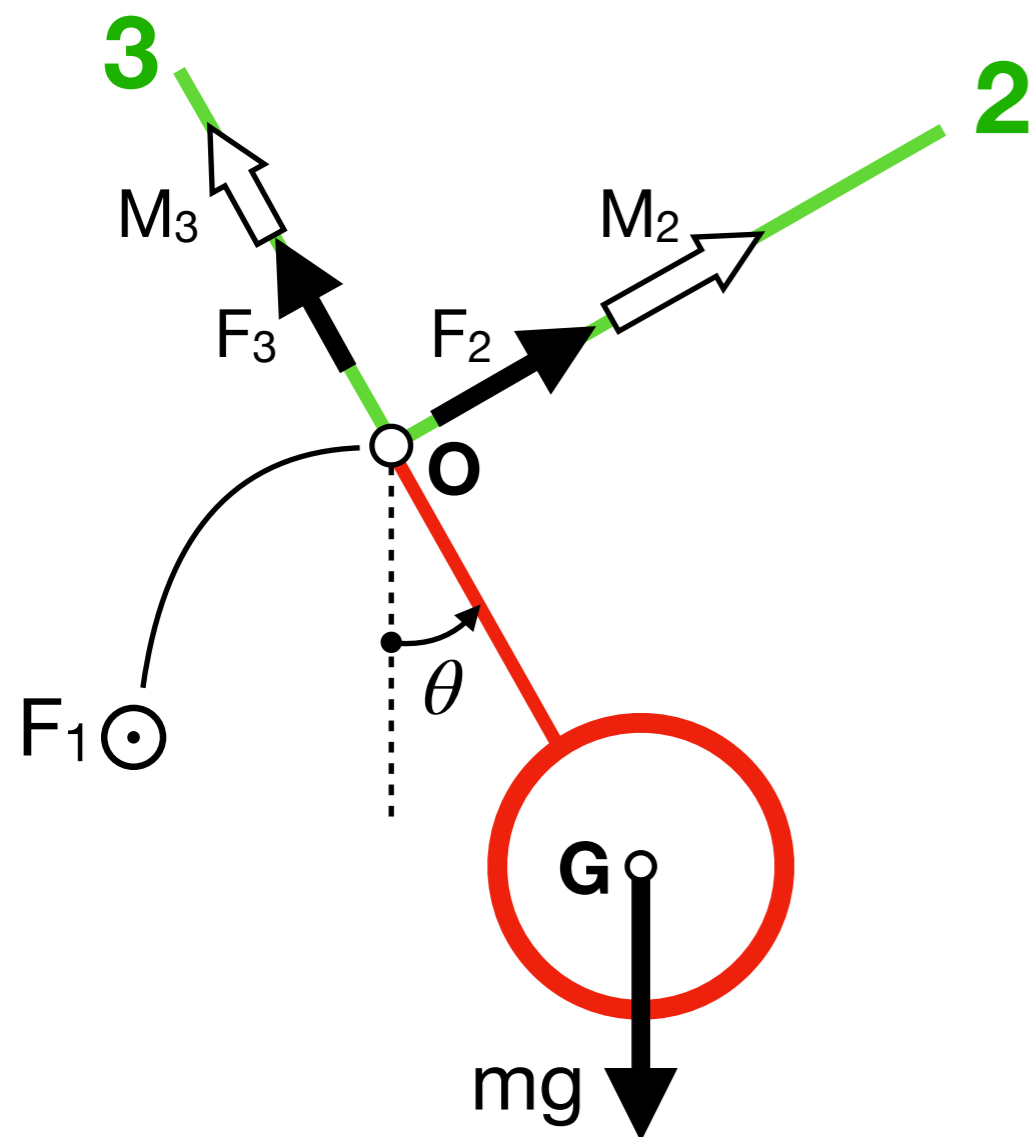
Sistema	Incògn.	Problema
Pèndol	5 ie, $\ddot{x}$ , $\ddot{\theta}$	<b>INDET</b>



Sistema	Incògn.	Problema
Pèndol	5 ie, $\ddot{x}$ , $\ddot{\theta}$	<b>INDET</b>
Pènd. + sup.	5 ie, $\ddot{x}$ , $\ddot{\theta}$	<b>INDET</b>

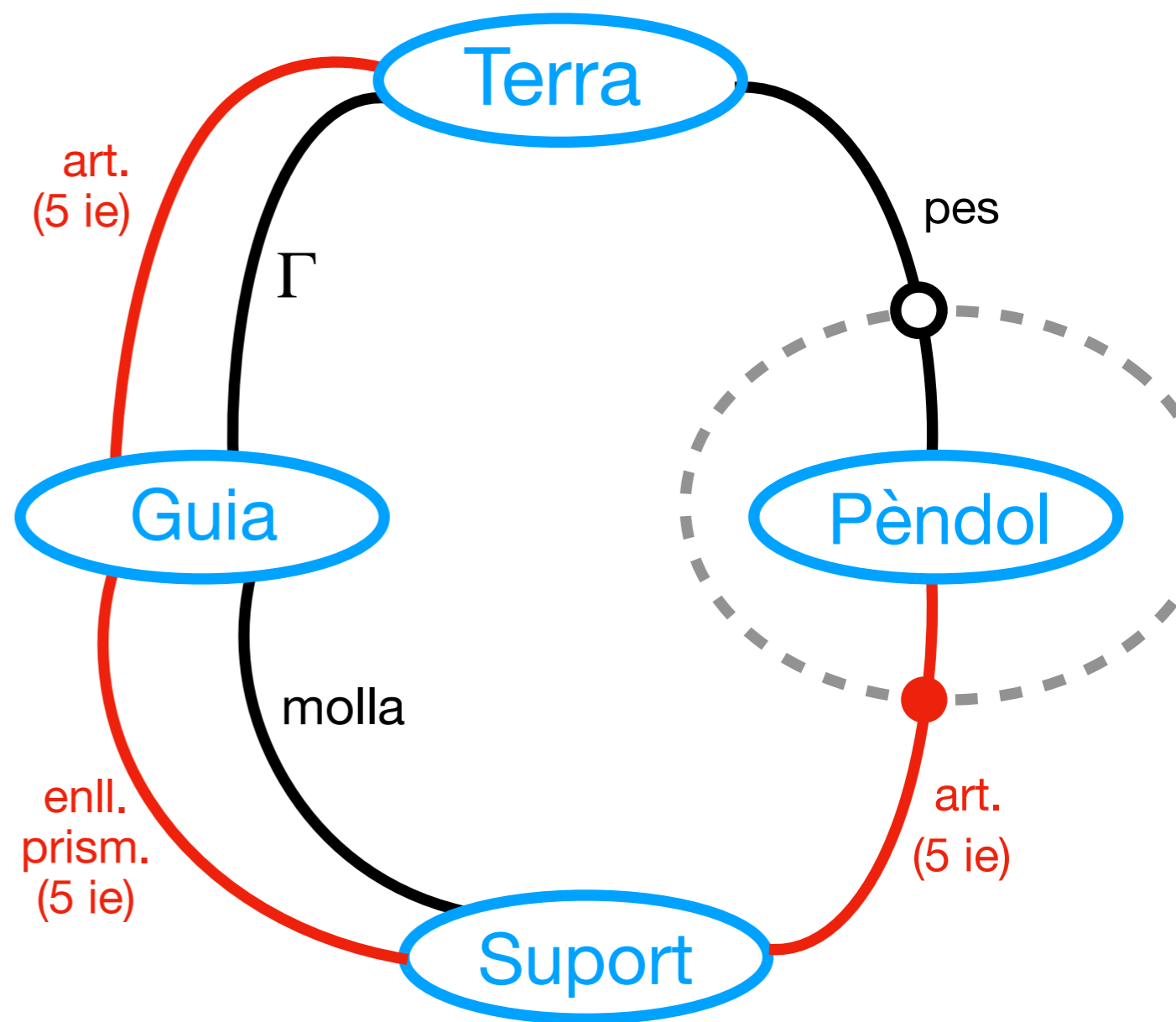
**Els altres sistemes tenen + incògnites !**

# Sist = Pèndol

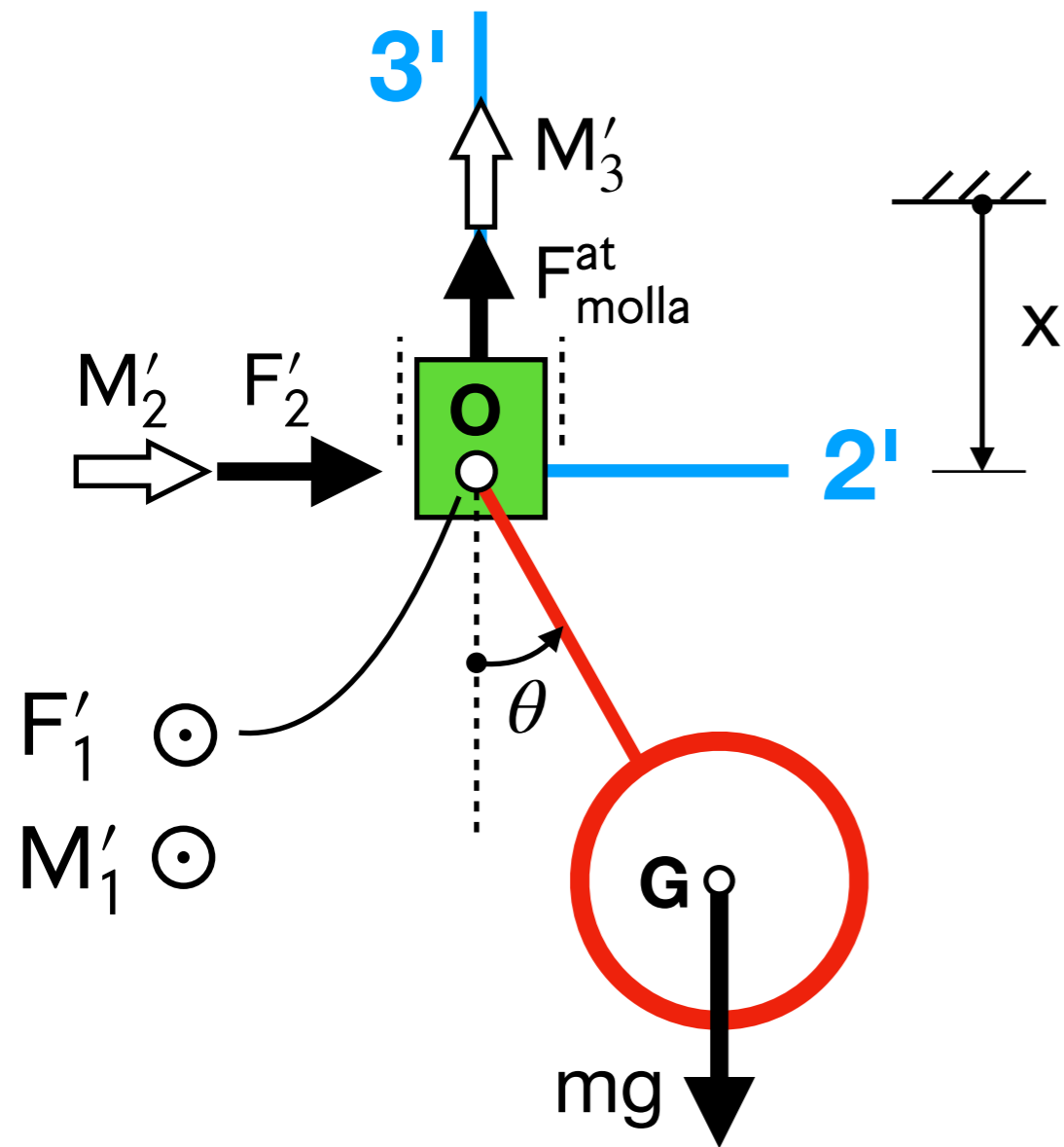


$$\left\{ \bar{\mathbf{F}}_{\text{Sup} \rightarrow \text{Pendol}} \right\}_{\text{B}} = \begin{Bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \\ \mathbf{F}_3 \end{Bmatrix}$$

$$\left\{ \bar{\mathbf{M}}_{\text{Sup} \rightarrow \text{Pendol}} (\mathbf{O}) \right\}_{\text{B}} = \begin{Bmatrix} 0 \\ M_2 \\ M_3 \end{Bmatrix} \leftarrow \begin{array}{l} \mathbf{TMC}(\mathbf{O}) ]_1 \\ \text{lliure de ie} \end{array}$$



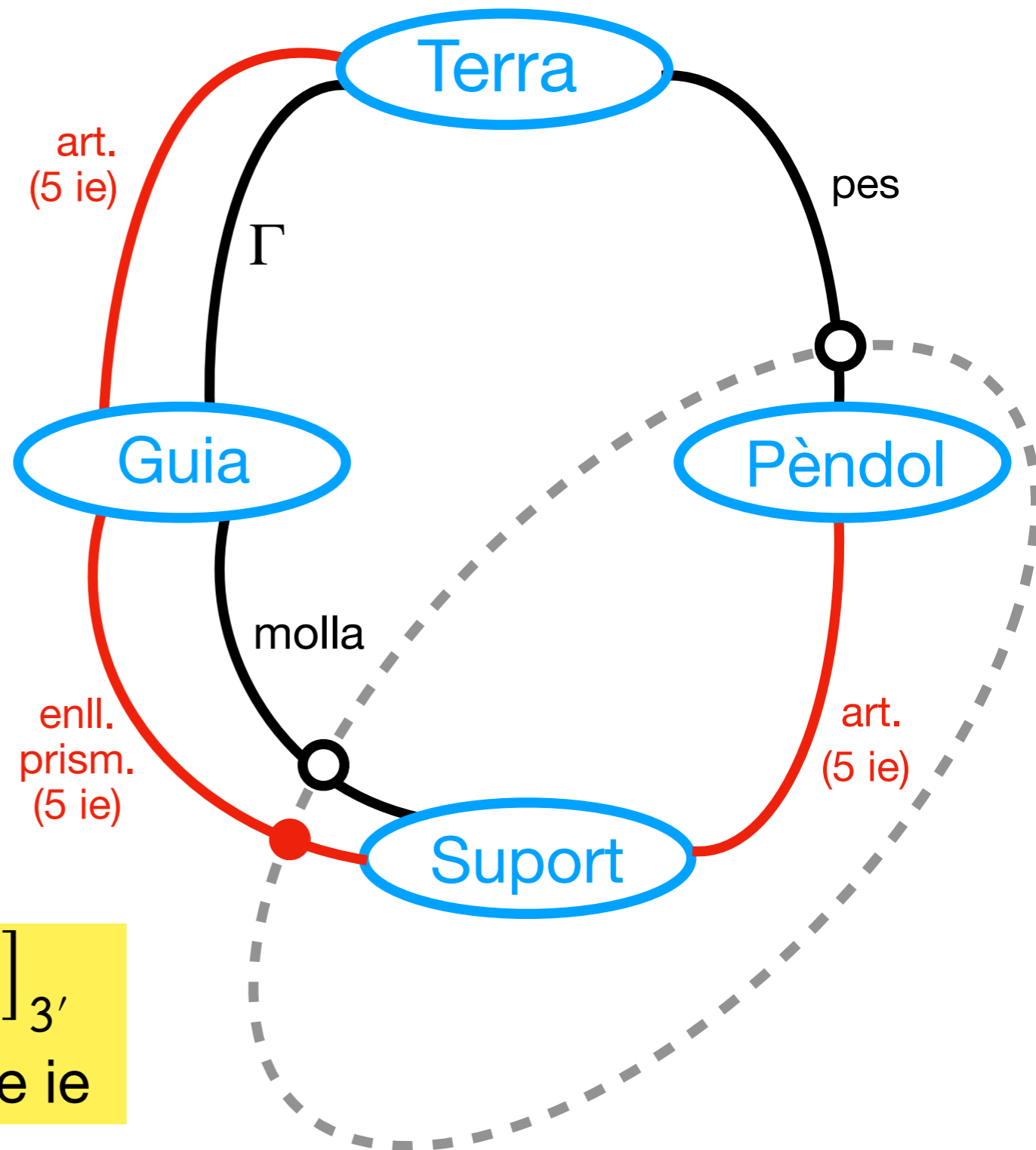
# Sist = Pèndol + Suport



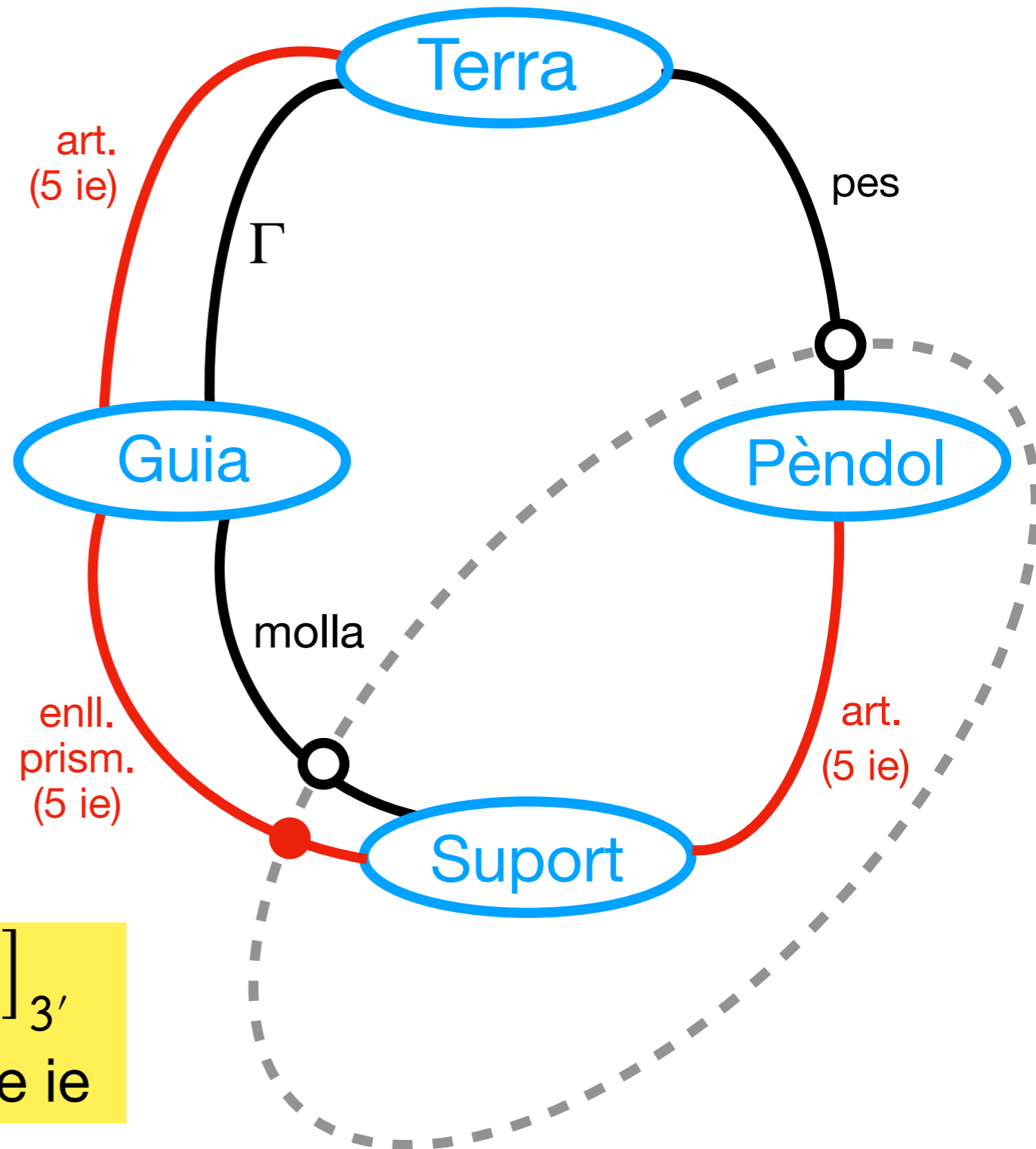
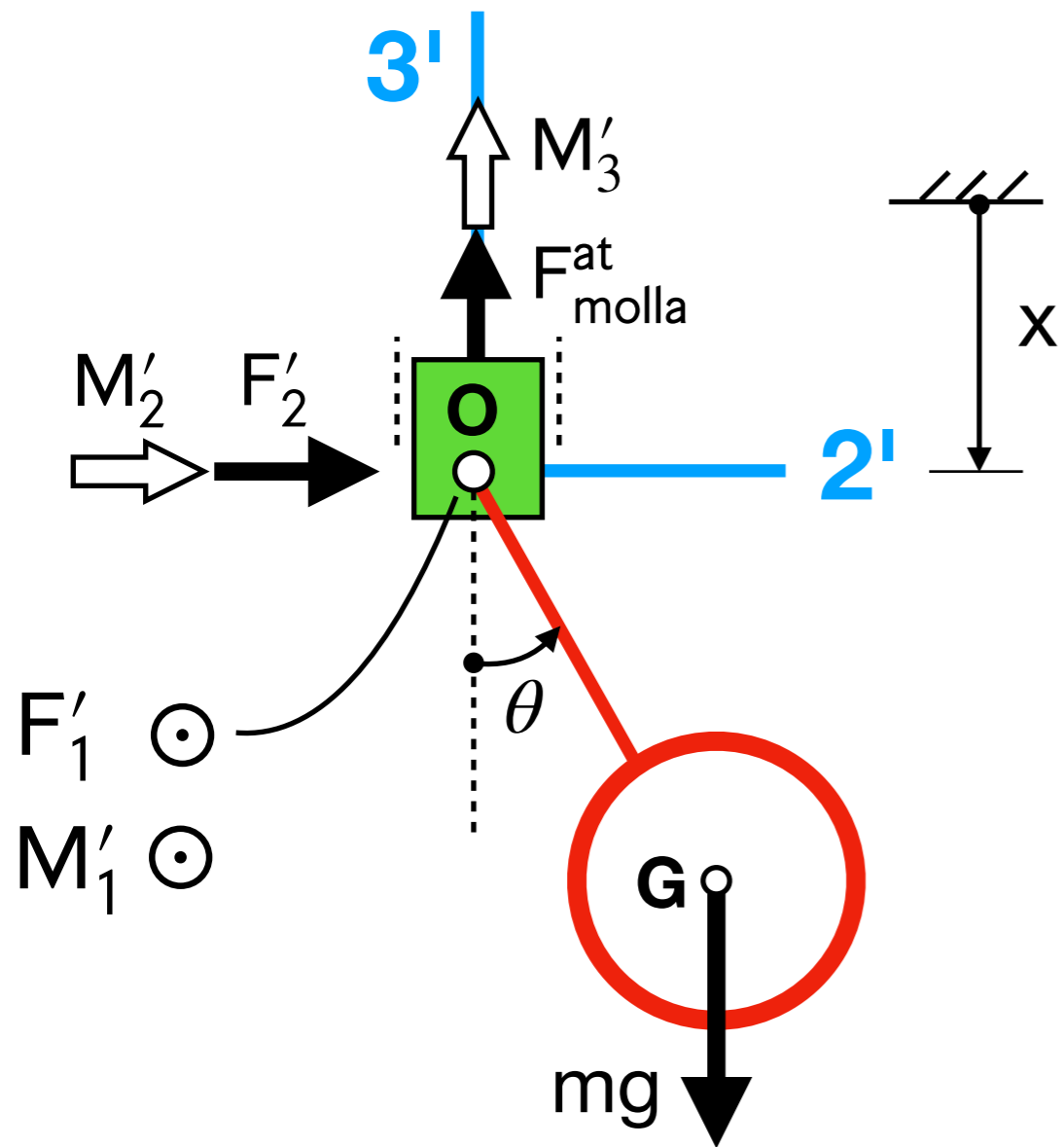
$$\left\{ \bar{F}_{\text{Guia} \rightarrow \text{Sup}} \right\}_{B'} = \begin{Bmatrix} F'_1 \\ F'_2 \\ 0 \end{Bmatrix}$$

$$\left\{ \bar{M}_{\text{Guia} \rightarrow \text{Sup}}(\mathbf{O}) \right\}_{B'} = \begin{Bmatrix} M'_1 \\ M'_2 \\ M'_3 \end{Bmatrix}$$

TQM]<sub>3'</sub>  
lliure de ie



# Sist = Pèndol + Suport

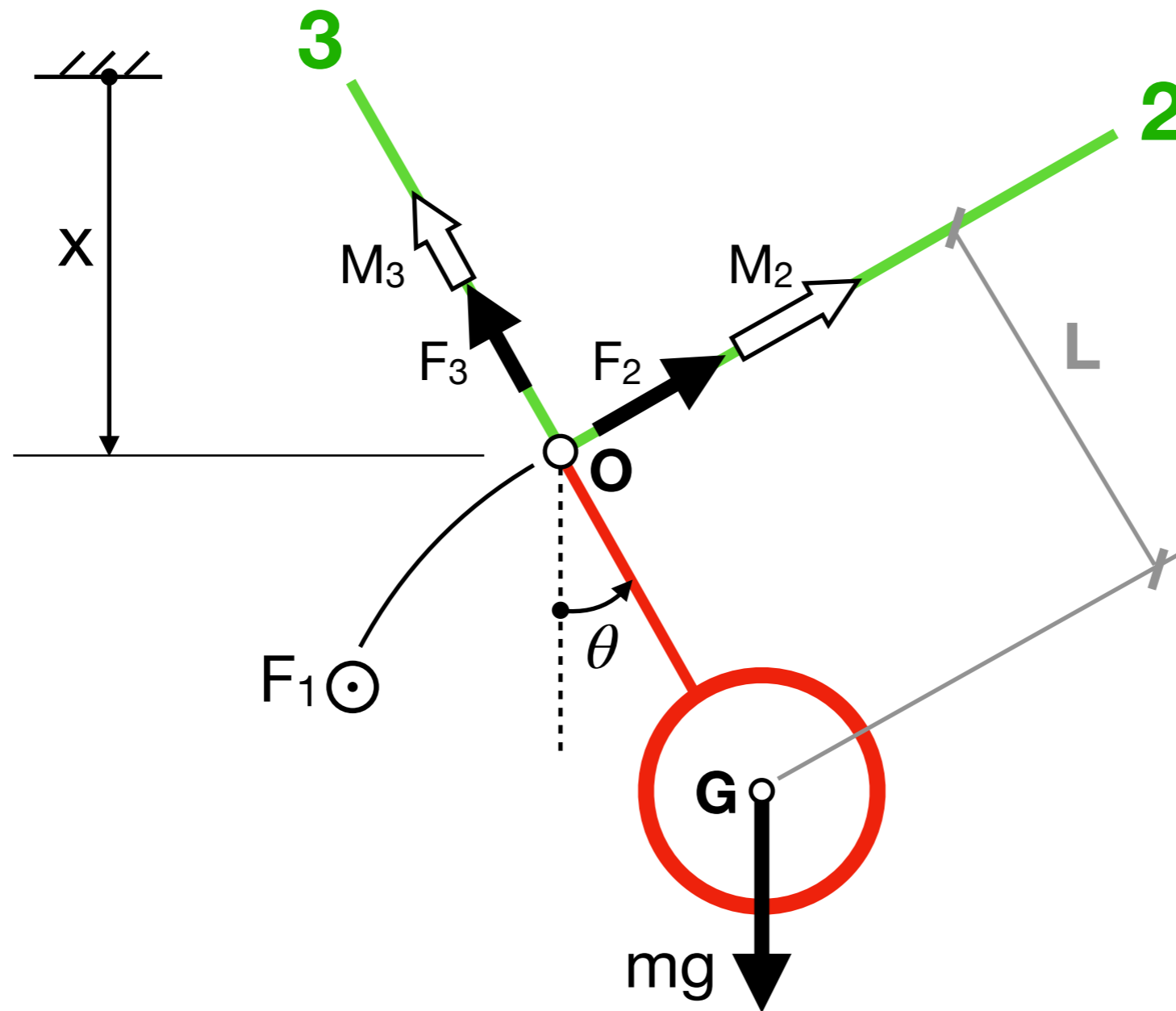


$$\left\{ \bar{\mathbf{F}}_{\text{Guia} \rightarrow \text{Sup}} \right\}_{B'} = \begin{Bmatrix} F'_1 \\ F'_2 \\ 0 \end{Bmatrix}$$

$$\left\{ \bar{\mathbf{M}}_{\text{Guia} \rightarrow \text{Sup}} (\mathbf{O}) \right\}_{B'} = \begin{Bmatrix} M'_1 \\ M'_2 \\ M'_3 \end{Bmatrix}$$

$\text{TQM}]_{3'}$   
lliure de ie

# TMC(O) ]<sub>1</sub> sobre SIST = Pèndol



# TMC(O) ]<sub>1</sub> sobre SIST = Pèndol

$$\left\{ \dot{\ddot{\mathbf{H}}}_{\text{RTO}}(\mathbf{O}) \right\}_{\text{B}} = \left\{ \begin{array}{l} I_{11} \ddot{\theta} + (I_{33} - I_{22}) \dot{\psi}_0^2 \sin \theta \cos \theta \\ (I_{11} + I_{22} - I_{33}) \dot{\psi}_0 \dot{\theta} \cos \theta \\ (I_{22} - I_{11} - I_{33}) \dot{\psi}_0 \dot{\theta} \sin \theta \end{array} \right\} \quad \text{(III)}$$

# Pas final

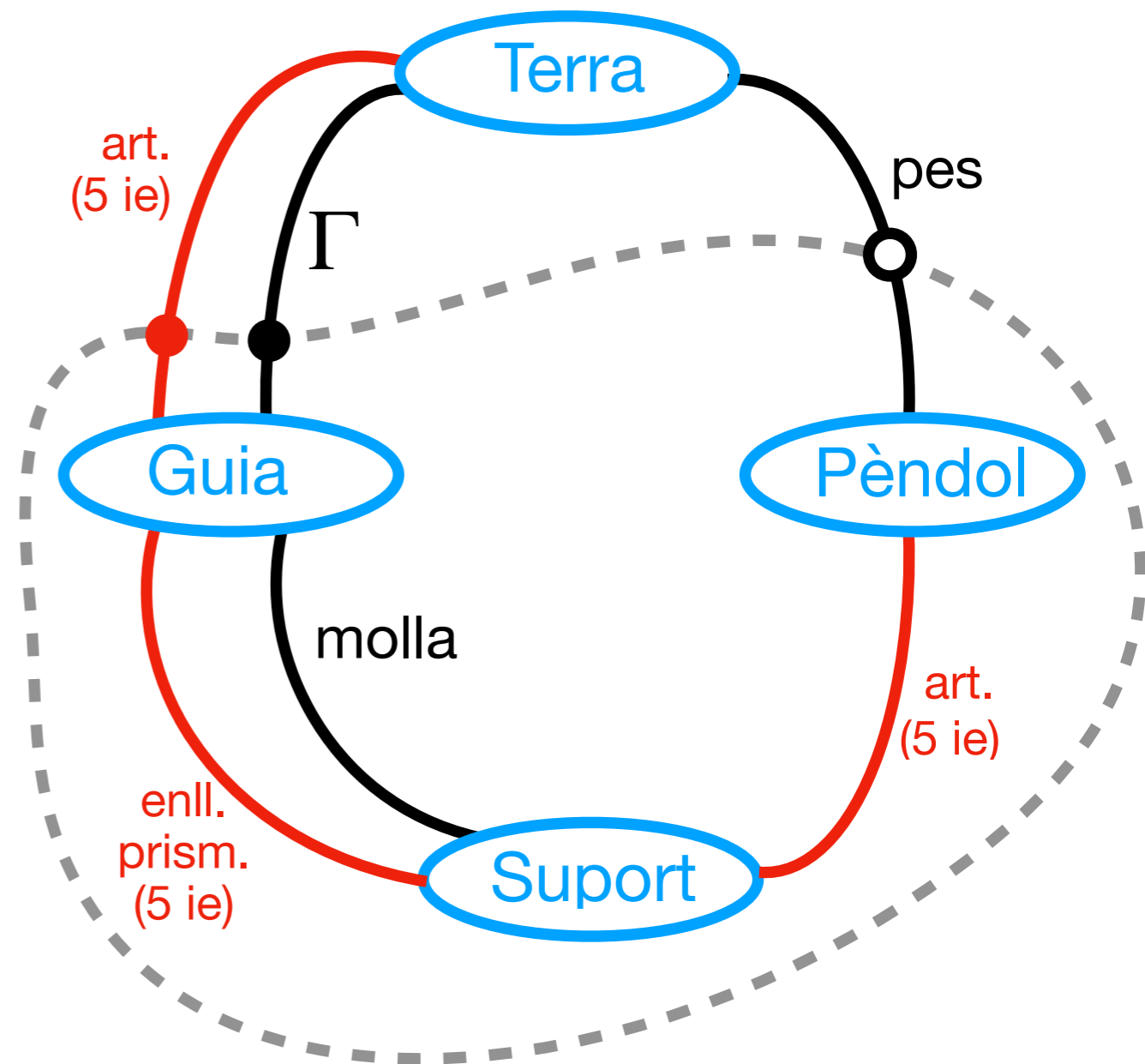
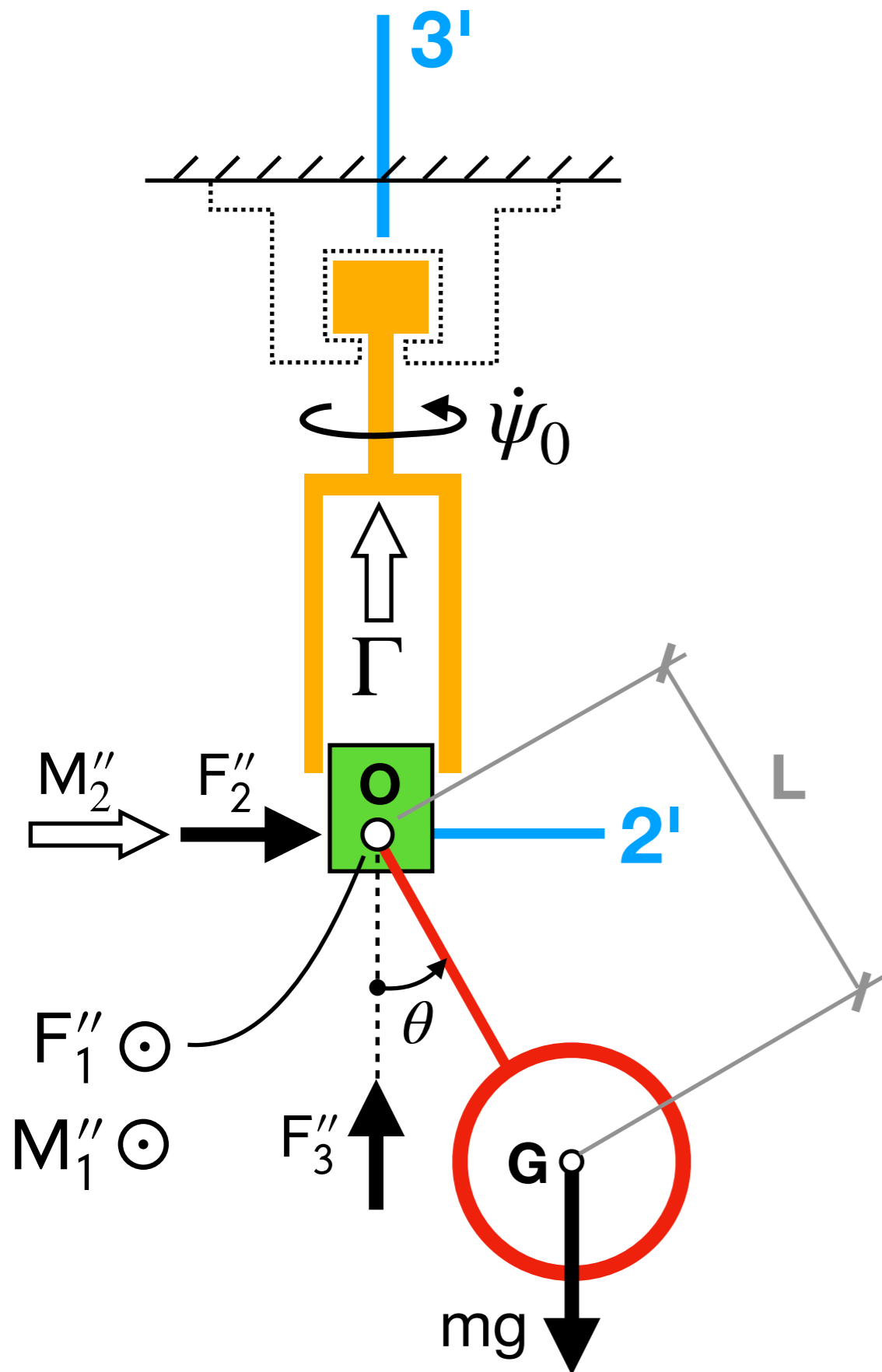
$$\begin{cases} (R^2 + L^2) \ddot{\theta} - (L \sin \theta) \ddot{x} = (L \dot{\psi}_0^2 \cos \theta - g) L \sin \theta \\ - (L \sin \theta) \ddot{\theta} + \ddot{x} = -\frac{k}{m} x + L \dot{\theta}^2 \cos \theta \end{cases}$$

Aillant  $\ddot{\theta}$  i  $\ddot{x}$

$$\begin{cases} \ddot{\theta} = F_1(\theta, \dot{\theta}, x, \dot{x}) & \text{Eq. mov. } \theta \\ \ddot{x} = F_2(\theta, \dot{\theta}, x, \dot{x}) & \text{Eq. mov. } x \end{cases}$$

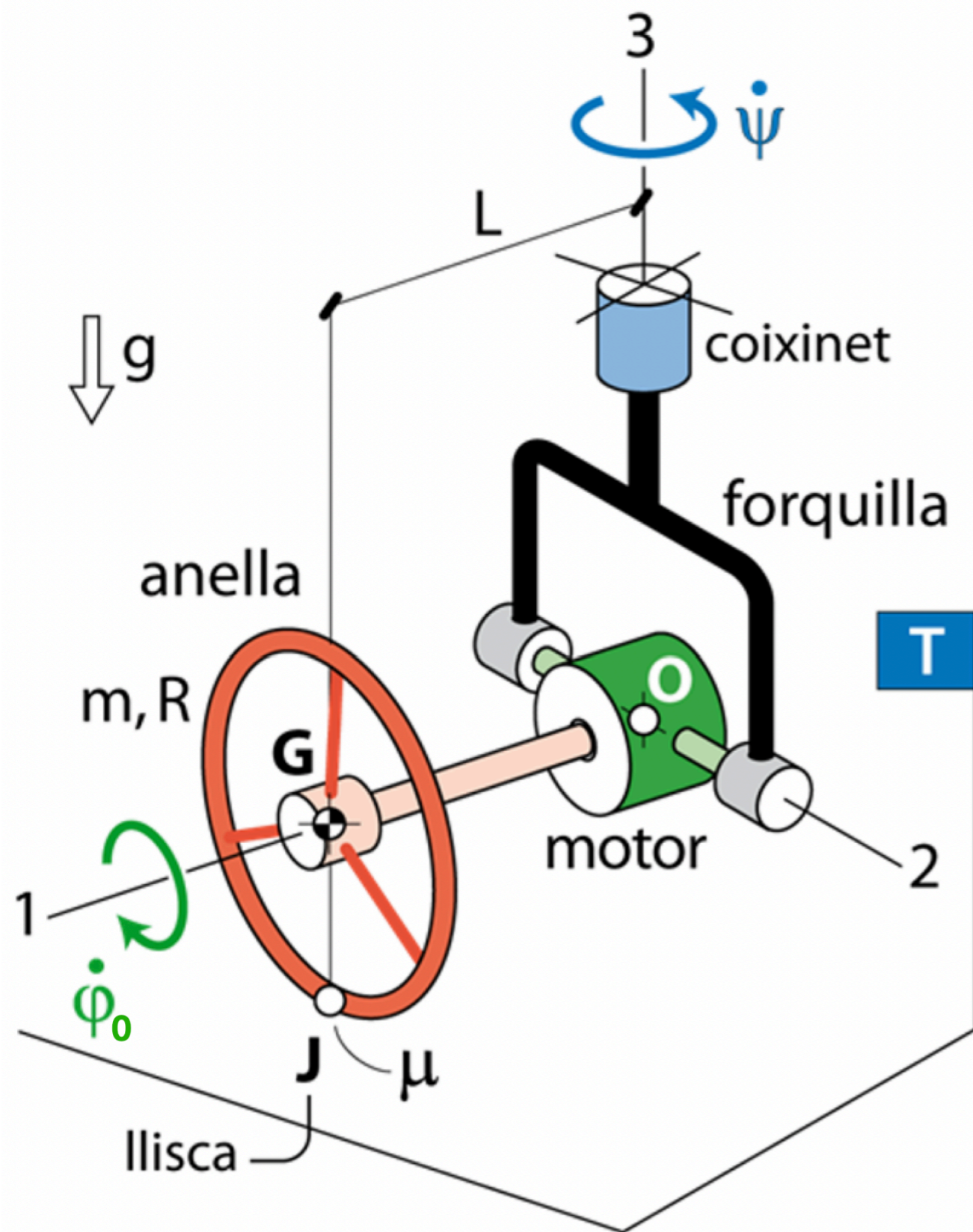
No cal  
que  
ho feu

# Parell motor per mantenir $\dot{\psi}_0 = ct$



# D'abans

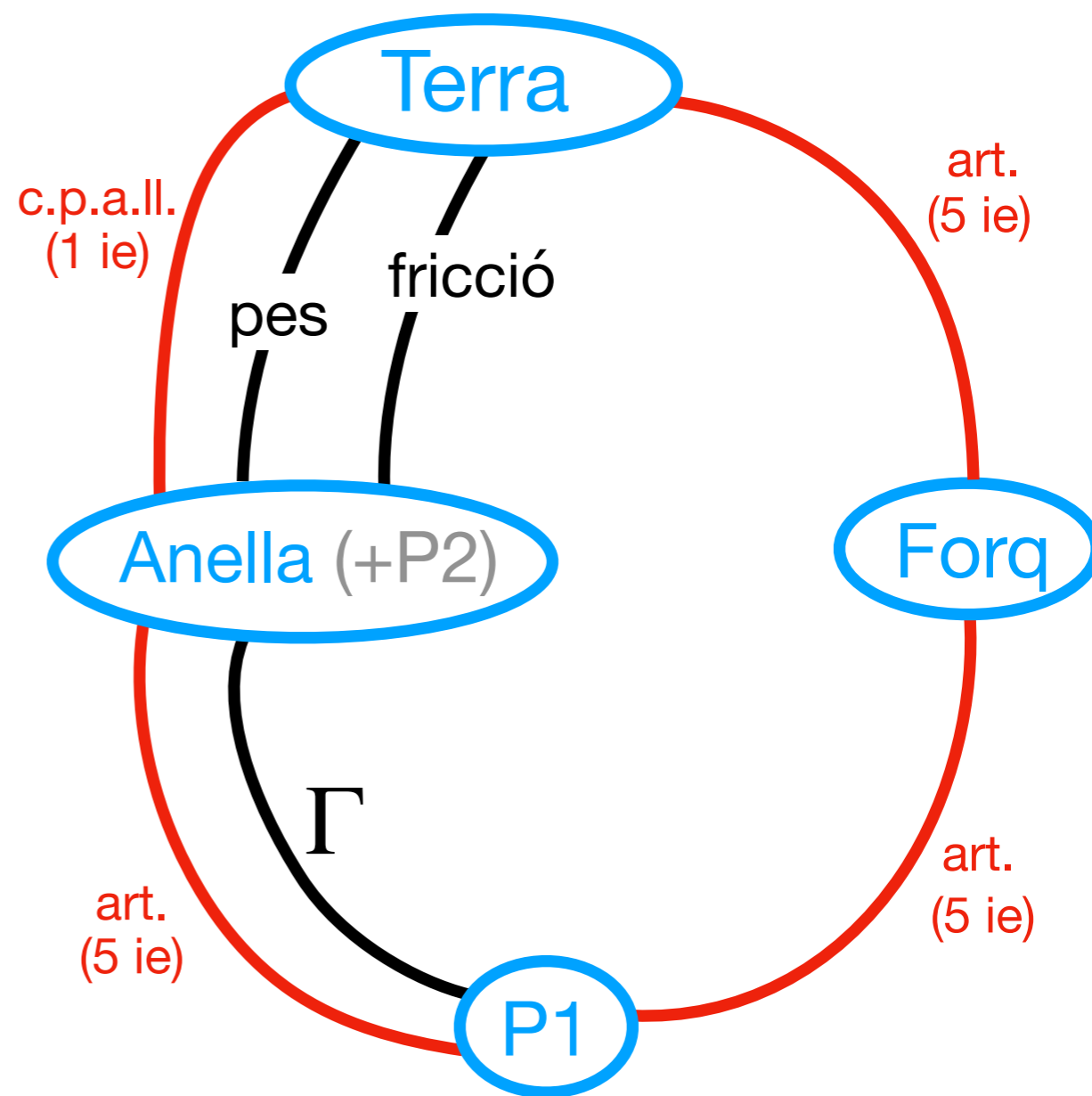
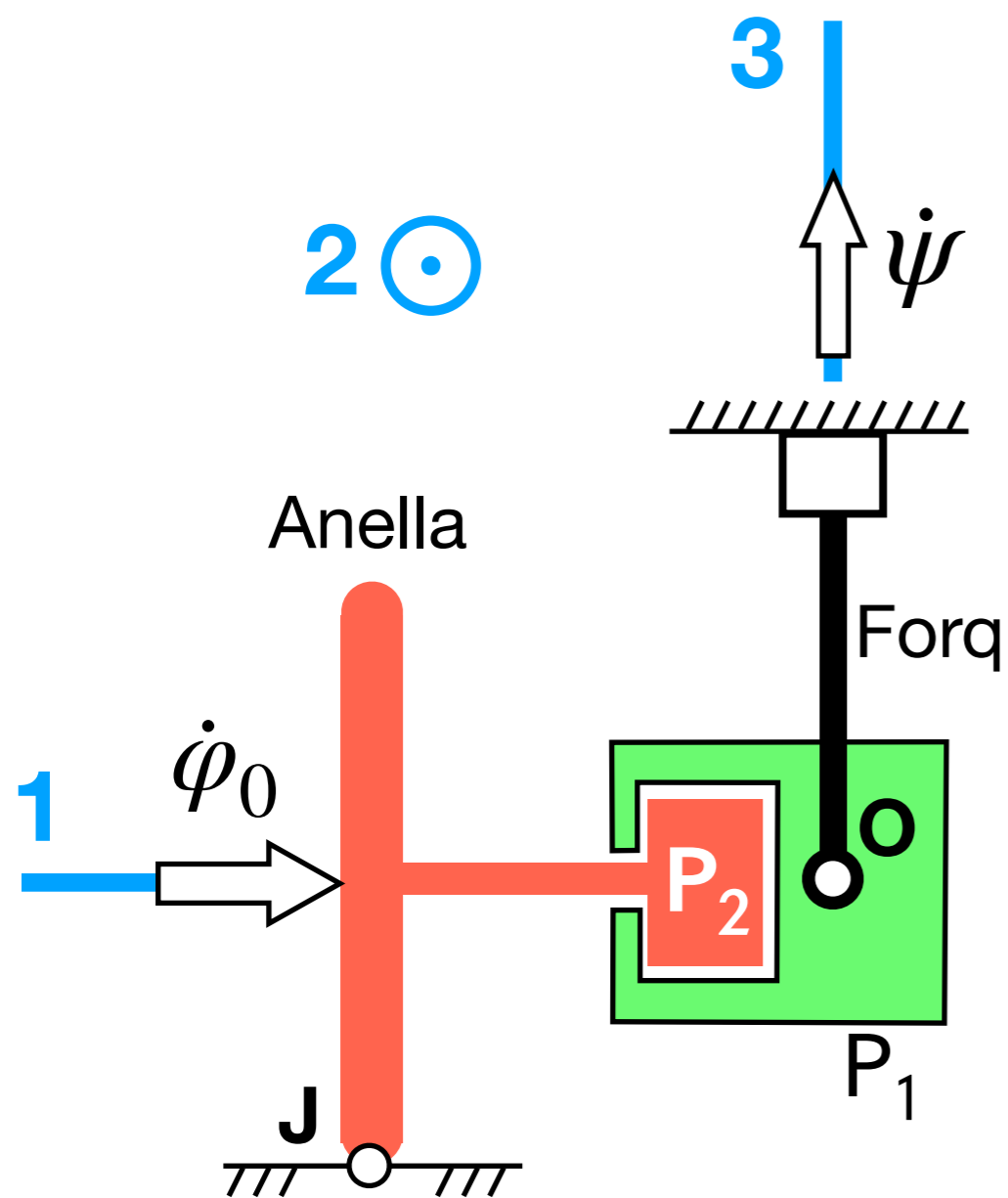
$$\left\{ \dot{\ddot{\mathbf{H}}}_{\text{RTO}}(\mathbf{O}) \right\}_{\text{B}} = \begin{Bmatrix} I_{11}\ddot{\theta} + (I_{33} - I_{22})\dot{\psi}_0^2 \sin\theta \cos\theta \\ (I_{11} + I_{22} - I_{33})\dot{\psi}_0 \dot{\theta} \cos\theta \\ (I_{22} - I_{11} - I_{33})\dot{\psi}_0 \dot{\theta} \sin\theta \end{Bmatrix} = \begin{Bmatrix} \dot{\mathbf{H}}_1 \\ \dot{\mathbf{H}}_2 \\ \dot{\mathbf{H}}_3 \end{Bmatrix}$$



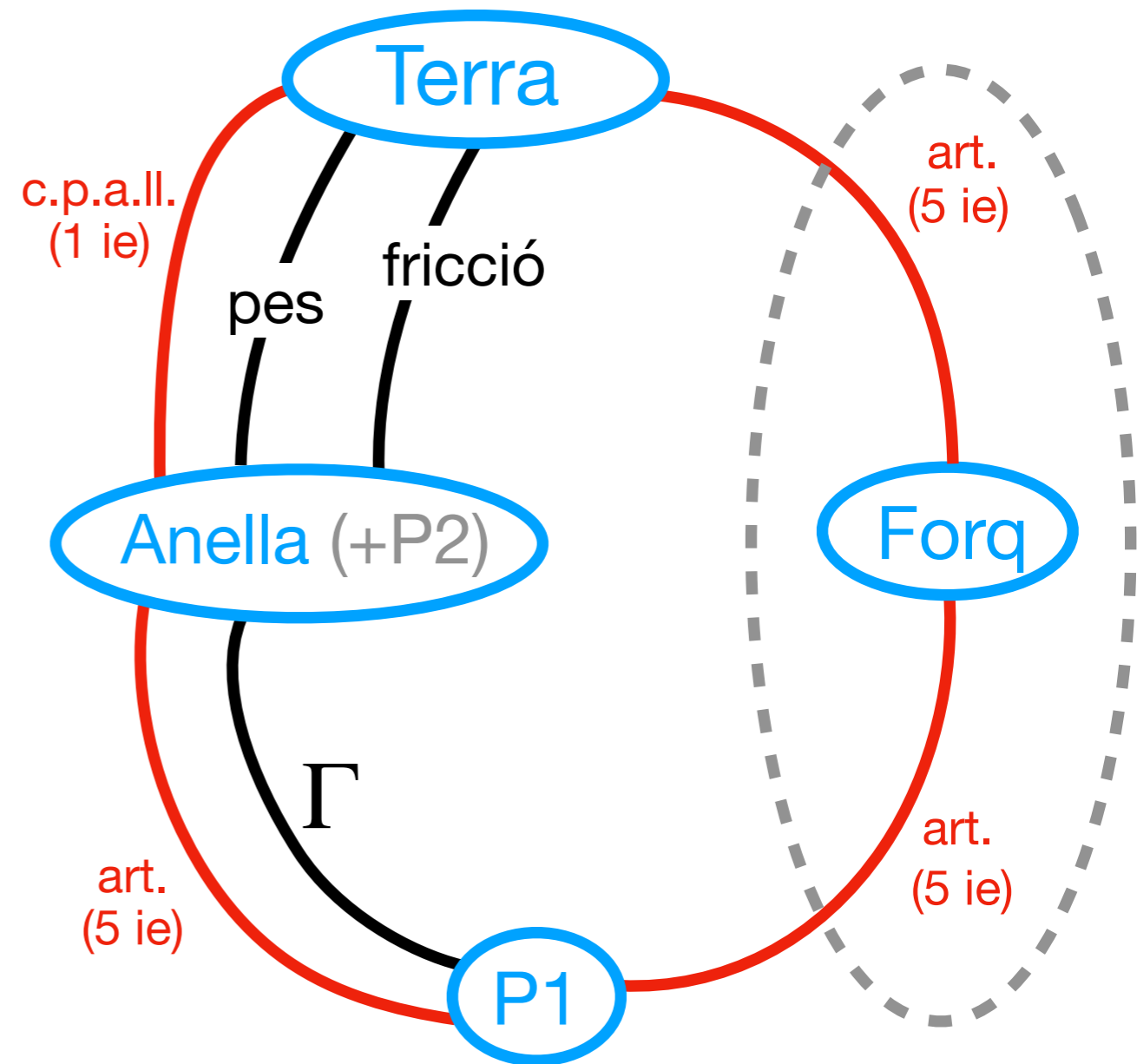
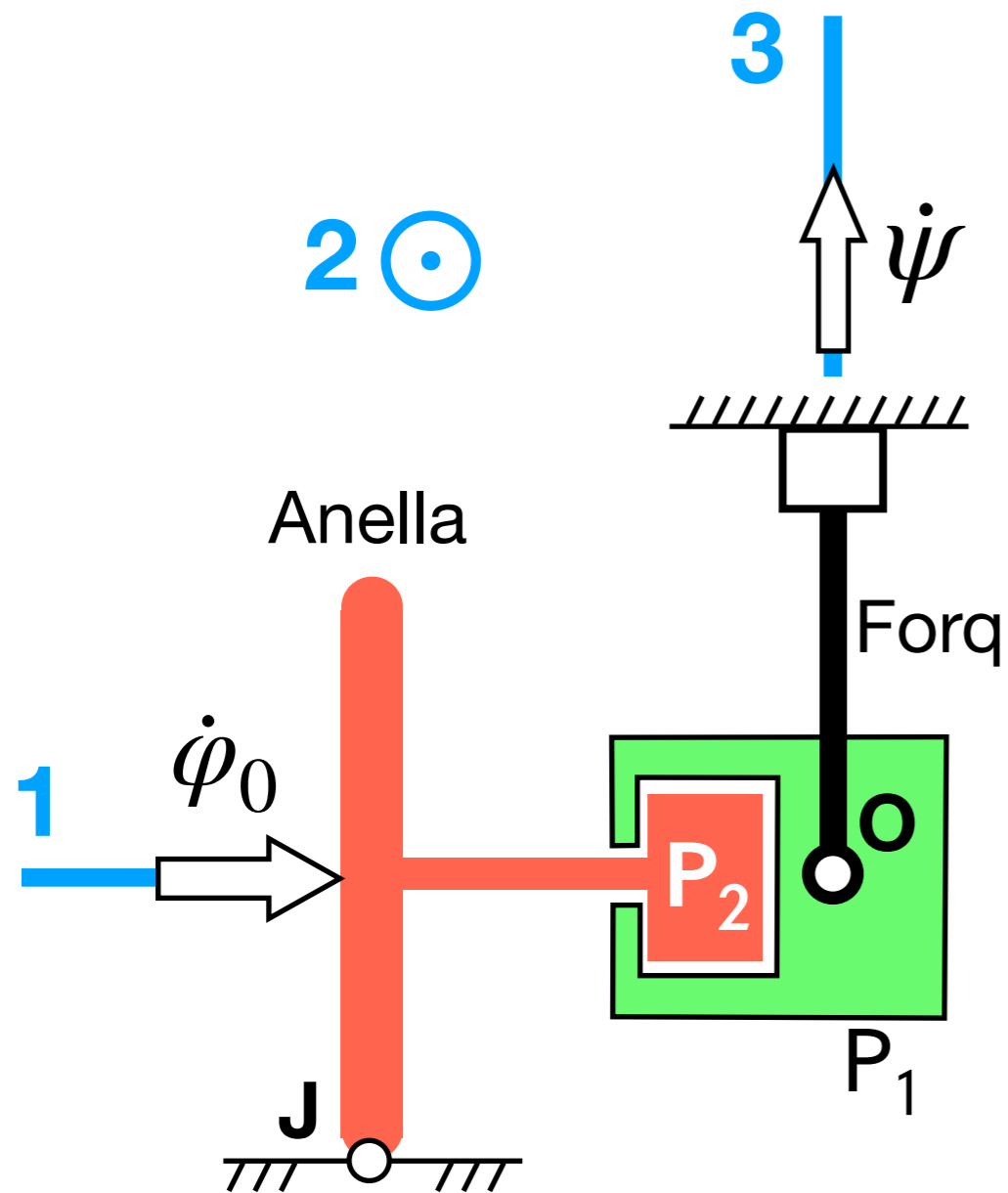
- GL sistema?
- DGI
- Caracterització de torsors?
- Eq. mov per a la coord.  $\psi$
- Força normal a  $J$
- Parell motor per mantenir  $\dot{\phi}_0$

Manté contacte puntual a  $J$

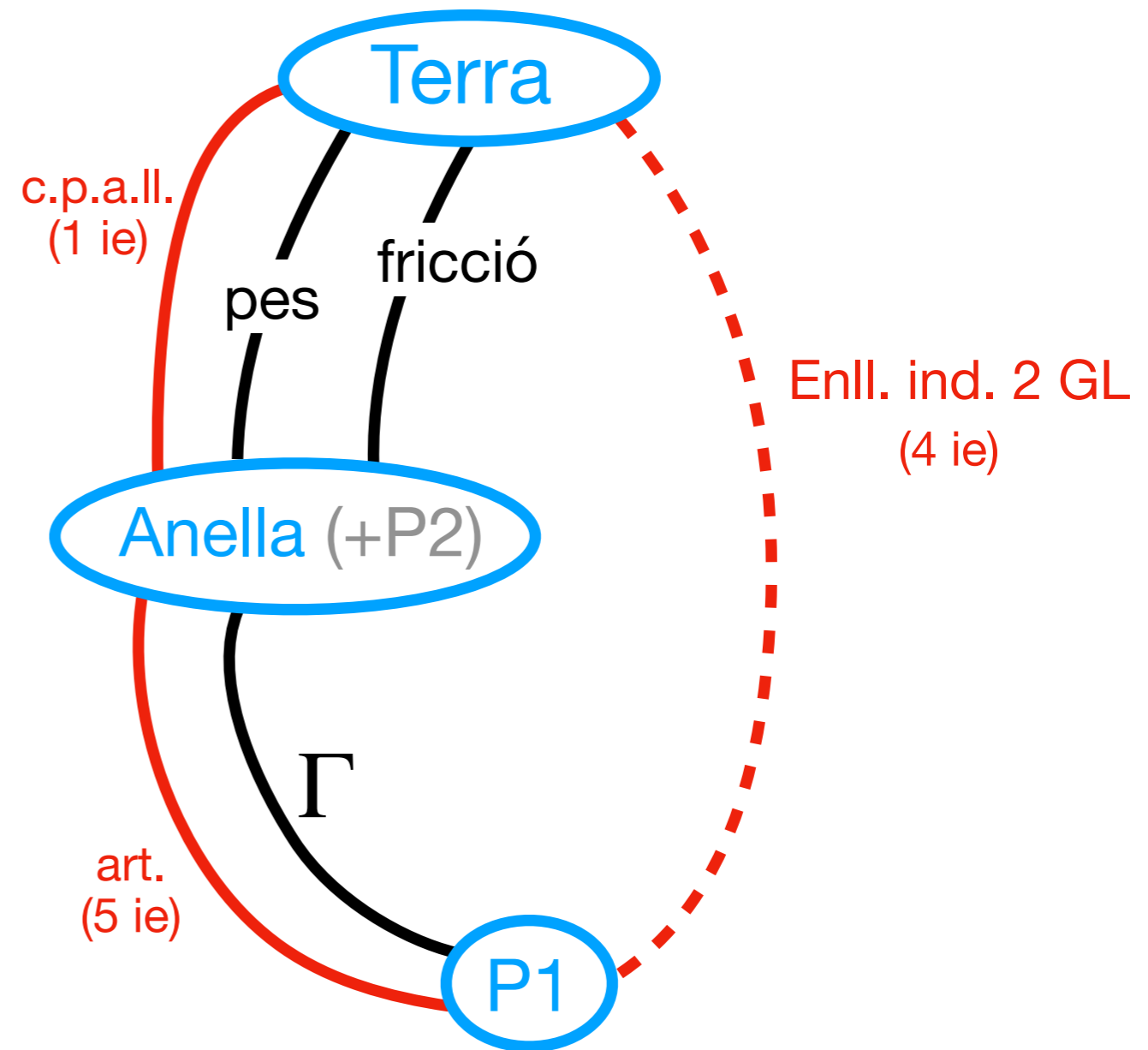
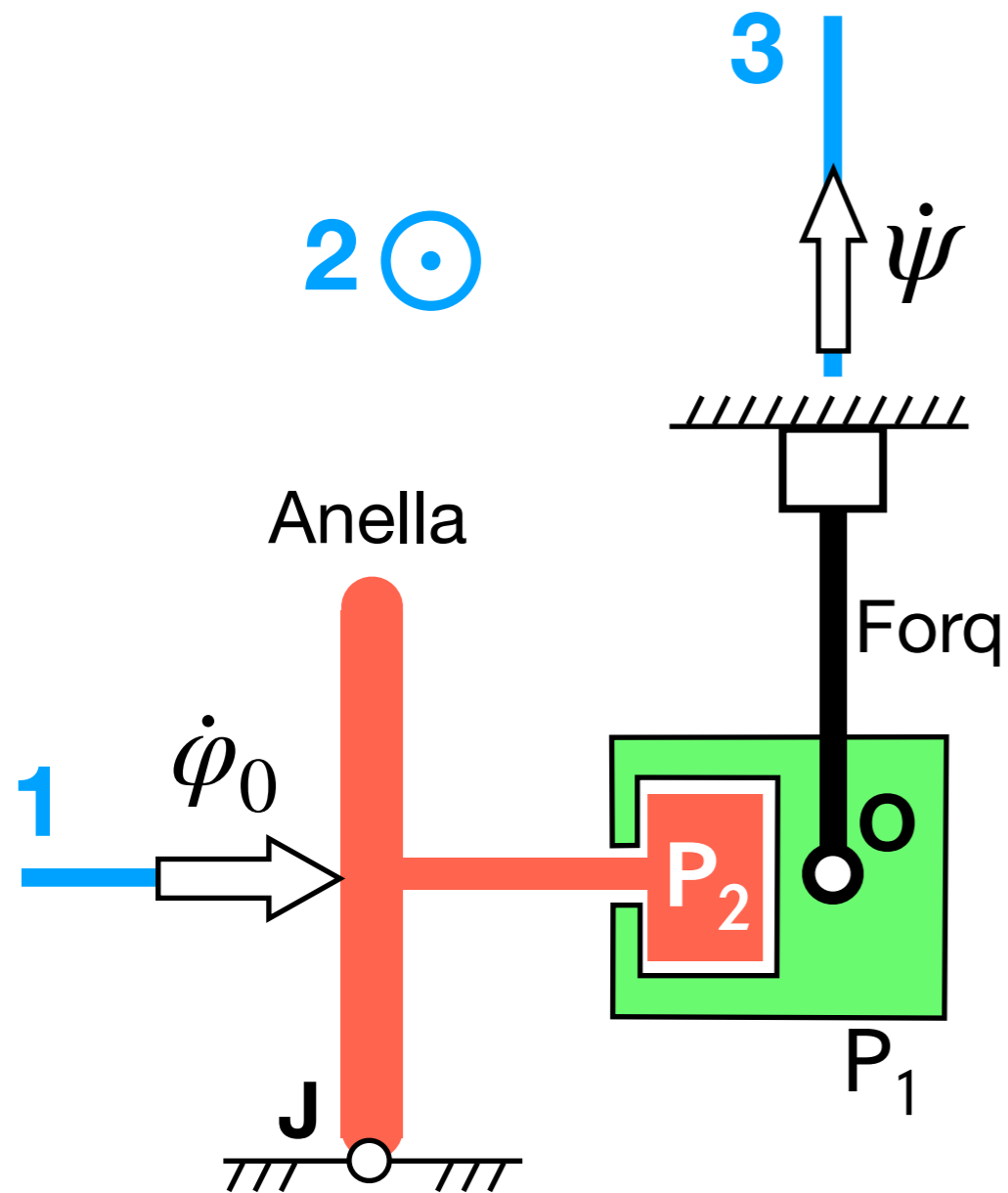
Motor manté  $\dot{\phi}_0 = \text{ct}$



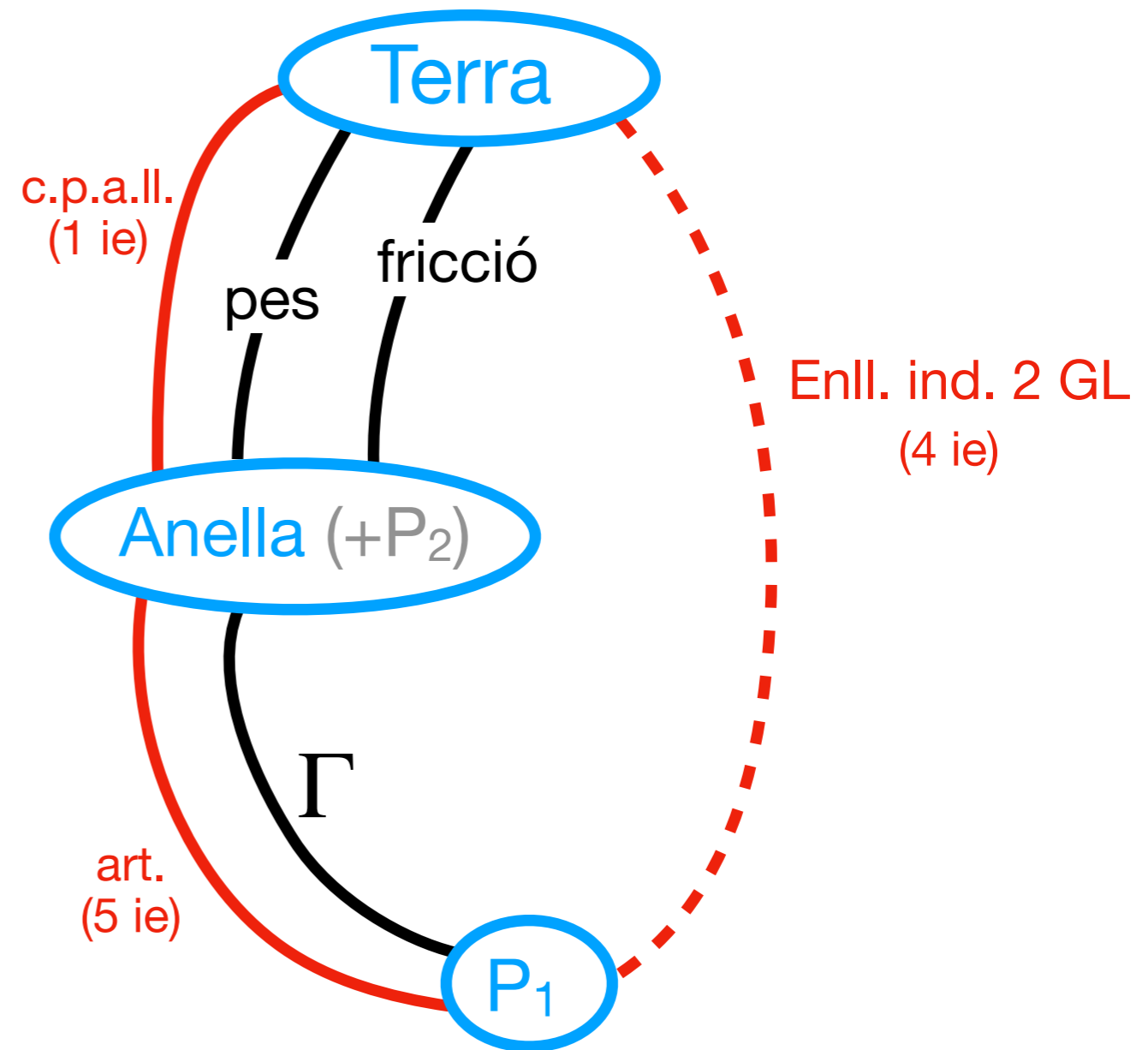
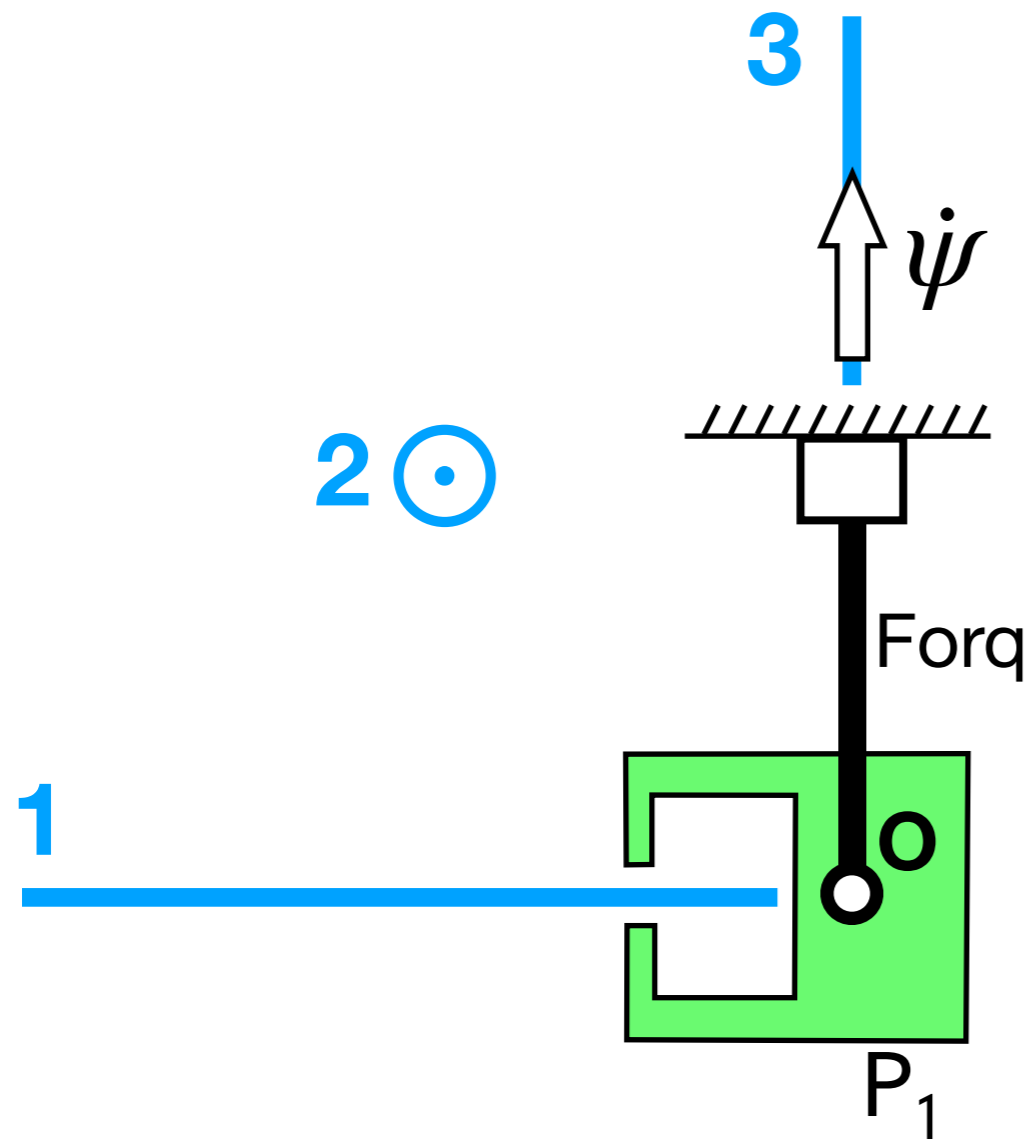
Forq és SAE  $\Rightarrow$  Substituim-la per l'enllaç indirecte  $T \rightarrow P_1$



Forq és SAE  $\Rightarrow$  Substituim-la per l'enllaç indirecte  $T \rightarrow P_1$



Forq és SAE  $\Rightarrow$  Substituim-la per l'enllaç indirecte  $T \rightarrow P_1$



Torsor d'enllaç indirecte

$$\left\{ \bar{\mathbf{F}}_{T \rightarrow (\text{forq}) \rightarrow \text{Sup}} \right\}_B = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \quad \text{O és fix a T}$$

$$\left\{ \bar{\mathbf{M}}_{T \rightarrow (\text{forq}) \rightarrow \text{Sup}} (\mathbf{O}) \right\}_{B'} = \begin{Bmatrix} M_1 \\ 0 \\ 0 \end{Bmatrix} \quad \text{Zeros en les rotacions permeses de } P_1 \text{ respecte de T}$$

# Full ruta per eq. del mov. $\psi$

$\psi$  afecta la cinemàtica de P1 i la de l'anella



Sistema ha d'incloure P1 o Anella o ambdós



Sistema	Incògn.	Problema
Anella	6 ie, $\Gamma$ , $\ddot{\psi}$	<b>INDET</b>
P1	9 ie, $\Gamma$ , $\ddot{\psi}$	<b>INDET</b>
Anella + P1	5 ie, $\ddot{\psi}$	<b>DET</b>



Triem SIST = Anella + P1

