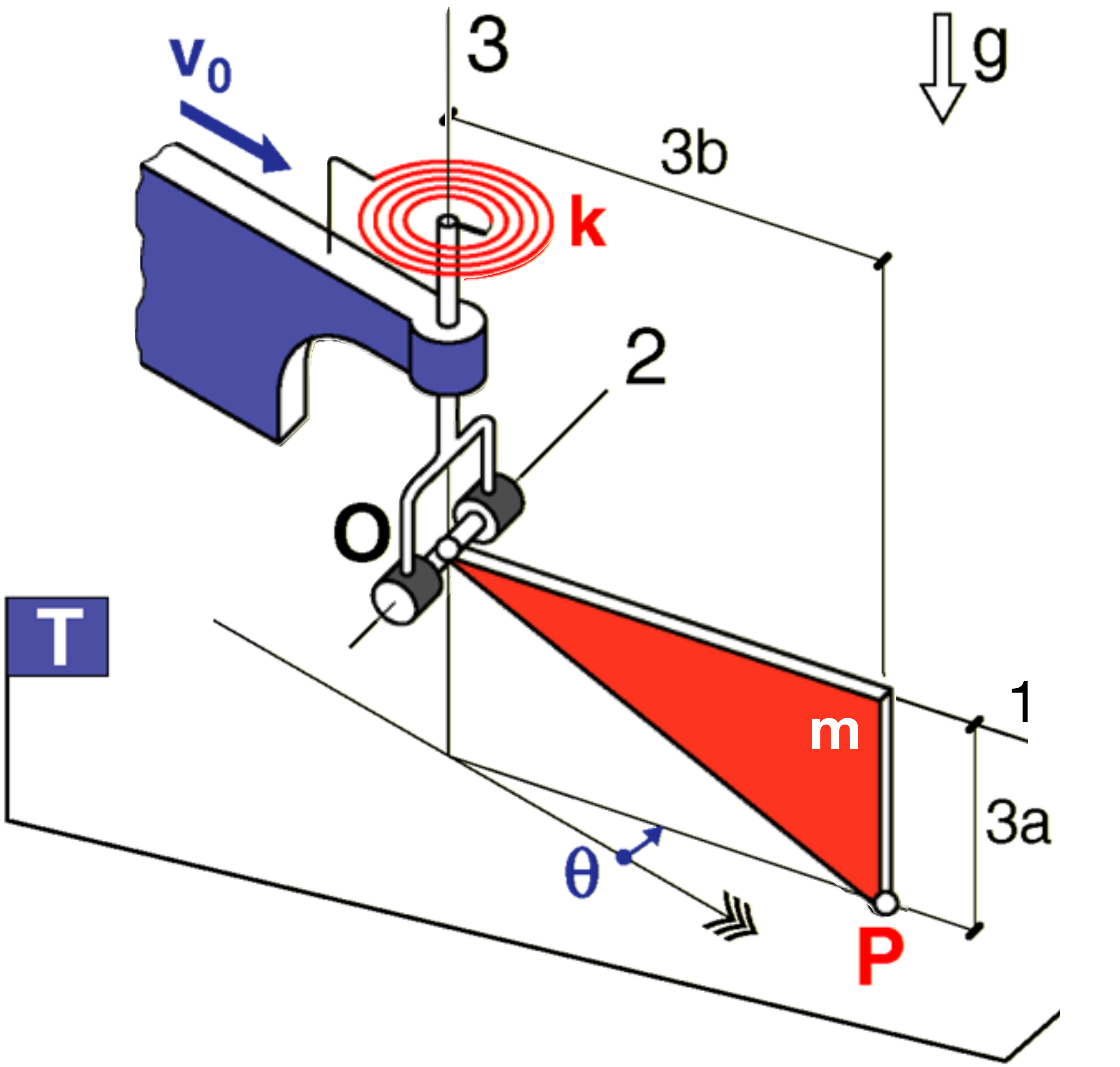
12P

Teoremes vectorials II

Exemples 3D

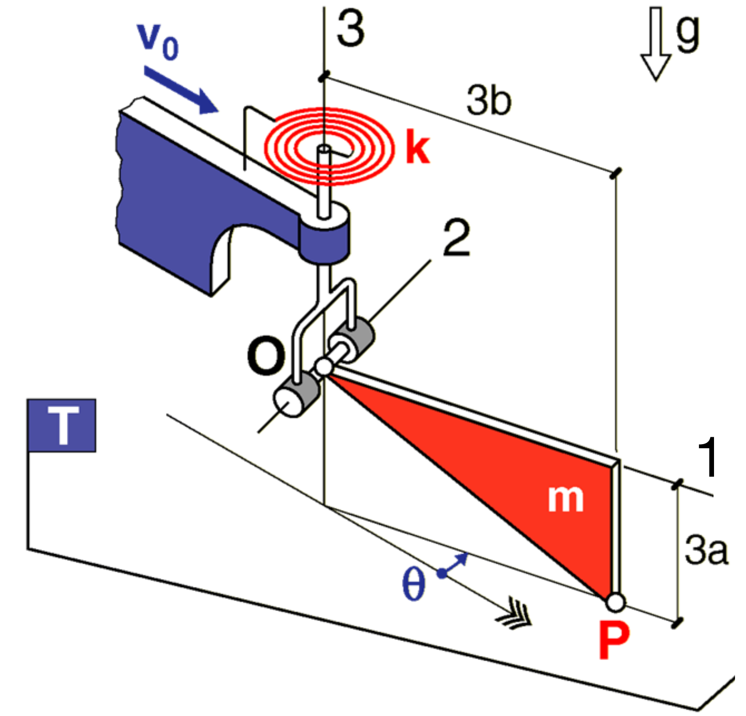


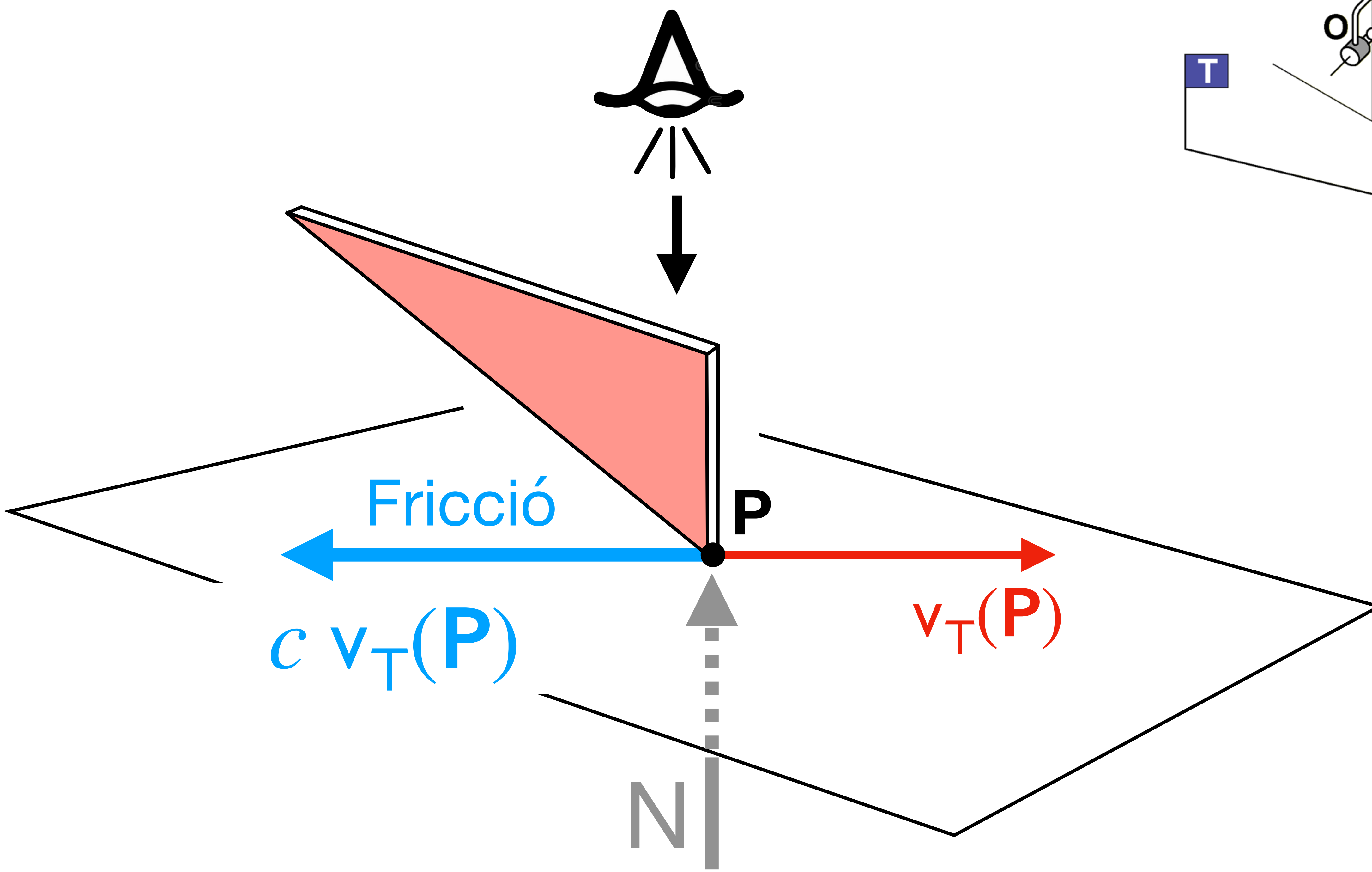
- Eq. mov. per a θ ?
- k_{min} per a que $\theta_{eq} = 0$ sigui **ESTABLE** ?

$\exists$ frec viscós T → **P**
(de coef c)

Per $\theta = 0$ la molla
està distesa

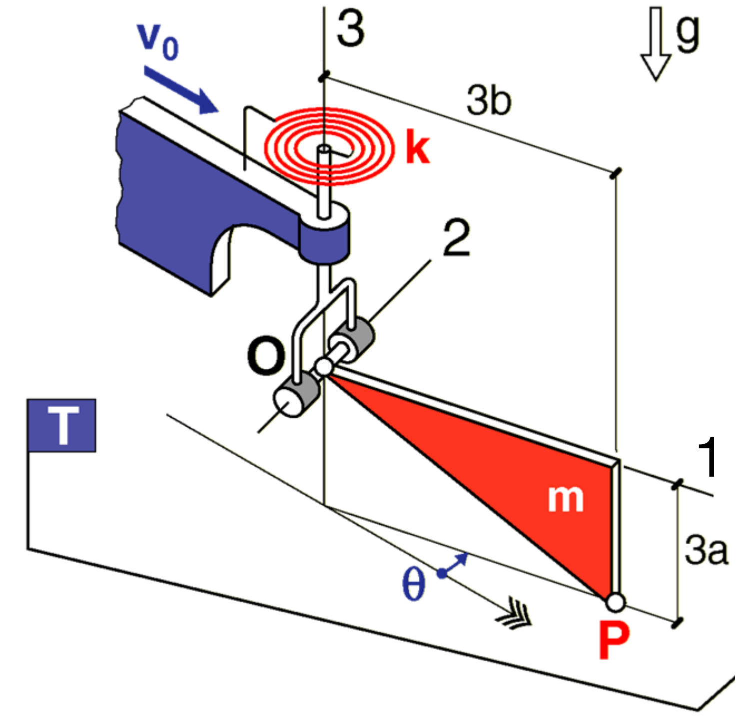
Força de frec viscós T → **P**

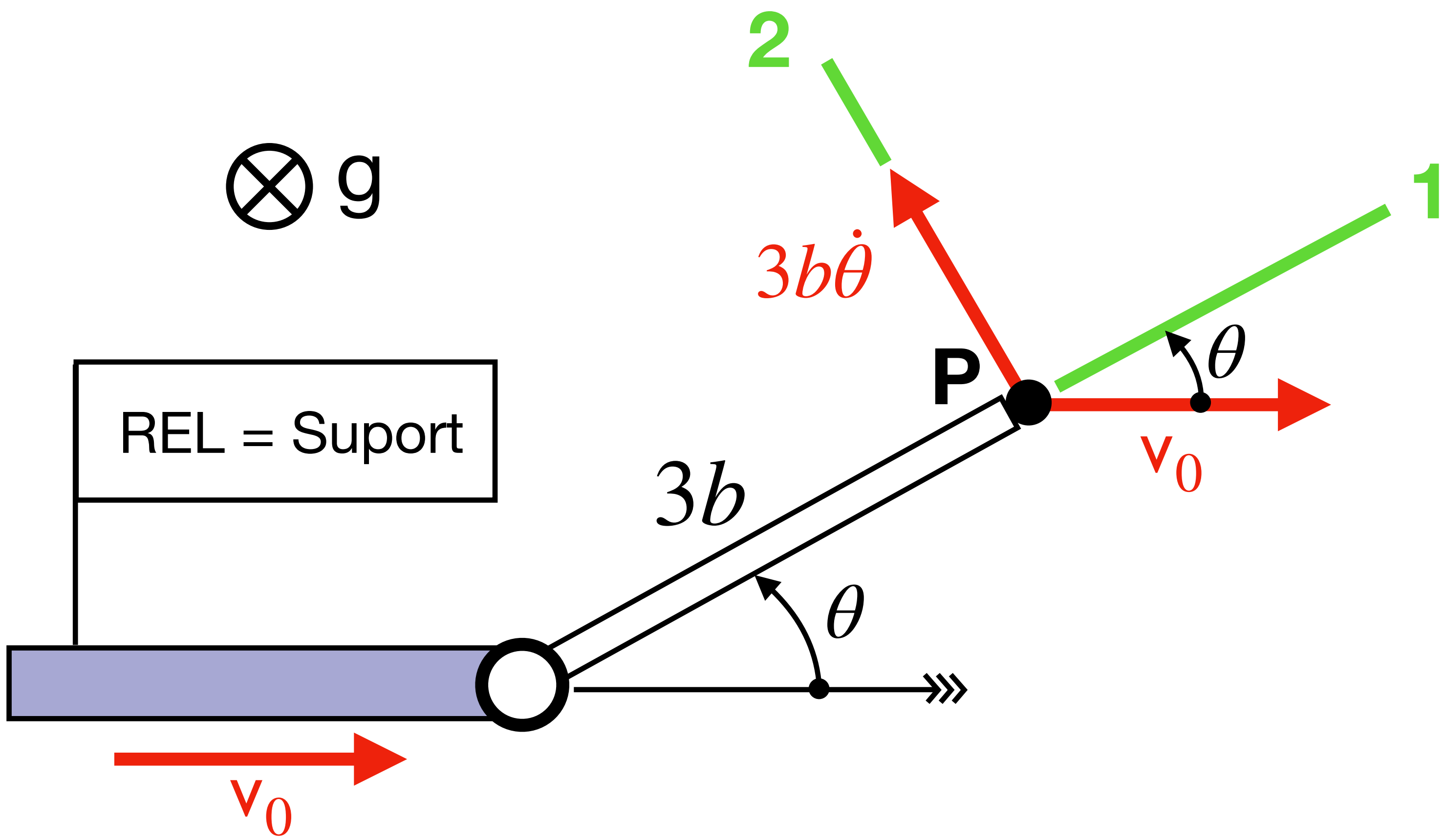




$$\boxed{\overline{\mathbf{F}}_{fv} = -\,c\,\overline{v}_T(\mathbf{P})}$$

Força de frec viscós T → **P**

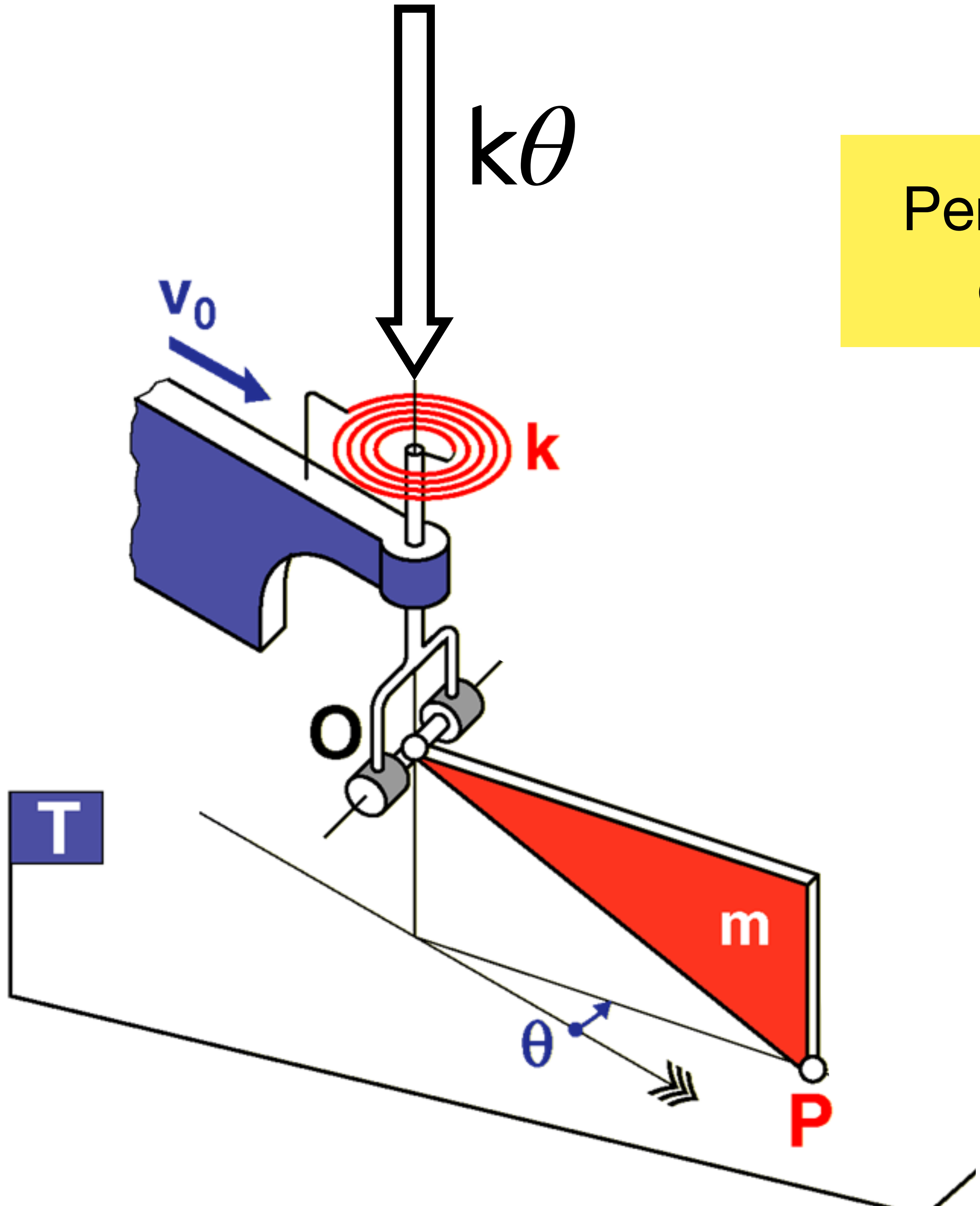




$$\bar{v}_T(\mathbf{P}) = \bar{v}_{REL}(\mathbf{P}) + \bar{v}_{ar}(\mathbf{P}) = \begin{Bmatrix} v_0 \cos\theta \\ -v_0 \sin\theta + 3b\dot{\theta} \\ 0 \end{Bmatrix}_{B=(1,2,3)}$$

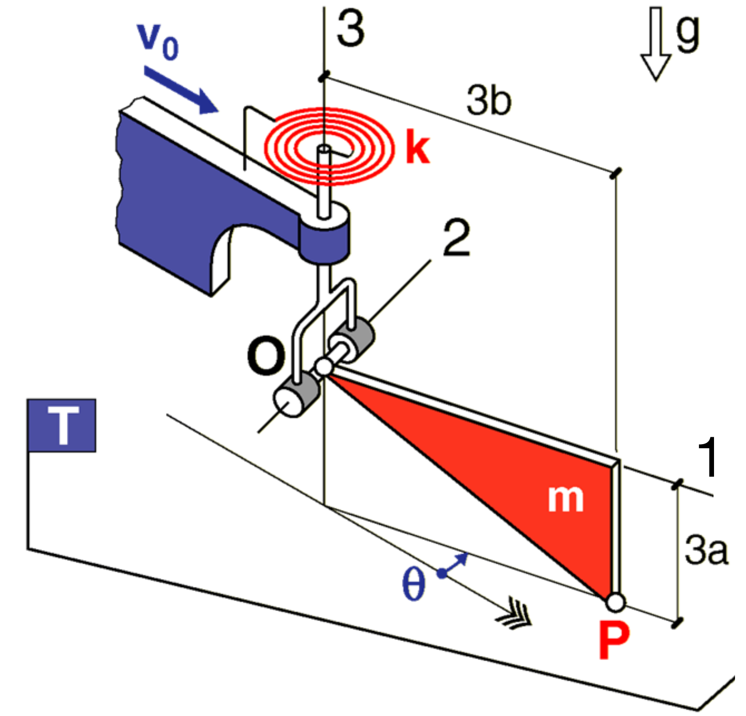
$$\bar{F}_{fv} = -\,c\;\bar{v}_T(\mathbf{P}) = \begin{Bmatrix} \underbrace{\boxed{-cv_0 \cos\theta}}_{F_{fv1}} \\ \underbrace{\boxed{cv_0 \sin\theta - 3cb\dot{\theta}}}_{F_{fv2}} \\ 0 \end{Bmatrix}_B$$

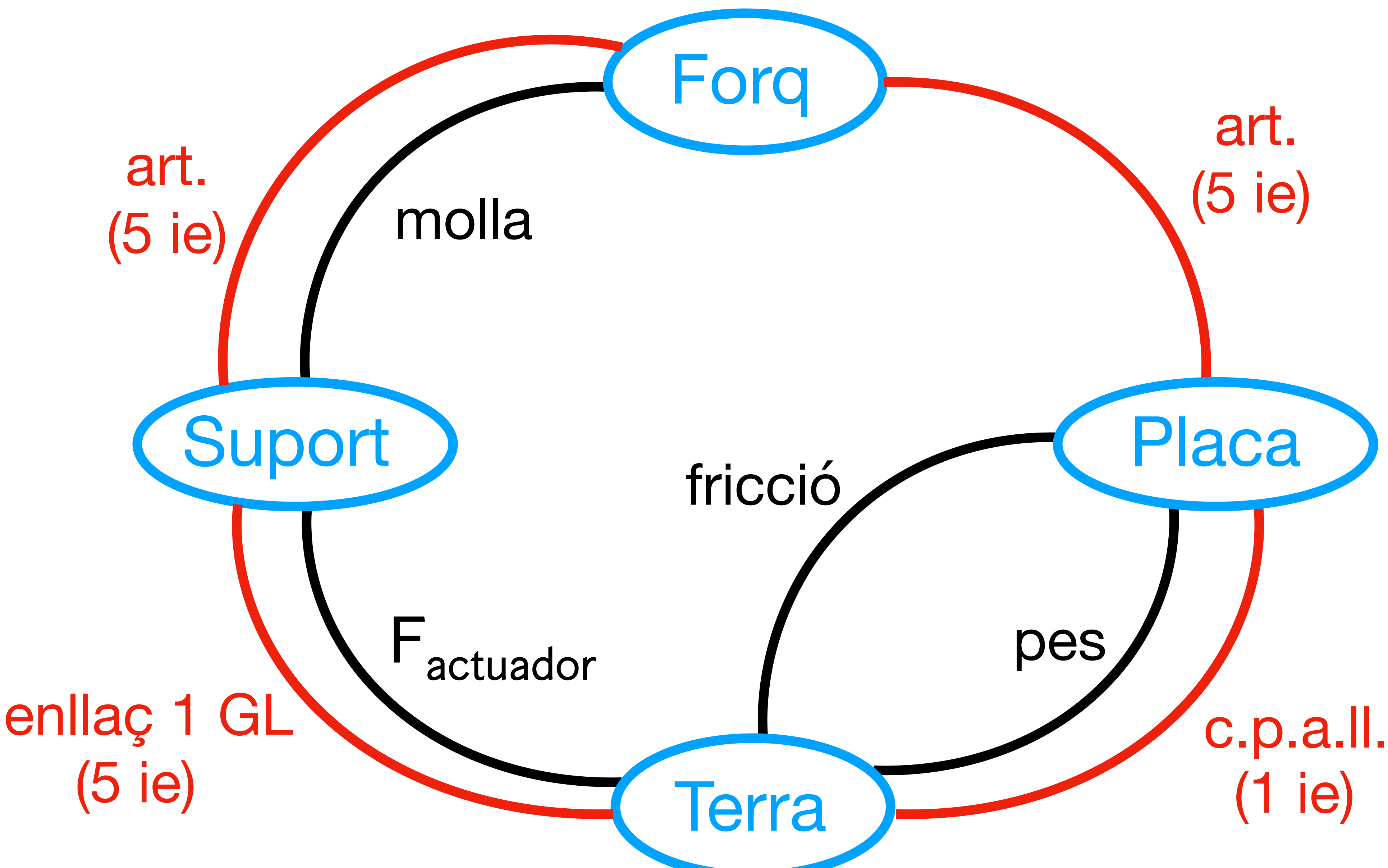
Parell molla torsional → forq



Per $\theta = 0$ la molla **està distesa**

DGI = Diagrama general d'interaccions

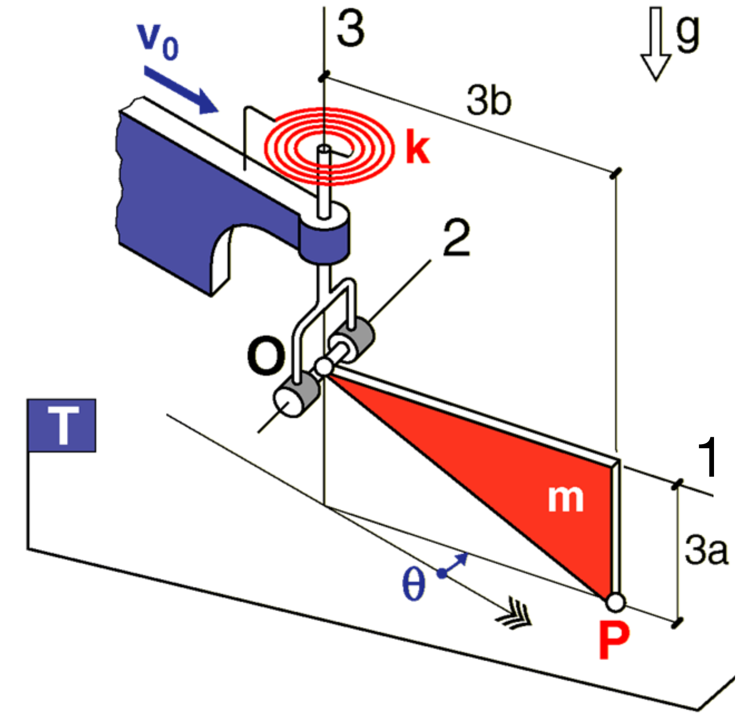


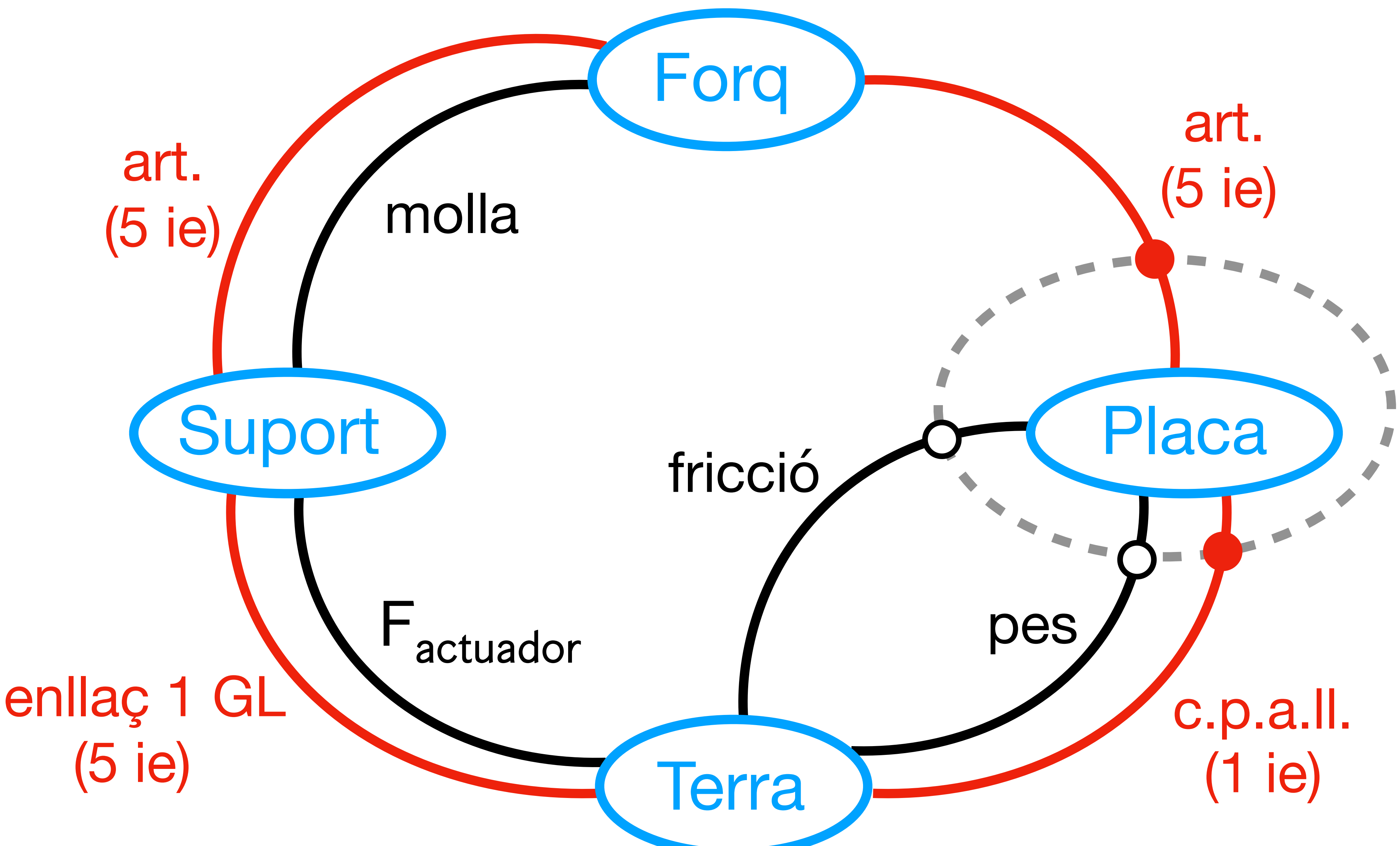


INDET

Sist = Placa

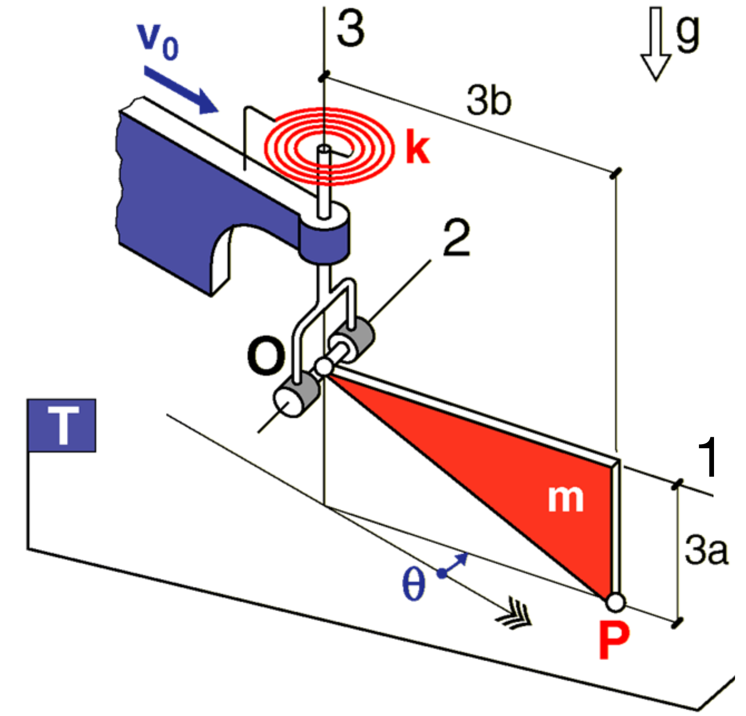
6 ie + $\ddot{\theta}$ = 7 incòg

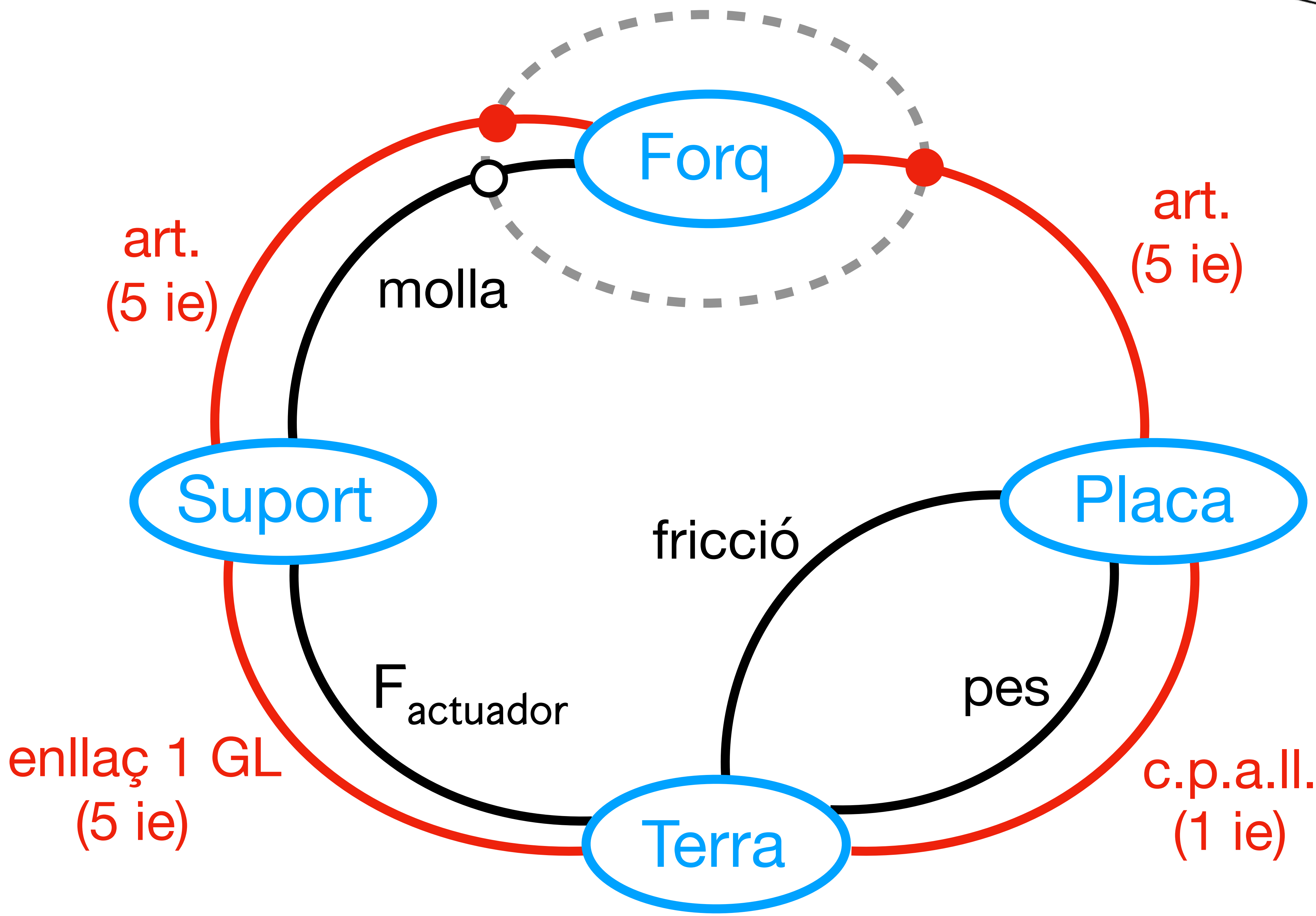




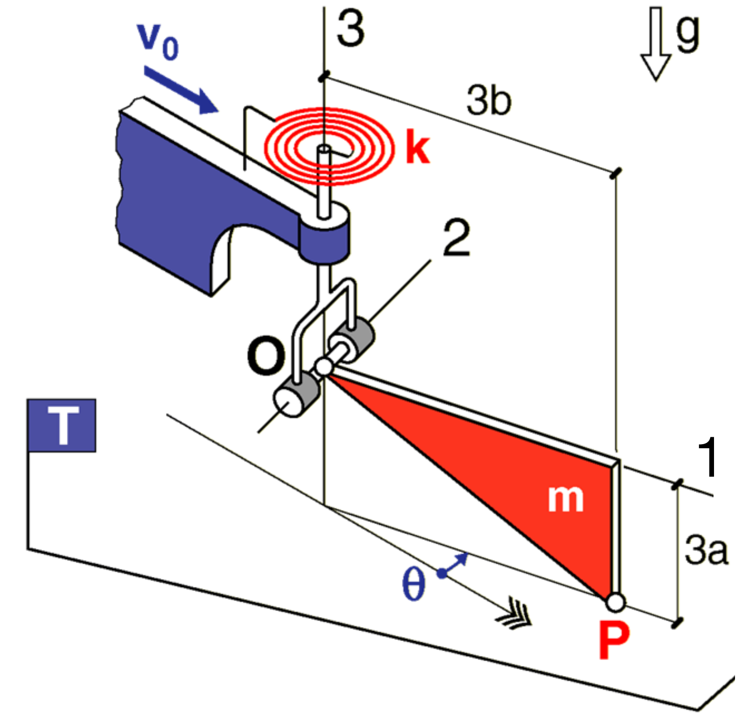
INDET

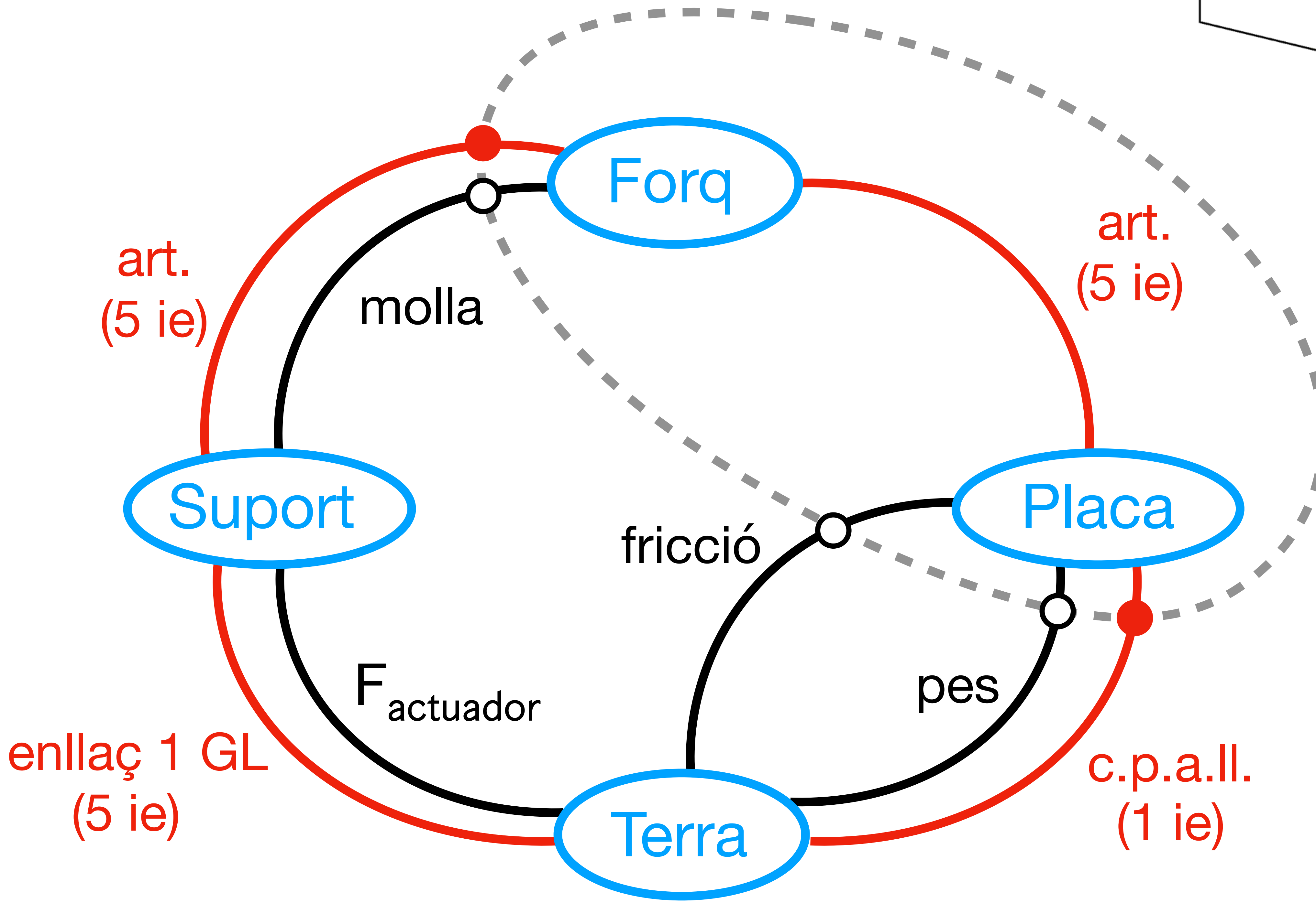
$$\left\{\begin{array}{l}\text{Sist = Forq}\\ 10\text{ ie} + \ddot{\theta} = \textbf{11 incòg}\end{array}\right.$$



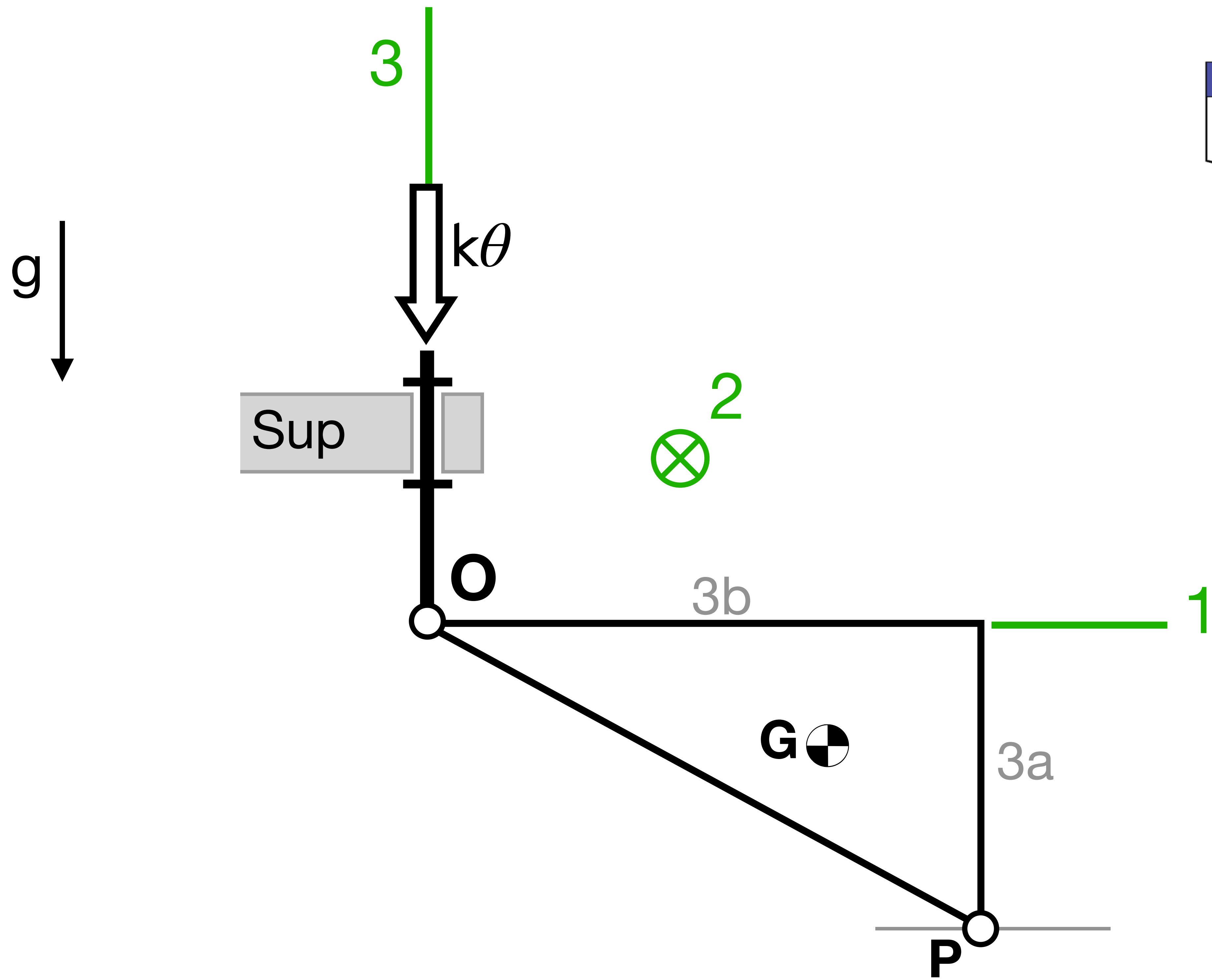


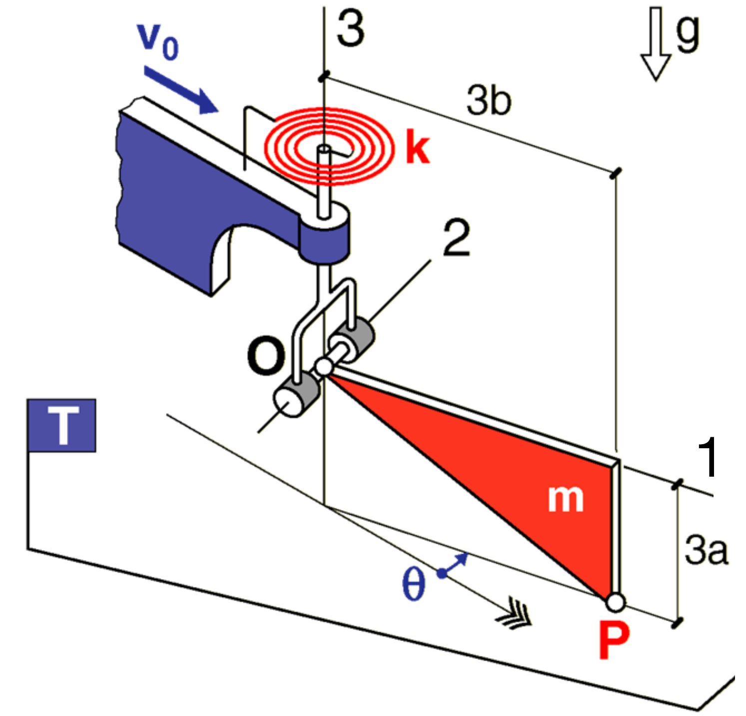
INDET $\left\{ \begin{array}{l} \text{Sist} = \text{Placa} + \text{Forq} \\ 6 \text{ ie} + \ddot{\theta} = \textbf{7 incòg} \end{array} \right.$





Forces sobre "Placa + Forq"

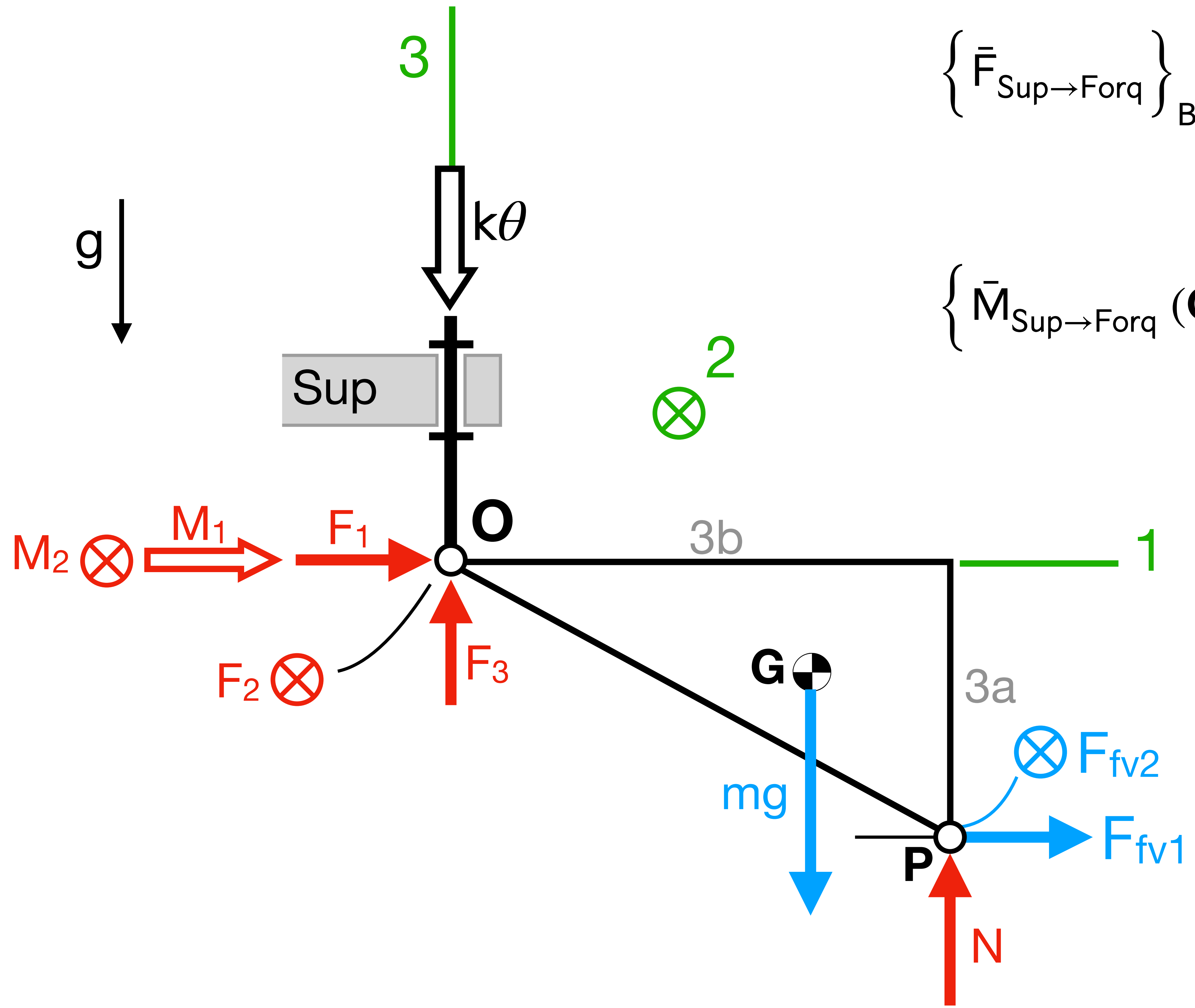




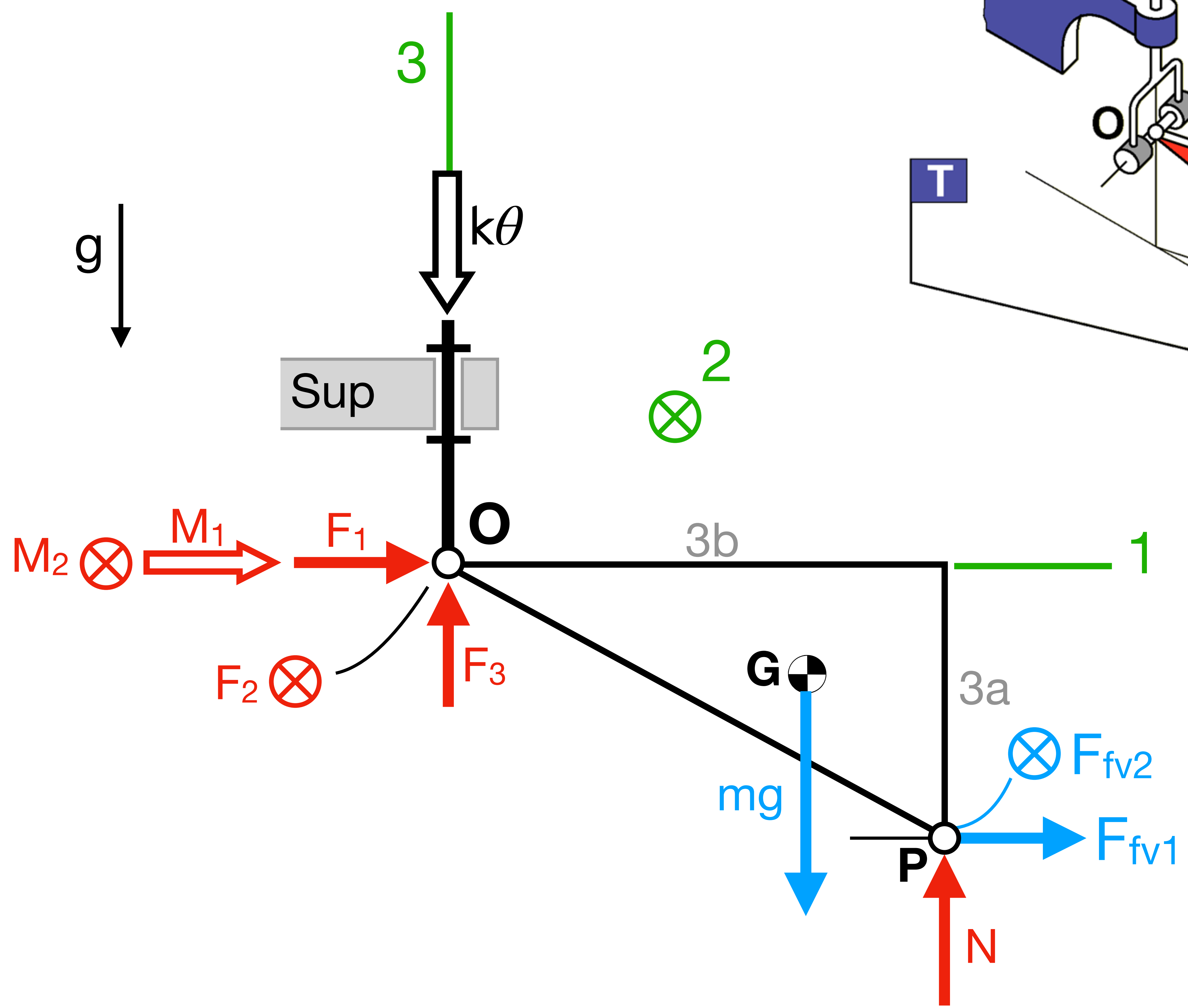
Forces sobre "Placa + Forq"

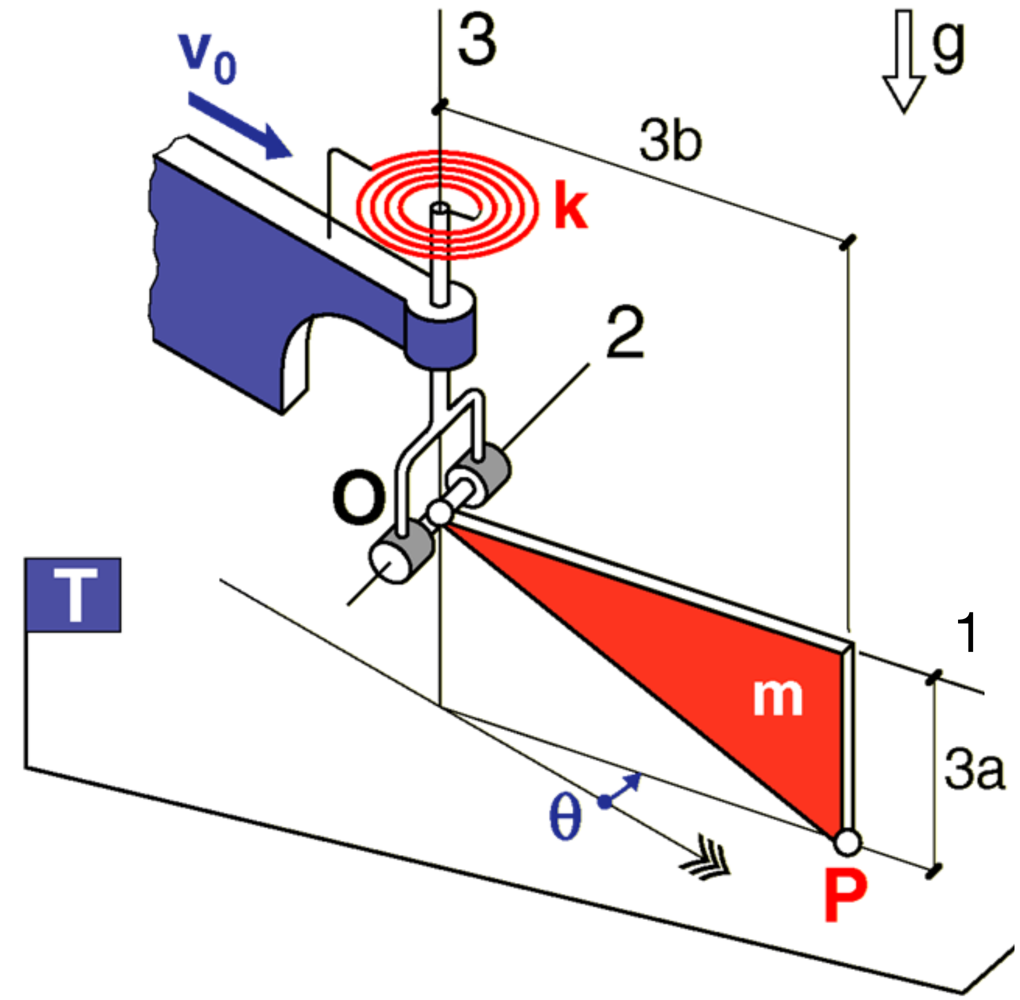
$$\left\{ \bar{F}_{Sup\rightarrow Forq} \right\}_B = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\left\{ \bar{M}_{Sup\rightarrow Forq}\,(\mathbf{O}) \right\}_B = \begin{Bmatrix} M_1 \\ M_2 \\ 0 \end{Bmatrix}$$



Forces sobre "Placa + Forq"





Anàlisi de l'estabilitat de $\theta_{eq} = 0$

3 passos

com al pèndol simple

$$I_{33}\,\ddot{\theta} + 9cb^2\,\dot{\theta} + k\,\theta - 3bc\mathrm{v}_0\,\sin\theta = 0$$

Obtenim EDO de l'error ε

$\theta = \theta_{\mathrm{eq}} + \varepsilon = \varepsilon$ en aquest exemple

$\dot{\theta} = \dot{\varepsilon}$

$\ddot{\theta} = \ddot{\varepsilon}$

$$I_{33}\,\ddot{\varepsilon} + 9cb^2\,\dot{\varepsilon} + k\,\varepsilon - 3bc\mathrm{v}_0\,\sin\varepsilon = 0$$

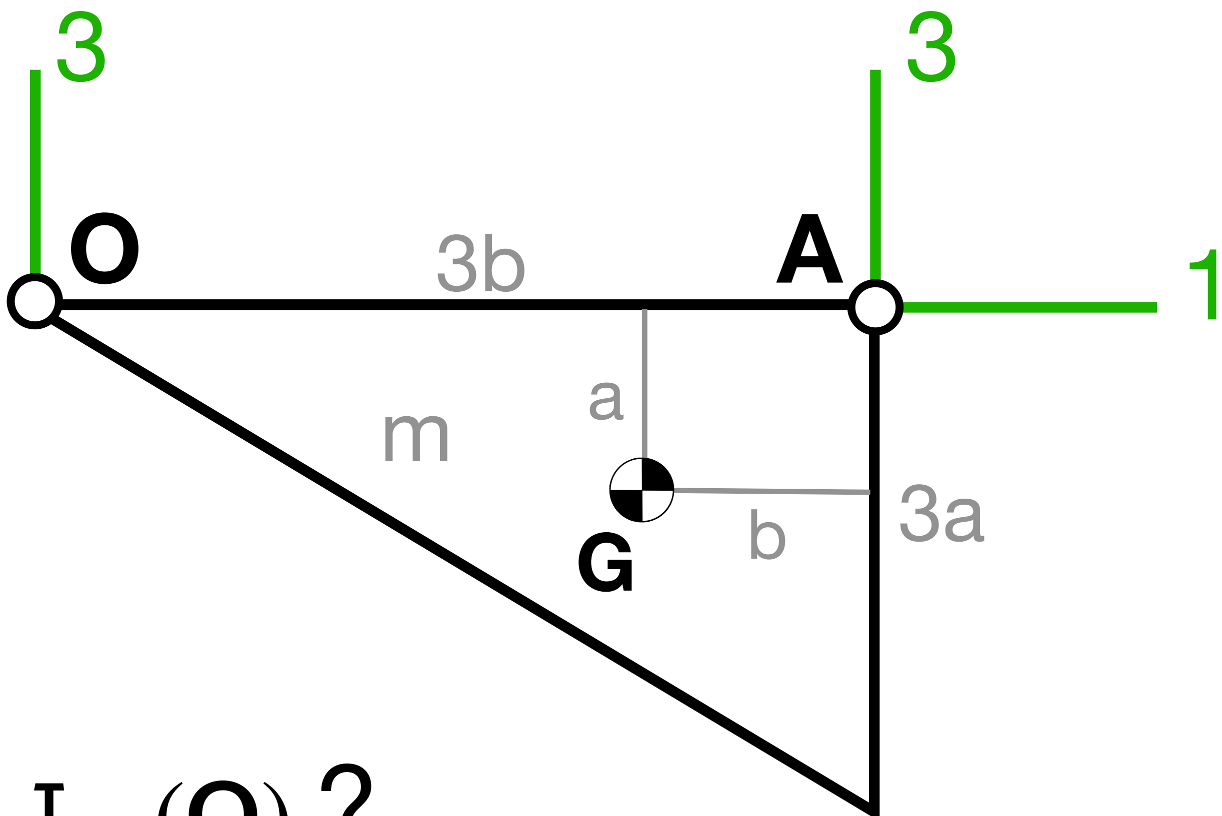
La linealitzem $\quad \sin\varepsilon \approx \varepsilon$

$$I_{33}\,\ddot{\varepsilon} + \underbrace{9cb^2}_{\mathbf{A}}\,\dot{\varepsilon} + \underbrace{(k - 3bc\mathrm{v}_0)}_{\mathbf{B}}\,\varepsilon = 0$$

$$\ddot{\varepsilon} = -\underbrace{\frac{B}{I_{33}}}_{\mathbf{K}}\varepsilon - \underbrace{\frac{A}{I_{33}}}_{\mathbf{C} > 0}\dot{\varepsilon}$$

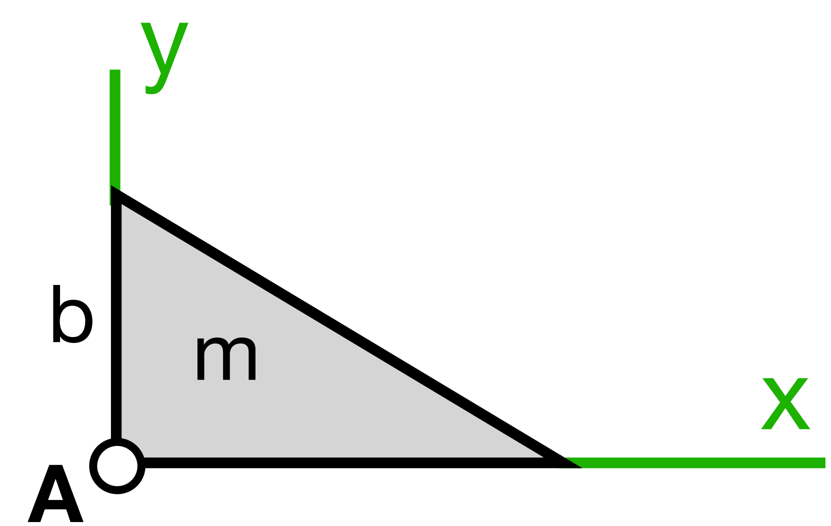
K > 0?

$$\mathbf{K} > 0 \iff \mathrm{B} > 0 \iff k > 3bc\mathrm{v}_0$$



$I_{33}(\mathbf{O})$?

Taules



$$I_{xx}(\mathbf{A}) = \frac{1}{6}mb^2$$

$I_{33}(\mathbf{A})$ de taules + **doble Steiner** per passar de **A** a **O**:

$$(a) \qquad I_{33}(\mathbf{O}) = I_{33}(\mathbf{G}) + I_{33}^{\oplus}(\mathbf{O})$$

$$(b) \qquad I_{33}(\mathbf{A}) = I_{33}(\mathbf{G}) + I_{33}^{\oplus}(\mathbf{A})$$

$$(a-b) \qquad I_{33}(\mathbf{O}) = I_{33}(\mathbf{A}) + I_{33}^{\oplus}(\mathbf{O}) - I_{33}^{\oplus}(\mathbf{A})$$

$$I_{33}(\mathbf{O}) = \frac{1}{6}m(3b)^2 + m(2b)^2 - mb^2 = \frac{9}{2}mb^2$$

DEURES

Determineu el valor de la normal N del terra sobre la placa en funció de les variables d'estat del sistema

Pista: apliqueu el full de ruta

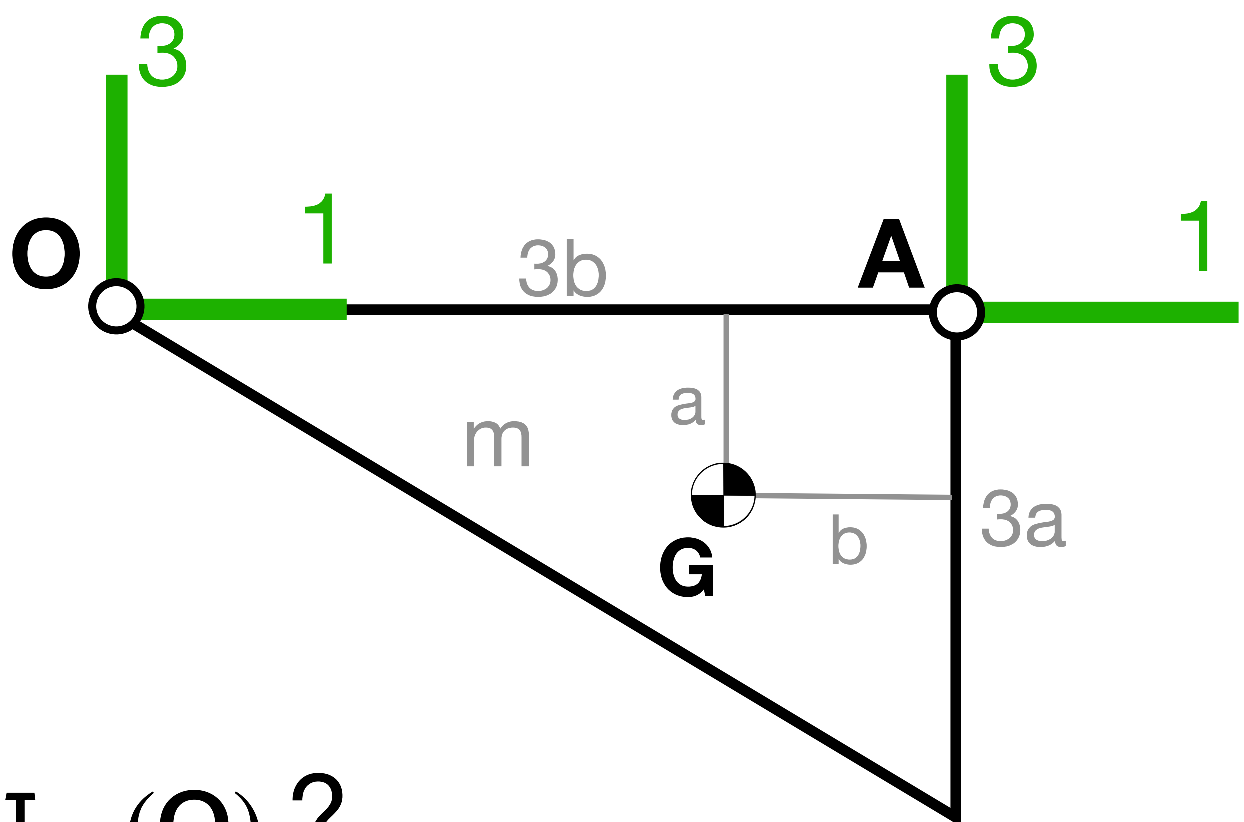
SISTEMA = Placa

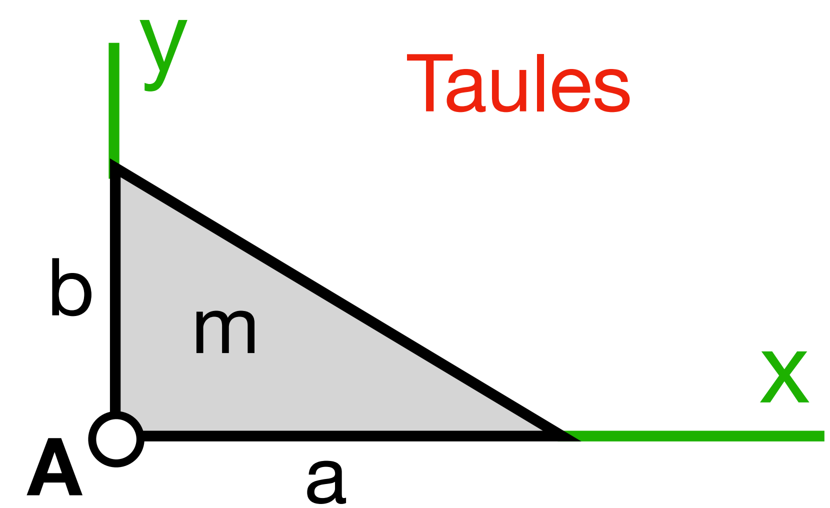
$\text{TMC}(\mathbf{O})\big]_2$ (aprofiteu el moment cinètic abans calculat per a l'eq. del mov.)

Solució:

$$N = \frac{2}{3}mg + c\frac{a}{b}v_0\cos\theta - \frac{3}{4}ma\dot{\theta}^2$$

Per calcular N us caldrà $\mathrm{I}_{13}(\mathbf{O})$:





$$\mathrm{I}_{xy}(\mathbf{A}) = -\frac{1}{12}\,m\,a\,b$$

Negatiu perquè la massa del trinagle és al primer quadrant de **A** (adapteu signe al vostre cas)

$\mathrm{I}_{13}(\mathbf{O})$?

$\mathrm{I}_{13}(\mathbf{A})$ de taules + **doble Steiner** per passar de **A** a **O**:

$$\mathrm{I}_{13}(\mathbf{O}) = \mathrm{I}_{13}(\mathbf{A}) + \mathrm{I}_{13}^{\oplus}(\mathbf{O}) - \mathrm{I}_{13}^{\oplus}(\mathbf{A})$$

$$\mathrm{I}_{13}(\mathbf{O}) = -\frac{1}{12}\,m\,(3a)\,(3b) + m\,2b\,a - \left(-m\,a\,b\right) = \frac{9}{4}\,m\,a\,b$$

Negatiu perquè tota la massa de la placa és al 3er quadrant de **A**

Positiu perquè la massa concentrada a **G** és al 4rt quadrant de **O**

Negatiu perquè la massa concentrada a **G** és al 3er quadrant de **A**

Surt positiu perquè tota la massa de la placa és al 4rt quadrant de **O**

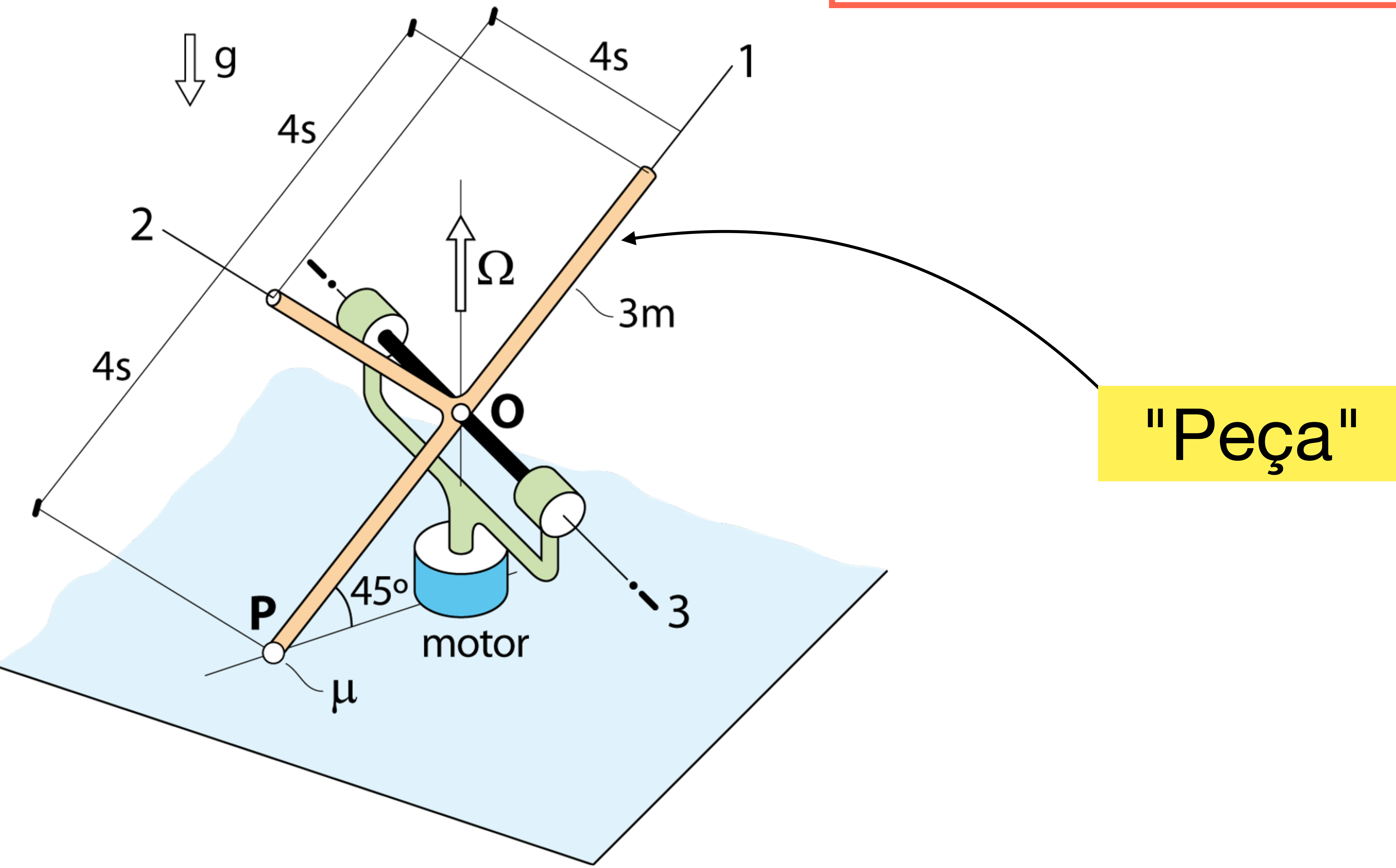
DEURES

Finalment, per practicar, calculeu tot el tensor d'inèrcia de la placa referit al punt O

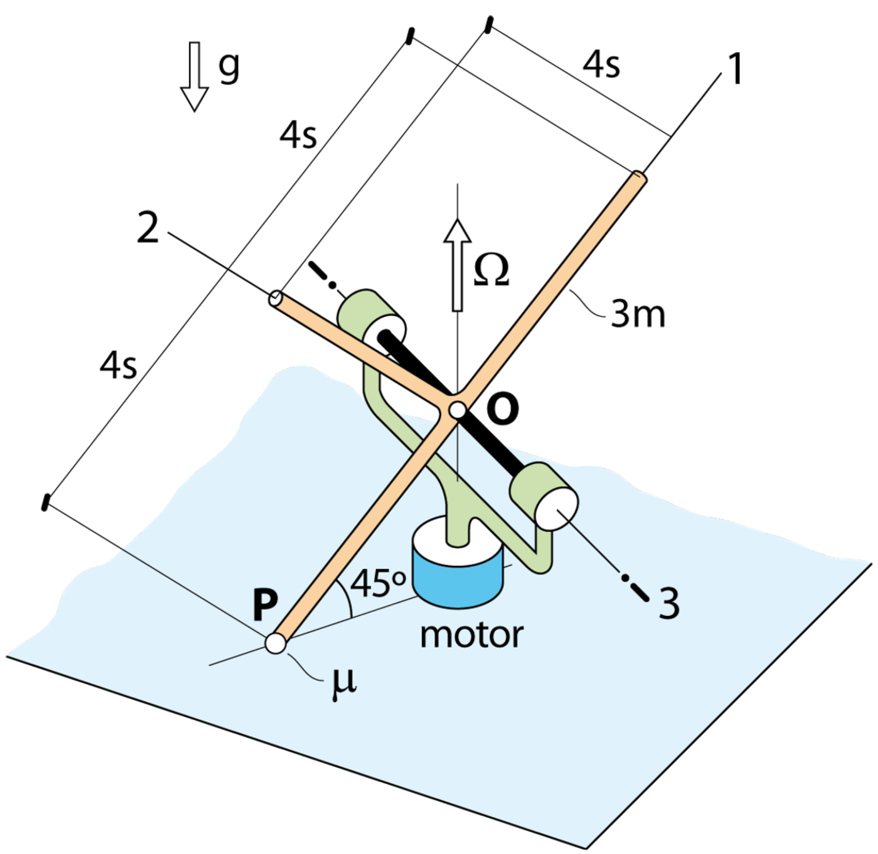
$$\Omega_{T}^{\text{forq}} = \Omega = \text{ct}$$

Ω_{critica} per pèrdua contacte a P?

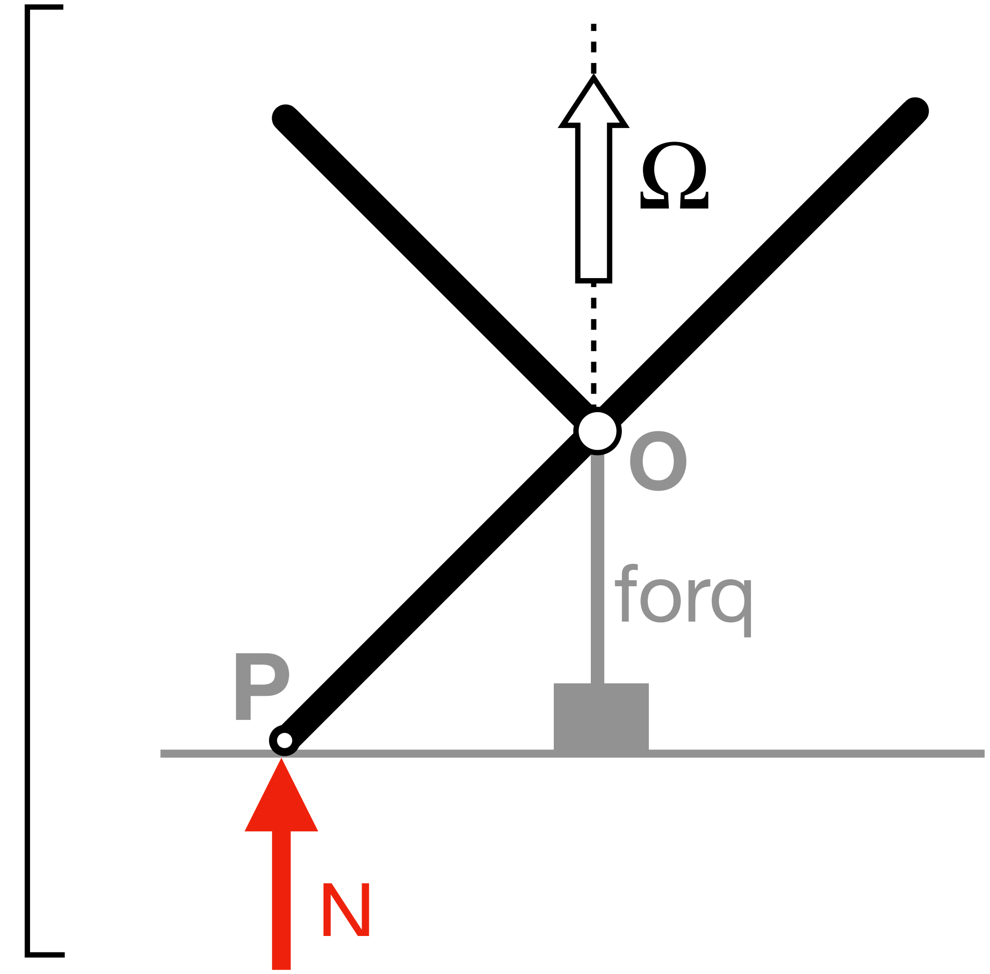
Eq. mov. quan $\Omega > \Omega_{\text{critica}}$



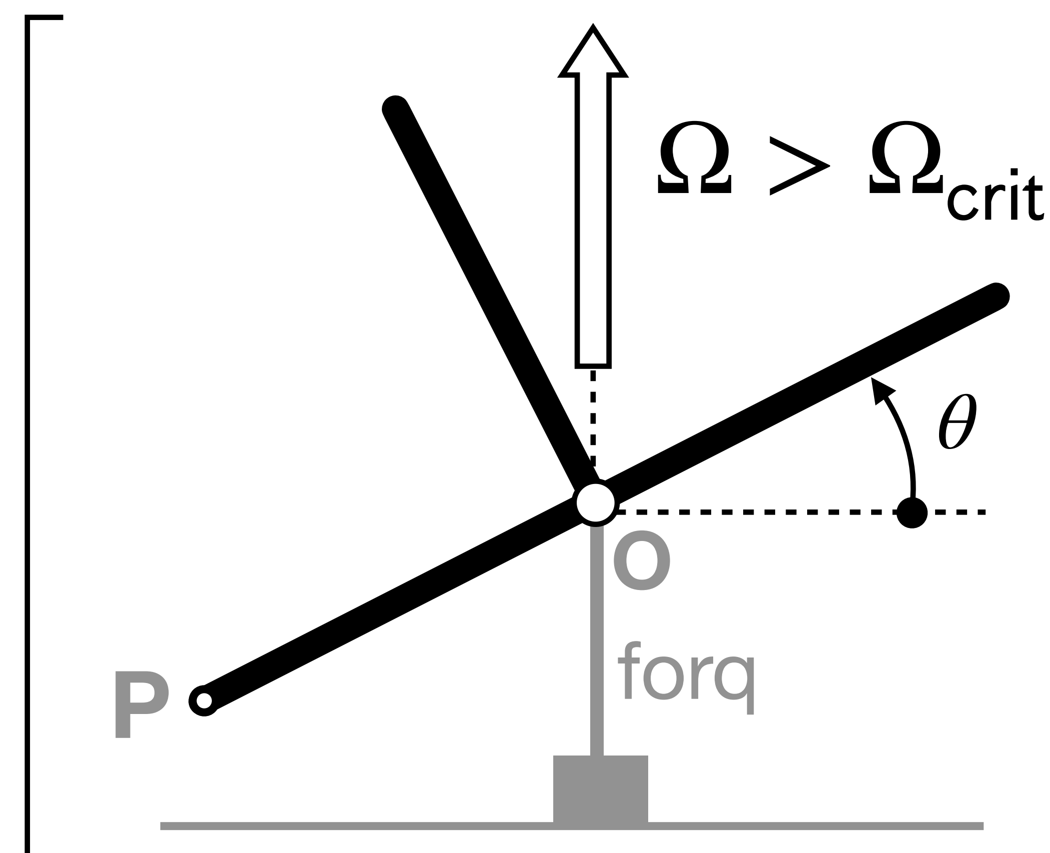
GL? 2 situacions!

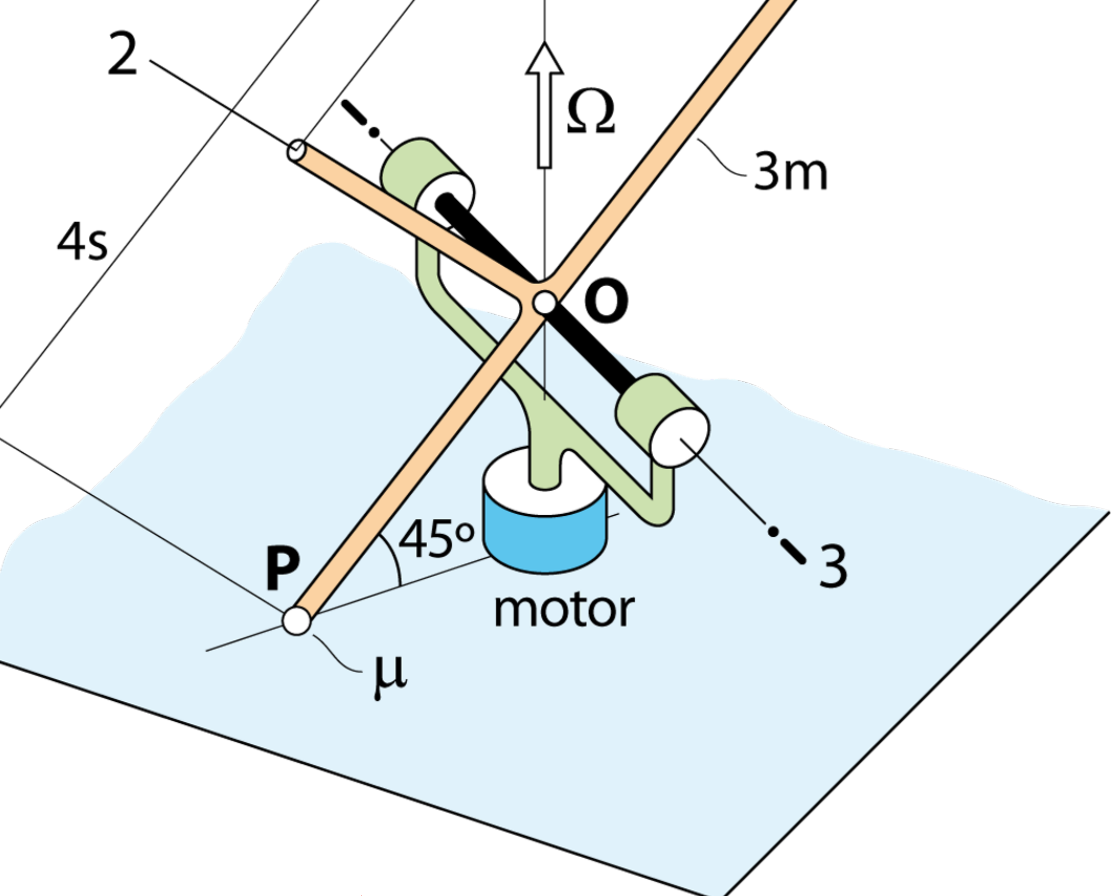


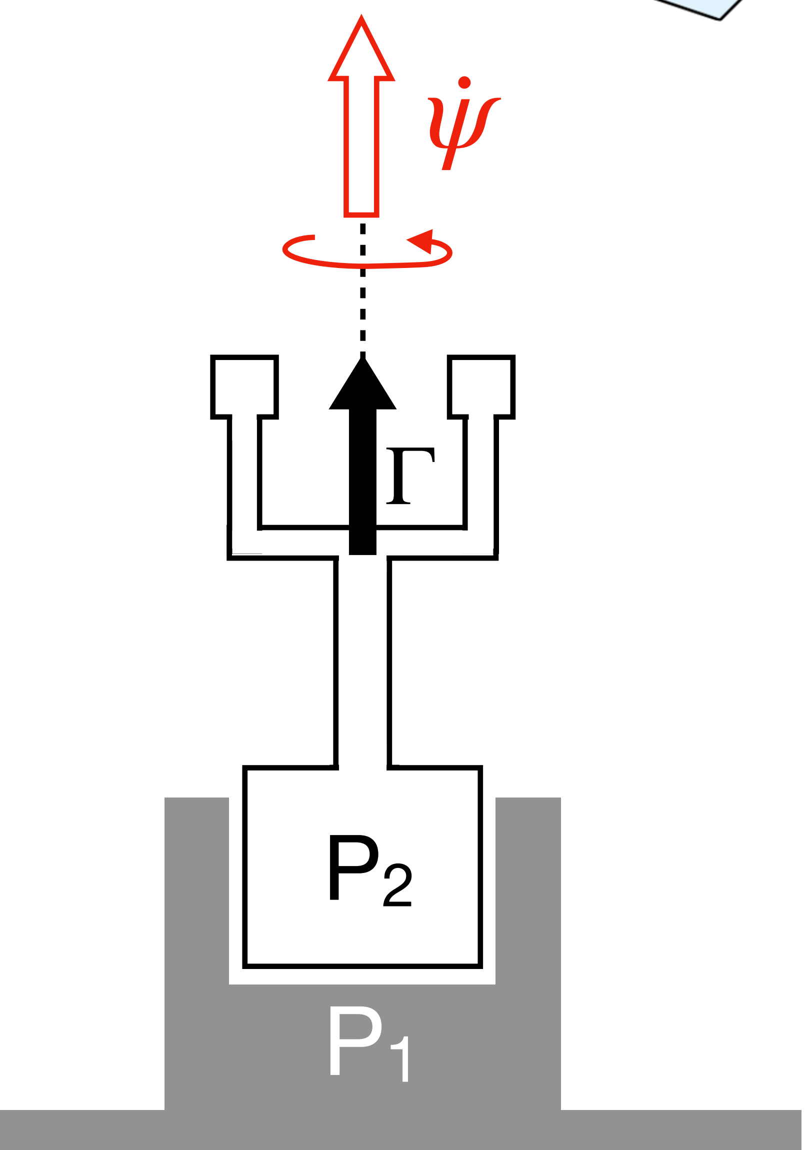
P manté contacte amb T



Contacte perdut







Recordeu

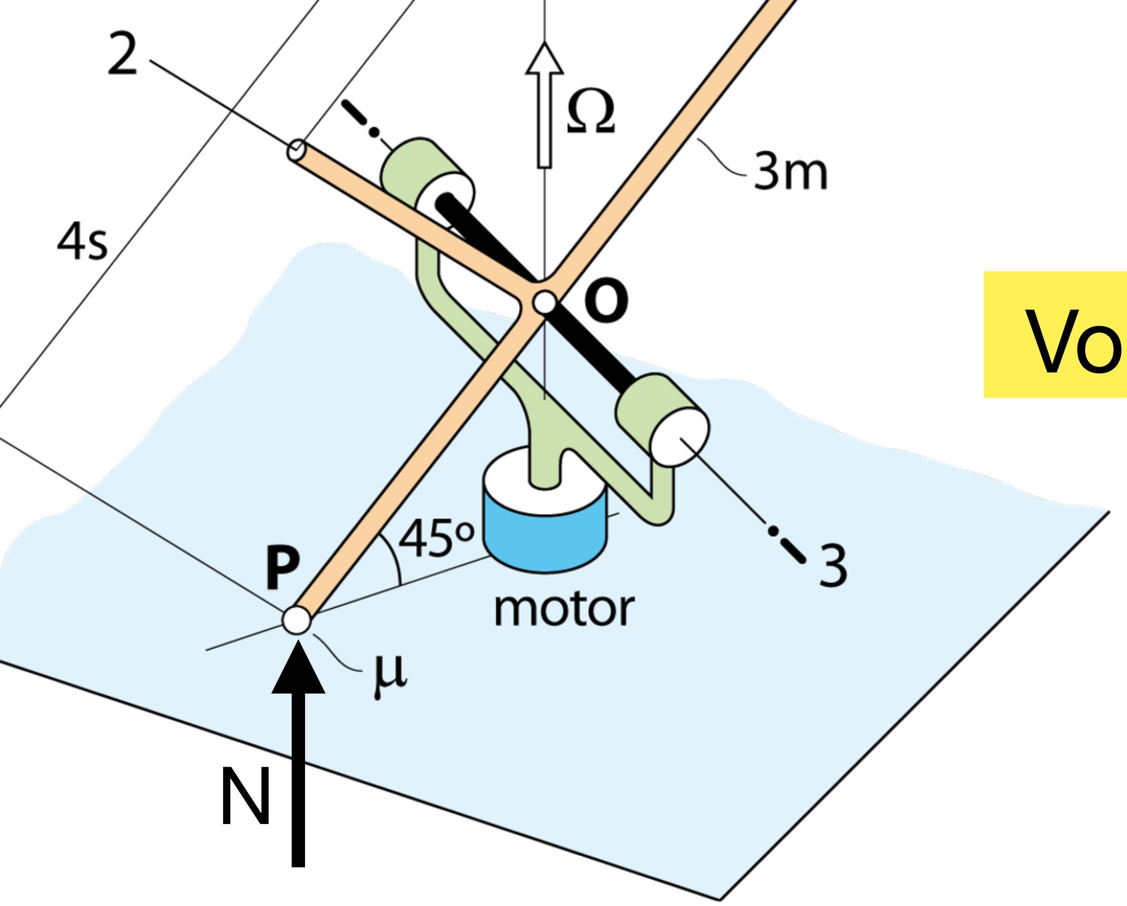
Γ conegut $\Rightarrow$ $\ddot{\psi}$ és incògnita

$\ddot{\psi}$ coneguda $\Rightarrow$ Γ és incògnita

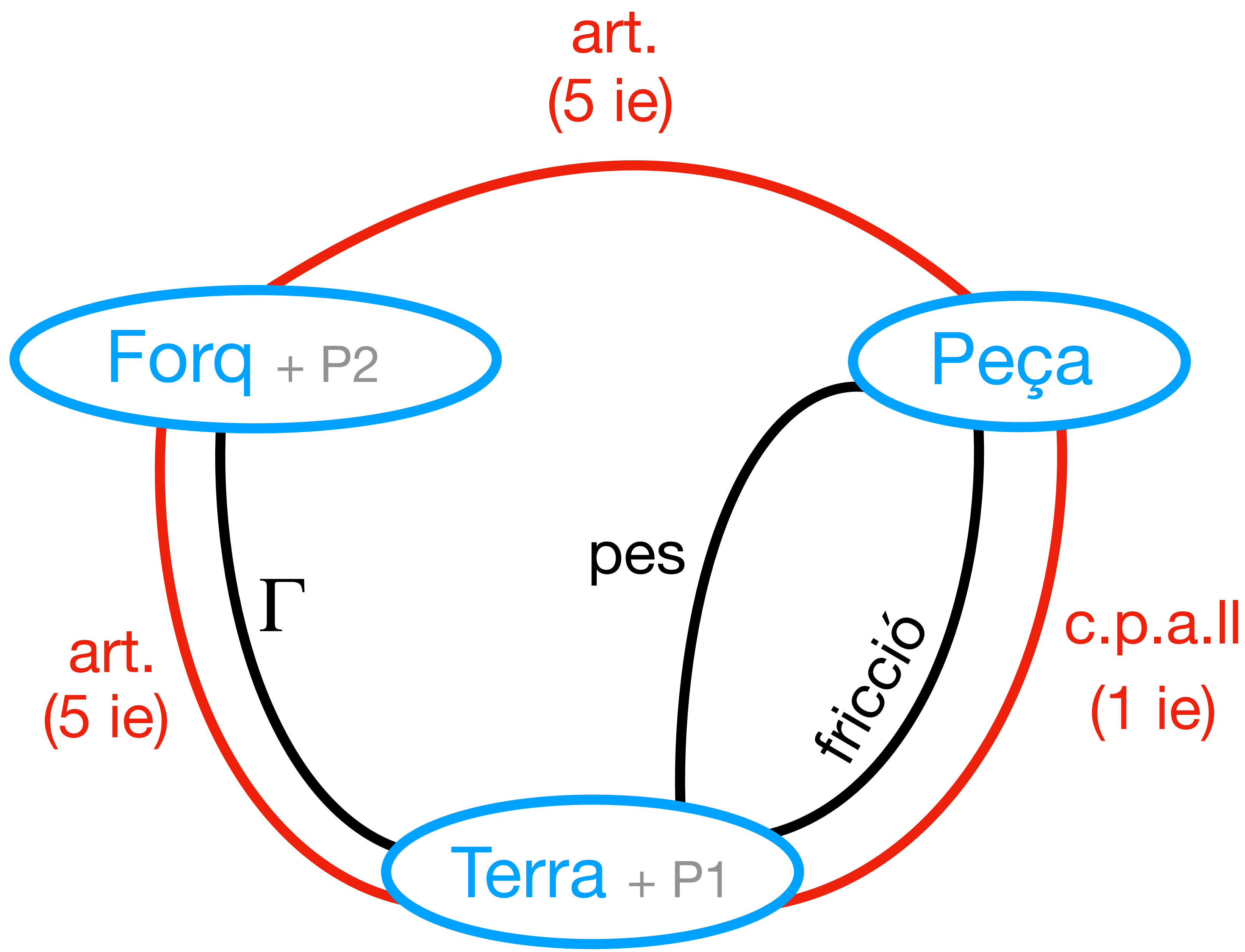
En aquest exercici

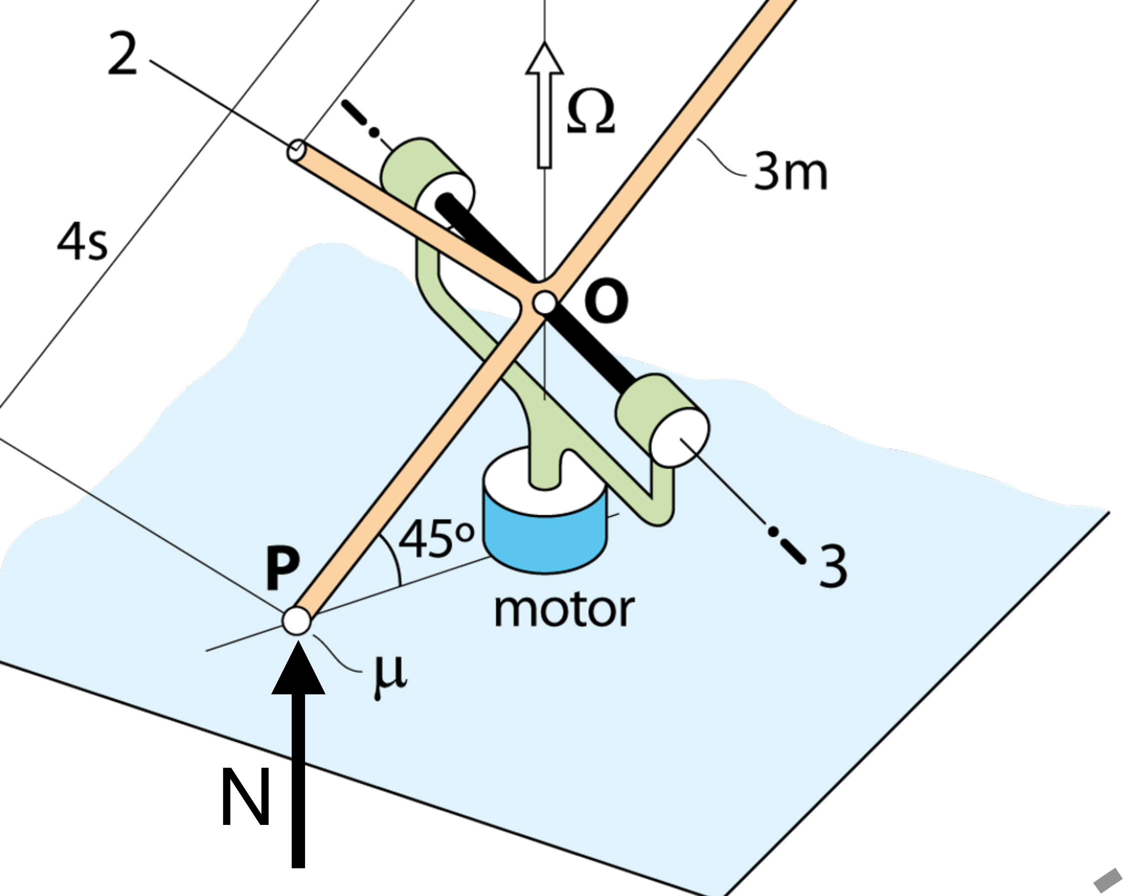
$\dot{\psi} = \Omega = ct \quad \Rightarrow \quad \ddot{\psi} = 0$ (coneguda)

Γ serà incògnita

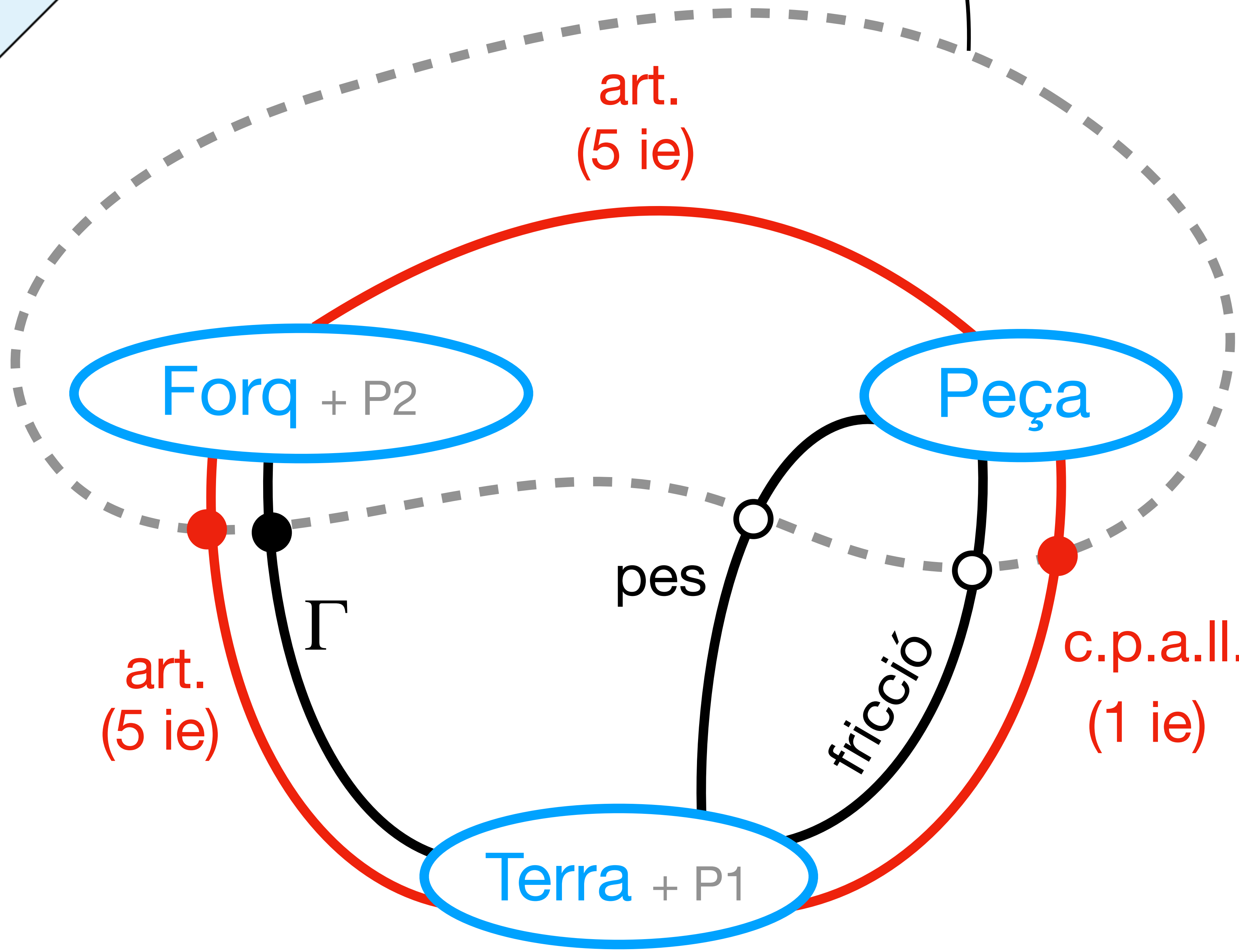


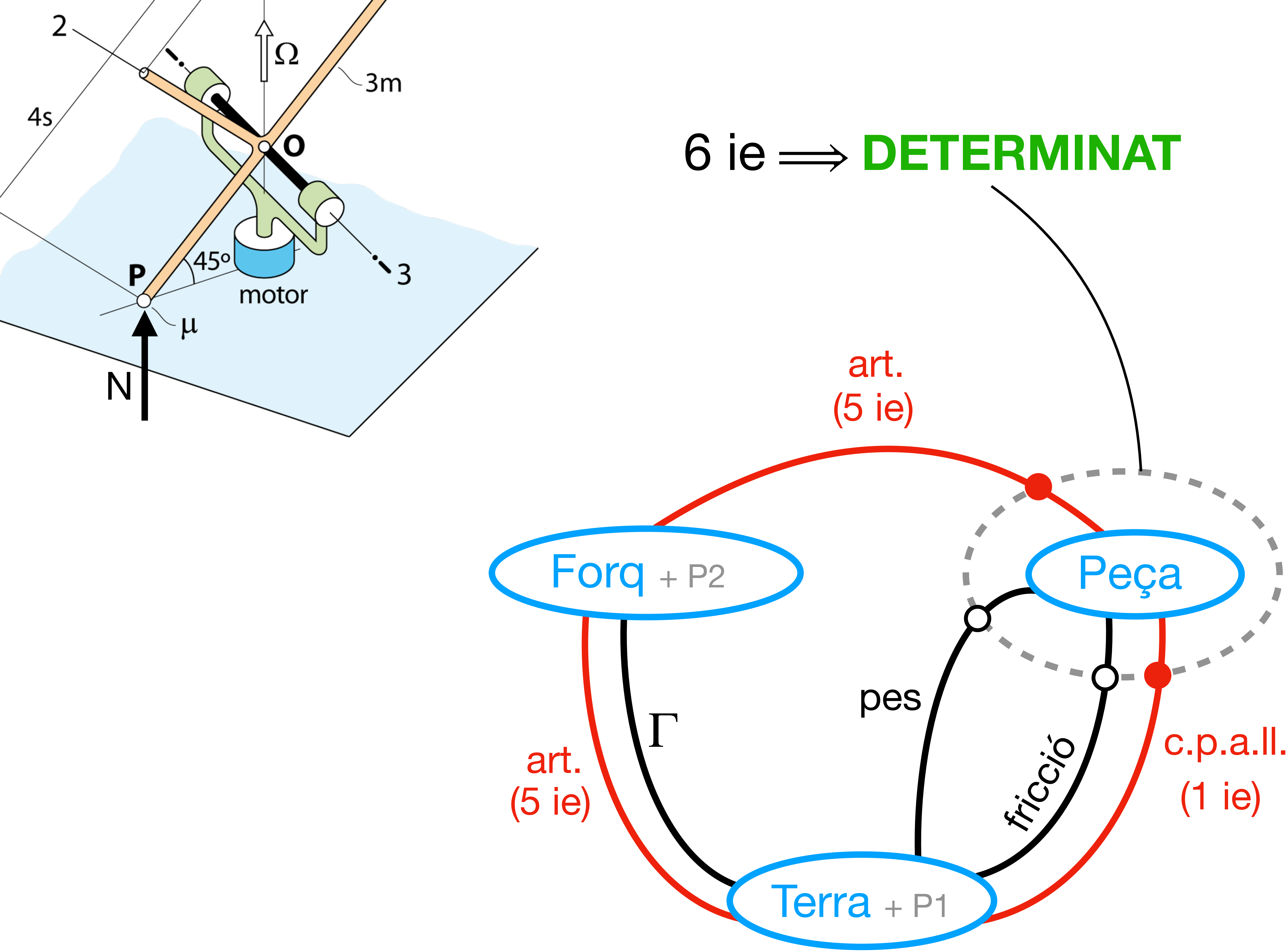
Volem N ⇒ SIST ha d'incloure la peça!

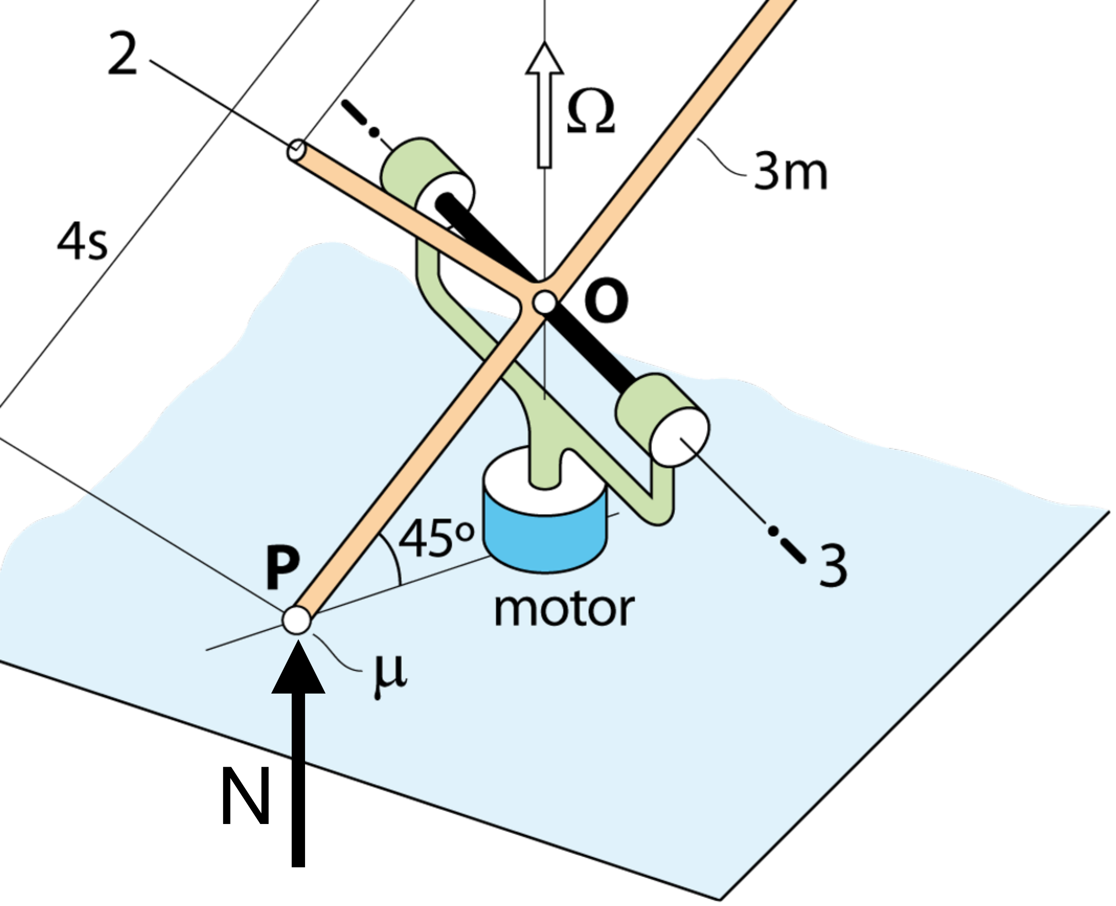




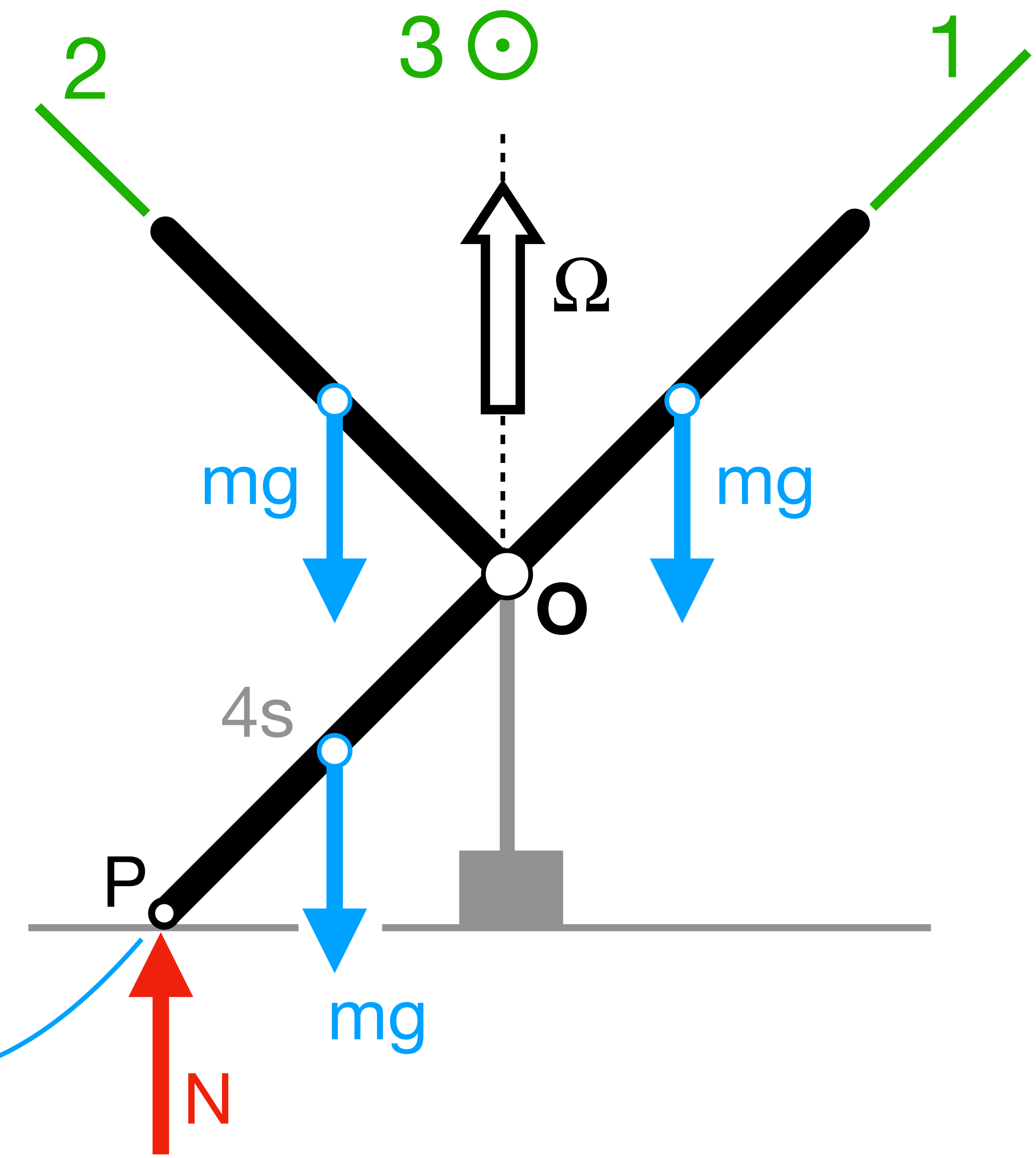
6 ie $+\ \Gamma \Longrightarrow$ **INDETERMINAT**




2
Ω
3m
4s
O
P
45º
motor
3
μ
N
6 ie ⟹ DETERMINAT
art.
(5 ie)
Forq + P2
Peça
pes
Γ
art.
(5 ie)
fricció
c.p.a.ll.
(1 ie)
Terra + P1



Forces sobre "Peça"



$$\left\{ \bar{\mathrm{F}}_{\mathrm{Forq}\rightarrow\mathrm{Peça}} \right\}_{\mathrm{B}} = \begin{Bmatrix} \mathrm{F}_1 \\ \mathrm{F}_2 \\ \mathrm{F}_3 \end{Bmatrix}$$

$$\left\{ \bar{\mathrm{M}}_{\mathrm{Forq}\rightarrow\mathrm{Peça}}\,(\mathbf{O}) \right\}_{\mathrm{B}} = \begin{Bmatrix} \mathrm{M}_1 \\ \mathrm{M}_2 \\ 0 \end{Bmatrix}$$

DEURES

Determineu

- Parell motor Γ per mantenir $\Omega = ct$
- Eq. mov. per al cas en que el contacte a P ja s'ha perdut ($\Omega > \Omega_{\text{critica}}$)