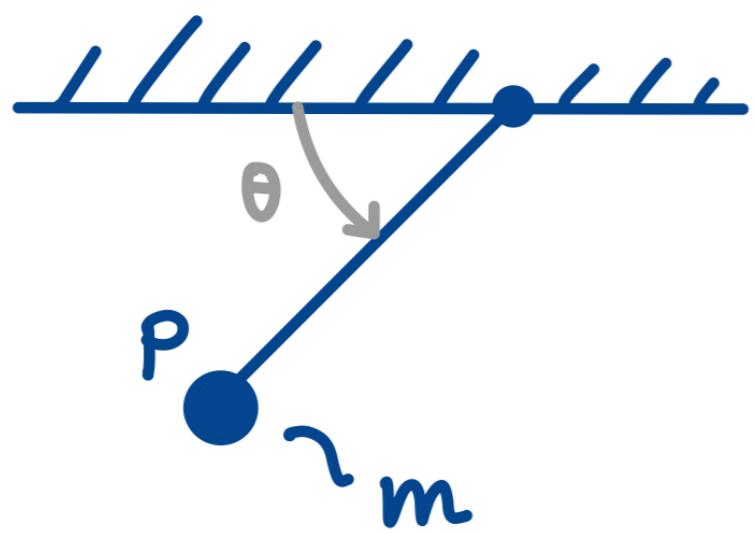
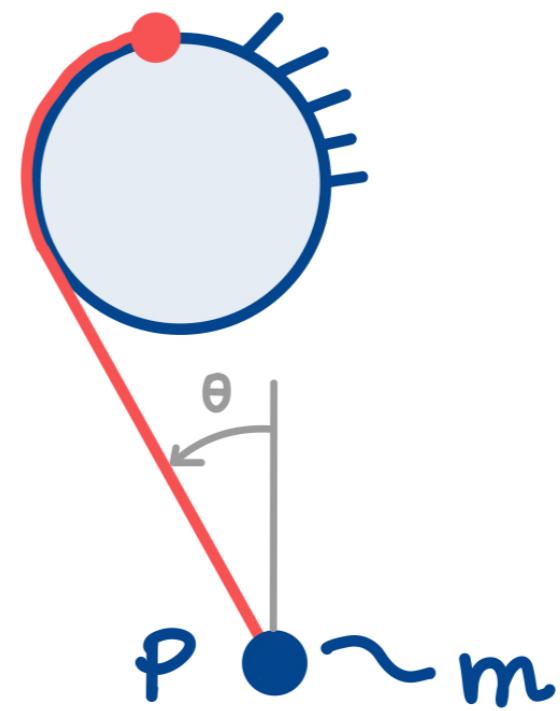


9P

Geometria de masses

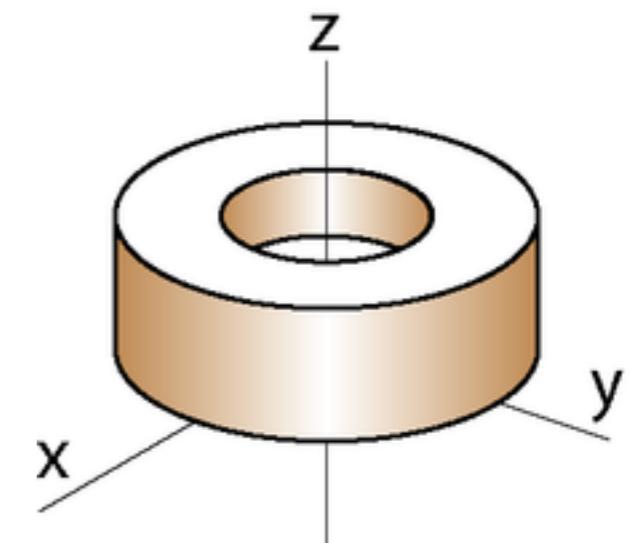
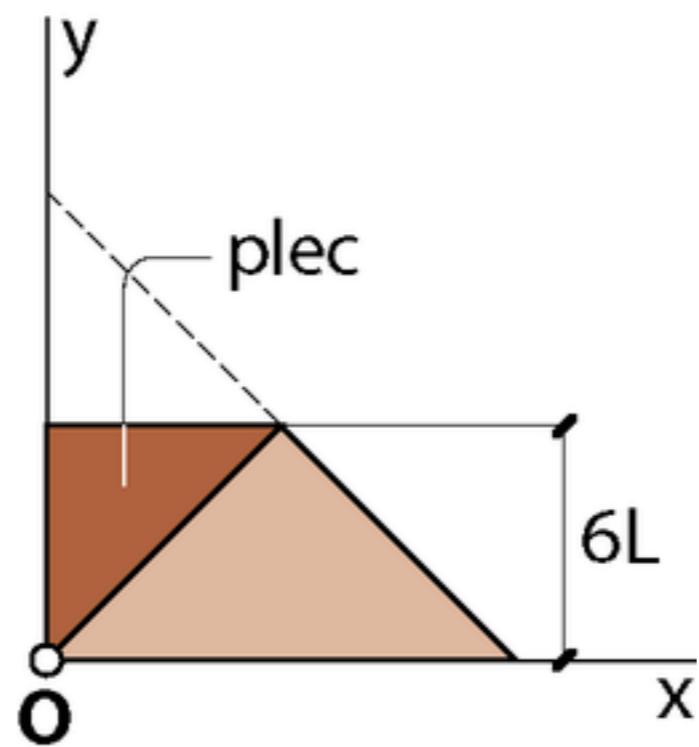
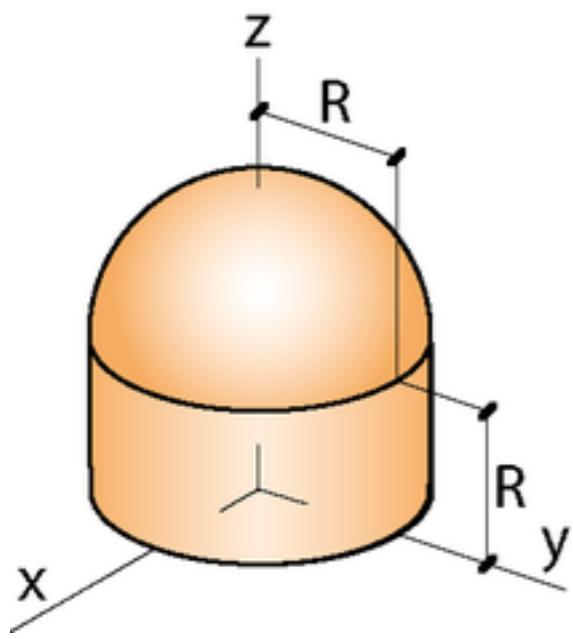


Pèndol simple



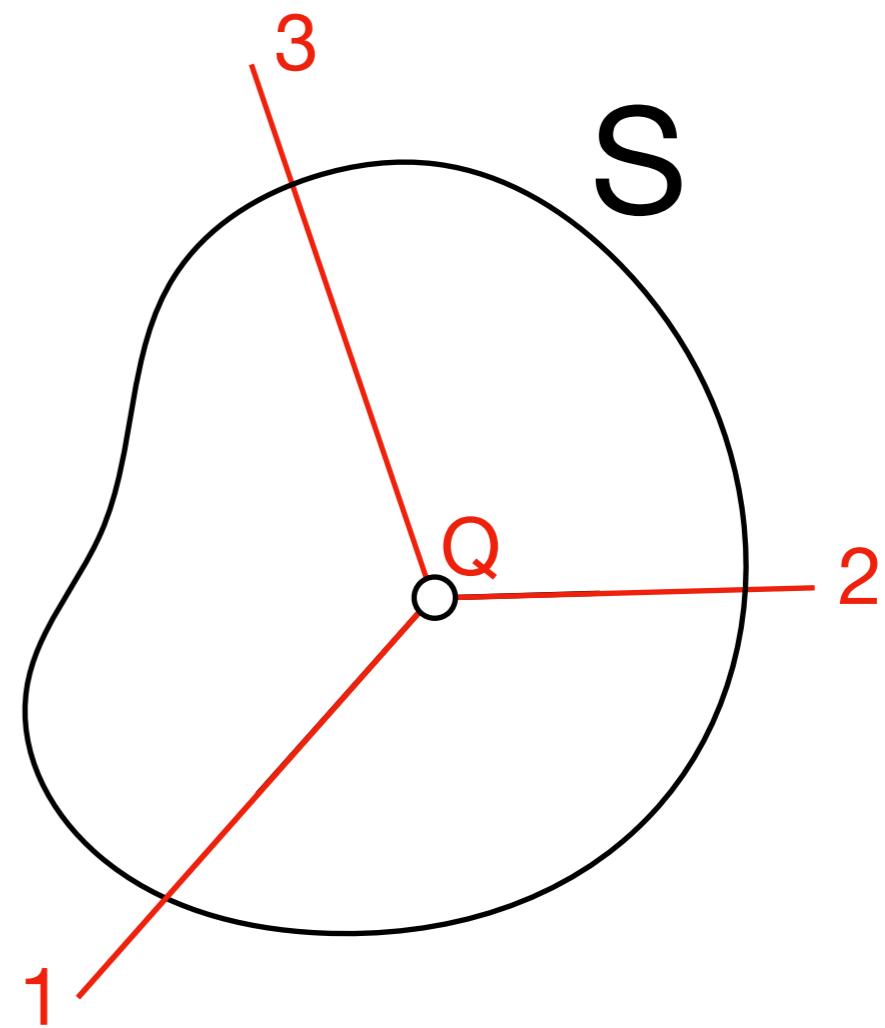
Pèndol sobre cilindre

Exm: D5.1 - D5.2 - D5.3 (Wikimec)



Tensor d'inèrcia de S a Q

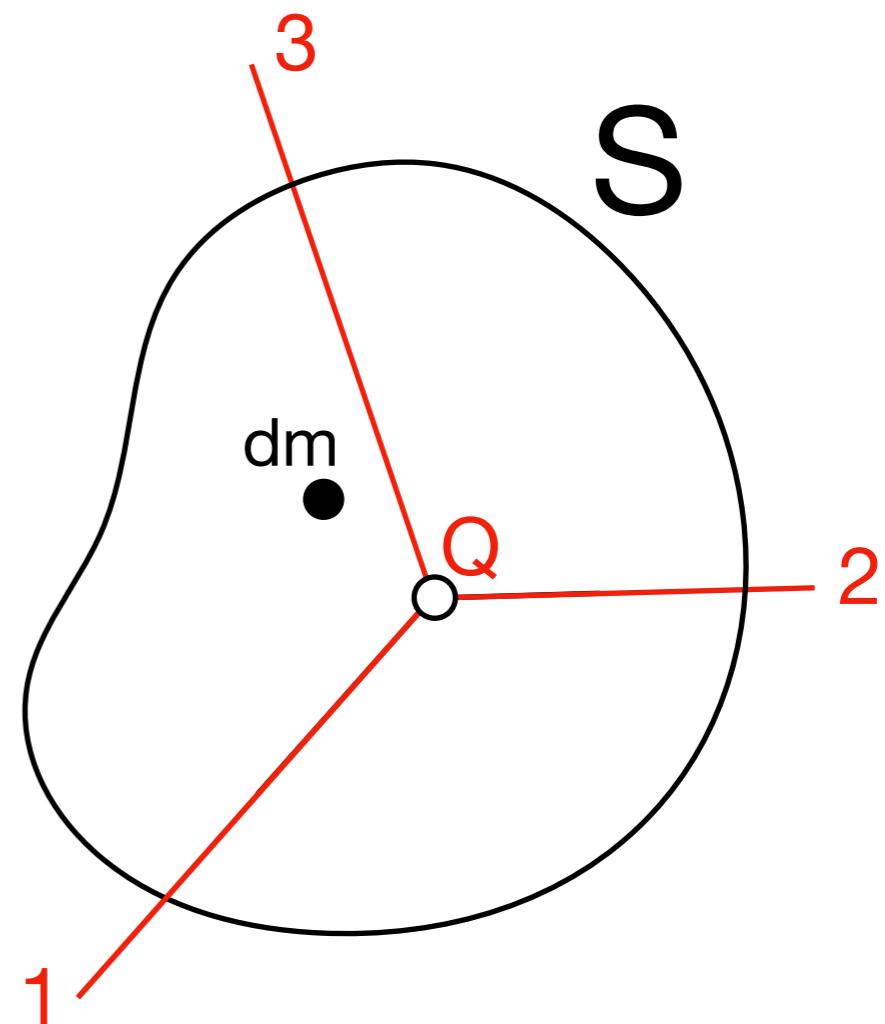
$$[\mathbb{I} (Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$



$$B = (1, 2, 3)$$

Tensor d'inèrcia de S a Q

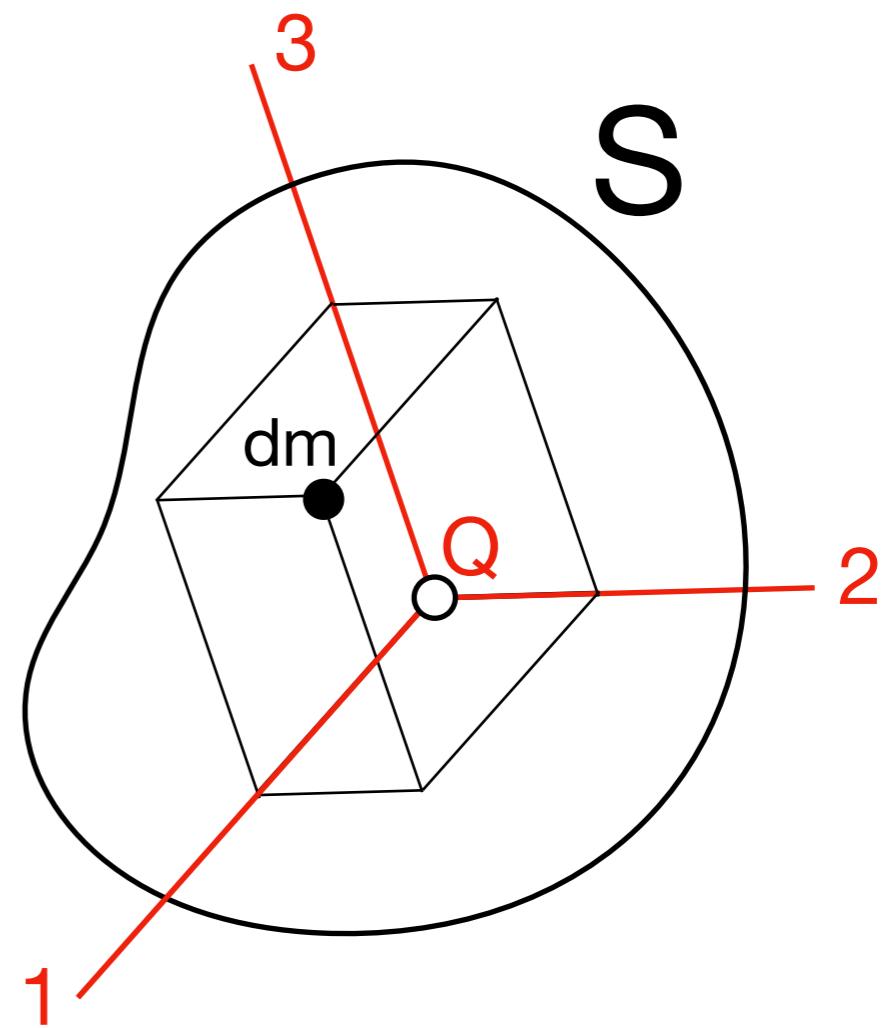
$$[\mathbb{I} (Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$



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Tensor d'inèrcia de S a Q

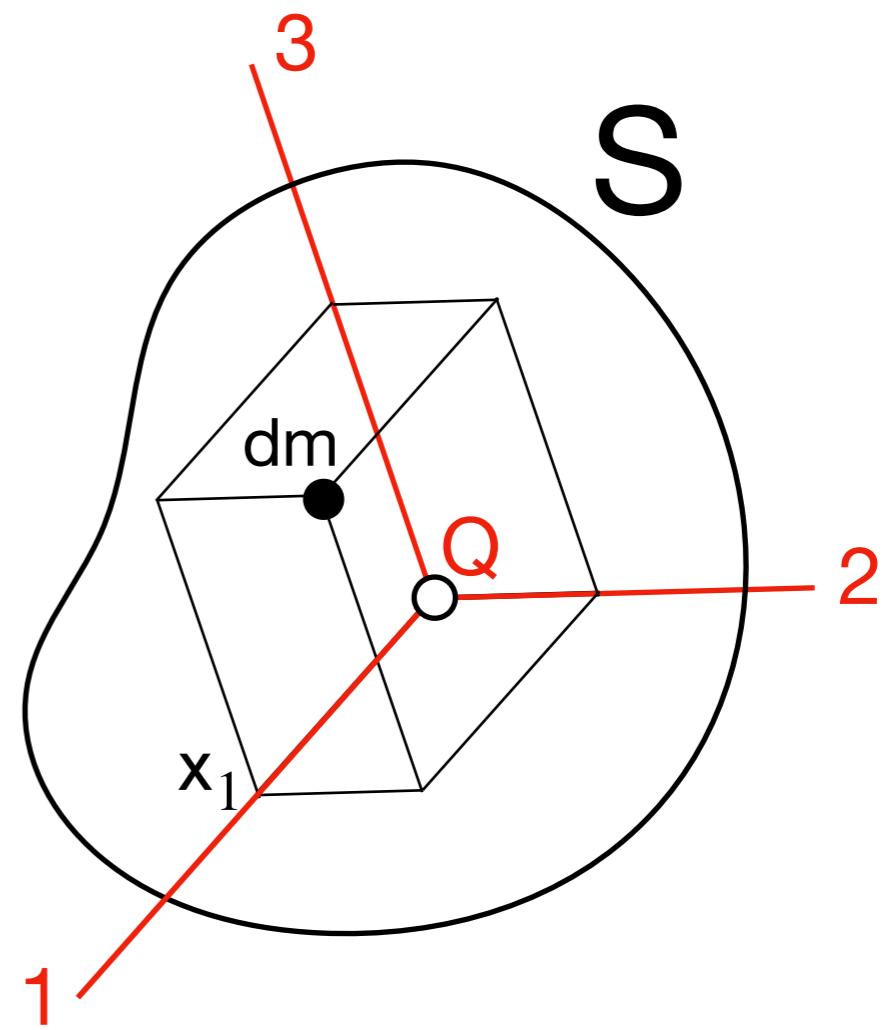
$$[\mathbb{I} (Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$



$B = (1, 2, 3)$

Tensor d'inèrcia de S a Q

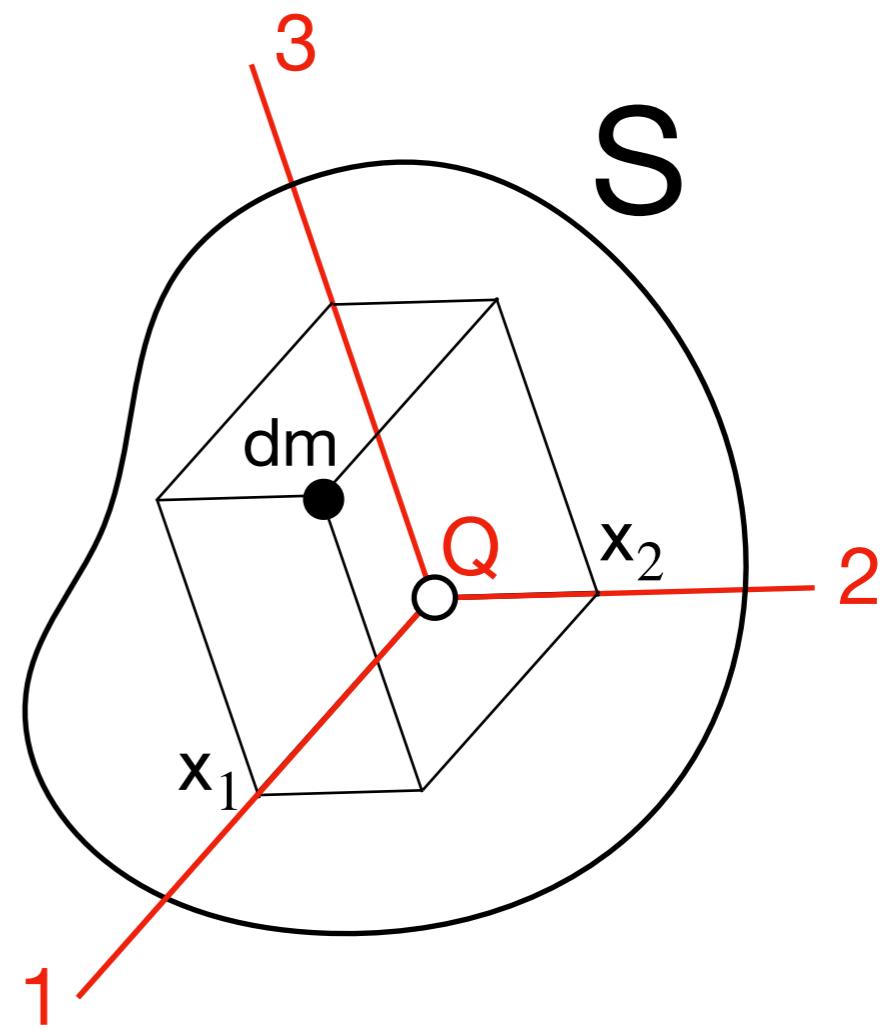
$$[\mathbb{I} (Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$



$B = (1,2,3)$

Tensor d'inèrcia de S a Q

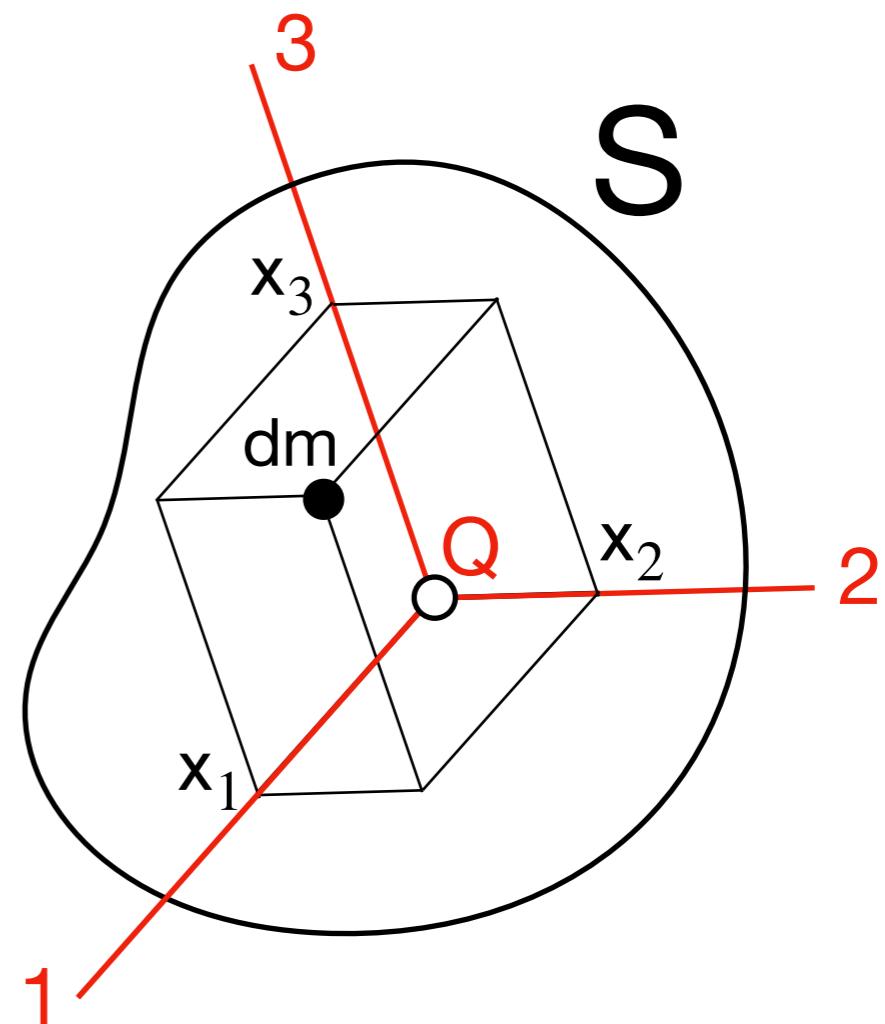
$$[\mathbb{I} (Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$



$B = (1, 2, 3)$

Tensor d'inèrcia de S a Q

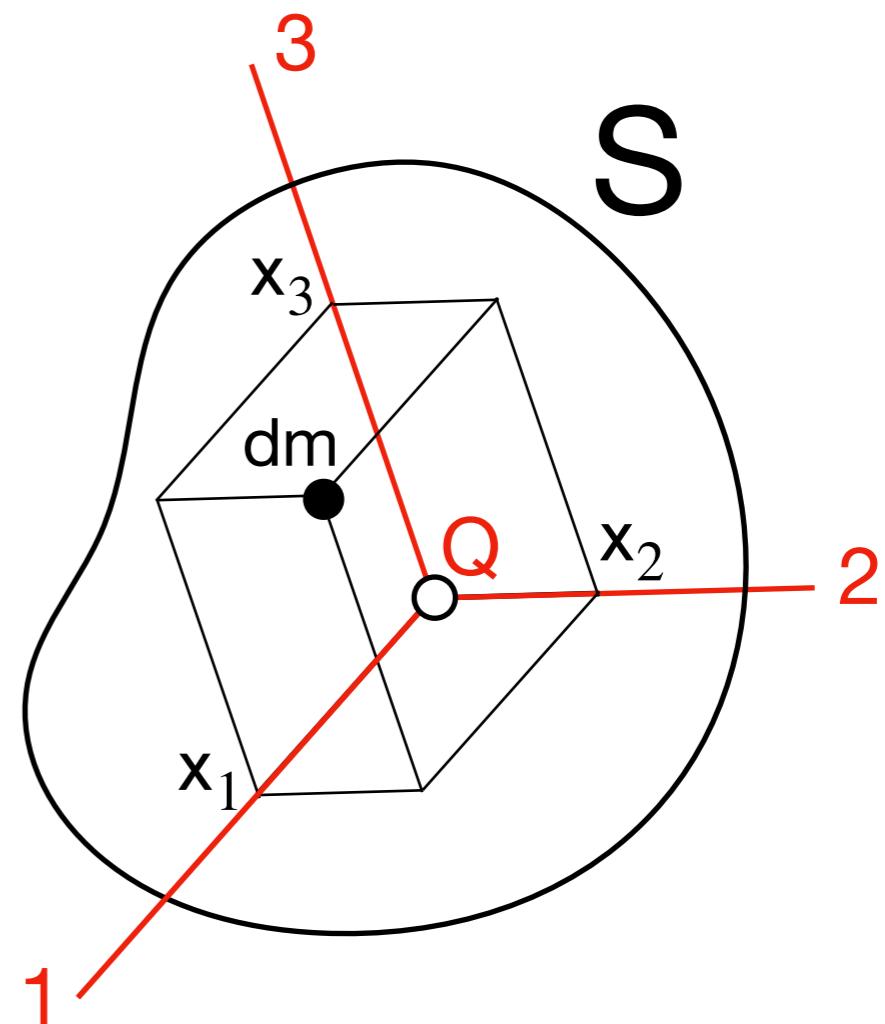
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$B = (1, 2, 3)$

Tensor d'inèrcia de S a Q

$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

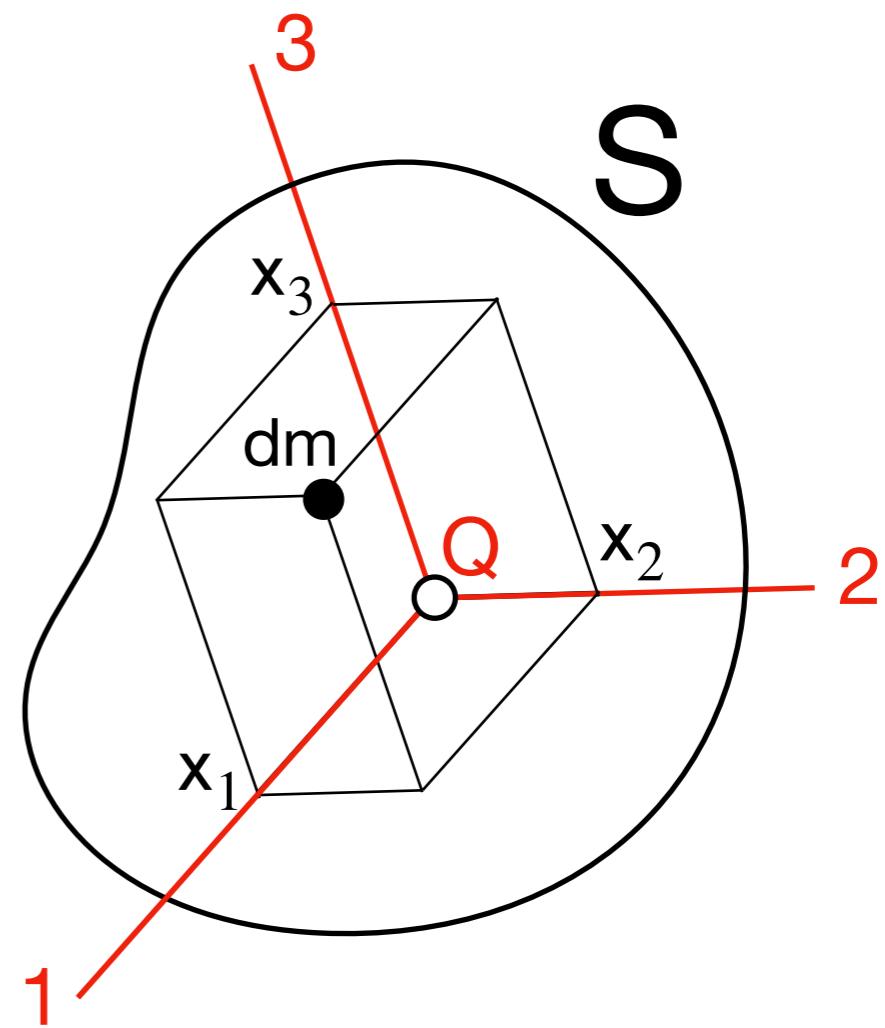


$$B = (1, 2, 3)$$

Tensor d'inèrcia de S a Q

$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

Moments d'inèrcia



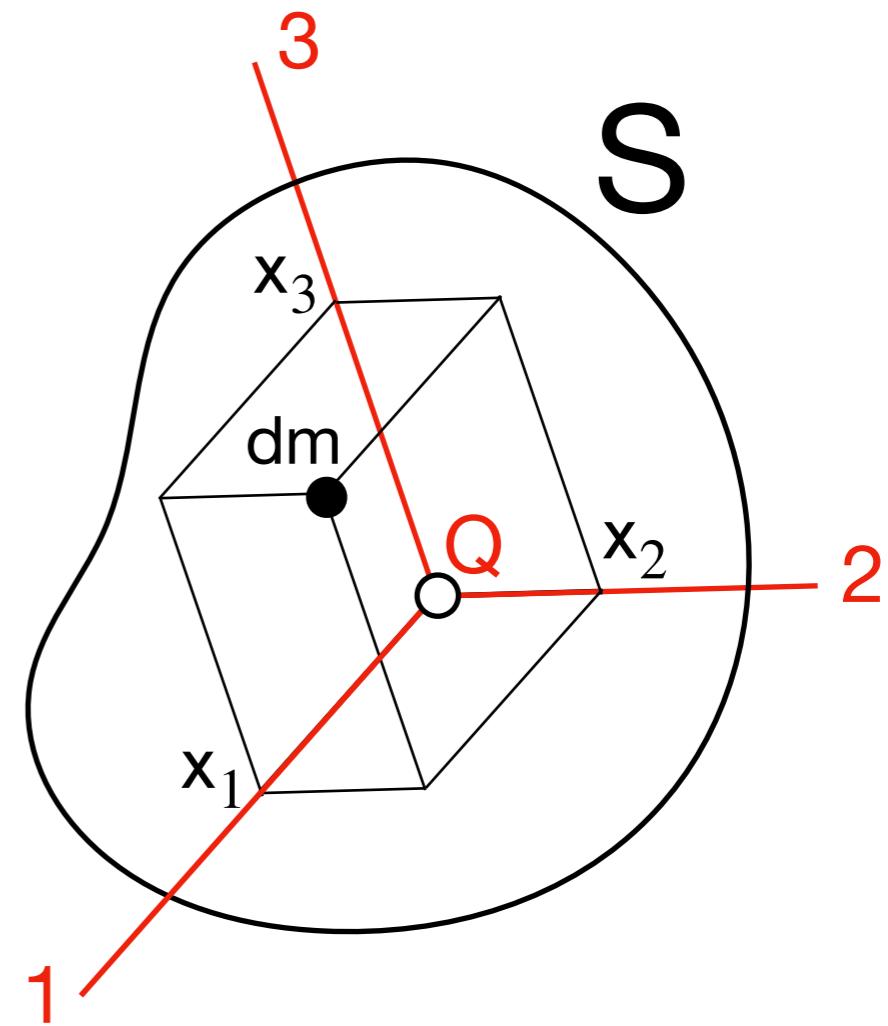
$$B = (1, 2, 3)$$

Tensor d'inèrcia de S a Q

$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

Moments d'inèrcia

$$I_{ii} = \int_S (x_j^2 + x_k^2) dm$$



$$B = (1, 2, 3)$$

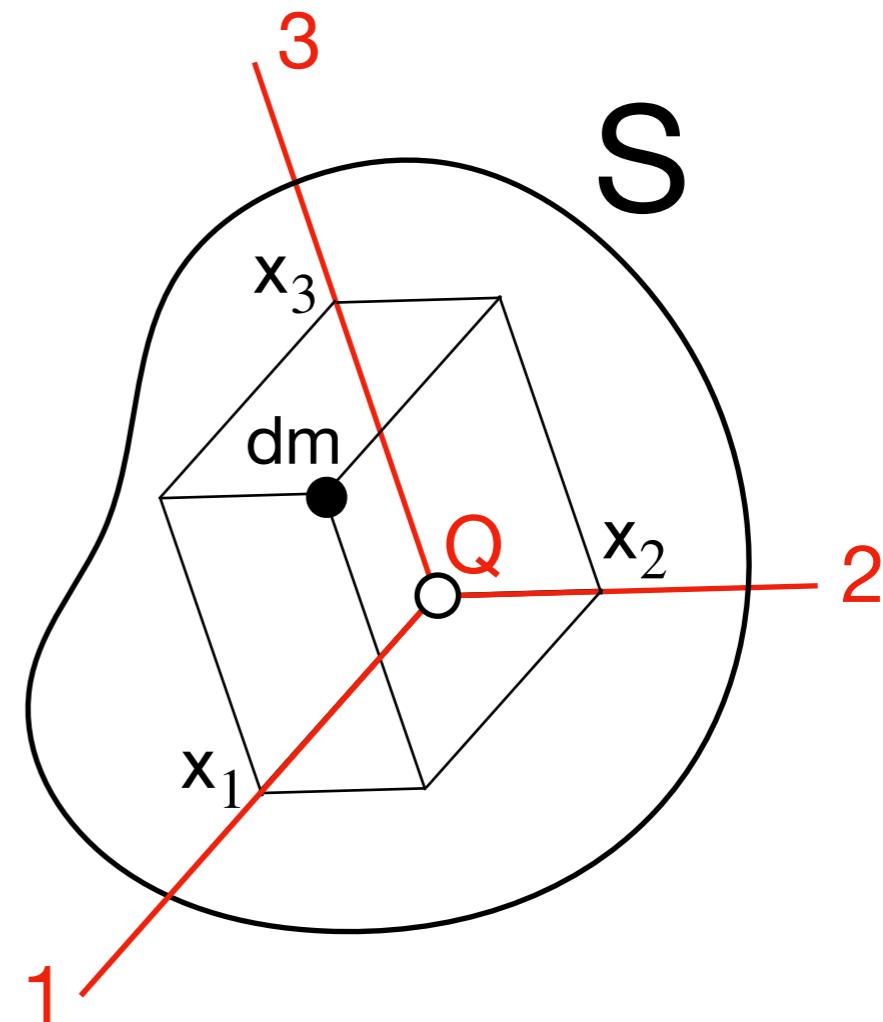
Tensor d'inèrcia de S a Q

$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

Moments d'inèrcia

$$I_{ii} = \int_S (x_j^2 + x_k^2) dm$$

Exm:



$$B = (1, 2, 3)$$

Tensor d'inèrcia de S a Q

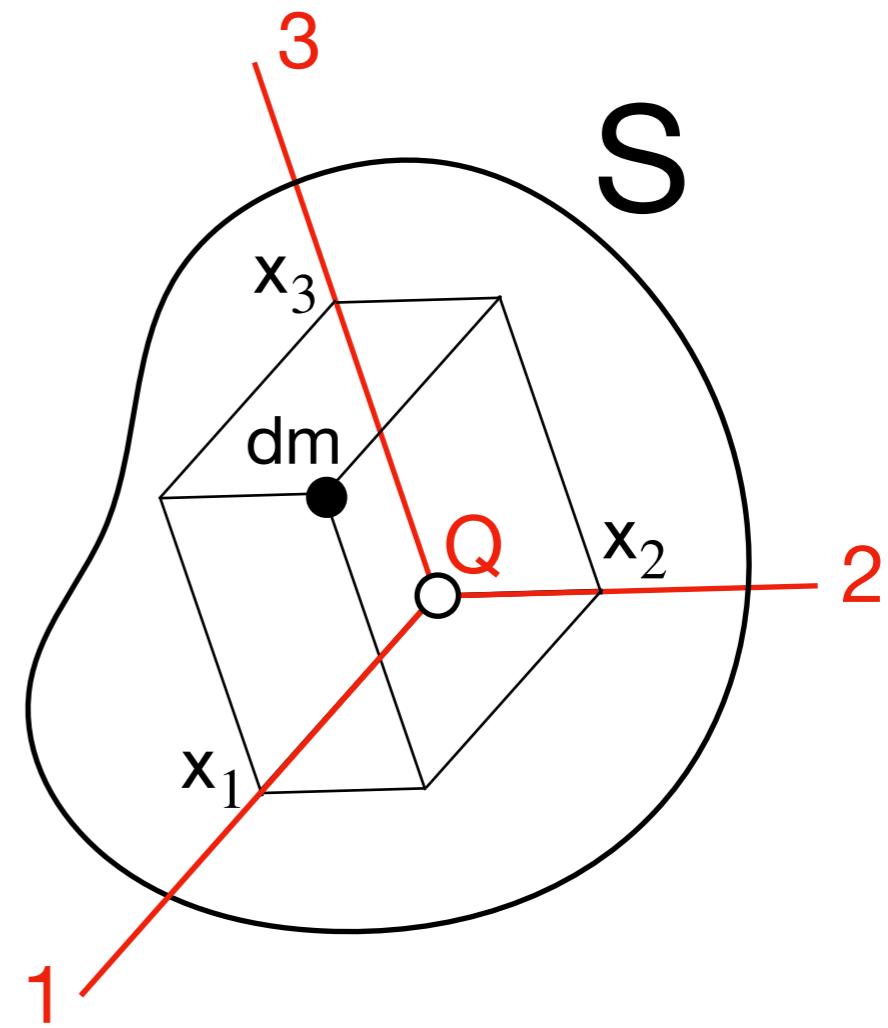
$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

Moments d'inèrcia

$$I_{ii} = \int_S (x_j^2 + x_k^2) dm$$

Exm:

$$I_{11} = \int_S (x_2^2 + x_3^2) dm$$



$$B = (1, 2, 3)$$

Tensor d'inèrcia de S a Q

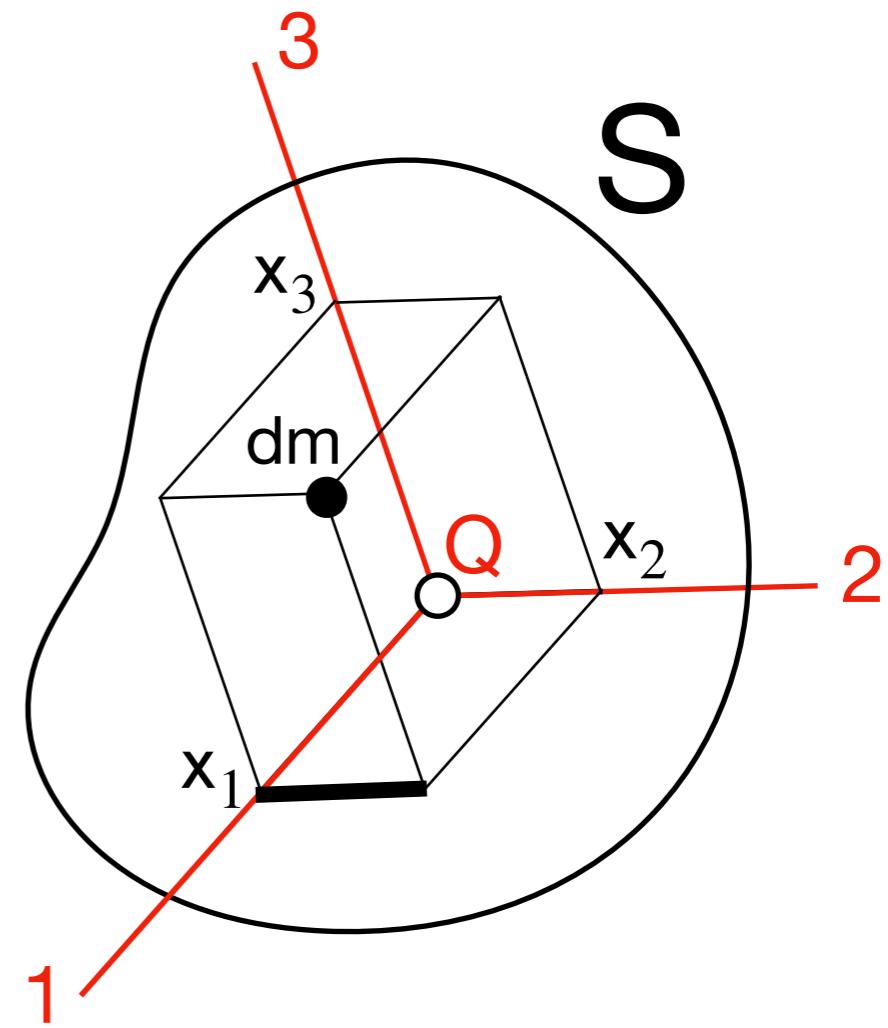
$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

Moments d'inèrcia

$$I_{ii} = \int_S (x_j^2 + x_k^2) dm$$

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$$B = (1, 2, 3)$$

Tensor d'inèrcia de S a Q

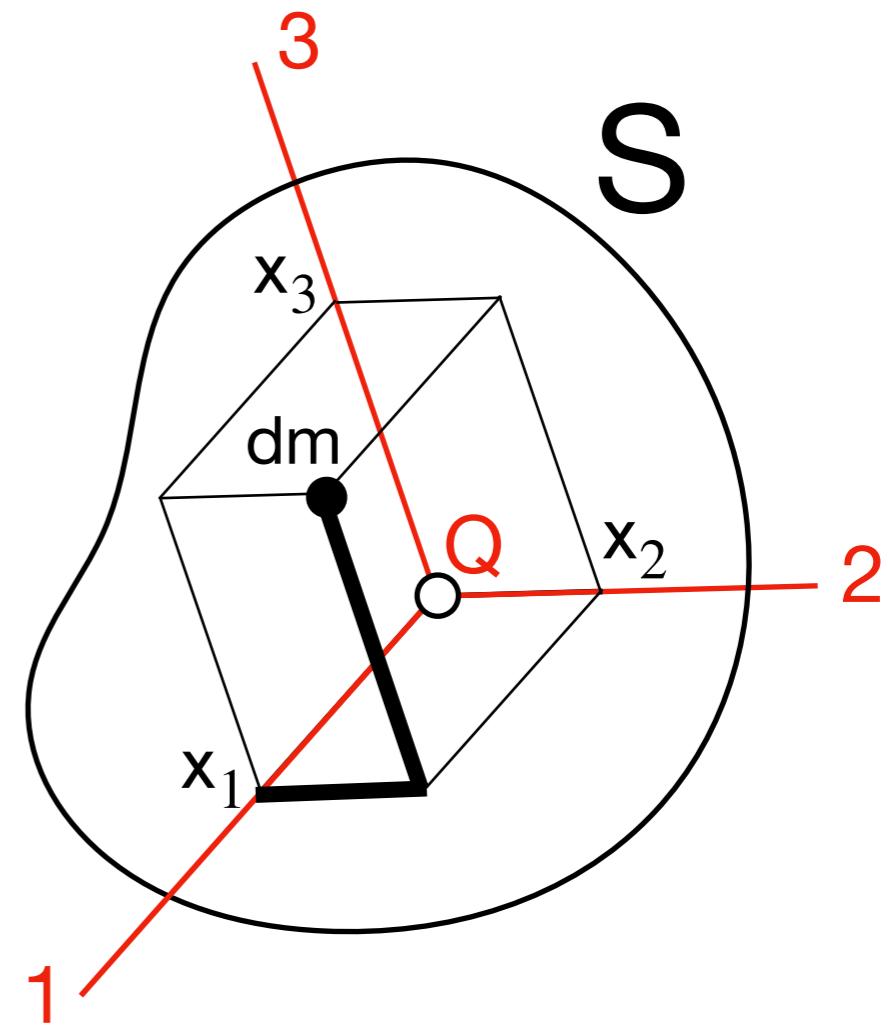
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Tensor d'inèrcia de S a Q

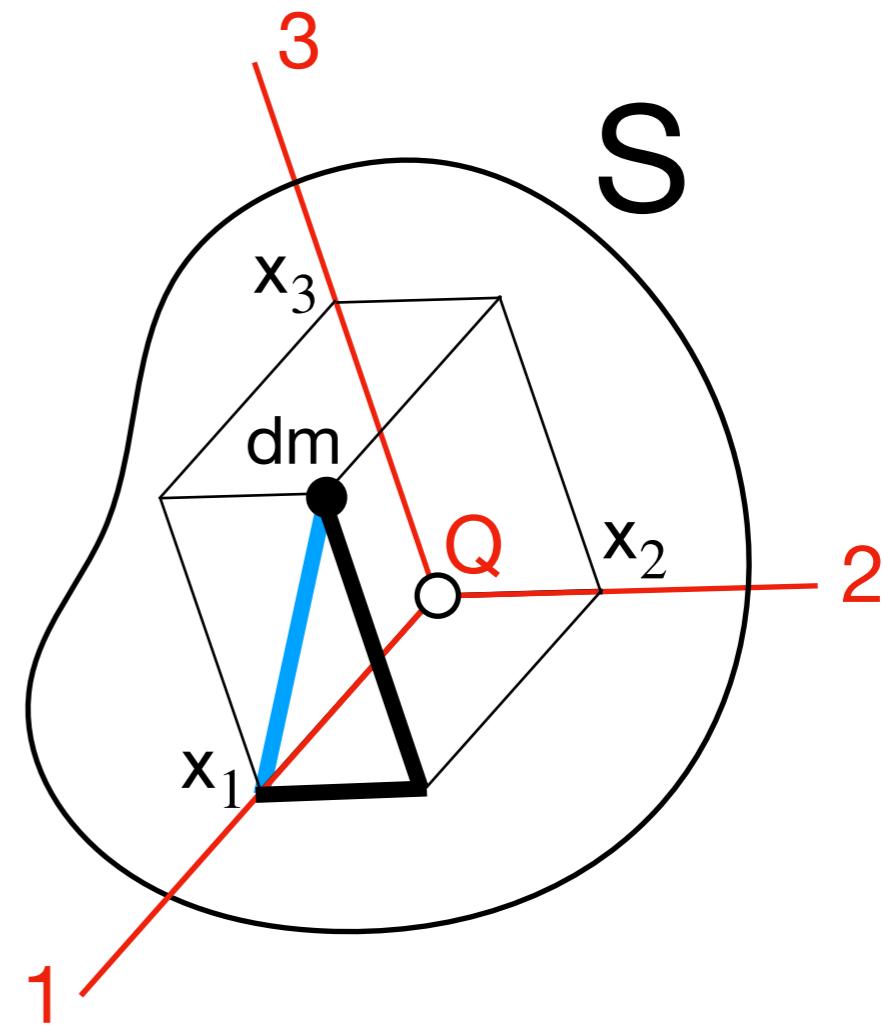
$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

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Tensor d'inèrcia de S a Q

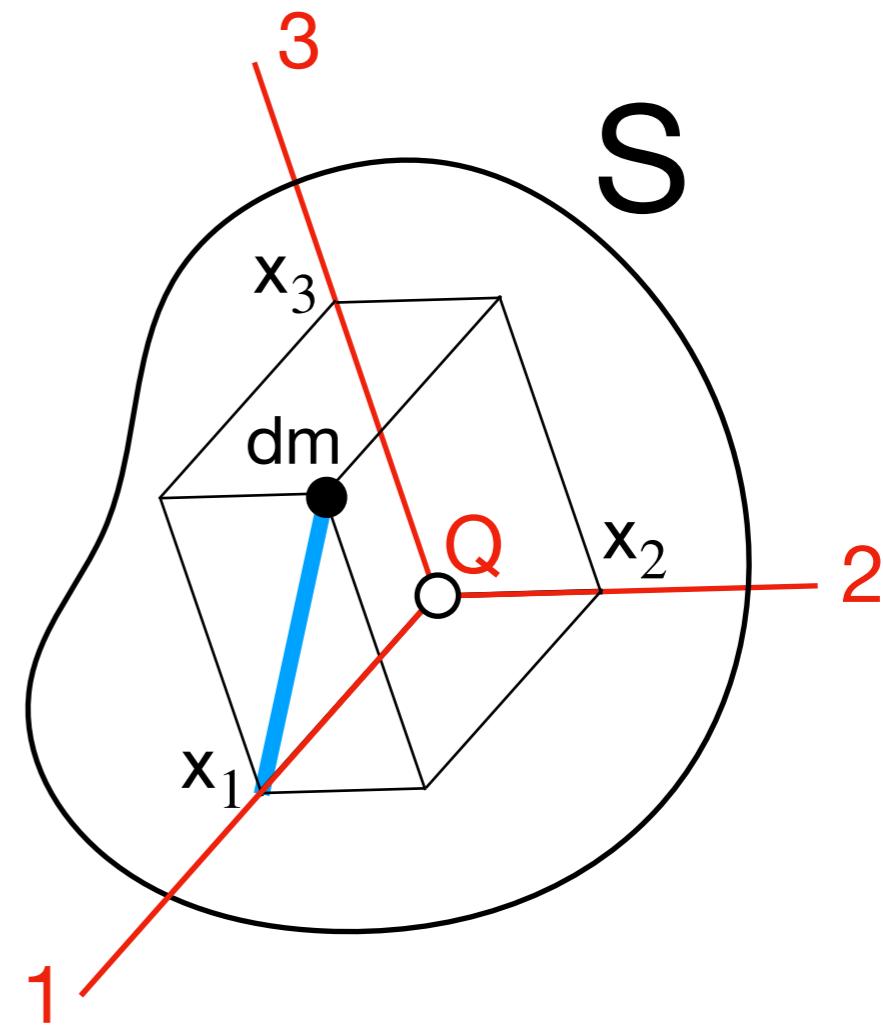
$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

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Tensor d'inèrcia de S a Q

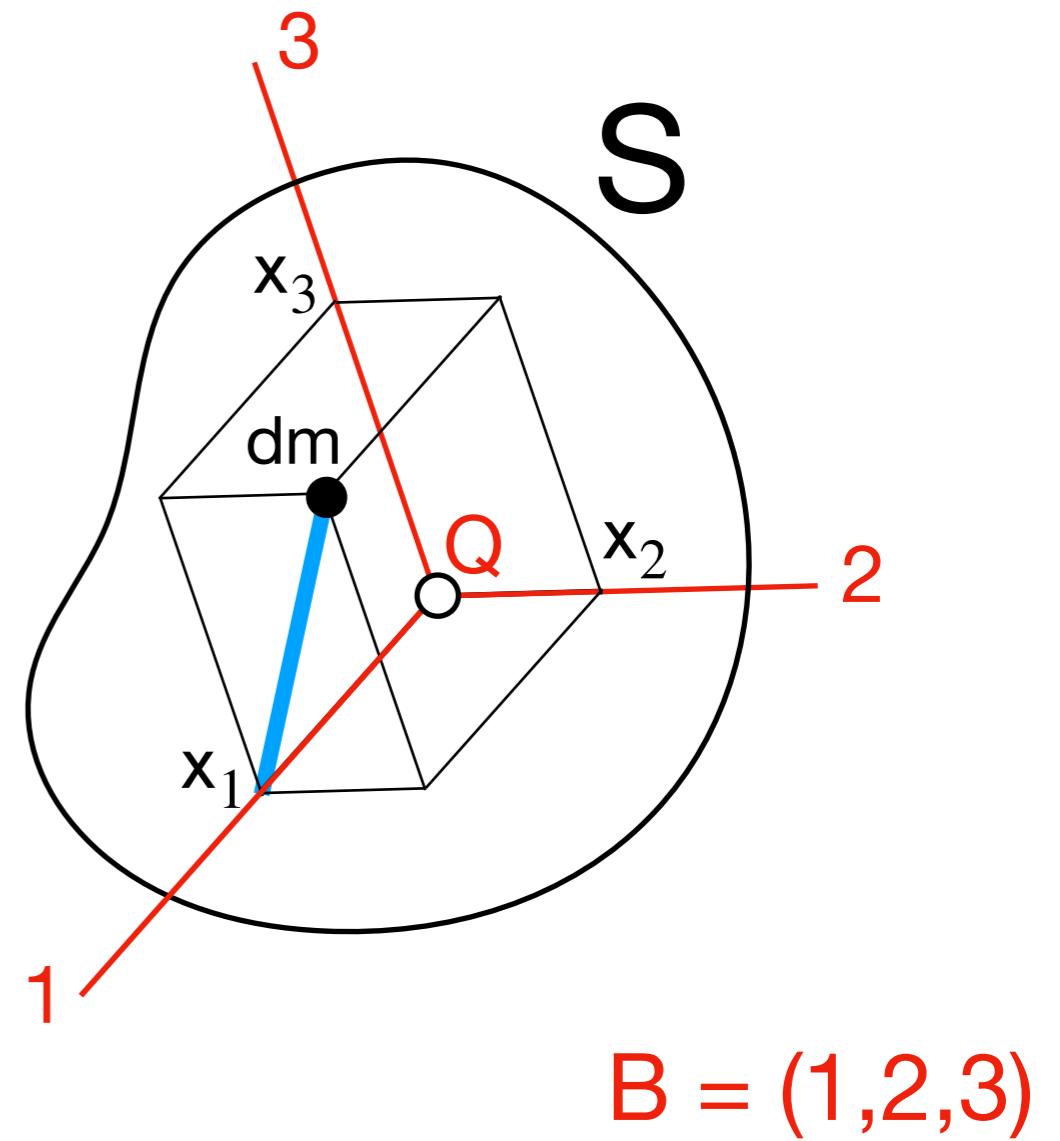
$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

Moments d'inèrcia

$$I_{ii} = \int_S (x_j^2 + x_k^2) dm$$

Exm:

$$I_{11} = \int_S \underbrace{(x_2^2 + x_3^2)}_{\text{dist a eix 1}^2} dm$$



Tensor d'inèrcia de S a Q

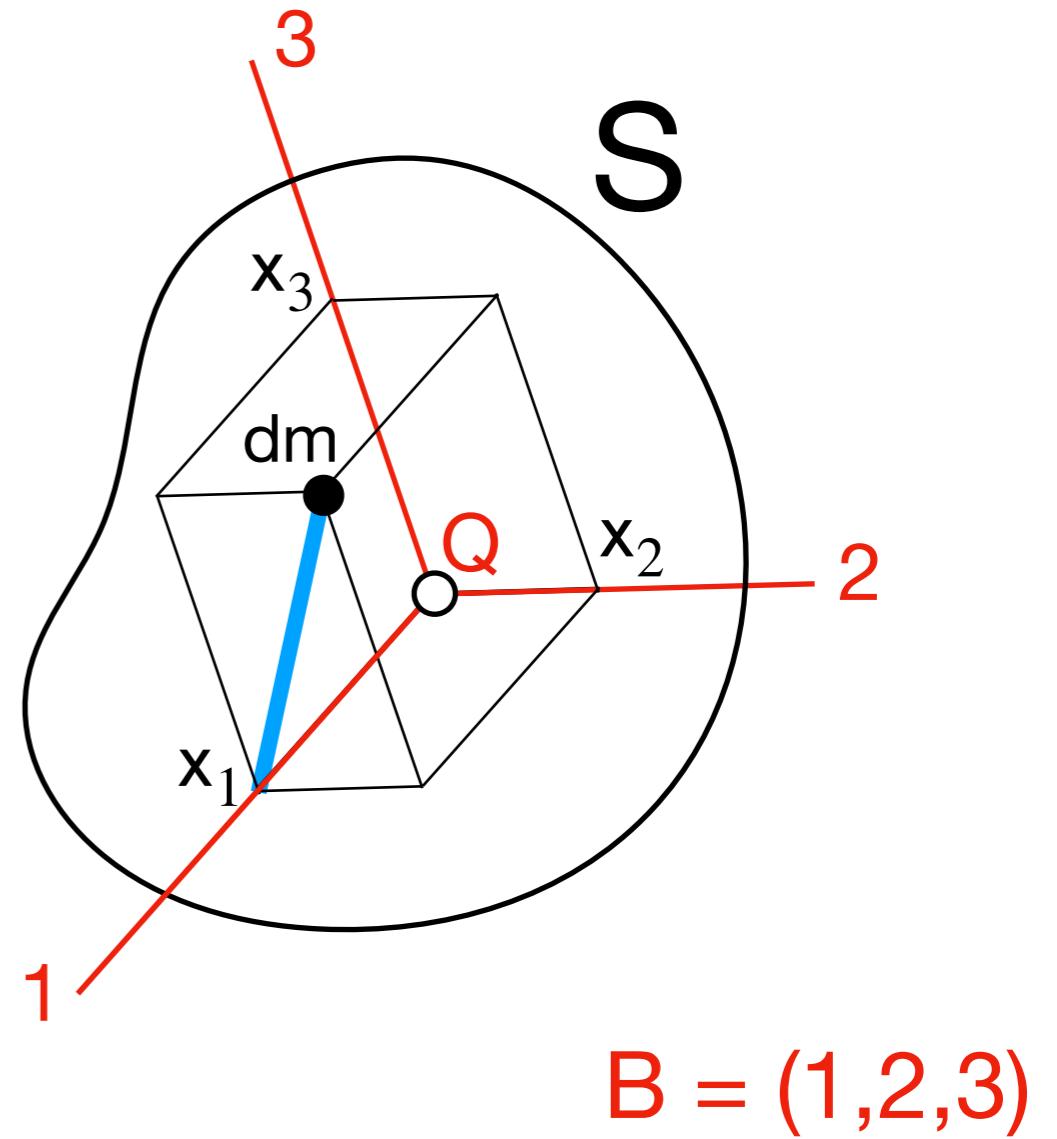
$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

Moments d'inèrcia

$$I_{ii} = \int_S \underbrace{\left(x_j^2 + x_k^2 \right)}_{\text{(dist a eix } i)^2} dm$$

Exm:

$$I_{11} = \int_S \underbrace{\left(x_2^2 + x_3^2 \right)}_{\text{(dist a eix 1)}^2} dm$$



B = (1,2,3)

Tensor d'inèrcia de S a Q

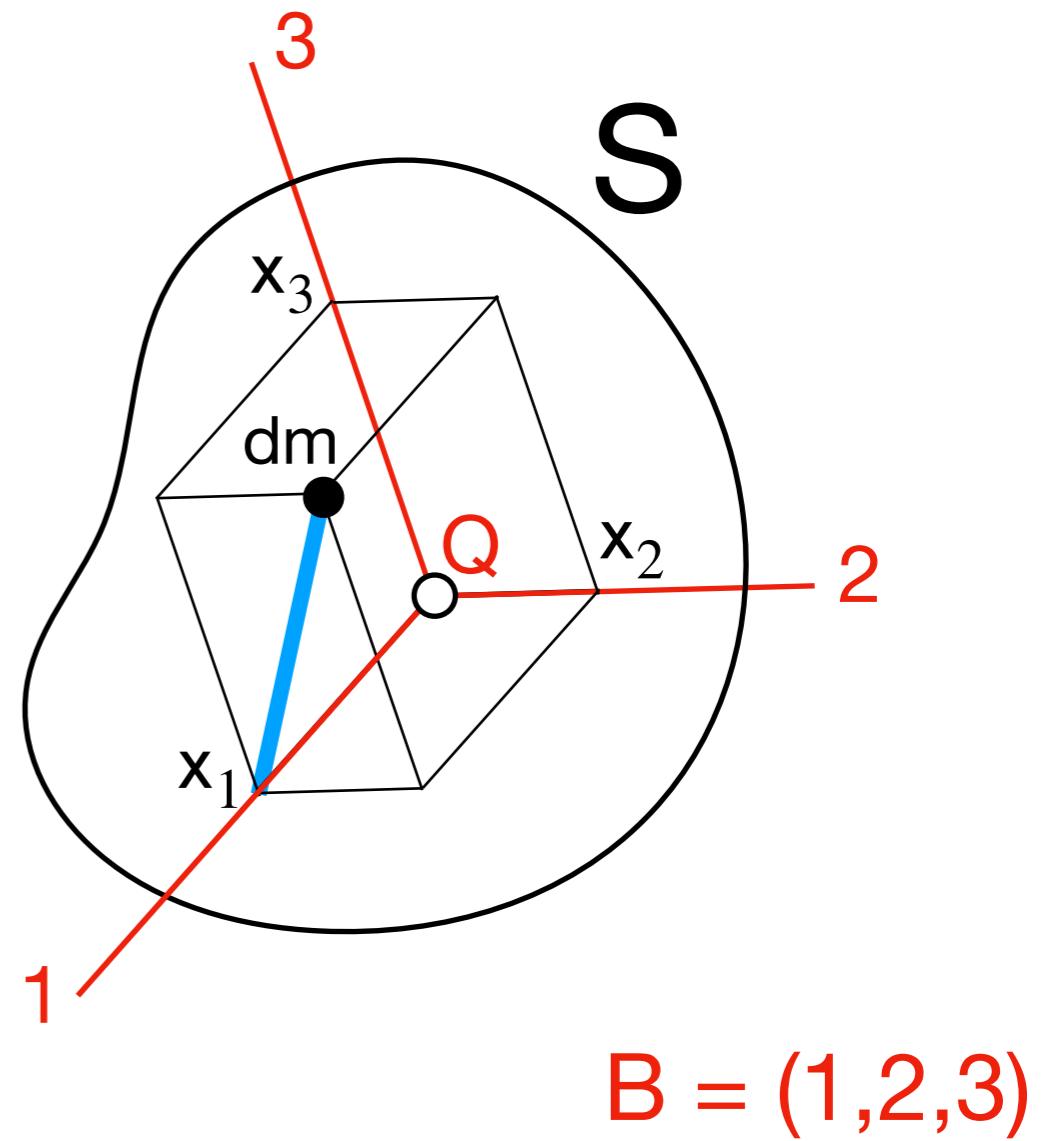
$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

Moments d'inèrcia

$$I_{ii} = \int_S \underbrace{\left(x_j^2 + x_k^2 \right)}_{\text{(dist a eix } i)^2} dm \geq 0$$

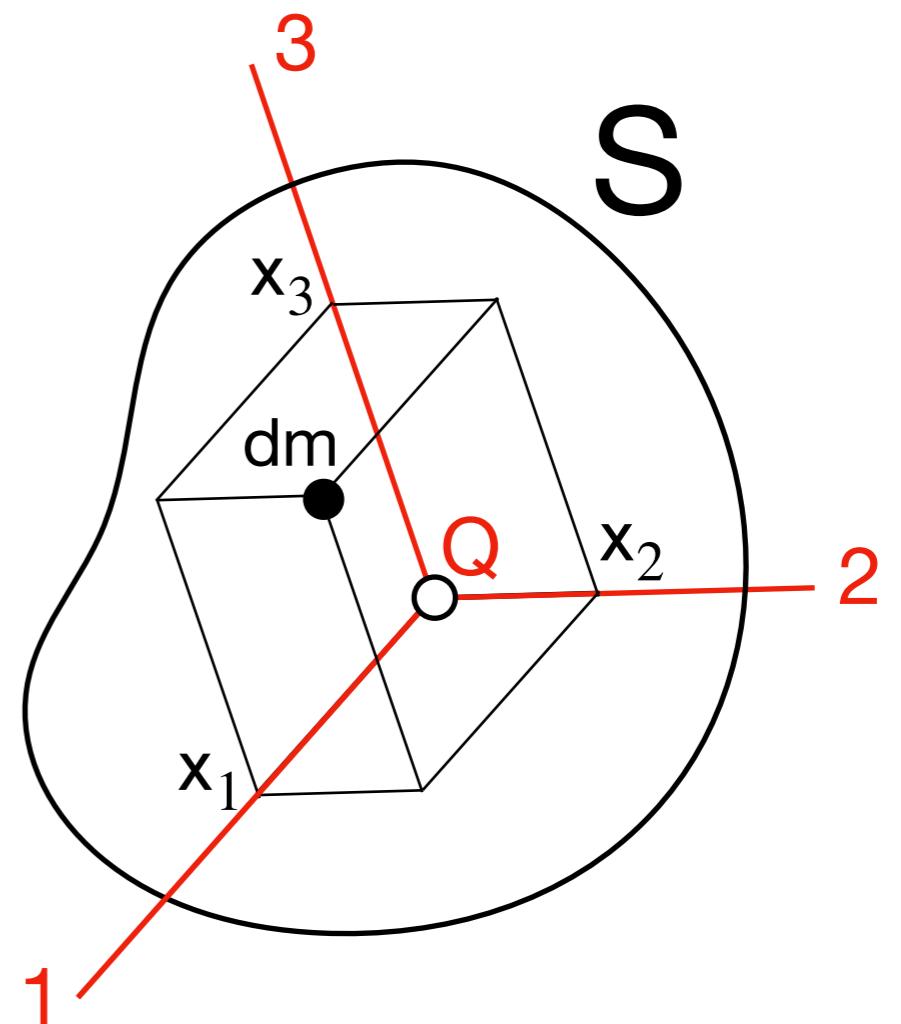
Exm:

$$I_{11} = \int_S \underbrace{\left(x_2^2 + x_3^2 \right)}_{\text{(dist a eix 1)}^2} dm$$



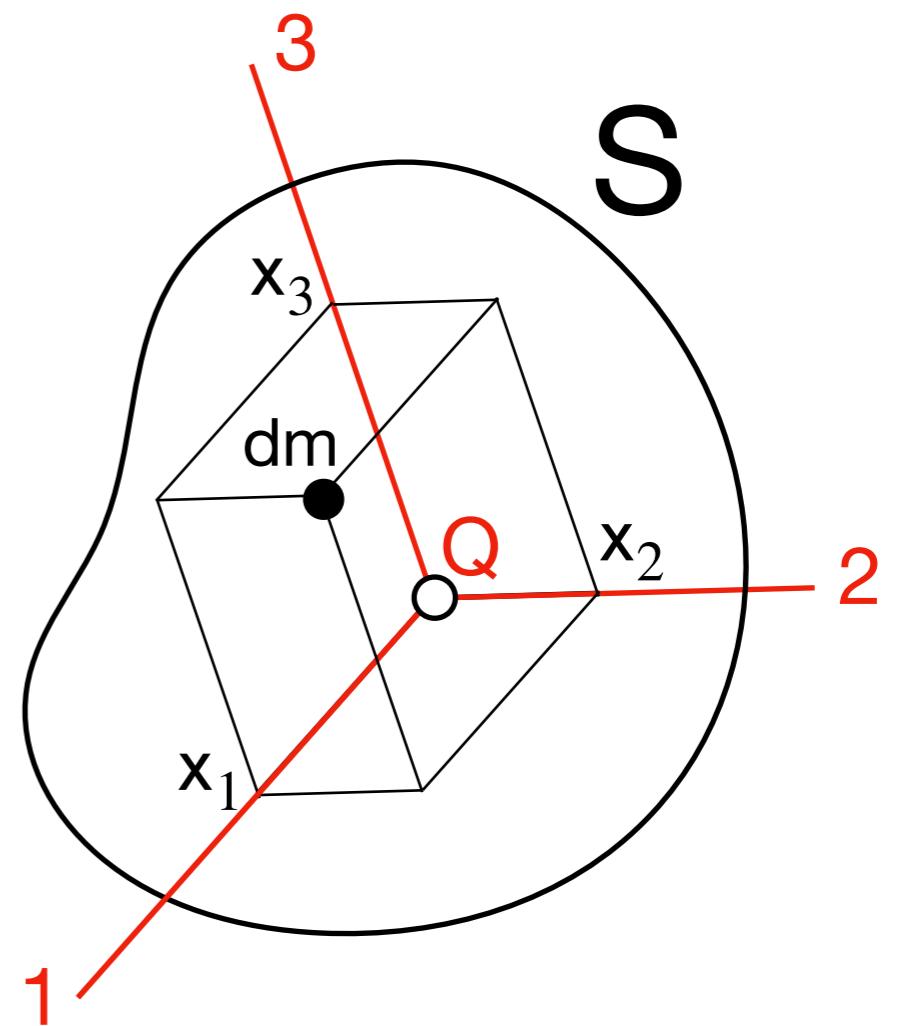
Tensor d'inèrcia de S a Q

$$[\mathbb{I} (Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$



Tensor d'inèrcia de S a Q

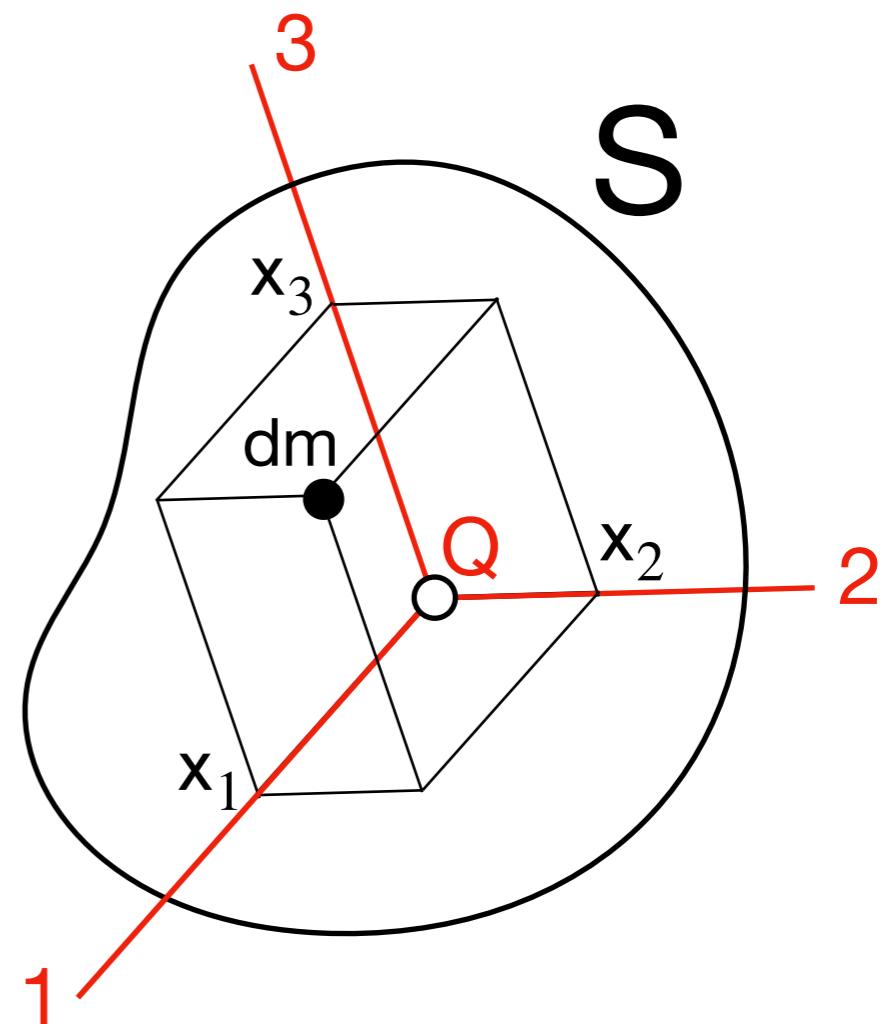
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Tensor d'inèrcia de S a Q

$$[\mathbb{I} (Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

Productes d'inèrcia

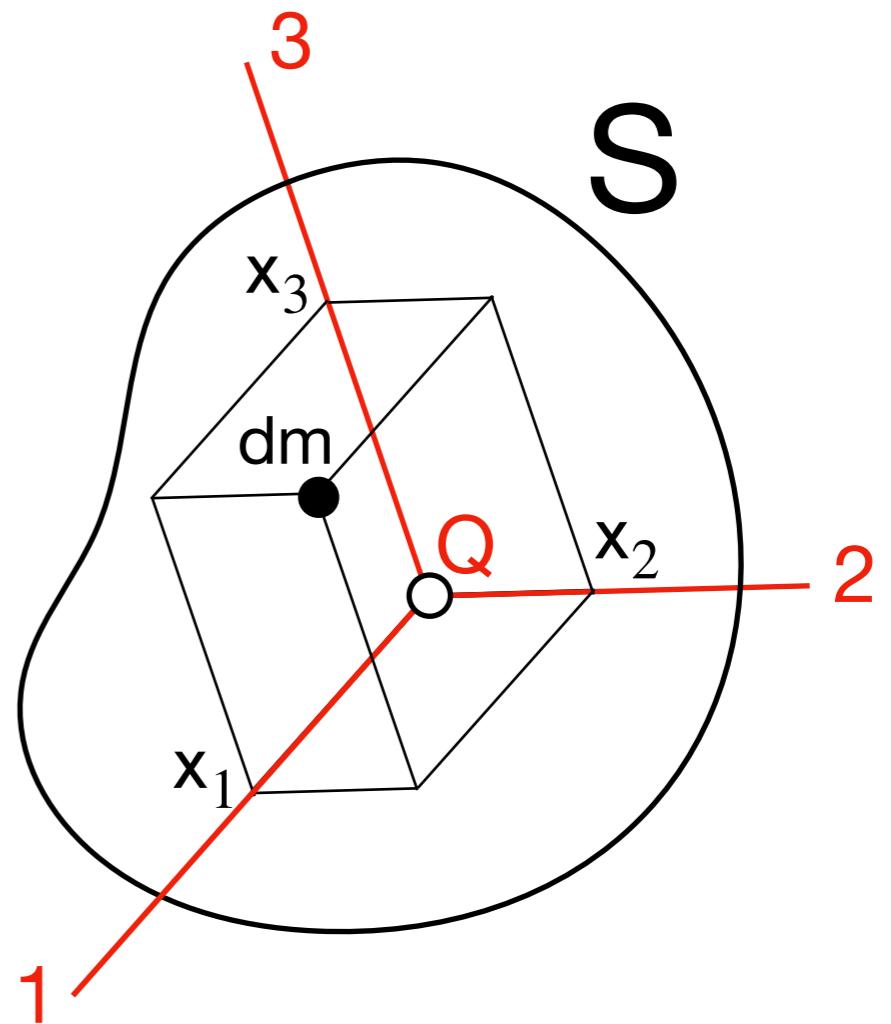


Tensor d'inèrcia de S a Q

$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

Productes d'inèrcia

$$I_{ij} = - \int_S x_i x_j dm$$



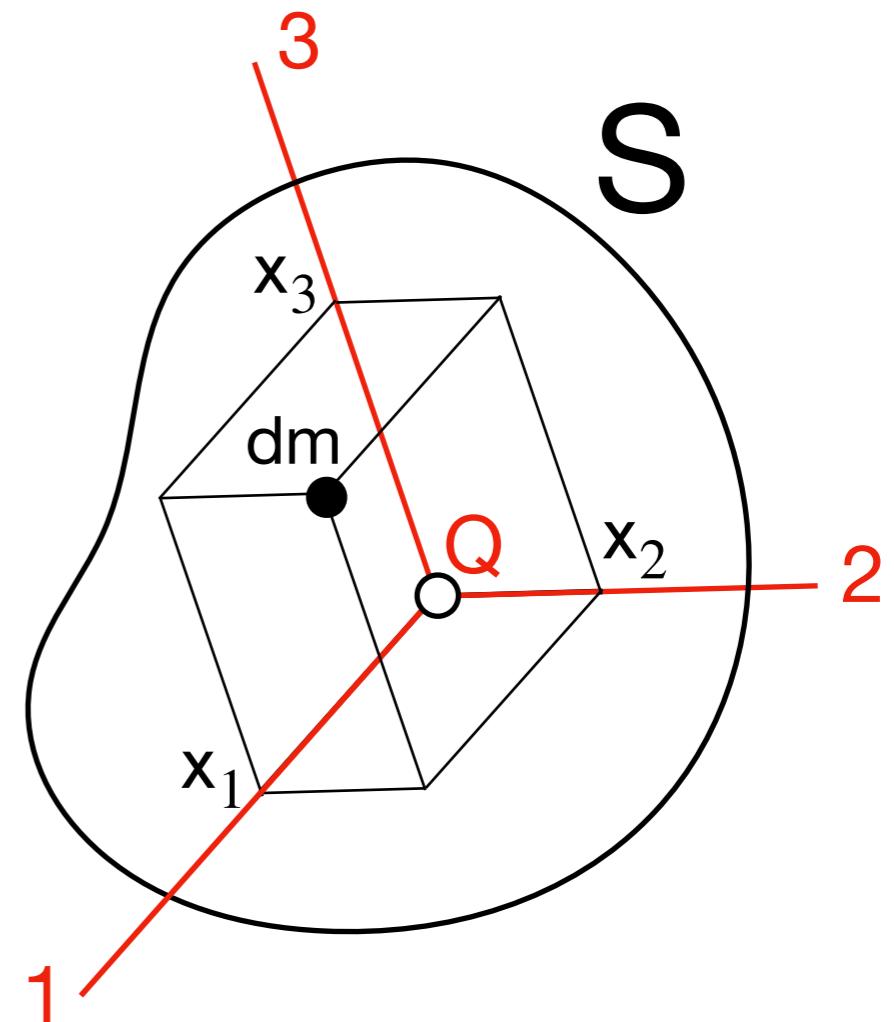
Tensor d'inèrcia de S a Q

$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

Productes d'inèrcia

$$I_{ij} = - \int_S x_i x_j dm$$

Exm:



Tensor d'inèrcia de S a Q

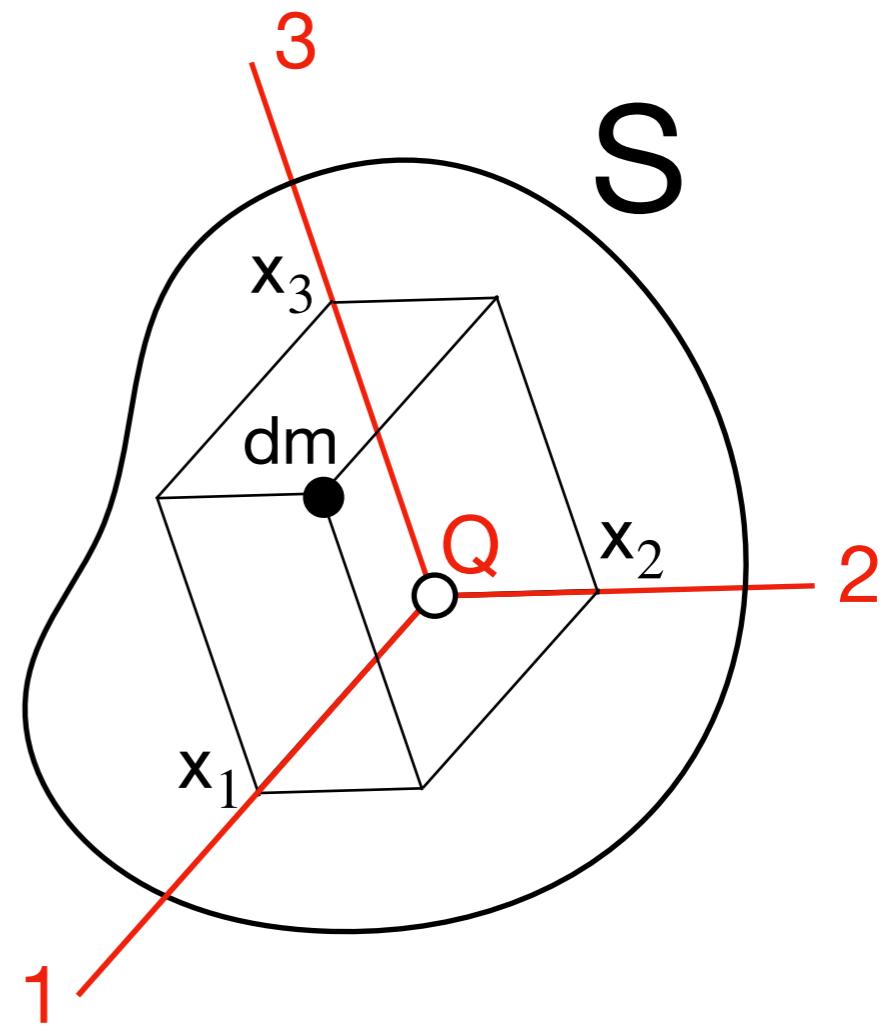
$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

Productes d'inèrcia

$$I_{ij} = - \int_S x_i x_j dm$$

Exm:

$$I_{12} = - \int_S x_1 x_2 dm$$



Tensor d'inèrcia de S a Q

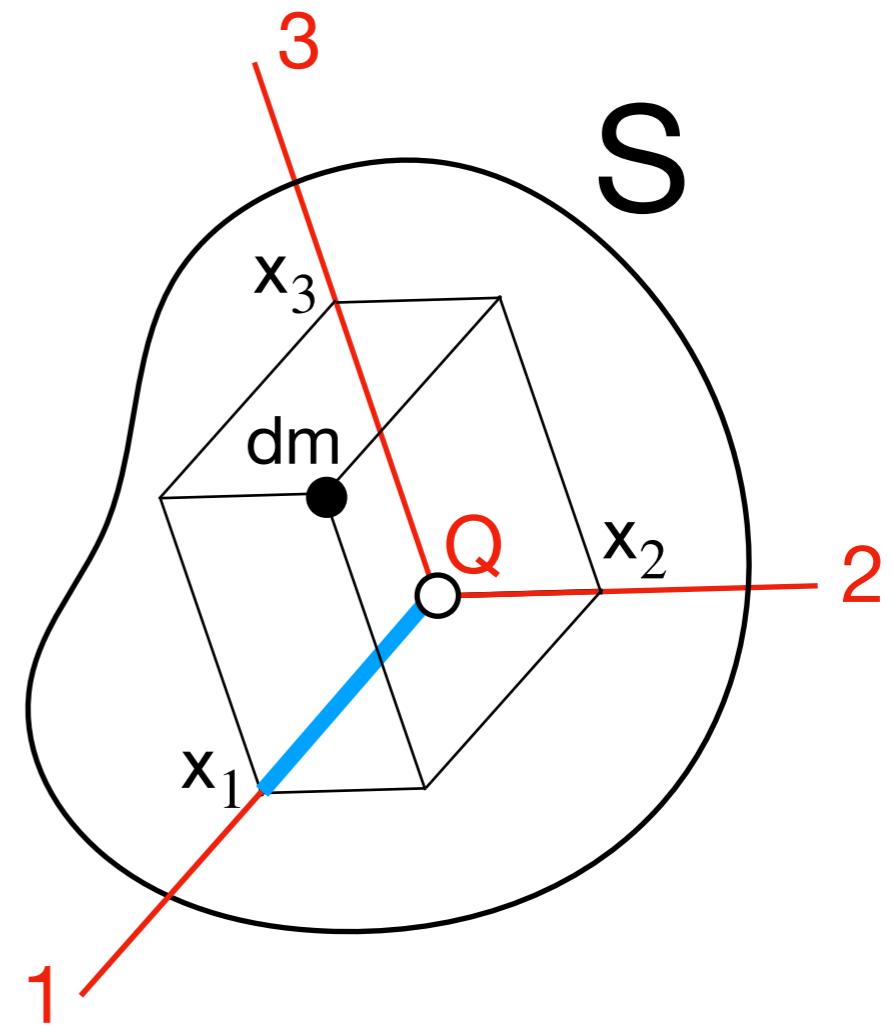
$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

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Tensor d'inèrcia de S a Q

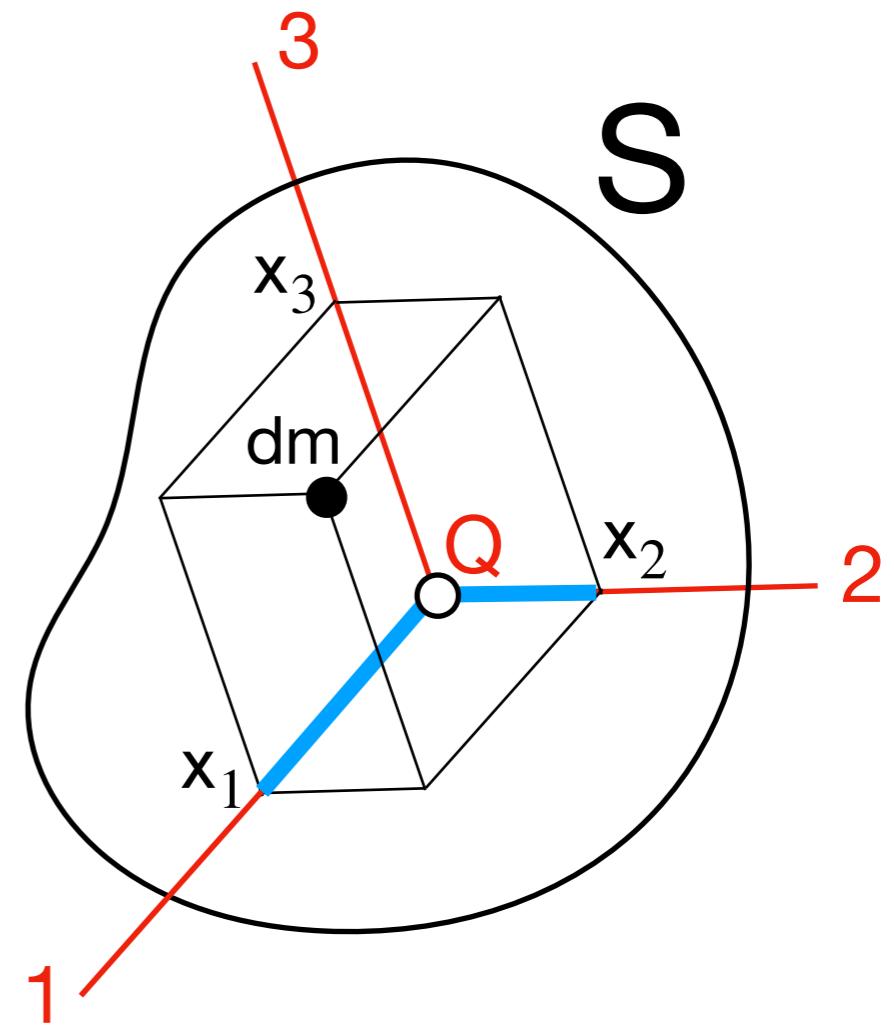
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Productes d'inèrcia

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Tensor d'inèrcia de S a Q

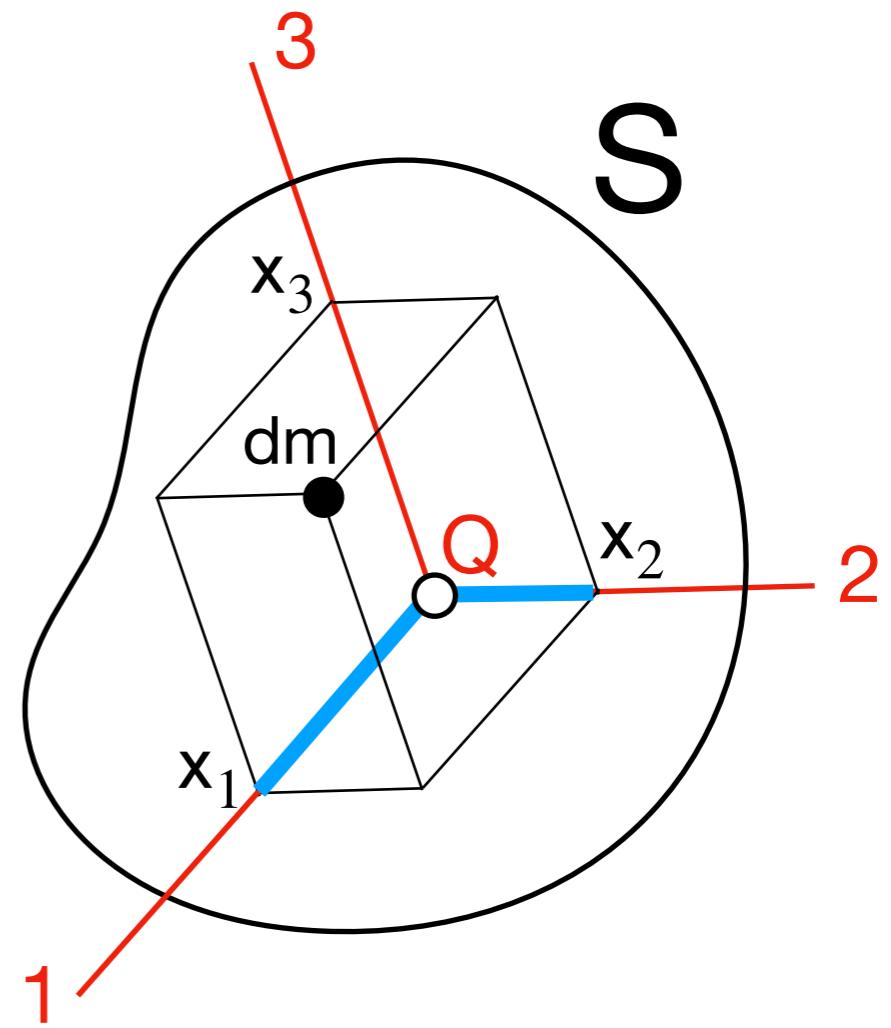
$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

Productes d'inèrcia

$$I_{ij} = - \int_S x_i x_j dm \quad (>0, <0, =0)$$

Exm:

$$I_{12} = - \int_S x_1 x_2 dm$$

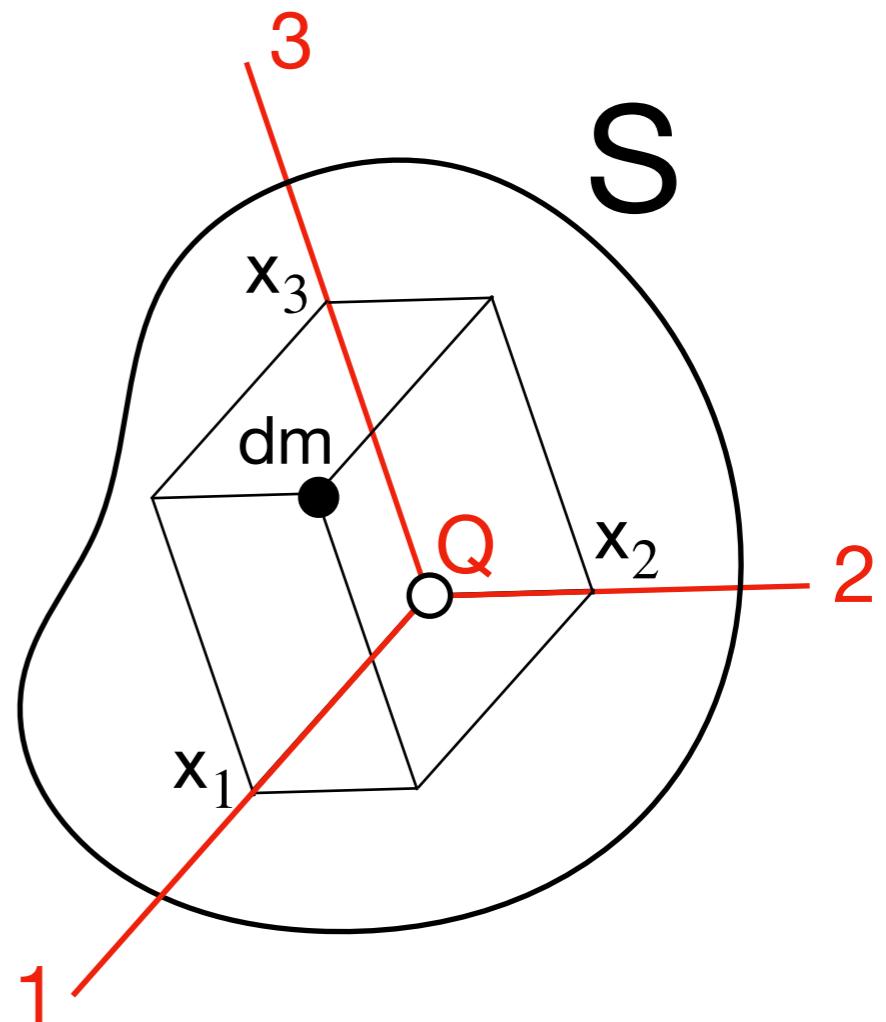


Tensor d'inèrcia de S a Q

$$[\mathbb{I}(Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

Sempre triarem B fixa a S

↓
tensor constant

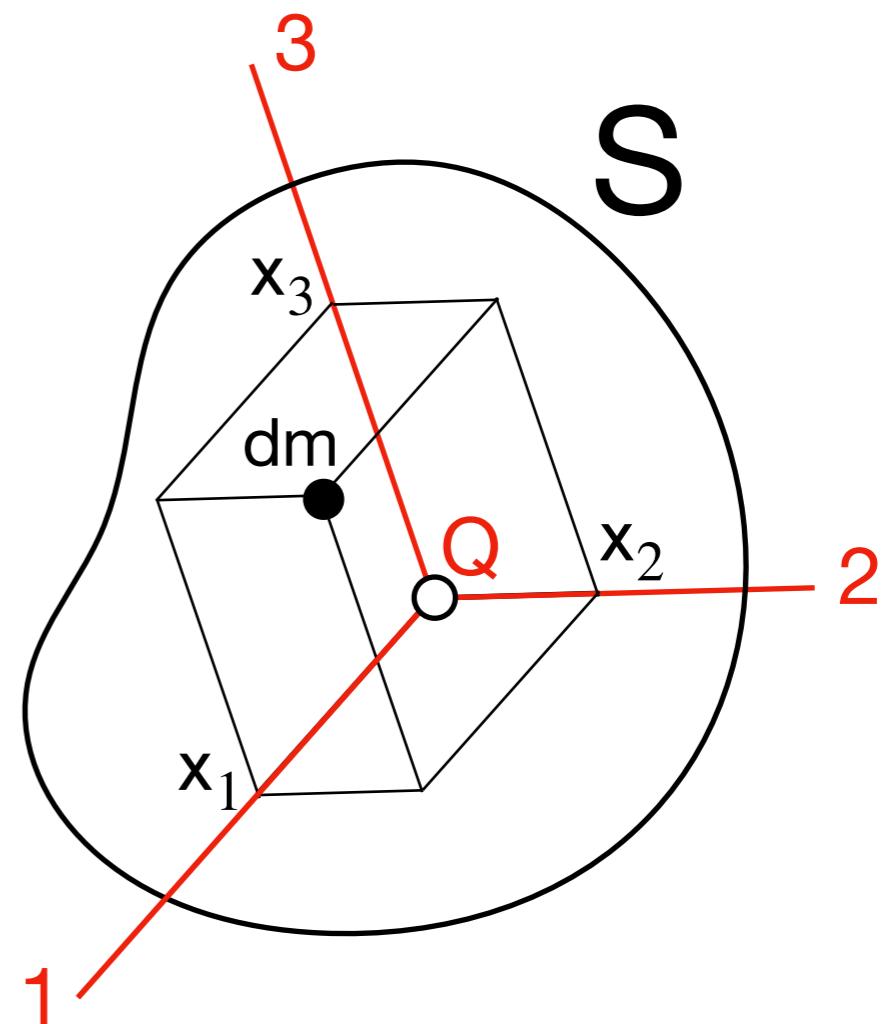


Tensor d'inèrcia de S a Q

$$[\mathbb{I} (Q)]_B = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$$

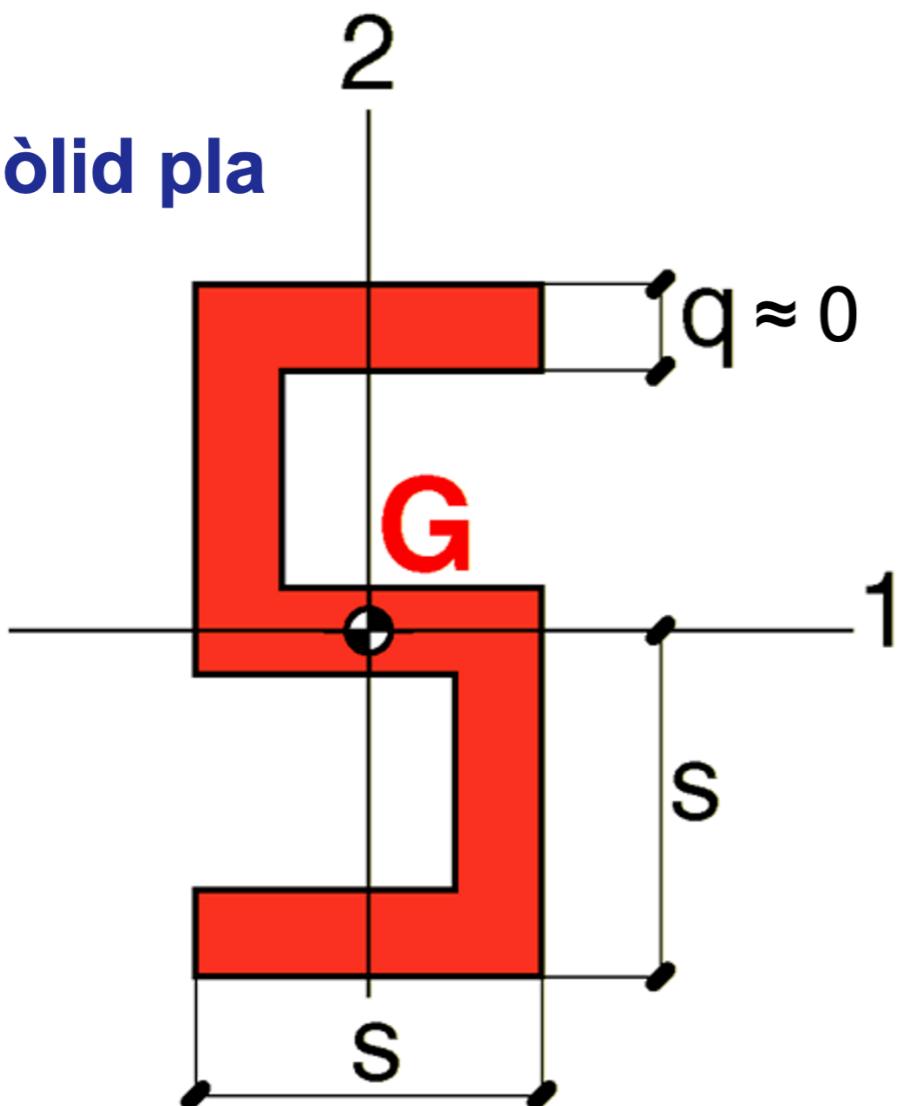
Aprendrem a construir-lo:

- Forma qualitativa
- Forma quantitativa

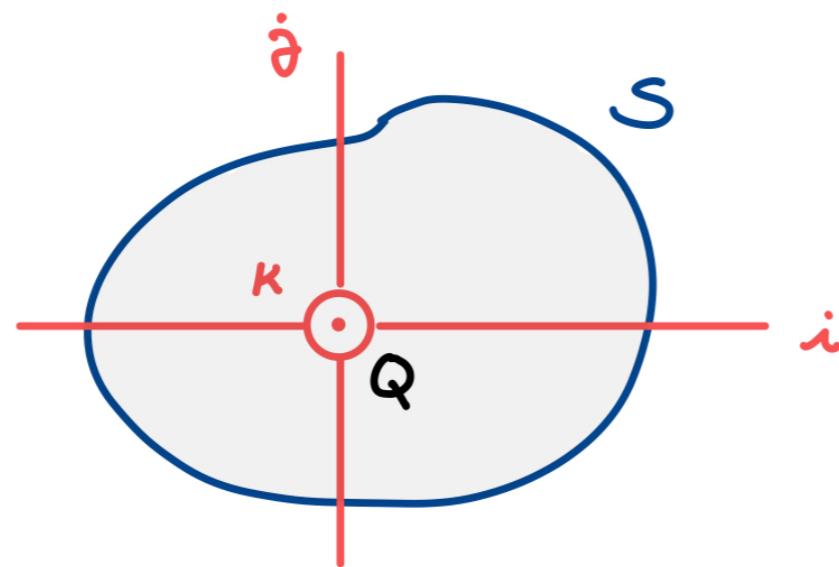


[II(G)] ?

sòlid pla



Si $S = \text{solid pla}$



$$B = (i, j, k)$$

es compleix:

$\forall Q \in S$ la dir. \perp a S es DPI

$$I_{kk} = I_{ii} + I_{jj}$$

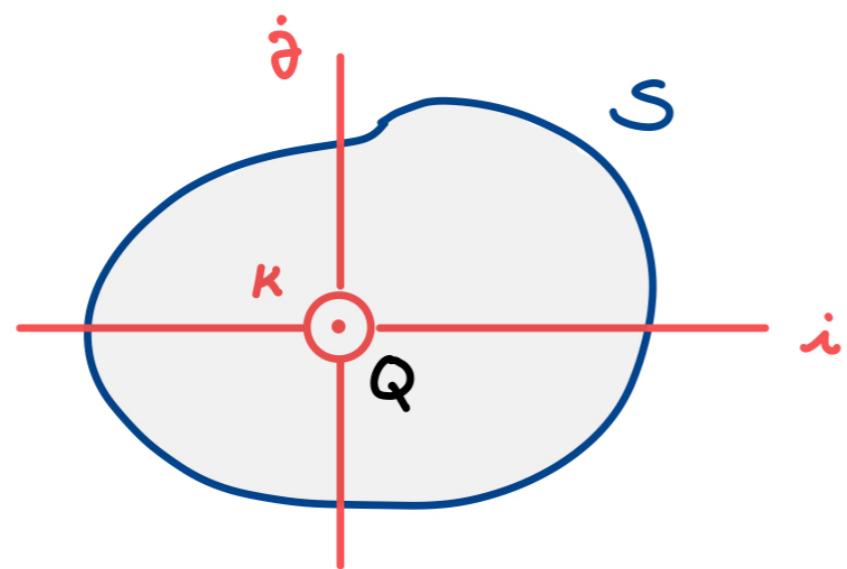
↪ És MPI

1

$$[\mathbb{II}(Q)]_B = \begin{bmatrix} I_{ii} & I_{ij} & 0 \\ I_{ij} & I_{jj} & 0 \\ 0 & 0 & I_{kk} \end{bmatrix}$$

The matrix has a circled entry $I_{ii} + I_{jj}$ at the bottom-right position.

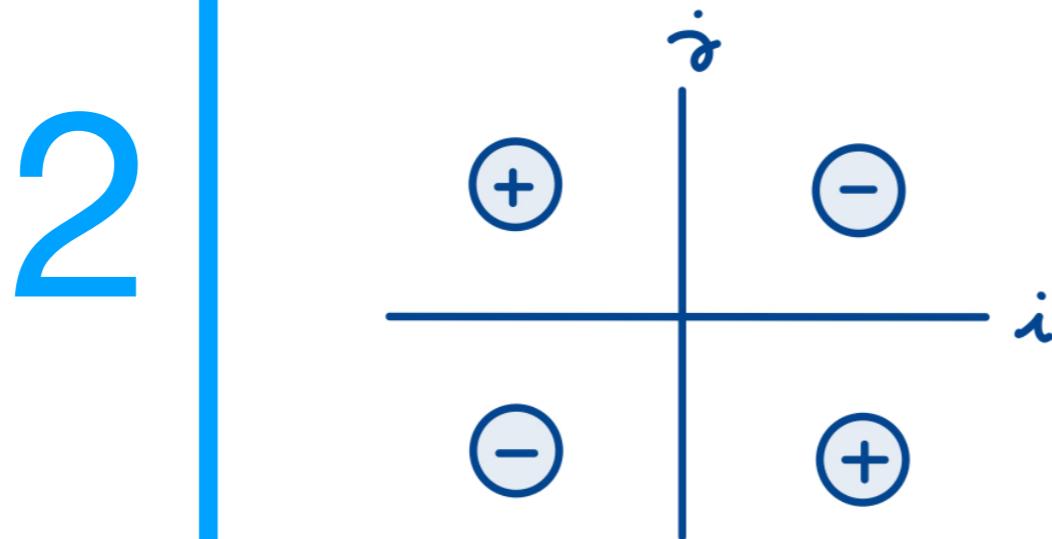
Si $S = \text{solid pla}$



$$B = (i, j, k)$$

es compleix:

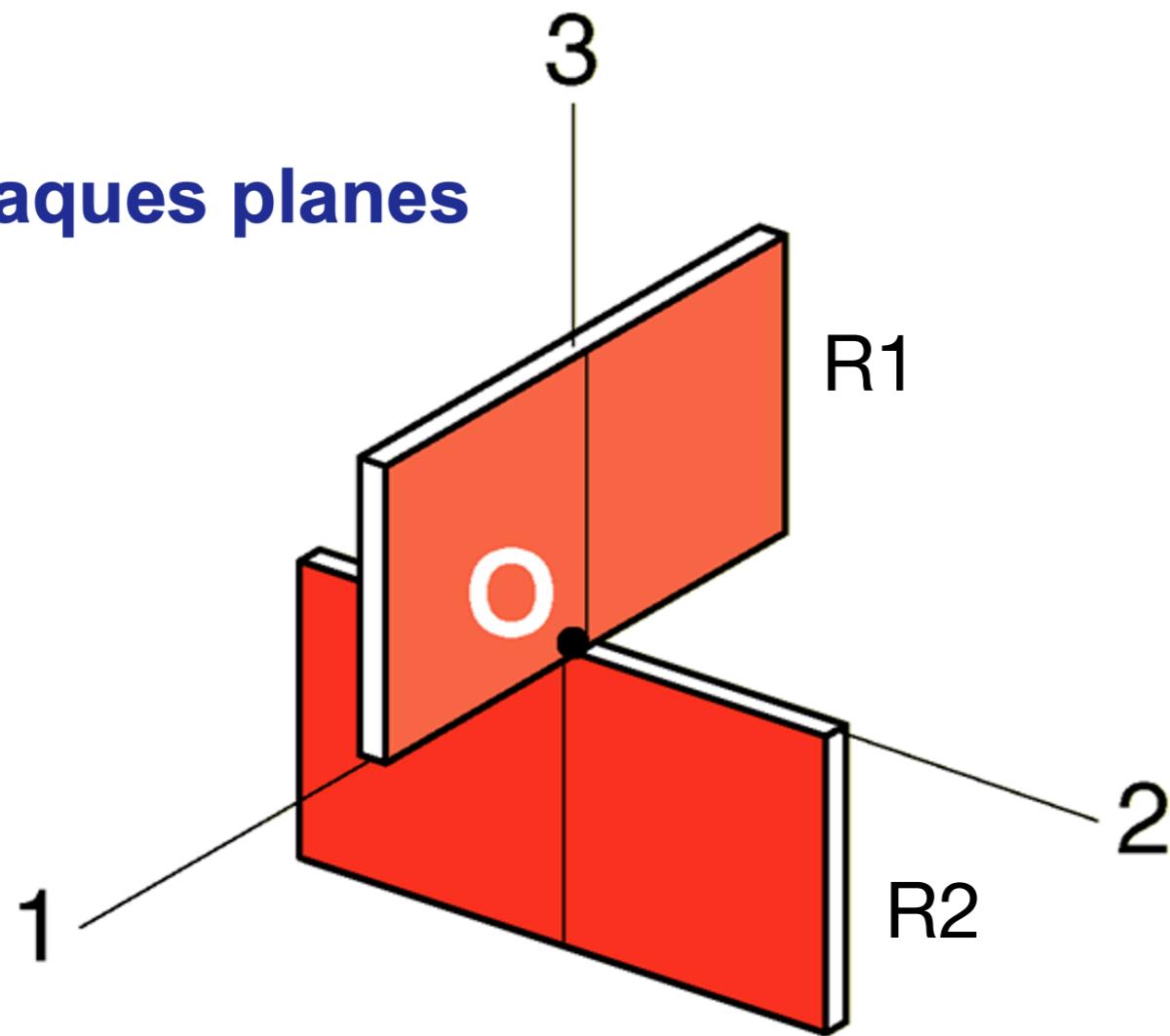
Signe contribució dm a I_{ij} :



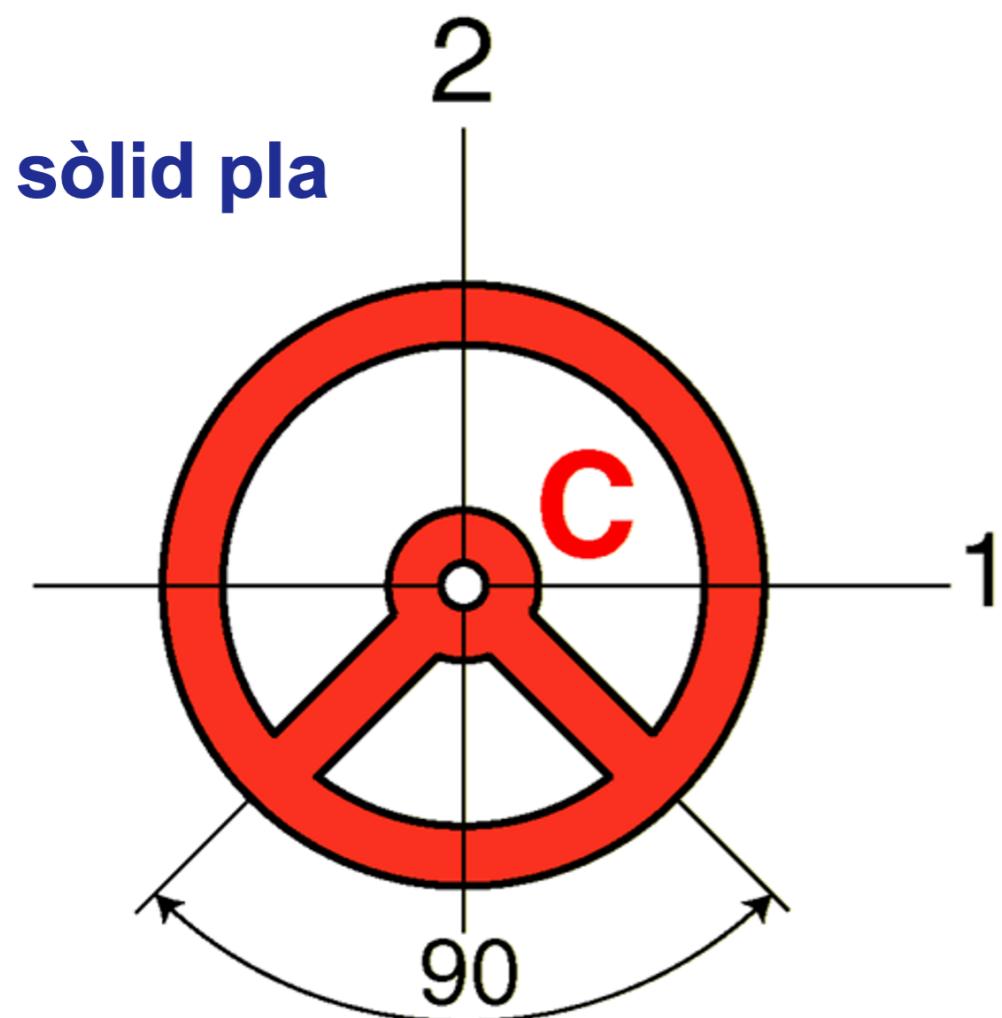
$$I_{ij} = - \int_S x_i x_j dm$$

[II(O)] ?
qualitatiu

plaques planes



[II(C)] ?
qualitatiu



Si per al punt O les direccions (i,j) són DPI
amb **mateix moment d'inèrcia**



$\mathbb{I}(O)$ és invariant a rotacions de la base
al voltant de l'eix k

$$\begin{bmatrix} I & & \\ & I & \\ & & I_{33} \end{bmatrix}$$

Invariant
rot **eix 3**

$$\begin{bmatrix} I & & \\ & I_{22} & \\ & & I \end{bmatrix}$$

Invariant
rot **eix 2**

$$\begin{bmatrix} I_{11} & & \\ & I & \\ & & I \end{bmatrix}$$

Invariant
rot **eix 1**

"Rotor simètric per O"

Si per al punt O les direccions (i,j) són DPI
amb mateix moment d'inèrcia



$\mathbb{I}(O)$ és invariant a rotacions de la base
al voltant de l'eix k

$$\begin{bmatrix} I & & \\ & I & \\ & & I_{33} \end{bmatrix}$$

Invariant
rot **eix 3**

$$\begin{bmatrix} I & & \\ & & I_{22} \\ & & I \end{bmatrix}$$

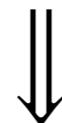
Invariant
rot **eix 2**

$$\begin{bmatrix} I_{11} & & \\ & I & \\ & & I \end{bmatrix}$$

Invariant
rot **eix 1**

"Rotor esfèric per O"

Si per al punt O les direccions (i,j,k) són
DPI amb mateix moment d'inèrcia

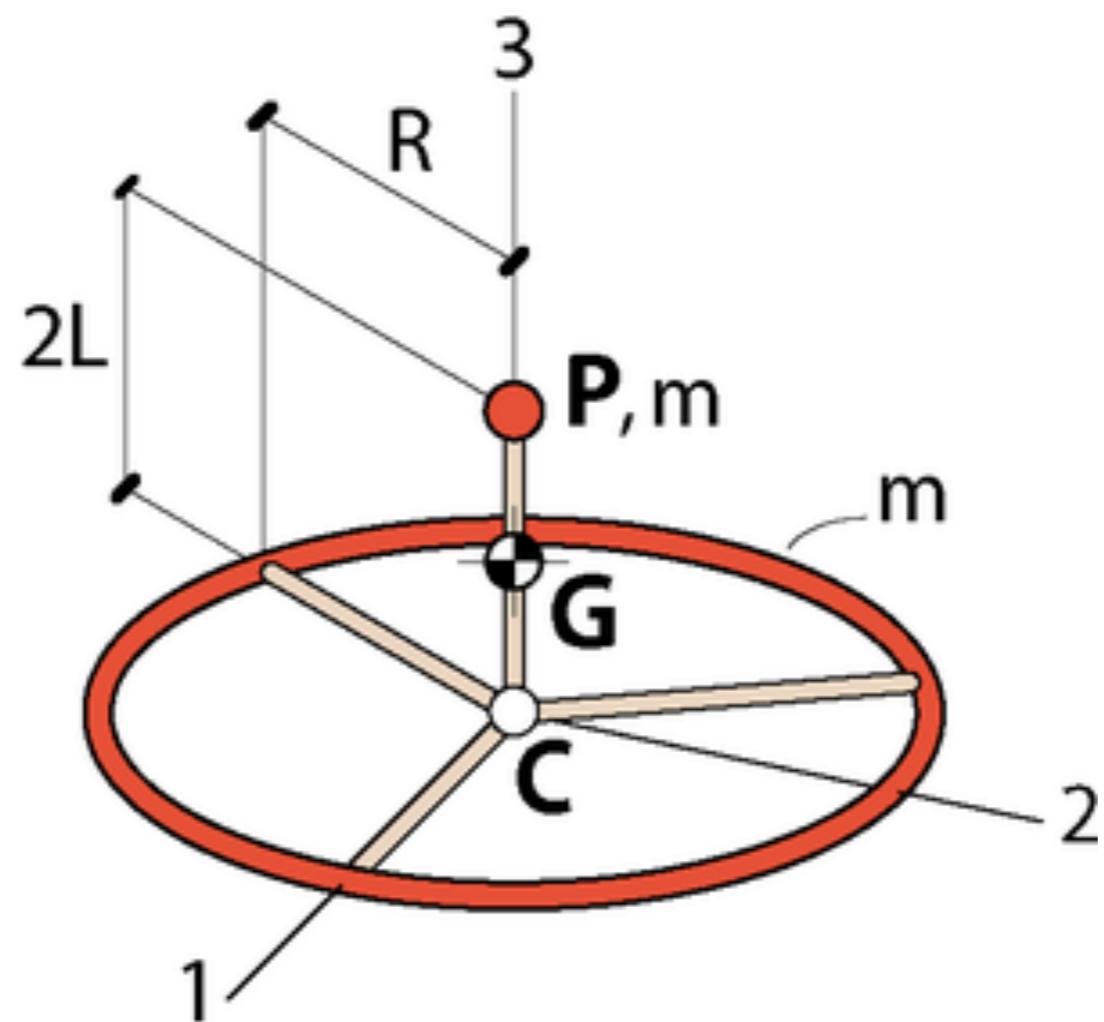


$\mathbb{I}(O)$ té la forma

$$\begin{bmatrix} I & & \\ & I & \\ & & I \end{bmatrix}$$

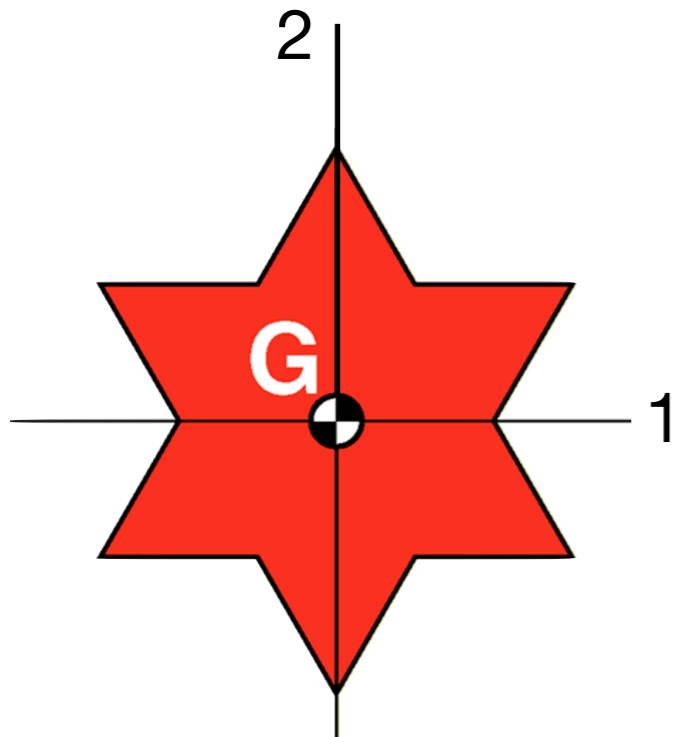
independently de la base triada

Exemple D5.9 - Wikimec



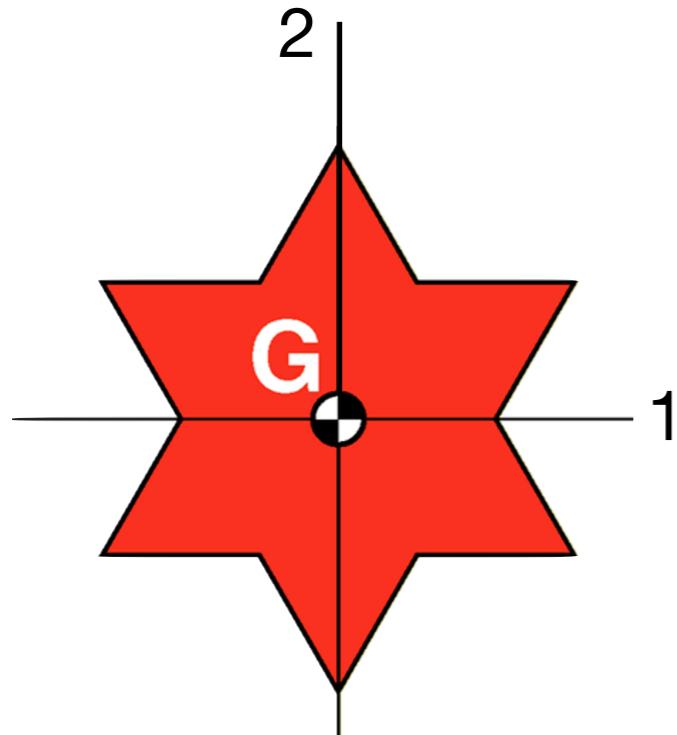
[II(G)] ?

qualitatiu



[II(G)] ?

qualitatiu

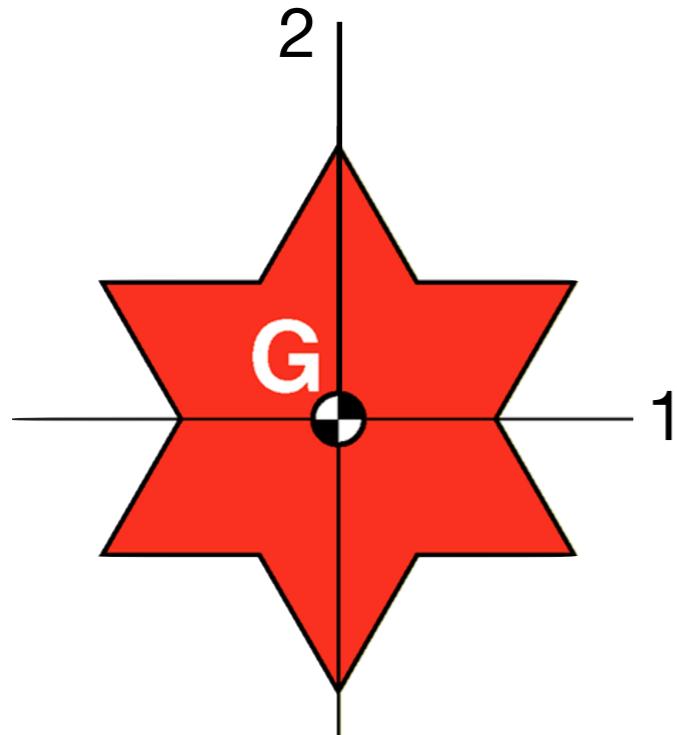


Sòlid pla i eix 2 de simetria:

$$[\mathbb{I}(G)]_B = \begin{bmatrix} I_{11} & & \\ & I_{22} & \\ & & I_{11} + I_{22} \end{bmatrix}$$

[II(G)] ?

qualitatiu



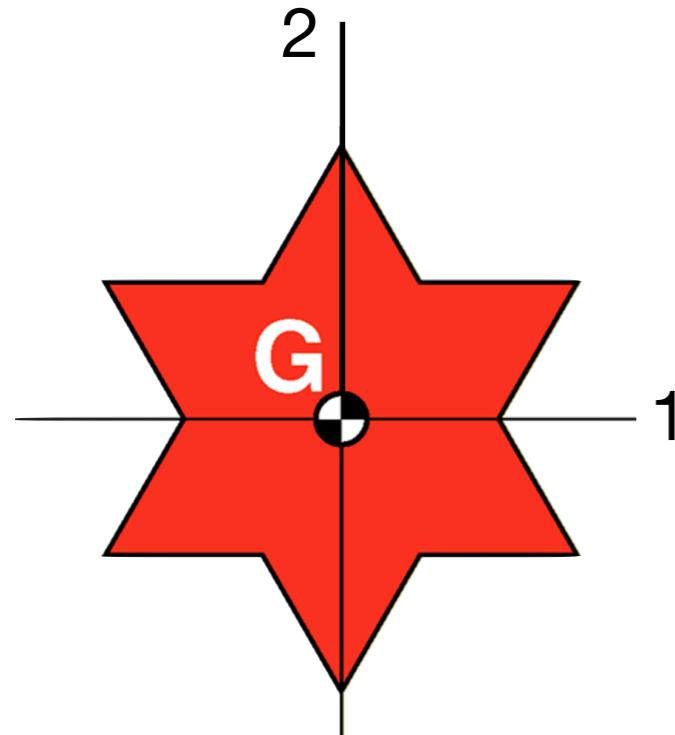
Sòlid pla i eix 2 de simetria:

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I_{11}, I_{22} ?

[II(G)] ?

qualitatiu



Sòlid pla i eix 2 de simetria:

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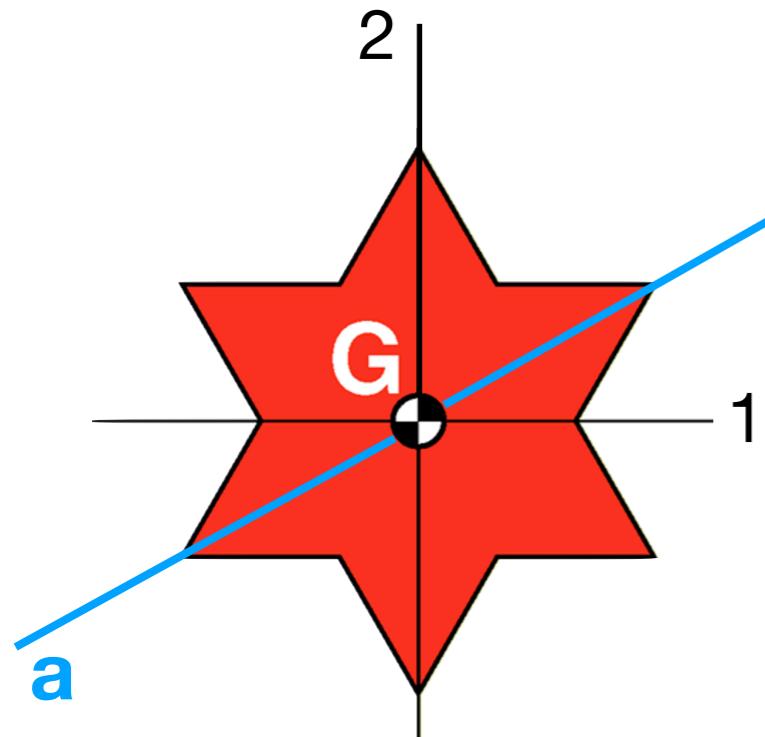
I_{11}, I_{22} ?

Per un punt O

Si 3 o més moments d'inèrcia
en un mateix pla (i,j) són iguals,
el sòlid és **rotor simètric a O**

[II(G)] ?

qualitatiu



Sòlid pla i eix 2 de simetria:

$$[\mathbb{I}(G)]_B = \begin{bmatrix} I_{11} & & \\ & I_{22} & \\ & & I_{11} + I_{22} \end{bmatrix}$$

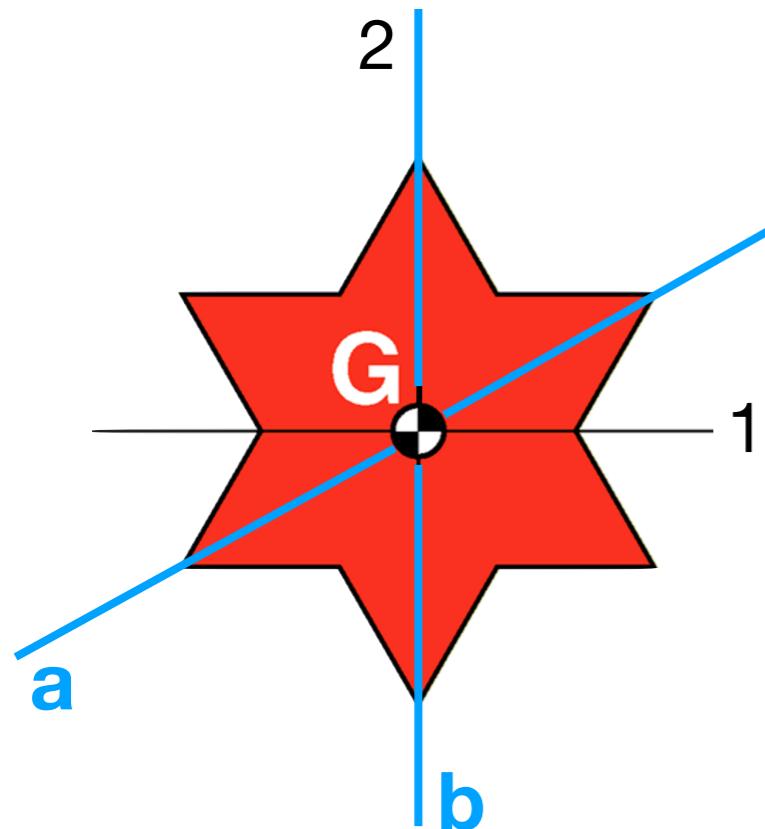
I_{11}, I_{22} ?

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[II(G)] ?

qualitatiu



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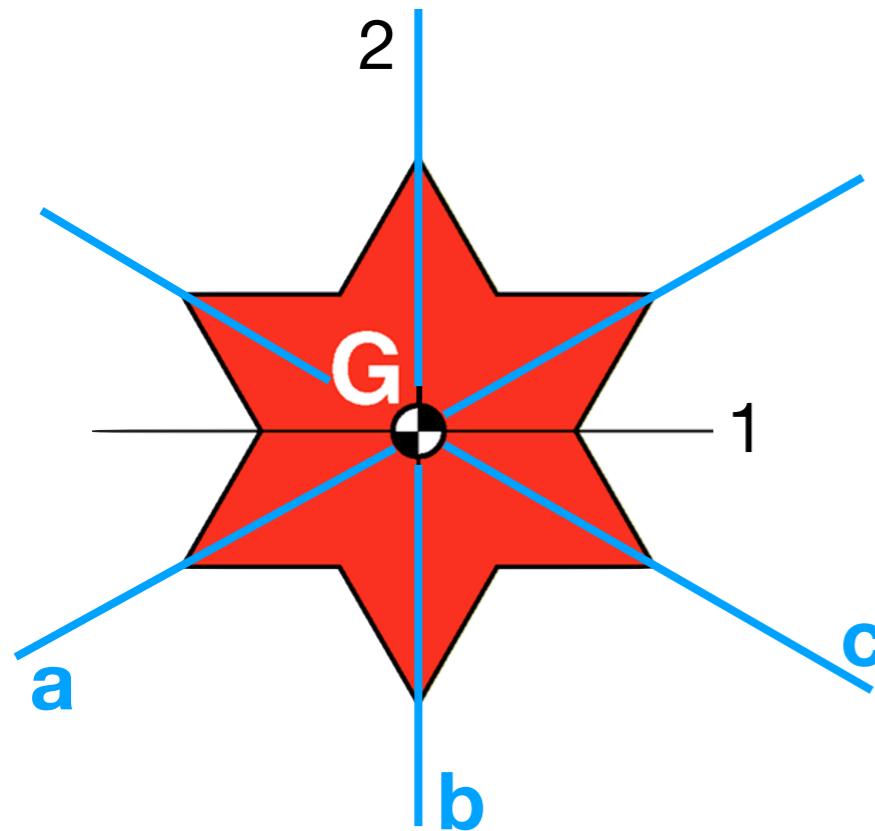
I_{11}, I_{22} ?

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qualitatiu



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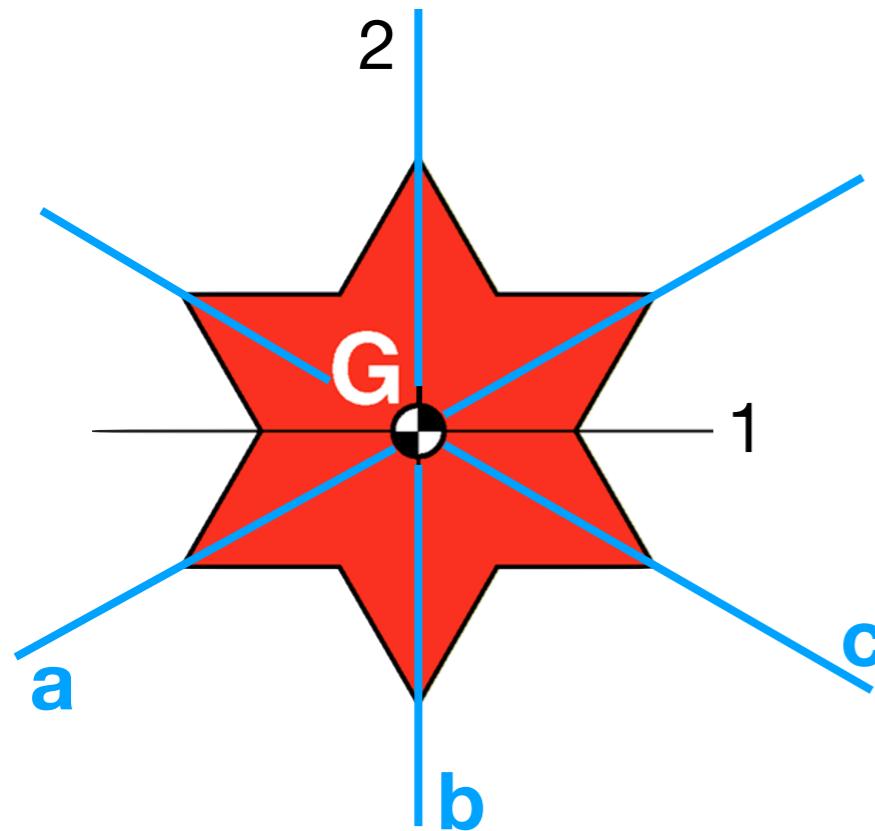
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qualitatiu



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I_{11}, I_{22} ?

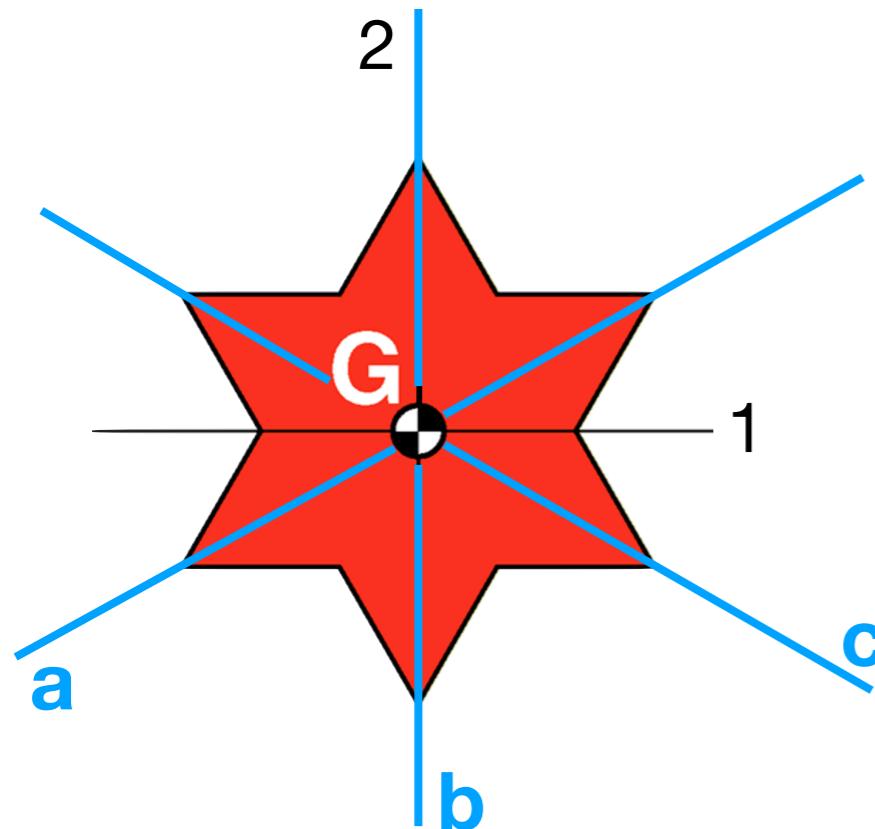
3 mom. inèrcia en pla (1,2) iguals

Per un punt O

Si 3 o més moments d'inèrcia
en un mateix pla (i,j) són iguals,
el sòlid és **rotor simètric a O**

[II(G)] ?

qualitatiu



Sòlid pla i eix 2 de simetria:

$$[\mathbb{I}(G)]_B = \begin{bmatrix} I_{11} & & \\ & I_{22} & \\ & & I_{11} + I_{22} \end{bmatrix}$$

I_{11}, I_{22} ?

3 mom. inèrcia en pla (1,2) iguals

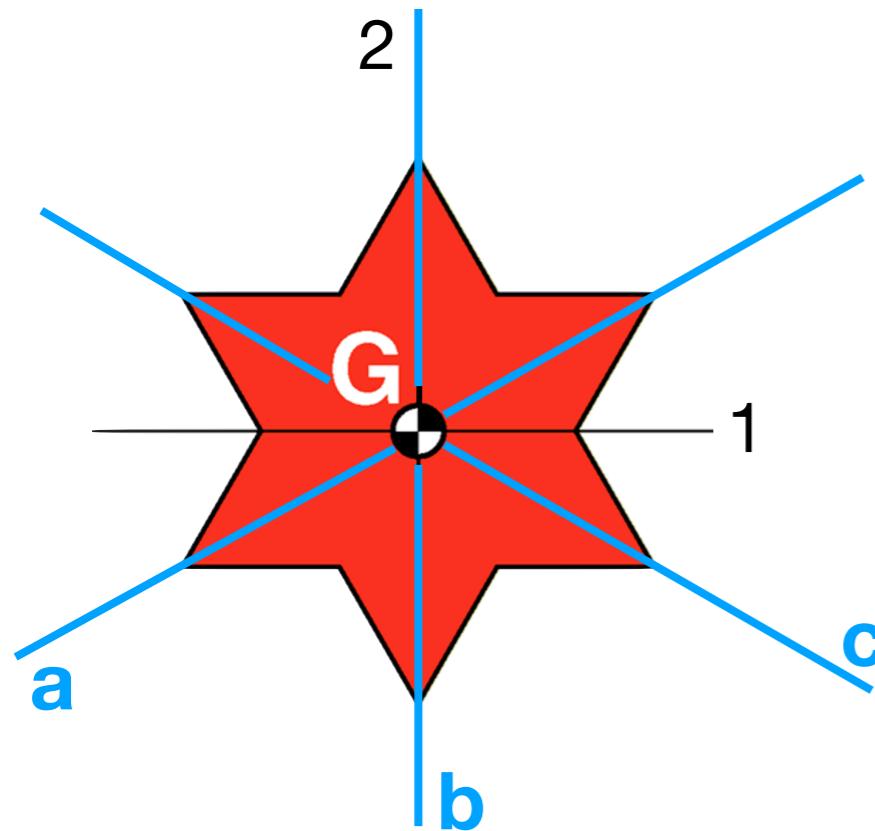
Rotor simètric a G

Per un punt O

Si 3 o més moments d'inèrcia
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[II(G)] ?

qualitatiu



Sòlid pla i eix 2 de simetria:

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I_{11}, I_{22} ?

3 mom. inèrcia en pla (1,2) iguals

Rotor simètric a G

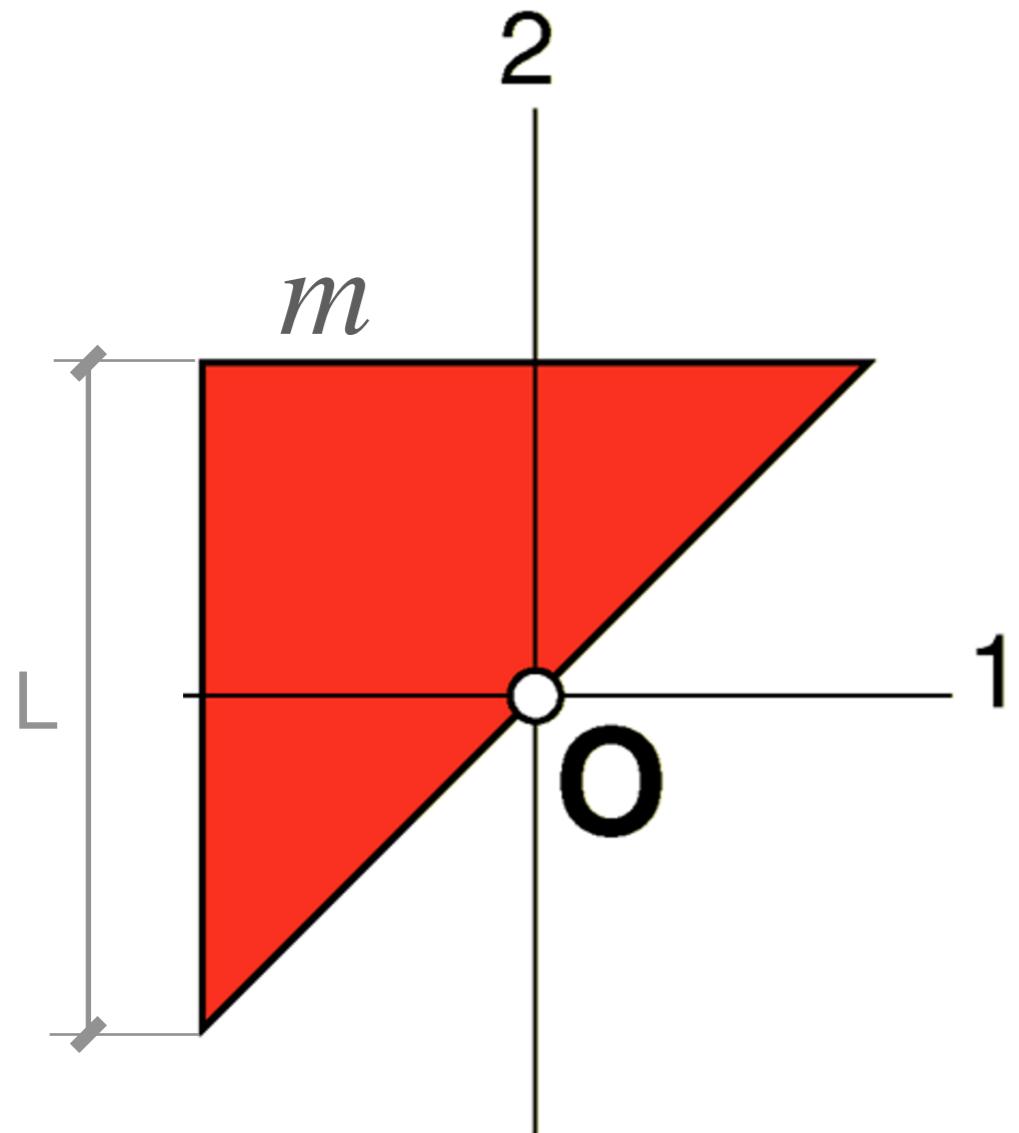
$$[\mathbb{I}(G)]_B = \begin{bmatrix} I & & \\ & I & \\ & & 2I \end{bmatrix}$$

Per un punt O

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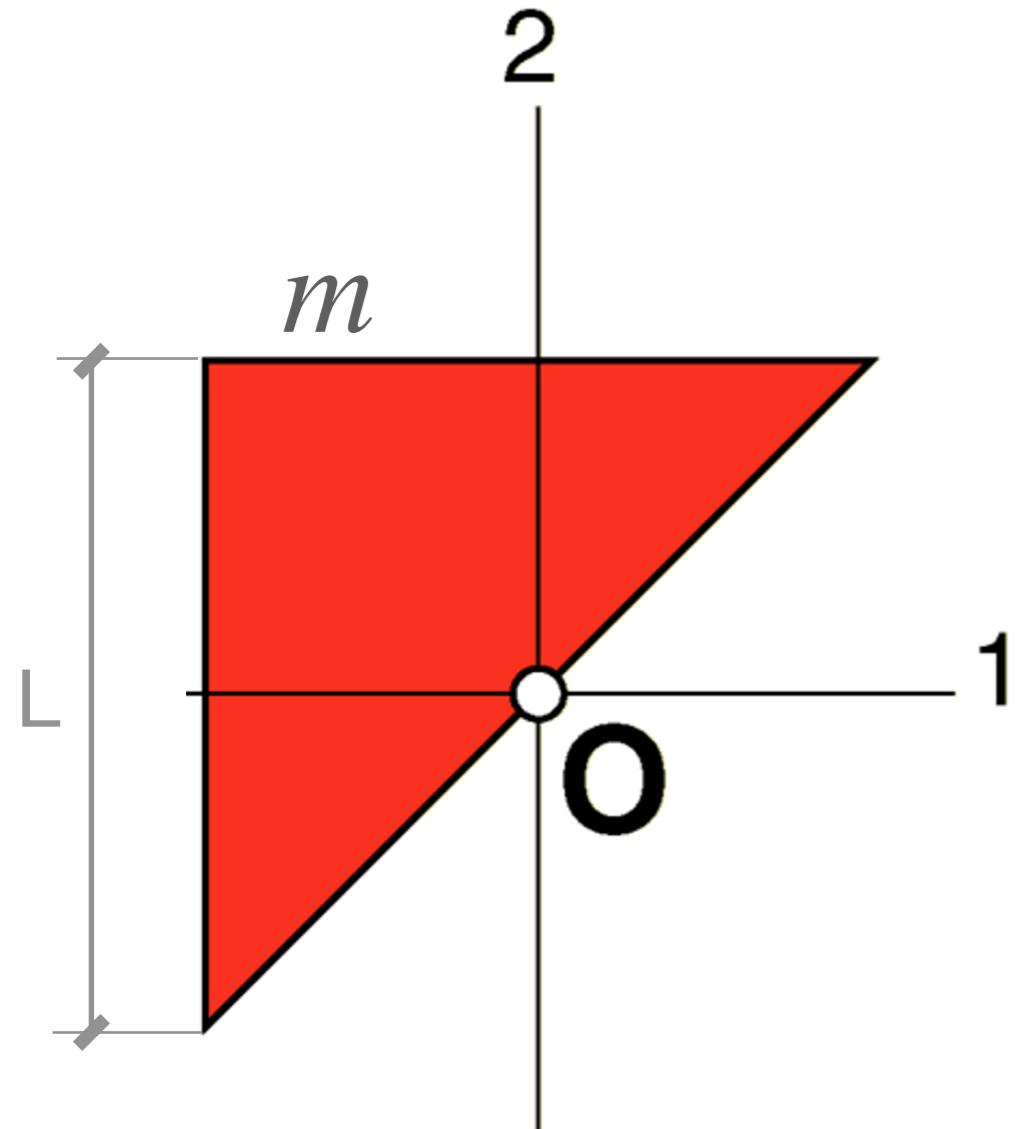
qualitatiu
quantitatiu



[II(O)] ?

qualitatiu
quantitatiu

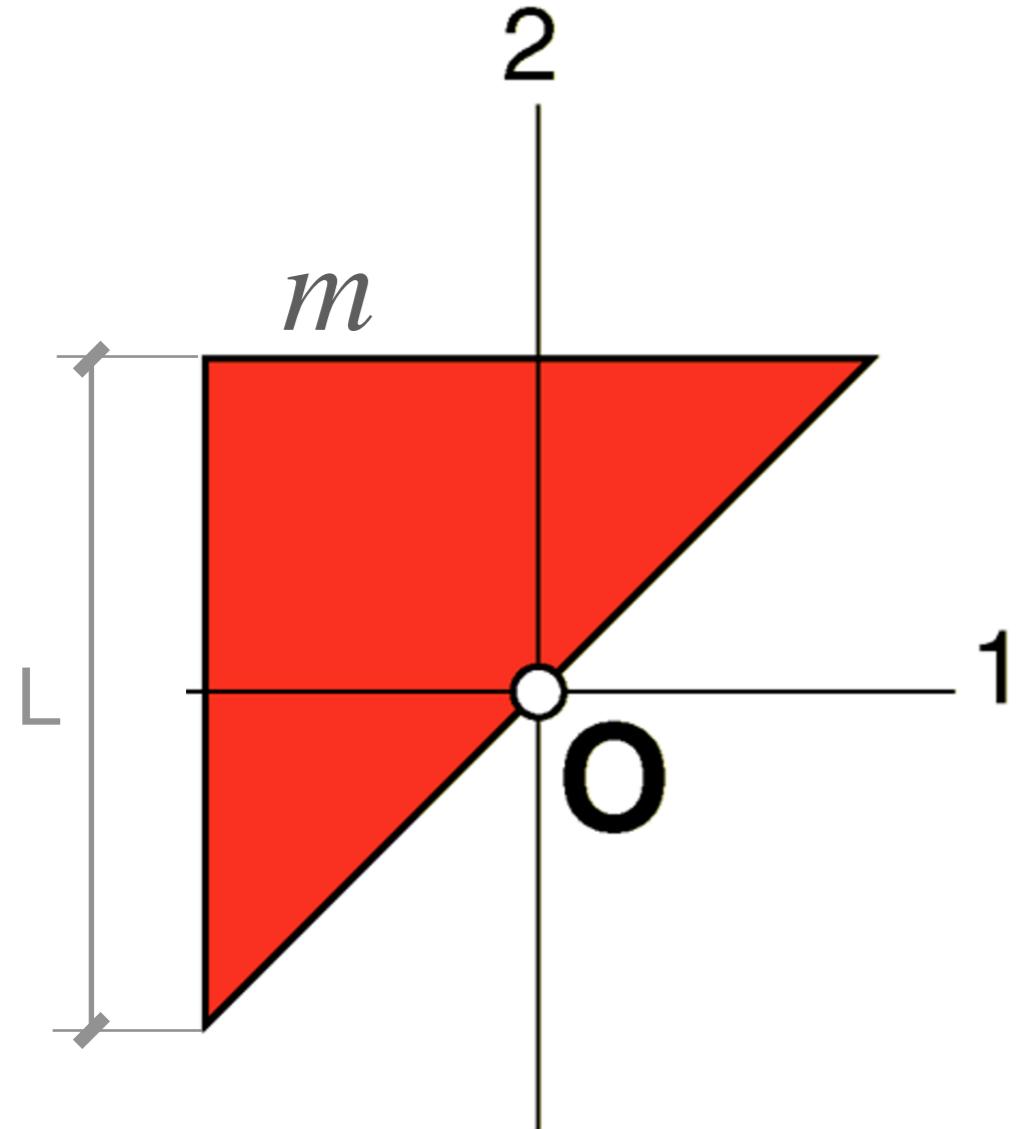
Fig. plana \implies 3 és DPI



[II(O)] ?

qualitatiu
quantitatius

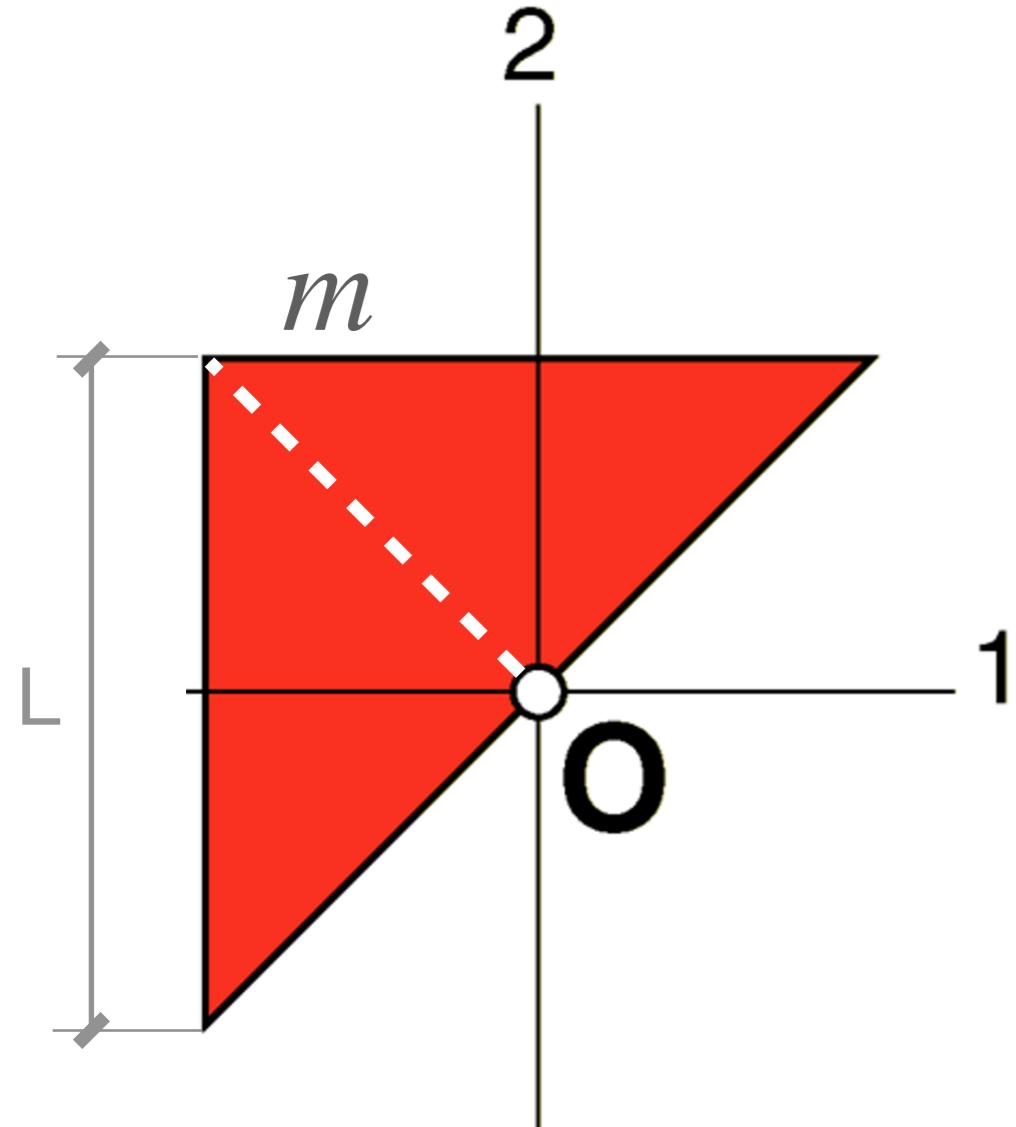
Fig. plana \implies 3 és DPI
 I_{12} ?



[II(O)] ?

qualitatiu
quantitatius

Fig. plana \implies 3 és DPI
 I_{12} ?

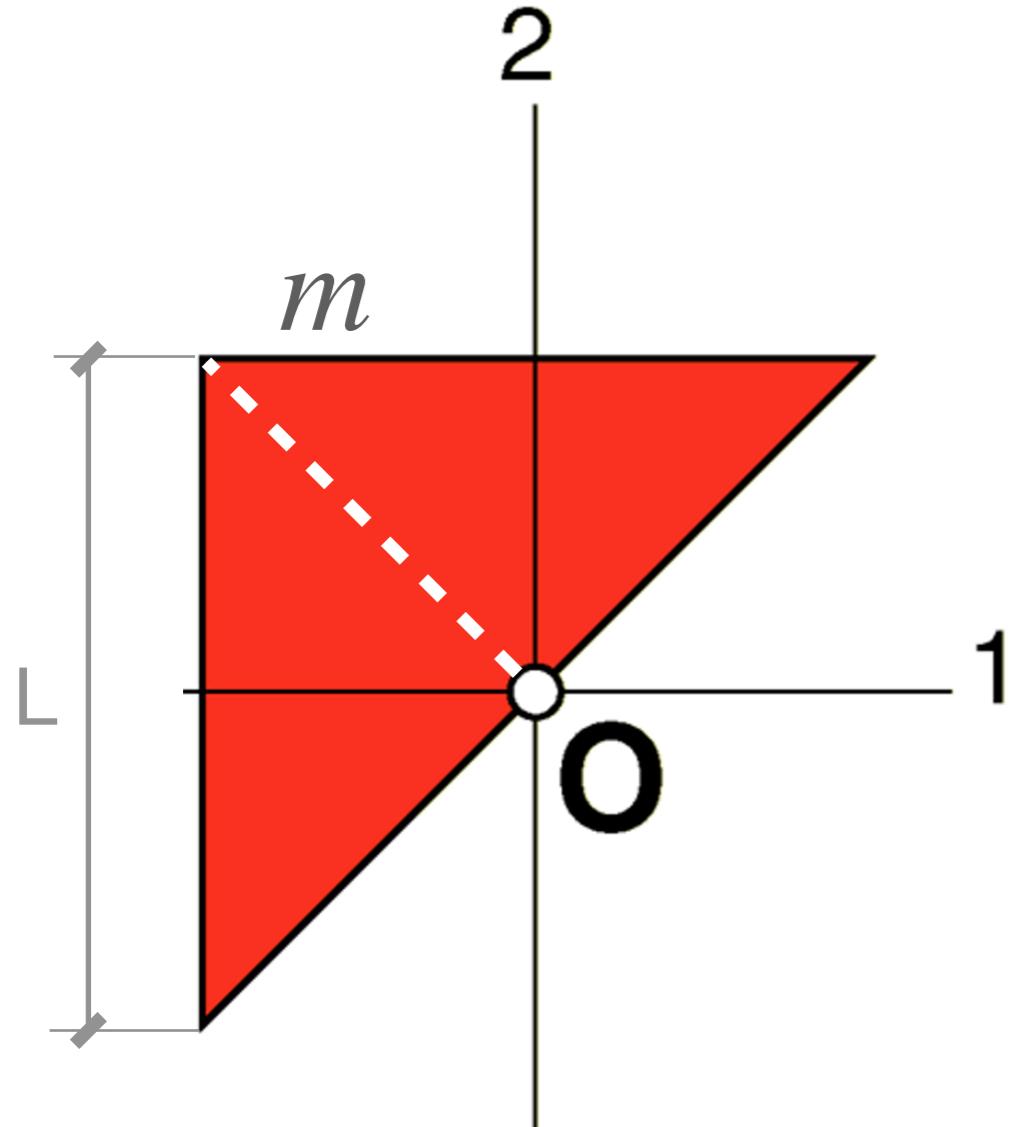


[II(O)] ?

qualitatiu
quantitatiu

Fig. plana \implies 3 és DPI

I_{12} ? \leftarrow És zero!



[$\mathbb{II(O)}$] ?

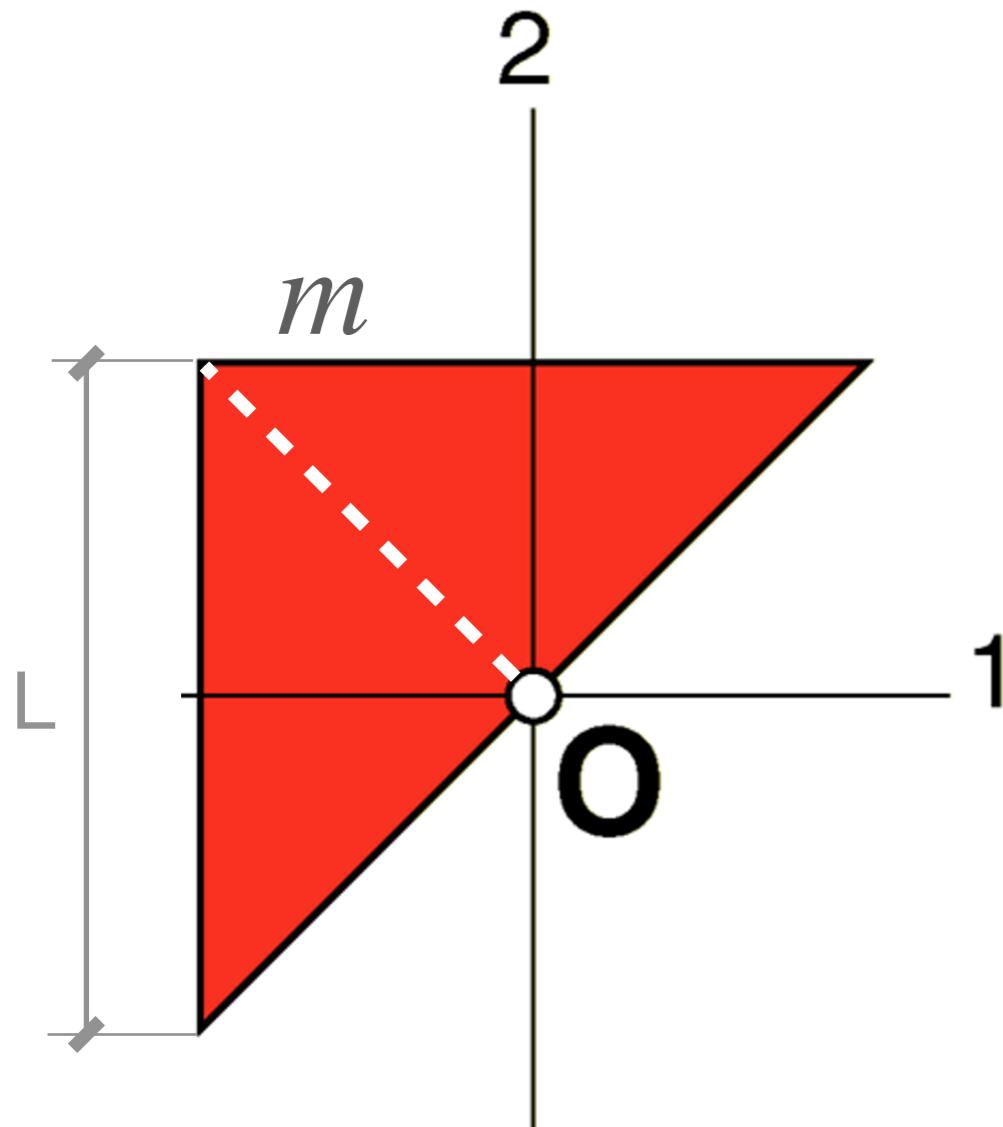
qualitatiu

quantitatius

Fig. plana \implies 3 és DPI

I_{12} ? \leftarrow És zero!

$$[\mathbb{II(O)}]_B = \begin{bmatrix} I_{11} & & \\ & I_{22} & \\ & & I_{11} + I_{22} \end{bmatrix}$$



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qualitatiu

quantitatius

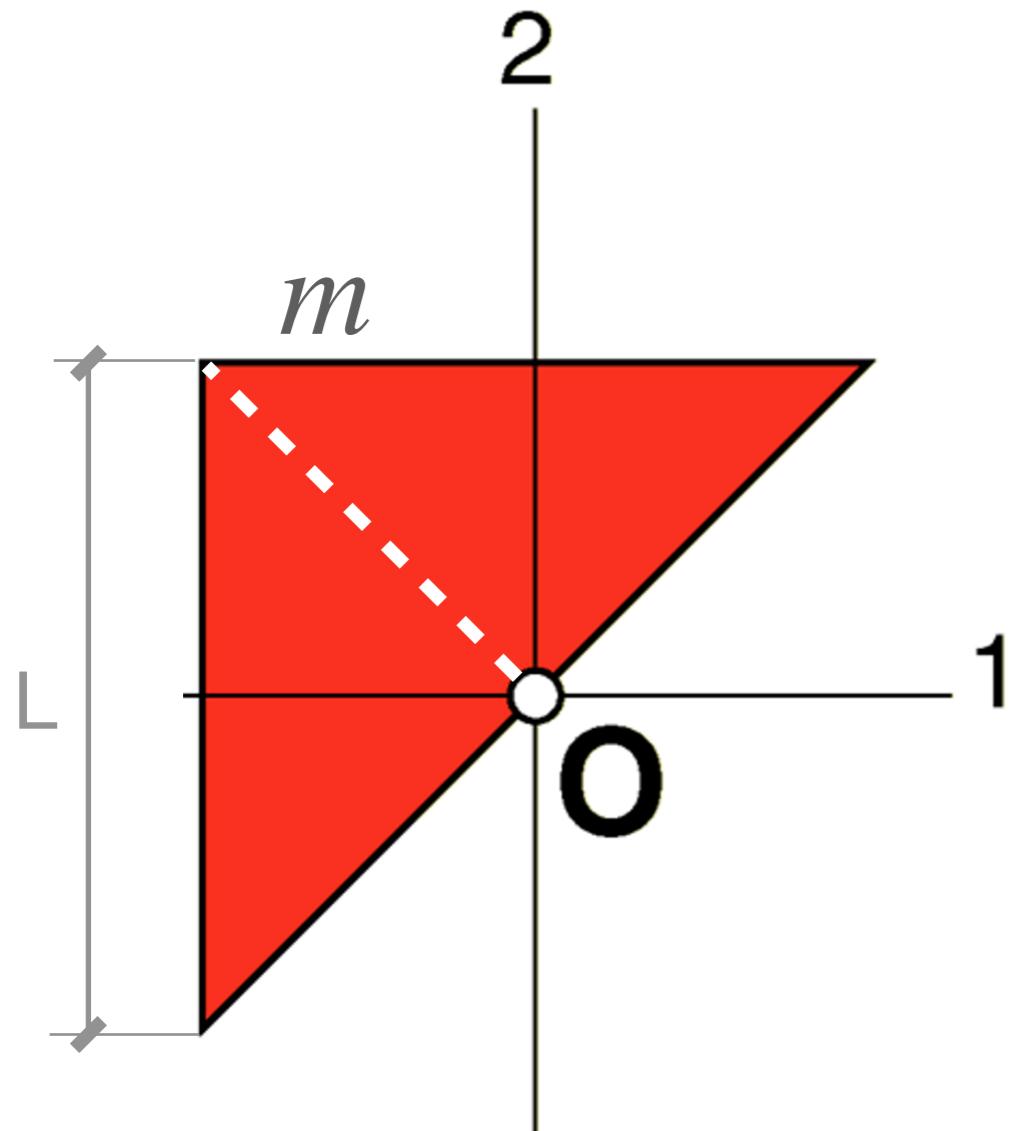


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I_{11}, I_{22} ?

[$\mathbb{II(O)}$] ?

qualitatiu

quantitatius

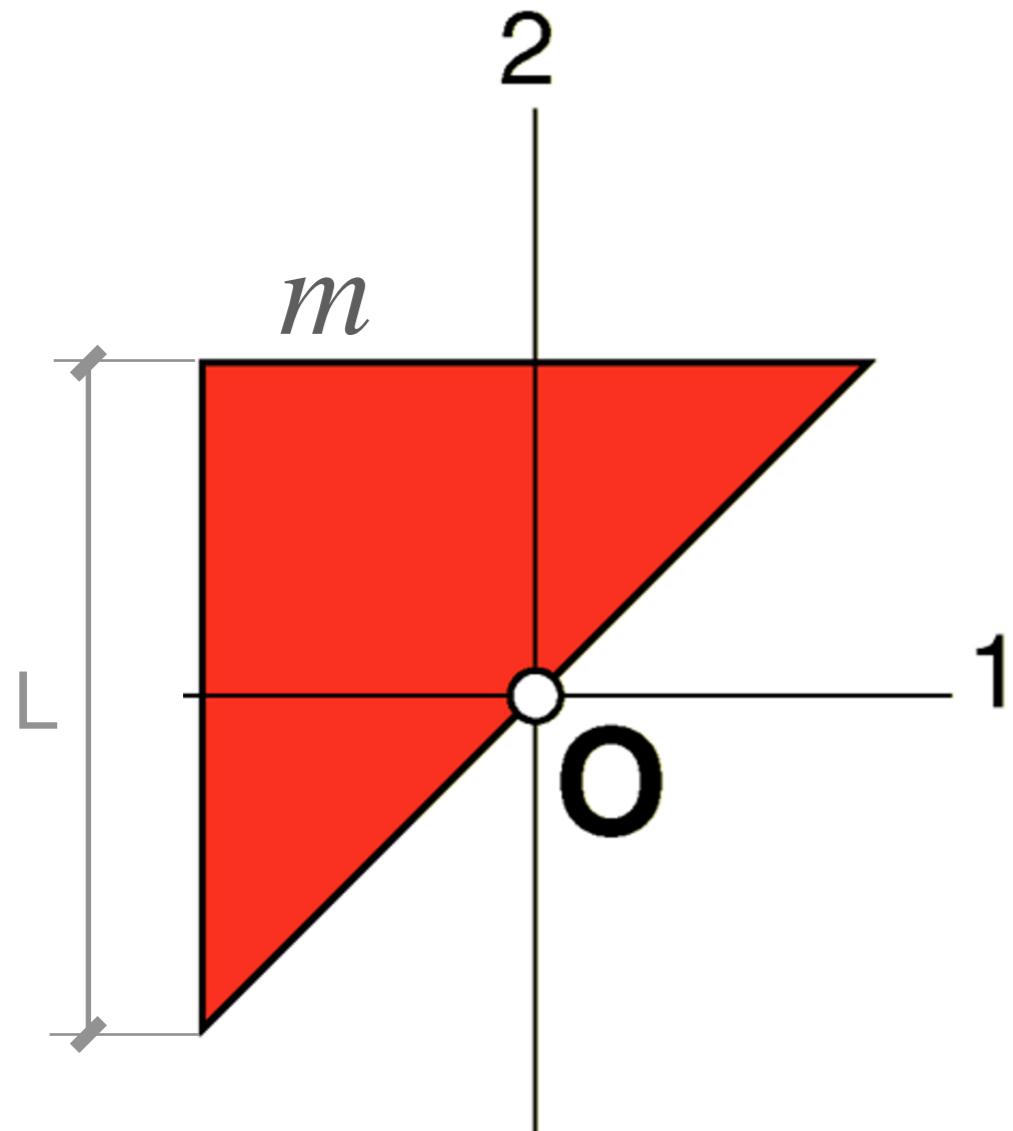


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quantitatius

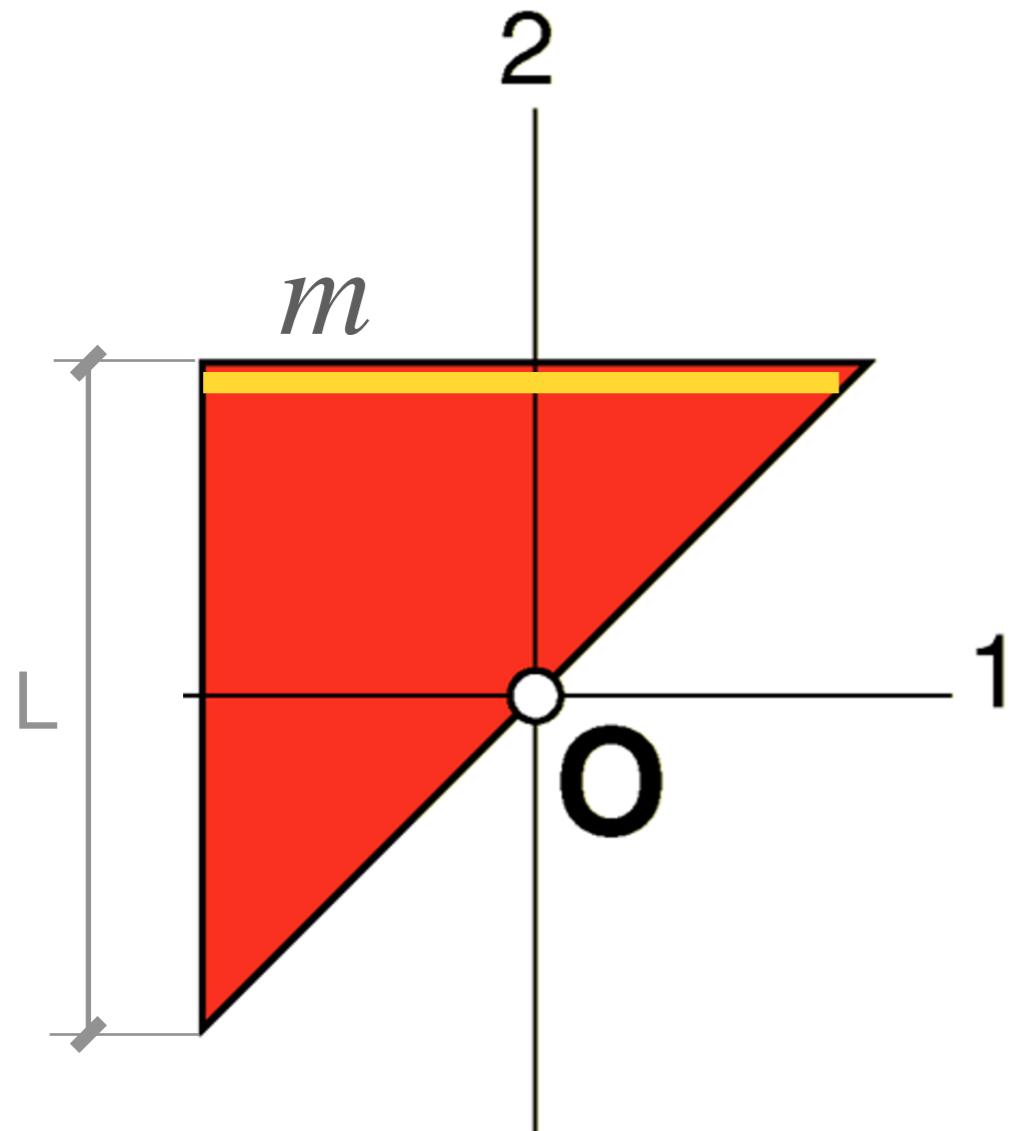


Fig. plana \implies 3 és DPI

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qualitatiu
quantitatius

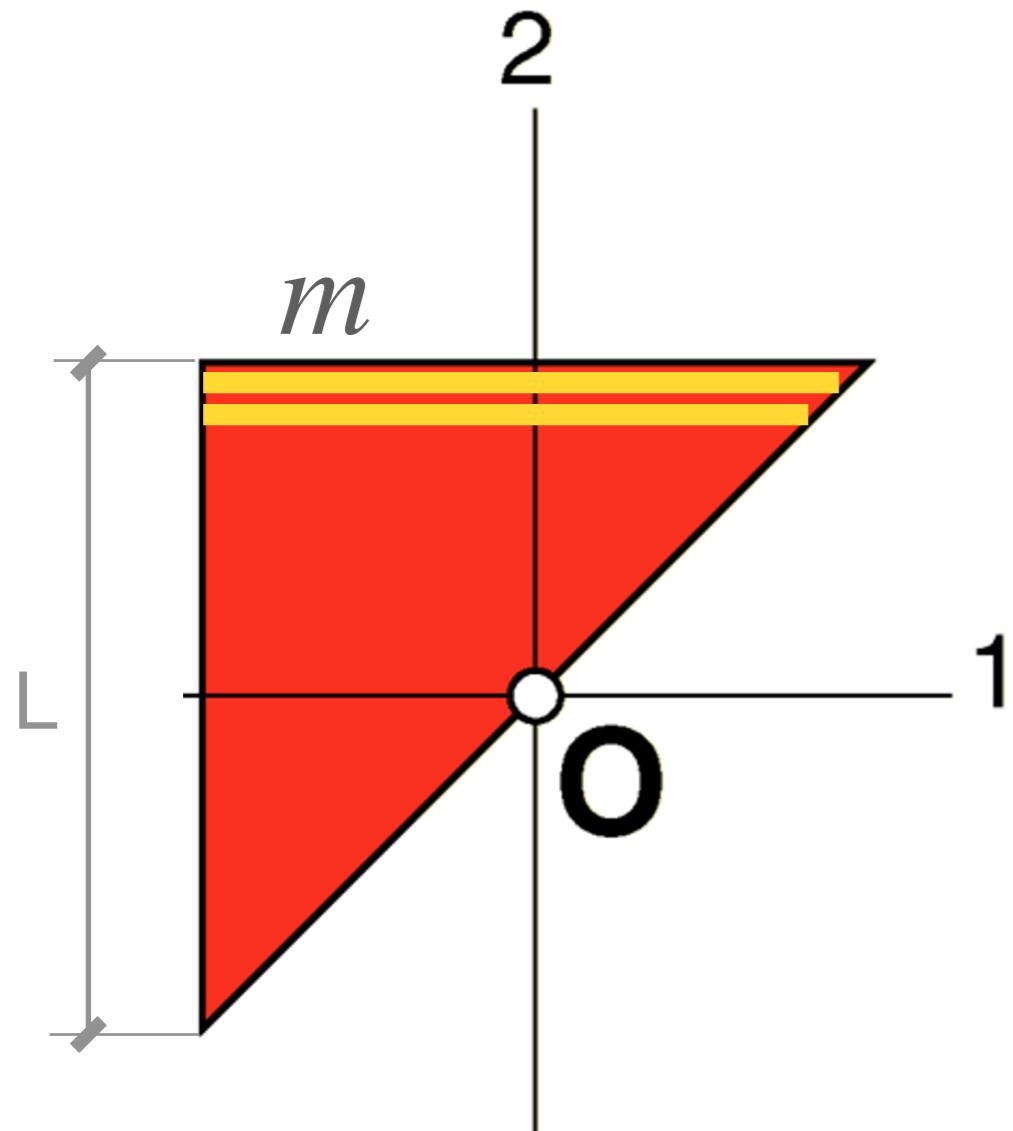


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quantitatius

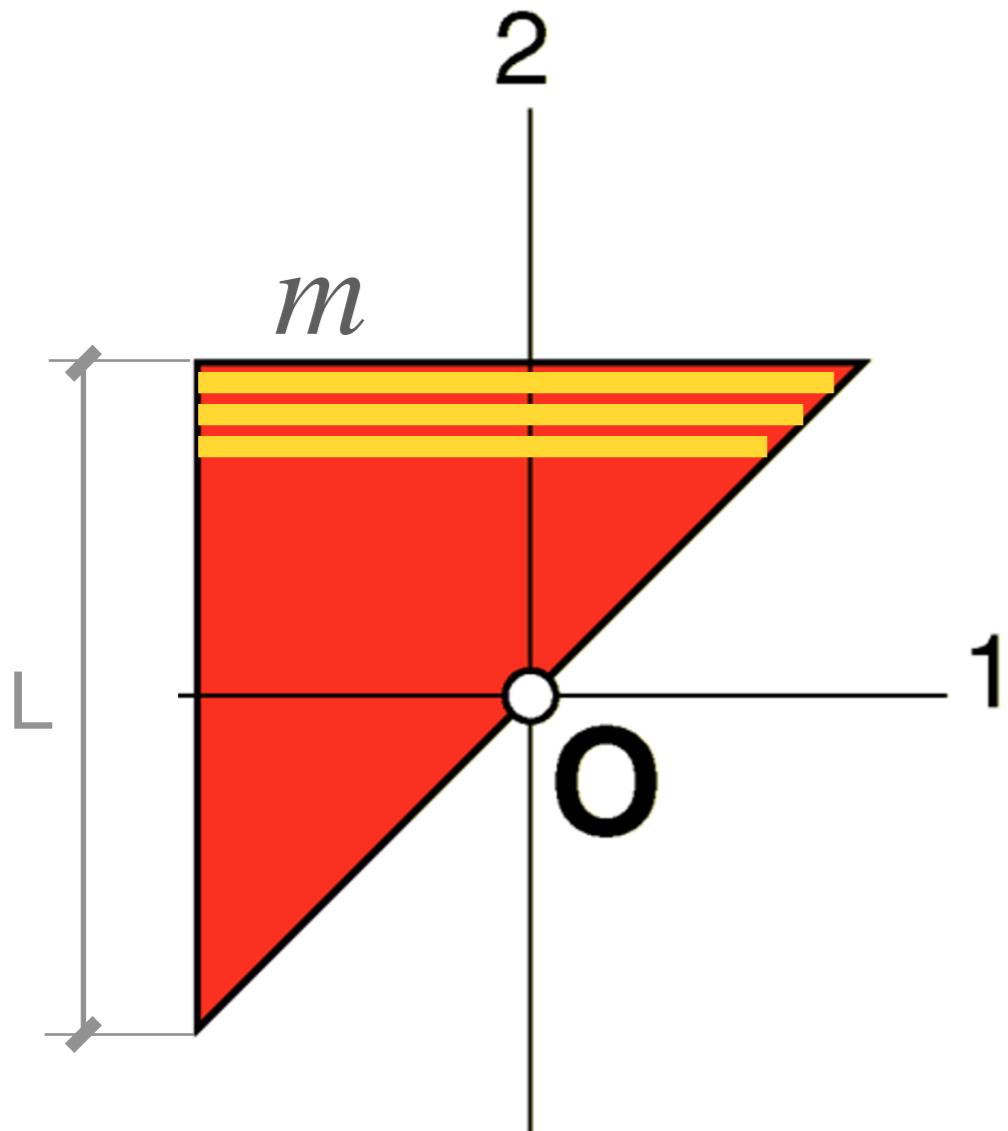


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qualitatiu

quantitatius

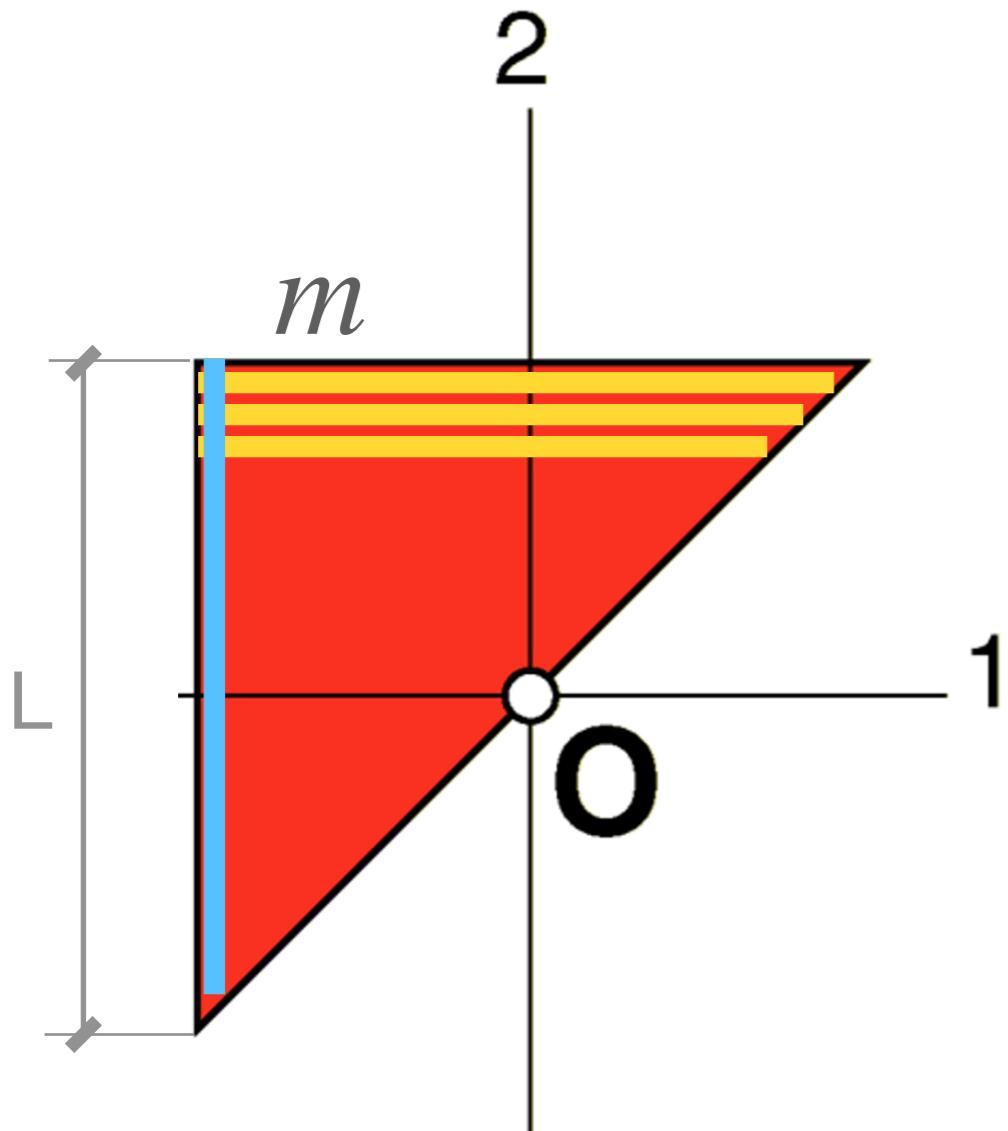


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qualitatiu
quantitatius

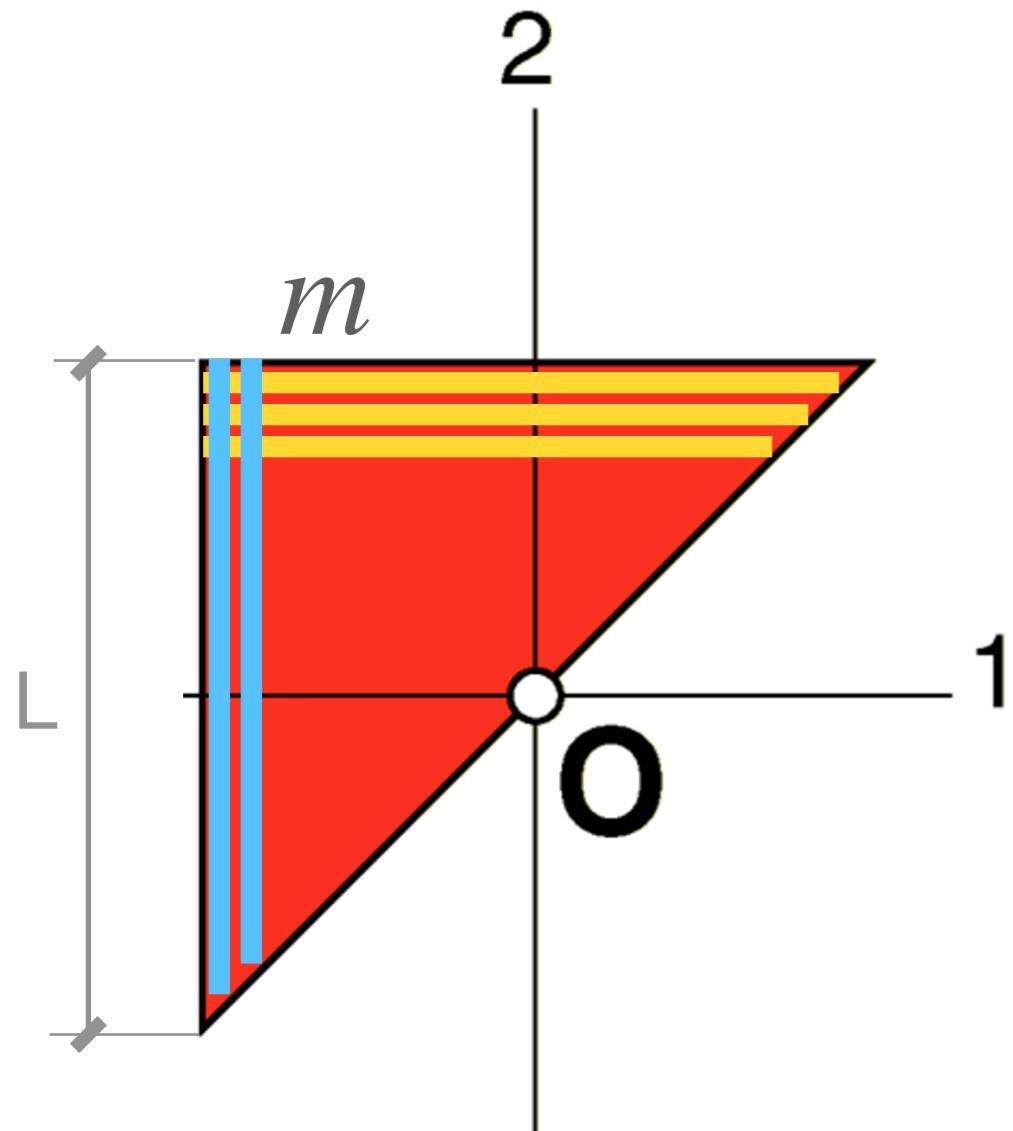


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qualitatiu

quantitatius

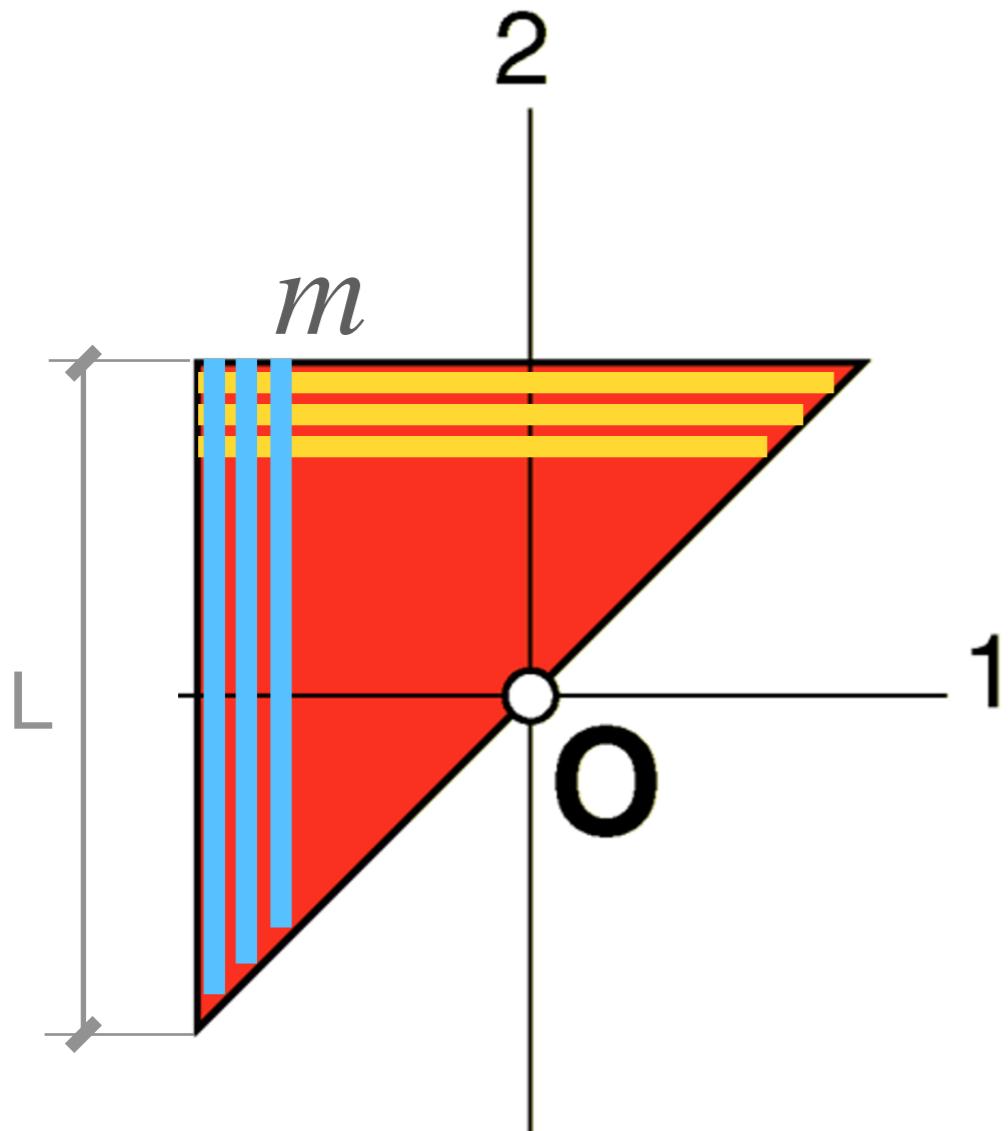


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qualitatiu

quantitatius

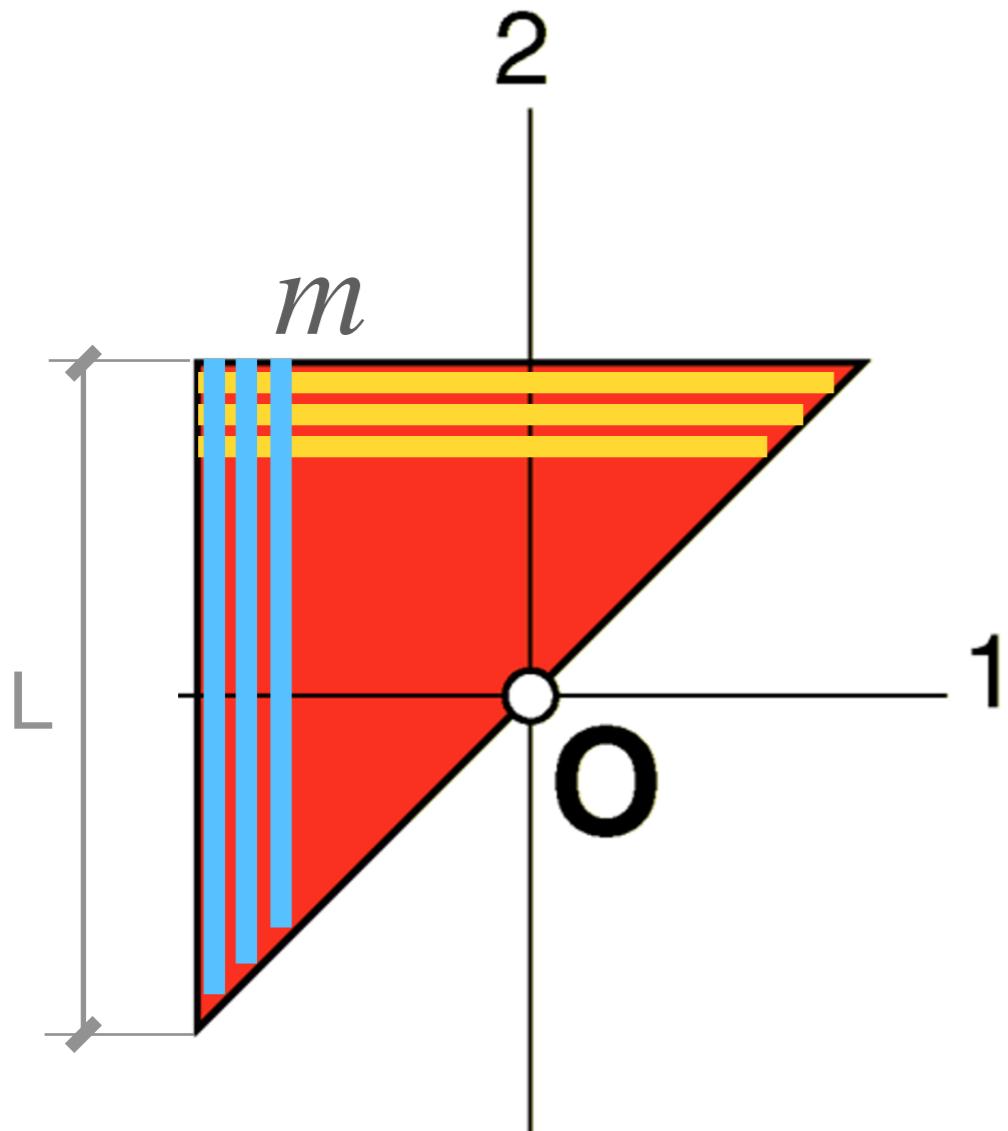


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[$\mathbb{II(O)}$] ?

qualitatiu

quantitatius

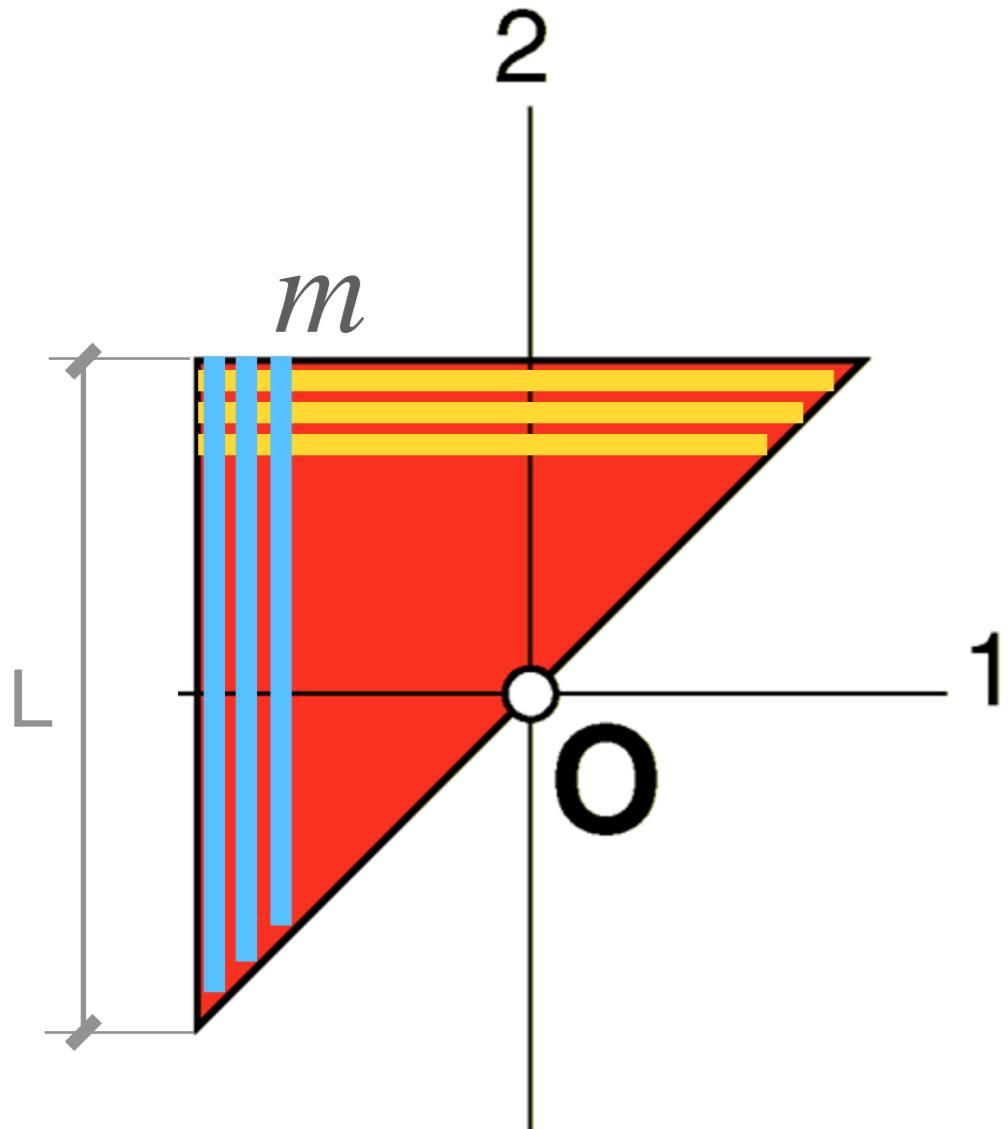


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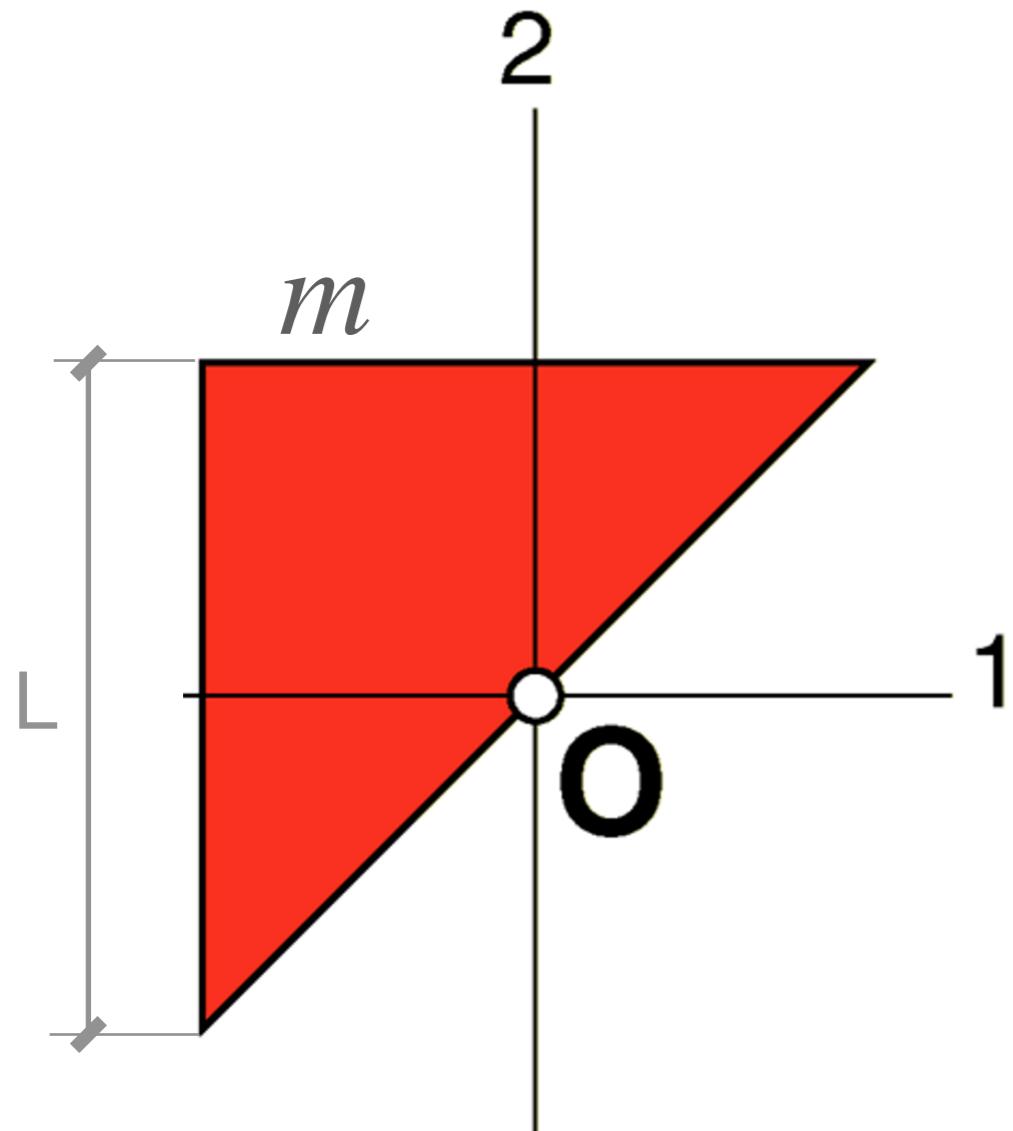
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Rotor simètric a O

[$\mathbb{II}(O)$] ?

qualitatiu
quantitatiu

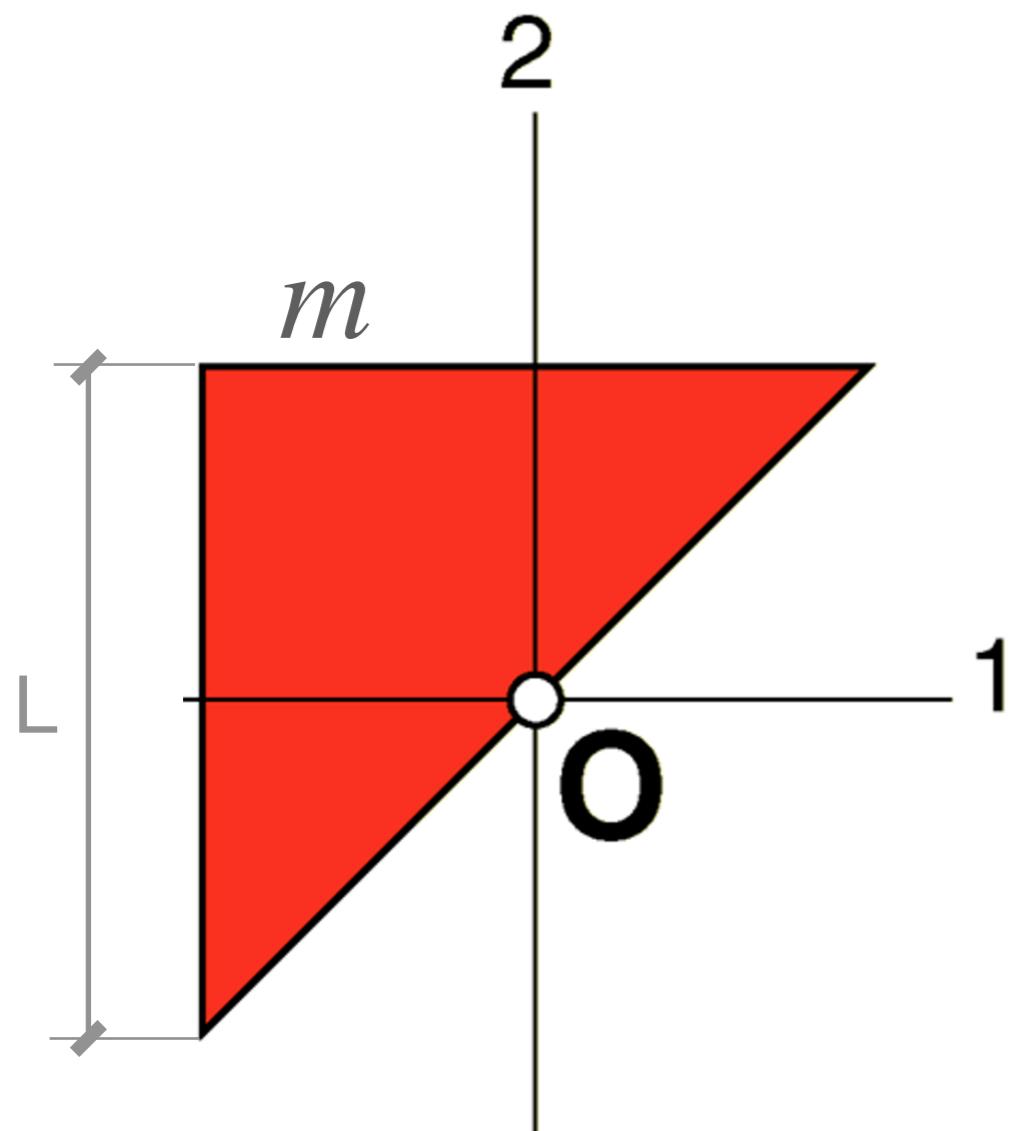


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Rotor simètric a O

[II(O)] ?

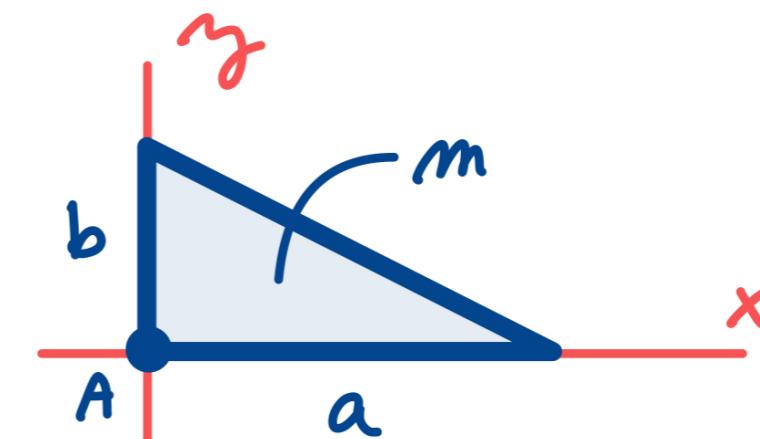
qualitatiu
quantitatiu



$$[I(O)]_B = \begin{bmatrix} I & & \\ & I & \\ & & 2I \end{bmatrix}$$

Rotor simètric a O

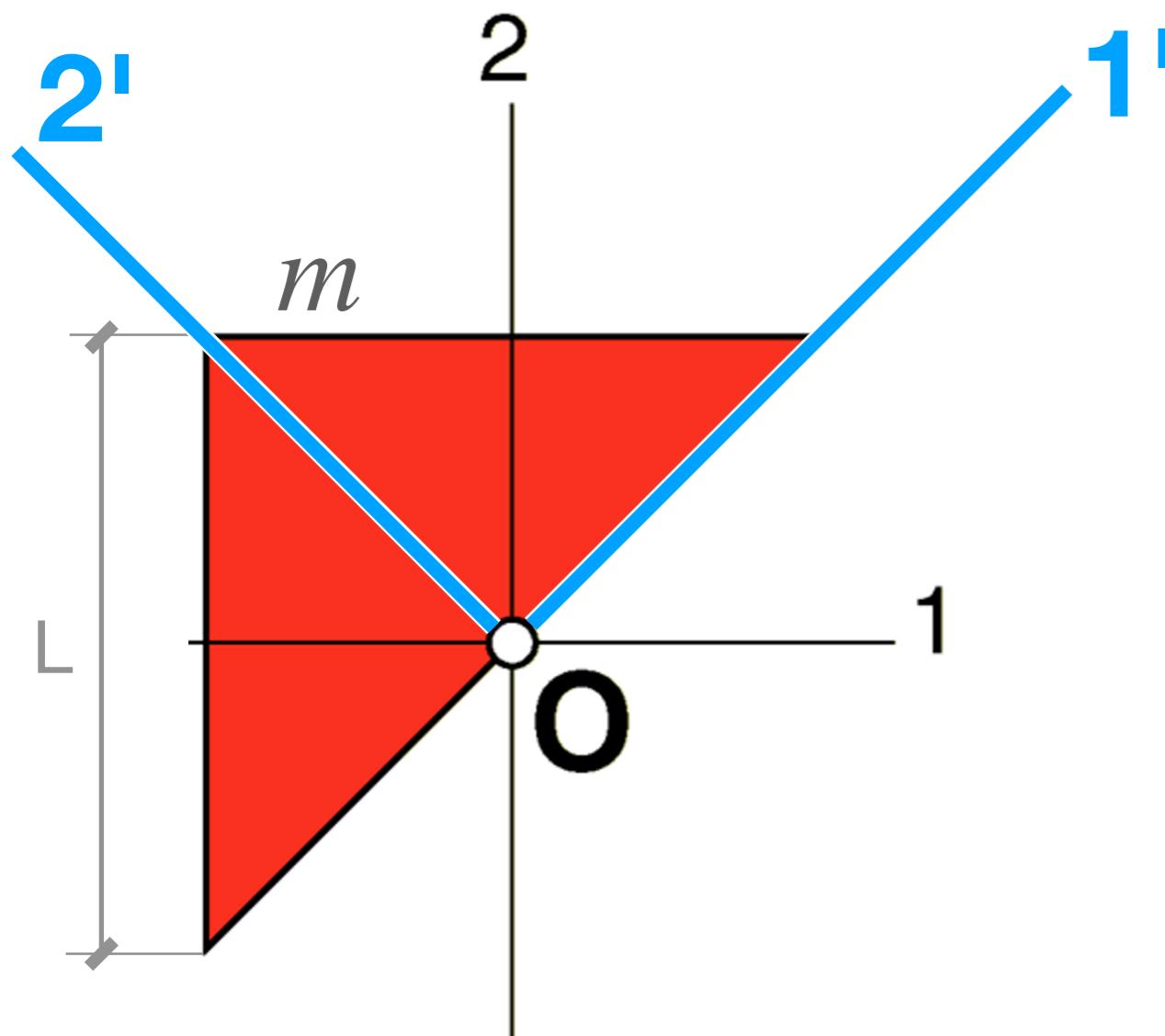
Taula



$$I(A) = \begin{cases} I_{xx} = \frac{1}{6}mb^2 \\ I_{xy} = -\frac{1}{12}mab \end{cases}$$

[II(O)] ?

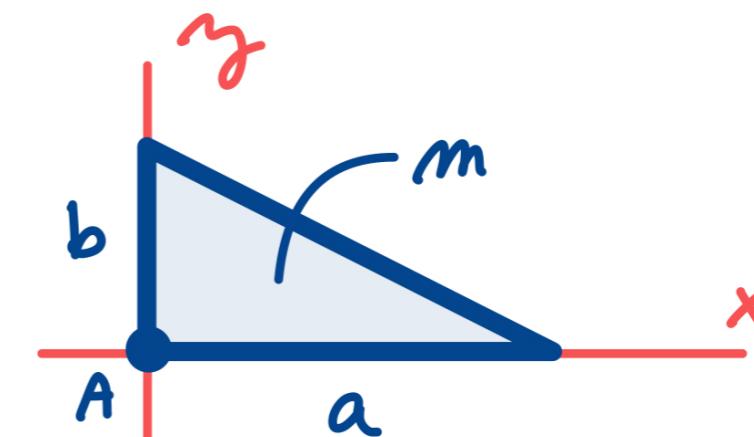
qualitatiu
quantitatiu



$$[I(O)]_B = \begin{bmatrix} I & & \\ & I & \\ & & 2I \end{bmatrix}$$

Rotor simètric a O

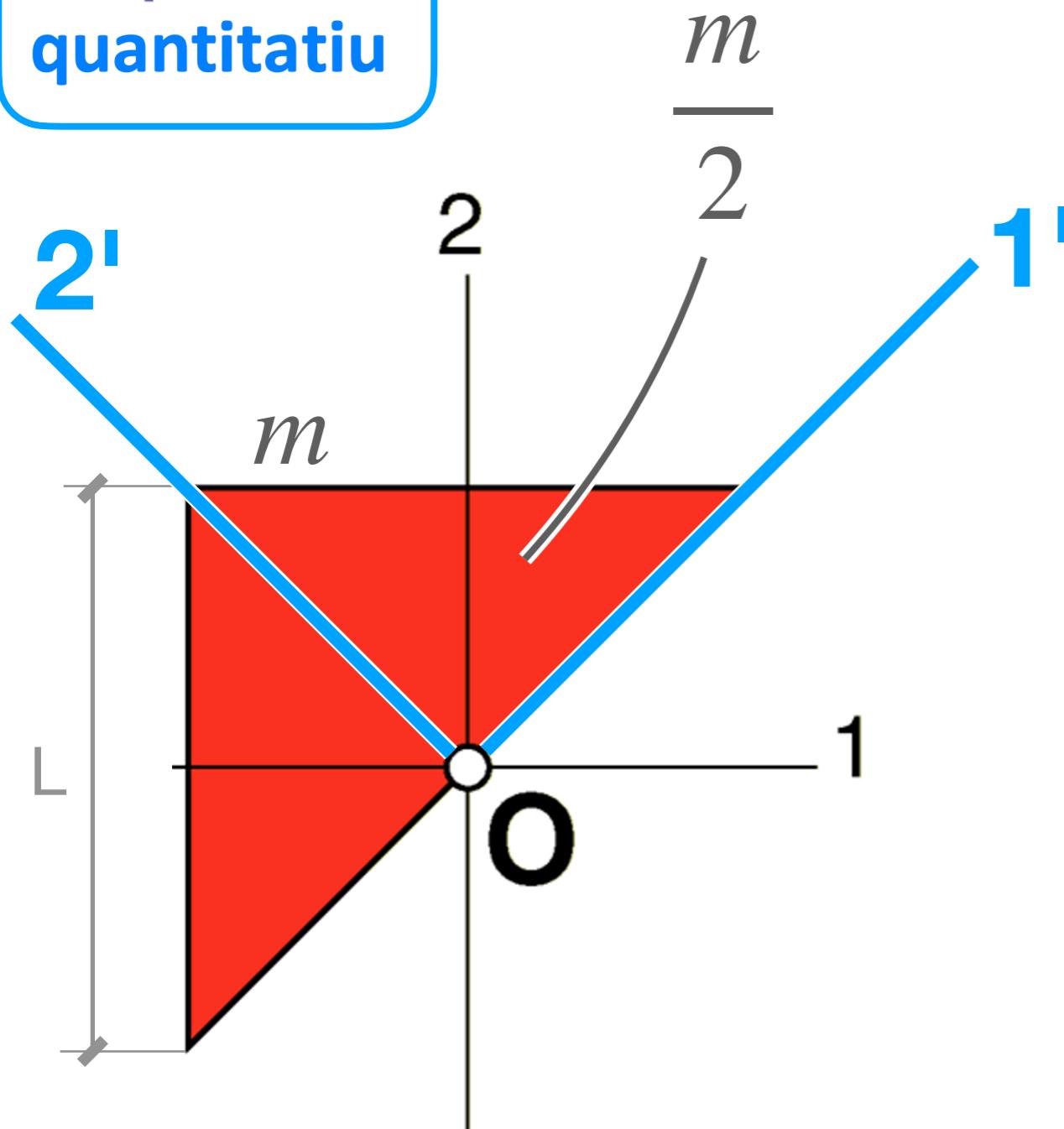
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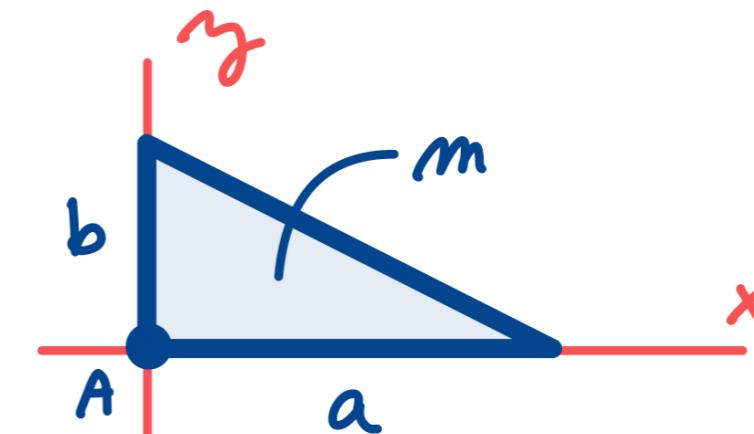
qualitatiu
quantitatiu



$$[I(O)]_B = \begin{bmatrix} I & & \\ & I & \\ & & 2I \end{bmatrix}$$

Rotor simètric a O

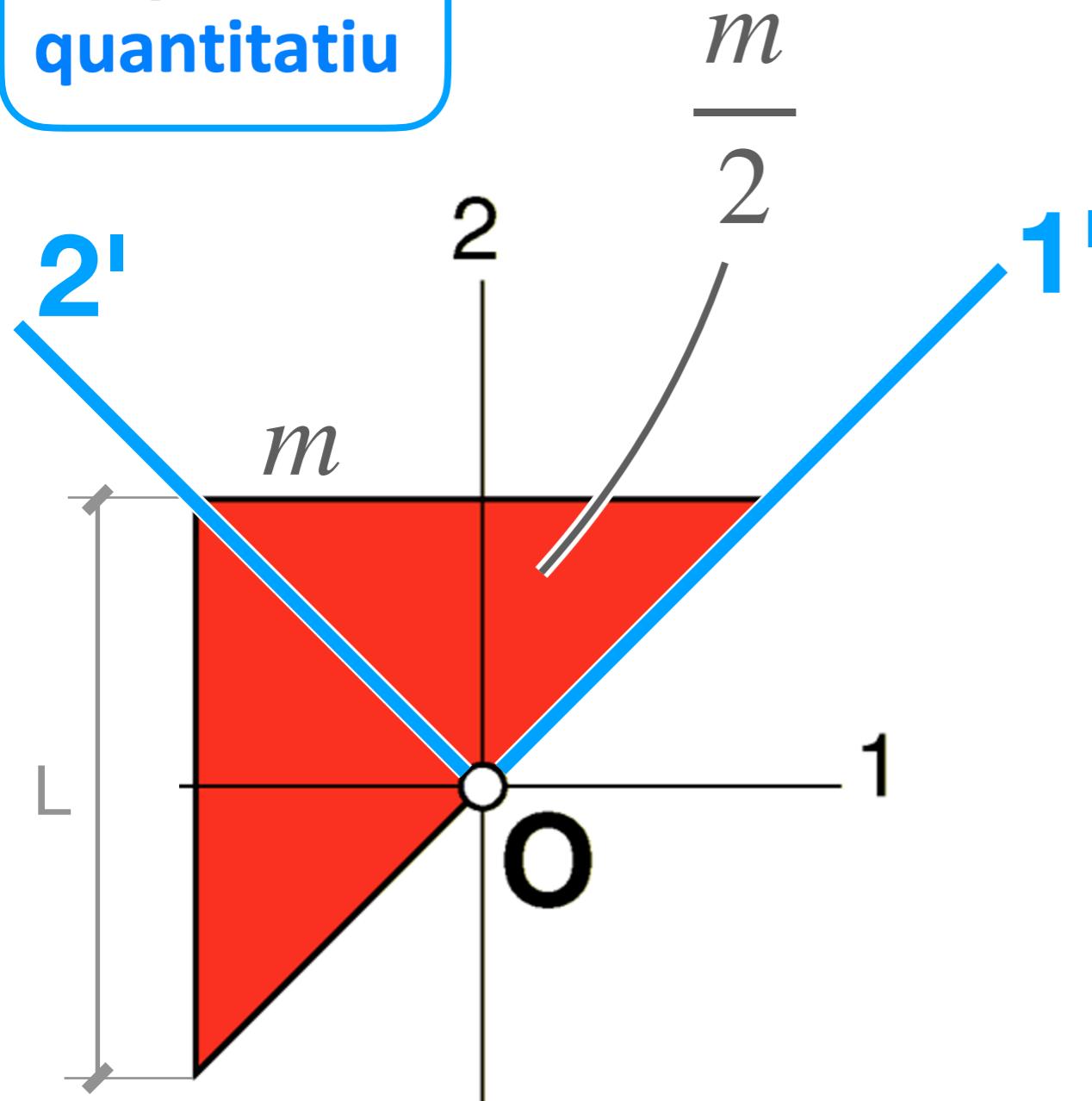
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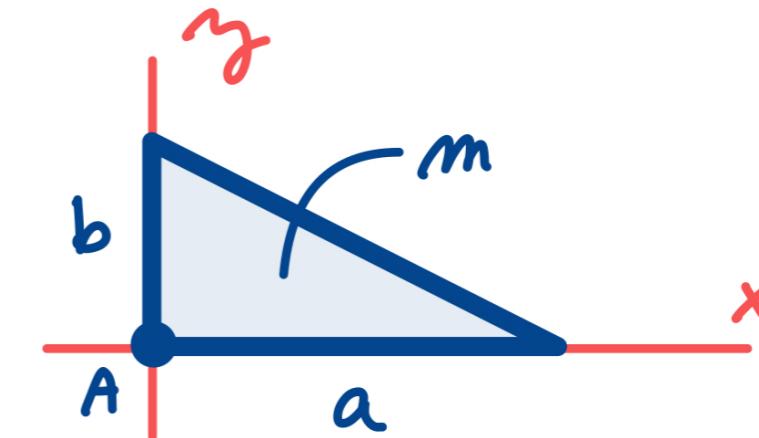
qualitatiu
quantitatiu



$$[I(O)]_B = \begin{bmatrix} I & & \\ & I & \\ & & 2I \end{bmatrix}$$

Rotor simètric a O

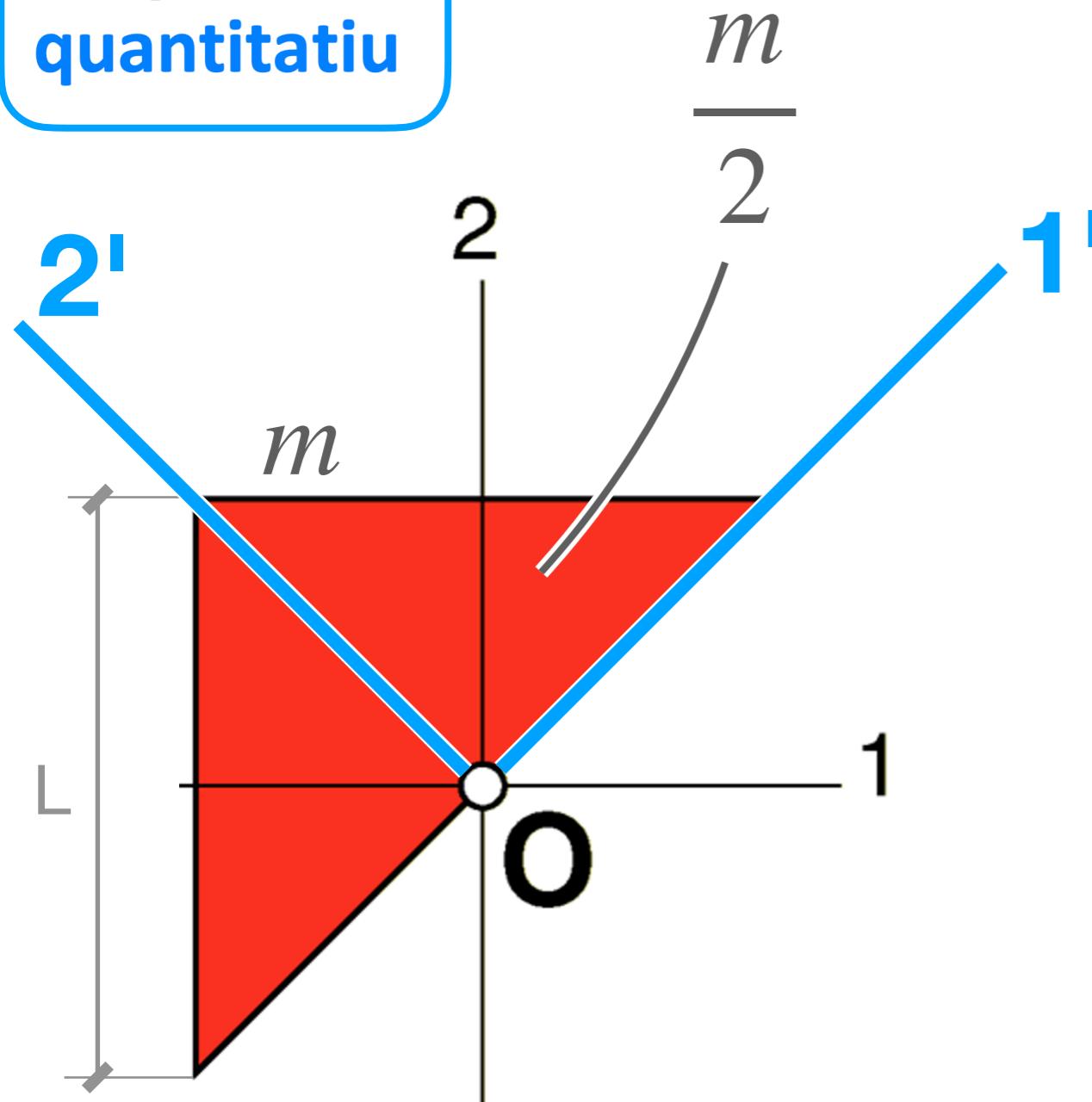
Taula



$$\mathcal{I}(A) = \begin{cases} \mathcal{I}_{xx} = \frac{1}{6}mb^2 \\ \mathcal{I}_{xy} = -\frac{1}{12}mab \end{cases}$$

[II(O)] ?

qualitatiu
quantitatiu

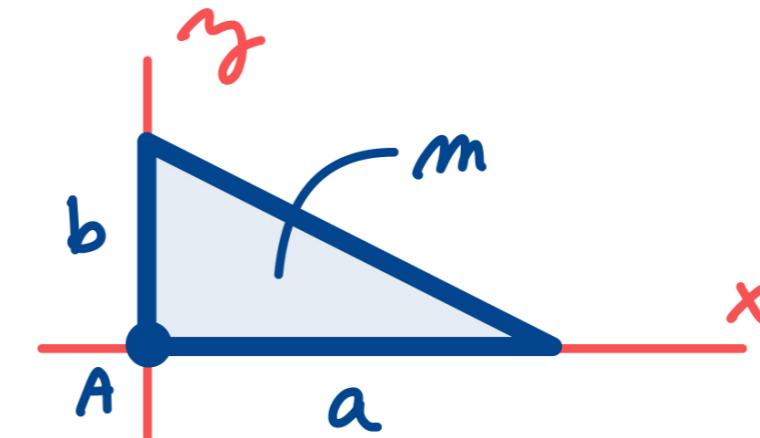


$$I = 2 \left[\frac{1}{6} \frac{m}{2} \left(\frac{L}{\sqrt{2}} \right)^2 \right]$$

$$[\mathbf{II(O)}]_B = \begin{bmatrix} I & & \\ & I & \\ & & 2I \end{bmatrix}$$

Rotor simètric a O

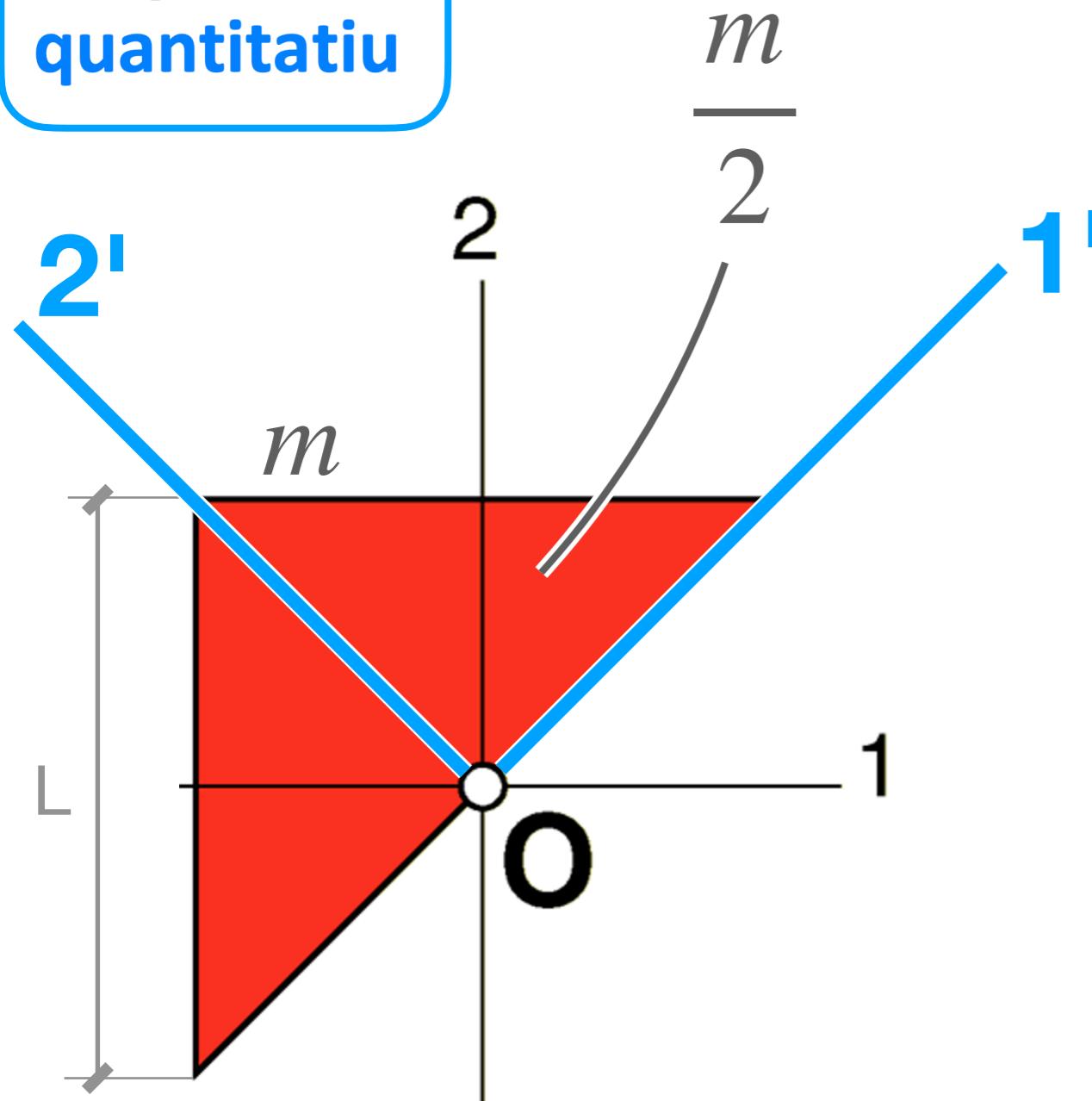
Taula



$$\mathbf{II(A)} = \begin{cases} I_{xx} = \frac{1}{6} mb^2 \\ I_{xy} = -\frac{1}{12} mab \end{cases}$$

[II(O)] ?

qualitatiu
quantitatiu

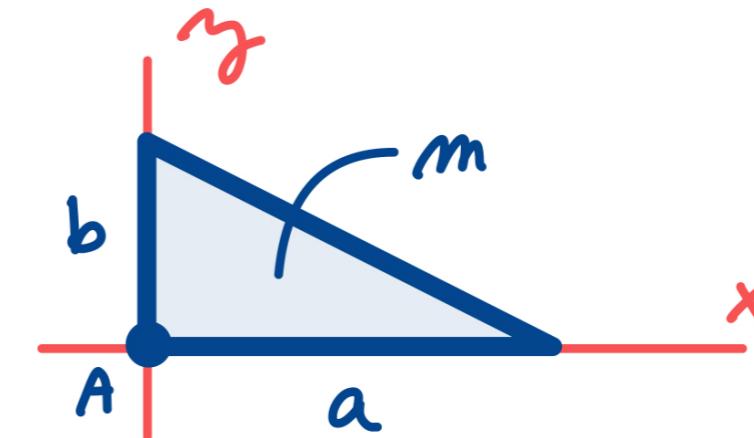


$$I = 2 \left[\frac{1}{6} \frac{m}{2} \left(\frac{L}{\sqrt{2}} \right)^2 \right] = \frac{mL^2}{12}$$

$$[\mathbb{II}(O)]_B = \begin{bmatrix} I & & \\ & I & \\ & & 2I \end{bmatrix}$$

Rotor simètric a O

Taula



$$\mathbb{II}(A) = \begin{cases} I_{xx} = \frac{1}{6} mb^2 \\ I_{xy} = -\frac{1}{12} mab \end{cases}$$

Teorema de Steiner

$$\underbrace{\underline{\mathbb{I}}(Q)}_{\text{Tensor per al punt } \mathbf{Q}} = \underbrace{\underline{\mathbb{I}}(G)}_{\text{Tensor per al centre d'inèrcia } \mathbf{G}} + \underbrace{\underline{\mathbb{I}}^\oplus(Q)}_{\text{Tensor per a } \mathbf{Q} \text{ suposant massa sòlid concentrada a } \mathbf{G}}$$

Teorema de Steiner

$$\underbrace{\underline{I}(Q)}_{\text{Tensor per al punt } Q} = \underbrace{\underline{I}(G)}_{\text{Tensor per al centre d'inèrcia } G} + \underbrace{\underline{I}^\oplus(Q)}_{\text{Tensor per a } Q \text{ suposant massa sòlid concentrada a } G}$$

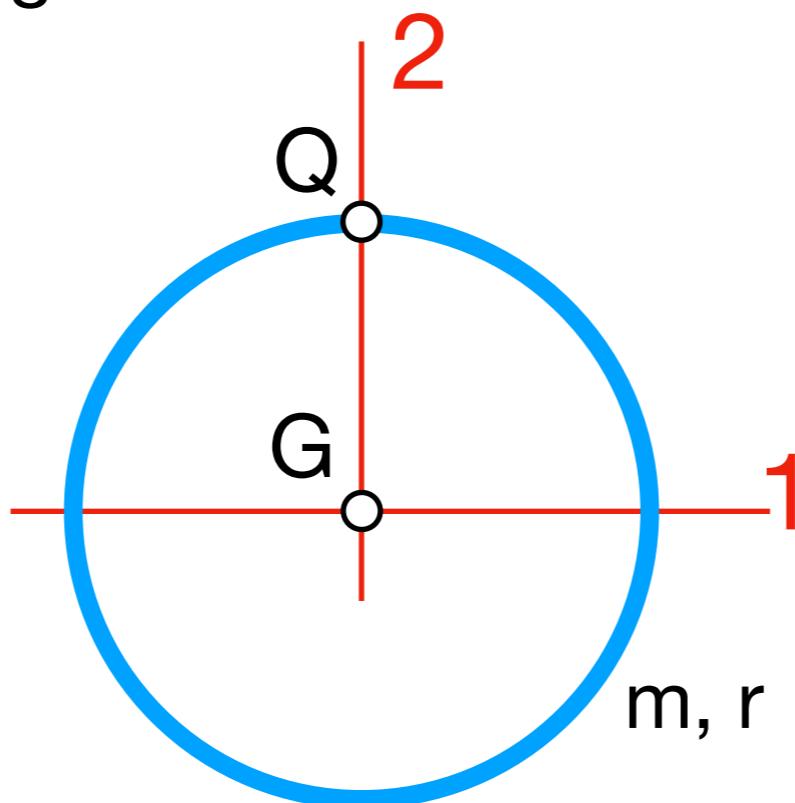
Tensor per al punt **Q**

Tensor per al centre d'inèrcia **G**

Tensor per a **Q** suposant massa sòlid concentrada a **G**

Exemple: anell homogeni

$\underline{I}(Q) ?$



Teorema de Steiner

$$\underbrace{\underline{\mathbb{I}}(Q)}_{\text{Tensor per al punt } Q} = \underbrace{\underline{\mathbb{I}}(G)}_{\text{Tensor per al centre d'inèrcia } G} + \underbrace{\underline{\mathbb{I}}^\oplus(Q)}_{\text{Tensor per a } Q \text{ suposant massa sòlid concentrada a } G}$$

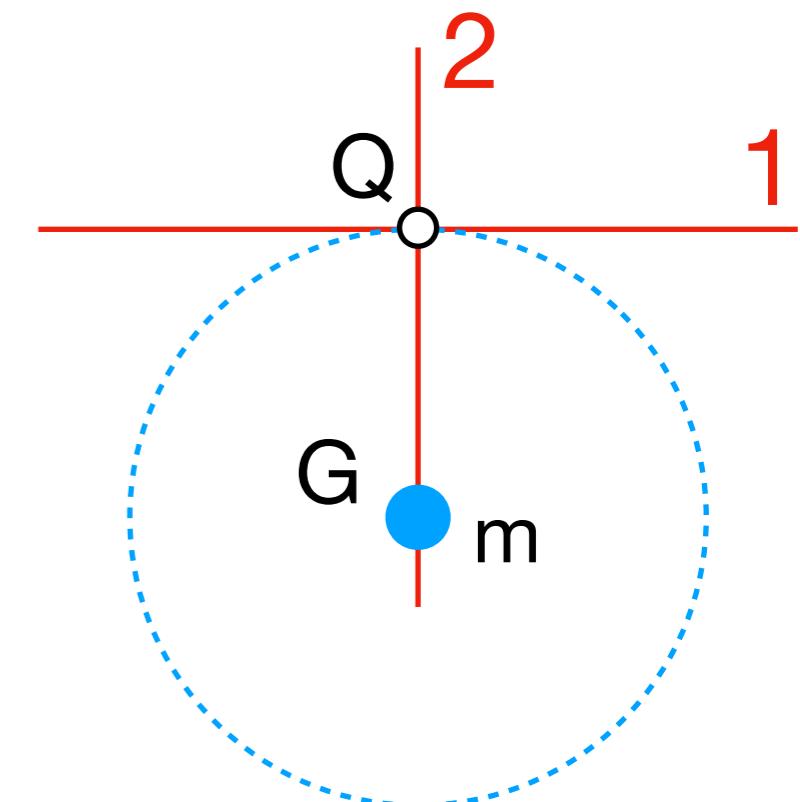
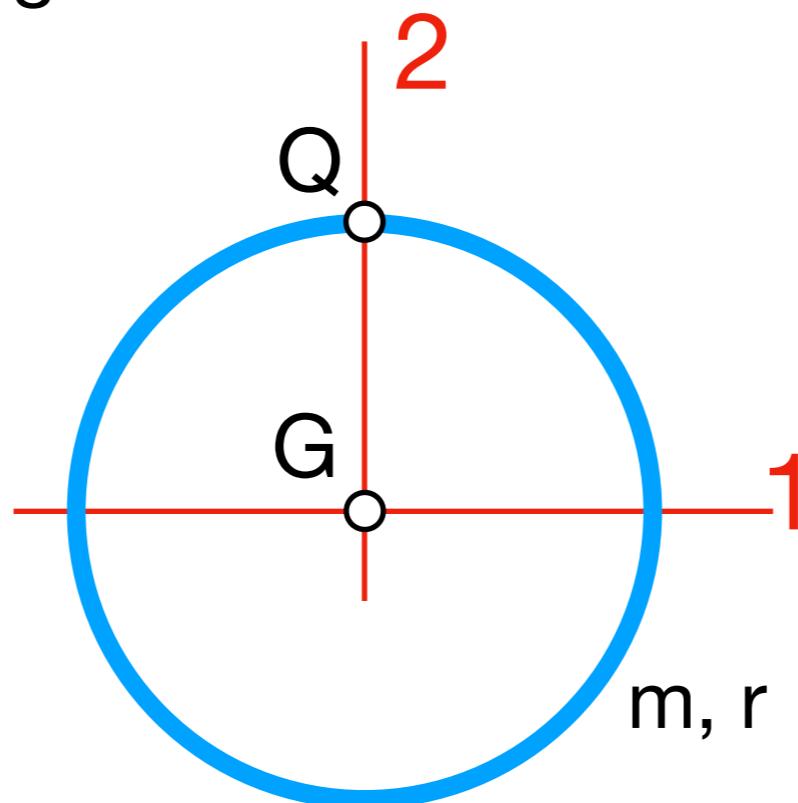
Tensor per al punt **Q**

Tensor per al centre d'inèrcia **G**

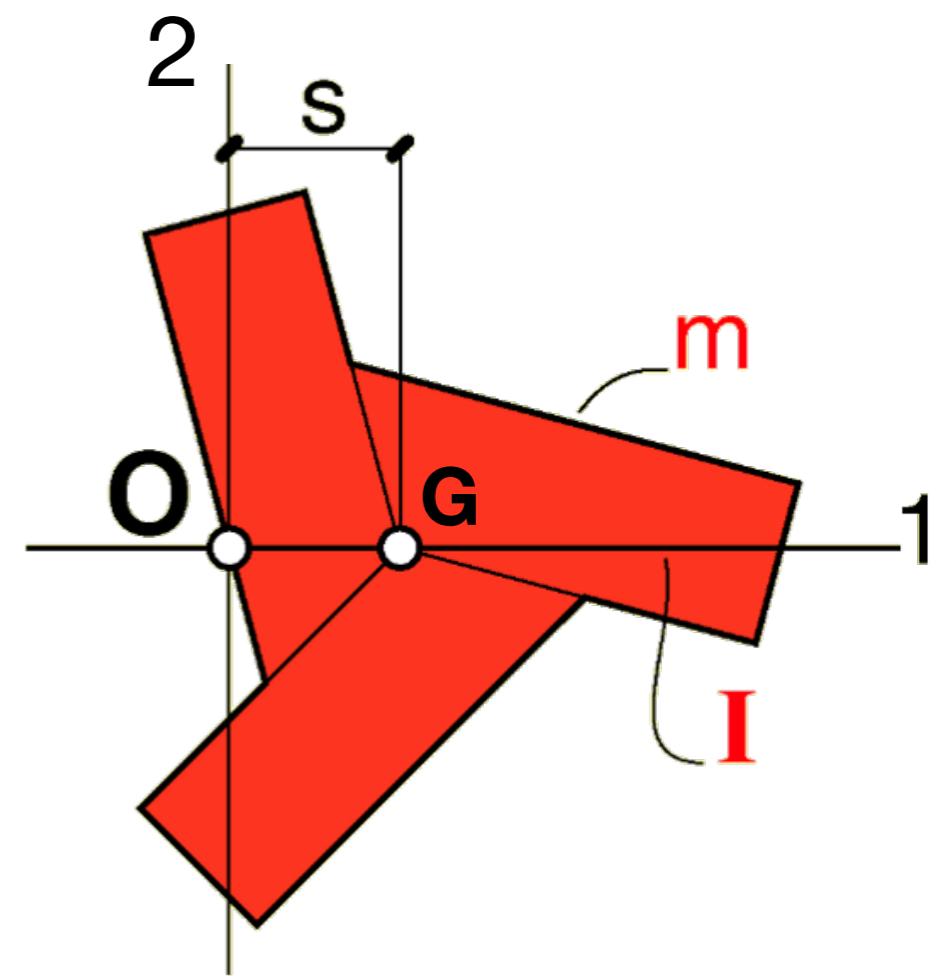
Tensor per a **Q** suposant massa sòlid concentrada a **G**

Exemple: anell homogeni

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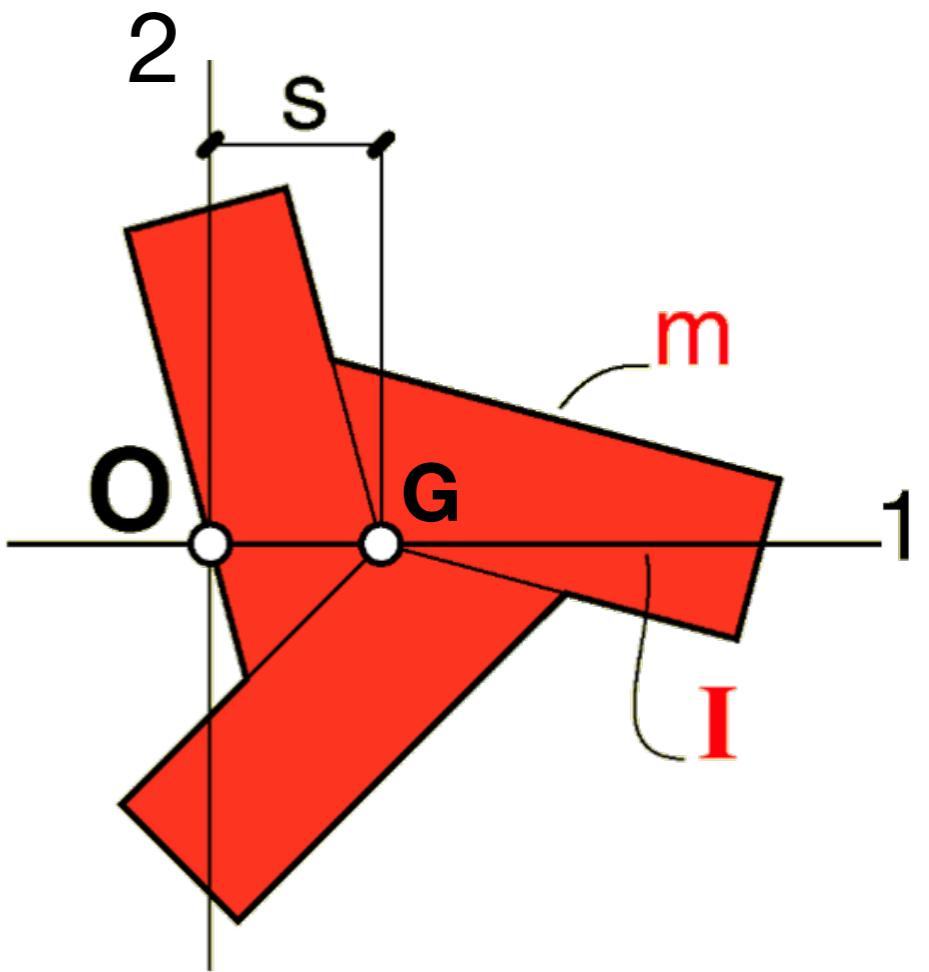


[II(O)] ?



Tensor a G és fàcil:

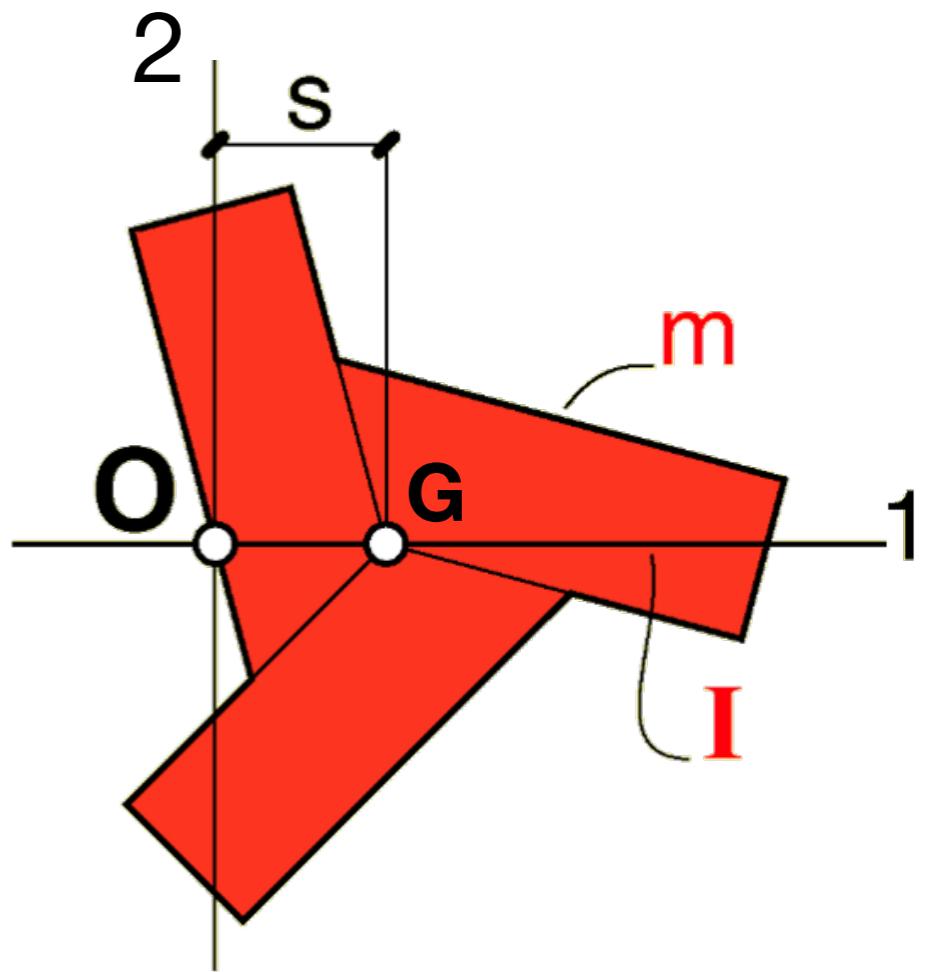
[II(O)] ?



Tensor a G és fàcil:

[II(O)] ?

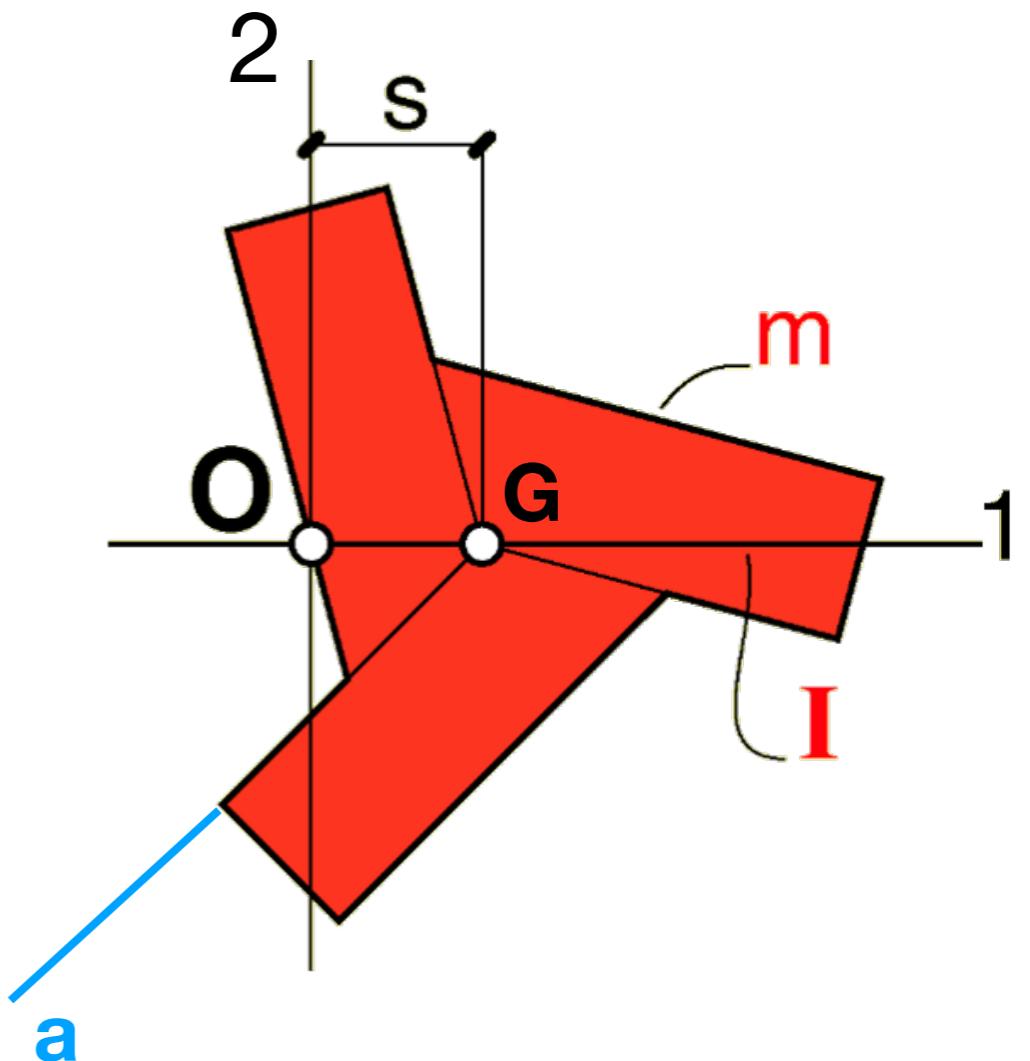
Sòlid pla \Rightarrow 3 és DPI



Tensor a G és fàcil:

[II(O)] ?

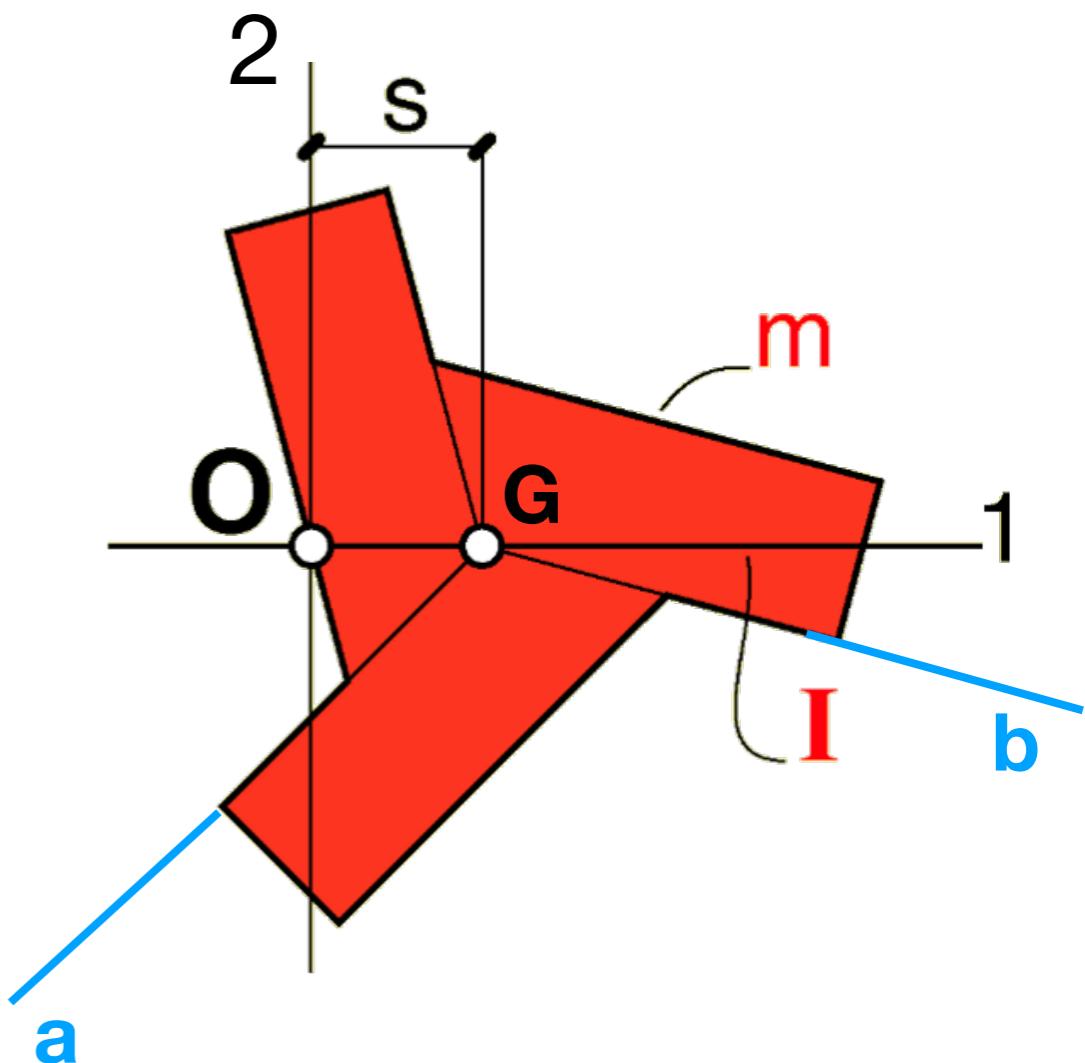
Sòlid pla \Rightarrow 3 és DPI



Tensor a G és fàcil:

[II(O)] ?

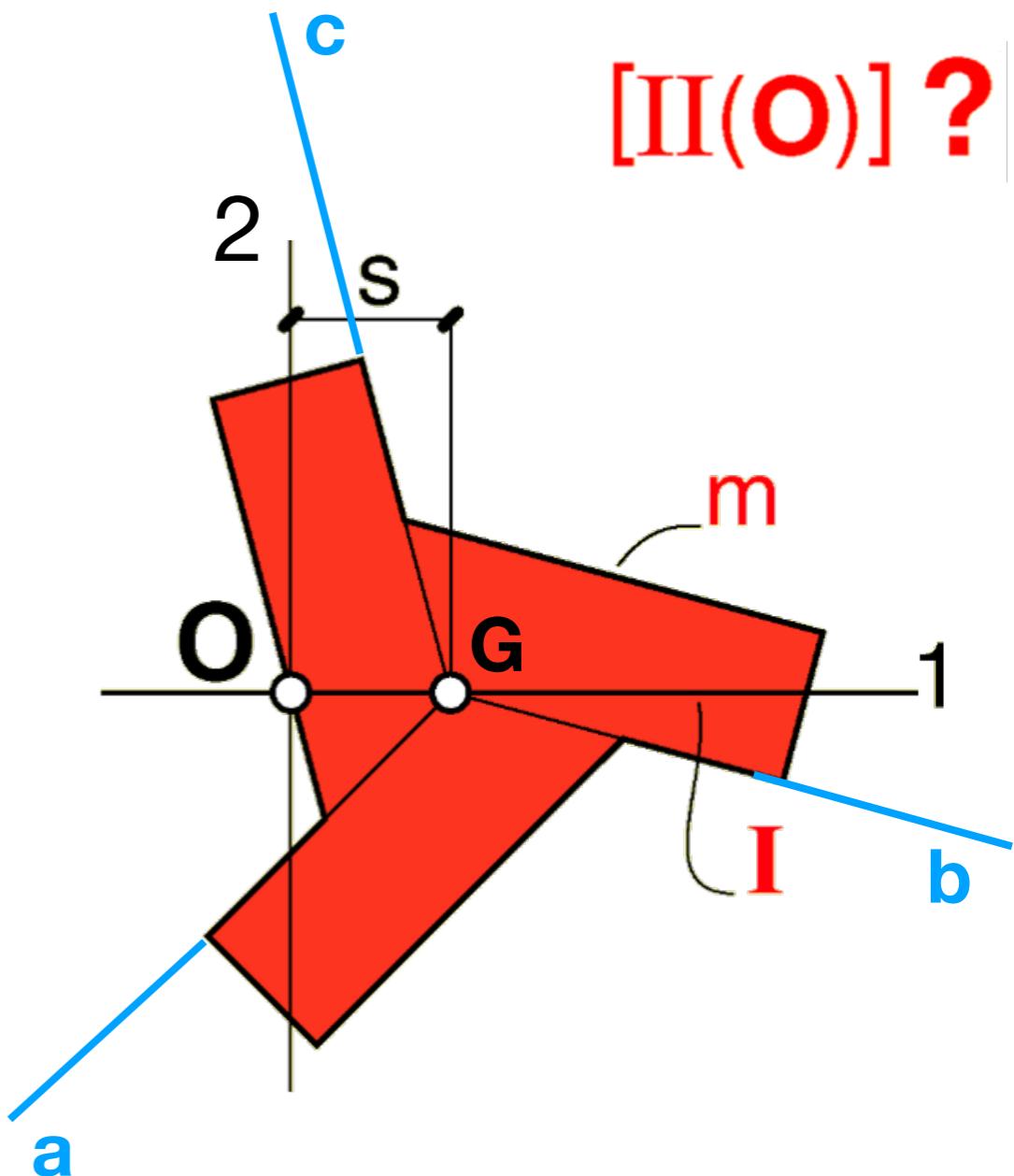
Sòlid pla \Rightarrow 3 és DPI



Tensor a G és fàcil:

Sòlid pla \Rightarrow 3 és DPI

[II(O)] ?



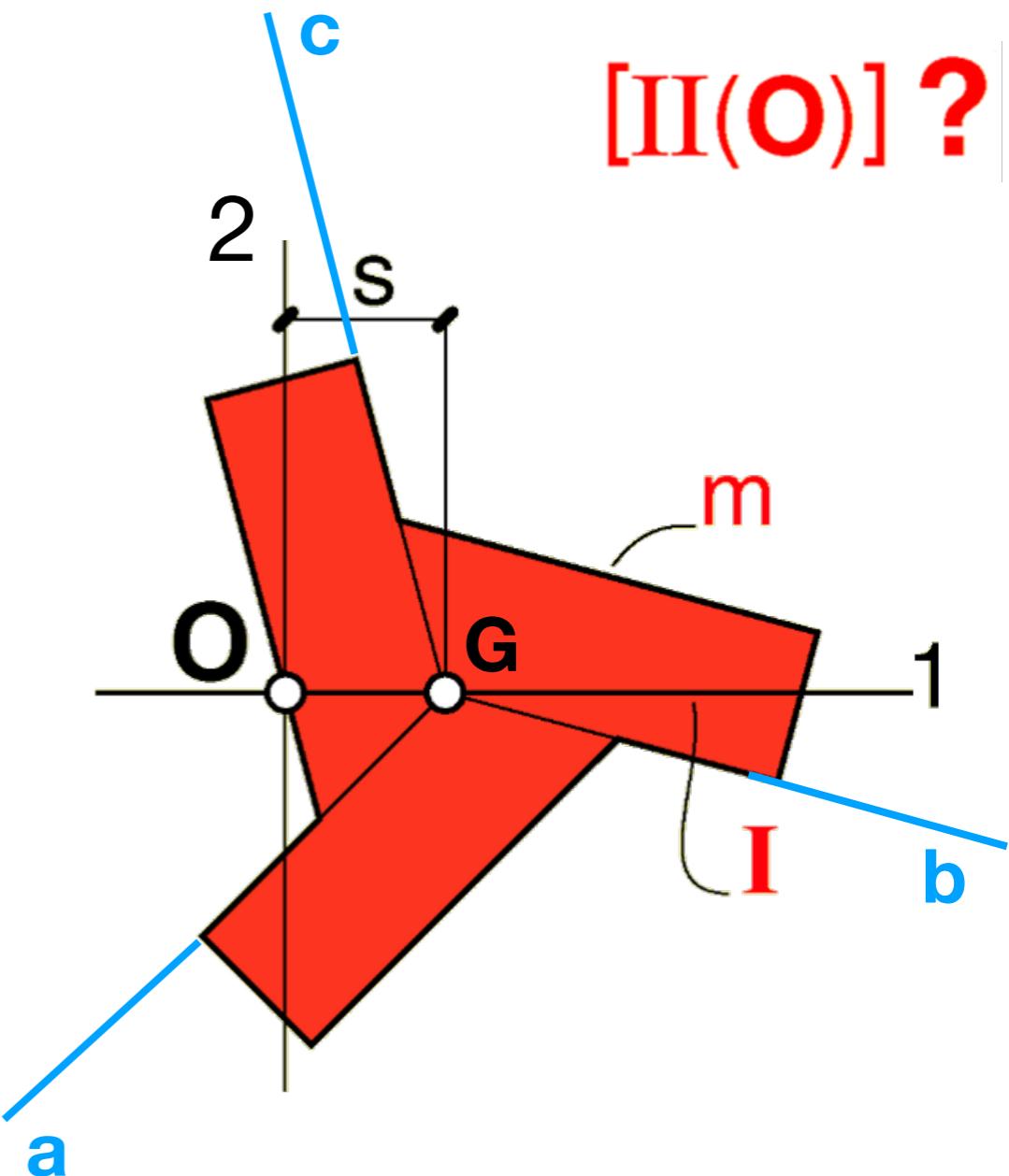
Tensor a G és fàcil:

[II(O)] ?

Sòlid pla \Rightarrow 3 és DPI

$$\underbrace{I_{aa} = I_{bb} = I_{cc}}_{} = I$$

rotor simètric per a G



Tensor a G és fàcil:

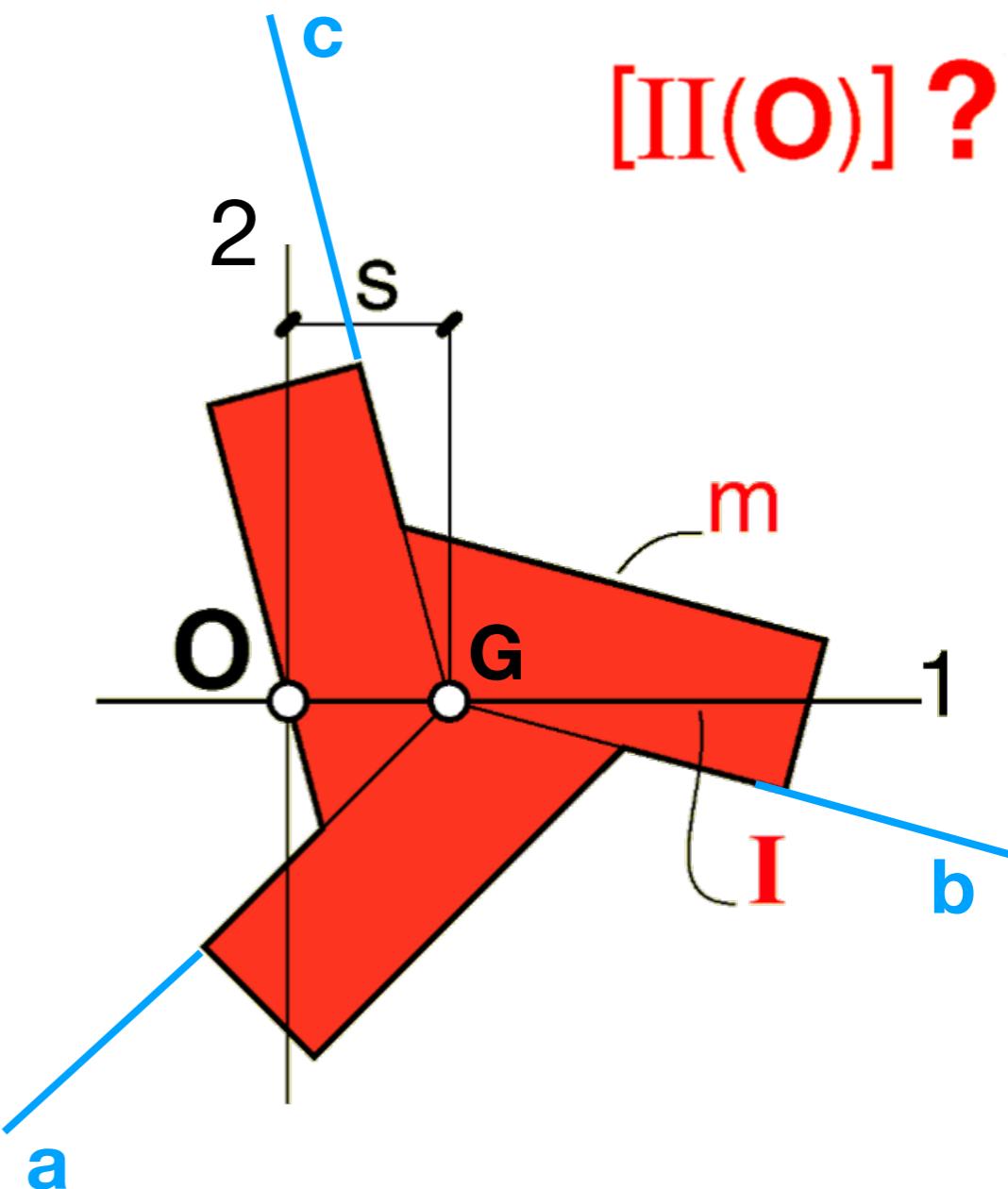
[II(O)] ?

Sòlid pla \Rightarrow 3 és DPI

$$\underbrace{I_{aa} = I_{bb} = I_{cc}}_{\text{rotor simètric per a } G}$$

rotor simètric per a G

$$[I(G)]_B = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 2I \end{bmatrix}$$



Tensor a G és fàcil:

[II(O)] ?

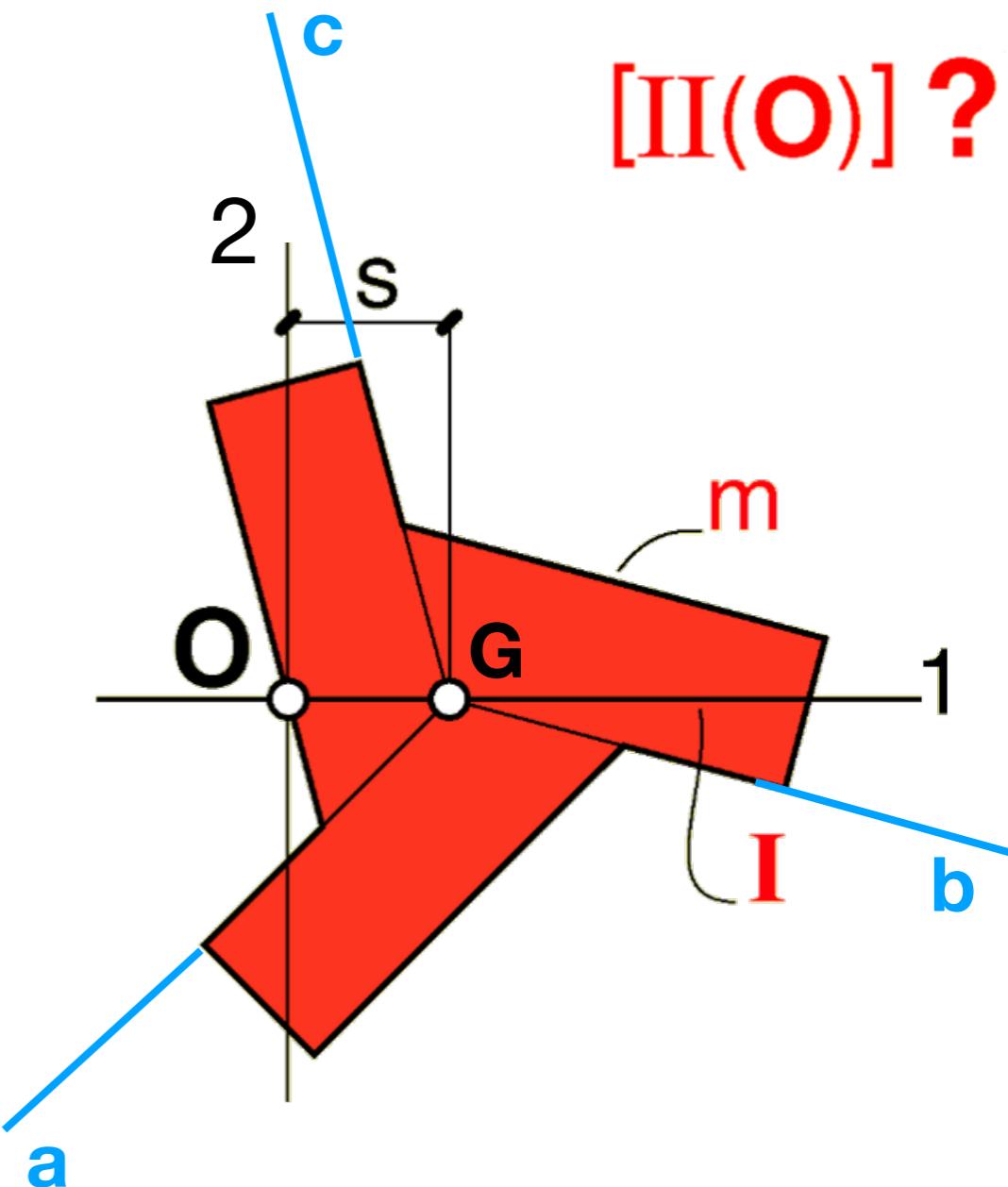
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Canvi a O via Steiner:



Tensor a G és fàcil:

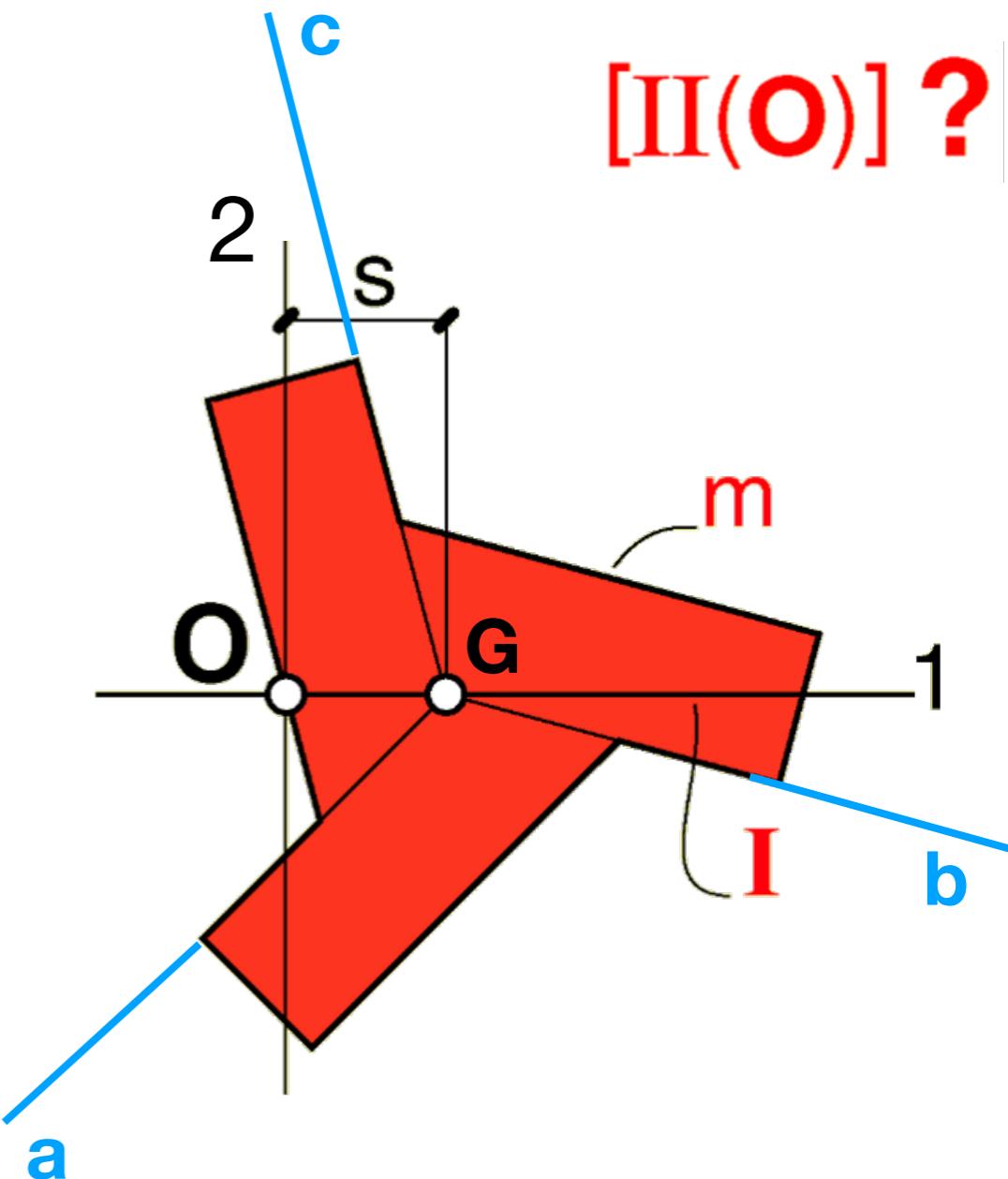
[II(O)] ?

Sòlid pla \Rightarrow 3 és DPI

$$\underbrace{I_{aa} = I_{bb} = I_{cc}}_{\text{rotor simètric per a } G} = I$$

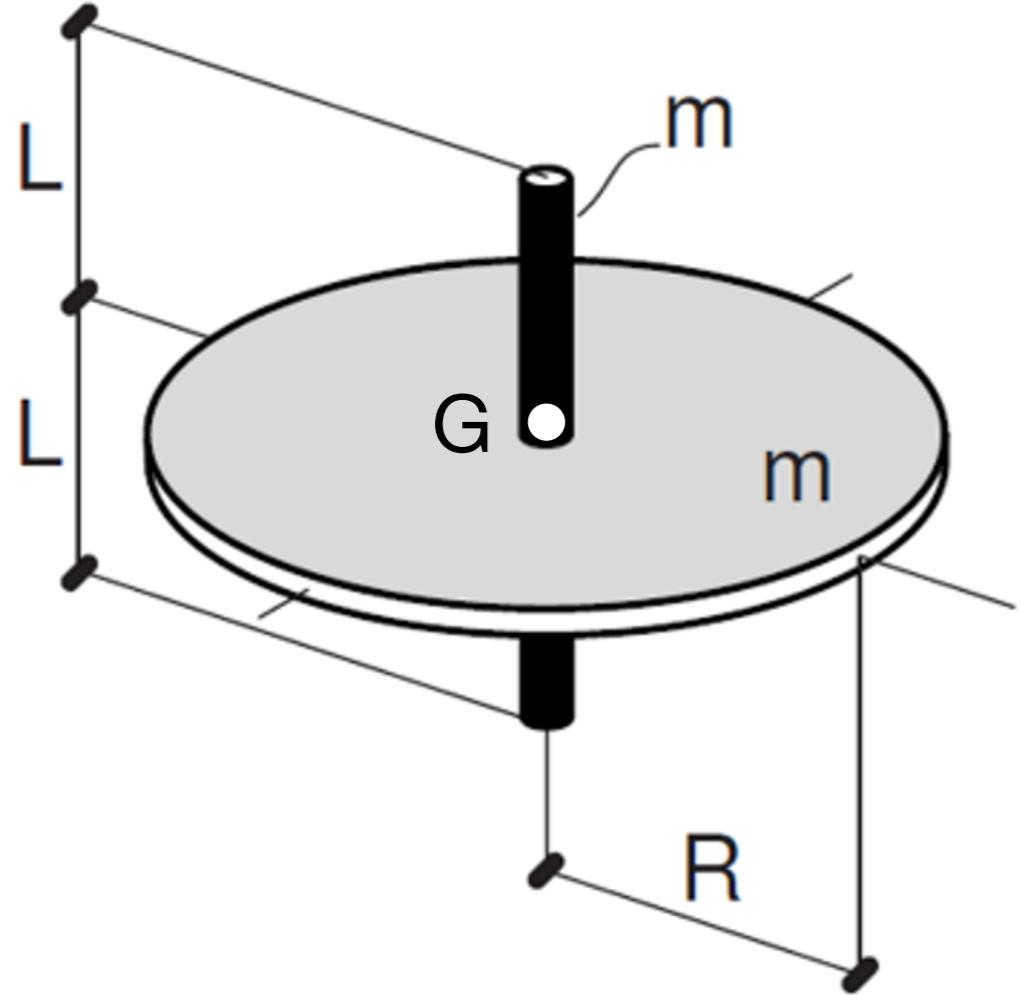
rotor simètric per a G

$$[I(G)]_B = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 2I \end{bmatrix}$$

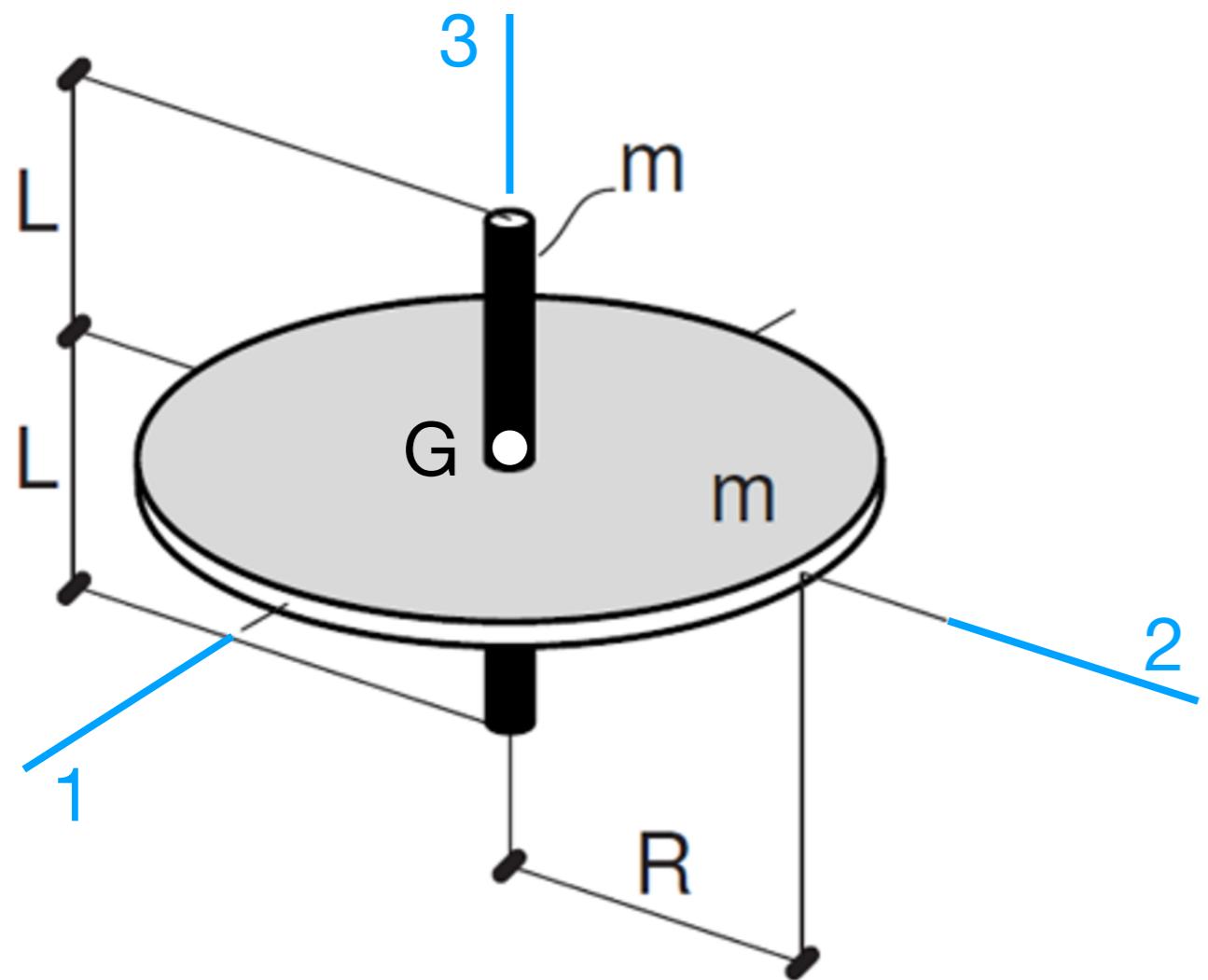


Canvi a O via Steiner:

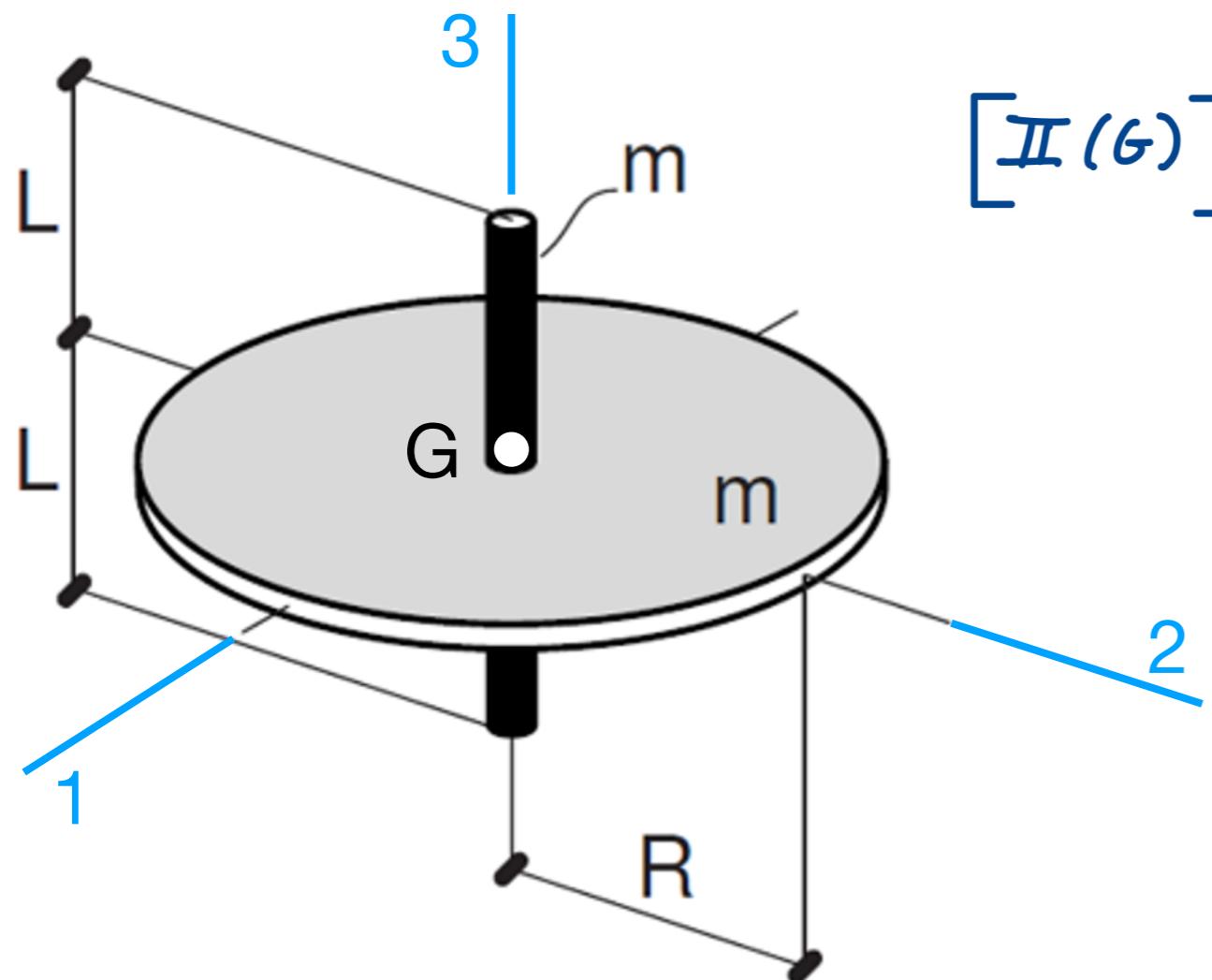
$$[I(O)]_B = \underbrace{\begin{bmatrix} I & I & 2I \end{bmatrix}}_{I(G)} + \underbrace{\begin{bmatrix} 0 & ms^2 & ms^2 \end{bmatrix}}_{I^\oplus(O)}$$



**Relació entre L i R
per tal que sigui
rotor esfèric a G ?**

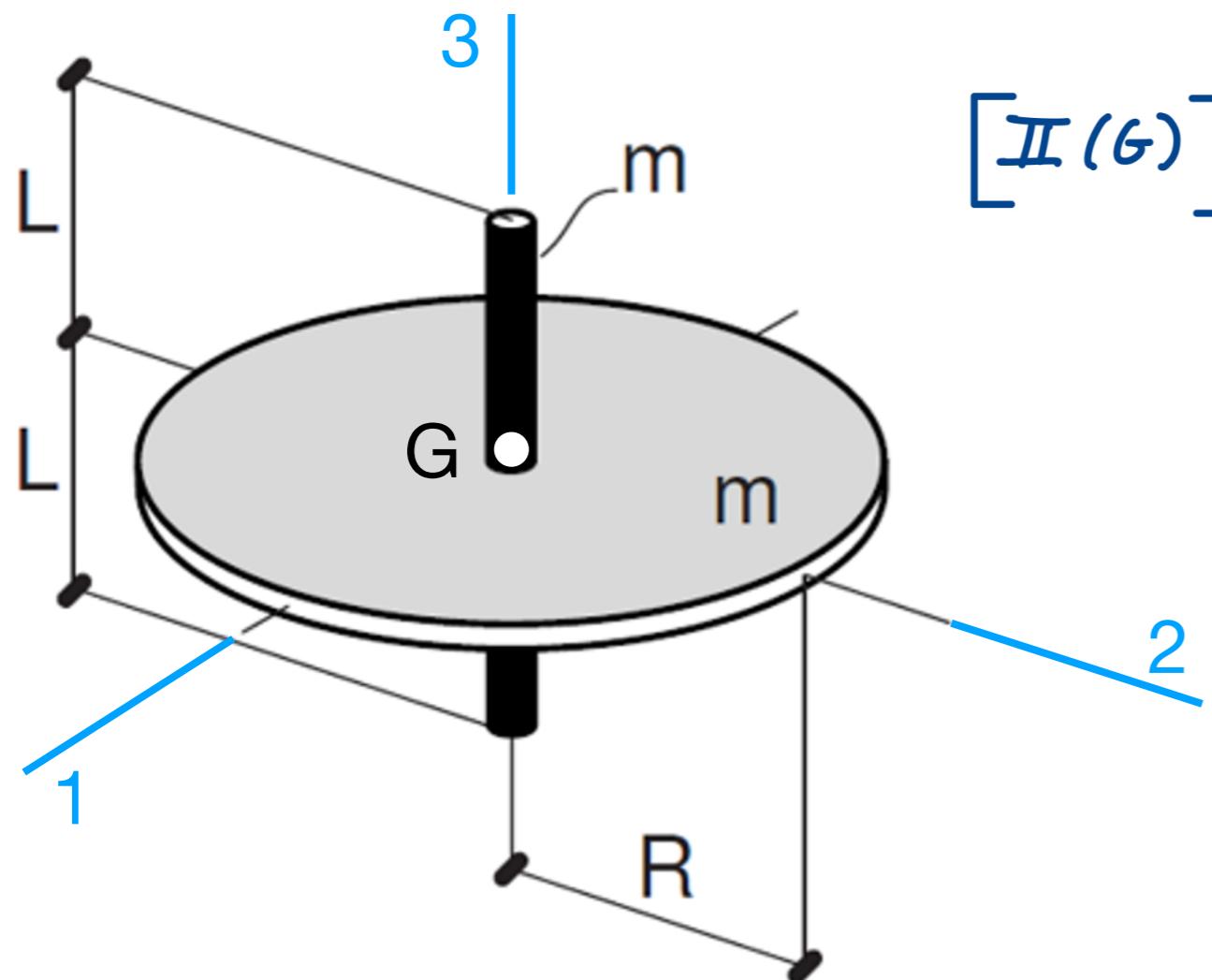


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per tal que sigui
rotor esfèric a G ?**

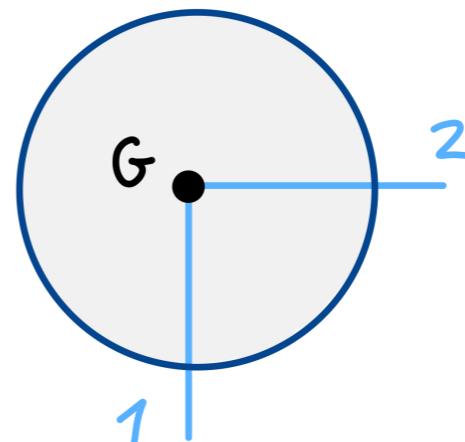


$$[\mathcal{I}(G)]_B = [\mathcal{I}_{disc}(G)]_B + [\mathcal{I}_{barra}(G)]_B$$

**Relació entre L i R
per tal que sigui
rotor esfèric a G ?**

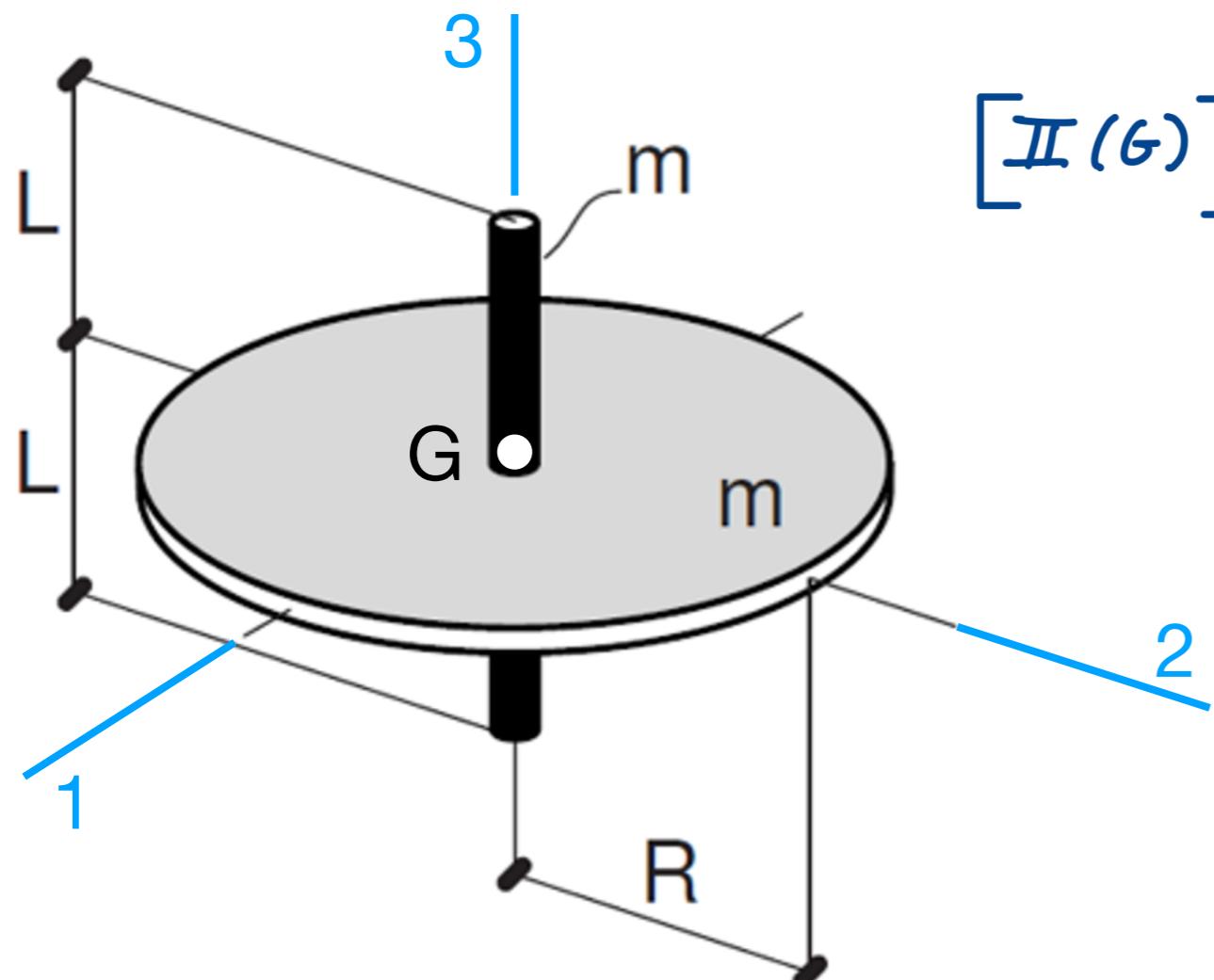


$$[\mathcal{I}(G)]_B = [\mathcal{I}_{disc}(G)]_B + [\mathcal{I}_{barra}(G)]_B$$

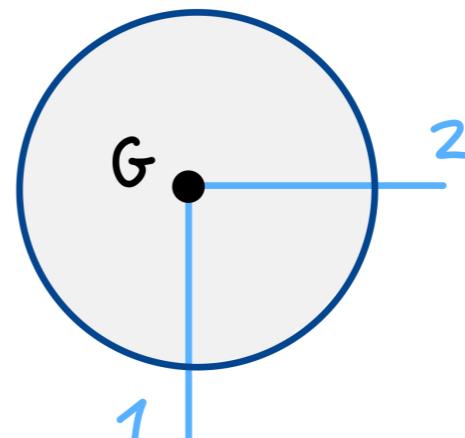


$$\begin{bmatrix} I' & & \\ & I' & \\ & & 2I' \end{bmatrix}$$

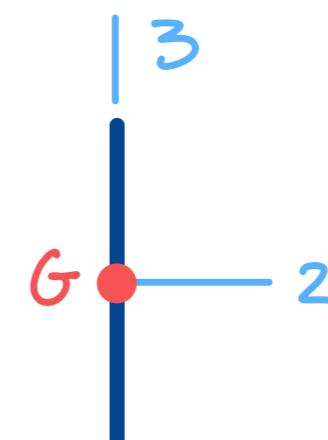
**Relació entre L i R
per tal que sigui
rotor esfèric a G ?**



$$[\mathcal{I}(G)]_B = [\mathcal{I}_{disc}(G)]_B + [\mathcal{I}_{barra}(G)]_B$$

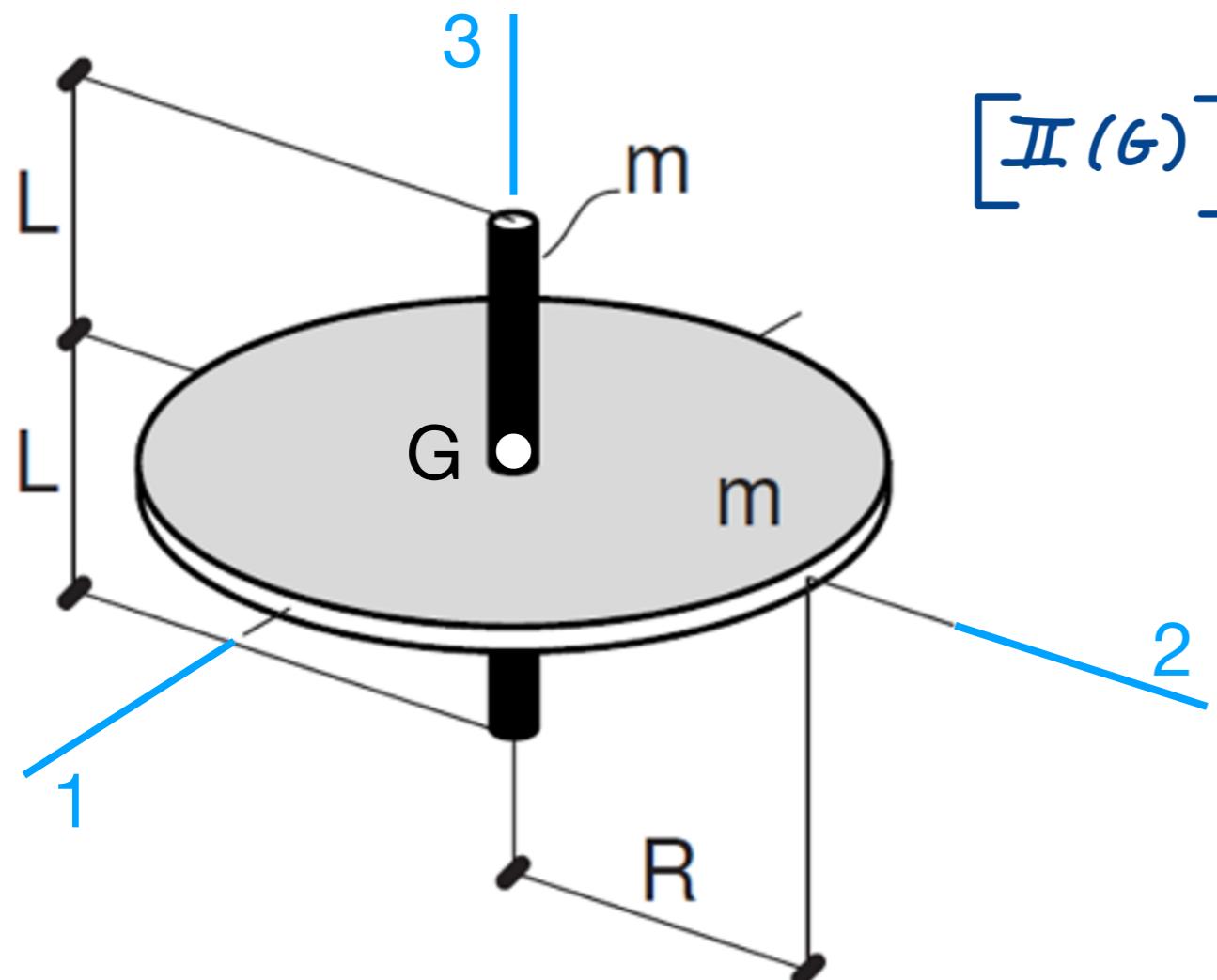


$$\begin{bmatrix} \mathcal{I}' & & \\ & \mathcal{I}' & \\ & & 2\mathcal{I}' \end{bmatrix}$$



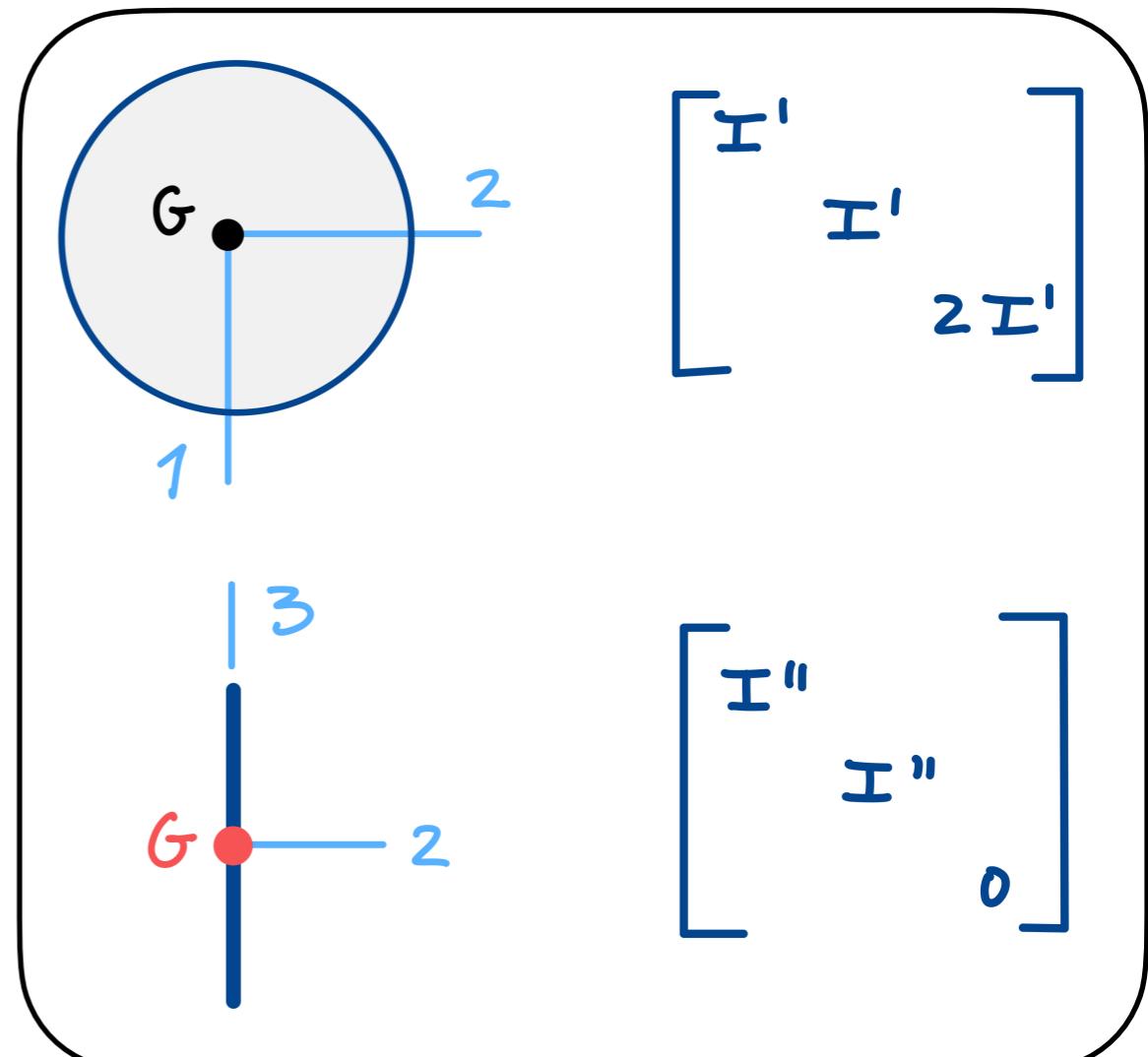
$$\begin{bmatrix} \mathcal{I}'' & & \\ & \mathcal{I}'' & \\ & & 0 \end{bmatrix}$$

**Relació entre L i R
per tal que sigui
rotor esfèric a G ?**

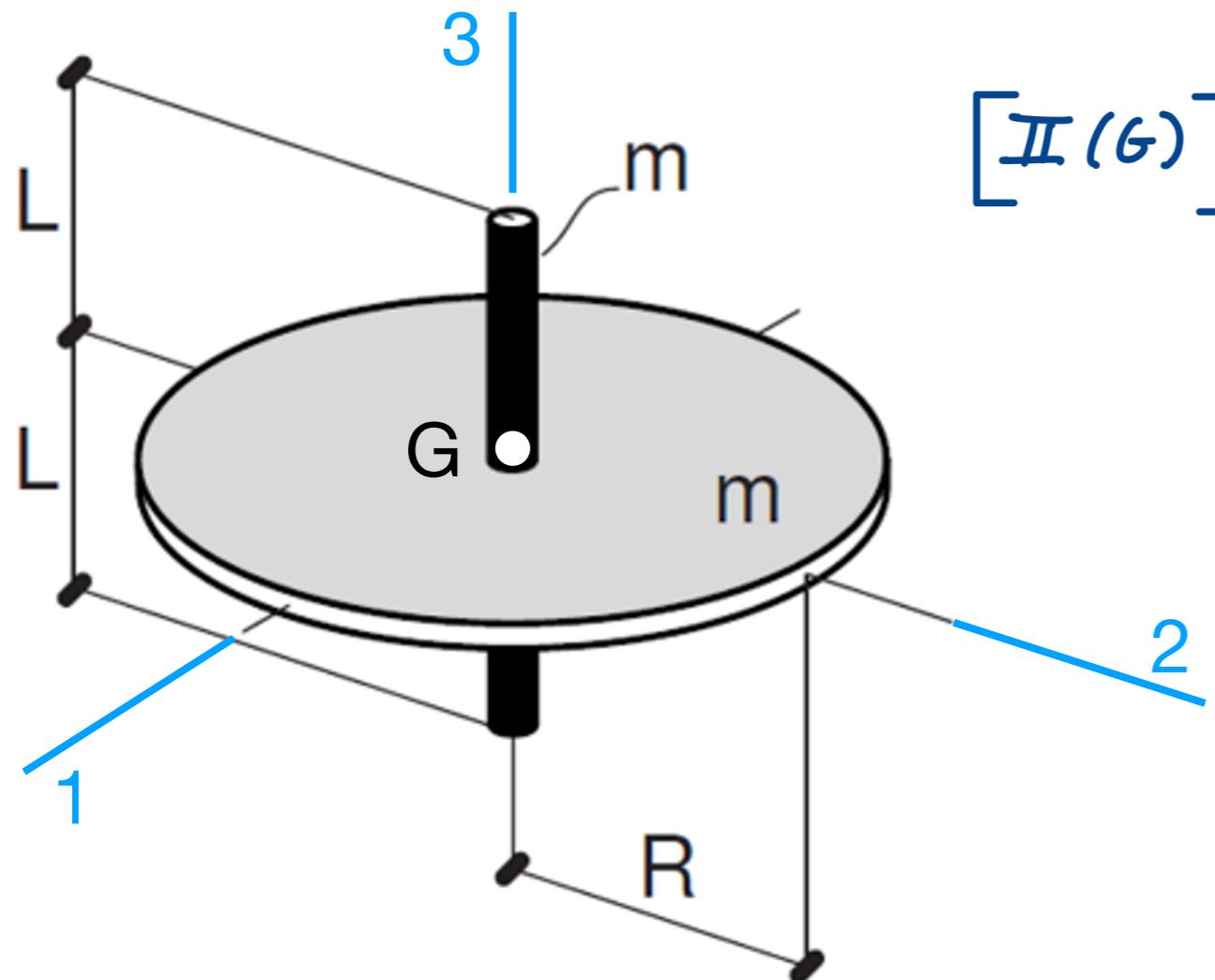


**Relació entre L i R
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$$[\mathcal{I}(G)]_B = [\mathcal{I}_{disc}(G)]_B + [\mathcal{I}_{barra}(G)]_B$$

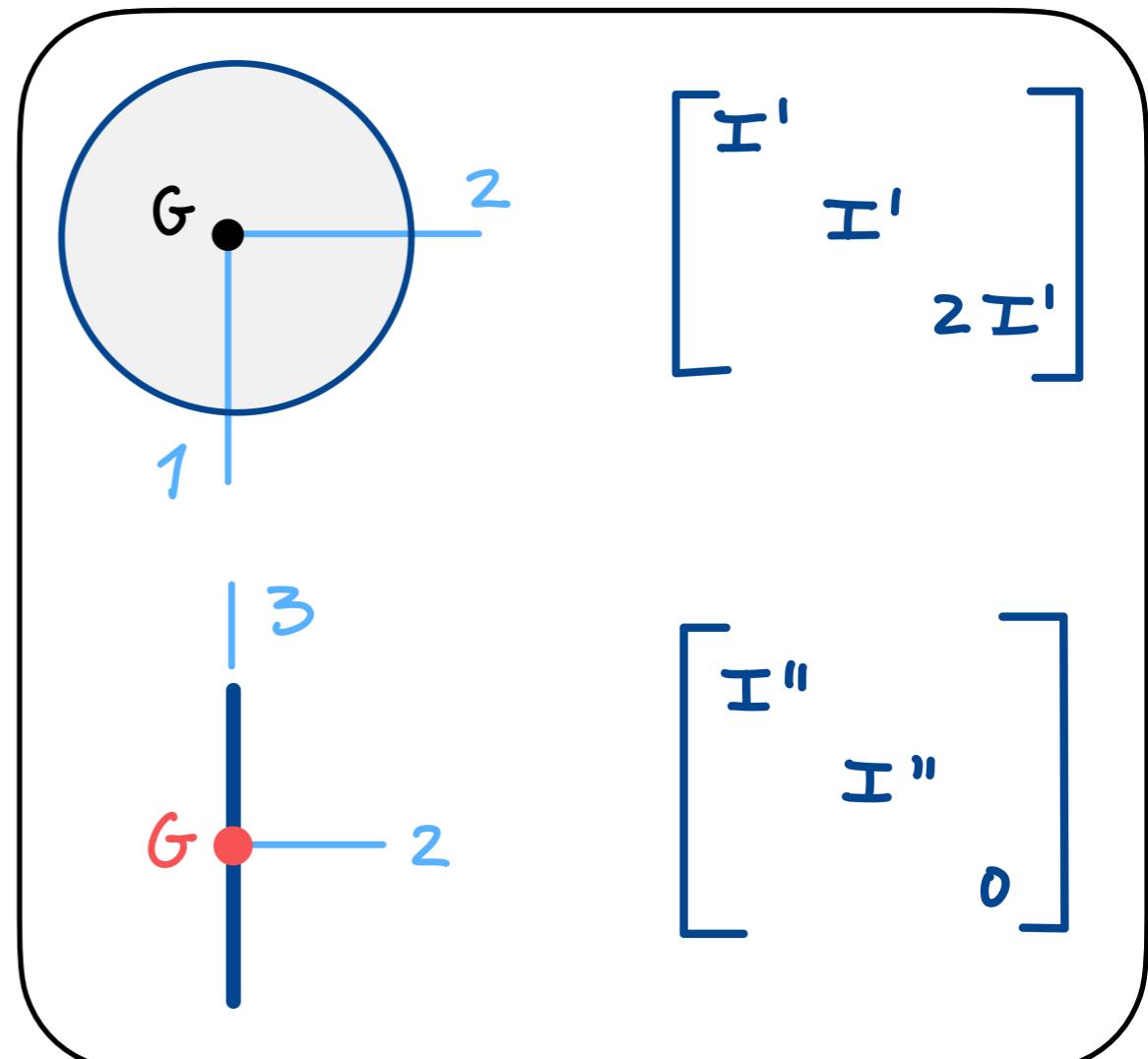


La suma és un tensor diagonal



**Relació entre L i R
per tal que sigui
rotor esfèric a G ?**

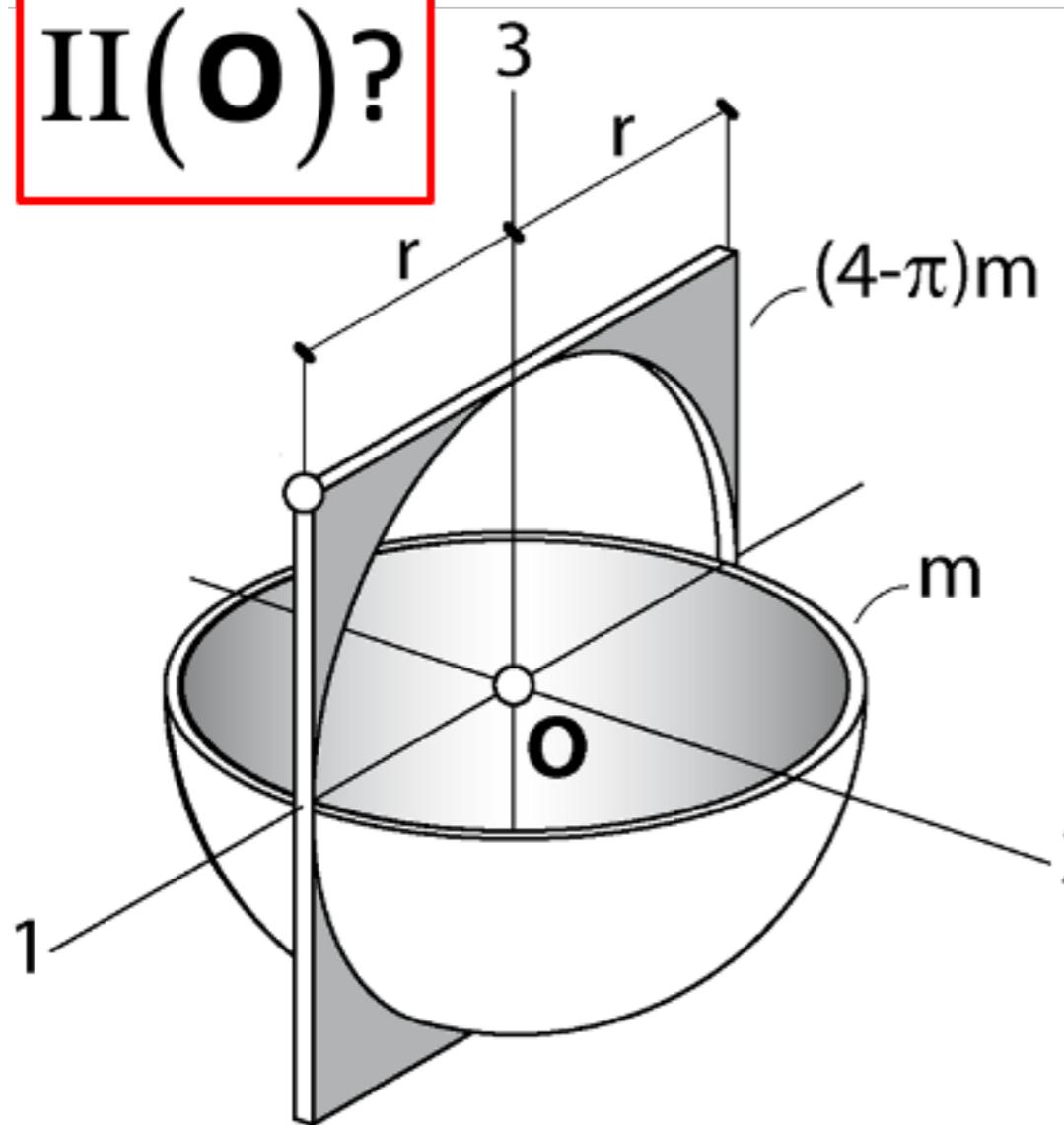
$$[\mathcal{I}(G)]_B = [\mathcal{I}_{disc}(G)]_B + [\mathcal{I}_{barra}(G)]_B$$



La suma és un tensor diagonal

Cal imposar que els 3 elements de la diagonal siguin iguals

II(**O**)?



Solució a 9P.pdf