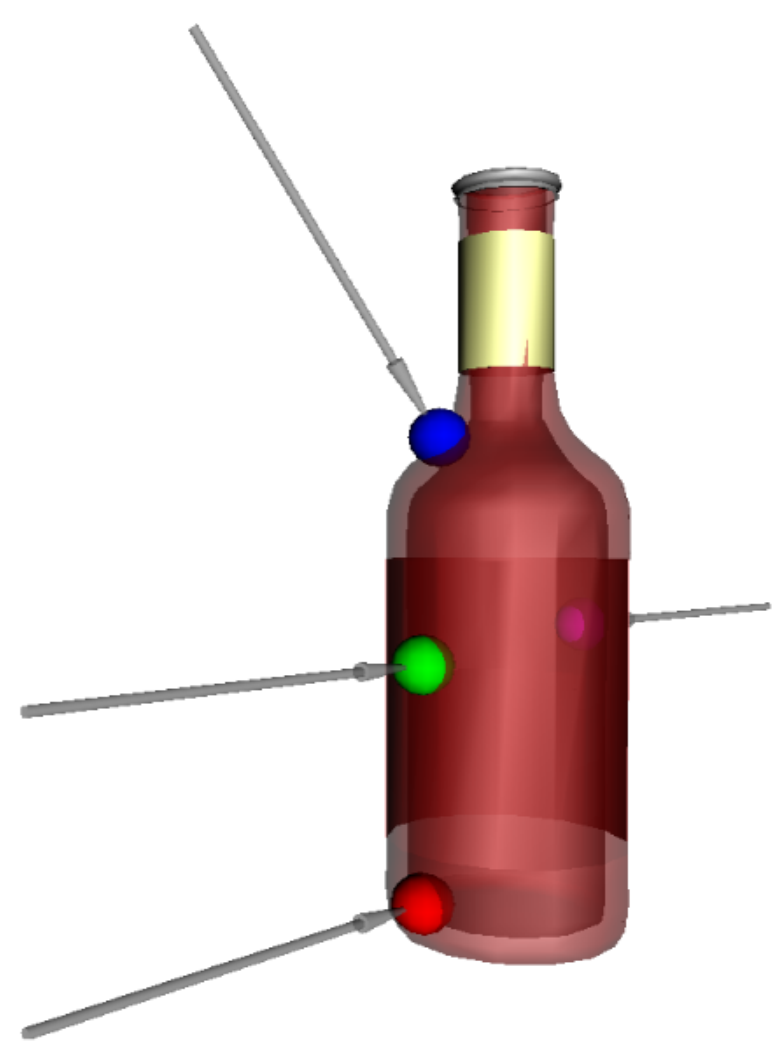


Finding All Valid Hand Configurations for a Given Precision Grasp

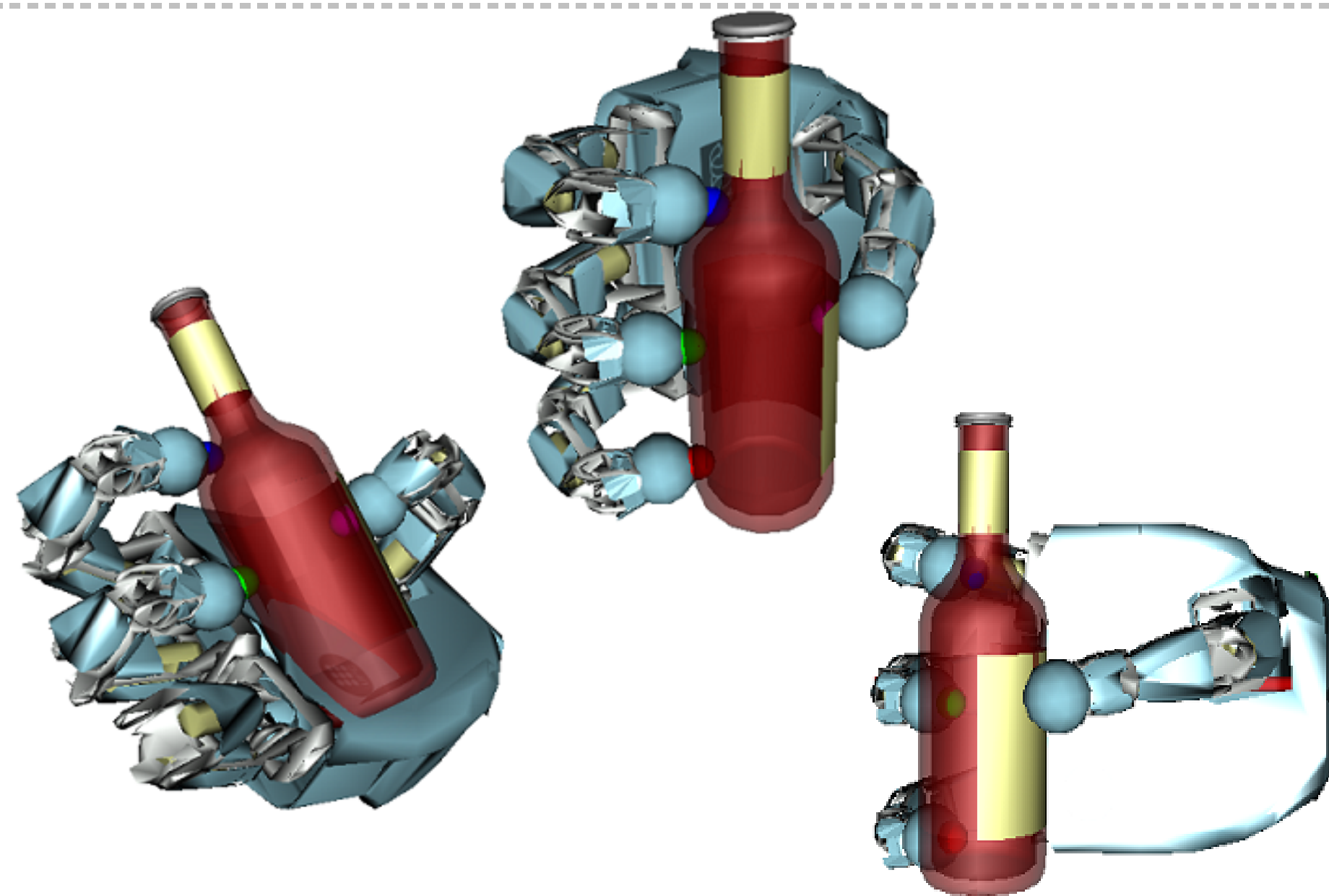
C. Rosales, J. M. Porta, R. Suárez and L. Ros

Problem statement

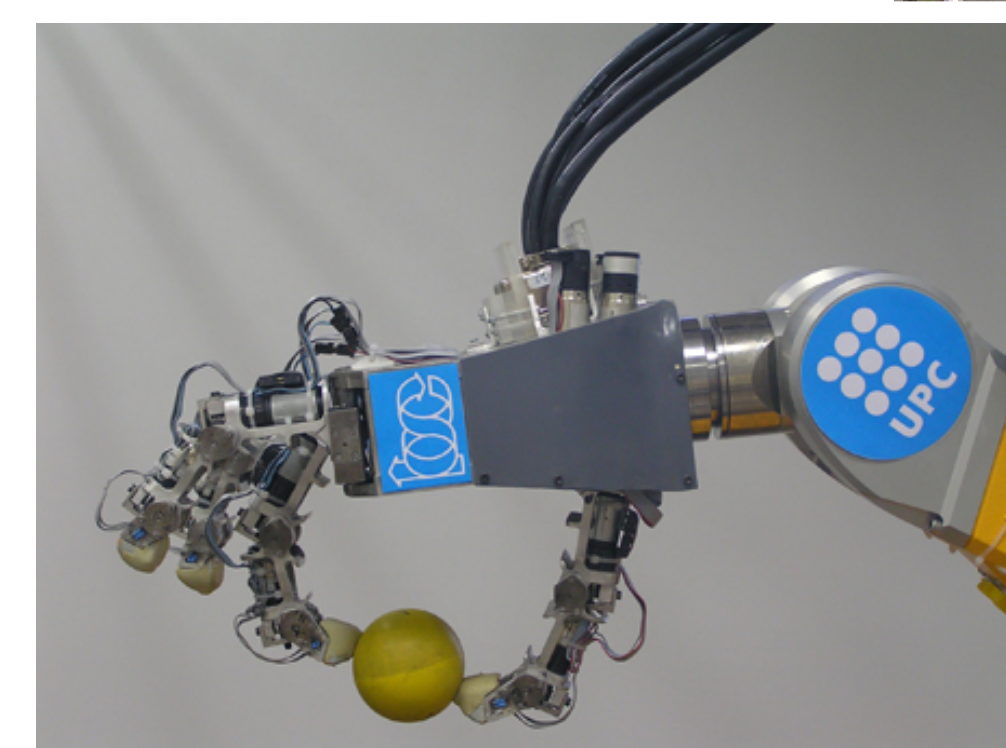


A precision grasp on an object:

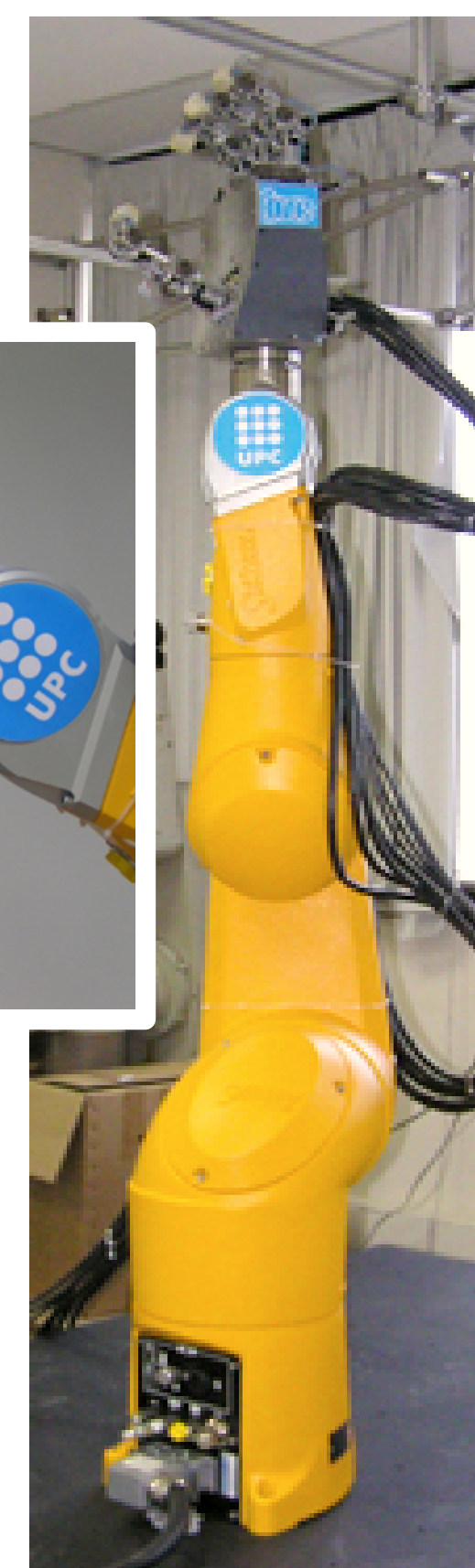
- Stable grasp
- Finger-to-point assignment
- Normal on contact point



Find all configurations of the hand-object system
Multiple configurations are possible, even infinite

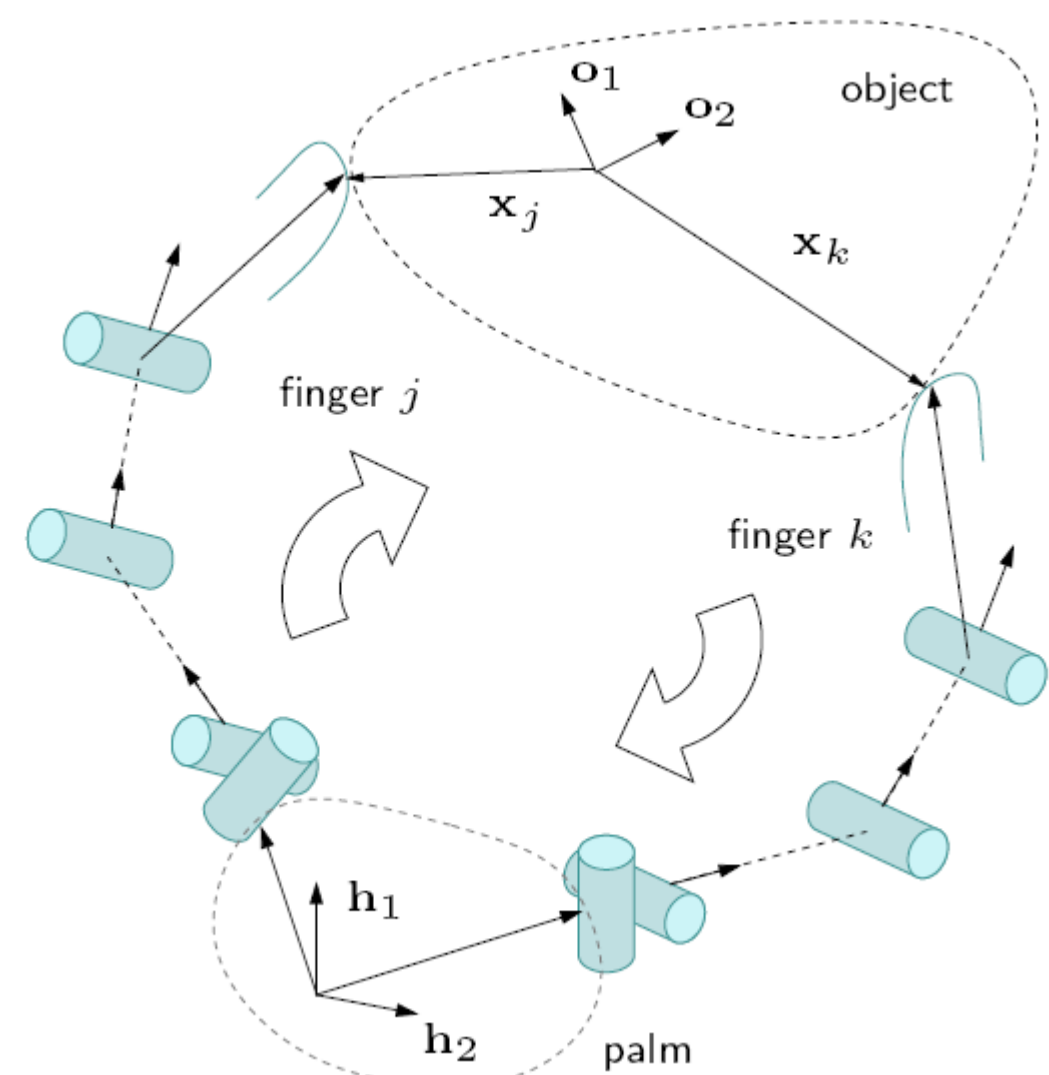


Robot hand MA-I mounted on a robot arm

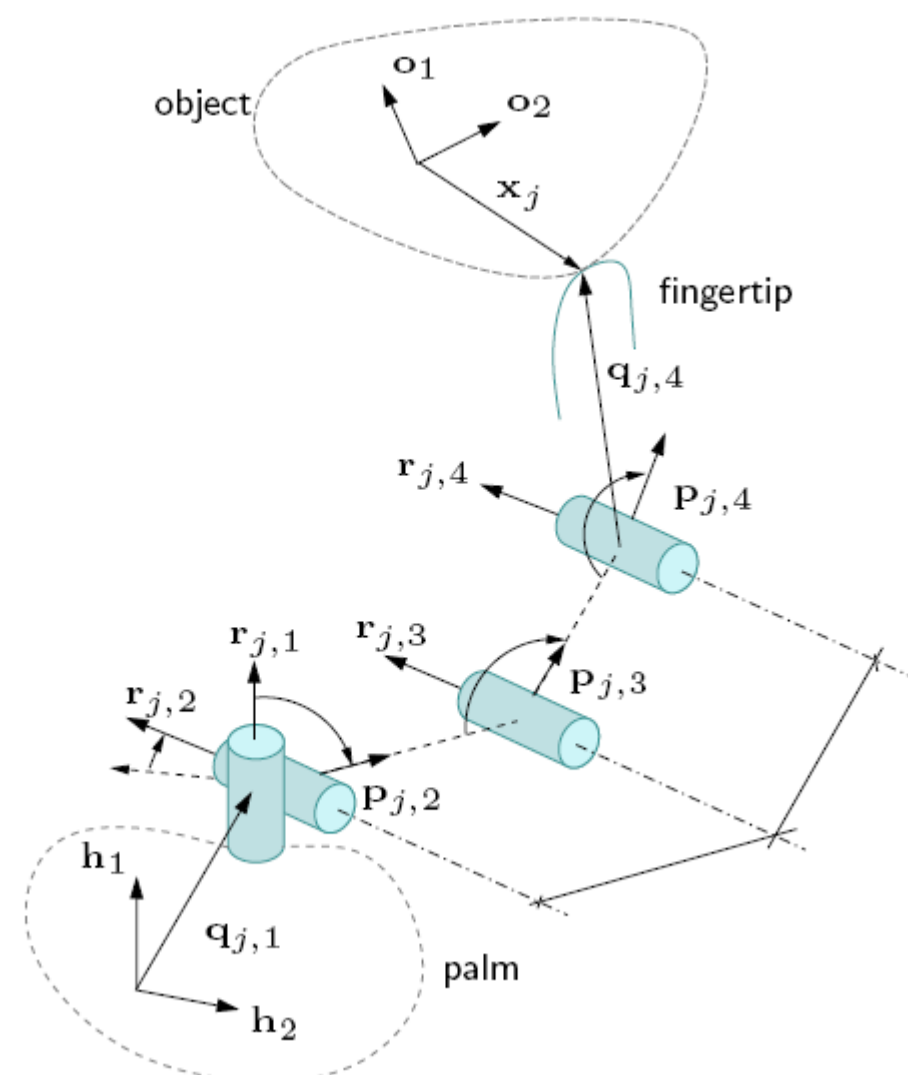


Formulation

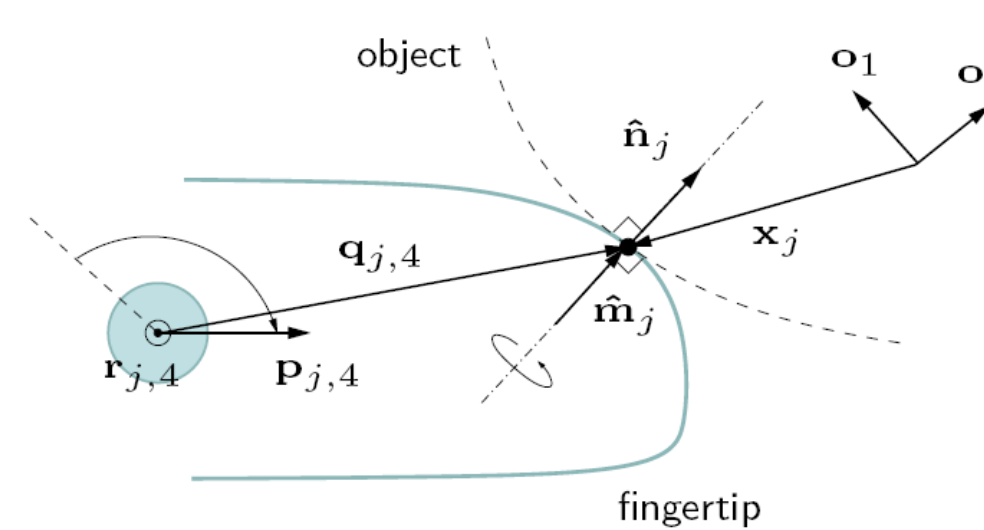
The variables are in global coordinates
Algebraic equations with linear, bilinear and quadratic monomials



I. Loop constraints



II. Finger kinematic and reference frame constraints



III. The contact point is fixed and modelled as a revolute joint

$$c = \cos(\phi) \implies c = \mathbf{u} \cdot \mathbf{v}$$

$$s = \sin(\phi) \implies s \cdot \mathbf{w} = \mathbf{u} \times \mathbf{v}$$

IV. Joint angles are limited by constraining their sines and cosines, defined by dot and cross products of finger vectors

Dimension of the solution

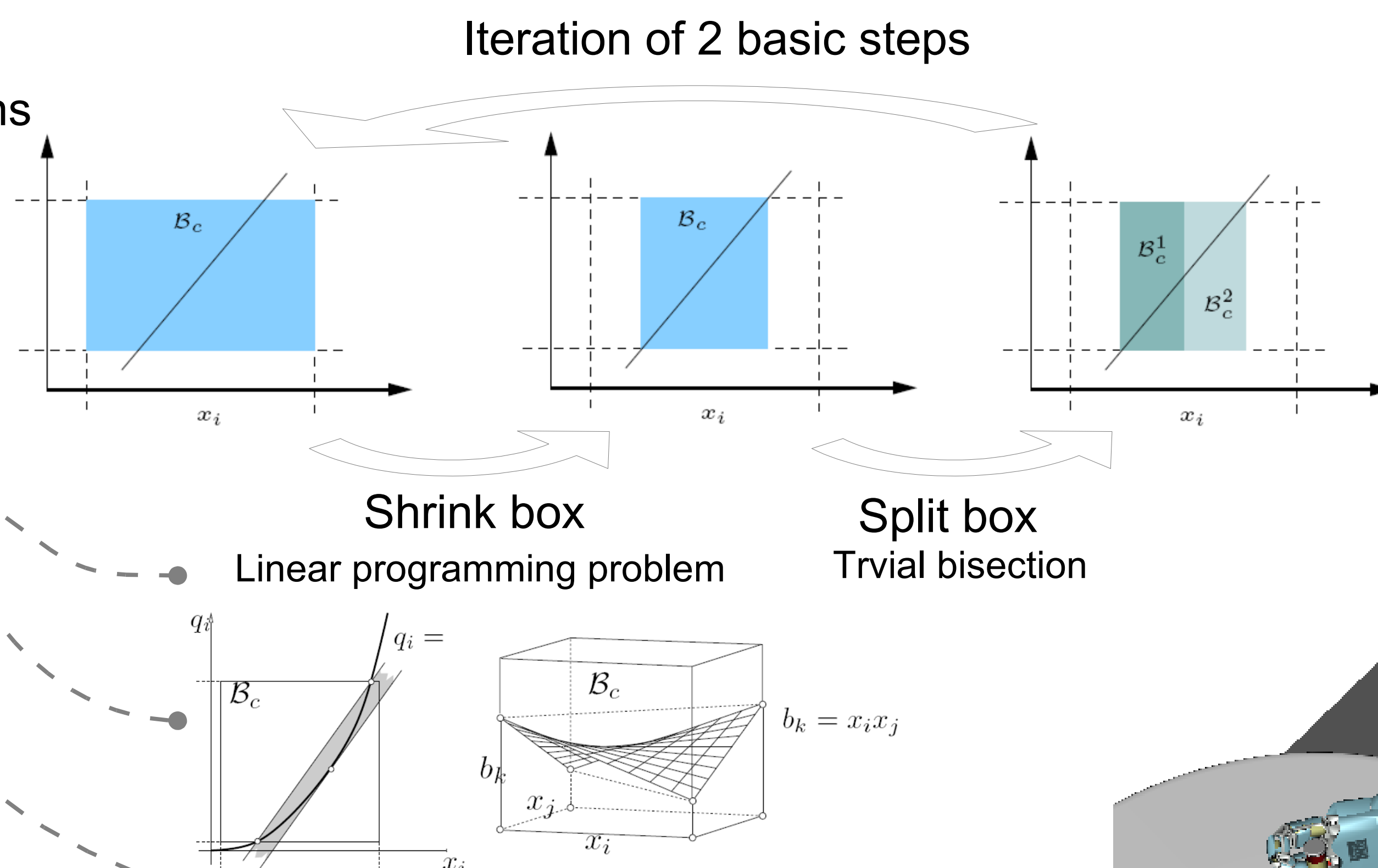
For n fingers:
 $f = 5n$ degrees of freedom
 $c = 6(n - 1)$ constraints
 By the Grübler-Kutzbach's criterion
 $d = f - c = 6 - n$

Numerical Solution

Branch-and-prune algorithm based on linear relaxations

Pre-processing step

$$\begin{aligned} x_i, x_i^2 \text{ and } b_k = x_i x_j & \quad L(\mathbf{x}) = 0 \\ x_i x_j & \quad Q(\mathbf{x}) = 0 \\ \text{initial system} \rightarrow \text{change of variables} \rightarrow \text{new system} & \quad B(\mathbf{x}) = 0 \end{aligned}$$



The configuration of the robot arm has a closed-form solution

Test cases

Snapshots of 1-dimensional solution subset

