Book: Basic Mathematics Author: Serge Lang

Contents

Chapter 1

Numbers

1.1 The Integers

No exercises in this section.

1.2 Rules for Addition

Justify each step, using commutativity and associativity in proving the following identities.

1. $(a + b) + (c + d) = (a + d) + (b + c)$
2. $(a + b) + (c + d) = (a + c) + (b + d)$
3. $(a - b) + (c - d) = (a + c) + (- b - d)$
4. $(a - b) + (c - d) = (a + c) - (b + d)$
5. $(a - b) + (c - d) = (a + d) - (c - b)$
6. $(a - b) + (c - d) = -(b + d) + (a + c)$
7. $(a - b) + (c - d) = -(b + d) - (-a - c)$
8. $((x + y) + z) + w = (x + z) + (y + w)$
9. $(x - y) - (z - w) = (x + w) - y + z$
10. $(x - y) - (z - w) = (x - z) - (w - y)$
11. Show that $- (a + b + c) = -a + (-b) + (-c)$.
12. Show that $-(a -b - c) = -a + b + c$.
13. Show that $-(a - b) = b - a$.

Solve for x in the following equations.

14. -2 + x = 4

15. 2 - x = 5

16. x - 3 = 7

17. -x + 4 = 1

18. 4 - x = 8

19. -5 - x = -2

20. -7 + x = -10

21. -3 + x = 4

22. Prove the **cancellation law for addition:**

$$\text{If } a + b = a + c, \text{then } b = c$$

23. Prove: If a + b = a, then b = 0

1.3 Rules for Multiplication

1. Express each of the following expressions in the form $2^n 3^n a^r b^s$, where m, n, r, s are positve integers.
   (a) $8a^2b^3(27a^4)(2^5ab)$
   (b) $16b^3a^2(6ab^4)(ab)^3$
   (c) $3^2(2ab)^3(16a^2b^5)(24b^2a)$
   (d) $24a^3(1ab^2)^3(3ab)^2$
   (e) $(3ab)^2(27a^3b)(16ab^5)$
   (f) $32a^4b^5a^3b^2(6ab^3)^4$

2. Prove:

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

3. Obtain expansion for $(a + b)^4$ and $(a - b)^4$ similar to the expansions for $(a + b)^3$ and $(a - b)^3$ of the preceding exercise.