Book: Basic Mathematics Author: Serge Lang

# Contents

# Chapter 1

# Numbers

## 1.1 The Integers

No exercises in this section.

## 1.2 Rules for Addition

Justify each step, using commutativity and associativity in proving the following identities.

1. (a + b) + (c + d) = (a + d) + (b + c)
2. (a + b) + (c + d) = (a + c) + (b + d)
3. (a - b) + (c - d) = (a + c) + (- b - d)
4. (a - b) + (c - d) = (a + c) - (b + d)
5. (a - b) + (c - d) = (a + d) - (c - b)
6. (a - b) + (c - d) = -(b + d) + (a + c)
7. (a - b) + (c - d) = -(b + d) - (-a - c)
8. ((x + y) + z) + w = (x + z) + (y + w)
9. (x - y) - (z - w) = (x + w) - y + z
10. (x - y) - (z - w) = (x - z) - (w - y)
11. Show that - (a + b + c) = -a + (-b) + (-c).
12. Show that -(a -b - c) = -a + b + c.
13. Show that -(a - b) = b - a.

Solve for x in the following equations.

14. -2 + x = 4
15. 2 - x = 5
16. x - 3 = 7
17. -x + 4 = 1
18. 4 - x = 8
19. -5 - x = -2
20. -7 + x = -10
21. -3 + x = 4
22. Prove the **cancellation law for addition:**

$$\text{If } a + b = a + c, \text{then } b = c$$

23. Prove: If a + b = a, then b = 0

## 1.3 Rules for Multiplication

1. Express each of the following expressions in the form $2^n 3^n a^r b^s$, where $m$, $n$, $r$, $s$ are positve integers.
   (a) $8a^2b^3(27a^4)(2^5ab)$
   (b) $16b^3a^2(6ab^4)(ab)^3$
   (c) $3^2(2ab)^3(16a^2b^5)(24b^2a)$
   (d) $24a^3(1ab^2)^3(3ab)^2$
   (e) $(3ab)^2(27a^3b)(16ab^5)$
   (f) $32a^4b^5a^3b^2(6ab^3)^4$
2. Prove:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

3. Obtain expansion for $(a+b)^4$ and $(a-b)^4$ similar to the expansions for $(a+b)^3$ and $(a-b)^3$ of the preceding exercise.

Expand the following expressions as sums of powers of $x$ multiplied by integers.

4. $(2-4x)^2$
5. $(1-2x)^2$
6. $(2x+5)^2$
7. $(x-1)^2$
8. $(x+1)(x-1)$
9. $(2x+1)(x+5)$
10. $(x^2+1)(x^2-1)$
11. $(1+x^3)(1-x^3)$
12. $(x^2+1)^2$
13. $(x^2-1)^2$
14. $(x^2+2)^2$
15. $(x^2-2)^2$
16. $(x^2-4)^2$
17. $(x^3-4)(x^3+4)$
18. $(2x^2+1)(2x^2-1)$
19. $(-2+3x)(-2-3x)$
20. $(x+1)(2x+5)(x-2)$
21. $(2x+1)(1-x)(3x+2)$
22. $(3x-1)(2x+1)(x+4)$
23. $(-1-x)(-2+x)(1-2x)$
24. $(-4x+1)(2-x)(3+x)$
25. $(1-x)(1+x)(2-x)$
26. $(x-1)^2(3-x)$
27. $(1-x)^2(2-x)$
28. $(1-2x)^2(3+4x)$
29. $(2x+1)^2(2-3x)$

30. The population of a city in 1910 was 50,000, and it doubles every 10 years. What will it be (a) in 1970 (b) in 1990 (c) in 2000?

31. The population of a city in 1905 was 100,00 and it doubles every 25 years. What will it be after (a) 50 years (b) 100 years (c) 150 years?

32. The population of a city was 200 thousand in 1915, and it triples every 50 years. What will be the population:

    (a) in the year 2215?

    (b) in the year 2165?

33. The population of a city was 25,000 in 1870, and it triples every 40 years. What will it be:

    (a) in 1990?

    (b) in 2030?