TRACTATUS DE MUSICA

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For Philippe Chanial
(1967–2024)

In the spirit of the gift

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That of demonstrating truths already found, and of elucidating them in such a manner that the proof of them shall be irresistible, is the only one that I wish to give; and for this I have only to explain the method which geometry observes in it; for she teaches it perfectly by her examples, although she may produce no discourse on it. And since this art consists in two principal things, the one in proving each proposition by itself, the other in disposing all the propositions in the best order, I shall make of it two sections, of which the one will contain the rules for the conduct of geometrical, that is, methodical and perfect demonstrations; and the second will comprehend that of geometrical, that is, methodical and complete order: so that the two together will include all that will be necessary to direct reasoning, in proving and discriminating truths, which I design to give entire.

Blaise Pascal

Contents

Prologue

Those who seek to understand how a single word can splinter into countless meanings need only trace the definitions of mathematics through time—watch how each age has carved its own understanding from the same stone. What emerges is not clarity, but a constellation of contradictions. Even Kant, who crowned mathematics as the cornerstone of his grand architecture, could not have imagined that the very act of defining it would render that cornerstone the most fragile. This paradox lay dormant in the shadows of certainty—until Kripke, with deliberate precision, drew it back into the light.

The nature of mathematics remains, even now, a territory we dare not fully explore—not for lack of courage, but for the vertigo it induces. Yet this uncertainty does nothing to diminish its strange power to structure thought. Here, at the intersection of necessity and mystery, we find them gathered: Llull, Bruno, Pascal, Spinoza, Newton, Darwin, Freud or Lacan— All those who were blessed by nature's generosity meet at the same point.

In approaching this confluence of mathematical structure and musical experience, this treatise employs a novel methodological tool: *Sync-Turing*, an evaluation method that measures the ability of humans and machines to utilize their communication capacities at maximum efficiency and high synchronization during interaction. This method can achieve harmonious temporal unity between two distinct, non-identical entities—that is, rhythm.

It is in this light that *Tractatus de Musica* presents itself as the first ever fully runnable book. Its runnable body is Tropus, a software-based instrument and a hybrid tool for theory and generative practice. Through this medium, users can actively engage with the *Tractatus*, which both develops new theoretical pathways and generates its own music. In this sense, the *Tractatus* becomes the first music book that continues to grow while producing its own music. (tropus.io)

Paris, October 2025

Introduction

Certain technical terms—such as frequency, timbre, harmonic motion, or perception—are not defined within the formal structure of this text. Their meanings are assumed to conform to standard scientific usage. This omission reflects not an oversight but a commitment to structural economy and methodological clarity.

Where concepts appear to carry multiple definitions, this multiplicity arises from the inherent capacity of any definition to proliferate into variation once perspective is taken as the criterion. Such plurality is not an inconsistency but a historical inevitability.

From a methodological standpoint, strict deductive uniformity has not been pursued rigidly. The nature of the phenomena under investigation, and the particular problems they present, require a diversity of demonstration types. The *Tractatus* tradition itself possesses no singular canonical form; like every literary genre, it evolves, adapting its conventions to the questions it confronts. Those aspects of this work that diverge from conventional structures—chosen deliberately rather than inadvertently—are best understood against this background.

This divergence finds its justification in historical considerations. As Kant demonstrated at the threshold of his epistemology, the criterion for distinguishing axioms from propositions is itself historical: what counts as analytic or synthetic depends upon the historically acquired knowledge of predicates we bring to any given subject. While this remains a general philosophical difficulty, it explains why certain components here appear under designations that their structural form might not strictly require.

Understanding these methodological decisions within their proper context should ease the work's reception, while preserving the systematic rigor demanded by the geometric method.

1

On the Physical Foundations of Sound

Definition 1: Physically speaking, a wave is a localized, time-dependent oscillatory variation in one or more physical quantities, which propagates through space and time, transferring energy without transporting matter.

Definition 2: There are many types of waves. A mechanical wave is a wave that propagates through the oscillation of particles in a material medium, the behavior of which depends on the medium's mechanical properties, such as elasticity and inertia.

Definition 3: There are three types of mechanical waves: transverse waves, longitudinal waves, and surface waves. A sound wave, distinctively, is a longitudinal wave that causes the medium to vibrate along or parallel to the wave's direction of travel.

Definition 4: A sound wave is a mechanical wave that propagates through a solid, liquid, or gas medium as a pressure disturbance, accompanied by longitudinal oscillations of particles within the medium. When the frequency lies within the range of human hearing (approximately 20 Hz to 20 kHz), it is called audible sound. Frequencies below this range are referred to as infrasound, while those above are classified as ultrasound.

Definition 5: A sinusoidal wave is a type of mechanical wave in which each point of the medium undergoes simple harmonic motion at a single frequency. In acoustics, such a wave corresponds to a pure tone, characterized by its single frequency and absence of overtones.

Axiom 1: A sound wave is not a physical substance, such as the medium itself, but a process that occurs within a physical medium, where energy propagates through the oscillations of the medium's particles. The medium itself serves as the substance, while the sound wave represents the dynamic energy transfer.

Axiom 2: A sinusoidal wave represents an idealized mathematical model used to approximate real-world wave phenomena, characterized by a smooth, periodic oscillation. It also functions as a basis in Fourier decomposition, enabling the reconstruction of arbitrary periodic functions through linear combination.

Axiom 3: Sinusoidal waves serve as a fundamental tool in the analysis of sound waves because any sound wave can be expressed as a sum of sinusoidal waves at different frequencies (via Fourier analysis).

Proposition 1: During a performance, each sound source generates both its fundamental frequency, the fundamental tone, and its harmonics (overtones).

Demonstration 1: A simple harmonic motion (SHM) is a theoretical model in which a point in a medium oscillates in a perfectly sinusoidal pattern. This motion is typically represented by a single sine wave in time and, when extended across space, it gives rise to a single-frequency wave known as a pure tone. This forms the foundation of the concept of a fundamental frequency in acoustics.

However, real-world acoustic events are rarely this simple. Instead of a single oscillation, most sound sources generate compound harmonic motions, where multiple frequencies coexist. These include the fundamental tone and its harmonics—whole-number multiples of the fundamental. This results not in a singular waveform, but in a superposition of waves.

This superposition gives rise to timbre, the qualitative tone color that distinguishes one sound source from another, even when pitch and loudness are held constant. Timbre is therefore not an abstract aesthetic property but a direct consequence of physical multiplicity in the waveform.

The identification of a fundamental frequency within such compound structures is controversial. Even with technical tools, it remains complex due to the entanglement of partials and the diversity of harmonic content. From a phenomenological standpoint, perception does not always align with physical stimulus.

Indeed, the auditory system often constructs singular percepts from physically plural inputs. This is especially evident in the phenomenon of the missing fundamental, where listeners perceive a pitch that is not present in the stimulus. Other examples include: Masking effects, where multiple coexisting frequencies collapse into one perceived event or render others inaudible; critical bandwidth studies, which show limits in frequency resolution; the perceptual construction of timbre, wherein tones containing dozens of partials are heard as single identities; neuroimaging data, indicating that the brain encodes frequencies not directly audible.

These findings suggest that the auditory object is not a direct reflection of physical structure, but a cognitive synthesis. Perceptual systems reduce physical plurality to intelligible singularity. While musical experience appears unified, it is underpinned by a multiplicity that remains active, even when not consciously perceived.

Corollary 1: The auditory phenomenon in question is not reducible to a singular frequency, but is constituted by a compound harmonic motion resulting from the superposition of multiple oscillatory components.

Corollary 2: The physical reality of sound, in all its spectral richness, exceeds what is immediately given to consciousness.

Corollary 3: Contrary to the illusion of particularity, sound is not a physical substance, nor is it a particular event. Sound, as an oscillation of a material medium, is a series of events — or more precisely, a process. This dynamic plurality unfolds over time, reinforcing the notion that sound is not a static object, but a temporal process.

Scholium 1: The decay of a vibrating string reveals the most elementary truth of wave mechanics: frequency remains constant while amplitude diminishes. The oscillation range contracts, but the wavelength is unaffected, since frequency is determined solely by the string's length, tension, and mass per unit length. Decay dissipates vibrational energy into heat and sound radiation.

In linear systems such as the idealized string, the wave equation under fixed boundary conditions admits only ascending harmonics: f, 2f, 3f... These emerge as necessary solutions. Subharmonics do not arise spontaneously; they require nonlinear coupling or external forcing.

Decay does not alter the harmonic order; it only suppresses higher modes more rapidly than lower ones. Thus, the spectrum perceived by the ear evolves toward its fundamental, giving the impression of a sound "darkening" as complexity fades.

What appears mechanically as amplitude loss is, in thermodynamic terms, the irreversible transformation of ordered vibrational energy into entropy, in accordance with the second law of thermodynamics. The coherent oscillation of the string dissolves into molecular disorder.

Proposition 2: Although the behavior of sound is relatively well-modeled within architectural acoustics — a field grounded in science and engineering — music is not confined to performance within such controlled spaces. In open or non-engineered acoustic environments, the processes of prediction and reproduction become fragile, unpredictable, and irreducibly singular.

Demonstration 2: When sound leaves controlled environments and enters open or non-engineered spaces, its behavior becomes highly sensitive to environmental variation.

Physical variables such as temperature, humidity, air pressure, and altitude alter the medium's density and thus the propagation of sound waves. Wind modifies both the direction and speed of transmission, introducing anisotropy and instability.

Beyond scalar variation, spatial complexity profoundly shapes acoustic behavior. The interaction of sound waves with heterogeneous surfaces — reflective, absorptive, irregular — causes phenomena such as reflection, diffraction, and scattering. Additionally, interference patterns emerge when waves from different sources overlap, producing evolving auditory textures that are neither stable nor reducible. The resulting acoustic landscape is shaped by both the materiality of space and the multiplicity of sources, rendering every environment acoustically unique and non-replicable.

Resonance, in this context, plays a pivotal role. It amplifies specific frequencies when structural and spatial conditions align, but those alignments are transient and sensitive to change. Resonance is not a fixed property of instruments or spaces but a dynamic co-production of medium and event. Poetically speaking, resonance transforms inert space into a responsive presence — a vibrating participant in the performance. This relational aspect underscores the singularity of musical acts outside engineered predictability.

From a physical standpoint, reproducing a sound event in such conditions would require the exact reconfiguration of all environmental, structural, and temporal variables — an impossible demand. Therefore, in open acoustic environments, the musical event is not only context-dependent but irreversibly singular.

Corollary 4: Every recording or live performance constitutes a singular acoustic event, irreproducible due to its temporal and spatial conditions. This structural indeterminacy is not a flaw but a constitutive feature of sound, endowing each auditory act with irreducible uniqueness.

Corollary 5: The shift from referring to a sound wave to simply 'sound' marks a conceptual transition from physical causality to experiential perception. This terminological move reflects not only the sensory complexity of audition, but also the implicit common-sense framework through which acoustic events are humanly grasped.

Corollary 6: In non-engineered environments, the medium ceases to be neutral. It acts, responds, and co-shapes the auditory event, turning the space itself into a participant in the acoustic interaction.

Corollary 7**:** The acoustic character of a space contributes physically and semantically to the event it hosts. What is heard is never divorced from where it is heard.

2

On the Musical Medium and Its Dynamics

Axiom 4: Music is produced through a medium.

Scholium 2: While it could be created in liquid or solid media, it is predominantly performed in a gaseous medium—a fact worth emphasizing. However, the medium is not restricted to air alone. The wide variety of musical instruments reveals that music can also be generated through solid and liquid media.

Axiom 5: Every art has its own material.

Proposition 3: The material of music is the given medium. As both actors and potential manipulators, we are inherently a component of this medium.

Demonstration 3: Sound requires a medium to propagate, and in the majority of musical contexts, this medium is air. The properties of the air—its density, temperature, and humidity—determine the speed and fidelity of sound transmission. As sound waves travel through this medium, every object, including the human body, acts as either an emitter, reflector, absorber, or diffuser of sound energy.

In vocal or wind-based performance, human body becomes a source of pressure waves that modulate the surrounding air. This modulation constitutes the primary generation of sound. At the same time, the performer remains embedded within the same medium, surrounded by air particles that carry, reflect, and transform these waves. Thus, the medium is not external to the musician—it includes them and is dynamically shaped by their presence.

Corollary 8: The performer is not separate from the medium but materially entangled with it. Every musical act is therefore a localized transformation within the medium itself, not merely an external manipulation of it.

Scholium 3: As Goethe remarked about the Italian carnivals, there is no distinction between the work, the stage, and the audience; everything transforms into part of a single whole in musical performance.

Proposition 4: A musician plays with a medium, plays through a medium, and ultimately plays a medium itself. No other art possesses such a privilege.

Demonstration 4: All musical performance entails interaction with the acoustic medium. When a violinist draws a bow across a string, the vibration is transmitted through the wooden resonance box and then into the surrounding air, which carries the sound to the listener. In this case, the musician manipulates a solid medium and indirectly excites a gaseous one.

Moreover, the transmission of sound occurs through the medium, which determines the direction, intensity, and coloration of the resulting sound. Reverb, absorption, and delay effects arise from the way the medium behaves, not merely from the instrument. Finally, in cases like singing or playing wind instruments, the performer directly excites the medium itself—air becomes both instrument and expression. The medium is therefore not just a channel but the very substance that is set into motion.

Corollary 9: Music is the only art form in which the medium is not just employed but physically set into motion to become the expressive substance.

Proposition 5: Because sound is a process rather than a physical substance (Corollary 3), every performance is inherently singular. The medium itself—and everything within it—serves as a potential manipulator, absorbing sound, amplifying it, and producing resonant harmonics.

Demonstration 5: Sound is a time-dependent disturbance in a medium, characterized by periodic fluctuations in pressure and density. As such, it does not exist as a stable entity but only as a transient propagation event. Unlike physical objects, sound cannot be stored in its original form—it must be continually re-generated under specific conditions.

The medium through which sound propagates not only carries the disturbance but alters it. Different surfaces and bodies within the medium absorb certain frequencies, reflect others, or cause resonance depending on their mass, texture, and spatial configuration. These interactions modify the waveforms in real time, producing harmonics, damping certain modes, and amplifying others.

Since the exact configuration of the medium—including temperature, humidity, geometry, materials, and position of bodies—cannot be perfectly reproduced, the resulting sound event is always unique. Therefore, even with identical inputs, the acoustic outcome of a performance remains singular due to the processual and manipulable nature of sound.

Corollary 10: Because sound is a transient event shaped by the contingent configuration of its medium, repetition does not entail reproduction. No musical event can be fully duplicated, even under controlled conditions.

Proposition 6: A performance gives rise to a compound and complex dynamic system, often exhibiting nonlinear behaviors, driven by the continuous interaction and feedback of multiple sound waves within the medium.

Demonstration 6: When multiple sound waves propagate simultaneously within a shared medium, their interactions result in dynamic interference patterns that evolve over time. These patterns are shaped by the initial conditions of each source, the properties of the medium, and the spatial configuration of the environment. Even small variations—such as a minor shift in a performer's position or a change in temperature—can produce measurable differences in the resulting acoustic field.

These wave interactions do not sum linearly. Instead, phenomena such as constructive and destructive interference, feedback loops, and localized resonance zones cause the system to exhibit nonlinear behavior. As waves reflect and refract off surfaces, they feed back into the space, altering subsequent wave propagation in real time. This recursive process introduces sensitivity to initial conditions, a hallmark of complex dynamic systems.

Therefore, a musical performance cannot be reduced to a fixed input-output mechanism. It operates as a self-modifying system where the behavior of the medium, the actions of the performer, and the acoustic feedback continually reshape each other in a coupled, evolving interaction.

Corollary 11: A performance constitutes a dynamically coupled system whose evolution is sensitive to initial and boundary conditions. As a result, musical form emerges not only from composition but also from the system's real-time behavior.

Corollary 12: Every time you listen to music, you are also listening to the medium itself—its behavior and its response to the event.

3

On Sound Recording and Reproduction

Proposition 7: Musical performances cannot be exactly replicated.

Demonstration 7: A music recording captures and potentially processes music. The efficacy of an electroacoustic system is often constrained by its weakest component, referred to as a bottleneck. Assuming the process is conducted under ideal conditions, the copy of a recording would represent an ideal rendition. Yet, even an ideal reproduction remains contingent upon the physical conditions of its actual playback.

Corollary 13: Thus, even under ideal conditions, reproducing a music recording is a unique process, dictated not by human participants, but by the one-time performance of devices under specific circumstances.

Corollary 14: The reproduction of music does not threaten the existence of music, owing to its inherent uniqueness in live performance.

4

On Silent Duration

Axiom 6: A musical composition includes intervals of time with no audible sound; these intervals are not absences, but integral components of the musical structure.

Proposition 8: Silent duration, as a determinant within the temporal framework, is a foundational element of musical construction. It coexists with sound in defining rhythm, phrasing, and musical identity.

Demonstration 8: In musical composition, silence serves to segment time functionally, though not physically—delimiting perceptual or structural units within the score. In both notation and performance, silence is not an omission but a precisely measured temporal quantity. Consider a piece in 4/4 time that begins with sound on the fourth beat; the initial three beats, although silent, are not extraneous or discardable. They are embedded within the compositional intention, marked into the rhythmic grid, and executed in performance with the same precision as any sounding measure. In this context, the musical work begins long before the first audible tone. Silent duration, therefore, occupies time without sound and operates not as a void, but as an active temporal placeholder—contributing structurally and semantically to the musical whole.

This concept is exemplified by John Cage's 4′33″, where no deliberate sound is produced by the performer, yet the composition is realized through structured silence. As in Cage's case, musical silence is of course not a physical absolute; ambient sounds, physiological noise, and room acoustics remain active, even in the absence of intentional signal. Regardless of the instrument—be it a piano, a broom, or the human voice—the essential point remains: silence, when temporally framed, is not absence but presence in latent form. This leads to the conclusion that silence must be understood as a measurable acoustic parameter; trying to play 4′33″ substituting the silent for sound would obviously be a categorical error.

To deny the inclusion of silent durations in a musical composition would be as absurd as a freight company refusing to ship the empty spaces inside the Eiffel Tower on grounds that "emptiness" carries no weight.

Corollary 15: A temporally measured silence cannot be replaced by any alternative indicator. Silence and sound are structurally complementary and categorically distinct; attempts to sonify silence are acoustically incoherent.

Corollary 16: Structured silence, as evidenced in Cage's 4′33″, functions as an active temporal and acoustic condition. It is not a negation of music but an intentional compositional act.

Scholium 4: While some may insist that rests, bar lines, or even notational scaffolding such as the staff (porte) should be considered primary musical elements, such assertions confuse structural support with acoustic content. The staff organizes pitch; rests organize time. But only silence and sound occupy a musical work.

Equating structured silence with non-musicality is akin to invoking Russell's teapot as a valid physical variable: if the teapot's hypothetical mass and trajectory were routinely included in orbital calculations, the analogy would stand. But they are not—and silence, unlike the teapot, actively shapes the acoustic outcome.

One might still object that tools such as the conductor's hand or the ticking of a metronome "replace" silence with an observable marker. However, these do not substitute for silence acoustically; they merely serve as temporal reference systems. The conductor does not emit sound, and the metronome—when external to the performance—is not part of the musical structure. Neither invalidates the role of silence as a musically measured yet acoustically latent duration.

Corollary 17: Ultimately, music is created through the articulation of both sound and silence. These two elements, inseparable in temporal structure, define the full range of musical expression.

Scholium 5: In many ritual contexts, silence functions not as absence but as a heightened presence, a suspension that frames sound. In Gregorian chant, long pauses between phrases were conceived as moments of divine receptivity, while in Japanese ma, silence is regarded as an active interval shaping perception as much as sound. In both cases, silence is not neutral—it charges the medium with expectation, intensifies listening, and reveals that music is as much about what is withheld as about what is given.

5

On Music, Nature, and Structured Play

Axiom 7: While sound is a physical phenomenon governed by natural laws, musical structure is not a spontaneous extension of nature but an intentional organization of sound, constructed through symbolic and functional systems.

Proposition 9: The structural validity of music does not require adherence to natural harmonic relations. Beyond the constraints of human auditory perception, music can incorporate any acoustic material—natural or artificial—so long as it is embedded within a system of functional relationships.

Demonstration 9: Musical systems such as the equal-tempered scale diverge from natural harmonic ratios in order to prioritize structural flexibility and modularity. These systems demonstrate that musical coherence can arise from internally consistent but non-natural frameworks.

Furthermore, auditory cognition operates effectively within such frameworks due to the adaptability of perception and the brain's ability to track relational patterns, not absolute physical laws. This shows that the validity of music lies in structured perception, not in natural origin.

Proposition 10: While music may engage the mind through numerical patterns—as Leibniz proposed in describing it as a “secret arithmetic of the soul”—this account captures only a portion of the phenomenon. The pleasure of listening to music is more deeply reinforced by perceptual mechanisms related to counting, periodicity, and structured repetition. This effect is associated with cognitive and affective patterns that frequently underlie ritual and pedagogical systems.

Demonstration 10: Repetition and structured acoustic patterns engage the human auditory system through neural entrainment, wherein brain activity synchronizes with external rhythmic stimuli. This process enhances rhythmic prediction and facilitates affective engagement by reinforcing temporal coherence. Tonal and rhythmic repetitions—such as motifs and phrase structures—activate auditory and motor-related reward circuits, strengthening attention and memory.

This capacity extends beyond humans. Studies in animal cognition have shown that certain species respond to acoustic repetition and temporal regularity. Zebra finches reproduce phrase-like sequences; humpback whales produce long-range patterned vocalizations; cockatoos and sea lions exhibit rhythmic entrainment. Chimpanzees and bonobos display periodic drumming behaviors that suggest rudimentary sensitivity to beat-based structuring. These examples indicate that sensitivity to acoustic repetition is supported by evolutionarily conserved perceptual mechanisms.

In human cultures, the pairing of sound and counting underlies pedagogical and ritual systems. Biblical psalmody, Qur'anic tilawah, Vedic mantra cycles, and Plato's Nomoi all use repetition to reinforce learning, memory, and emotional resonance. Rhythmic patterning in these contexts functions as a cognitive scaffold, binding affective and symbolic meaning to structured auditory input.

Therefore, the perceptual effect of counting and repetition in music rests on measurable neurophysiological mechanisms, validated by cross-species parallels and deeply embedded in human cultural practices. Even in cases where rhythm becomes irregular or highly complex, it persists as a temporal reference for the listener and performer alike. The "kırık hava" usul in Turkish music and the intricate tala cycles of Indian classical music stand as concrete examples of such enduring rhythmic frameworks.

Corollary 18: What we call music often emerges when structured repetition interacts with bodily and cognitive expectations, producing patterns that evoke memory, anticipation, and closure.

Proposition 11: Music is a free play constructed between physical constraints and conventional rules. The moment a listener recognizes the framing of this play—whether via tonality, rhythm, or timbre—they enter a perceptual space shaped by both natural hearing and social convention.

Demonstration 11: Musical systems are constructed between two classes of constraint: physical limits, such as auditory range, resonance characteristics of instruments, and propagation of sound in a medium; and conventional structures, such as tuning systems, scale hierarchies, metric patterns, and stylistic norms.

When a listener or performer identifies cues that frame an acoustic event as music—such as meter, pitch structure, or contextual performance—they begin to interpret sound within a rule-based symbolic system. Arbitrary or natural sounds are thus transformed into musical events by being subjected to constraints that define meaningful variation, hierarchy, and form.

This framing converts raw acoustic material into intelligible musical structures through the convergence of physical plausibility and social intelligibility —a connection so fundamental that Plato warned how changes in musical modes inevitably transform the laws of the state itself.

Scholium 6: Consistent with Aristotle's conception, our engagement with the medium can be refined through a progressive play with it. Georgian polyphonic music stands as one of the finest examples of this historical endeavor: men become builders by building, and lyre-players by playing the lyre.

Definition 6: Music is a form of free play in which the agent plays with a medium, plays through a medium, and ultimately plays a medium itself—using both sound and silence.

6

On Harmony and Freedom

Proposition 12: The historical evolution of music does not justify the claim that it possesses a fixed or essential nature.

Demonstration 12: Art is fundamentally a domain of poiesis—human creative construction. As in other arts, essentialist views contradict the very nature of artistic activity. Since musical forms change radically across cultures and epochs, the appeal to an immutable musical essence must remain unfounded and irrelevant.

Scholium 7: Countless examples could be given to demonstrate this point. During fieldwork in Africa, an ethnomusicologist Polo Vallejo reported the following encounter. When asked by a Gogo child about his profession, he replied in general terms that he was a music teacher. The child, only five years old, laughed and objected: “Can music ever be taught?” In the child’s community, music was present from the earliest years, embedded in every stage of life, and never regarded as something requiring separate instruction. The objection of the little Gogo child was thus both simple and profound.

Scholium 8: The fact that major streaming platforms such as Spotify are depersonalizing music through AI-generated artists does not diminish its original force. To object to machine-made music is no less absurd than objecting to the invention of a new instrument. Anyone who masters these tools properly has, today, only a handful of things they truly cannot produce in music—harmonies, modes, rhythmic patterns, and the like are already at hand. From this perspective, the future is most exciting precisely in musical terms: somewhere on the horizon, from a place not yet visible, music is arriving—the music of the future—and we have every reason to follow where it leads.

Proposition 13: There may be, in principle, an infinite number of harmonic systems, each potentially valid within its own artistic context.

Demonstration 13: Harmony arises from the pursuit of coherence between sound and silence. This coherence need not align with physical laws or acoustic proportions. Even when acoustically grounded (e.g., harmonic series), harmonic relations do not necessarily support melodic progression, nor must they operate linearly or proportionally.

Harmony may instead be founded on e.g., rhythm, structured silence, spatial placement, or symbolic contrast. The assumption that harmony must follow natural causality has long been challenged—by figures such as Ernst Levy, Schoenberg and Collier. As musical thinking expanded beyond audible durations and pitch hierarchies, harmony came to encompass non-melodic, non-proportional, and even extra-acoustic structures.

Theories of harmony include sociological and theological interpretations, where harmonic coherence may emerge from contrasts between structured and unstructured sound, sacred and profane content, or conceptual layers rather than tones. Harmony becomes a system of relationships—not necessarily between frequencies, but between perceptual or symbolic domains.

This is evidenced in systems such as the Rast maqam, which eschews equal temperament; Bartók's axis system, which uses tonal pivot points instead of key centers; and Ligeti's microtonal textures, which abandon symmetry while maintaining sonic coherence. In these systems, harmony is constructed through asymmetry, contrast, and distribution—rather than geometric reflection.

Harmony may also be constructed from mathematical operations such as ratios, lattices, and geometrical configurations, or from non-mathematical logics such as mythology, metaphor, or theological symbolism. For instance, Gérard Grisey's spectral works organize pitch spirally around overtone structures instead of tonal centers. Harry Partch's two-dimensional just-intonation lattices position pitches across a planar grid, generating harmonic relations spatially rather than hierarchically. These show that harmony may arise from geometrical structures, while other works employ metaphorical or symbolic schemes detached from acoustic necessity.

In contemporary practice, harmony may also function as a relation between conceptual layers—not merely between pitches. It may mediate the interplay of structured and unstructured sound, or negotiate tensions between sacred and profane, central and marginal. This expansion does not dilute harmony, but reaffirms its expressive and structural plasticity.

Scholium 9: Descartes, in the *Compendium Musicae*, sought to found music on numerical ratios. For him, harmony was the ordered application of proportion, and its effect lay in the clarity of this order upon the mind. Such a view presupposes one rational system of consonance. The assertion here, that harmony may exist in a plurality of forms, thus stands in contrast to the Cartesian ideal of a singular and necessary order.

Scholium 10 : Rousseau denied that harmony arose from nature. In the *Dictionnaire de musique* and in the *Essai sur l'origine des langues,* he presented it as an invention of culture, a convention shaped by history rather than a direct extension of sensation. For Rousseau, harmony belongs not to physics but to society; its legitimacy rests on use, not on nature. Insisted upon, it may even suffocate the very melody it was meant to serve.

7

On Signification, Variability, and Representation

Axiom 8: The mimetic challenge complicates the formation of a musical grammar, highlighting the difficulty of representing and reproducing musical experience with fidelity.

Axiom 9: A musical sign can be said to exist only when a functional articulation occurs between a signifier and a signified.

Proposition 14: A musical signified may, in principle, exhibit infinite variability depending on the content and context.

Demonstration 14: The scope of what counts as musical content has expanded dramatically in recent decades. Through the practice of sonification, structured data drawn from non-musical domains is translated into sound, allowing us to hear what we typically only visualize or compute. For example, astronomical phenomena—such as star pulsations, black hole oscillations, or Hubble imagery—have been turned into immersive audio experiences. In a similar fashion, the bioelectrical signals of plants are measured via electrodes and converted into MIDI-based ambient compositions. DNA sequences can be mapped onto musical scales, producing sonic representations of genetic patterns. Visual data like portraits or satellite imagery are scanned pixel by pixel and rendered as melodies. Even climate statistics, financial charts, mathematical constants like π, or bodily movement are now being interpreted through sound. These practices reflect a shift in perceptual orientation: the act of listening is no longer limited to what the ear naturally perceives, but is extended to encompass abstract and symbolic structures as well. In this way, music becomes a vehicle for reconfiguring how we engage with the world, by listening to what we otherwise only see or measure.

Corollary 19: Any possible content—visual, biological, astronomical, or mathematical etc.—can, in principle, be rendered into music, as a signified, provided it is organized in a temporally coherent auditory structure.

Proposition 15: A musical signifier, by contrast, must be constituted as a sound or silence organized within a temporal measure.

Demonstration 15: As developed in prior sections, music arises through the organized interplay of sound and silence within a medium. These sounds or silences must carry a musical function—they must participate in a temporal structure capable of sustaining perceptual coherence.

If an element (such as a noise, pause, or interval) is not placed within such a structure, it does not act as a musical signifier.

Axiom 10: As in natural languages, musical writing constitutes a secondary mediation—an abstract representation of performative experience. Like in Barthes's semiotic analysis, a musical signifier may adopt additional external forms (notation, graphic scores, digital symbols).

Proposition 16: Musical meaning cannot be reduced to grammatical structures modeled on natural language. Music, as a form of free play, is not obliged to fulfill syntactic expectations. This is even more pronounced in its capacity to engage directly with formal and symbolic domains.

Demonstration 16: In its cultural forms, music has been associated with virtue, social prestige, ritual play, and pure aesthetic pleasure. Moreover, music often serves as a mode of interacting with abstract structures—a role classically described in Plato's Timaeus, where music becomes an extension of mathematics, dialectics, and metaphysical order.

Musical meaning therefore may resist capture by grammatical systems.

Corollary 20: No art is required to operate within the grammatical constraints of language. Music, in particular, maintains expressive autonomy through its structural freedom.

Proposition 17: A totality composed solely of the external forms of musical signifiers cannot be regarded as music.

Demonstration 17: A visual arrangement of musical notation, such as a drawing made from notes, lacks musical function unless it produces a temporally organized sonic experience. Notation, in the absence of audition, remains potential rather than actualized music, for no semiotic articulation occurs between signifier and signified.

Epilogue

Orpheus, Once Again: The Medium That Plays Back

Just as Apollo tamed Dionysus, did Pythagoras tame Orpheus? When the magical dimension of music is reduced to pure mathematics, it becomes not only impoverished, but also detached from musical reality. If music can be reduced to mathematics, what we achieve is merely mathematics itself. Without serving a musical function in a playful activity, such a reduction must be considered meaningless and misguided. By contrast, practices such as algoraves demonstrate how mathematical structures can serve genuine musical functions through live coding, where human–algorithm interaction becomes an instrument of real-time compositional play rather than mere exercises in computation. Yet, even at the very moment when music is said to be "killed" by mathematics—especially in today's age of machine-generated music—musical performance remains an unrepeatable event and the most playful gift the universe offers us. Our play with the medium is unique and singular. Though we know nothing about who Orpheus was, what he looked like, or what he stood for, the fact that his power remains undiminished speaks volumes about the magical and indestructible nature of music. So let them go ahead and kill the only form of free play in which the agent plays with a medium, plays through a medium, and ultimately plays a medium. Mathematics may rule over music, yet music plays with it.

Indeed, the limited knowledge we have about Orpheus and his enduring mastery of mystery—his influence persisting through time despite having minimal signifiers—offers us the wisdom necessary to engage with the unrepeatable. Despite the rigid measure of our era's Pythagorean spirit, it is this mortal, playful, and unrepeatable tie we form with the medium that both draws us, like Odysseus, into the Realm of the Sirens and saves us from it. As a subject of freedom and knowledge, a musician, like Orpheus on his return from Hades, may choose to look back. Yet a musician's happiness and aesthetic joy lie in continuing forward without looking back—embracing the fleeting beauty of what cannot be repeated.

There are musicians who play the medium itself as an instrument (SaReGaMa pillars of Nada Brahma or Selbstgebaute Musik). The playability of the medium is sometimes discovered through a simple act, such as casual cough echoing in a corridor or an underpass, resulting in the medium being engaged as a one-time spontaneous instrument. This interaction occurs due to specific seasonal conditions, the presence of objects—possibly in motion—made of particular materials within the medium. Consequently, it is highly probable that the same medium will not produce identical responses on another occasion.

A person can be confined to a dungeon without any tools, yet—unlike many other arts— music requires, in principle, nothing else there. The body and the surrounding environment can quickly become musical instruments. Unlike all other arts, music can play the physical reality of the dungeon itself as an event. Sisyphus finds his stone and becomes one with it in this play.

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