Lab 5-Thermoelectric effects

June 27, 2012

1 Object

To study the properties of two thermoelectric effects; the Seebeck effect and the Peltier effect

2 Introduction

A thermoelectric module was used to observe the Seebeck and Peltier effects. Both occur due to a relationship between net electron movement, and a temperature gradient across a conductor. If a temperature difference is established across a homogenous conductor, there will be a net diffusion of electrons down the temperature gradient. This diffustion of electrons, (and hence charge), leads to an electromotive force proportional to the magnitude of the temperature difference given by,

$$V_1 = S_1 \Delta T \tag{1}$$

where S_1 is the proportionality coefficient; a constant that depends on the material being used.

If two conductors, A and B, have different proportionality coefficients S_1 and S_2, the electromotive force given by eq(1) will be different if ΔT is the same. If a temperature gradient $T_1 \to T_2$ is established between two junctions of A and B (Fig.1), then the difference in electromotive force will create a potential difference. This is known as the Seebeck effect,

$$V_1 - V_2 = (S_1 - S_2)(T_1 - T_2)$$

The Seebeck coefficient (S_{12}) is the difference in proportionality coefficients of the two conductors,

$$\Delta V = S_{12} \Delta T \tag{2}$$

If a direct electric current is passed through the junction of two conductors with different Seebeck coefficients, heat will flow across the junction. This is known as the Peltier effect. The rate of heat flow, H, is related to the magnitude of current being passed through the junction, I, and the Peltier Coefficient, $\pi_{12} = S_{12}T$,

$$H = \frac{dQ}{dt} = \pi_{12} I \tag{3}$$

A thermoelectric module was used to observe the Seebeck and Peltier effects. It was made from multiple bismuth telluride semiconductor units layered between two ceramic sheets. The Bi_2Te_3 crystal is not an effective conductor, but can be chemically doped to become an n-type, or p-type semiconducter. The process of chemical doping involves replacing atoms in the crystal lattice with atoms that have more (for n-type) or less (for p-type) valence electrons, without changing the crystal geometry. An n-type semiconductor is doped so that it contains extra electrons in the crystal which are able to move through the crystal without

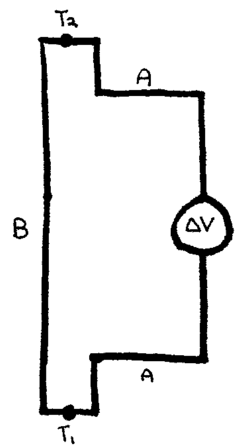

Figure 1: Overview of the Seeback effect[2}

difficulty. A p-type semiconductor is doped so that it lacks some bonding electrons in the crystal lattice, known as “holes”. When a current is passed through, it appers that the holes are moving in the direction of the current. This is caused by electrons filling in the holes in the opposite direction to electric current, and leaving openings behind[1].

The thermoelectric module used multiple units of p-type and n-type bismuth telluride. Figure 2 shows a simplified view for a one-unit module. When an electric current is passed through the module. The electrons in the n-type semiconductor move opposite to the direction of current, and the holes in the p-type semiconductor move in the same direction as the current. Both of these movments result in a net flow of heat from the hot side to the cold side due to the Peltier effect. If a temperature gradient is established across the module, the electrons in the n-type semiconductor diffuse from the hot end to the cold, and the holes in the p-type semiconductor move from the cold end to the hot. Both of these movments create a potential difference due to the Seebeck effect.

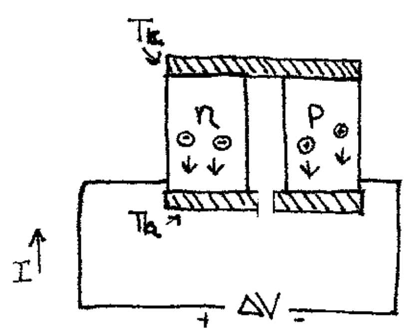

Figure 2: Electron/hole movment related to heat transfer and electric current in a simple thermoelectric module using p-type and n-type semiconductors[3]

The thermoelectric module used had a thermal resistance, R_{th}, that opposes the flow of heat, and an electrical resistance, R_m, that opposes the flow of electric current. The electrical resistance leads to production of heat, known as Joule heating. The thermal resistance can be determined when the cold block is at thermal equilibrium and the module has been turned off. R_{th} is related to the temperature gradient between the heat sink, T_h and cold block ,T_c, and to the power used to heat the cold block, P, by the equation,

$$R_{th} = \frac{T_c - T_s}{P} \tag{4}$$

To determine the rate of heat flow through the module, equation 3 must be corrected to account for thermal resistance and heat lost due to electrical resistance (see question #2). The rate of heat flow, H, from the cold block through the module is given by,

$$H = S_{12}T_cI_m - \frac{1}{2}I_m^2R_m + \frac{T_c - T_h}{R_{th}} \tag{5}$$

WhereI_m is the current through the module.

The coefficient of performance, $C.O.P$, is a measurement of the effectiveness of a heat pump. It is related to the rate of heat flow and to the power delevered to the module by the equation,

$$C.O.P = \frac{H}{I_mV_m} \tag{6}$$

where V_m is the voltage across the module.

3 Procedure

A. The Seebeck Effect

The temperature of the heat sink was held constant by running tap water through it. The power supply was adjusted so that approximately $20\,W$ was dissipated in the heating resistor. The temperature of the cold block, temperature of the heat sink, and potential difference across the module, were recorded simultaneously until the system approached thermal equilibrium (Table 2). Measurments were taken approximately every time the cold block had increased in temperature by $0.5°C$. The relationship between potential difference and temperature gradient between the cold block and heat sink was used to determine the Seebeck coefficient (Fig 4).

When the system reached thermal equilibrim the temperature of the cold block, temperature of the heat sink, resistor voltage and resistor current were recorded. This data was used to determine the thermal resistance of the module using equation 5.

B. The Peltier Effect

The power delivered to the module and the power delivered to the heating resistor were adjusted so that the temperature of the cold block was $1°C$ less than the temperature of the heat sink, and not changing. Once this equilbrium was established, the temperature of the heat sink, temperature of the cold block, current through the resistor, voltage across the resistor, current through the the module and voltage across the module was recorded. Measurements were recorded with a module power supply from $2.0W$ to $7.0W$ in $0.5W$ Increments. Temperatures were kept as constant as possible throughout the data collection.

A graph of heat flowrate as a function of module current was created and compared to the theoretical curve predicted by equation 5. A graph of coefficient of performance as a function of module current was created.

4 Apparatus

Table 1: Summary components used in apparatus

Component	Device	Number
Resistor Voltmeter	8600A Digital Multimeter	UVic PHYS #825
Module Voltmeter	8000A Digital Multimeter	UVic PHYS #805
Resistor Ammeter	#1 Armaco Ammeter	UVic PHYS #1010
Module Ammeter	#1 Armaco Ammeter	UVic PHYS #713
Resistor Power Supply	Regulated Power Supply	UVic PHYS #724
Heat-Sink Thermometer	Tracable Calibration Control Co.	serial-72176747
Cold-Block Thermometer	Tracable Calibration Control Co.	serial-72176747
Variac-Transformer	Variac Variable Transformer	UVic PHYS #746
Electric Resistance Module	Heating-Resist thermoelectric effect	serial-386170
Module Power Supply	unnamed (part of previous comp)	#83

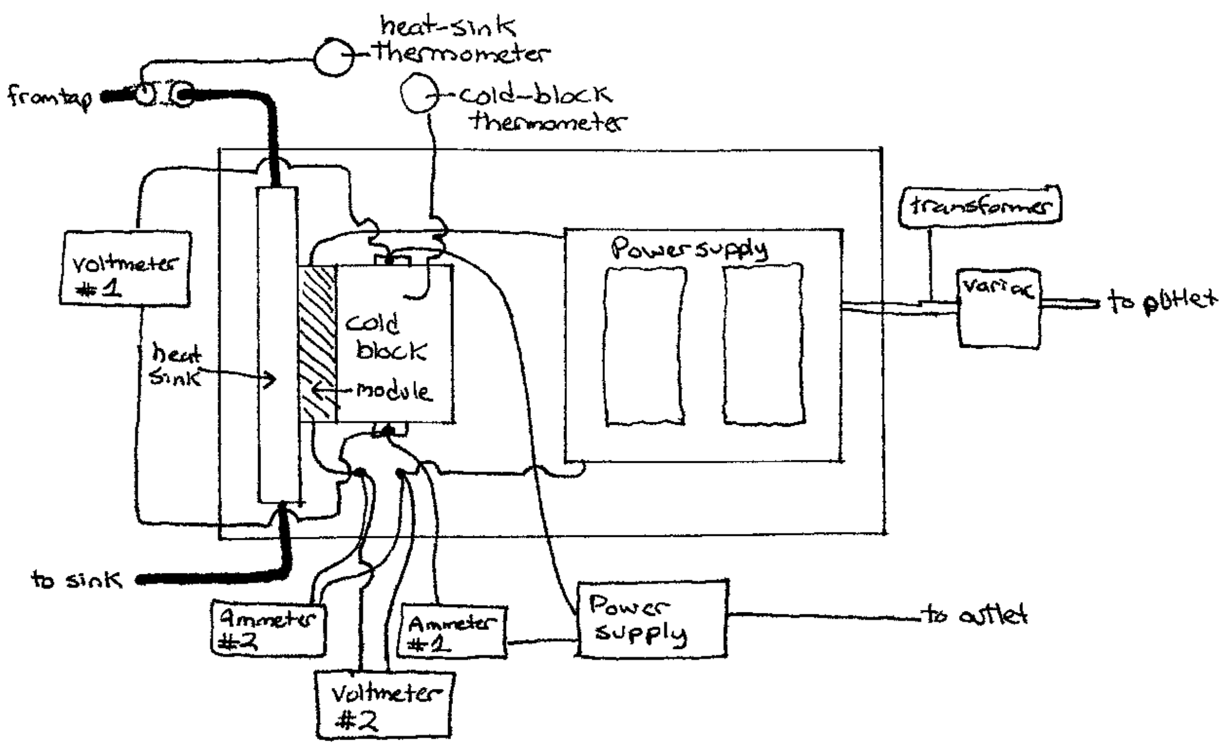

Figure 3: Summary of apparatus used for the experiment

5 Data

Table 2: Part A: Measured values for temperature of heat sink, T_h ($°C \pm 0.05°C$), temperature of cold block, T_c ($°C \pm 0.05°C$), and potential difference, ΔV ($V \pm 0.001V$), for a heating resistor at aproximatly 20W.

$T_h\,(°C)$	$T_c\,(°C)$	ΔV	$T_h\,(°C)$	$T_c\,(°C)$	ΔV	$T_h\,(°C)$	$T_c\,(°C)$	ΔV
16.1	18.9	0.043	16.2	43.0	0.308	16.3	57.6	0.479
16.1	19.5	0.051	16.3	44.4	0.323	16.3	58.0	0.484
16.1	20.0	0.067	16.3	45.0	0.329	16.3	58.5	0.489
16.1	21.0	0.078	16.3	46.0	0.344	16.3	59.0	0.495
16.1	22.0	0.092	16.3	47.0	0.358	16.3	59.5	0.501
16.1	23.0	0.101	16.3	48.0	0.369	16.3	60.5	0.510
16.1	24.0	0.101	16.3	49.0	0.381	16.3	60.8	0.513
16.1	25.0	0.111	16.3	50.0	0.393	16.3	61.0	0.515
16.1	26.0	0.120	16.3	51.0	0.405	16.3	61.2	0.518
16.1	27.0	0.132	16.3	51.5	0.410	16.3	61.6	0.522
16.1	28.0	0.140	16.3	52.0	0.417	16.3	62.0	0.528
16.1	29.0	0.149	16.3	52.5	0.422	16.3	62.5	0.535
16.1	30.0	0.159	16.3	53.0	0.428	16.3	63.0	0.542
16.1	31.0	0.170	16.3	53.5	0.434	16.3	63.3	0.546
16.1	32.0	0.183	16.3	54.0	0.440	16.3	63.6	0.550
16.2	33.0	0.195	16.3	54.2	0.443	16.3	63.9	0.554
16.2	34.0	0.208	16.3	54.4	0.444	16.3	64.2	0.558
16.2	35.0	0.222	16.3	54.8	0.449	16.3	64.5	0.562
16.2	36.0	0.232	16.3	55.0	0.451	16.3	64.8	0.565
16.2	37.0	0.245	16.3	55.5	0.456	16.3	65.1	0.569
16.2	38.0	0.255	16.3	56.0	0.462	16.3	65.4	0.573
16.2	39.0	0.267	16.3	56.4	0.468	16.3	65.7	0.578
16.2	40.0	0.278	16.3	56.8	0.471	16.3	66.4	0.587
16.2	42.0	0.297	16.3	57.3	0.476	16.3	66.5	0.588

At thermal equilibrium the temperature of the heat sink was $T_h = 16.3°C \pm 0.05°C$. Temperature of the cold block was $T_c = 66.5°C \pm 0.05°C$, The resistor voltage was $V_R = 6.56V \pm 0.005V$. The resistor current was $I_R = 3.20A \pm 0.01A$.

Table 3: Part B: Heat sink temperature $T_h(°C \pm 0.05°C)$, cold block temperature$(°C \pm 0.05°C)$, resistor voltage $(V \pm 0.005V)$, resistor current $(A \pm 0.01A)$, module voltage$(V \pm 0.005V)$ and module resistance $(A \pm 0.01A)$. Calculated values of heat flow rate (H) and coefficient of performance $(C.O.P)$

T_h	T_c	V_R	I_R	V_M	I_M	$H\,(W)$	$C.O.P$
14.9	13.9	1.99	1.00	0.300	0.86	2.309 ± 0.043	8.95 ± 0.25
15.0	14.1	2.21	1.12	0.333	0.93	2.568 ± 0.044	8.29 ± 0.21
15.3	14.5	2.44	1.25	0.377	1.07	3.041 ± 0.044	7.54 ± 0.16
15.5	14.2	2.66	1.34	0.436	1.18	3.243 ± 0.044	6.30 ± 0.12
15.5	14.4	2.85	1.43	0.495	1.45	4.049 ± 0.044	5.64 ± 0.09
15.6	14.5	3.00	1.51	0.547	1.60	4.531 ± 0.044	4.78 ± 0.07
15.6	14.7	3.15	1.59	0.592	1.63	4.664 ± 0.044	4.83 ± 0.07
15.7	14.8	3.30	1.65	0.686	1.86	5.328 ± 0.044	4.18 ± 0.05
15.7	14.7	3.50	1.76	0.731	2.23	6.324 ± 0.044	3.88 ± 0.04
15.7	14.7	3.64	1.83	0.828	2.30	6.517 ± 0.044	3.42 ± 0.03
15.8	14.8	3.72	1.87	0.874	2.43	6.875 ± 0.045	3.24 ± 0.03

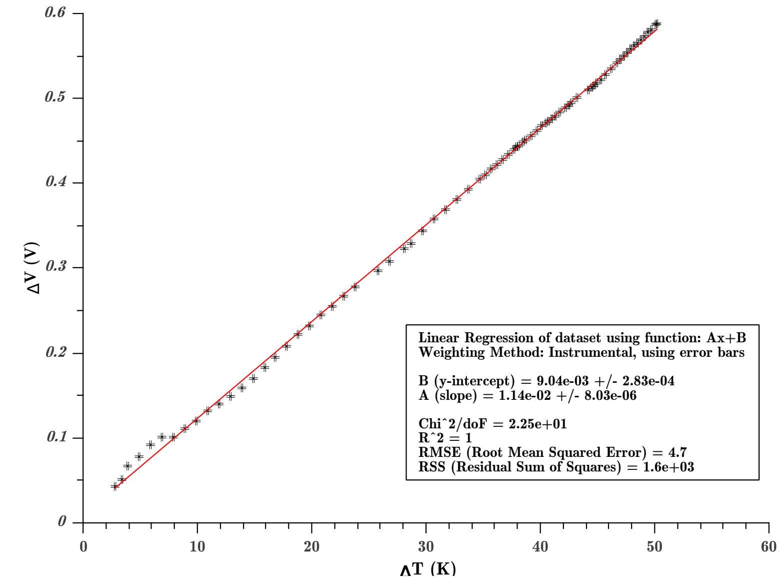

Figure 4: Linear relationship between temperature gradient and potential difference.

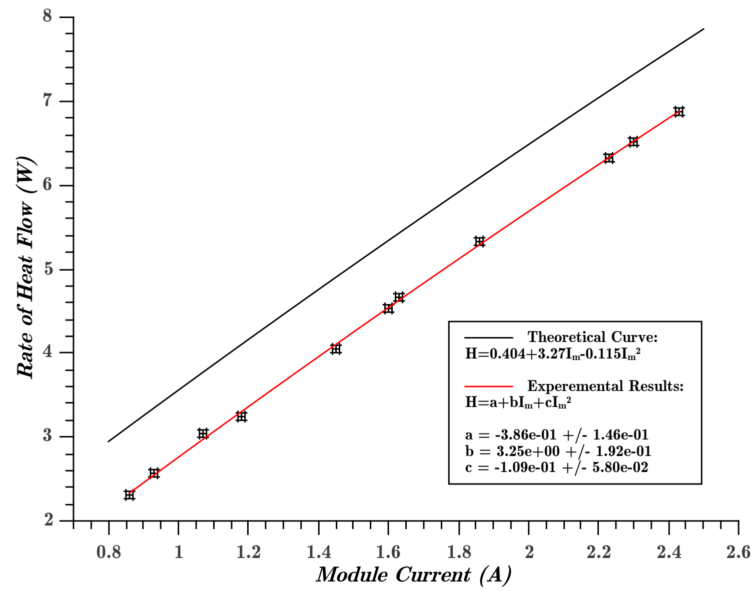

Figure 5: Theoretical and experimentally determined rate of heat flow as a function of module current

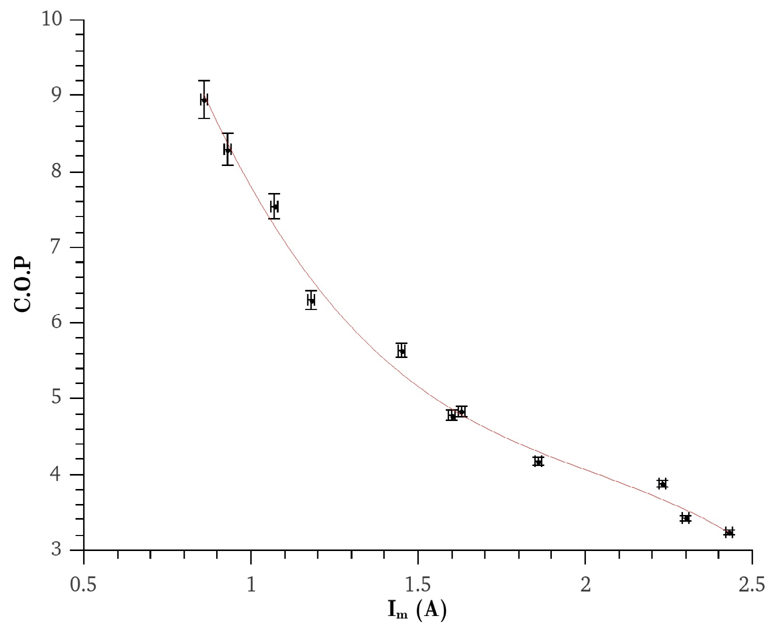

Figure 6: Coefficient of performace as a function of module current.

6 Calculations

Temperature difference between heat sink and cold block,

$$\Delta T = T_c - T_h = 19.5^\circ C \pm 0.005^\circ C - 16.1^\circ C \pm 0.005^\circ C = 3.4^\circ C \pm \left(\sqrt{0.005^2 + 0.005^2}\right)\ ^\circ C = 3.44^\circ C \pm 0.007^\circ C \quad (7)$$

Average ΔT was $32.17\,K$ and the average ΔV was $0.3758\,V$

Standard deviation for the observed values of ΔT with $N-1$ weighting:

$$\sigma_{\Delta T} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\Delta T_i - \Delta\bar{T}\right)^2} = \sqrt{\frac{1}{75}\sum_{i=1}^{76}(\Delta T_i - 32.17\,K)^2} = 14.48\,K$$

Similarily, $\sigma_{\Delta V} = 0.1652\,V$

Correlation coefficient:

$$r_{\Delta T \Delta V} = \frac{\sum_{i=1}^{n}(\Delta T_i - \Delta\bar{T})(\Delta V_i - \Delta\bar{V})}{(n-1)\sigma_{\Delta T} s_{\Delta V}} = 0.9996$$

Slope of the graph

$$m = r_{\Delta T \Delta V}\frac{\sigma_{\Delta V}}{\sigma_{\Delta T}} = 0.9996 \cdot \frac{0.1652\,V}{14.48\,K} = 1.14 \times 10^{-2} V K^{-1}$$

Since the measurement uncertainty, ($\delta_V = 0.001\,V$) was the same for each measurement, the uncertanty in slope can be calculated from

$$\sigma_m = \frac{\delta_V}{\sigma_x\sqrt{N-1}} = \frac{0.001\,V}{14.48\,K\sqrt{75-1}} = 8.03^{-6}VK^{-1}$$

So the Seebeck coefficient is $S_{12} = 1.14\times10^{-2}VK^{-1}\ \pm\ 8.03^{-6}VK^{-1}$

Thermal resistance of the module:

$$R_{th} = \frac{T_c - T_h}{P} = \frac{T_c - T_h}{I_m V_m} = \frac{(66.5\ \pm\ 0.05 - 16.3\ \pm\ 0.05)K}{(6.56\,V\ \pm 0.005\,V)(3.20\,A\ \pm\ 0.01A)} = \frac{50.2\,K\ \pm\ \sqrt{0.05^2+0.05^2}K}{20.99\,W\ \pm\ \sqrt{(0.076\%)^2+(0.310\%)^2}}$$

$$R_{th} = \frac{50.2\,K\ \pm\ 0.139\%}{20.99\,W\ \pm\ 0.310\%} = 2.39\,K{\cdot}W^{-1} \pm \sqrt{0.139\%^2+0.310\%^2} = 2.39\,K{\cdot}W^{-1} \pm 0.350\% = 2.39\,K{\cdot}W^{-1} \pm 0.01\,K{\cdot}W^{-1}$$

Rate of heat flow

$$H = S_{12}T_cI_m - \frac{1}{2}I_m^2R_m + \frac{T_c - T_h}{R_{th}}$$

$$= \left(1.14\times10^{-2}VK^{-1} \pm 0.07\%\right)(286.9K \pm 0.02\%)\,(0.86A{\pm}1.12\%) - \frac{(0.86A \pm 1.12\%)^2(0.23\Omega)}{2} + \frac{(13.9-14.9)K \pm 7.0\%}{2.39\,K\cdot W^{-1}\ \pm\ 0.350\%}$$

$$= 2.813W \pm \sqrt{0.07\%^2+0.02\%^2+1.12\%^2} - 0.0851W \pm 1.16\% - 0.418W \pm \sqrt{7.0\%^2+0.350\%^2}$$

$$= 2.813W \pm 1.17\% - 0.0851W \pm 1.16\% - 0.418W \pm 7.0\%$$

$$= 2.813W \pm 0.033W - 0.0851W \pm 0.001W - 0.418W \pm 0.029W$$

$$= 2.813W \pm \sqrt{0.033^2+0.001^2+0.029^2}W$$

$$H = 2.309W \pm 0.043W$$

All values of H are summarized in Table 3.

Coefficient of performance:

$$C.O.P = \frac{H}{I_mV_m} = \frac{2.309W \pm 1.86\%}{(0.86A \pm 1.12\%)(0.300V \pm 1.67\%)} = 8.95 \pm \sqrt{1.86\%^2+1.12\%^2+1.67\%^2} = 8.95 \pm 2.79\%$$

$$C.O.P = 8.95 \pm 0.25$$

All values for C.O.P are summarized in Table 3.

7 Discussion

The slope of Figure 4 corresponds to the Seebeck coefficient for the thermoelectric module (because of the linear relationship $\Delta V = S_{12}\Delta T$). Using the ordinary least squares method, the Seebeck coefficient was determined to be $S_{12} = 1.14\times10^{-2}VK^{-1} \pm 8.03^{-6}VK^{-1}$. Once thermal equilibrium had been reached, the thermal resistance of the module was determined to be $2.39\,K\cdot W^{-1}\ \pm\ 0.01\,K\cdot W^{-1}$ by using equation 4.

It was important that the module was in thermal equilibrium before measurements were taken for the Peltier effect because the rate of heat flow was not measured directly. Equation 4 was used to calculate the rate of heat flow. The power delivered to the heating resistor and the power delivered to the thermoelectric module both had to be set at a level where they had no net effect on the temperature gradient between the heat sink and the cold block. These two measurements of power were used to indirectly measure the heat flow from the cold block through the module. A source of error was an inconsistent heat sink temperature when obtaining the data for the empirical curve (Table 3). The temperature difference was kept as close to 1K as possible, but the values for T_h were inconsistent, and T_c had to be adjusted accordingly. This temperature change was slow enough that it was still possible to approach thermal equilbrium for each data point.

The theoretical H vs I_m curve was determined to be the function $H_{the} = -0.115I_m^2 + 3.27I_m + 0.404$ and the experiment data fit the function $H = -(0.109 \pm 0.058)I_m^2 + (3.25 \pm 0.19)I_m - 0.386 \pm 0.146$ (Fig 5). The first and second degree terms of the theoretical function were within the error bounds of the first and second degree terms of empirical function. The constant term however was diferent in the theoretical curve than in the empirical curve. The theoretical curve predicted a higher heat flow from the cold block through the module than was observed, by a constant amount approximately equal to $0.7W$. This suggested that there was a process independent of the module current that was limiting the rate of heat flow from the cold block through the module. The uncertainty in the Seebeck coefficient for the module was ignored when determining the theoretical curve for H vs I_m. This was a source of error that was not taken into account in the data analysis.

The coefficient of performance is important because it relates the heat output to the work input of the thermoelectric module,. The results suggested that the coefficient of performance drops nonlinerally as current is increased. The maximum coefficient of performance was observed with a module current of $0.86A \pm 0.01A$, however there was no peak in the data to suggest that this was the maximum C.O.P for the module.

8 Conclusions

The Seebeck coefficient of the module was determined to be $S_{12} = 1.14 \times 10^{-2}VK^{-1} \pm 8.03^{-6}VK^{-1}$. The thermal resistance of the module was determined to be $2.39\,K\cdot W^{-1} \pm 0.01\,K\cdot W^{-1}$. A quadratic relationship between H and $I_m$was determined to be $H = -(0.109 \pm 0.058)I_m^2 + (3.25 \pm 0.19)I_m - 0.386 \pm 0.146$ and a lower value of H was observed than was predicted by the theoretical curve. The coefficient of performance was determined to drop nonlinerally with increasing current.

9 Questions

1.

$$H = S_{12}T_cI_m - \frac{1}{2}I_m^2R_m + \frac{T_c - T_h}{R_{th}}$$

The first term is the rate of heat flow for the peltier effect $H = \pi_{12}I$ where $\pi_{12} = S_{12}T$. The direction of the heat flow from the Peltier effect depends on the direction of current. The second term is to account for the dissipation of Joule heat caused by electrical resistance in the module. The I_m term is squared and therefore the heat is independent to the direction of current. This means half of the value must be subtracted from the rate of heat flow because there will be no net heat flow caused by this Joule Heating. The third term accounts for the thermal resistance to the flow of heat, caused by the properties of the material, and the temperature gradient between them.

2.

Electrical resistance given by Ohm's law:

$$R = \frac{V}{I}$$

Thermal resistance given by formula 8.6:

$$R_{th} = \frac{T_c - T_h}{P} = \frac{\Delta T}{P}$$

The difference in temperature ΔT can be viewed as analogous, to voltage ΔV because both are differences in potential. Just as electric current flows from high potential to low, heat (Q) flows from high temperature to low. So heat can be viewed as analogous to current. Formula 8.6 can be written as

$$R_{th} = \frac{\Delta T}{Q}$$

This makes sense because R_{th} was determined where the the rate of heat flow and power dissipated were in equilibrium. Furthermore, $P = \frac{dQ}{dt}$, which is again analogous to $I = \frac{dq}{dt}$.

References

1. Lauren Hill, Unknown Title, http://www.emsb.qc.ca/laurenhill/science/mosfet.pdf
2. http://en.wikipedia.org/wiki/File:Seebeck_effect_circuit_2.svg
3. CFD Simulations of a thermoelectric solar cavity receiver, www.pre.ethz.ch/teaching/topids/?id=52