

Lab 1- Ratio of Specific Heat Capacities

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1 Object

To determine the ratio of the specific heats of air, carbon dioxide and argon.

2 Introduction

The first law of thermodynamics states that the heat supplied to a system is the sum of the work done plus the increase in internal energy.

$$dU = dQ - dW \quad (1)$$

If process has no transfer of heat to or from the system, the process is called adiabatic, and

$$dU = -dW \quad (2)$$

The dimensionless unit γ is the ratio between specific heat at constant pressure and the specific heat at constant volume,

$$\gamma \equiv \frac{c_p}{c_v} \quad (3)$$

The accepted values for γ at 20°C is 1.400 for normal air, 1.667 for argon gas and 1.288 for carbon dioxide gas^[1]. This value is

γ is an important quantity when considering an adiabatic process. In an adiabatic process involving an ideal gas the relationship between pressure and volume is satisfied by,

$$\begin{aligned} PV^\gamma &= \text{constant} \\ \ln P + \gamma \ln V &= \text{constant} \end{aligned} \quad (4)$$

The change in pressure is given by the derivative of the relationship described in eq4.

$$\Delta P = -\frac{\gamma P \Delta V}{V} \quad (5)$$

The value of γ can be experimentally determined by measuring the pressure change as a ball oscillates inside a vertical tube placed on top of an otherwise sealed, inextensible container. If the ball has the same diameter as the tube, no air can escape and the pressure and volume of the system change with the movement of the ball. If the movement of the ball is reasonably quick, the process can be approximated as an adiabatic process^[2]. If the vertical tube has a cross sectional area, A , and the

ball has mass m , then the equilibrium pressure, P_{eq} of the system can be determined by adding the pressure applied by the ball, to the atmospheric pressure, P_0 .

$$P_{eq} = P_0 + \frac{mg}{A} \quad (6)$$

If the ball moves from its equilibrium position by a distance x , the pressure difference will exert a net force on the ball. Since the system is assumed to be adiabatic, the pressure difference is satisfied by eq5. The difference in this force is given by Newton's second law,

$$\begin{aligned} A\Delta P &= -m \frac{d^2x}{dt^2} \\ -A \frac{\gamma P \Delta V}{V} &= -m \frac{d^2x}{dt^2} \end{aligned} \quad (7)$$

The change in volume of the system is directly proportional to the area of the cylinder and the vertical displacement of the ball ($\Delta V = -Ax$). Therefore eq 7 can be written as,

$$\begin{aligned} -A \frac{\gamma P (-Ax)}{V} &= -m \frac{d^2x}{dt^2} \\ \frac{A^2 \gamma P}{V} x &= -m \frac{d^2x}{dt^2} \end{aligned} \quad (8)$$

The volume of the glass cylinder is much less than the volume of the system and so P can be approximated by P_{eq} . This approximation allows eq8 to be approximated as a simple harmonic motion equation. Therefore γ can be determined by measuring the period of oscillation. Solving the simple harmonic equation for γ yields,

$$\gamma = \frac{64mV}{r^2 P_{eq} T^2} \quad (9)$$

Where r is the radius of the cylinder.

The damping effect caused by the air resistance of the oscillating ball must be considered when determining the period of oscillation. The effect of damping on the period will be negligible as long as

$$\frac{k^2 T_1^2}{4} \ll 4\pi^2 \quad (10)$$

where k is the damping ratio. Since the damped motion follows an exponential decay, the amplitude of two oscillations can be used to determine the left hand side of eq10, by the relation

$$\frac{k^2 T_1^2}{4} = \left(\frac{\ln A_{(n)} - \ln A_{(n+l)}}{l} \right)^2$$

Where $A_{(n)}$ is an initial amplitude and $A_{(n+l)}$ is the amplitude after l oscillations.

3 Procedure

A spherical ball in a cylindrical glass tube (precision ground and cleaned) was placed at the top of an inextensible glass container and sealed through a rubber stopper. The glass container was hooked up to a sensor that measured the pressure inside the container. The sensor was hooked up to a computer to allow continuous data to be recorded. The steel ball was allowed to drop through the glass tube, and the resulting oscillatory motion was recorded by measuring the pressure over time. This process was repeated with Argon gas and carbon dioxide gas inside of the container.

The time between n oscillations was recorded for each trial run, and the average period for each trial was determined by dividing this time by n . The average period for each gas was determined by averaging 10 values for period (two from each gas). For each gas, the amplitude of the first and seventh oscillation was recorded and the results were used to determine whether the damping effect of the gas affected the period of oscillation (see eq10).

4 Apparatus

Table 1: Summary of components used in apparatus

Component	Device	Number
Pressure sensor	Vernier software and tech gas pressure sensor	GPS-BTA
Lab Pro Sensor	Vernier Lab Pro Sensor	UVic Phys #25
Analysis Software	Vernier Lab Pro 3	
Argon Gas	Praxair compressed argon	
CO ₂ Gas	Praxair compressed CO ₂	
Barometer	Mercury Barometer	UVic room #
Glass Container	10.75 $L \pm 0.05 L$ glass jar	
Glass Tube	Precision ground glass tube	
Steel Ball	Precision ground steel ball	

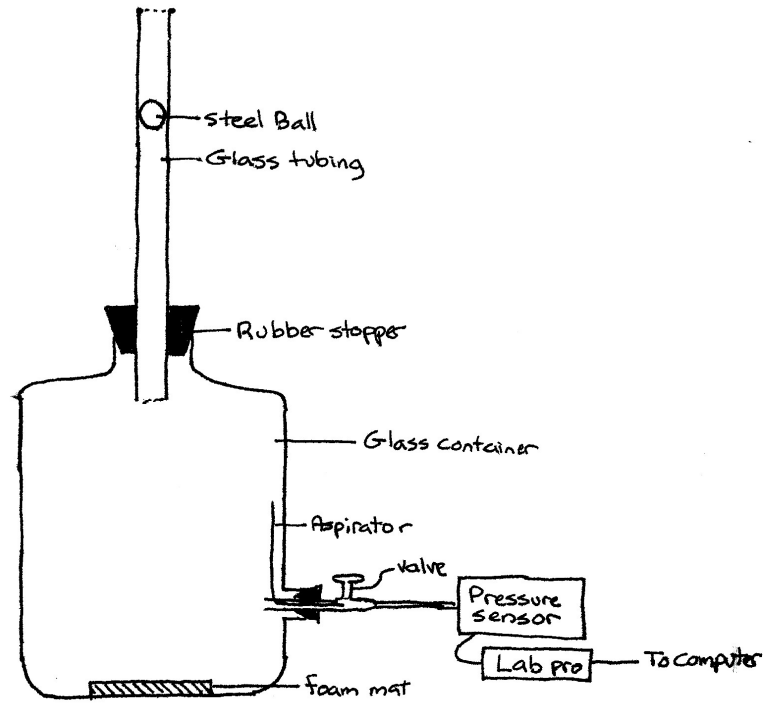


Figure 1: Summary of apparatus used in experiment

5 Data

Mass of steel ball: $16.5\text{ g} \pm 0.05\text{ g}$

Outside diameter of steel ball: $16.01\text{ mm} \pm 0.02\text{ mm}$

Inside diameter of glass tube: $16.03\text{ mm} \pm 0.02\text{ mm}$

Temperature of room: $22.2^\circ\text{C} \pm 0.1^\circ\text{C}$

Atmospheric pressure: $752.6\text{ mmHg} \pm 0.2\text{ mmHg}$

Table 2: Amplitude at first and seventh oscillation for air, argon and carbon dioxide

Gas	$A_1(\text{kPa})$	$A_7(\text{kPa})$
Air	0.686	0.250
Argon	0.591	0.115
CO ₂	0.602	0.125

Table 3: Experimental results for normal air

Initial Time (s)	Final Time (s)	ΔT (s)	Number of oscillations	Average period	Uncertainty
0.80	14.30	13.50	12	1.125	0.003
1.40	11.45	10.05	9	1.117	0.004
0.10	12.45	12.35	11	1.123	0.003
0.70	14.15	13.45	12	1.121	0.003
1.05	10.00	8.95	8	1.119	0.004
0.50	9.50	9.00	8	1.125	0.004
1.35	11.45	10.10	9	1.122	0.004
1.90	10.90	9.00	8	1.125	0.004
0.55	8.40	7.85	7	1.121	0.005
1.10	8.95	7.85	7	1.121	0.005
Average				1.122	0.004

Table 4: Experimental results for Argon

Initial Time (s)	Final Time (s)	ΔT (s)	Number of oscillations	Average period (s)	Uncertainty
0.65	6.90	6.25	6	1.042	0.006
0.15	7.45	7.30	7	1.043	0.005
0.30	8.65	8.35	8	1.044	0.004
0.85	7.00	6.15	6	1.025	0.006
0.55	8.80	8.25	8	1.031	0.004
1.05	10.35	9.30	9	1.033	0.004
0.65	6.85	6.20	6	1.033	0.006
0.15	5.30	5.15	5	1.030	0.007
0.55	5.70	5.15	5	1.030	0.007
1.00	4.10	3.10	3	1.033	0.012
Average				1.034	0.006

Table 5: Experimental results for CO₂

Initial Time (s)	Final Time (s)	ΔT (s)	Number of oscillations	Average period	Uncertainty
1.40	6.05	4.65	4	1.163	0.008
2.00	6.60	4.60	4	1.150	0.008
1.05	6.80	5.75	5	1.150	0.007
1.65	6.25	4.60	4	1.150	0.009
1.20	5.85	4.65	4	1.163	0.009
1.75	6.40	4.65	4	1.163	0.009
0.40	6.20	5.80	5	1.160	0.007
1.00	6.75	5.75	5	1.150	0.007
0.40	6.10	5.70	5	1.140	0.007
0.90	6.80	5.90	5	1.180	0.007
Average				1.157	0.008

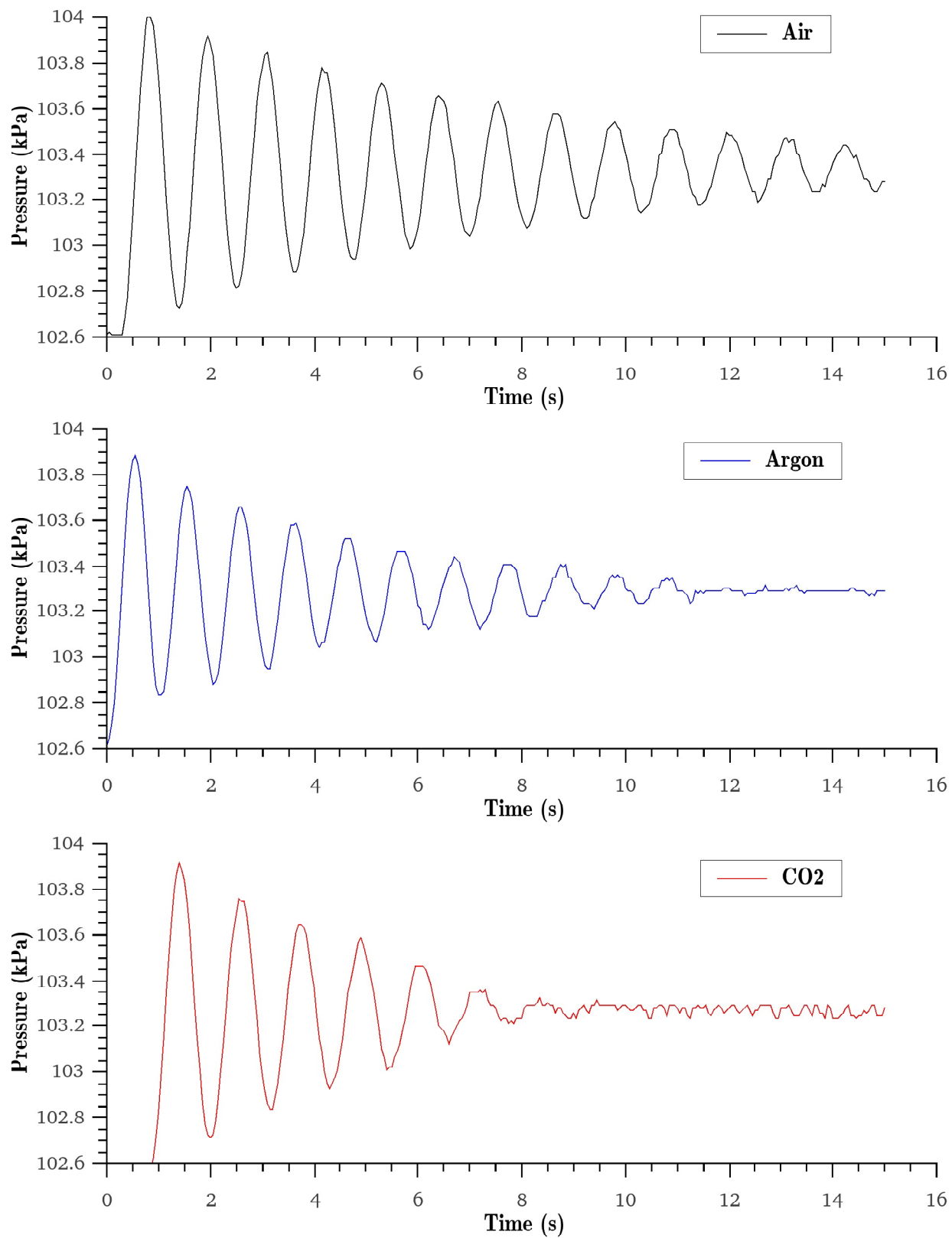


Figure 2: Pressure difference as a function of time for a steel ball oscillating in air, argon and carbon dioxide.

6 Calculations

The period for each trial run (example is air, trial 1).

$$T = \frac{t - t_0}{n_{osc}} = \frac{14.30s \pm 0.025s - 0.80s \pm 0.025s}{12}$$

$$T = \frac{13.5s \pm \sqrt{(0.025s)^2 + (0.025s)^2}}{12} = \frac{13.5s \pm 0.262\%}{12} = 1.125s \pm 0.262\%$$

$$T = 1.125s \pm 0.003s$$

Other calculations summarized in Table3 , Table 4, and Table 5

One standard deviation was used as the uncertainty of the average period (example is air)

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$

$$\sigma = \sqrt{\frac{1}{10-1} \sum_{i=1}^{10} (x_i - 1.122s)^2}$$

$$\sigma = 0.004s$$

Average period for air: $\bar{T}_{air} 1.122s \pm 0.004s$

Average period for argon: $\bar{T}_{Ar} = 1.034s \pm 0.006s$

Average period for CO₂: $\bar{T}_{CO_2} = 1.157s \pm 0.008s$

Equilibrium pressure (air)

$$P_{eq} = P_0 + \frac{mg}{A} = P_0 + \frac{mg}{\pi r^2}$$

$$= 752.6mmHg \pm 0.2mmHg + \frac{(16.5g \pm 0.005g)(9.81m/s^2)}{\pi \left(\frac{16.03mm \pm 0.02mm}{2} \right)^2}$$

$$= (752.6mmHg \pm 0.027\%) (133.32Pa \cdot mmHg^{-1}) + \frac{(0.0165kg \pm 0.03\%)(9.81m/s^2)}{\pi \left(\frac{0.01603m \pm 0.12\%}{2} \right)^2}$$

$$= 1.00337 \cdot 10^5 Pa \pm 0.027\% + 802.03Pa \pm \sqrt{(0.03\%)^2 + (0.12\%)^2}$$

$$= 1.00337 \cdot 10^5 Pa \pm 26.67Pa + 802.03Pa \pm 1.029Pa = 1.0114 \cdot 10^5 Pa \pm \sqrt{(26.67Pa)^2 + (1.029Pa)^2}$$

$$P_{eq} = 1.0114 \cdot 10^5 Pa \pm 26.8Pa$$

Calculation for γ

$$\gamma_{air} = \frac{64mV}{r^2 P_{eq} \bar{T}_{air}^2} = \frac{64 (0.0165kg \pm 0.03\%) (10.75L \pm 0.05L)}{(0.00802m \pm 0.12\%)^2 (1.0114 \cdot 10^5 Pa \pm 26.8Pa) (1.122s \pm 0.004s)^2}$$

$$= \frac{64 (0.0165kg \pm 0.03\%) (10.75L \pm 0.47\%)}{(0.00802m \pm 0.12\%)^2 (1.0114 \cdot 10^5 Pa \pm 0.028\%) (1.122s \pm 0.36\%)^2}$$

$$= 1.386 \pm \sqrt{(0.03\%)^2 + (0.047\%)^2 + (0.12\%)^2 + (0.028\%)^2 + (0.36\%)^2} = 1.386 \pm 0.28\%$$

$$\gamma_{air} = 1.386 \pm 0.004$$

$$\gamma_{Ar} = 1.635 \pm 0.019$$

$$\gamma_{CO_2} = 1.304 \pm 0.016$$

Consistency check (air)

$$|\gamma_{experimental} - \gamma_{theoretical}| = |1.386 - 1.400| = 0.014 > 0.004 \therefore \text{Inconsistent}$$

Argon was also inconsistent, CO_2 was consistent.

Percent difference (air)

$$\frac{|\gamma_{experimental} - \gamma_{theoretical}|}{\gamma_{theoretical}} \cdot 100 = \frac{|1.386 \pm 0.004 - 1.400|}{1.400} \cdot 100 = 1.00\% \pm 0.29\%$$

Argon percent difference $1.92\% \pm 0.59\%$

Verifying equation 1.13

$$\begin{aligned} \frac{k^2 \bar{T}_{Air}^2}{4} &= \left(\frac{\ln A_n - \ln A_{(n+l)}}{l} \right)^2 \\ &= \left(\frac{\ln(0.686kPa) - \ln(0.250kPa)}{7} \right)^2 \\ &= 0.021 \\ 0.021 &\ll 4\pi^2 \end{aligned}$$

$$\begin{aligned} \frac{k^2 \bar{T}_{Ar}^2}{4} &= \left(\frac{\ln(0.591kPa) - \ln(0.115kPa)}{7} \right)^2 \\ &= 0.013 \\ 0.013 &\ll 4\pi^2 \end{aligned}$$

$$\begin{aligned} \frac{k^2 \bar{T}_{CO_2}^2}{4} &= \left(\frac{\ln(0.602kPa) - \ln(0.125kPa)}{7} \right)^2 \\ &= 0.050 \\ 0.050 &\ll 4\pi^2 \end{aligned}$$

7 Discussion

The period of oscillation for the steel ball in air was $\bar{T}_{air} = 1.122s \pm 0.004s$. In argon gas the period was $\bar{T}_{Ar} = 1.034s \pm 0.006s$ and in CO_2 it was $\bar{T}_{CO_2} = 1.157s \pm 0.008s$. One standard deviation of from the mean was used as the experimental error. Other forces such as friction between the sides were not accounted for in the error, although the values of $k^2 T^2/4$ were low enough that the effect of damping on the period was determined to be negligible. The equilibrium pressure ($P_{eq} = 1.0114 \cdot 10^5 Pa \pm 26.8 Pa$) was determined indirectly by adding the theoretical pressure applied by weight of the the ball over the cross sectional area of the tube, to the air pressure of the laboratory. This indirect calculation of equilibrium pressure may have resulted in a significant source

of error which was not accounted for in the calculations because only the instrument measurement uncertainty was carried through the calculations.

The value for γ_{air} was determined to be 1.386 ± 0.004 . This is inconsistent with the accepted value (1.400). There was a $1.00\% \pm 0.29\%$ difference from the accepted value for γ_{air} . The value for γ_{Ar} was determined to be 1.635 ± 0.019 . This is inconsistent with the accepted value for γ_{Ar} (1.667). There was a $1.92\% \pm 0.59\%$ difference from the accepted value for γ_{Ar} . The value of γ_{CO_2} was determined to be 1.304 ± 0.016 . This value is consistent with the accepted value for γ_{CO_2} . The percent difference between experimental and accepted values of γ for air and argon were both relatively small. Only error in measurement was taken into account throughout the calculations, and this resulted in fairly small reported error. The data appears to be inconclusive, and many more tests would have to be carried out in order to obtain a more accurate measurement for the average period (and hence γ).

8 Conclusion

The period of oscillation were, $\bar{T}_{air} = 1.122s \pm 0.004s$, $\bar{T}_{Ar} = 1.034s \pm 0.006s$, and $\bar{T}_{CO_2} = 1.157s \pm 0.008s$. The equilibrium pressure was determined to be $1.0114 \cdot 10^5 Pa \pm 26.8 Pa$. γ_{air} was determined to be 1.386 ± 0.004 which is inconsistent with the accepted value (1.400). A $1.00\% \pm 0.29\%$ difference from the accepted value of γ_{air} was observed. γ_{Ar} Was determined to be 1.635 ± 0.019 which is inconsistent with the accepted value of 1.667. A $1.92\% \pm 0.59\%$ difference from the accepted value of γ_{Ar} was observed. γ_{CO_2} was determined to be 1.304 ± 0.016 which is consistent with the accepted value. The value of $k^2 T^2 / 4$ was determined to be 0.021 for air, 0.013 for argon and 0.050 for carbon dioxide. All three values were significantly less than the value of $4\pi^2$, so the effects of damping on period were considered negligible.

9 Question

The ratio of specific heat depends on the degrees of freedom, f for the gas by the equation

$$c_v = \frac{f}{2} R$$

For an ideal monoatomic gas there are 3 degrees of freedom and therefore

$$c_v = \frac{3}{2} R, c_p = \frac{5}{2} R, \gamma = 1.67$$

Argon is a monoatomic gas and the accepted value for γ (at SATP) is very similar to that of an ideal monoatomic gas. For an ideal diatomic gas there are 5 degrees of freedom, so $\gamma = 1.40$. Air is mostly composed of diatomic gases (nitrogen and oxygen) and so the accepted value for γ_{air} (at SATP) is again very similar. Similarly, carbon dioxide is a triatomic gas, which have 9 degrees of freedom and the accepted value for γ_{CO_2} is very similar to that of an ideal triatomic gas.

10 References

1. Cengel, Y., Boles, M., (2011). Thermodynamics, an engineering approach. 7th Ed. Mc Graw Hill. ISBN 0-07-352932-X
2. UVic Physics 217 Lab Manual. Lab 1-Ratio of specific heat capacities.