Determination of free air gradient using relative gravimeter measurements.

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ABSTRACT

The free air gravity gradient at the University of Calgary was estimated by determining the vertical gravity gradient in the Earth Sciences tower. Relative gravity measurements were recorded on each floor of the tower using a Worden Master Relative Gravimeter. Several gravity measurements were made on each floor and the elevation of the floor was measured. The average gravity measurement on each floor and the elevation of the floor was used to compute the change in gravity with elevation in the building. This value was used as an estimate of the free air gravity gradient. The free air gradient was determined to be $-0.2860\,\mathrm{mGal/m}$. This was a 7.28 % difference from the expected value of $-0.3085\,\mathrm{mGal/m}$. The free air gravity gradient and possible sources of errror are discussed in this paper.

THEORY

When performing a gravity survey it is common to apply an elevation correction to the measured gravity data. This is because the expected value of gravity varies with elevation and therefore elevation changes in the survey may mask underlying anomalies. The elevation correction generally consists of two separate corrections: the Bouguer plate correction and the free air correction.

The Bouguer plate correction accounts for the difference in mass due to extra material between the survey point and the reference elevation. For an above ground survey this correction is applied by assuming that the change in elevation is due to an infinite slab with a thickness equal to the change in elevation relative to the reference elevation. Long and Kaufmann (2013) derive a formula for the Bouguer plate correction

$$\Delta g_{\mathrm{BP}} = 2\pi G\rho b \tag{1}$$

where G is the gravitational constant, and b is the change in elevation. The Bouguer plate correction is not neccesary if gravity measurements are made above ground level because there is no extra mass to account for other than the mass of the air. The change in gravity due to the mass of air is on the order of μGal over a distance of 1000 m (Lowrie, 2007), and therefore negligible for most surveys.

The free air correction accounts for the change in gravity measurements due to a change in elevation. It ignores the effect of excess mass, which is accounted for in the Bouguer plate correction (Lowrie, 2007). The free air correction is applied by assuming the rate at which gravity measurements change with respect to elevation. This rate of change is known as the free air gravity gradient. Its value can be derived using Newton's law of gravitation. The gravitational acceleration at a distance r from the Earth's centre of mass is given by the equation

$$g(r) = \frac{GM_e}{r^2} \tag{2}$$

where M_e is the mass of the Earth (5.972×10^{24}kg) and G is the gravitational constant ($5.674 \times 10^{-11}\mathrm{Nm}^2/\mathrm{kg}^2$). The free air correction can be approximated by taking the derivative of equation 2 with respect to r, and substituting in the radius of a sphere with the same volume as the earth (6371 km). The result is

$$\frac{dg}{dr} = -\frac{2GM_e}{r^3} \approx -0.3083 \text{ mGal/m} \tag{3}$$

Equation 2 does not account for the centrifugal acceleration due to the rotation of the Earth, or the variation of its radius with latitude due to its non-spherical shape. A more complex derivation can be performed by taking the derivative of an equation that accounts for these factors. This more accurate relationship (Burger et al., 2006) is

$$\frac{dg}{dr} = -0.3086 - 0.00023 \cos(2\theta) + 0.00000000z \;\; [\text{mGal/m}] \tag{4}$$

where θ is the latitude and z is the elevation in metres. The University of Calgary has an approximate latitude of 51.08° and an elevation of 1111 m. Using these values in equation 4 results in an expected free air gradient of -0.3085 mGal/m. This value is similar to the result from equation 3.

If a gravimeter is placed in one position, its readings will vary over time. Part of this variation is due to a small long-period fluctuation in the readings of the gravimeter known as instrument drift (Burger et al., 2006). The instrument drift can be monitored by making repeated measurements at an established base station over the course of the gravity survey. For some older gravimeters, the drift may be non-linear over a time period of several hours, and therefore more frequent base station measurements must be made (Long and Kaufmann, 2013). If the instrument drift is assumed to be linear in the time between the base station measurements, the drift correction can be computed using the formula

$$\Delta g_{\text{drift}}(t) = \frac{g_2 - g_1}{t_2 - t_1}(t - t_1) \tag{5}$$

where g_2 and t_2 are the gravity and time measurements at the end of the loop, g_1 and t_1 are the measurements at the beginning of the loop, and t is the time of the measurement being corrected.

There are other temporal changes to gravimeter readings over time, such as tidal effects. Tidal effects create a difference in gravity measurements due to the relative position of the sun and the moon. These effects are generally corrected by leaving an absolute gravimeter at a base station and recording frequent measurements. They may also be corrected by computing the expected tidal effect using computer software (Burger et al., 2006). If the survey is performed over a time interval less than 2 h the tidal effect is small and approximately linear. In such a case the tidal correction can be included in the drift correction (eqn 5) with a small loss in precision Burger et al. (2006).

METHODS

Survey Methods

The gravity survey was conducted in the Earth Sciences Building at the University of Calgary. The building had ten floors above ground level and a basement. The basement was used as the base station for instrument drift measurements. A Worden Master Relative Gravimeter was used for all relative gravity measurements. An overview of the meter controls is shown in figure 1. The gravimeter had a dial constant of .1025 to convert the dial readings into units of mGal.

At each station, the base plate was set approximately 2 m from a stairwell door so it was in the same relative location on each floor. The gravimeter was levelled on the baseplate first longitudinally by turning the two longitudinal level screws simultaneously, and then transversely by turning the transverse level screws.

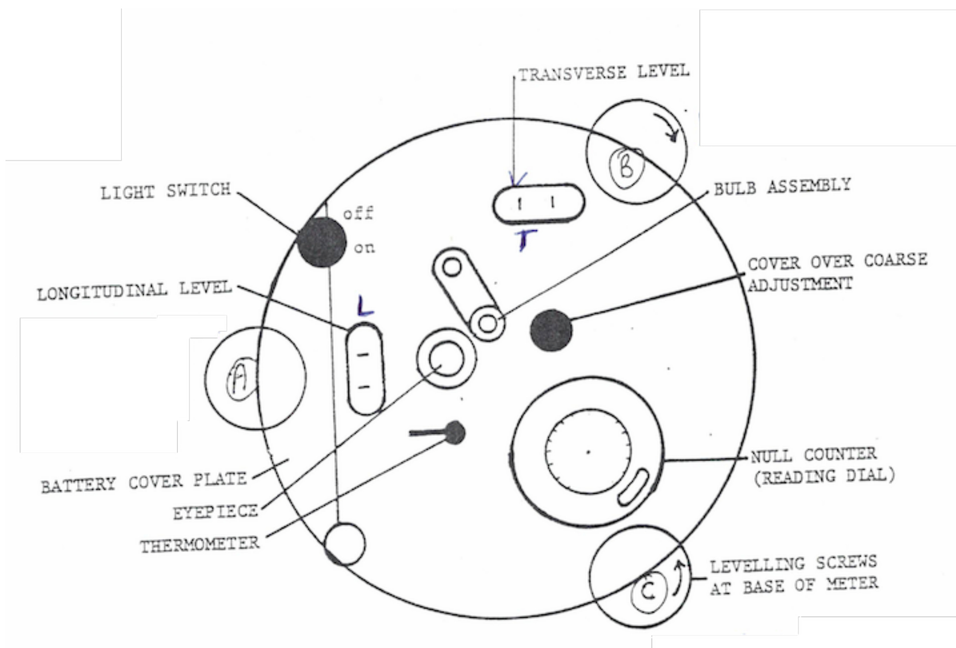

Figure 1: Overview of the Worden Master relative gravimeter controls.

After levelling, the meter light was switched on and the adjustment dial was rotated until the the light beam was in the central hairline of the eyepiece. The beam was observed for at least 30 s to ensure there were no errors due to medium-period oscillations. The reading was determined using the vernier scale on the instrument. Three measurements were made at each station. Gravity readings were taken on each floor starting with the basement and working up. After the third, seventh, and tenth floor, a separate set of base station measurements were recorded in order to compute the instrument drift.

The distance between each floor was recorded using a 30 m tape measure.

Data Processing

The MATLAB_2015 software package was used for all data processing. The recorded data for the gravity measurements, measurement times, floor number, and the distance between each floor was imported into MATLAB. The basement floor was set at an elevation of 0 m and the elevation of each floor was computed by adding the distance between each floor to the elevation of the previous floor.

```
%compute height of each floor
floor_height = zeros(11,1);

for i = 1:10
    floor_height(i+1) = floor_height(i)+floor_separation(i+1);
end
```

The gravity readings were multiplied by the dial constant to obtain the readings in units of mGal. The relative gravity readings were determined by subtracting the first measurement (made in the basement) from all other measurements. The *datenum()* function was used to convert the time measurements to seconds, and the relative time was determined by subtracting the time of the first measurement from all other readings. A time series plot of the raw gravity measurements was created.

Drift corrections were applied to the gravity readings using equation 5. The base station measurements were extracted from the data, and plotted as a time series. The gravity and time data was averaged for each set of base station measurements. These averaged values were added to the plot. The rate of instrument drift between each of these sets was determined by finding the slope of the line segments connecting the averaged data points. The result was a set of three drift rates and their starting reference time. The drift correction was applied to the gravity readings by multipling the correct drift rate by the difference between the measurement time and the reference time.

```
  %find the dg_drift over each linear segment
dg_drift = zeros(3,1);
for i = 1:3
    dg_drift(i) = (g_drift_avg(i+1)-g_drift_avg(i))/(t_drift_avg(i+1)-t_drift_avg(i));
end

%% apply drift correction to data
g_rel_corr = g_rel*0; %initialize a new vector for drift corrected gravity data
for i = 1:size(g_rel_corr)
    if floor_reading(i) <= 3 && floor_reading(i)> 0
        g_rel_corr(i) = g_rel(i)-dg_drift(1)*(t(i)-t_drift_avg(1));
    elseif floor_reading(i) <= 7
        g_rel_corr(i) = g_rel(i)-dg_drift(2)*(t(i)-t_drift_avg(2));
    else
        g_rel_corr(i) = g_rel(i)-dg_drift(3)*(t(i)-t_drift_avg(3));
    end
end
```

The drift-corrected gravity measurements were plotted as a function of elevation. The *polyfit()* function was used to fit a first-order polynomial to the data and its slope was used as an estimate of the free air gravity anomaly.

RESULTS

The raw gravity measurements are shown in figure 2. The time series showed an overall decrease in relative gravity measurements over time. These results were expected because the survey began at the basement level, and measurements were made on higher floors with increasing time. The decrease in gravity measurements over time was a result of increasing elevation as predicted by equation 3. There was an overall decrease of approximately 12 mGal over the course of the two hour survey. The time series had four prominent maxima which corresponded to where the base station measurements were repeated.

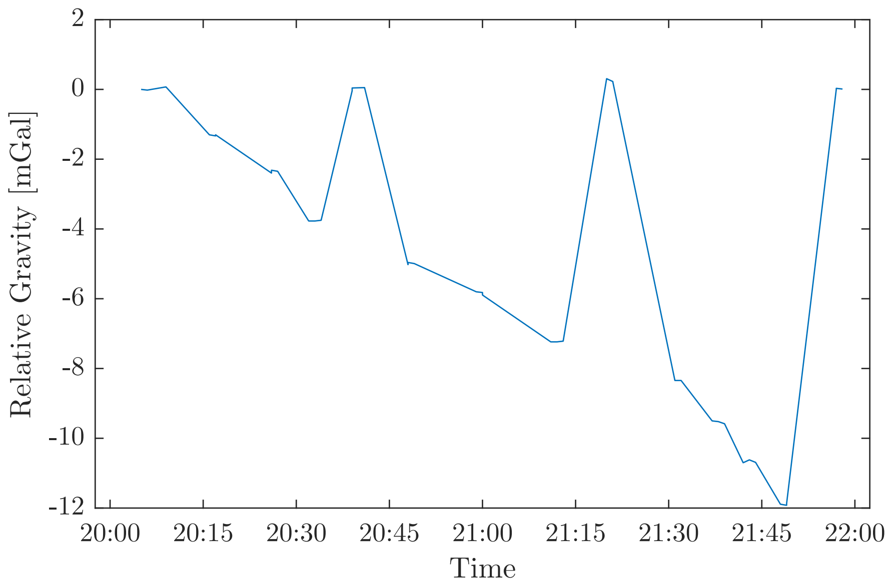

Figure 2: Time series plot of relative gravity measurements. The relative gravity measurements decreased over time. The prominent spikes in the time series was due to the repeated base station measurements.

The base station measurements and the average for each set are show in figure 3. The drift rates between each of the base station measurements are summarized in table 1. There was no recorded drift between the first and second set of base station measurements. The average recording during the third base station measurement was approximately 0.25 mGal higher than it was during the other three sets of base station measurements. This lead to a drift correction that was significant for measurements made from the fourth to the tenth floor. The overall profile of the drift correction curve was highly irregular, and did not suggest the drift was linear over the course of the survey. This could have lead to an error in the drift-corrected gravity measurements because it was assumed that the drift was linear. This non-linearity in the base station measurements may have been caused by temporal changes in the recorded gravity due to external factors other than instrument drift.

Tidal variations often cause a change in observed gravity over time. However, because the survey was performed over a 2 h period, the tidal variations should have been small and approximately

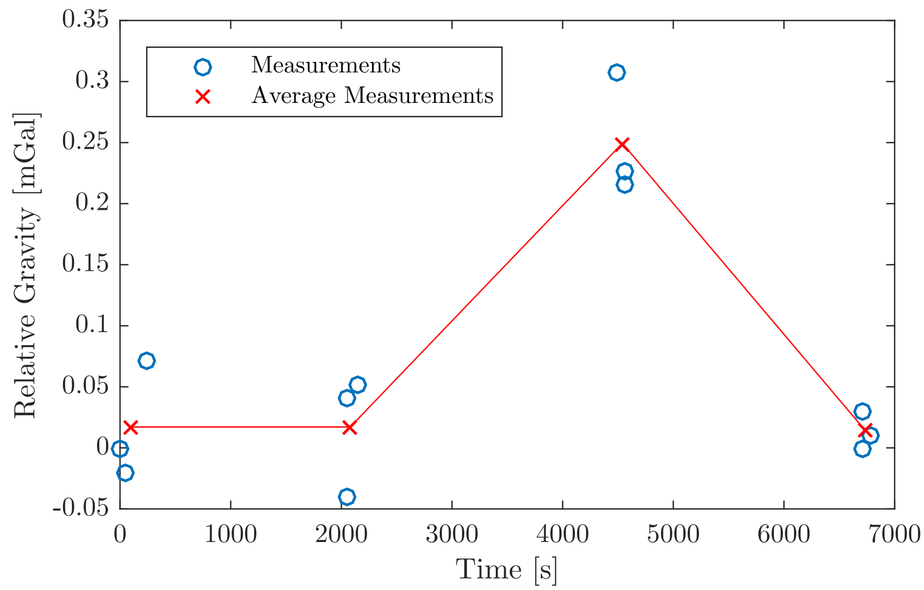

Figure 3: Time series plot of base station measurements. The average value for each set of base station measurements is shown in red. The slope of the three segments of the red line was used as the drift rate for the drift corrections. There was no significant drift between the first and second base station measurements. The relative readings increased by approximately 0.2 mGal between the second and third set of measurements, and then decreased by approximately 0.2 mGal between the third and fourth set of measurements.

linear. This suggests that the non-linear shape of the drift correction curve was likely not due to tidal variations, and using the standard drift technique to correct for both tidal effects and drift effects should have only produced a small error in the gravity measurements. It is also possible that the non-linear shape of the drift correction curve was due to a change in the mass distribution of the building over time. The Earth Sciences building had three elevators, each containing a massive counterweight. It is possible that the alignment of the counterweights affected the base station readings.

The averaged gravity readings at each station are summarized in table 2. The data showed a decrease in gravity of approximately 1 mGal per floor. Figure 4 shows the average measurements plotted as a function of the elevation above the basement floor. The data showed a clear linear decrease in gravity with elevation. This linear shape suggested that the vertical gravity gradient was constant throughout the survey. This was expected because equation 4 predicts a constant vertical gravity gradient for relatively small elevation changes, because its third term is negligible for small elevation changes and its second term is constant for constant latitude.

The slope of the polynomial fit to the data by a linear regression was used as an estimate of the free air gradient. It was determined to be -0.2860 mGal/m. This value was less than the expected value of -0.3085 mGal/m with a 7.28 % error. Some of this error was likely due to the mass of the building. The free air correction ignores the effect of the mass underlying an increase in elevation. However, this mass could have significant effects on the measured gravity value.

To estimate the effect that the mass of the building could have on the resuting free air gradient, a simple model of the building was examined. Equation 1 shows that a concrete slab with thick-

Time Interval [s]	Floor Interval	Drift Rate [mGal/s]
0 - 2080	1 - 3	0.00
2080 - 4540	4-7	9.444×10^{-5}
4550 - 6740	8-10	-1.072×10^{-4}

Table 1: Summary of drift rates used for drift corrections of gravity data. The drift rate was used to correct all measurements that fell in the corresponding time range. The corresponding floor range is also shown. There was no recorded drift for measurements made between the 1st and 3rd floors. There was a positive drift measured between the 4th and 7th floors. There was a negative drift measured between the 8th and 10th floors.

Floor	Elevation [m]	Relative Gravity [mGal]
0	0	-0.047
1	4.26	-1.312
2	9.21	-2.354
3	13.20	-3.765
4	17.20	-5.041
5	21.21	-5.952
6	25.17	-7.413
7	29.17	-8.638
8	33.16	-9.425
9	37.16	-10.527
20	41.17	-11.713

Table 2: Average relative gravity measurement at each floor and the elevation of the floor relative to the basement (floor 0). The data showed a decrease in gravity of approximately 1 mGal per floor. Each floor was separated by an approximate distance of 4 m.

ness 0.2 m below the gravimeter would lead to an increase in measured gravity of approximately 0.0201 mGal. The gravity effect of an infinite slab depends only on the thickness of the slab, and not on the distance from the slab (Burger et al., 2006). Therefore, on the tenth floor, the combined gravity effect of ten concrete slabs would be approximately 0.201 mGal. This would lead to an underestimation of the free air gradient by approximately 0.005 mGal/m. Although the geometry of the building would require a much more complicated correction to obtain the true free air gradient, this approximation demonstrated that the effect of the mass of the building could have a significant effect on the measured gradient. Additionally, there was no terrain correction applied to the gravity measurements. The terrain correction always decreases the measured gravity, which could have lead to an underestimation of the gravity measurements. This may have contributed to the underestimation of the free air gravity gradient.

The theoretical gravity gradient is based on a reference spheroid of the earth. This spheroid does not account for any regional variations in the geiod. Some of the error in the results may have been due to a regional variation in the geoid that was not accounted for when calculating the expected value of the free air gradient.

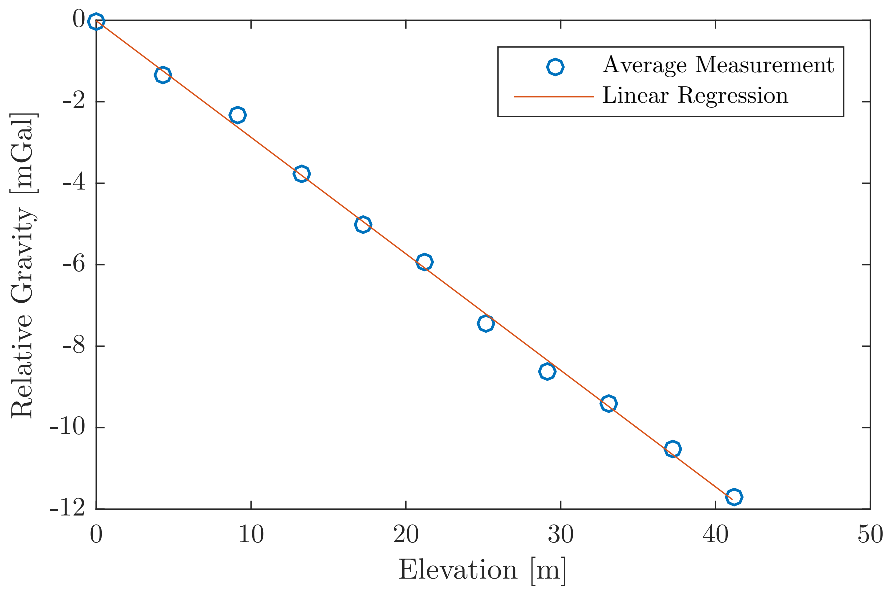

Figure 4: Drift-Corrected relative gravity measurements plotted against elevation. The data showed a clear decreasing trend in gravity measurements with increasing elevation. A linear regression was used to fit a first order polynomial to the data and had a slope of $-0.2860\,\mathrm{mGal/m}$.

CONCLUSION

The free air gravity gradient at the University of Calgary was determined to be $-0.2860\,\mathrm{mGal/m}$. This result was lower than the expected value of $-0.3085\,\mathrm{mGal/m}$. The underestimation was likely due to the gradient being measured in a building rather than in free air. The mass of the building likely contributed to the error in the gravity gradient. The surrounding terrain may have also led to an error in the gravity gradient. It was also possible that the theoretical (expected) gradient was incorrect due to regional variations in the geoid.

REFERENCES

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