Lab 5-The Sterling Cycle

July 25, 2012

1 Object

To study the thermodynamic properties of the Stirling cycle; its application as a heat engine

2 Theory

The Stirling cycle is a thermodynamic cycle that can used in a heat engine to provide mechanical work. It is a reversible cycle, meaning it can be followed against the thermal gradient if mechanical work is suppled. Fig.1 describes the four steps of an idealized sterling cycle. The first step (a-b) is an isochoric temperature increase. The second step (b-c) is an isothermal volume increase. The third step (c-d) is an isochoric temperature decrease. The fourth step (d-a) an isothermal volume decrease.

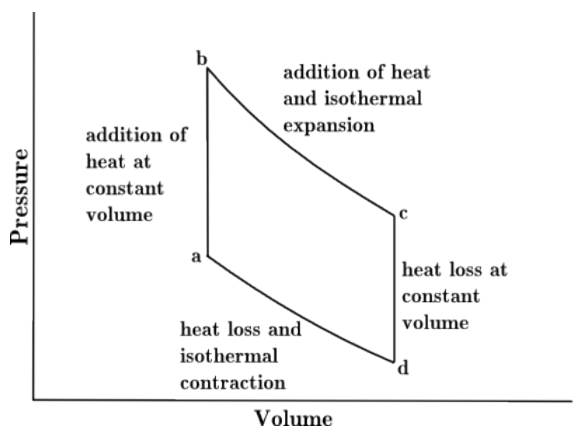

Figure 1: The four steps of an idealized Stirling cycle. Adapted from Lee and Sears, 1963

A reversible regenerator can be used in the cycle to recover the heat lost in the isochoric temperature decrease (c-d) and add it to the isochoric temperature increase (a-b). With a regenerator, the efficiency of an idealized Stirling cycle is the same as it is for the Carnot cycle. Carnot's theorem shows that is the maximum possible thermal efficiency for any heat engine[1]. The thermal efficiency of a heat engine can be determined by comparing the power input to the engine, to the thermodynamic power output. This power output is determined by the the work, which is based on the integral of a P-V diagram around a closed loop(see question 2).

3 Procedure

A beta-type Sterling engine with one vertical cylinder was used (described in apparatus) . The slide wire potentiometer was calibrated by recording the voltage at the maximum ($310\,cm^3$), and minimum ($155\,cm^3$) cylinder volume. The engine was started and allowed to run for 10 minutes with the heater current around 11A to stabilize the P-V cycle. Measurements of the pressure and volume in the cylinder was recorded with a frequency of 100 measurements per second. Each data set was collected for the approximate length of one full cycle (0.23s). Five different data sets representing 5 full engine cycles were recorded. Graphical Analysis software was used to plot the pressure as a function of volume (P-V diagram) for each cycle, the software was used to find the area enclosed by the curve.

The mechanical power output was measured by loading the engine with a braking force to the point where it nearly stalled. The braking force was applied by wrapping a copper band around a spindle attached to the flywheel, and applying a force to each end (by hand). The two force were measured using spring scales. The movement of the lower piston as a function of time was recorded on logger pro while the braking force was being applied. This function was used to determine the frequency of rotation of the engine.

4 Apparatus

The cylinder was completely sealed so the amount of gas within it remained constant. The engine had an electric heat coil at the top of the cylinder, and simple water-cooled heat sink at the bottom. Enclosed within the cylinder was a displacement piston attached by a push rod to a crankshaft to provide a moving thermal barrier between the top and bottom of the cylinder, while still allowing gas to flow freely within it. At the bottom of the cylinder was driving piston attached to the crankshaft by a push rod. The the movement of the driving piston changed the total volume of the cylinder. A a slide-wire potentiometer was fixed to the bottom of the driving piston to determine the volume by sending a voltage reading to a Lab-pro sensor, attached to a computer running logger pro software.

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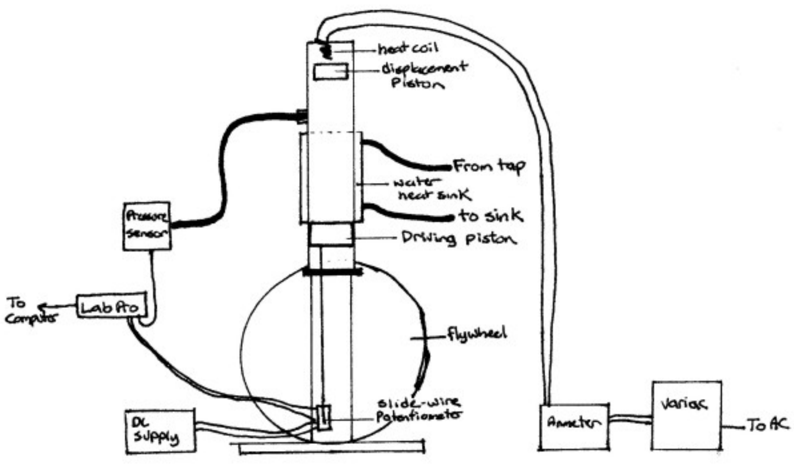

Figure 2: Summary of apparatus used in the experiment

Ammeter- UVic Physics #27, DC power supply- UVic Physics #448, Variac- UVic Physics #110

5 Data

5.1 Part I

A potential difference of $10.98V \pm 0.2V$ and a current of$11.1A \pm 0.1A$ was measured across the heat coil. The area inside the P-V curves is summarized in Table 1.An example P-V diagram is shown in Fig3. The time for a complete engine cycle was $0.23\,s\ \pm 0.005\,s$

Table 1: Area enclosed by P-V diagram

	net work ($kPa \times cm^3$)	net work (Joules)
	5094	5.094
	4987	5.987
	5097	5.097
	5195	5.195
	5210	5.210
average	5117	5.117
σ	90.22	0.37

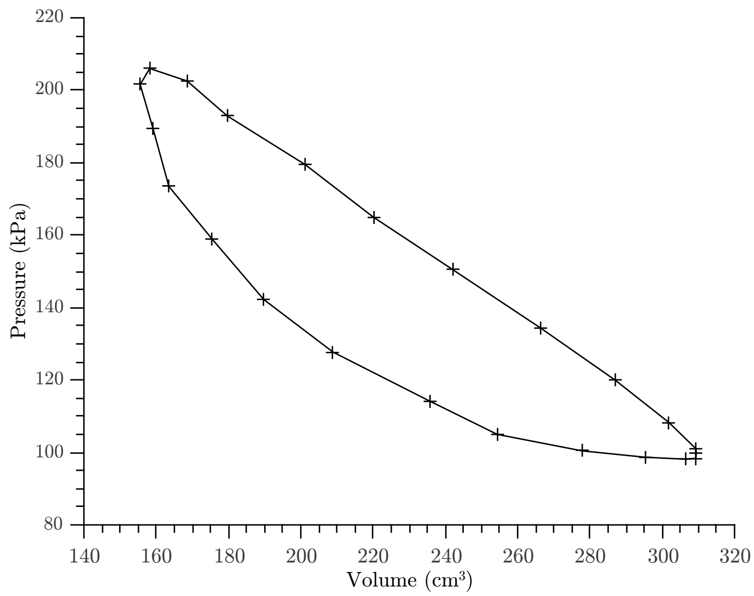

Figure 3: P-V diagram for sterling engine

5.2 Part II

The diameter of the spindle was $d = 25.26\,mm\ \pm\ 0.02\,mm$. The spring-scale reading of the forces applied to the copper strap were $F_1 = 1.43\,kg\ \pm\ 0.025\,kg$, and $F_2 = 0.60\,kg\ \pm\ 0.025\,kg$. F The time for two complete cycles with the engine loaded was $1.32\,s\ \pm\ 0.01\,s$.

6 Calculations

The area enclosed by the P-V diagram gives the net work for the cycle (See Question 1). The net work obtained from the P-V diagrams was measured in $kPa \times cm^3$which can be divided by 1000 to give work in Joules (See1). This gave an average thermodynamic work of $5.177\,J\ \pm\ 0.38\,J$.

The thermodynamic power generated by the engine is:

$$P_{Thermo} = \frac{\triangle W}{\triangle t} = \frac{5.177\,J\,\pm\,0.38\,J}{0.23\,s\,\pm\,0.005\,s} = \frac{5.177\,J\,\pm\,7.34\%}{0.23\,s\,\pm\,2.17\%} = 23\,W \pm (\sqrt{7.34^2 + 2.17^2})\% = 23\,W \pm 7.65\% = 23\,W \pm 2\,W$$

The power input is given by Joule's law P=IV:

$$P_2 = (10.98\,V\,\pm\,0.2\,V)(11.1\,A\,\pm\,0.1\,A) = (10.98\,V\,\pm\,1.82\%)(11.1\,A\,\pm\,0.90\%)$$

$$P_2 = 121.9\,W\,\pm\,(\sqrt{1.82^2 + 0.90^2})\% = 121.9\,W\,\pm\,2.03\% = 121.9\,W\,\pm\,2.47\,W$$

The thermodynamic efficiency is the ratio of the the thermodynamic power to the power input:

$$\eta = \frac{P_{Thermo}}{P_2} = \frac{23\,W\,\pm\,7.65\%}{121.9\,W\,\pm\,2.03\%} = 0.19\,\pm\,(\sqrt{7.65^2 + 2.03^2})\% = 0.19\,\pm\,7.92\%$$

$$\eta_{Thermo} = 0.19\,\pm\,0.02$$

6.1 Part II

The tension on each side of the strap is determined by converting the reading on the spring scale into Newtons:

$$F_1 = (1.43\,kg \pm 0.025\,kg)(9.81 m/s^2) = (1.43\,kg \pm 1.75\%)(9.81\,m/s^2) = 14.0\,N \pm 1.75\% = 14.0\,N \pm 0.2\,N$$

$$F_2 = (0.60\,kg \pm 0.025\,kg)(9.81\,m/s^2) = (0.60\,kg \pm 4.16\%)(9.81\,m/s^2) = 5.89\,N \pm 4.16\% = 5.89 N \pm 0.2\,N$$

The braking force F is the difference in tensions:

$$F = F_1 - F_2 = 14.0\,N\,\pm\,0.2\,N - 5.89 N\,\pm\,0.2\,N = 8.1\,N\,\pm\,\sqrt{0.2^2 + 0.2^2} N = 8.1\,N\,\pm\,0.3\,N$$

The frequency of rotation under load was:

$$f = \frac{2}{1.32\,s\,\pm\,0.01\,s} = \frac{2}{1.32\,s\,\pm\,0.76\%} = 1.51\,s^{-1}\,\pm\,0.76\% = 1.51\,s^{-1}\,\pm\,0.012\,s^{-1}$$

The radius of the spindle was:

$$r = \frac{(2.526 \cdot 10^{-2}\,m\,\pm\,2.0\,\cdot 10^{-5}\,m)}{2} = \frac{2.526 \cdot 10^{-2}\,m\,\pm\,0.08\%}{2} = 1.263{\cdot}10^{-2}\,m \pm 0.08\% = 1.263{\cdot}10^{-2}\,m \pm 1.0\,{\cdot}10^{-5}\,m$$

Using the frequency of rotation of the engine under load, the two tensions in the strap and the radius of the spindle, the mechanical output power was determined (see question 2).

$$P_1 = F \cdot 2\pi f \cdot r = (8.11\,N\,\pm\,0.3\,N) \cdot 2\pi(1.51\,s^{-1}\,\pm\,0.012\,s^{-1}) \cdot (1.263 \cdot 10^{-2}\,m\,\pm\,1.0\,\cdot 10^{-5}\,m)$$

$$= (8.11\,N\,\pm\,3.69\%) \cdot 2\pi(1.51\,s^{-1}\,\pm\,0.75\%) \cdot (1.263 \cdot 10^{-2}\,m\,\pm\,0.08\%)$$

$$= 0.97\,W\,\pm\,(\sqrt{3.69^2 + 0.75^2 + 0.08^2})\% = 2.38\,W\,\pm\,3.77\% = 0.97\,W\,\pm\,0.04\,W$$

The mechanical output efficiency is the ratio of the the mechanical output to the power input

$$\eta_{Mech} = \frac{P_1}{P_2} = \frac{0.97\,W\,\pm\,3.69\%}{121.9\,W\,\pm\,2.03\%} = 0.008\,\pm\,(\sqrt{3.69^2 + 2.03^2})\% = 0.008\,\pm\,4.21\% = 0.008\,\pm\,0.0003$$

7 Discussion

The stages of the observed cycle were not as well defined as they are in an idealized Stirling cycle (Fig 1.). The overall shape of the P-V diagram for the cycle (Fig 3.) can be described as "quasi-elliptical". The first stage, ideally an isochoric temperature increase, was recognized in the observed cycle as a very steep slope. This indicated a large temperature change with a relatively small volume change (quasi-isochoric). The second stage, ideally an isothermal expansion, was recognized in the observed cycle as an almost completely straight line, with little change in slope. The constant slope indicated an increase in temperature in the first part of the expansion, followed by a decrease in temperature in the second. The change in temperature was relatively small when compared with the change in pressure and volume (quasi-isothermal). The third stage, ideally an isochoric temperature decrease, was more difficult to see in observed cycle. There was still evidence for a quasi-isochoric temperature change, but the P-V curve almost approached a point between the compression and expansion stages. This indicated that the stage was not as well represented in the observed cycle. The fourth stage, ideally an isothermal compression, was recognized in the observed cycle as a large change in volume and pressure with small change in temperature (quasi-isothermal). This part of the curve appears to better approximate an isotherm than the straight did in stage two. The approximate location of these four stages on the P-V curve is annotated in (Fig3).

The thermodynamic power ($23\,W \pm 2\,W$) was much greater than the mechanical output power ($0.97\,W \pm 0.04\,W$), and therefore the thermodynamic efficiency (0.19 ± 0.02) was much greater than the mechanical output efficiency (0.008 ± 0.0003). The thermal efficiency was greater than the mechanical efficiency because of power lost to the mechanical movement of the engine (friction). The engine also had to use a lot of mechanical power to do the work to keep the engine running, which further decreased the mechanical power output.

There was a notable source of error when determining the net torque applied by the braking force. It was assumed that these two braking forces were applied at an angle π to each other, and that they were applied perpendicular to the spindle's axis of revolution. This assumption led to an error which was not accounted for in the calculation for net torque (implicitly calculated when determining the mechanical output power).

Another source of error was encountered when using Graphical Analysis to compute the area enclosed by the P-V diagram. The software required a perfectly closed path to compute the integral. As the observed paths were not completely closed, it was necessary to alter one data point from each data set in order to close the loop. Similarly, the computer joined the each observation to the next by a straight line, leaving more uncertainty in the area calculation. This source of error could be reduced by increasing the number of cycles observed, or by recording observations at a higher frequency.

8 Conclusions

The average thermodynamic work per cycle was $5.177\,J \pm 0.38\,J$.The power input to the engine was $121.9\,W \pm 2.47\,W$. The thermodynamic power output of the engine was $23\,W \pm 2\,W$, giving the engine a thermal efficiency of 0.19 ± 0.02. The mechanical power output with a $19.9\,N \pm 0.3\,N$ braking force applied to the spindle, was $0.97\,W \pm 0.04\,W$ giving the engine a mechanical efficiency of 0.008 ± 0.0003.

9 Questions

1.

The volume was on the x-axis, and measured in cm^3. The pressure was on the y-axis and measured in kPa. This gives pressure as a function of volume.

P-v work is defined as $dW = pdV$. That is, the change in work equals the pressure times the change in volume. Therefore integrating both sides would give work as a function of pressure and volume, from an initial volume to a final. In our case this is a closed loop so we would get

$$W = \oint pdV$$

The integral of a curve taken on these set of axis gives the units kPacm^3. This is 1000 times the work in Joules:

$$1\,kPa \cdot cm^3 \frac{1\,m^3}{10^6\,cm^3} \frac{1000\,Pa}{1kPa} = 0.001\,Pa \cdot m^3 = 0.001\,J$$

2.

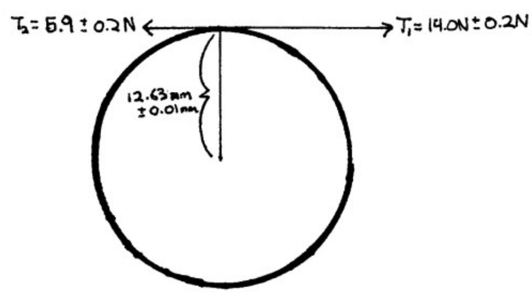

Figure 4: The Free body diagram

The two forces applied by the braking wire (See Fig.4) apply a net torque on the spindle given by

$$\tau_{net} = T_2 r - T_1 r = r(T_2 - T_1)$$

This is because the forces are parallel, but in opposite directions, and tangential to the flywheel (perpendicular to the radius). This torque was constant for the full revolution.

The work done by a torque is given by $W = \tau\theta$, where θ is the angular displacement[2]. In this case we were measuring for one full revolution, so $\theta = 2\pi$. Combining these gives

$$W = \pi r(T_2 - T_1)$$

Power is the work done per unit time, so we divide by the period of one revolution, which is equivalent to multiplying by the frequency. This gives

$$P = Wf = \pi r(T_2 - T_1)f$$

And finally if F=$T_2 - T_1$, this becomes the equation in the lab manual

$$P = F \cdot 2\pi f \cdot r$$

References

1. Lee and Sears, (1963). Thermodynamics - an introductory text for engineering students. Addison and Westley

2. http://www.engineeringtoolbox.com/work-torque-d_1377.html