Fluid Mechanics

Exercises II: KINEMATICS

1. A two-dimensional steady flow has velocity components $u = y$ and $v = x$. Show that the streamlines are rectangular hyperbolas

$$x^2 - y^2 = const. \tag{1}$$

   Sketch the flow pattern and convince yourself that it represents an irrotational flow in a 90 degree corner.

2. The velocity components in an unsteady plane flow are given by

$$u = x/(1+t) \tag{2}$$

   and

$$v = 2y/(2+t). \tag{3}$$

   Describe the path lines and the streamlines. Note that the path lines are found by following the motion of each particle, that is, by solving the differential equations $dx/dt = u(\mathbf{x}, t)$ and $dy/dt = v(\mathbf{x}, t)$, subject to $\mathbf{x} = \mathbf{x}_0$ at $t = 0$.

3. Consider the simple plane flow given by

$$u_1 = x_1/(1+t) \tag{4}$$
$$u_2 = x_2 \tag{5}$$
$$u_3 = 0 \tag{6}$$

   Compute the velocity gradient matrix, strain and rotation tensors. Compute divergence and vorticity of the given field.