

Numerical Methods I

Root finding 3

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`/afs/ictp/public/g/ggiulian/WORLD`



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Secant method

- ❖ Like most of the root finding problems, origin can be traced back to Babylonians and Egyptian (Rhynd Papyrus)
- ❖ Today formulation a modification of the Newton method using a first order approximation to the first derivative



<https://www.jstor.org/stable/pdf/10.4169/amer.math.monthly.120.06.500.pdf>



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Secant method: when

When does it work?

- We can estimate an interval $[x_0, x_1]$ of the x axis where the root is expected to be
- The function f can be evaluated at any generic point in the interval and nowhere inside $f(a) == f(b)$ for (a, b) in $[x_0, x_1]$



Secant method: how

How does it work?

- Construct the line joining the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$

$$y(x) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} (x - x_1) + f(x_1)$$

- Find the intersection with the x axis, x_2

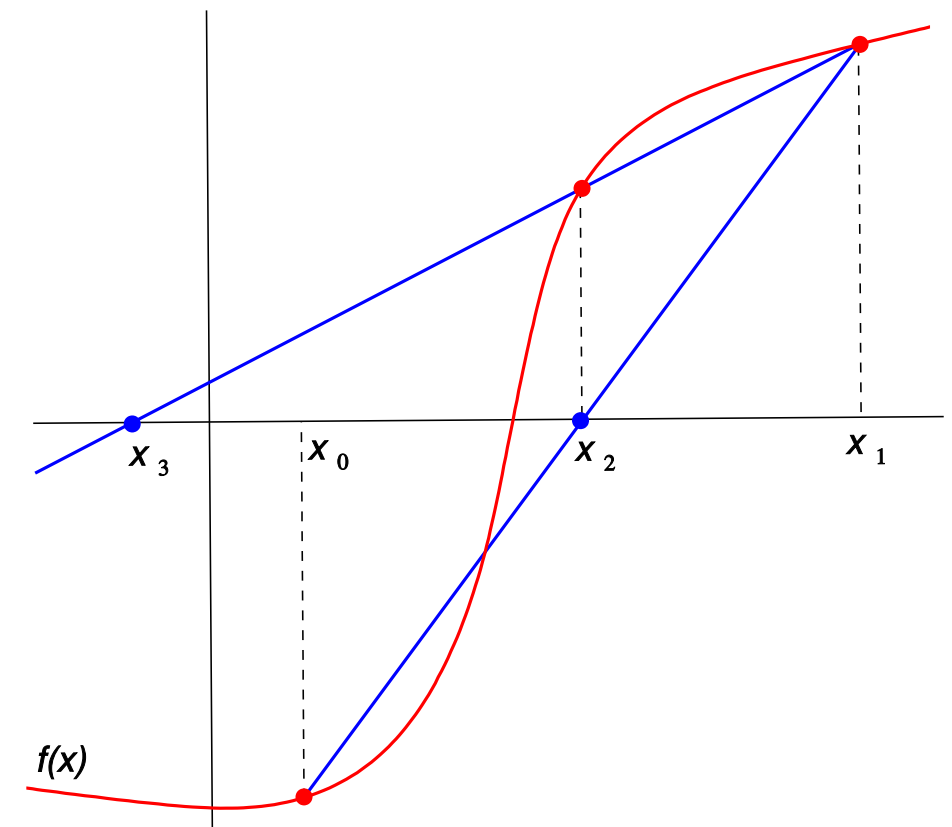
$$y=0 \rightarrow x_2 = x_1 - f(x_1) \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

- Repeat the process using now x_1 and x_2 until convergence when the distance less ε

Secant algorithm

Given:

- The function f
- The initial estimate of x_0 and x_1
- The error tolerance $\varepsilon > 0$



- 1) Compute f in x_0 and x_1
- 2) If $(\text{abs}(f(x_1) - f(x_0)) < \text{tiny}(x_0))$ exit for algorithm failure
- 3) Compute $x_2 = x_1 - f(x_1) (x_1 - x_0) / (f(x_1) - f(x_0))$
- 4) If $(\text{abs}(x_2 - x_1) < \varepsilon)$, stop and accept x_2 as root.
- 5) Set $x_0 = x_1$, $x_1 = x_2$ and iterate from point 1

Pros and Cons

The advantage of this method is that

- ❖ The order of the convergence is ... the golden ratio. (!)
- ❖ It does not require the computation of the derivative f'

However, there are also some disadvantages which are

- We need to bracket the root
- For the algorithm to converge, we need good estimates for the bracketing interval.
- It does not always converge as the bisection or false position and it is not as fast as the Newton method.

Pathological cases

