

Numerical Methods I

Root finding 2

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Newton-Raphson method

- ❖ First described in the work of Sir Isaac Newton as a method for finding the root of polynomial
- ❖ Generalized by Thomas Simpson (1740) as an iterative method for solving nonlinear equations using calculus



Newton method: when

When does it work?

- We have a first estimate of the root
- We can evaluate BOTH the function f and its first derivative at generic points in an interval of the x axis where the root is expected to be

Newton method: how

How does it work?

- Start with initial estimate of x_0 such that

$$\begin{aligned}y &= f(x_0) \\ y' &= f'(x_0)\end{aligned}$$

- Compute the new estimate x_1 as

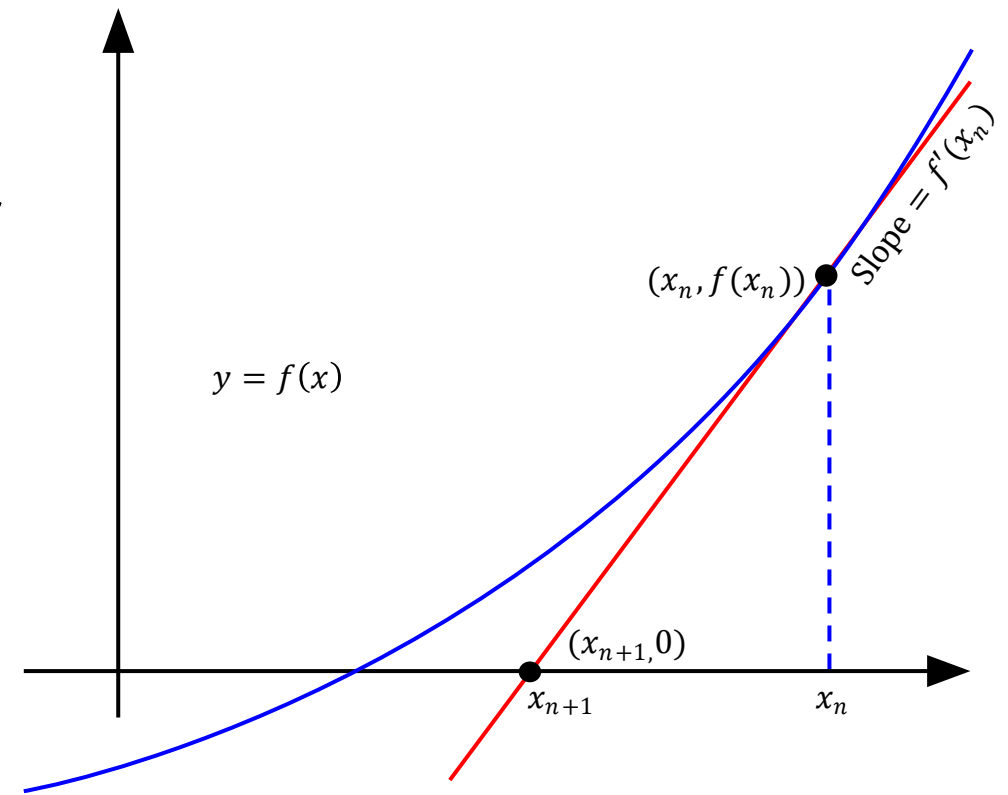
$$x_1 = x_0 - \frac{y}{y'}$$

- Iterate until the distance between x_0 and x_1 is less than ϵ

Newton algorithm

Given:

- The function f and derivative f'
- The initial estimate x_0
- The error tolerance $\varepsilon > 0$



- 1) Compute f, f' in x_0
- 2) If $(\text{abs}(f'(x_0)) < \text{tiny}(x_0))$, stop for algorithm failure.
- 3) Compute $x_1 = x_0 - f(x_0)/f'(x_0)$
- 4) If $\text{abs}(x_1 - x_0) < \varepsilon$, accept x_1 as the solution, else iterate with $x_0 = x_1$

Pros and Cons

The advantage of this method is that

- ❖ We need only a single estimate x_0
- ❖ Fast convergence nearby the root.

However, there are also some disadvantages which are

- It requires the analytical form of both f , f'
- We must place initial estimate near the expected root
- The derivative cannot be zero near the root

Pathological cases

